EXTRA DIMENSIONS

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INTRODUCTION

- Motivations for extra dimensions:

1. Solve problems of the SM:

   hierarchies (Planck-weak and flavor), dark matter...

2. Occur in String theory (Dienes' lectures)

3. Dual description of strongly coupled $4D$ theories

   ...

- References:

1. Dienes, this TASI and

   http://scipp.ucsc.edu/ haber/tasi_proceedings/dienes.ps

2. Csaki, hep-ph/0404096

3. Sundrum, hep-th/0508134

4. Kribs, hep-ph/0605325

5. KA's course: http://physics.syr.edu/courses/

   PHY880.06Spring/phy880_spring06.htm

GOALS OF LECTURES

Theory + Phenomenology:

- Extra dimensions appear as modes from $4D$ point of view
  (a la particle in $1D$ box):

Lightest $\equiv$ SM

Heavier ones (KK's) $\rightarrow$

Variety of signals in high-energy (real production) and low-energy experiments (virtual effects)

Especially if TeV scale to explain Planck-weak hierarchy!

OUTLINE

1. Basics of KK Decomposition in Flat Spacetime:
   Fermion Chirality from Orbifold

2. Solution to Flavor puzzle, Flavor *Problem*

3. Solution to Flavor Problem:
   Large Brane Kinetic Terms

4. Electroweak Precision Tests

5. Relation to Warped Spacetime and
   Collider Phenomenology

LECTURE 1

BASIC KK DECOMPOSITION

Real Scalar field:

$$S_{5D} \;=\; \int d^4x \int dy[\left(\partial^M \Phi\right)\left(\partial_M \Phi\right) - M^2 \Phi\Phi] \quad (1)$$

- Compacfity on a circle (S^1):

  $-\infty < y < \infty$ with $y \equiv y + 2\pi R$ or

  $0 \le y \le 2\pi R$ ($y = 0$ same as $y = 2\pi R$)

Periodic boundary condition: $\Phi(y = 2\pi R) = \Phi(y) \Rightarrow$

$$\Phi \;=\; \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{n=+\infty} \phi^{(n)}(x) e^{iny/R} \qquad (2)$$

Substitute into S_{5D}, use orthogonality of profiles:

$$S_{4D} \;=\; \int d^4x \sum_n \left[\left(\partial_\mu \phi^{(n)}\right)\left(\partial^\mu \phi^{(n)}\right) - \left(M^2 + \frac{n^2}{R^2}\right) \phi^{(n)} \phi^{(n)} \right] \qquad (3)$$

$4D$ **POINT OF VIEW**

Tower of $4D$ fields: **Kaluza-Klein (KK) modes**

$\phi^{(n)}$ with mass^2: $m_n^2 = M^2 + n^2/R^2$

(n^2/R^2 from ∂_5 acting on profile)

(Fig. 1)

Lightest or zero-mode ($n = 0$) has mass M

(massless only for $M = 0$)

KK modes start at $\sim 1/R$: **compactification scale**

(for $M \ll 1/R$)

Generalize to δ circles of same radius:

$m_n^2 = M^2 + \Sigma_{i=1}^{\delta} n_i^2/R^2$

- Signature of extra dimension from $4D$ point of view:

  appearance of infinite tower of KK modes

  Lightest (zero)-modes $\equiv$ SM

  $+$

  Heavier ones (KK's)

ORBIFOLD

Circle is (smooth) manifold: no special points

"Mod out" manifold by discrete symmetry $\rightarrow$ orbifold

- S^1/Z_2: discrete identification:

  $y \leftrightarrow -y$ in addition to $y \equiv y + 2\pi R$

Physical/fundamental region/domain:

$y = 0$ to $y = \pi R$

(or $y = 0$ to $y = 2\pi R$: $y = \pi R$ to $y = 2\pi R$ *not* independent)

(Fig. 2)

- Endpoints ($y = 0, \pi R$) do *not* transform under Z_2

  (fixed points)

  not identified with each other either by S^1 or Z_2

  (unlike $y = 0, 2\pi R$ on circle)

KK DECOMPOSITION ON ORBIFOLD

Rewrite KK decomposition in terms of even/odd

functions:

$$\Phi(x,y) \;=\; \frac{1}{\sqrt{2\pi R}}\phi^{(0)} + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}}\phi_{\pm}^{(n)} \cos ny/R \;(\sin ny/R) \tag{4}$$

with $\phi_{\pm}^{(n>0)} = 1(i)/\sqrt{2}\left(\phi^{(n)} \pm \phi^{(-n)}\right)$

- Require S_{5D} to be invariant under $y \longrightarrow -y$

  assign (intrinsic) parity transformation to Φ:

$$\Phi(x,-y) \;=\; P\Phi(x,y) \tag{5}$$

  with $P = \pm 1$ (even/odd)

Sets $\phi_{\mp}^{(n>0)} = 0$ for $P = \pm 1$ (Fig. 3)

$\phi^{(0)} = 0$ for $P = -1$

- Summary of orbifold (useful for fermion/gauge fields):

  Reduce number of modes by 2

  Remove zero-mode for odd case

FERMIONS ON CIRCLE

Representation of $5D$ Clifford algrebra:

$$\{\Gamma_M, \Gamma_N\} \;=\; 2\eta_{MN} \tag{6}$$

provided by Dirac matrices

$$\Gamma_\mu = \gamma_\mu \;\; \Gamma_5 = -i\gamma_5 \tag{7}$$

- Smallest (irreducible) representation has 4 components (cf. 2-component Weyl spinor in $4D$)

$$S_{5D} \;=\; \bar{\Psi}\left(i\partial_M \Gamma^M - M\right)\Psi \tag{8}$$

Plug $\Psi_{\alpha=1-4} = \Sigma_n\, \psi_\alpha^{(n)} e^{iny/R}$

$$S_{4D} \;=\; \sum_n \bar{\psi^{(n)}}\left(i\gamma_\mu\partial^\mu - M - in/R\right)\psi^{(n)} \tag{9}$$

- Tower of Dirac (4-component) spinors from $4D$ point of view:

  $m_n^2 = M^2 + n^2/R^2$ (Fig. 4)

FERMION CHIRALITY PROBLEM

For $M = 0$, non-chiral massless modes:

$\psi^{(0)}_{\alpha=1-4} \sim [\psi^{(0)}_L(\alpha = 1,2), \psi^{(0)}_R(\alpha = 3,4)]$

in Weyl representation of Dirac matrices:

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix} \tag{10}$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{11}$$

$$\sigma_\mu = (\sigma_{i=1..3}, 1) \tag{12}$$

- L and R identical under gauge symmetry $\rightarrow$

  can't be SM (chiral fermions):

  LH (RH) doublets/singlets under $SU(2)_L$

CHIRALITY FROM ORBIFOLD

- Choose Ψ_L even $\longrightarrow$ Ψ_R odd:

$$\bar{\Psi}\Gamma^5\partial_5\Psi \ \ni \ \Psi_L^\dagger\partial_5\Psi_R \tag{13}$$

For $M = 0$:

$$\Psi_{L,R} \ \sim \ \sum_n \psi_{L,R}^{(n)} \cos ny/R \ (\sin ny/R) \tag{14}$$

- Massless-mode only for Ψ_L (even)

(Fig. 5)