Why the Standard Model isn't Perfect

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Pre-SUSY 2011, Lecture 2

The Standard Model Works

- Any discussion of the Standard Model has to start with its success
- This is unlikely to be an accident!
- But it's not perfect
  - Chimney plot: Consistency of the Standard Model
  - Unitarity constraints
  - Higgs mass renormalization

Is the Standard Model Self-Consistent?

- M_h is a free parameter in the Standard Model
- Can we derive limits on the basis of consistency?
- Consider a scalar potential:

$$V = \frac{M_h^2}{2} h^2 + \frac{\lambda}{4} h^4$$

- This is potential at electroweak scale
- Parameters evolve with energy in a calculable way

High Energy Behavior of λ

- Renormalization group scaling

$$\boxed{16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 + 12\lambda g_t^2 - 12 g_t^4 + (gauge)}$$

$$t \equiv \log\left(\frac{Q^2}{\mu^2}\right) \qquad g_t = \frac{M_t}{v}$$

- *Large λ (Heavy Higgs):* self coupling causes λ to grow with scale
- *Small λ (Light Higgs):* coupling to top quark causes λ to become negative

Lower Bound on M_h

- Use $\lambda = M_h^2/(2v^2)$
- Quartic coupling finite: $1/\lambda(Q) > 0$ gives upper limit on M_h

$$M_h^2 < \frac{32\pi^2 v^2}{9\log\left(\frac{Q^2}{v^2}\right)}$$

- Assume theory is valid to 10^{16} GeV
  - Gives upper limit on $M_h < 180$ GeV

Does Spontaneous Symmetry Breaking Happen?

- $\lambda(\Lambda) > 0$ gives lower bound on M_h

$$M_h^2 > \frac{3v^2}{2\pi^2} g_t^2 \log\left(\frac{\Lambda^2}{v^2}\right)$$

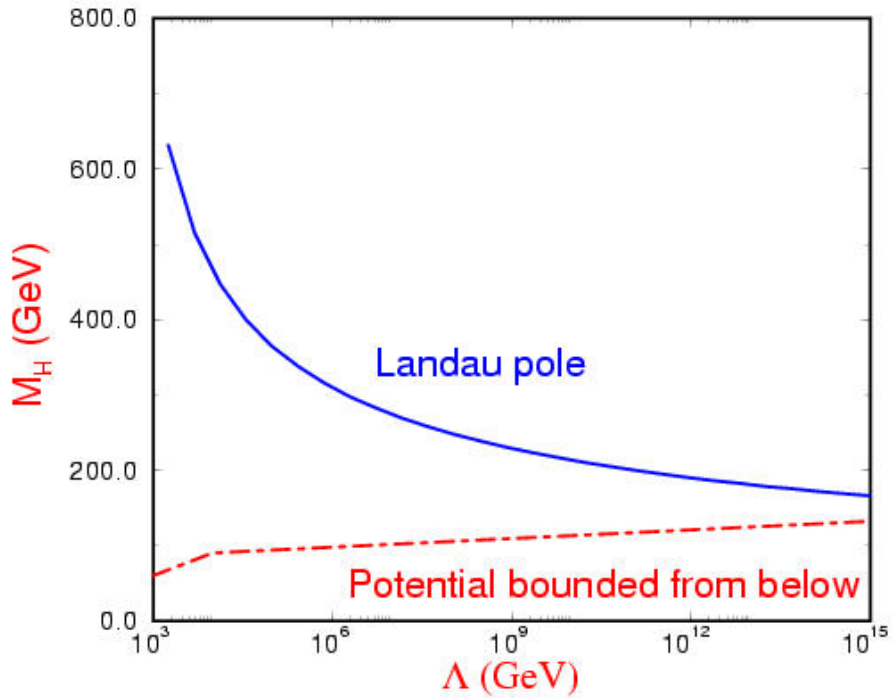
- If Standard Model valid to 10^{16} GeV

$$M_h > 130\ GeV$$

- For any given scale, Λ, there is a theoretically consistent range for M_h

Bounds on SM Higgs Boson

- If SM valid up to Planck scale, only a small range of allowed Higgs Masses



But don't precision measurements require a light Higgs?

- Higgs mass limits from precision measurements assume Standard Model
  - The Standard Model is an effective low energy theory valid to scale Λ

$$L = L_{SM} + \sum \frac{c_i}{\Lambda^2} O_i^{\,6} + \ldots..$$

We don't know the high scale physics!

Unitarity

- Idea: Use unitarity to limit parameters of theory
- Compute J=0 partial wave for scattering amplitude

$$a_0^0 = \frac{1}{16\pi s}\int dt\,|A|$$

- Optical theorem requires $a_0^0 < 1/2$

Rates which grow with energy always violate unitarity at some energy scale

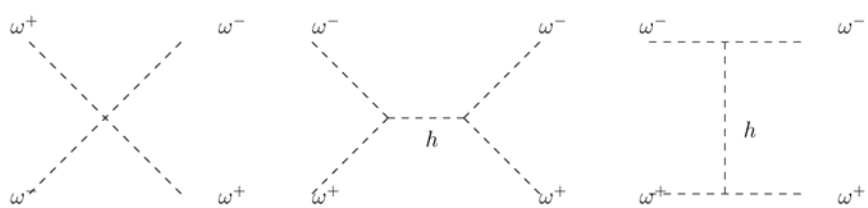
Example: $W^+W^- \to W^+W^-$

- Recall scalar potential (Include Goldstone Bosons in Feynman gauge)

$$V = \frac{M_h^2}{2} h^2 + \frac{M_h^2}{2v} h\left(h^2 + z^2 + 2\omega^+\omega^-\right) + \frac{M_h^2}{8v^2}\left(h^2 + z^2 + 2\omega^+\omega^-\right)^2$$

- Consider Goldstone boson scattering: $\omega^+\omega^- \to \omega^+\omega$

$$iA(\omega^+\omega \to \omega^+\omega^-) = -2i\frac{M_h^2}{v^2} + \left(-i\frac{M_h^2}{v}\right)^2 \frac{i}{t - M_h^2} + \left(-i\frac{M_h^2}{v}\right)^2 \frac{i}{s - M_h^2}$$



Remember this is a real process measured in vector boson fusion

$\omega^+\omega^- \to \omega^+\omega^-$

- Two interesting limits:
  - s, t >> M_h^2

$$A(\omega^+\omega^- \to \omega^+\omega^-) \to -2\frac{M_h^2}{v^2}$$

$$a_0^0 \to -\frac{M_h^2}{8\pi v^2}$$

  - s, t << M_h^2

$$A(\omega^+\omega^- \to \omega^+\omega^-) \to -\frac{u}{v^2}$$

$$a_0^0 \to -\frac{s}{32\pi v^2}$$

$$A(\omega^+\omega^- \to \omega^+\omega^-) = A(W_L^+W_L^- \to W_L^+W_L^-) + O(M_W^2/s)$$

Use Unitarity to Bound Higgs

$$\left|\mathrm{Re}(a_l)\right| \leq \frac{1}{2}$$

- High energy limit:

$$a_0^0 \to -\frac{M_h^2}{8\pi v^2}$$

M_h < 800 GeV

- Heavy Higgs limit:

$$a_0^0 \to -\frac{s}{32\pi v^2}$$

E_c ~1.7 TeV

$\to$ New physics at the TeV scale

Problems with the Higgs Mechanism

- We often say that the SM cannot be the entire story because of the quadratic divergences of the Higgs Boson mass
- But don't we just renormalize the mass using dimensional regularization ????
- Higgs mass has large sensitivity to high scale physics (hierarchy problem)
- $M_h \rightarrow 0$ doesn't increase symmetry of theory
  - Nothing protects Higgs mass from large corrections

Symmetry and the fermion mass

- $\delta m_F \approx m_F$
  - $m_F=0$ quantum corrections vanish
  - $m_F=0$ Lagrangian is invariant under
    - $\Psi_L \rightarrow e^{i\theta_L}\Psi_L$
    - $\Psi_R \rightarrow e^{i\theta_R}\Psi_R$
  - $m_F \rightarrow 0$ increases the symmetry of the theory
  - Yukawa coupling (proportional to mass) breaks symmetry and so corrections $\approx m_F$

$$L = \overline{\Psi}(i\partial)\Psi - \left(\frac{m_F}{v}\overline{\Psi}_L\Psi_R\phi + h.c.\right) + \ldots$$

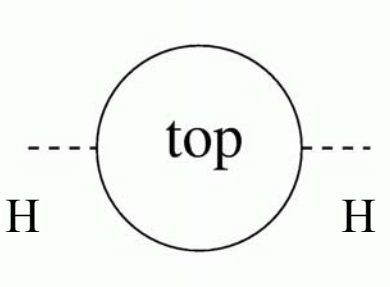
$$= \overline{\Psi}_L(i\partial)\Psi_L + \overline{\Psi}_R(i\partial)\Psi_R - \left(\frac{m_F}{v}\overline{\Psi}_L\Psi_R\phi + h.c.\right) + \ldots$$

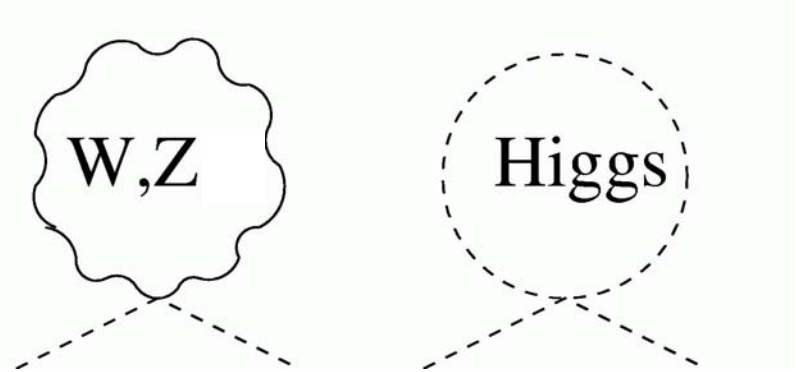
Scalar Masses

- Higgs mass has large sensitivity to high scale physics (hierarchy problem)
- $M_h \rightarrow 0$ doesn't increase symmetry of theory
  - Nothing protects Higgs mass from large corrections

$$\delta M_H^2 \approx -\left(\frac{\Lambda}{0.7\ \mathrm{TeV}}\, 200\ \mathrm{GeV}\right)^2$$

Why 1 TeV (the Terascale)?





- Higgs mass grows with high scale, Λ (*a priori* $\Lambda = M_{pl}$)

$M_H \leq 200$ GeV requires $\Lambda \sim$ TeV

Points to 1 TeV as scale of new physics

What physics can be lurking?

- We are about to probe a new energy scale at the LHC
- There may very well be new physics in the electroweak sector
- Let's review what we really know
- The Standard Model provides an excellent description of physics at the Tevatron energy
- BUT…..we haven't seen the Higgs

Consider the Standard Model as an effective low energy theory

Trouble…..

- Much possible new physics is excluded at the TeV scale
  - Look at possible dimension 6 operators
    - Many more operators than shown here
  - Limits depend on what symmetry is violated

New operators	Experimental limits
$\frac{(\bar{d}s)(\bar{d}s)}{\Lambda^2}$	$\Lambda > 1000\ TeV$
$\frac{m_b(\bar{s}\sigma_{\mu\nu}F^{\mu\nu}b)}{\Lambda^2}$	$\Lambda > 50\ TeV$
$\frac{(h^+D_\mu h)^2}{\Lambda^2}$	$\Lambda > 5\ TeV$
$\frac{(D^2h^+D^2h}{\Lambda^2}$	$\Lambda > 5\ TeV$

New Physics typically must be at scale Λ > 5 TeV

No Higgs?

- Remember, Higgs is used to unitarize the SM
- Unitarity violated at 1.7 TeV without a Higgs
- This sets the scale for something new
- Construct the Standard Model without a Higgs

Higgs is only piece we haven't seen experimentally

Standard Model Revisited

- Scalar sector described by SU(2) doublet

$$\Phi = \frac{(v+h)}{\sqrt{2}} e^{iw^a \sigma^a / v} \qquad V = \frac{\lambda}{4}\left[Tr\left(\Phi^+ \Phi\right) - \frac{v^2}{2} \right]^2$$

- Scalar potential invariant under global $SU(2)_L$ x $SU(2)_R$ symmetry:

$$\Phi \rightarrow L \Phi R^+$$

- Global symmetry broken by $< \Phi >= \frac{v}{\sqrt{2}}$

Higgsless Standard Model

- Construct effective Lagrangian with 2 derivatives describing Goldstone bosons with global $SU(2)_L$ x $SU(2)_R$ symmetry
- This construction is expansion in powers of E^2/Λ^2

$$L = \frac{v^2}{4} Tr\left[\partial_\mu \Sigma \partial^\mu \Sigma^+\right]$$

$$\Sigma = e^{jw^a \sigma^a / v}$$

Looks a lot like the Higgs field Φ with only the Goldstone bosons

Higgsless Standard Model

Gauge theory:

$$L = \frac{v^2}{4} Tr\left[D_\mu \Sigma D^\mu \Sigma^+\right] + (kinetic)$$

$$D_\mu \Sigma = \partial_\mu \Sigma - igW_\mu^a \frac{\sigma^a}{2}\Sigma + ig'B_\mu \Sigma \frac{\sigma_3}{2}$$

This is SM with massive gauge bosons

- At $O(E^2/\Lambda^2)$ gauge couplings identical to SM
- Since no Higgs, unitarity violated TeV scale

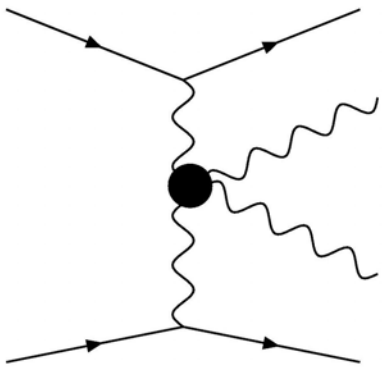
Higgsless Standard Model

- Add $O(E^4/\Lambda^4)$ operators
  - Contributions from $O(E^2/\Lambda^2)$ operators at one loop generate infinities (SM is not renormalizable without Higgs)
  - These infinities absorbed into definitions of $O(E^4/\Lambda^4)$ operators
  - Can do this at every order in the energy expansion
- Coefficients are unknown but limited by precision measurements
- The $O(E^4/\Lambda^4)$ terms will change 3 and 4 gauge boson interactions

WW scattering at LHC

- Four gauge boson interactions are sensitive to unitarity violating physics (Vector boson fusion)

- Look for W^+W^-, ZZ, $Z\gamma$, $W\gamma$ pair production in vector boson fusion



No resonance ⇨
Counting experiment ⇨
Hard

Supersymmetric Models as Alternative to Standard Model

Many New Particles:

- Spin ½ quarks $\Rightarrow$ spin 0 squarks
- Spin ½ leptons $\Rightarrow$ spin 0 sleptons
- Spin 1 gauge bosons $\Rightarrow$ spin ½ gauginos
- Spin 0 Higgs $\Rightarrow$ spin ½ Higgsino

Unbroken supersymmetry $\Rightarrow$ degenerate masses of partners

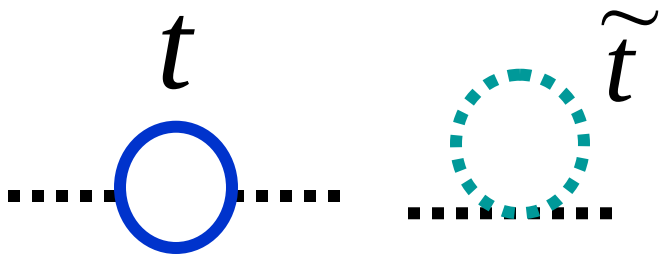
SUSY must be a broken symmetry

Supersymmetric Theories

- Predict many new undiscovered particles (>29!)
- Very predictive models
  - Can calculate particle masses, interactions, everything you want in terms of a few parameters
  - Solve naturalness problem of Standard Model
- Any Supersymmetric particle eventually decays to the lightest supersymmetric particle (LSP) which is stable and neutral (assuming R parity)
  - Dark Matter Candidate

SUSY….Our favorite model

- Quadratic sensitivity to high scale physics cancelled automatically if SUSY particles at TeV scale
- Cancellation result of ***supersymmetry***, so happens at every order

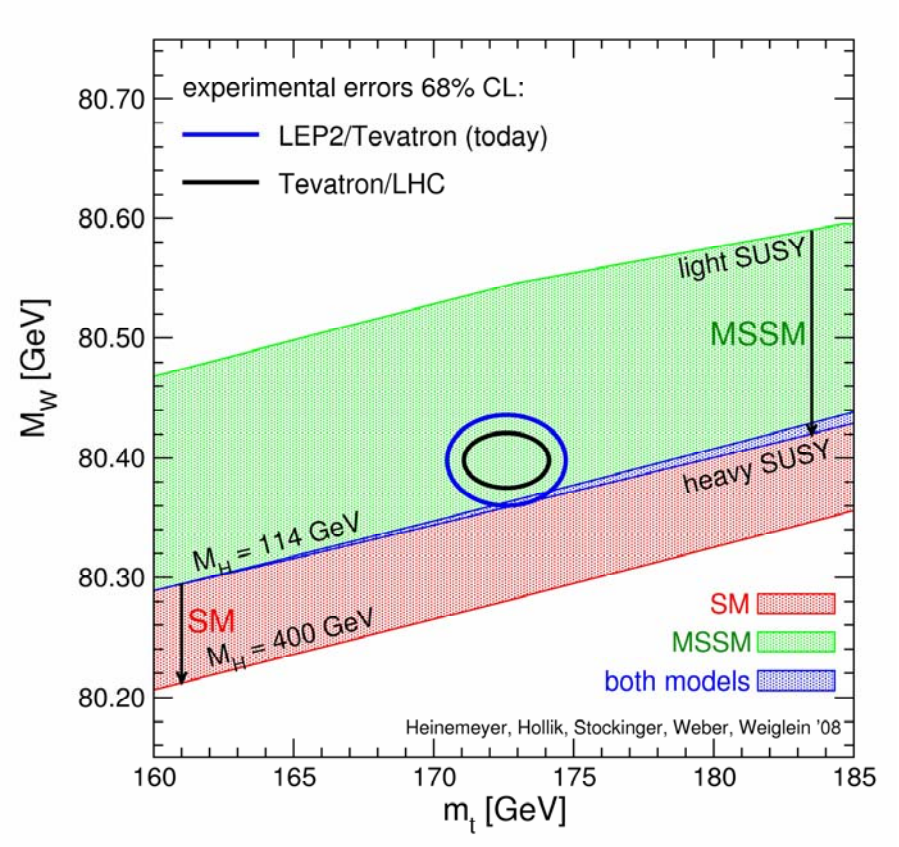


$$\delta M_h^{\ 2} \approx (....) G_F \Lambda^2 (M_t^{\ 2} - M_{\tilde{t}}^{\ 2})$$

- Stop mass should be TeV scale

Supersymmetry (MSSM version)

- Good agreement with EW measurements if SUSY masses are 1-2 TeV



General 2 Higgs Doublet Model

- 6 free parameters, plus a phase

$$
\begin{aligned}
V(H_1,H_2) \quad &= \lambda_1 (H_1^{\ +} H_1 - v_1^2)^2 + \lambda_2 (H_2^{\ +} H_2 - v_2^2)^{\ 2} \\
&+ \lambda_3 \left[(H_1^{\ +} H_1 - v_1^2) + (H_2^{\ +} H_2 - v_2^2) \right]^2 \\
&+ \lambda_4 \left[(H_1^{\ +} H_1)(H_2^{\ +} H_2) - (H_1^{\ +} H_2)(H_2^{\ +} H_1) \right] \\
&+ \lambda_5 \left[\mathrm{Re}(H_1^{\ +} H_2) - v_1 v_2 \cos\xi \right]^2 \\
&+ \lambda_6 \left[\mathrm{Im}(H_1^{\ +} H_2) - v_1 v_2 \sin\xi \right]^2
\end{aligned}
$$

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}$$

- W and Z masses just like in Standard Model $M_W^2 = \dfrac{g^2 (v_1^2 + v_2^2)}{2}$
- ρ parameter:

$$\rho = \frac{M_W}{M_Z \cos\theta_W} = 1$$

ρ=1 for any number of Higgs doublets or singlets

Higgs Potential Restricted in SUSY Models

- Two Higgs doublets with opposite hypercharge

$$H_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \qquad H_! = \begin{pmatrix} \phi_1^{0*} \\ -\phi_1^- \end{pmatrix}$$

- Quartic couplings fixed by supersymmetry
- H_1 has Y=-1/2, H_2 has Y=+1/2 [$Q_{em}=(T_3+Y)/2$]
- SUSY Potential has “F” and “D” terms:

$$V_F = |\mu|^2 \left(|H_1|^2 + |H_2|^2 \right)$$

$$V_D = \frac{1}{2} \sum_a \left(g_a H_i^* T^a H_i \right)^2$$

$$U(1): \quad \left[\frac{g'}{2} \left(|H_2|^2 - |H_1|^2 \right) \right]^2$$

$$SU(2): \quad \left[\frac{g}{2} \left(H_1^{i*} \sigma_{ij}^a H_1^j + H_2^{i*} \sigma_{ij}^a H_2^j \right) \right]^2$$

Gauge couplings / generators

SUSY Potential

- SU(2) identity: $\sigma^a_{ij}\sigma^a_{kl} = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}$

$$V_D = \frac{g^2}{8}\left(4\left|H_1^* \cdot H_2\right|^2 - 2\left(H_1^* \cdot H_1\right)\left(H_2^* \cdot H_2\right) + \left(\left|H_1\right|^2\right)^2 + \left(\left|H_2\right|^2\right)^2\right) + \frac{g'^2}{8}\left(\left|H_2\right|^2 - \left|H_1\right|^2\right)^2$$

- Scalar potential

$$\boxed{V = \left|\mu\right|^2\left(\left|H_1\right|^2 + \left|H_2\right|^2\right) + \frac{g^2 + g'^2}{8}\left(\left|H_2\right|^2 - \left|H_1\right|^2\right)^2 + \frac{g^2}{2}\left|H_1^* \cdot H_2\right|^2}$$

- V is positive definite: minimum at $\langle V\rangle = \langle H_1\rangle = \langle H_2\rangle = 0$

No electroweak symmetry breaking!

The MSSM Philosophy

- Add all soft (dimension 3 or less) terms allowed
  - They don't introduce quadratic Λ^2 contributions
- Potential has 3 free parameters (1 of which is fixed by M_W)

$$V = \left(m_1^2 + |\mu|^2\right) H_1 H_1^+ + \left(m_2^2 + |\mu|^2\right) H_2 H_2^+ - m_{12}^2 (\varepsilon_{ab} H_1^a H_2^b + h.c.)$$
$$+ \left(\frac{g'^2 + g^2}{8}\right)\left(H_1 H_1^+ - H_2 H_2^+\right)^2 + \frac{g^2}{2}\left|H_1 H_2^+\right|^2$$

Gauge Couplings

- If m_{12}=0, potential is positive definite and no symmetry breaking

Minimize Potential

- Minimize potential

$$\frac{\partial V}{\partial h_1^0} = 2\left(m_1^2 + |\mu|^2\right)h_1^0 - 2m_{12}^2 h_2^{0*} + \frac{g^2 + g'^2}{2} h_1^0\left(\left|h_1^0\right|^2 - \left|h_2^0\right|^2\right) = 0$$

$$\frac{\partial V}{\partial h_2^0} = 2\left(m_2^2 + |\mu|^2\right)h_2^0 - 2m_{12}^2 h_1^{0*} - \frac{g^2 + g'^2}{2} h_2^0\left(\left|h_1^0\right|^2 - \left|h_2^0\right|^2\right) = 0$$

- Minimization conditions:

$$m_{12}^2 = \frac{\left(m_1^2 + m_2^2 + 2\mu^2\right)\sin 2\beta}{2}$$

$$\mu^2 = \frac{m_2^2 - m_1^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{g^2 + g'^2}{4}\left(v_1^2 + v_2^2\right)$$

$$\frac{M_Z^2}{2}$$

μ is naturally of $O(M_Z)$

EWSB and SUSY Models

- Electroweak symmetry broken by vevs

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

- W gets mass, $M_W^2 = g^2(v_1^2 + v_2^2)/2$
- 5 Physical Higgs bosons, h^0, H^0, $H^\pm$, A^0
- 2 free parameters, typically pick

$$M_A, \tan\beta = v_2/v_1$$

- Predict M_h, M_H, $M_{H\pm}$

$$M_A^2 = m_{12}^2 (\tan\beta + \cot\beta)$$

$$M_{H^\pm}^2 = M_A^2 + M_W^2$$

Neutral Higgs Masses

$$M_{h,H}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{\left(M_A^2 + M_Z^2\right)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

- $M_h < M_Z \cos 2\beta$
- Theory implies light Higgs boson!
- Neutral Higgs mass matrix diagonalized with mixing angle α

$$\cos 2\alpha = -\cos 2\beta \left(\frac{M_A^2 - M_Z^2}{M_H^2 - M_h^2}\right)$$

Theoretical Upper Bound on M_h

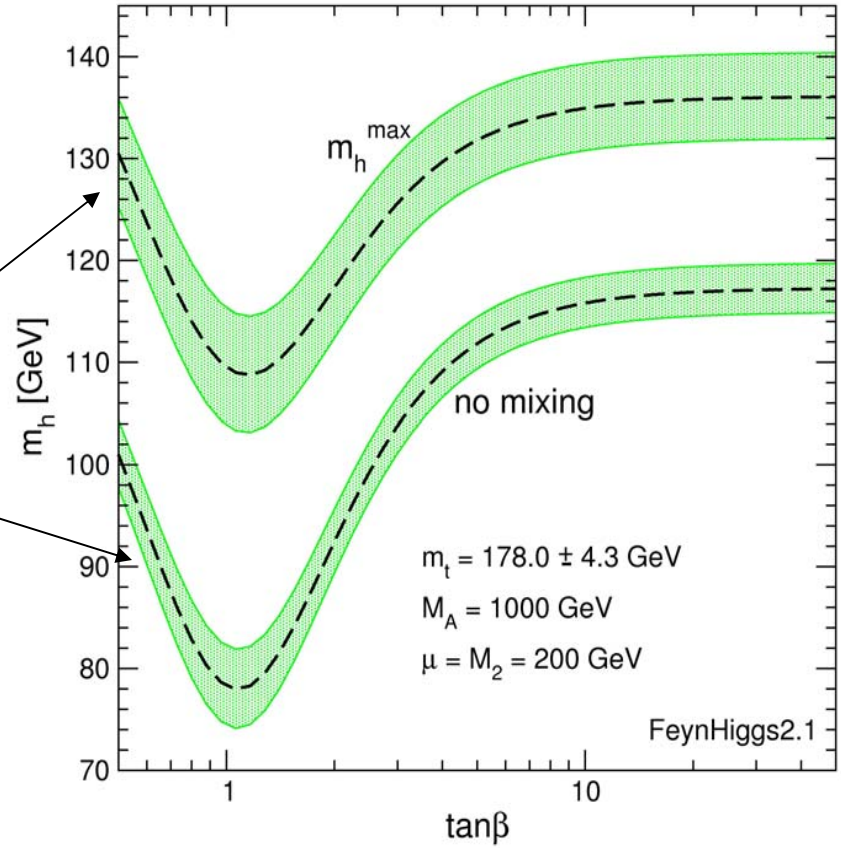
- At tree level, $M_h < M_Z$
- Large corrections $O(G_F m_t^2)$
  - Predominantly from stop squark loop

$$M_h^2 \le M_Z^2 \cos^2 2\beta + \frac{3G_F m_t^4}{\sqrt{2}\pi^2 \sin^2 \beta} \ln\left[\frac{\tilde{m}_t^2}{m_t^2}\right]$$

- Stop mass should be TeV scale for naturalness

Theoretical Upper Bound on M_h

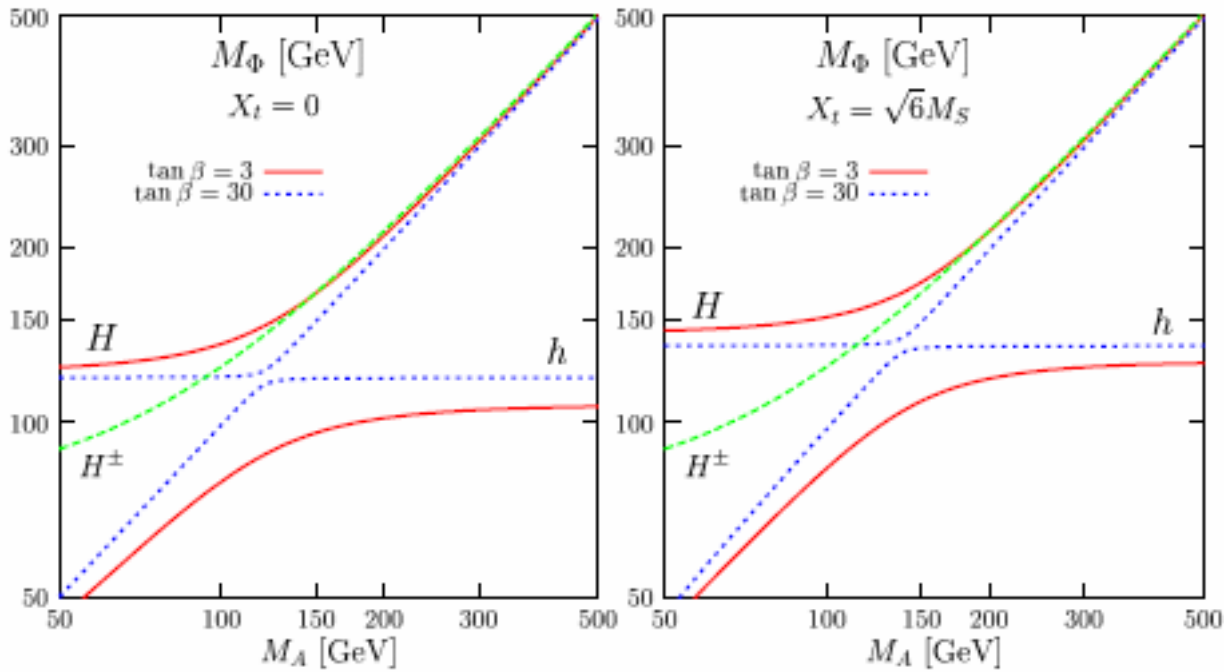
Upper bound on lightest neutral Higgs boson mass with m_{stop} = 1 TeV



- M_t^4 enhancement
- Logarithmic dependence on stop mass

Higgs Masses in MSSM

$$M_{H^\pm}^2 = M_A^2 + M_W^2$$



Large M_A: Degenerate A, H, $H^\pm$ and light h

Find Higgs Couplings

- Higgs-fermion couplings:

$$L = -\frac{gm_d}{2M_W \cos\beta}\bar{d}d\left(H\cos\alpha - h\sin\alpha\right) + \frac{igm_d \tan\beta}{2M_W}\bar{d}\gamma_5 dA$$
$$-\frac{gm_u}{2M_W \sin\beta}\bar{u}u\left(H\sin\alpha + h\cos\alpha\right) + \frac{igm_d \cot\beta}{2M_W}\bar{u}\gamma_5 uA$$

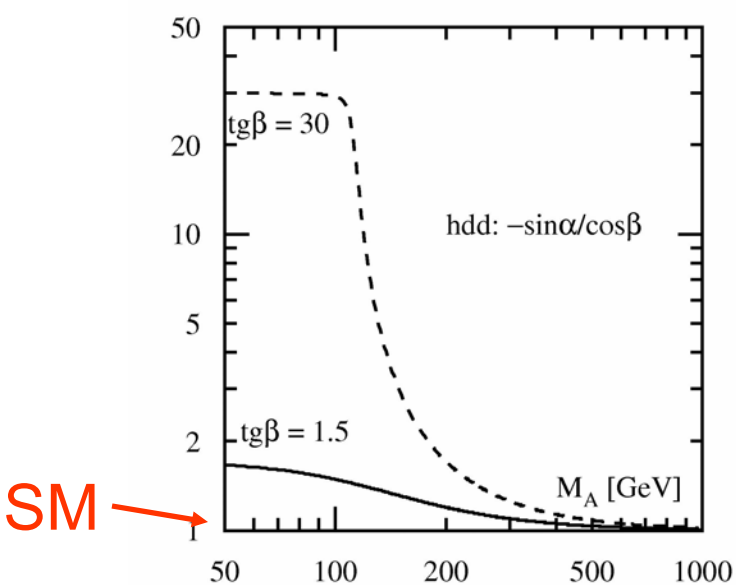
- Couplings given in terms of α, β
- Can be very different from Standard Model
- No new free parameters

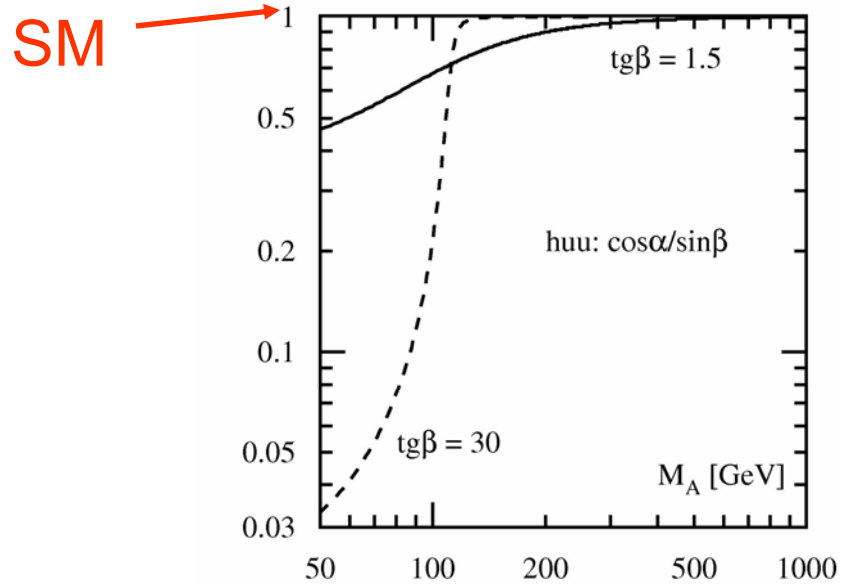
Higgs Couplings Different from SM

Lightest Neutral Higgs, h

Couplings to d, s, b enhanced at large tan β for moderate M_A

Couplings to u, c, t suppressed at large tan β for moderate M_A





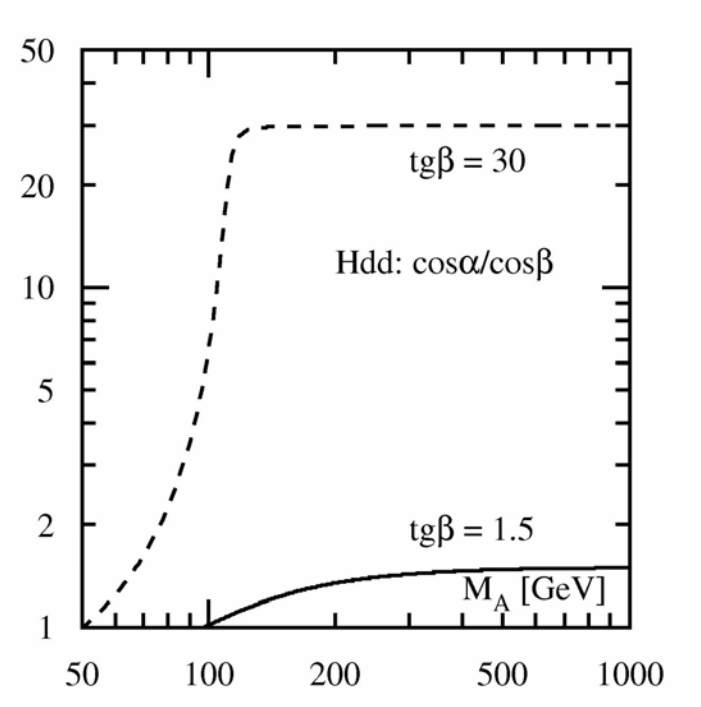
Decoupling limit: For $M_A \rightarrow \infty$, h couplings go to SM couplings

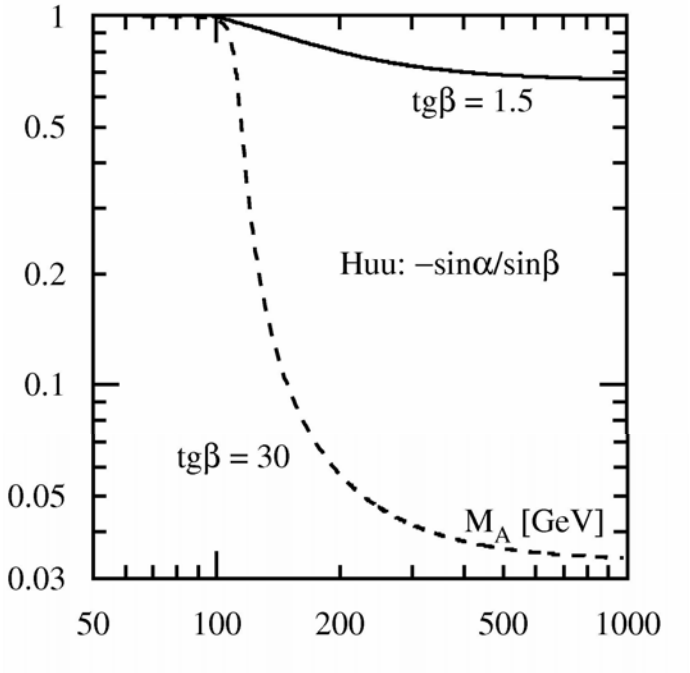
Higgs Couplings in SUSY

Heavier Neutral Higgs, H

Couplings to d, s, b enhanced at large tan β

Couplings to u, c, t suppressed at large tan β



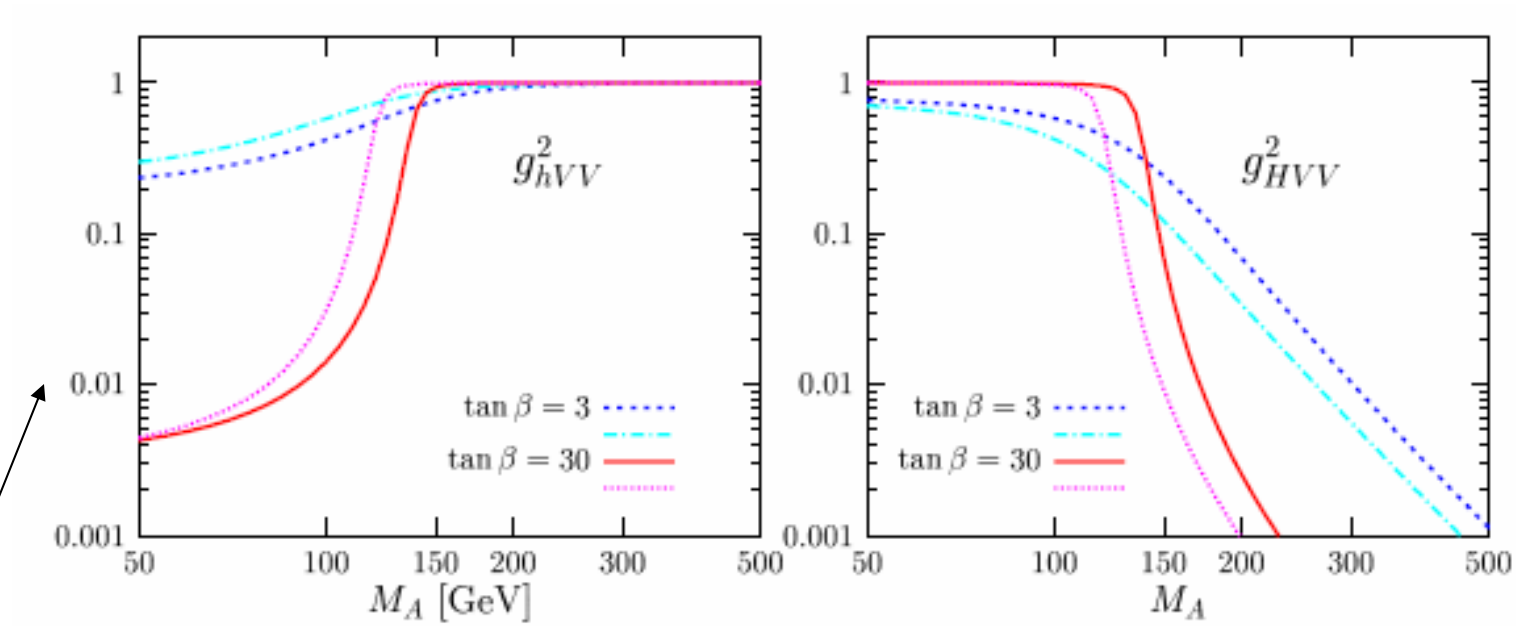


Gauge Boson Couplings to Higgs

- $g_{hVV}^2+g_{HVV}^2=g_{hVV}^2(SM)$
- Vector boson fusion and Wh production always suppressed in MSSM

$$\frac{g_{hVV}}{g_{h,smVV}} = \sin(\beta-\alpha)$$

$$\frac{g_{HVV}}{g_{h,smVV}} = \cos(\beta-\alpha)$$



Normalized to SM couplings

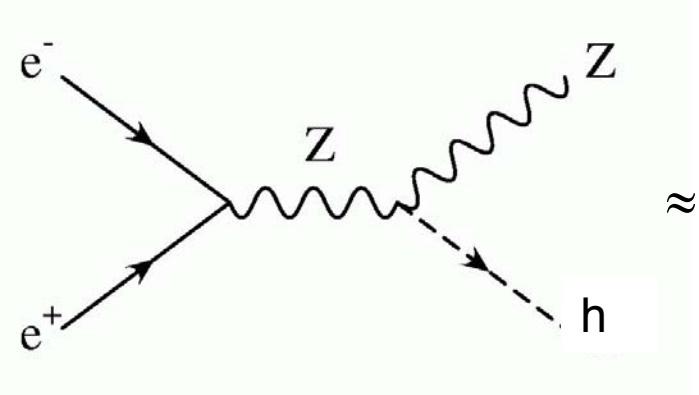
Tools

- Calculate SM and MSSM Higgs branching ratios:
  - HDECAY
  - http://people.web.psi.ch/spira/hdecay/

- Calculate MSSM Higgs masses and Higgs branching ratios:
  - FEYNHIGGS
  - http://www.feynhiggs.de/

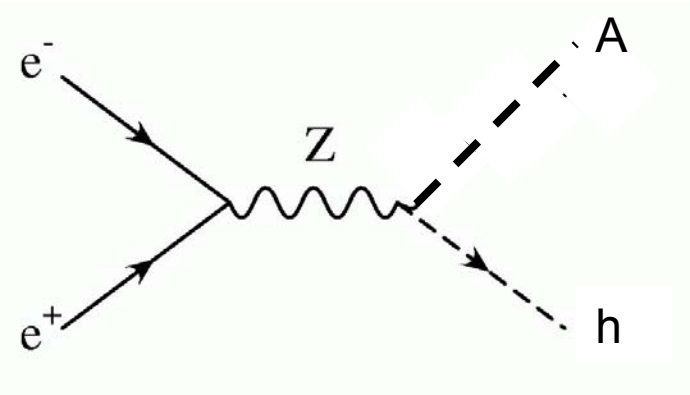
Both of these programs are very easy to use!

Limits from LEP



$$\approx \cos(\alpha - \beta)$$

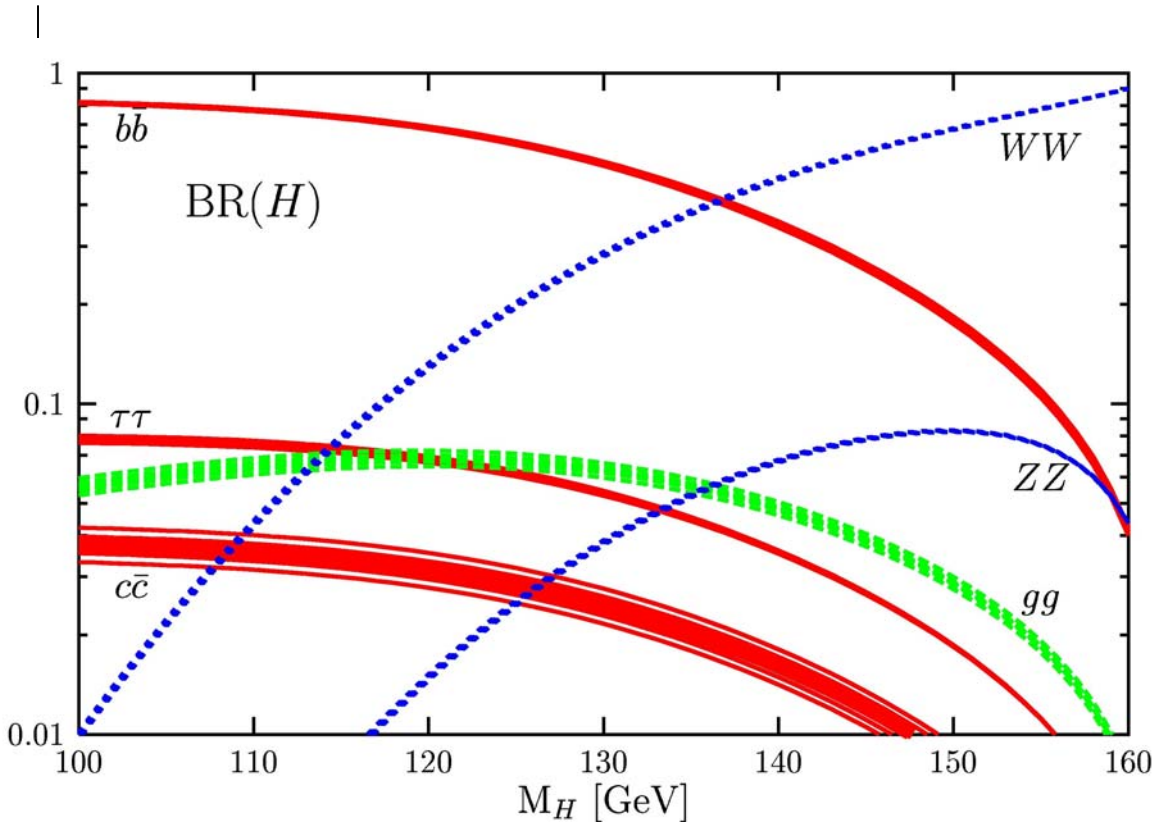
Complementary processes



$$\approx \sin(\alpha - \beta)$$

Remember Higgs Decays in SM

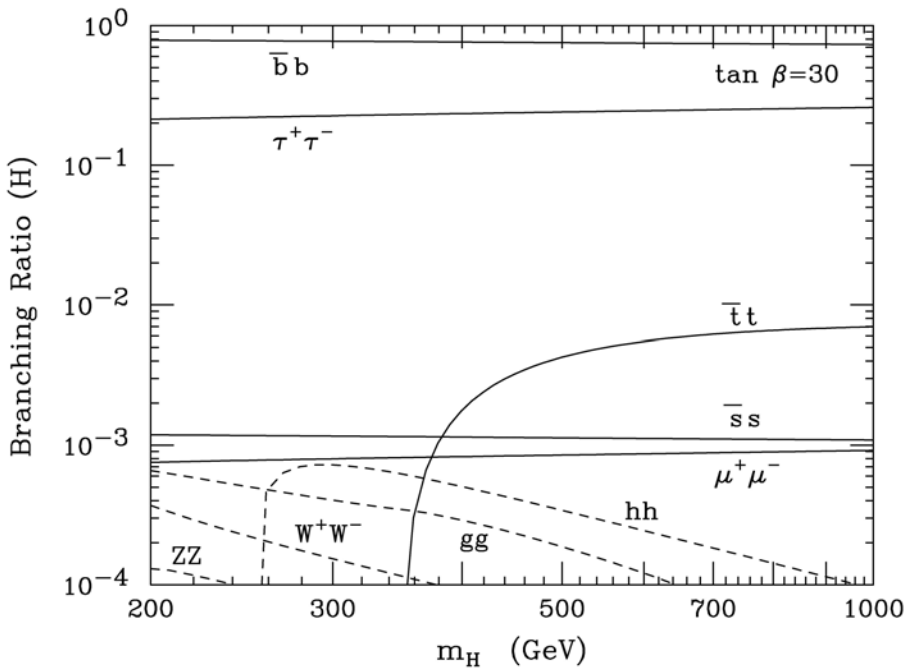
- SM: Higgs branching rates to $b\bar{b}$ and $\tau^+\tau^-$ turn off as rate to W^+W^- turns on (M_h > 160 GeV)



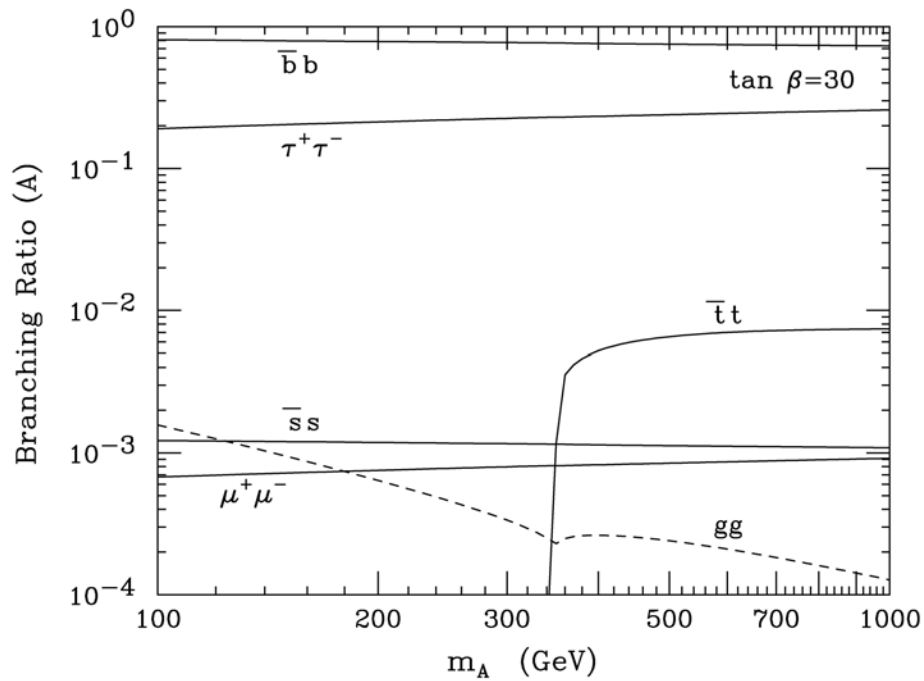
Higgs Decays Changed at Large tan β

- MSSM: At large tan β, rates to $\bar{b}b$ and $\tau^+\tau^-$ large

Heavy H^0 MSSM BRs

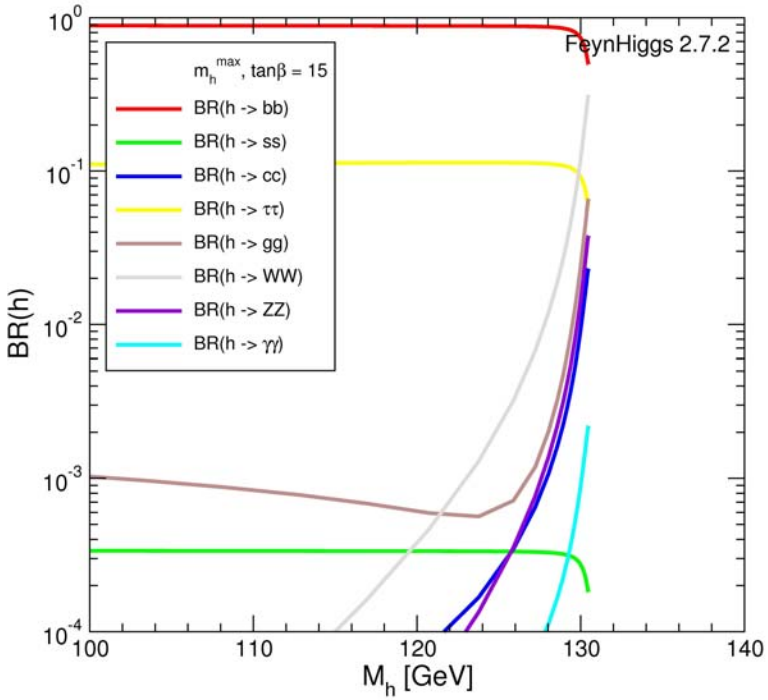


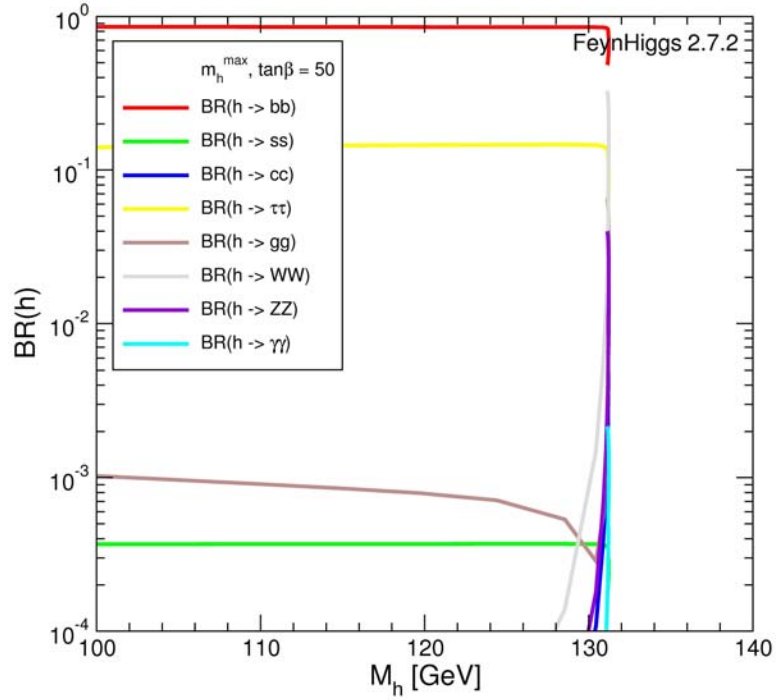
A^0 MSSM BRs



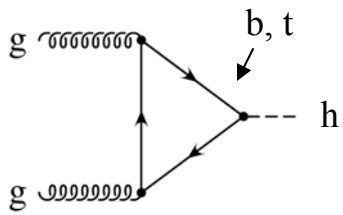
Rate to $b\bar{b}$ and $\tau^+\tau^-$ almost constant in MSSM for H, A

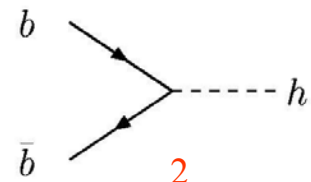
Light Higgs Branching Ratios





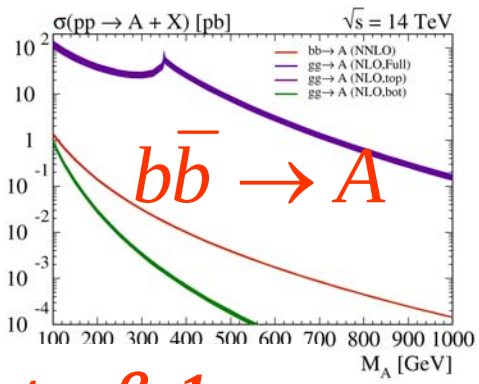
Large tanβ Changes Relative Importance of Production Modes



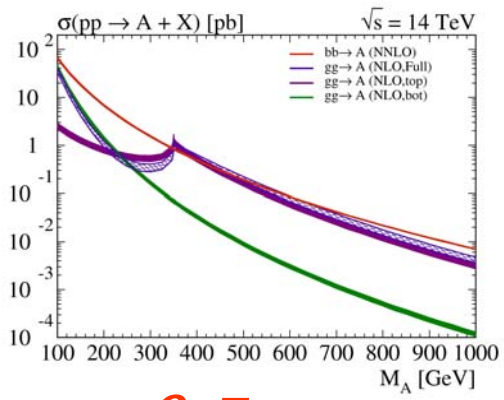


$$\sigma_{gg} = \frac{1}{M_h^2}\left(c_1 \cot^2\beta + c_2 \frac{m_b^2}{M_h^2} + c_3 \frac{m_b^4}{M_h^4}\tan^2\beta\right)$$

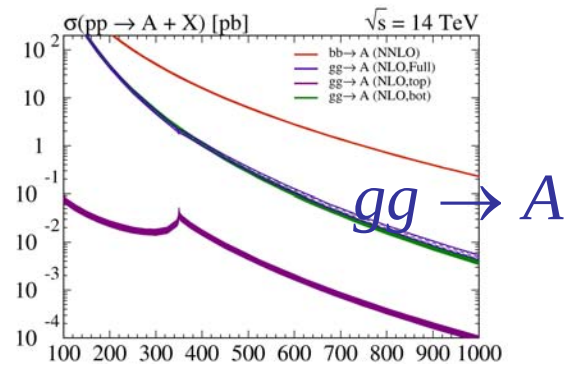
$$\sigma_{bb} = \frac{m_b^2}{M_h^4} c_4 \tan^2\beta$$



tanβ=1



tanβ=7

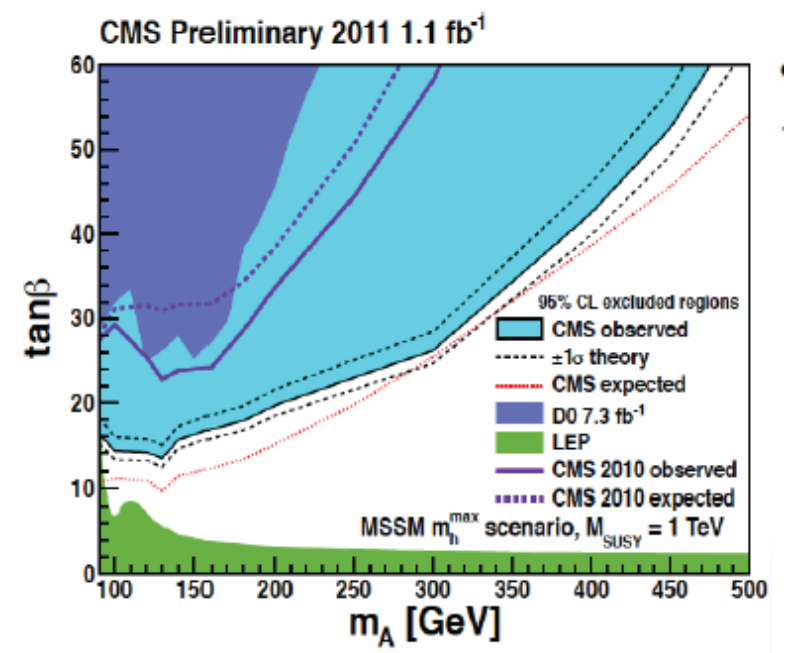


tanβ=40

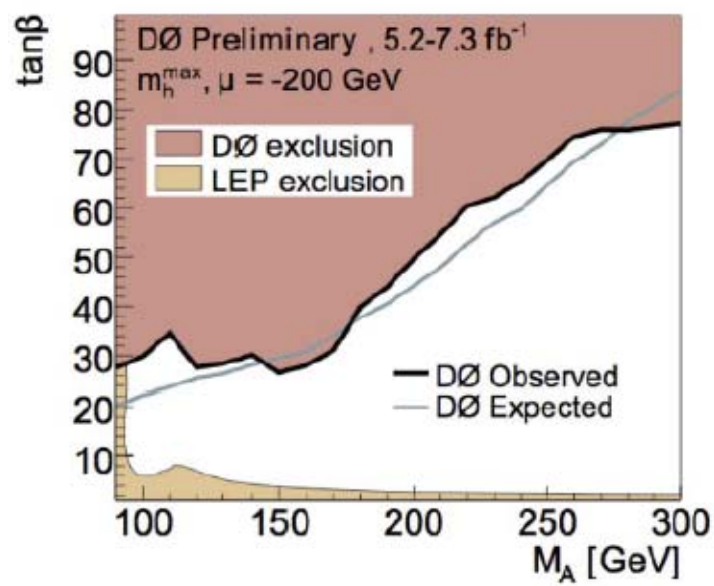
tanβ ≥ 7, $b\bar{b}$ production mode larger than gg

New Higgs Discovery Channels in SUSY

bbφ coupling enhanced for large tanβ



Combination of $H^{\pm}$ and $(b)\Phi^0 \rightarrow (b)\tau^+\tau$



Combination of $b\Phi^0 \rightarrow b\tau\tau$ and $b\Phi^0 \rightarrow bbb$

Many Possibilities Beyond SUSY

- Add singlet Higgs and try to evade LEP bounds
- Two Higgs doublets, but not SUSY
  - Same spectrum as SUSY
  - Must measure Higgs couplings
- Little Higgs Models
  - Have extended gauge sectors and new charge 2/3 quarks

Effective Lagrangian approach needed to study EWSB sector if no new particles found at LHC

Possibilities at the LHC

- We find a light Higgs with SM couplings and nothing else
  - How to answer our questions?
- We find a light Higgs, but it doesn't look SM like
  - Most models (SUSY, Little Higgs, etc) have other new particles
- We don't find a Higgs (or any other new particles)
  - How can we reconcile precision measurements?
  - This is the hardest case

Happy Higgs Hunting!