EXTRA DIMENSIONS

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LECTURE 5

References: Sundrum, hep-th/0508134 and

http://www-conf.slac.stanford.edu/ssi/2005/lec_notes/Sundrum1/default.htm

(+ "...Sundrum2..." and "...Sundrum3...")

SUMMARY OF MODEL

SM gauge and fermion in bulk, Higgs on/near brane

Solution to flavor hierarchy:

no flavor problem with large brane kinetic terms

(on *non*-Higgs brane/with light fermions)

Electroweak precision tests satisfied for $m_{KK} \sim$ TeV

with custodial isospin symmetry

UNANSWERED QUESTIONS

1. $m_{KK} \sim$ TeV $\ll M_{Pl.}$?

2. Large brane kinetic terms: mild hierarchy?

   Arbitrary: why only at $y = 0$

- Answered by *warped* geometry

3. Why does EWSB occur?

   $m_H^2 \ll M_{Pl}^2$ due to $m_H^2 \sim m_{KK}^2$?

4. Higgs localized on/near brane?

- Answered by Higgs as A_5 + warped geometry

WARPED EXTRA DIMENSION (RS1)

Bulk cosmological constant (CC) + brane tensions (CC's):

$$\begin{aligned} S_{5D} &= \int d^4x dy \sqrt{-\det G}\left(M_5^3 \mathcal{R}_5 - \Lambda\right) \\ S_{brane\,1,\,2} &= \int d^4x \sqrt{-\det g_{1,\,2}} T_{1,\,2} \end{aligned} \tag{1}$$

where $g_{\mu\nu\,1,\,2}(x) = G_{\mu\nu}(x, y = 0, \pi R)$

With 2 fine-tunings:

$$T_1 = -T_2 = 24 k M_5^3, \tag{2}$$

where $\Lambda = 24 k^2 M_5^3$, obtain flat, but y-dependent $4D$ metric:

$$(ds)^2 = e^{-2ky} \eta_{\mu\nu} (dx)^\mu (dx)^\nu + (dy)^2 \tag{3}$$

- Slice of AdS_5: warp factor, e^{-ky}

$4D$ GRAVITY

$4D$/zero-mode graviton: $g^{(0)}_{\mu\nu}(x) \approx \eta_{\mu\nu} + h^{(0)}_{\mu\nu}(x)$

Plug into $5D$ action:

$$S_{4D,\,\text{eff}} \equiv M_{Pl}^2 \int d^4x \sqrt{-\det\, g^{(0)}}\, \mathcal{R}_4[g^{(0)}] \qquad (4)$$

$$M_{Pl}^2 \sim \frac{M_5^3}{k}\left(1 - e^{-2k\pi R}\right)$$

$$\sim \frac{M_5^3}{k} \text{ for } kR \gg 1 \qquad (5)$$

Choose $k \lesssim M_5$ (higher curvature terms small) $\rightarrow$

$\left(k \lesssim\right) M_5 \lesssim M_{Pl} \sim 10^{18}$ GeV

- $4D$ graviton localized near $y = 0$ (Planck/UV brane):

  profile $\sim e^{-2ky}$

SOLUTION TO PLANCK-WEAK HIERARCHY (I)

$4D$ Higgs localized on $y = \pi R$ brane (TeV/IR brane):

$$S_{\text{Higgs}} = \int d^4x \sqrt{-\det g_{\text{ ind.}}} [g^{\mu\nu}_{\text{ind.}} \partial_\mu H \partial_\nu H - \lambda \left(|H|^2 - v_0^2 \right)^2] \quad (6)$$

using $g^{\text{ind.}}_{\mu\nu} = G_{\mu\nu}(y = \pi R) = g^{(0)}_{\mu\nu} e^{-2k\pi R} \rightarrow$

$$S_{\text{Higgs}} = \int d^4x \sqrt{-\det g^{(0)}} [e^{-2k\pi R} g^{(0)\,\mu\nu} \partial_\mu H \partial_\nu H - e^{-4k\pi R} \lambda \left(|H|^2 - v_0^2 \right)^2] \quad (7)$$

SOLUTION TO PLANCK-WEAK HIERARCHY (II)

Rescale for canonical normalization:

$H \equiv \hat{H} e^{k\pi R} \rightarrow$

$$S_{\text{Higgs}} = \int d^4x \sqrt{\det\, g^{(0)}} [g^{(0)\,\mu\nu} \partial_\mu \hat{H} \partial_\nu \hat{H} - \lambda \left(|\hat{H}|^2 - v_0^2 e^{-2k\pi R} \right)^2] \quad (8)$$

Weak scale/Higgs vev "warped-down" to $\sim$ TeV from Planck scale if

$$k\pi R \sim \log (M_{Pl}/\text{TeV}) \quad (9)$$

Radius stabilization using bulk scalar (Goldberger, Wise):

$M^2/k^2 \sim O(1/10)$

- Exponential hierachy from $O(10)$ hierarchy in $5D$ theory!

(Only 1 tuning remains, same as for usual $4D$ CC)

ANALOGY WITH EXPANDING UNIVERSE

$3D$ space expands with time

$4D$ space-**time** expands with moving along 5^{th} dimension

EFFECTIVE $4D$ MASS SCALE DEPENDS ON POSITION

Consider $y \sim y_0$:

$$(ds)^2_{y\sim y_0} \sim e^{-2ky_0}\eta_{\mu\nu}(dx)^\mu(dx)^\nu + (dy)^2 \quad (10)$$

Rescale: $\hat{x} \equiv e^{-ky_0}x$, $\hat{m}_{4D} \equiv e^{ky_0}m_{4D}$

$$(ds)^2_{y\sim y_0} \sim \eta_{\mu\nu}(d\hat{x})^\mu(d\hat{x})^\nu + (dy)^2 \quad (11)$$

"Flat" metric in terms of $\hat{x} \rightarrow$

$\hat{m}_{4D} \sim m_{5D}$ (no warping) $\rightarrow$

$m_{4D} \sim e^{-ky_0}m_{5D}$

SUMMARY OF RS1

- Master equation:

$$M_{4,\,\text{eff.}}(y) \;\sim\; M_{5,\,\text{fund.}} \times e^{-ky}\;(\text{warp factor})$$

Graviton ($y \sim 0$):

$$M_{\,\text{Pl.}} \sim M_{5D,\,\text{fund.}} \;\Rightarrow M_{5D,\,\text{fund.}} \sim 10^{18}\text{GeV} \tag{12}$$

- Higgs sector ($y \sim \pi R$):

$$M_{\,\text{weak}} \quad\sim\quad M_{5,\,\text{fund.}} \times e^{-k\pi R}$$
$$\Rightarrow M_{\text{weak}} \sim \text{TeV IF } k\pi R \sim \log\left(M_{\,\text{Pl.}}\;/\text{TeV}\right) \sim 30$$

SIMILAR TO FLAT TEV-SIZE EXTRA DIMENSION WITH LARGE BRANE TERMS

RS1: entire SM on brane

SM in bulk (Higgs on TeV brane):

flavor hierarchy $\sim$ as before

- All KK's localized near IR brane (automatically, with*out* actual brane terms) $+$
  $m_{KK} \sim k e^{-k\pi r_c} \sim$ TeV $\not\sim 1/r_c$

Solving wave equation in curved spacetime

intuition: KK's can minimize their mass by living near IR brane (all mass scales warped down)

- "Mimics" large brane kinetic terms of flat geometry
  $+\ m_{KK} \ll M_{Pl.}$ from warping

Solution to flavor problem $+$ electroweak precision tests

$\sim$ as before

UNIFICATION OF SPINS: HIGGS AS A_5

$SU(2)$ on S^1: $A_\mu^{(n)}$ "eat" $A_5^{(n)}$

zero-mode A_5: adjoint (charged)

Doublet fermion with Yukawa coupling $\sim g$:

$\bar{\Psi}_L A_5 \Psi_R$

(Gauge-Yukawa unification)

LOOP-INDUCED MASS FINITE

Naively, quadratically divergent:

$m^2_{A_5^{(0)}} \sim g_4^2/\left(16\pi^2\right)\Lambda_{UV}^2$?

From $5D$ point of view, $5D$ gauge invariance protects it

- $m^2_{A_5^{(0)}}$ cut-off by R^{-1}:

$$m^2_{A_5^{(0)}} \sim \frac{g_4^2}{16\pi^2} R^{-2} \tag{13}$$

(see Sundrum for calculation with $U(1)$: only fermion loop)

Intuitively: "regular" scalar till $E \sim R^{-1}$, part of gauge field for $E \gtrsim R^{-1} \rightarrow$ no loop contribution from these energies (cancellation among KK modes)

What did we gain relative to regular scalar (not A_5, say on brane)?

What is Λ_{UV}?

$5D$ THEORY NON-RENORMALIZABLE

Coupling constant dimensionful $\rightarrow$

loop expansion grows with energy: $g_5^2 E/\left(16\pi^2\right)$

Cut-off when it's ~ 1:

$$\begin{aligned} \Lambda_{UV} &\sim \frac{16\pi^2}{g_5^2} \\ &\sim \frac{16\pi}{g_4^2} R^{-1} \end{aligned} \tag{14}$$

- $m^2_{A_5^{(0)}}$ suppressed relative to regular scalar by $\sim (\Lambda_{UV} R)^2 \sim \left(16\pi/g_4^2\right)^2$

RADIATIVE SYMMETRY BREAKING (HOSOTANI MECHANISM)

$SU(2) \overset{\langle A_5^{(0)} \rangle}{\to} U(1)$ on S^1:

fermion loops $m_{A_5}^2 < 0$

gauge loops of opposite sign:

fermions win if more in number

Cartoon of SM:

$M_{W^\pm} \sim R^{-1}$

W_3 ("photon") massless

$M_{\psi^{(0)}} \sim R^{-1}$ (top quark)

- Not realistic:
  1. $1/R \gtrsim 100$ GeV from colliders:

     fine-tune fermion vs. gauge $\longrightarrow M_{W^\pm} \ll R^{-1}$

  2. no chirality

TOWARDS REALISTIC HIGGS AS A_5 (I)

S^1/Z_2 (for chirality):

A_μ of $SU(2)$ even $\rightarrow$ A_5 odd:

lose zero-mode (which is adjoint, need doublet)

Enlarge gauge group: $SU(3) \rightarrow SU(2) \times U(1)$ by BC

$$\begin{pmatrix} \\ 3 \\ \\ \end{pmatrix} \rightarrow P \begin{pmatrix} \\ 3 \\ \\ \end{pmatrix}$$

$$P = \begin{pmatrix} + & & \\ & + & \\ & & - \end{pmatrix} \tag{15}$$

$$\begin{pmatrix} \\ 8 \\ \\ \end{pmatrix} \rightarrow P^\dagger \begin{pmatrix} \\ 8 \\ \\ \end{pmatrix} P = \begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix} \tag{16}$$

$\rightarrow SU(2) \times U(1)$ A_μ even,

$SU(3)/[SU(2) \times U(1)]$ A_μ odd

$\rightarrow SU(3)/[SU(2) \times U(1)]$ A_5 even, doublet!

TOWARDS REALISTIC HIGGS AS A_5 (II)

$SU(2) \times U(1) \to U(1)$ by A_5 vev

Triplet fermion $\to$ zero-modes:

LH $SU(2)$ doublet + RH singlet

$$\Psi_L = \begin{pmatrix} \Psi_L^D & + \\ \Psi_L^S & - \end{pmatrix}$$
$$\Psi_R = \begin{pmatrix} \Psi_R^D & - \\ \Psi_R^S & + \end{pmatrix} \qquad (17)$$

with $\Psi_L^D A_5 \Psi_R^S$

(a la SM!)

REALISTIC HIGGS AS A_5 IN WARPED EXTRA DIMENSION

- $A_5^{(0)}$ automatically localized near TeV brane $\rightarrow$

  good SM Higgs candidate!

Add extra $U(1)$ for correct charges and custodial isospin

Ψ_L^D and Ψ_R^S have opposite M (not from $\langle A_5^{(0)} \rangle$) $\rightarrow$

LH near $y = 0$, RH near $y = \pi R$ (or vice versa)

Obtain LH and RH zero-modes from different bulk multiplets:

A_5 couples same multiplet?

Mix multiplets at $y = 0$...