

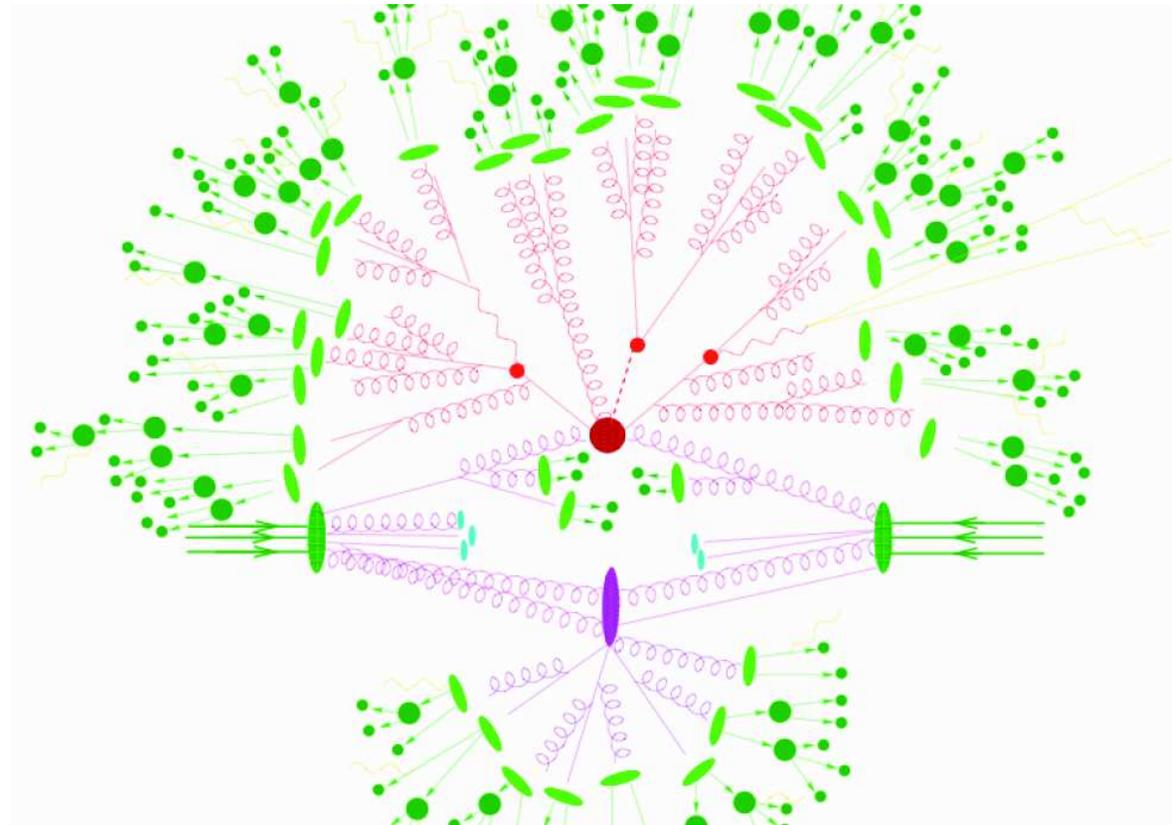
PARTON SHOWER + NLO: A POSITIVE-WEIGHT HARDEST EMISSION GENERATOR

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- Basics of Shower Monte Carlo programs
- The POWHEG formalism
- First applications
- Conclusions



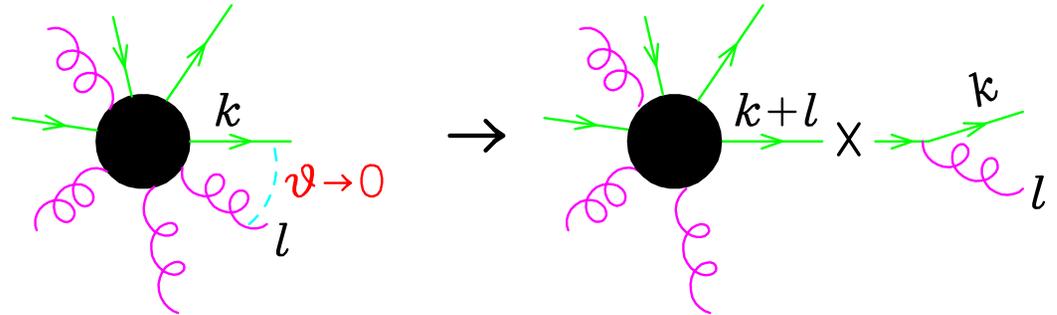
Motivations

- In high-energy collider physics not many questions can be answered without a Shower Monte Carlo (SMC).
- After LEP, QCD testing is less important. With LHC, QCD modeling is a primary issue.
- SMC models have long been neglected in theoretical physics: time to look back at them. And many physicist are looking at them, nowadays (Frixione & Webber; Soper & Nagy; Giele, Kosower & Skands; ...)
- Thinking in terms of shower algorithms gives us an easy to grasp, intuitive understanding of complex QCD phenomena (and a practical way to verify our ideas).
- Shower algorithms summarize most of our knowledge in PQCD: infrared cancellations, Altarelli-Parisi equations, soft coherence, Sudakov form factors. All have a simple interpretation in terms of shower algorithms

Shower basics: collinear factorization

QCD emissions are **enhanced** near the **collinear limit**

Cross sections factorize near collinear limit



$$|M_{n+1}|^2 d\Phi_{n+1} \implies |M_n|^2 d\Phi_n \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\varphi}{2\pi}$$

$$d\Phi_{n+1} = d\Phi_n d\Phi_r \quad d\Phi_r \doteq dt dz d\varphi$$

$$t : (k+l)^2, p_T^2, E^2\theta^2 \dots$$

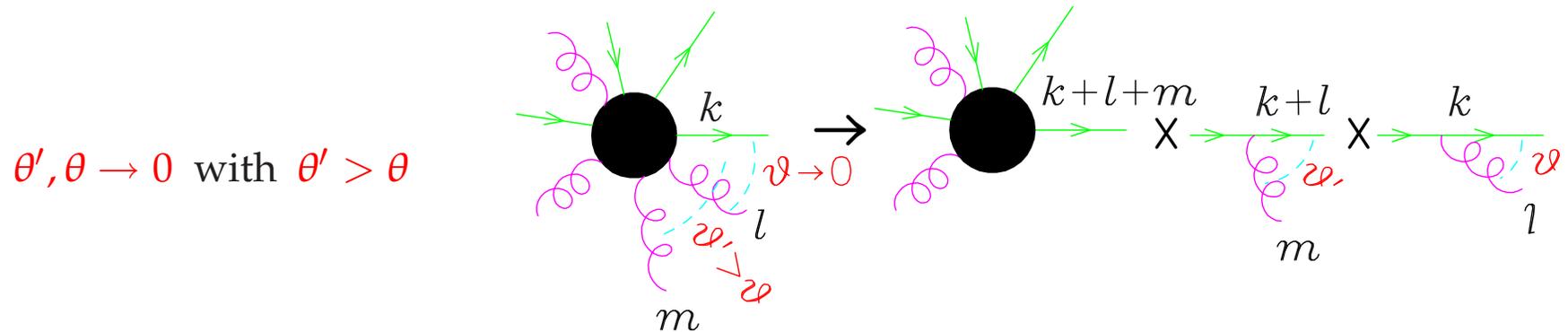
$$z = k^0 / (k^0 + l^0) : \text{energy (or } p_{\parallel} \text{ or } p^+) \text{ fraction of quark}$$

$$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z} : \text{Altarelli-Parisi splitting function}$$

(ignore $z \rightarrow 1$ IR divergence for now)

Shower basics: collinear factorization

If another gluon becomes collinear, **iterate** the previous formula



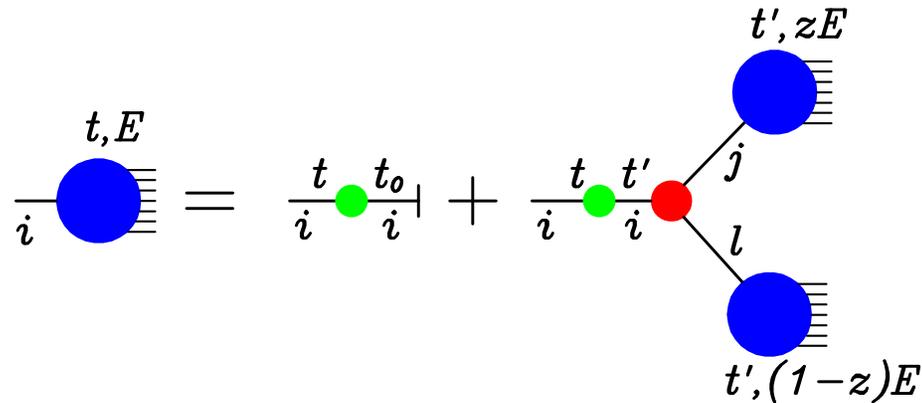
$$\begin{aligned}
 |M_{n+1}|^2 d\Phi_{n+1} &\implies |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi} \frac{dt'}{t'} P_{q,qg}(z') dz' \frac{d\varphi'}{2\pi} \\
 &\quad \times \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\varphi}{2\pi} \theta(t' - t)
 \end{aligned}$$

Collinear partons can be described by a factorized integral ordered in t .

For m collinear emissions

$$\int_{t_{\min}}^{t_{\max}} \frac{dt_1}{t_1} \int_{t_{\min}}^{t_1} \frac{dt_2}{t_2} \cdots \int_{t_{\min}}^{t_{m-1}} \frac{dt_m}{t_m} \propto \frac{\log^m \frac{t_{\max}}{t_{\min}}}{m!} \approx \frac{\log^m \frac{Q^2}{\Lambda^2}}{m!}, \quad \Lambda \approx \Lambda_{\text{QCD}}$$

Final Recipe

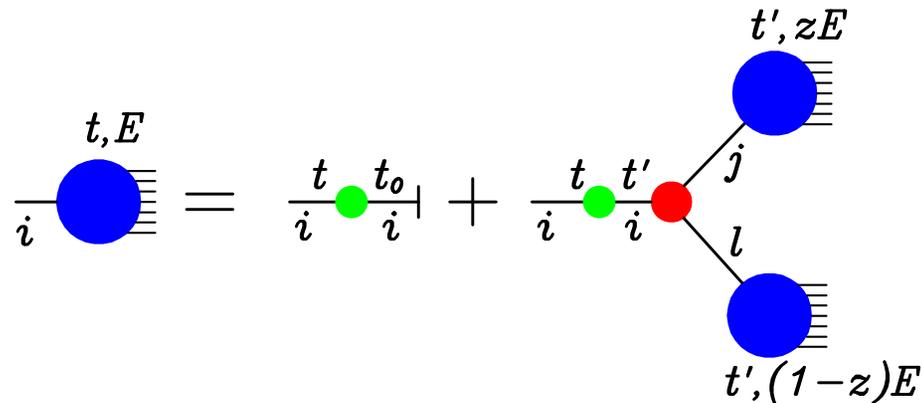


$$\mathcal{S}_i(t, E) = \Delta_i(t, t_0) \langle \mathbb{I} | + \sum_{(jl)} \int_{t_0}^t \frac{\alpha_s(t')}{2\pi} \frac{dt'}{t'} \int dz \int \frac{d\varphi}{2\pi} \Delta_i(t, t') \mathcal{S}_j(t', zE) \mathcal{S}_l(t', (1-z)E)$$

- consider all **tree graphs**.
- assign values to the radiation variables $\Phi_r(t, z \text{ and } \varphi)$ to **each vertex**.
- at each vertex, $i \rightarrow jk$, include a factor

$$\frac{dt}{t} dz \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{d\varphi}{2\pi}$$

Final Recipe



- include a factor $\Delta_i(t_1, t_2)$ to each internal parton i , from hardness t_1 to hardness t_2 .

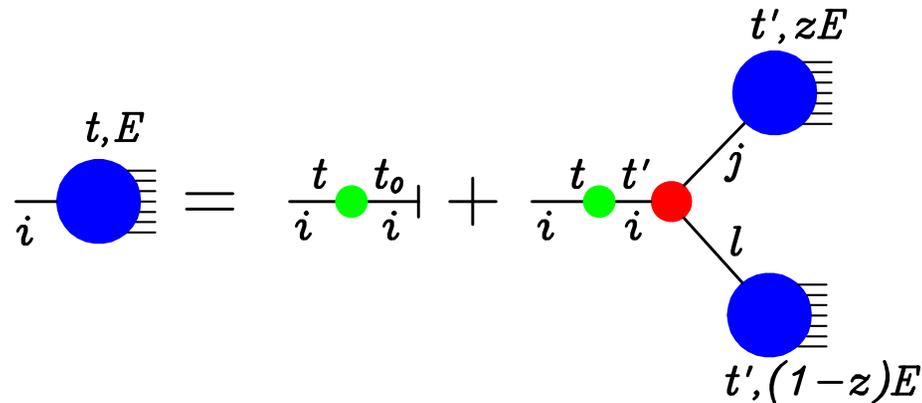
$$\Delta_i(t_1, t_2) = \exp \left[- \sum_{(jl)} \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_S(t)}{2\pi} \int dz P_{i,jl}(z) \int \frac{d\varphi}{2\pi} \right]$$

The weights $\Delta_i(t_1, t_2)$ are called **Sudakov form factors**. They resum all the **dominant virtual corrections** to the tree graph (in the collinear approximation).

Notice that $0 < \Delta < 1$. So Δ can be interpreted as the **non-emission probability**.

- include a factor $\Delta_i(t, t_0)$ on final lines ($t_0 = \text{IR cutoff}$)

First branching



The probability of the **first branching** is independent of subsequent branchings because of KLN cancellation. It is given by

$$\frac{\alpha_S(t')}{2\pi} \frac{dt'}{t'} P_{i,jk}(z) dz \frac{d\varphi}{2\pi} \Delta_i(t, t')$$

Integrating in $dz, d\varphi$, summing over jl , the t' distribution is

$$\Delta_i(t, t') \frac{\alpha_S(t')}{2\pi} \frac{dt'}{t'} \int \sum_{(jk)} P_{i,jk}(z) dz \frac{d\varphi}{2\pi} = -d\Delta_i(t, t')$$

i.e. the distribution is **uniform** in the **Sudakov form factor**!

Shower algorithm

- generate a uniform random number $0 < r < 1$
- solve the equation $\Delta_i(t, t') = r$ for t'
- if $t' < t_0$ stop here (final state line)
- generate z, jk with probability $P_{i,jk}(z)$, and $0 < \varphi < 2\pi$ uniformly
- restart from each of the two branches, with hardness parameter t' .

Accuracy: soft divergences and double-log regions

$z \rightarrow 1$ ($z \rightarrow 0$) region problematic. In fact, for $z \rightarrow 1$, $P_{qq}, P_{gg} \propto 1/(1-z)$

Choice of hardness variable t makes a difference

| | | | | |
|-------------|------------|------------------|--|--|
| virtuality: | $t \equiv$ | $E^2 z(1-z)$ | $\overbrace{\theta^2}^{2(1-\cos\theta)}$ | |
| p_T^2 : | $t \equiv$ | $E^2 z^2(1-z)^2$ | θ^2 | |
| angle: | $t \equiv$ | $E^2 \theta^2$ | | |

$$\text{virtuality : } z(1-z) > t/E^2 \implies \int \frac{dt}{t} \int_{\sqrt{t}/E}^{1-\sqrt{t}/E} \frac{dz}{1-z} \approx \frac{\log^2 \frac{t}{E^2}}{4}$$

$$p_T^2 : z^2(1-z)^2 > t/E \implies \int \frac{dt}{t} \int_{t/E}^{1-t/E} \frac{dz}{1-z} \approx \frac{\log^2 \frac{t}{E^2}}{2}$$

$$\text{angle : } \implies \int \frac{dt}{t} \int_0^1 \frac{dz}{1-z} \approx \log t \log \Lambda$$

Sizable difference in double-log structure!

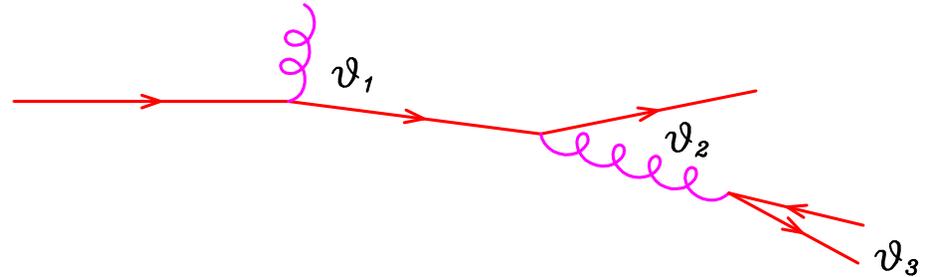
Angular ordering

Mueller (1981) showed that **angular ordering** is the correct choice

$$\frac{d\theta}{\theta} \frac{\alpha_s(p_T^2)}{2\pi} P(z) dz$$

$$\theta_1 > \theta_2 > \theta_3 \dots$$

$$p_T^2 = E^2 z^2 (1-z)^2 \theta^2$$



$\alpha_s(p_T^2)$ for a correct treatment of charge renormalization in **soft region**.

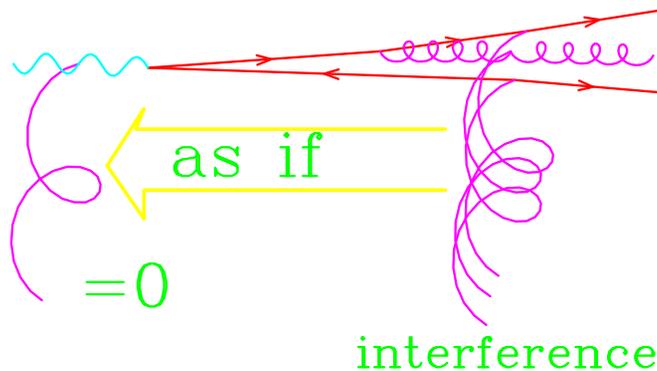
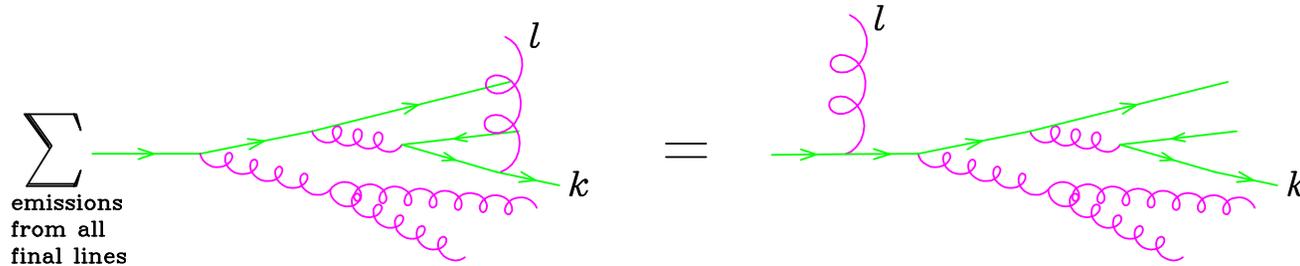
$$\Delta_i(t, t') = \exp \left[- \int_{t'}^t \frac{dt}{t} \int_{\sqrt{\frac{t_0}{t}}}^{1-\sqrt{\frac{t_0}{t}}} dz \frac{\alpha_s(p_T^2)}{2\pi} \sum_{(jk)} P_{i,jk}(z) \right]$$

$$\approx \exp \left\{ - \frac{c_i}{4\pi b_0} \left[\log \frac{t}{\Lambda^2} \log \frac{\log \frac{t}{\Lambda^2}}{\log \frac{t_0}{\Lambda^2}} - \log \frac{t}{t_0} \right]_{t'}^t \right\} \quad (c_q = C_F, c_g = 2C_A)$$

Sudakov dumping stronger than any power of t .

Color coherence

Soft gluons emitted at **large angles** from final-state partons add **coherently**



- angular ordering accounts for soft gluon interference.
- intensity for **photon** jets = 0
- intensity for gluon jets = C_A instead of $2C_F + C_A$

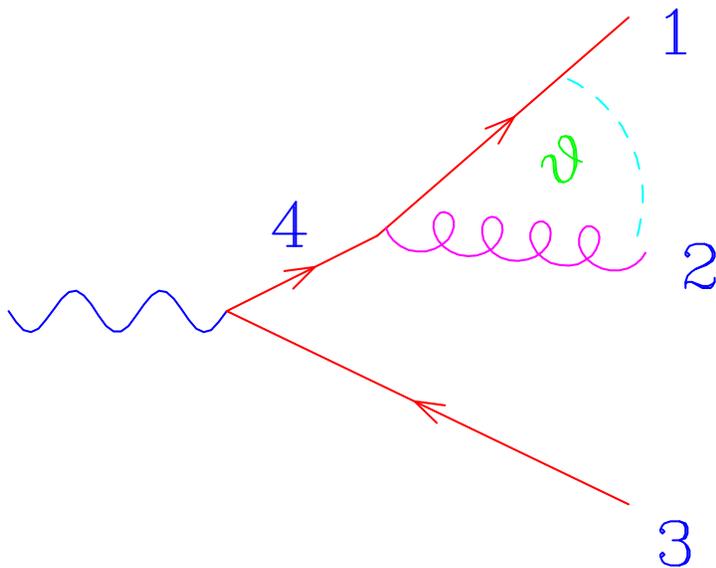
In angular ordered shower Monte Carlo, **large-angle soft emission** is generated **first**.
Hardest emission (i.e. highest p_T) **happens later**.

POWHEG

The POWHEG (Positive-Weight Hardest Emission Generator) method [Nason, hep-ph/0409146] deals with **two main issues**:

1. transform an **angular-ordered shower** into a shower where the **hardest emission** happens **first**
 - generate first event with hardest emission
 - generate all subsequent emissions with a p_T veto equal to the hardest emission p_T
 - pair up the partons that are nearest in p_T
 - generate an angular-ordered shower associated with the paired parton, stopping at the angle of the paired partons (**truncated shower**)
 - generate all subsequent **vetoed showers**
2. include **exact NLO** cross section

Example of truncated shower: e^+e^-



- nearby partons: 1 and 2
- truncated shower: 1 and 2 pair, from θ up to a maximum angle. The truncated shower reintroduces coherent soft radiation from 1 and 2 at angles larger than θ (angular-ordered shower Monte Carlo programs generate those earlier).
- 1 and 2 shower from θ to cutoff
- 3 showers from maximum to cutoff

Truncated showers not yet implemented.

No evidence of effects from their absence up to now [Nason and Ridolfi, hep-ph/0606275], [Latunde-Dada, Gieseke and Webber, hep-ph/0612281]

Reaching the NLO accuracy

$$\Phi_{n+1} = \Phi_{n+1}(\Phi_n, \Phi_r) \quad d\Phi_{n+1} = d\Phi_n d\Phi_r \quad d\Phi_r = dt dz \frac{d\varphi}{2\pi}$$

$$d\sigma^{\text{LO}} = B(\Phi_n) d\Phi_n \left[\Delta^{\text{LO}}(t_{\min}) + \int d\Phi_r \Delta^{\text{LO}}(t) \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \right]$$

$$d\sigma^{\text{NLO}} = \bar{B}(\Phi_n) d\Phi_n \left[\Delta_{t_{\min}}^{\text{NLO}} + \int d\Phi_r \Delta_t^{\text{NLO}} \frac{R(\Phi_{n+1})}{B(\Phi_n)} \right]$$

$$\bar{B}(\Phi_n) = B(\Phi_n) + \underbrace{V(\Phi_n)}_{\text{infinite}} + \underbrace{\int d\Phi_r R(\Phi_{n+1})}_{\text{infinite}}$$

FINITE!

$$\Delta^{\text{LO}}(t) = \exp \left[- \int_t d\Phi'_r \frac{\alpha_s}{2\pi} P(z') \frac{1}{t'} \right] \quad \Delta_t^{\text{NLO}} = \exp \left[- \underbrace{\int d\Phi'_r \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta(t' - t)}_{\text{FINITE because of } \Theta \text{ function}} \right]$$

with $t' = p_T(\Phi_n, \Phi'_r)$ = transverse momentum of the emitted parton.

If we **expand** the shower expression in α_s , we obtain a result that is **accurate** at the NLO level.

POSITIVE if \bar{B} is positive!

Mathematical tricks

- ✓ To **generate** the underlying **Born variables** (Φ_n), distributed according to $\bar{B}(\Phi_n)$, use programs like BASES/SPRING, that after a **single integration**, can generate points distributed according to the **integrand function**.
- ✓ Use the **veto technique** and the **highest- p_T bid** procedure, to generate the **radiation variables**, distributed according to $d\Delta_i(t, t')$.

These tricks are well known to Monte Carlo people.

We have collected a few of them on the appendix of our forthcoming paper.

POsitive-Weight Hardest Emission Generator

POWHEG is a **method** (NOT only a program)

- ✓ it is **independent** from **parton-shower** programs
- ✓ it can use **existing NLO results**
- ✓ it generates events with **positive weights**
- ✓ **NLO accuracy** for **integrated quantities**
- ✓ LL-collinear + double-log (soft-collinear) + large- N_c -soft-log accurate on first radiation (for 3 external colored partons, it is exact at NLL accuracy). It should be easy to extend to more colored partons.

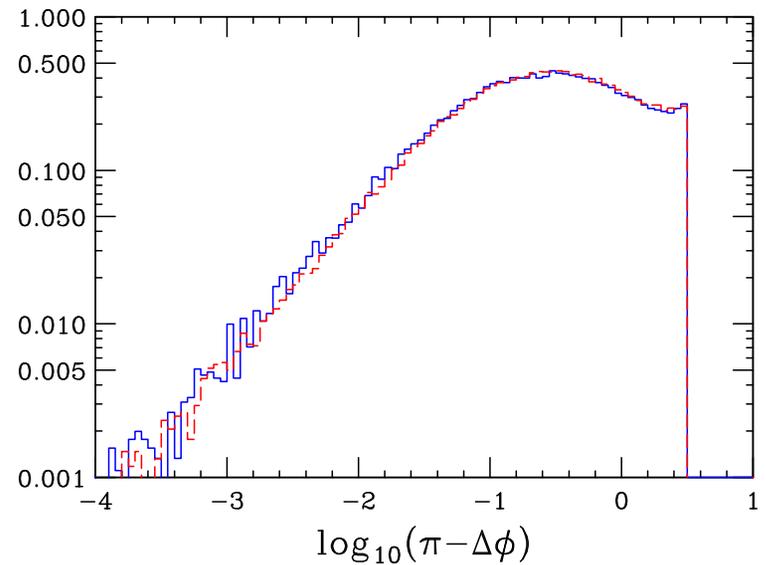
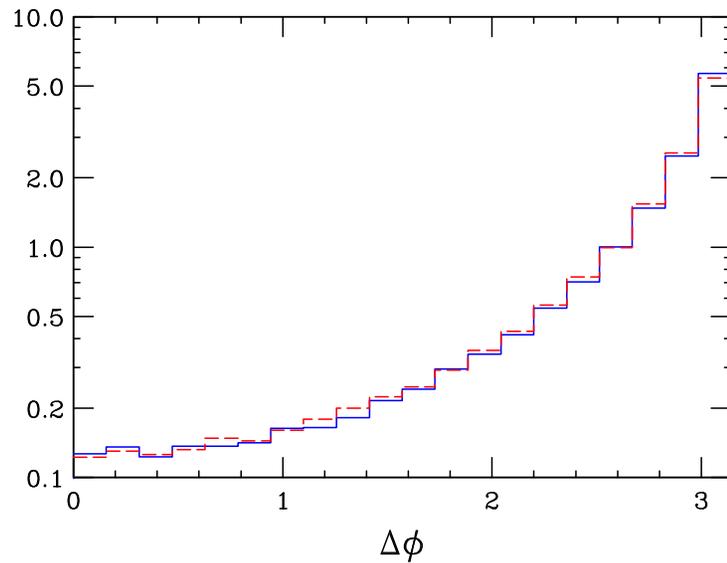
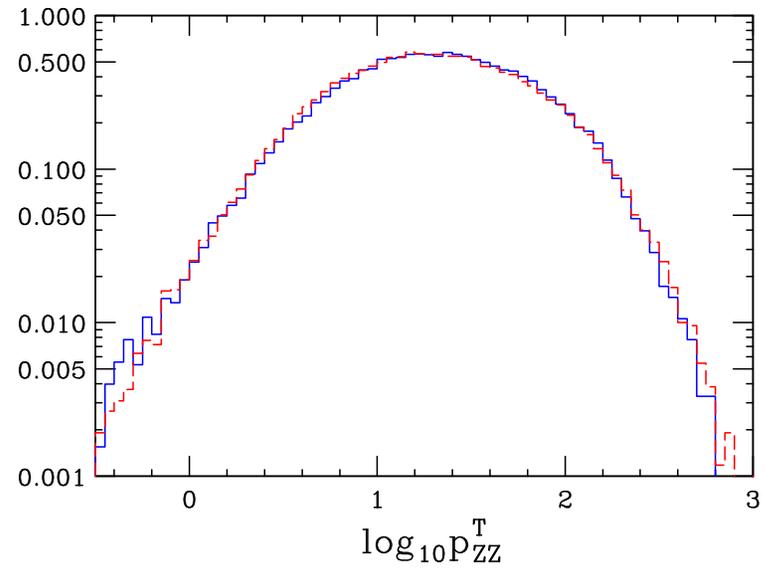
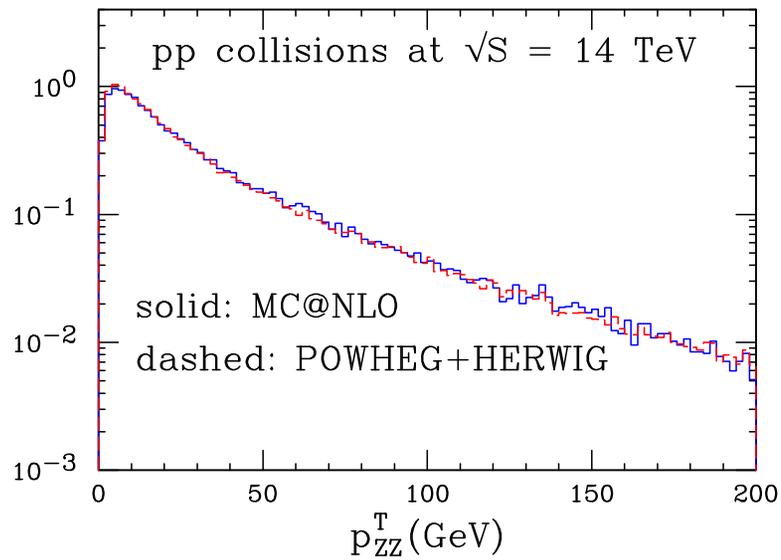
It has already been **successfully** used by

- [Nason and Ridolfi hep-ph/0606275] in **ZZ production**
- [Latunde-Dada, Gieseke and Webber, hep-ph/0612281] in **e^+e^- to hadrons**
- [Frixione, Ridolfi and Nason] in **heavy-quark production**. To appear soon.

ZZ production

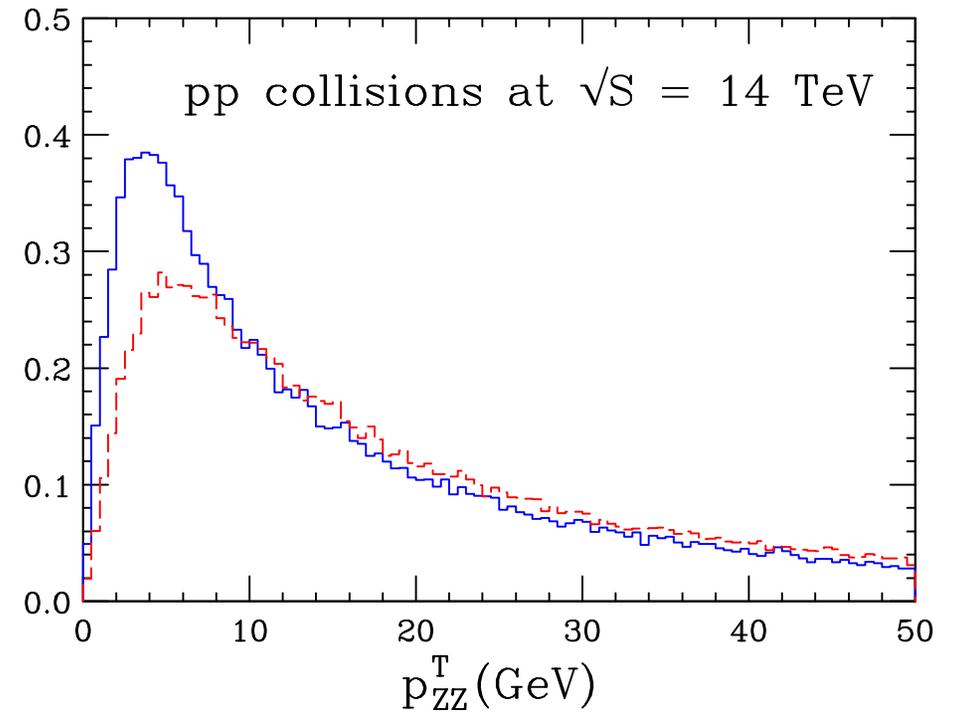
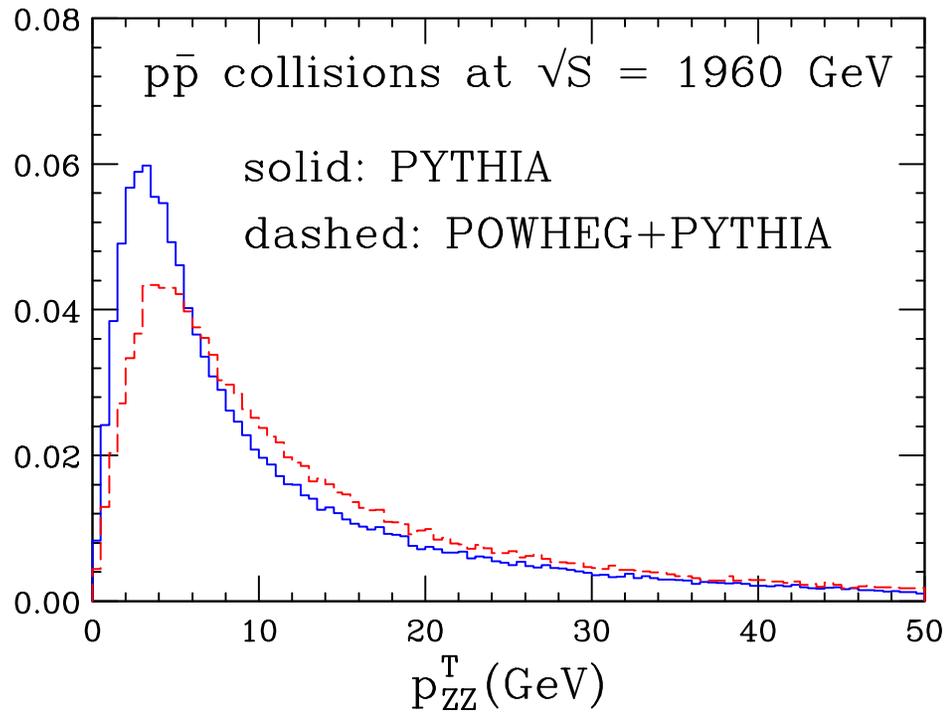
- ✓ Results for both Tevatron and LHC
- ✓ POWHEG interfaced with both **PYTHIA** and **HERWIG**
- ✓ **No truncated shower** for now
- ✓ **No significant difference** from **MC@NLO**

POWHEG + HERWIG vs MC@NLO



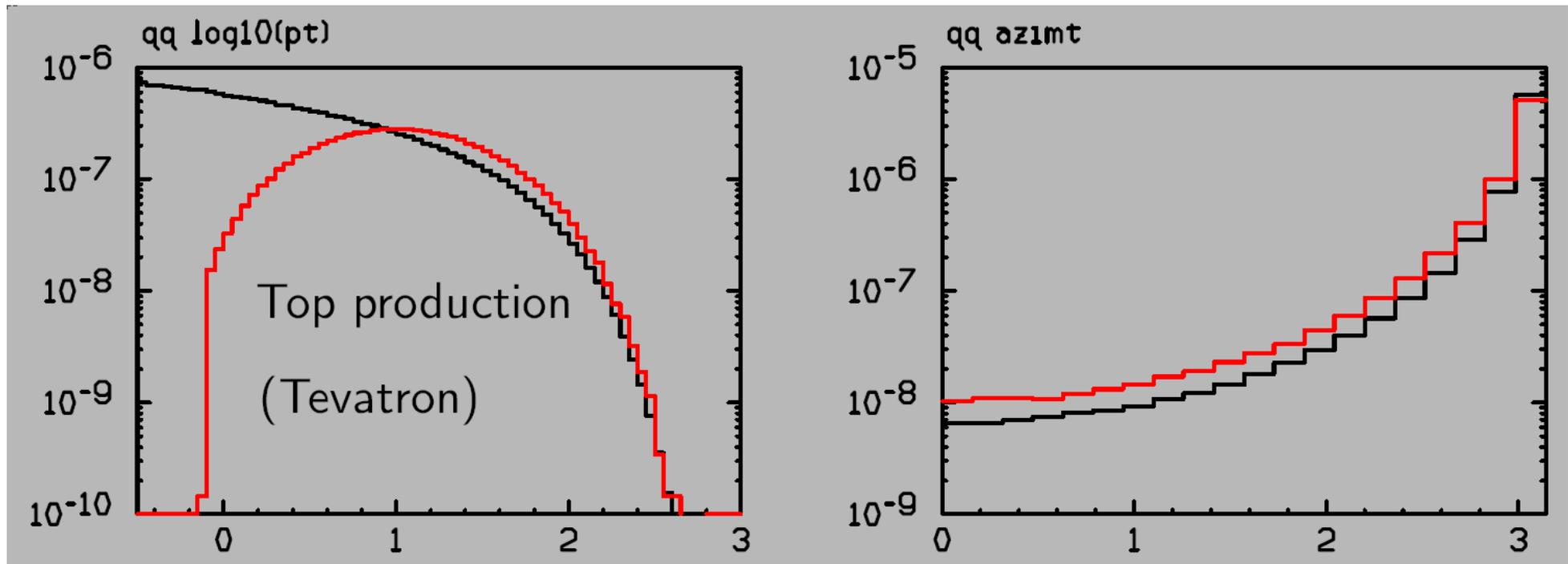
[Nason and Ridolfi hep-ph/0606275]

POWHEG + PYTHIA vs PYTHIA



[Nason and Ridolfi hep-ph/0606275]

$t\bar{t}$ production



red: POWHEG

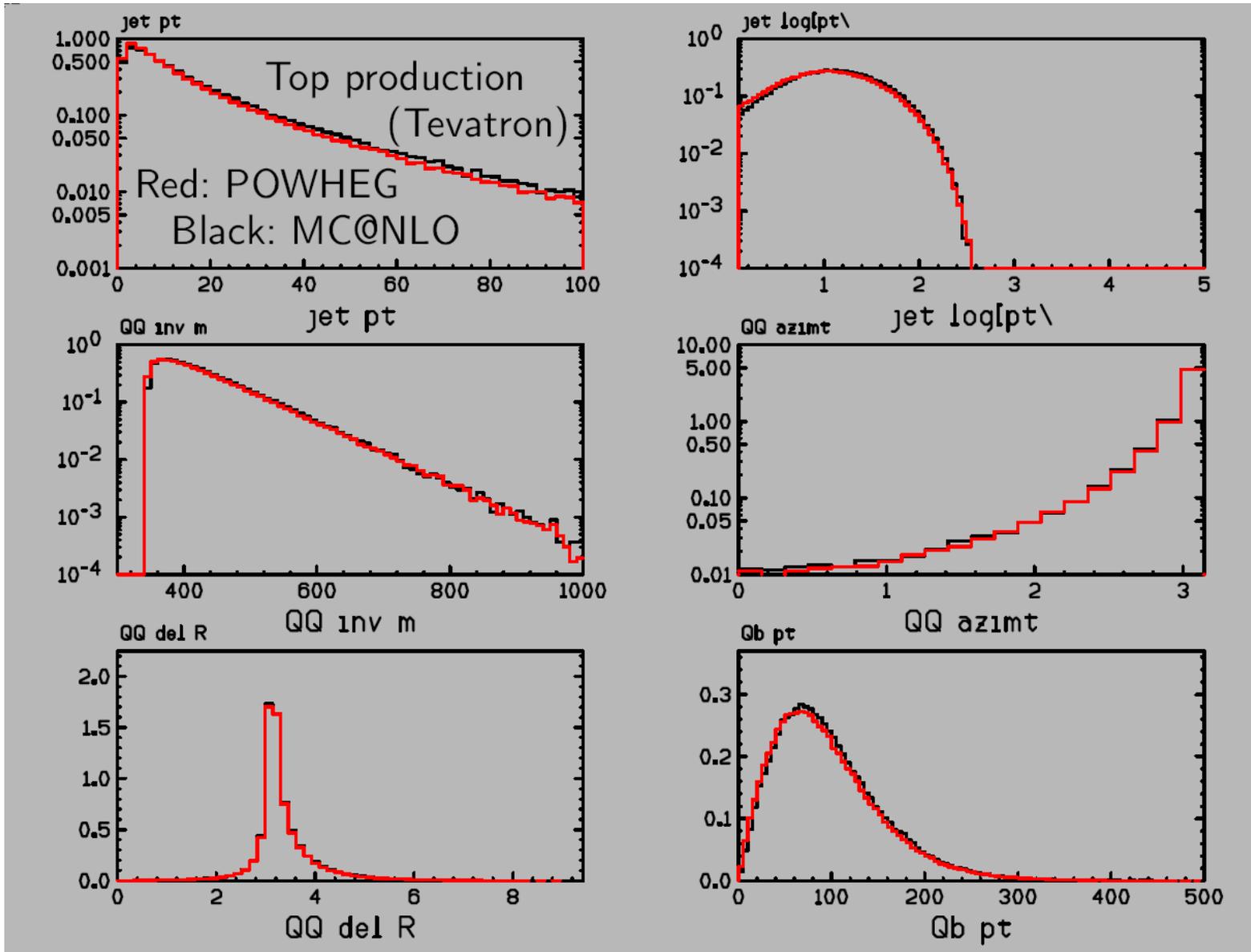
black: NLO

[Frixione, Nason and Ridolfi]

Many more plots on Paolo Nason's talk in Frascati, 23 October 2006.

<http://www.le.infn.it/mcws/>

$t\bar{t}$ production



ALPGEN vs MC@NLO: $t\bar{t} + 1$ jet

[M. Mangano, M. Moretti, F. Piccinini & M. Treccani, hep-ph/0611129]

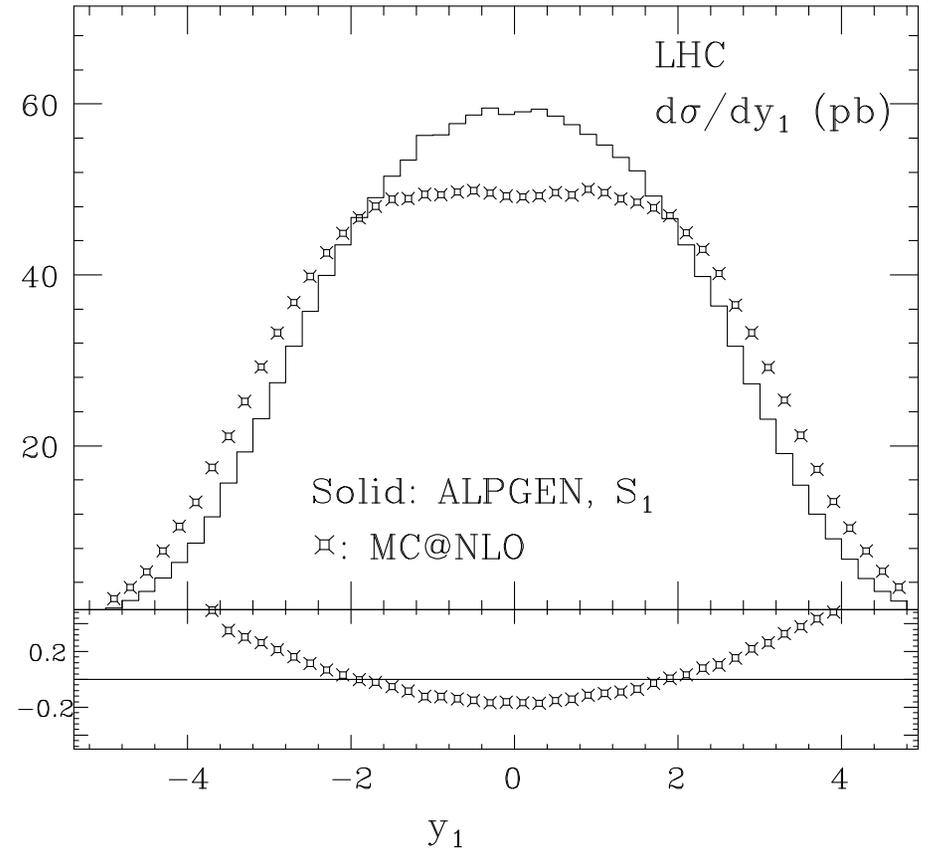
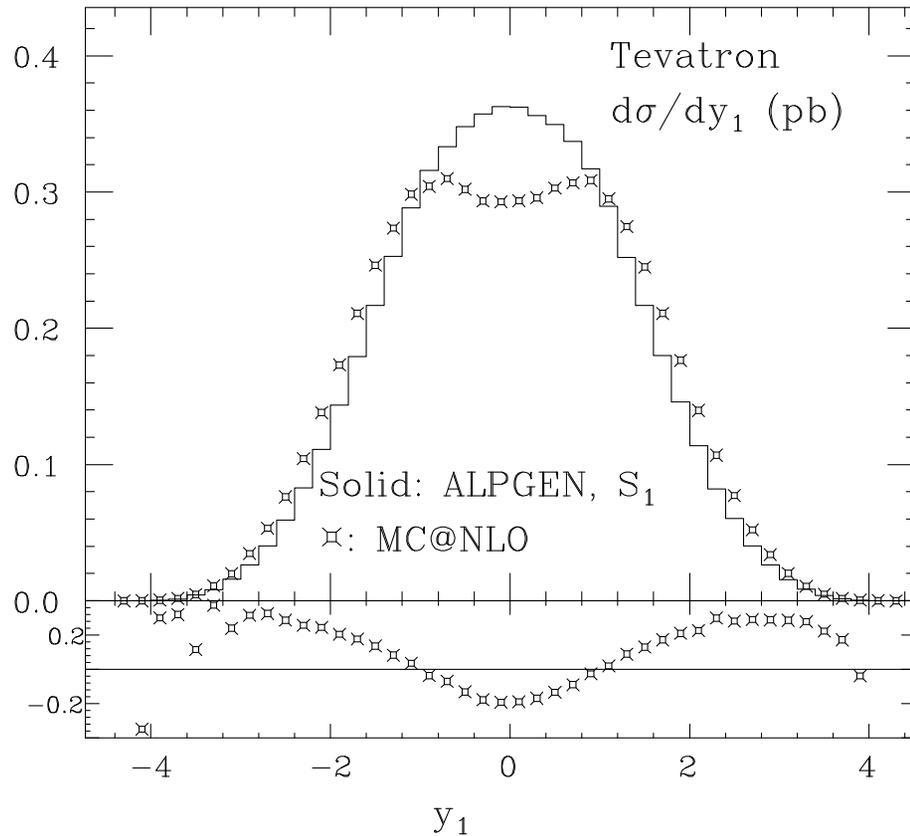
ALPGEN

- Generation: $P_{\min}^T = 30$ GeV, $\Delta R = 0.7$
- Matching: $E_{\min}^T = 30$ GeV, $\Delta R = 0.7$

Jet definitions

- **Tevatron:** $E_{\min}^T = 15$ GeV, $\Delta R = 0.4$, K factor = 1.45
- **LHC:** $E_{\min}^T = 20$ GeV, $\Delta R = 0.5$, K factor = 1.57

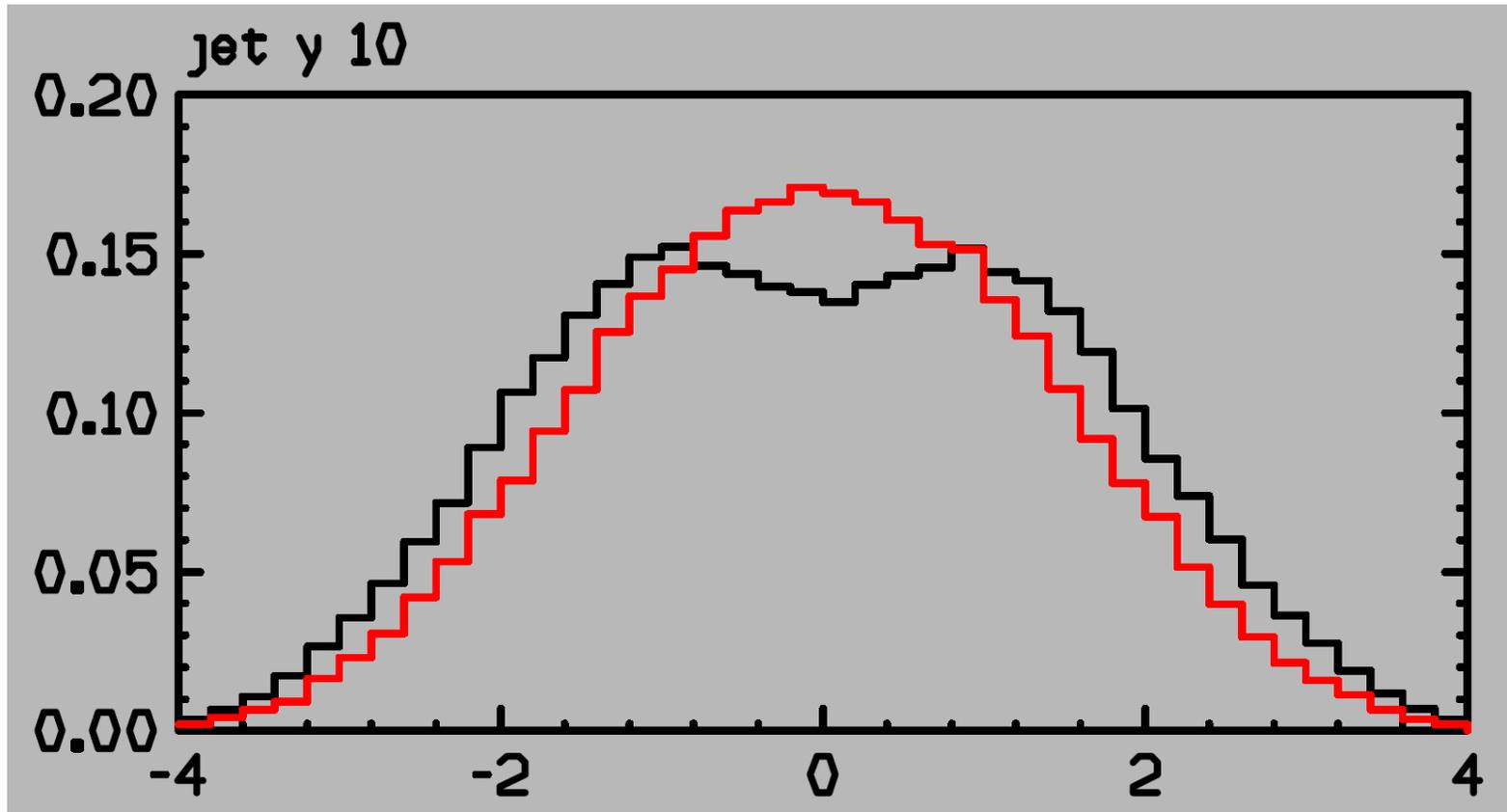
ALPGEN vs MC@NLO: $t\bar{t} + 1$ jet



Rapidity y_1 of the leading jet (highest p_T).

Different shapes both at Tevatron and at the LHC

POWHEG: rapidity of the leading jet



red: POWHEG

black: MC@NLO

Agreement between ALPGEN and POWHEG

Work in progress

POWHEG is fully general and can be applied to **any subtraction framework**.

We [Frixione, Nason and C.O.] are now working on providing any user with **all the formulae and ingredients** to implement an **existing NLO** calculation in the **POWHEG formalism**.

We have looked in details at POWHEG in the two subtraction schemes:

- the FKS ([Frixione, Kunszt, Signer](#))
- the CS ([Catani, Seymour](#)).

The paper will appear very soon (it is already 81 pages long!!).

Strategy

- ✓ Shower Monte Carlo programs to do the final shower already exist
- ✓ Most of them implement a p_T veto
- ✓ Most of them comply with a standard interface to hard processes, the so called **Les Houches Interface (LHI)**

SO...

- construct a POWHEG for a NLO process. Output on **LHI**
- if needed, construct a generator capable to add truncated showers to events from the **LHI**. Output again on **LHI**
- use standard Shower Monte Carlo to perform the p_T vetoed final shower from the event on **LHI**.

Thanks to the Higgs WG

- ✓ VBF Higgs production almost finished in MC@NLO
- ✓ VBF Higgs production is the next in line for POWHEG