The Evil that is Rooting

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- Are rooted staggered quarks a sensible approach to lattice QCD?

Disagreement boils down to:

“ugly” or “absurd”

Outline

- review chiral behavior of 3 flavor QCD
- what rooted staggered quarks get wrong
- what are rooted staggered quarks
- why they get it wrong
- advocated escape: lots of wrong things might all cancel out
  - non-unitary, non-local, unproven

Three flavor QCD

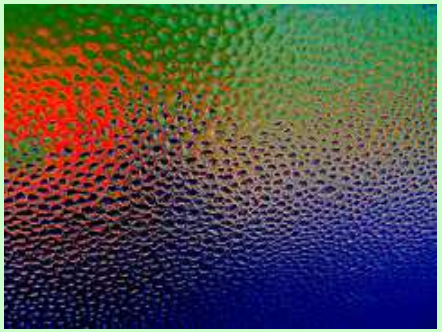
- up, down, strange quarks
- fields ψ^u, ψ^d, ψ^s
- masses m_u, m_d, m_s

Arguments based on analyticity in the mass parameters

- rooting introduces singularities where none expected

Theory has a quark condensate $\langle \overline{\psi}\psi \rangle \neq 0$

- $\langle \overline{\psi}_L{}^a \psi_R^b \rangle = v\, \Sigma^{ab}$ $\quad a, b$ "flavor" indices
- $\Sigma \in SU(3)$
- $SU(3) \times SU(3)$ chiral symmetry
  - $\Sigma \to g_L \Sigma g_R^\dagger \qquad g_L, g_R \in SU(3)$
  - spontaneously broken
- Non-perturbative



Jeff Stevens

Pseudoscalar mesons are small fluctuations about the condensate

- $\Sigma \sim \Sigma_0\, e^{i\lambda\cdot\pi(x)/f_\pi}$

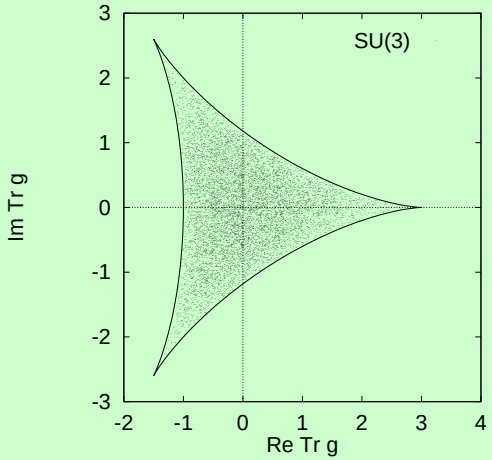
Low energy physics modeled by effective Lagrangian

- $L = \frac{f_\pi^2}{4} \mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) - v \ \mathrm{Re} \ \mathrm{Tr}(\Sigma M)$
- mass matrix $M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$

  - explicitly breaks $SU(3) \times SU(3)$ symmetry
  - selects physical vacuum Σ_0
  - Goldstone boson masses $M_\pi^2 \sim m_q$

Vacuum Σ_0 maximizes $\mathrm{Re\ Tr}\ M\Sigma$

- for real masses this is
  - sometimes unique; Σ_0 real
  - sometimes doubly degenerate
  - $M \propto -I \qquad \Rightarrow \qquad \Sigma_0 = e^{\pm 2\pi i/3}$

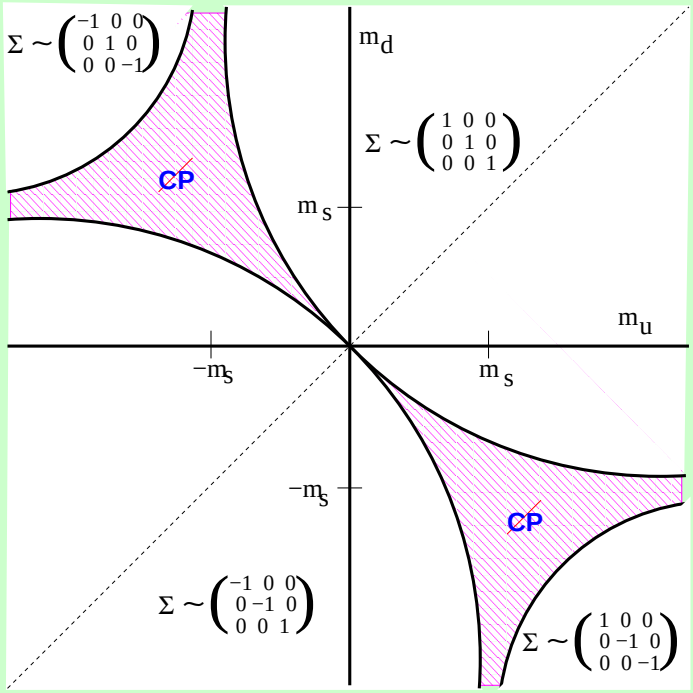


10,000 random SU(3) matrices

- complex vacuum signals spontaneous CP violation
  - $\langle \pi_0 \rangle \neq 0$

When I'm working on a problem, I never think about beauty. I think only how to solve the problem. But when I have finished, if the solution is not beautiful, I know it is wrong. -- R. Buckminster Fuller

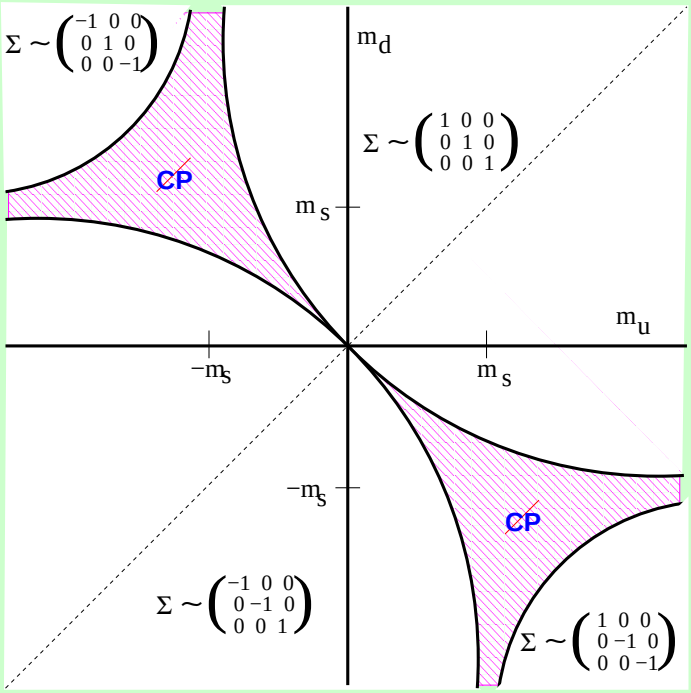
Fix $m_s > 0$, vary m_u and m_d



Phase boundaries at $m_u m_d \pm m_u m_s \pm m_d m_s = 0$

- where π_0 mass vanishes

Optimism, n. The doctrine or belief that everything is beautiful, including what is ugly. -- Ambrose Bierce, The Devil's Dictionary



No structure along m_d axis away from origin

- phase transition line shifted away from axis
- vacuum stabilized by other quarks
- $m_{\pi_0}^2 \sim \frac{m_u+m_d}{2} + O(m_q^2)$

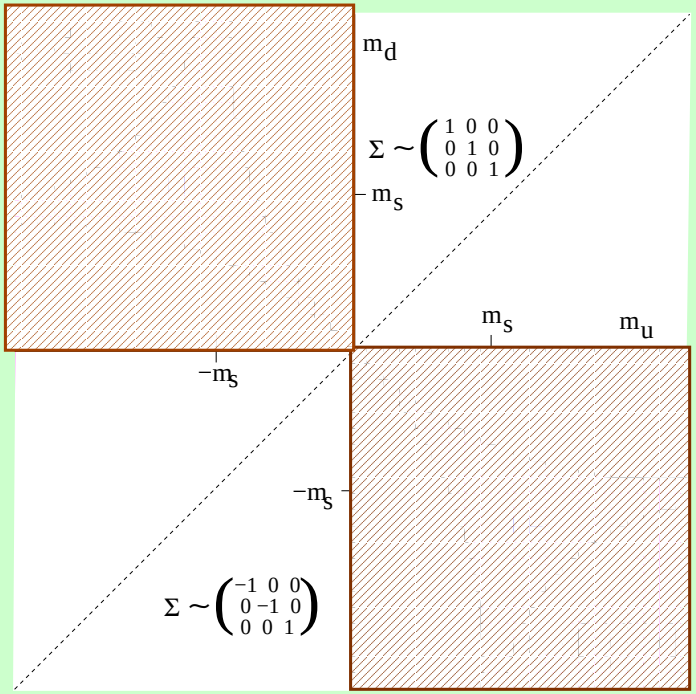
Keeping mass term diagonal, physics remains invariant under

- $M \to e^{i\theta_3\lambda_3 + i\theta_8\lambda_8} M$
- $SU(3)$ has rank 2
- 2 neutral non-strange Goldstone bosons: π_0, η
  - $m_P^2 \sim m_q$

Anomaly: not invariant under $U(1)$ rotation $M \to e^{i\theta} M$

- rotation changes the strong CP angle
- η' mass of order Λ_{qcd}
  - does not vanish with m_q
  - non-perturbative

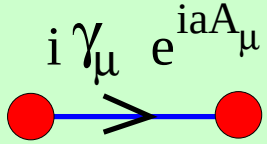
Rooted staggered fermions at finite a qualitatively different



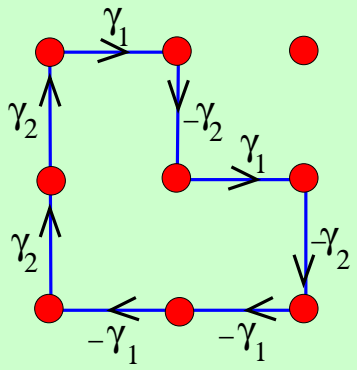
- massless state along m_d axis
- blocks continuation to regions with CP violation
- three neutral non-strange Goldstone bosons, not two
- independent symmetry under complex rotation of any quark mass

What are staggered fermions?

- start with naive fermions, γ_μ for each hop in direction μ
  - $\gamma_\mu p_\mu \rightarrow \gamma_\mu \frac{\sin(a p_\mu)}{a}$



- poles whenever components of momentum are 0 or π/a
  - 16 “doublers”
  - different chiralities since $\frac{d}{dp}\sin(p)|_{p=\pi} = -1$
- exact naive chiral symmetry maintained
  - actually a flavored symmetry of the doublers



In a closed fermion loop

- each factor of γ_μ appears an even number of times
  - product proportional to the identity
  - four spinor components of ψ are independent
- exact $SU(4)$ symmetry Karsten and Smit (1981)

I like nonsense, it wakes up the brain cells. Fantasy is a necessary ingredient in living, It's a way of looking at life through the wrong end of a telescope. Which is what I do, And that enables you to laugh at life's realities. -- Dr. Seuss

Staggered fermions project out one component per site $\psi \to P\psi$

$$P = \tfrac{1}{4}\left(1 + i\gamma_1\gamma_2(-1)^{x_1+x_2} + i\gamma_3\gamma_4(-1)^{x_3+x_4} + \gamma_5(-1)^{x_1+x_2+x_3+x_4}\right)$$

- reduces 16 doublers to 4
- still have exact chiral symmetry $m \to e^{i\theta\gamma_5} m$
- OK: still a flavored symmetry among the doublers

Beware of hidden tastes

The root of all evil

- replace fermion determinant $|D|$ with $|D|^{1/4}$
- hope to reduce effect of four doublers to one
  - BUT: maintains the exact $U(1)$ chiral symmetry

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Storms make trees take deeper roots. -- Dolly Parton

Three flavor QCD

- use rooting for each flavor
- exact $U(1)$ chiral symmetry for each flavor
- three commuting symmetries
- three neutral Goldstone bosons, not two
- incorrect theta dependence

Technical essence

't Hooft vertex

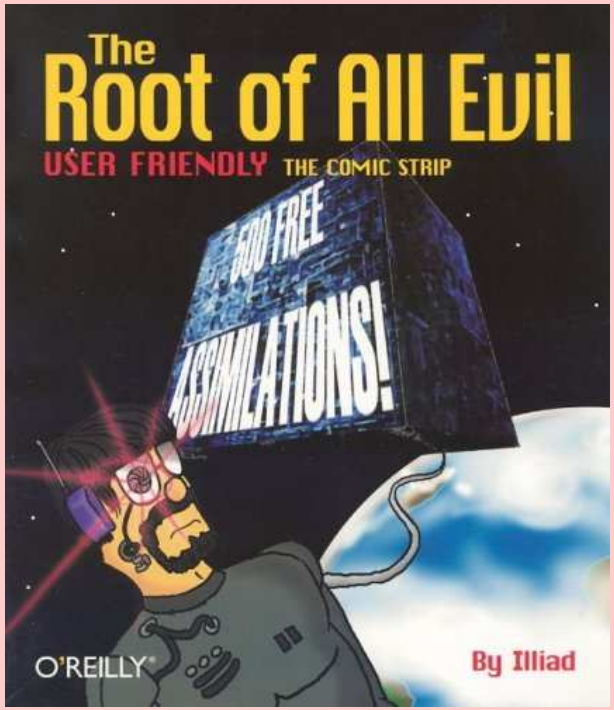
- N_f flavors give $2N_f$-fermion effective interaction
  - non-perturbative
  - related to gauge field topology
  - represents the anomaly
- 4 flavors: an octa-linear interaction $\sim (\overline{\psi}\psi)^4$
  - chiral symmetry OK since two tastes of each chirality
- 1 flavor: bi-linear interaction $\sim \overline{\psi}\psi$
  - effective mass shift
  - breaks all chiral symmetry

Rooting cannot generate the correct vertex

- forbidden by the exact symmetry

Proposed ‘‘escape’’

- unrooted theory 4 ‘‘tastes’’ per flavor
  - 12 quarks overall
  - 144 pseudoscalar mesons
  - 48 neutral, non-strange
- at finite a these might survive rooting

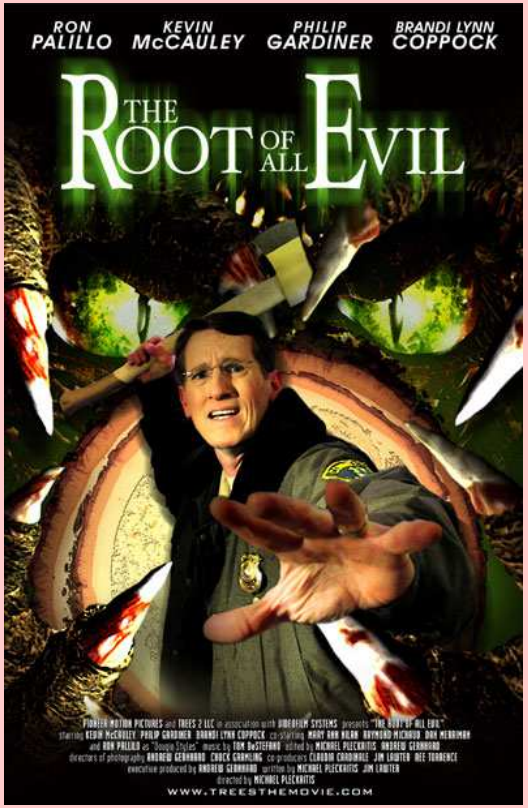


Conjecture:

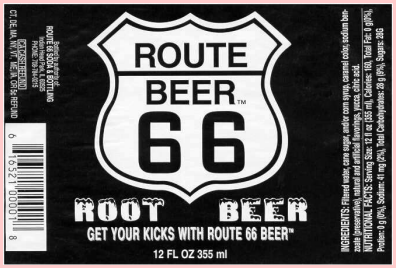
- the extra states cancel in continuum limit
  - including the extra Goldstone boson
- theta recovered by reweighting
  - use overlap for topology

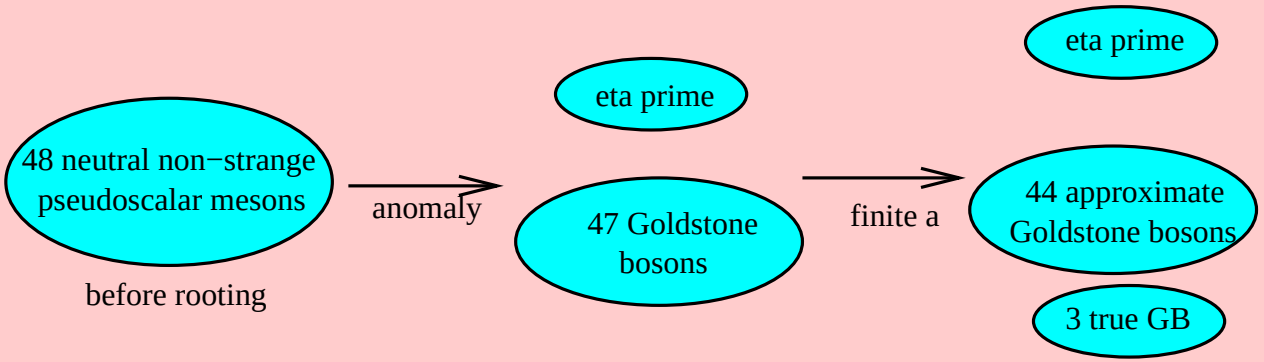
Consider composite propagator $\langle \overline{u}\gamma_5 u(x) \quad \overline{u}\gamma_5 u(y) \rangle$

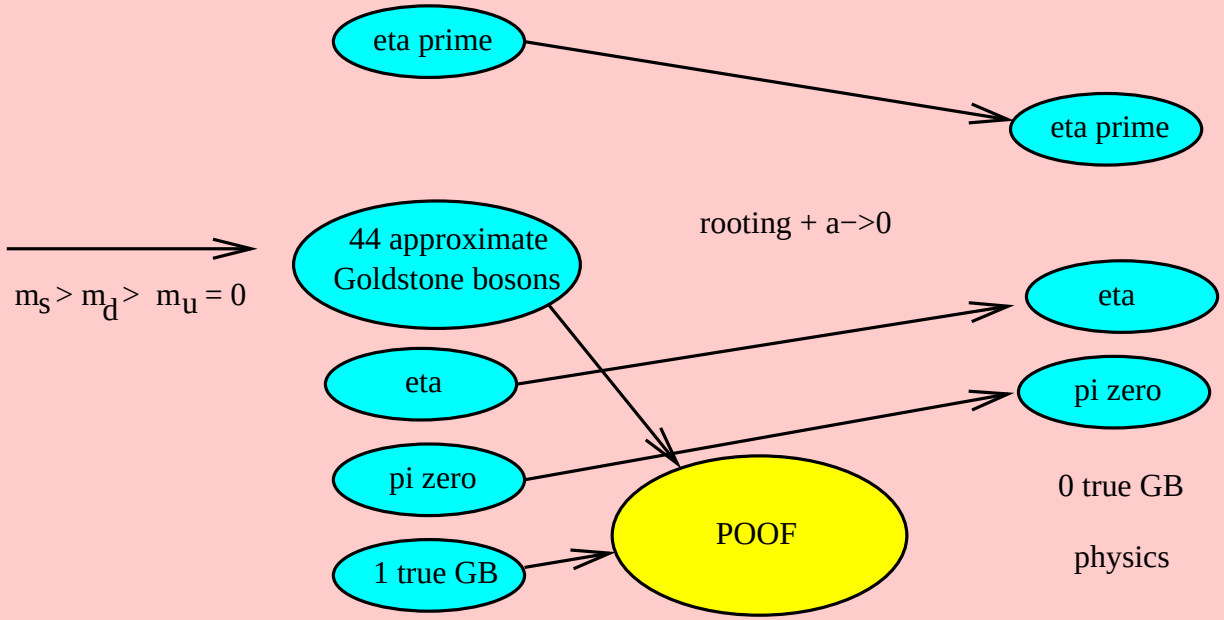
- couples to all 48 neutral non-strange mesons
- expect 48 poles
  - including the extra Goldstone boson
- rooting might give some negative residues
  - conjecture 45 poles cancel
- only $\pi_0,\ \eta,\ \eta'$ should survive



The proposed rooting route







A symmetry for any finite a that disappears for $a = 0$?

- usually we want to restore symmetries as $a \to 0$

Not unitary:

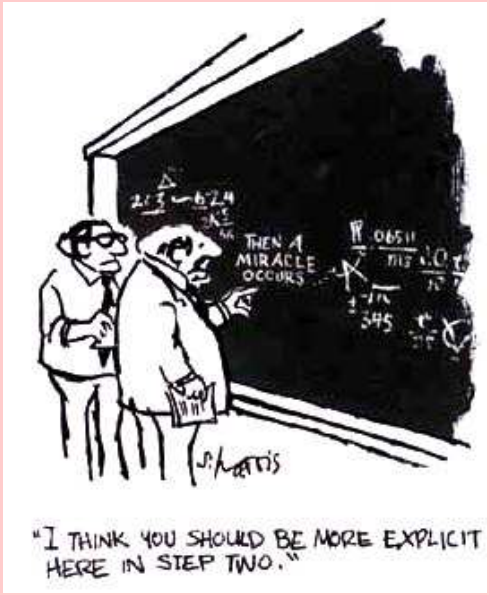
- some production cross sections must be negative

Not local at finite a:

- extra Goldstone boson

Unproven

- "trust us"
- intelligent design?



Issues absent with Wilson, domain wall, overlap

Even if OK, $3 << 48$: huge lattice artifacts expected in singlet sector

‘‘Ugly?’’ ‘‘Absurd?’’ ‘‘Obscene?’’

Ginseng on a billboard in Shanghai

Maybe a movie?