Neutrino Oscillation in Vacuum

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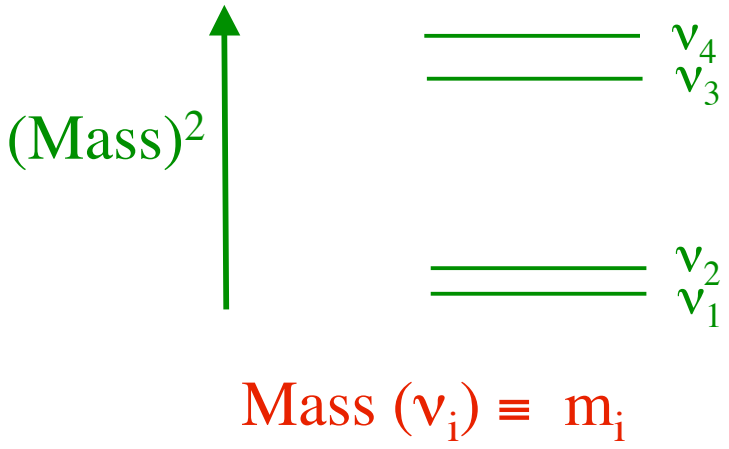
Let Us Assume Neutrino Masses and Leptonic Mixing

Neutrino mass —

There is some spectrum of 3 or more neutrino mass eigenstates ν_i:



Mass (ν_i) ≡ m_i

Leptonic mixing —

$\ell_e \equiv e,\ \ell_\mu \equiv \mu,\ \ell_\tau \equiv \tau$

When $W^+ \rightarrow \ell_\alpha^+ + \nu_\alpha$,

$e, \mu, \text{ or } \tau$

the produced neutrino state $|\nu_\alpha\rangle$ is

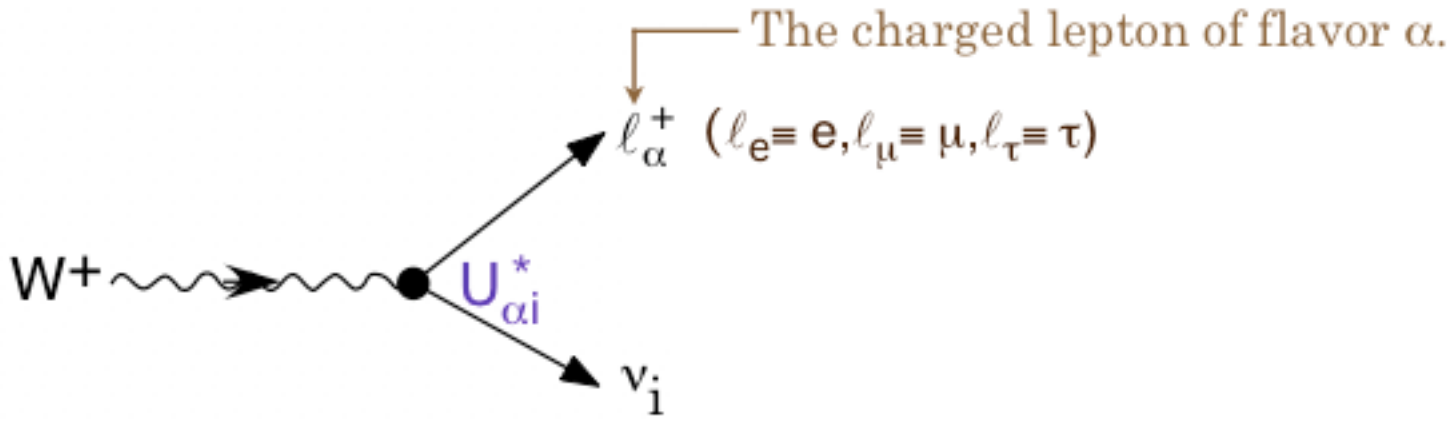
$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \ .$$

Neutrino of flavor α

Leptonic Mixing Matrix

Neutrino of definite mass m_i

Another way to look at W decay:



A given ℓ_α^+ can be accompanied by *any* ν_i.

$$\text{Amp}(W^+ \rightarrow \ell_\alpha^+ + \nu_i) = U^*_{\alpha i}$$

The neutrino state $|\nu_\alpha>$ produced together with ℓ_α^+

is $|\nu_\alpha> = \sum_i U^*_{\alpha i} |\nu_i>$.

According to the Standard Model, extended to include neutrino mass and leptonic mixing —

- The number of different ν_i is the same as the number of different ℓ_α (3).
- The mixing matrix U is 3 x 3 and unitary:

$$UU^\dagger = U^\dagger U = 1.$$

Some models include "sterile" neutrinos — neutrinos that experience none of the known forces of nature except gravity.

In such models, there are $N > 3$ ν_i, and U is N x N, but still unitary.

The Standard Model (SM) description of neutrino *interactions* (not masses or leptonic mixing) is well-confirmed.

We will assume it is true, and extend it to include mixing.

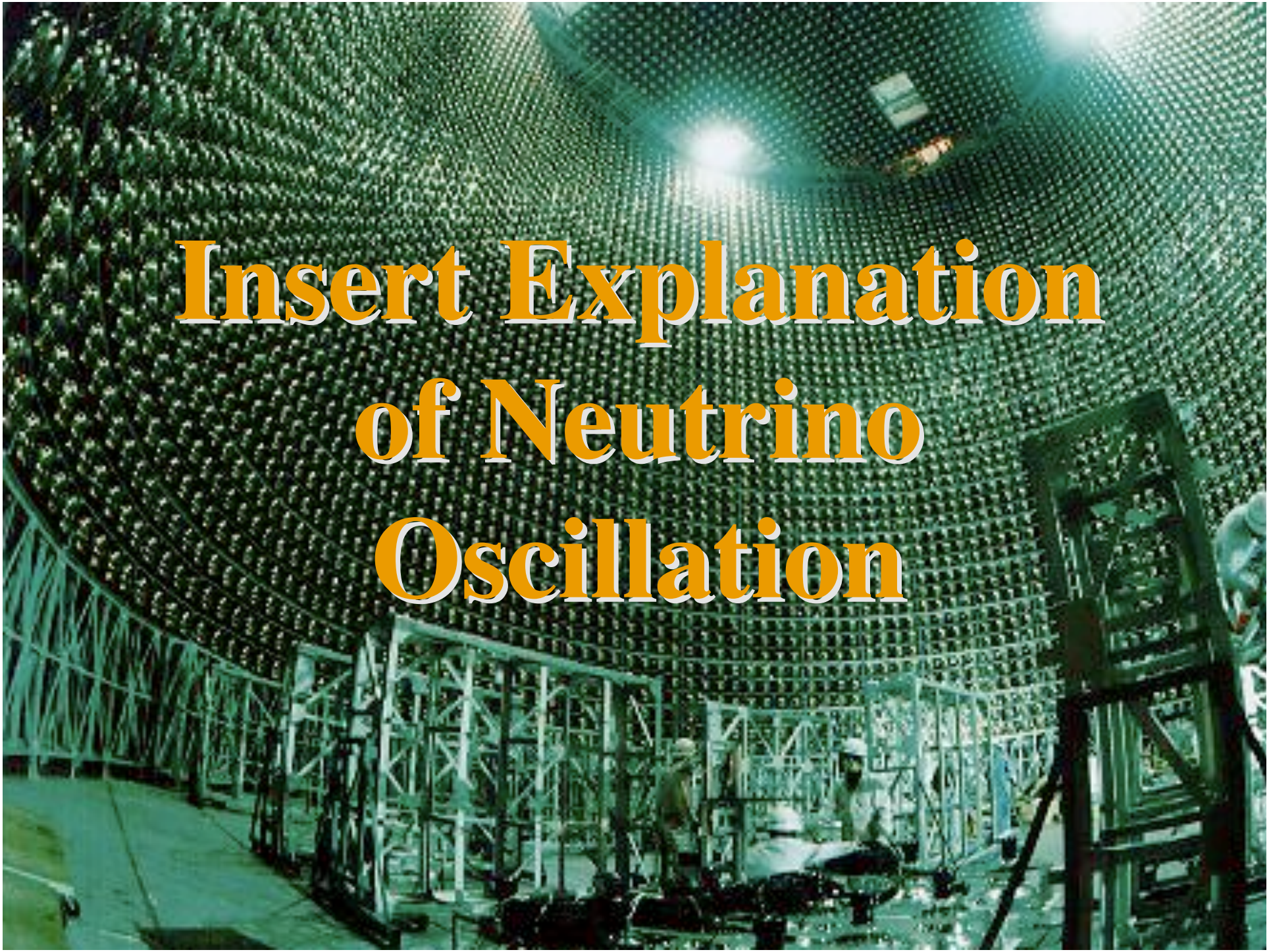
For the lepton couplings to the W boson, we then have —

Left-handed

$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \bar{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \bar{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

Taking mixing into account


Insert Explanation
of Neutrino
Oscillation