EXTRA DIMENSIONS

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LECTURE 2

ZERO-MODE FERMION PROFILES

$M = 0$: massless (chiral) mode has *flat* profile $\rightarrow$

no resolution of flavor hierrachy (need to put

hierarchies in $5D$ Yukawa couplings)

Bare mass term: $\bar{\Psi}\Psi = \Psi_L^\dagger \Psi_R + h.c.$ breaks Z_2

Couple fermions to Z_2-odd scalar with potential:

$$\begin{aligned} \mathcal{L}_{5D} \; = \; & \bar{\Psi}\left(i\partial_M \Gamma^M - h\Phi\right)\Psi + \\ & \left(\partial_M \Phi\right)^2 - \lambda\left(\Phi^2 - V^2\right)^2 \end{aligned} \qquad (1)$$

$V(\Phi)$ forces vev for Φ in-between endpoints ("bulk")

clashes with $\Phi = 0$ at endpoints

(Georgi, Grant and Hailu, hep-ph/0007350;

Kalpan and Tait, hep-ph/0110126)

- approximately kink-anti-kink profile for scalar vev

  effectively, adding Z_2-odd mass for fermion

  (via spontaneous breaking of Z_2)

  (Fig. 6)

KK DECOMPOSITION WITH BULK FERMION MASS

$$S_{5D} \;=\; \bar{\Psi}[i\partial_M \Gamma^M + M\epsilon(y)]\Psi \tag{2}$$

Eigenmodes are no longer *single* $\sin$ or $\cos$:

linear combinations of basis functions

GENERAL PROCEDURE FOR KK REDUCTION (I)

Scalar field: $\Phi(x, y) = \Sigma_n \, \phi^{(n)}(x) f_n(y)$

Plug into S_{5D}

$$S_{5D} \; = \; \int d^4x \int dy [\left(\partial^M \Phi\right) \left(\partial_M \Phi\right) - M^2 \Phi \Phi] \quad (3)$$

require that *after* $\int dy$, we get

$$S_{4D} \; = \; \int d^4x \sum_n [\left(\partial_\mu \phi^{(n)}\right) \left(\partial^\mu \phi^{(n)}\right) - \left(M^2 + \frac{n^2}{R^2}\right) \phi^{(n)} \phi^{(n)}] \quad (4)$$

$\rightarrow$ interpret $\phi^{(n)}$'s as particles

(KK modes) from $4D$ point of view

GENERAL PROCEDURE FOR KK REDUCTION (II)

(1) Matching ∂_μ kinetic terms: orthonormality condition

$$\int dy f_n^*(y) f_n(y) \;=\; 1 \qquad (5)$$

(2) Matching mass terms in S_{4D} to

mass term $+\ \partial_5$ in S_{5D} gives differential equation:

$$\partial_y^2 f_n(y) - M^2 f_n^2(y) \;=\; -m_n^2 f_n^2(y) \qquad (6)$$

- Eigenvalue problem: solve for m_n and $f_n(y)$

$m_n^2 \geq M^2 \to f_n(y) \sim e^{\pm i\sqrt{m_n^2 - M^2}y}$

Periodicity $\to \sqrt{m_n^2 - M^2} = n^2/R^2$

$\to m_n^2 = M^2 + n^2/R^2$

(as before)

(what about $m_n^2 < M^2$:

cannot satisfy continuity of derivative at $y = 0, \pi R$)

EXPONENTIAL PROFILES FOR FERMION ZERO-MODES

Plug $\Psi_{L,R} = \psi^{(n)}(x) f_{L,R\,n}(y)$ into S_{5D}:

$$[-\partial_5 + M\epsilon(y)] f_{L\,n} \;=\; m_n f_R \tag{7}$$

$$[\partial_5 + M\epsilon(y)] f_{R\,n} \;=\; m_n f_L \tag{8}$$

$\cos$ or $\sin$ are solutions for $M = 0$, not for $M \neq 0$

(On circle, M has no $\epsilon(y) \rightarrow f_{L,R\,n} \sim e^{iny/R}$)

Zero-mode ($m_n = 0$) for $M \neq 0$ ($m_n \neq 0$ difficult):

$$f_{L\,0}(y) \;=\; N e^{-My} \tag{9}$$

(N is normalization factor: see homework 1)

$f_{R\,0} \sim e^{-My}$ clashes with vanishing at $y = 0, \pi R \rightarrow$ no zero-mode

Discontinuity in derivative of $f_{L\,0}$ at $y = 0, \pi R$ (Fig. 7): matches $\epsilon(y)$ (cf. scalar case)

- $M \neq 0$ still gives massless-mode (unlike scalar)

SOLUTION TO FLAVOR PUZZLE

Coupling to Higgs localized on $y = \pi R$ brane (Fig. 8):

$$\int d^4 x H \Psi_L \Psi'_R \lambda_{5D} \;\rightarrow\; \lambda_{5D} H \psi_L^{(0)} \psi_R'^{(0)} \times f_{L\,0}(\pi R) f_{R\,0}(\pi R) \tag{10}$$

where Ψ_L, Ψ' are $SU(2)_L$ doublets/singlets

$$\lambda_{4D} \;\sim\; \lambda_{5D} e^{-\left(M+M'\right)\pi R} \tag{11}$$

Setting λ_{5D} same for d, s and also $M = M'$

(up to small dependence of normalization on M's)

$$\frac{m_d}{m_s} \;\sim\; e^{-2\Delta M \pi R} \tag{12}$$

$$\sim\; 1/100 \tag{13}$$

$\rightarrow \Delta M \equiv M_s - M_d \sim 2$ in units of $1/(\pi R)$ suffices

- No hierarchies in $5D$/fundamental parameters

  (M, λ_{5D}),

  but still hierarchies in $4D$ couplings

BASIC CONCEPTS

- $5D$ field $\rightarrow$

  KK modes from $4D$ point of view

  with profiles in extra dimension

- Profiles: solutions to wave equation in $5D$ space-time

- Coupling of particles $\propto$ overlap of profile in extra dimension

GAUGE FIELD ON CIRCLE

$$S_{5D} = \int d^4x dy \frac{1}{4} \mathcal{F}_{MN} \mathcal{F}^{MN} \quad (14)$$
$$= \int d^4x dy \frac{1}{4} \left(\mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} + \mathcal{F}_{\mu 5} \mathcal{F}^{\mu 5} \right) \quad (15)$$

with

$$\mathcal{A}_M = \mathcal{A}_\mu + \mathcal{A}_5 \quad (16)$$

Plug $\mathcal{A}_{\mu,\,5} = \Sigma_n \, A^{(n)}_{\mu,\,5} f_{\mu,\,5\;n}(y)$ into S_{5D}

(~ scalar, up to Lorentz index and gauge fixing)

On circle, both components have zero-modes (Fig. 9)

- Unification of spins:

  massless $4D$ scalars from gauge fields

Extra long range force (if $A_5^{(0)}$ massless) $\rightarrow$ ruled out

acquires mass from loop corrections $\rightarrow$

not a robust problem

(unlike fermions)

GAUGE FIELD ON ORBIFOLD

Get rid of zero-mode using orbifold:

$$\mathcal{F}_{\mu 5} = \partial_\mu \mathcal{A}_5 - \partial_5 \mathcal{A}_\mu \tag{17}$$

Two choices (Fig. 10):

(i) $\mathcal{A}_\mu$ even (zero-mode $\equiv$ SM gauge boson) $\longrightarrow$

$\mathcal{A}_5$ odd (no zero-mode)

or

(ii) $\mathcal{A}_\mu$ odd,

$\mathcal{A}_5$ even (zero-mode is Higgs?)

KK DECOMPOSITION FOR GAUGE FIELD ON ORBIFOLD

For $\mathcal{A}_{\mu,\,5}$ even (odd):

$$f_{\mu\,0} = \frac{1}{\sqrt{2\pi R}}\,(\text{flat}) \tag{18}$$

$$f_{\mu\,n}(y) = \frac{1}{\sqrt{\pi R}}\cos ny/R \tag{19}$$

$$f_{5\,n}(y) = \frac{1}{\sqrt{\pi R}}\sin ny/R \tag{20}$$

$A_\mu^{(n\neq 0)}$ "eats" $A_5^{(n)}$ to form massive spin-1 gauge boson

(a la longitudinal $W \sim$

unphyiscal component of Higgs):

$$\mathcal{F}^2_{\mu 5} \ni \partial_\mu \mathcal{A}_5 \partial_5 \mathcal{A}^\mu \tag{21}$$

$$\sim \sum_n A_\mu^{(n)} \partial^\mu A_5^{(n)} \partial_y f_{\mu\,n}(y) \tag{22}$$

(like $W_\mu \partial^\mu H \langle H \rangle$ in SM)

COUPLINGS OF GAUGE *ZERO*-MODE

- Universal (guaranteed by $4D$ gauge invariance):

$$\begin{aligned}\int d^4x dy \bar{\Psi} \Gamma^M \left(\partial_M + g_5 \mathcal{A}_M \right) \Psi \ \ni \ & \sum_n \bar{\psi_L^{(n)}} A_\mu^{(0)} \gamma^\mu \psi_L^{(n)} \times \\ & \int dy f_{L\,n}^2 \frac{g_5}{\sqrt{2\pi R}} \quad (23) \\ = \ & ... g_4 \left(\text{for all } n \right) \quad (24)\end{aligned}$$

with

$$g_4 \left(\text{or } g_{SM} \right) \ = \ \frac{g_5}{\sqrt{2\pi R}} \qquad (25)$$

COUPLINGS OF GAUGE *KK* MODES

- Couplings of zero-mode fermions to gauge KK modes *non*-universal (Fig. 11):

$$g(n, M) \;=\; g_5 \int dy \left(N e^{-My}\right)^2 \times f_{\mu\, n}(y) \tag{26}$$

$$\equiv \; g_4 \times a(n, M) \tag{27}$$

$a \sim O(1)$ (see homework 1)

FLAVOR *PROBLEM* FROM GAUGE KK

Flavor diagonal, but non-universal couplings

in interaction/weak basis:

$$g_4 \left(\bar{d}_{L\,\mathsf{weak}} \; \bar{s}_{L\,\mathsf{weak}} \right) \begin{pmatrix} a_d & 0 \\ 0 & a_s \end{pmatrix} \gamma^\mu A_\mu^{(n)} \begin{pmatrix} d_{L\,\mathsf{weak}} \\ s_{L\,\mathsf{weak}} \end{pmatrix} \tag{28}$$

$\longrightarrow$ Flavor violation *after* rotation to mass basis:

$$\begin{aligned} &\ldots g_4 D_L^\dagger \mathsf{diag}\left(a_d, a_s\right) D_L \;\ni\; g_4 \left(a_s - a_d\right) \left(D_L\right)_{12} \times \\ &\bar{d}_{L\,\mathsf{mass}} \gamma^\mu A_\mu^{(n)} s_{L\,\mathsf{mass}} \end{aligned} \tag{29}$$

where D_L is unitary transformation to go from

interaction/weak basis to mass basis

(for down-type quarks)

Contribution to $K - \bar{K}$ mixing amplitude vs. SM

(GIM-suppressed):

$$\frac{g_4^2}{M_{KK}^2} \left(a_s - a_d\right)^2 \left(D_L\right)_{12}^2 \;\sim\; \frac{g_4^4}{16\pi^2} \frac{m_c^2}{M_W^4} \left(V_{us} V_{ud}\right)^2 \tag{30}$$

$$\left(a_s - a_d\right) \sim O(1/10) \Rightarrow M_{KK} \gtrsim 20 \text{ TeV} \tag{31}$$

$\longrightarrow$ tension with solution to Planck-weak hierarchy