TASI 2006:
Extra Dimensions
Homework 1

1 Zero-Mode Fermion and $4D$ Yukawa Coupling

Show that normalized profile for zero-mode fermion is (lecture 2):

$$f(y) \;=\; \sqrt{\frac{M}{e^{2M\pi R}-1}}e^{My} \tag{1}$$

where $0 \leq y \leq 2\pi R$. Assuming that the Higgs field is localized at $y = \pi R$, we see that $M < 0$ to obtain small fermion wavefunction at location of Higgs and hence small $4D$ Yukawa couplings for light fermions (1st and 2nd generations). So, we can neglect $e^{M\pi R}$ compared to 1 whereever appropriate.

The zero-mode ($4D$ or SM) Yukawa coupling in terms of $5D$ Yukawa coupling: $\int dyd^4x\delta(y)\lambda_{5D}H\Psi_L\Psi'_R$, where Ψ_L is $SU(2)_L$ doublet and Ψ'_R is $SU(2)_L$ singlet is:

$$\lambda_{4D} \;\approx \lambda_{5D}Me^{2M\pi R} \tag{2}$$

and the $4D$ mass of fermion is

$$m \;\approx\; \lambda_{4D}v, \tag{3}$$

where, for simplicity, we assume equal $5D$ masses, M, for doublet and singlet fermions.

2 Coupling of Zero-mode Fermion to Gauge KK mode: No Brane Kinetic Terms

The profile for n^{th} gauge KK mode ($m_n = n/R$) is:

$$f_n(y) \;=\; \frac{1}{\sqrt{\pi R}}\cos\left(m_n y\right) \tag{4}$$

Calculate the coupling of zero-mode fermion to gauge KK modes in terms of the coupling of zero-mode gauge field (i.e., SM gauge coupling), $g_4 \equiv g_5/\sqrt{2\pi R}$:

$$g\left(n, M\right) \;=\; g_4 a\left(n, M\right) \tag{5}$$

You should obtain:

$$a(n, M) \;\approx\; \sqrt{2}\frac{4M^2}{4M^2+\left(n/R\right)^2} \tag{6}$$

Use $m_{d,s} = 1$ MeV, 100 MeV and the Higgs vev $v \approx 100$ GeV. Assume, for simplicity, that $\lambda_{5D}M = 1$ for both s, d – otherwise, we have to solve a transcendental equation to obtain M (given the $4D$ Yukawa coupling). Calculate the $5D$ masses $M_{s,d}$ and show that $a(1, M_s) - a(1, M_d) \approx 0.1$.

Compare $K - \bar{K}$ mixing from KK Z exchange as in lecture 2

$$\frac{g_Z^2}{m_{KK}^2} \left[a\left(1, M_s\right) - a\left(1, M_d\right) \right]^2 (\text{mixing angle})^2 \tag{7}$$

to the SM amplitude

$$\frac{g_2^4}{16\pi^2} \frac{m_c^2}{M_W^4} (\text{mixing angle})^2 \tag{8}$$

to obtain bound on m_{KK} of ≈ 20 TeV, using $g_Z \approx 0.75$ and $g_2 \approx 0.65$ for the SM Z and $SU(2)_L$ gauge couplings.

It turns out that another observable called ϵ_K (which is the imaginary/CP-violating part of the above $K - \bar{K}$ mixing amlitude) gives a stronger bound on KK mass scale of ~ 100 TeV.