Mixing, $\cancel{CP}$, and Masses

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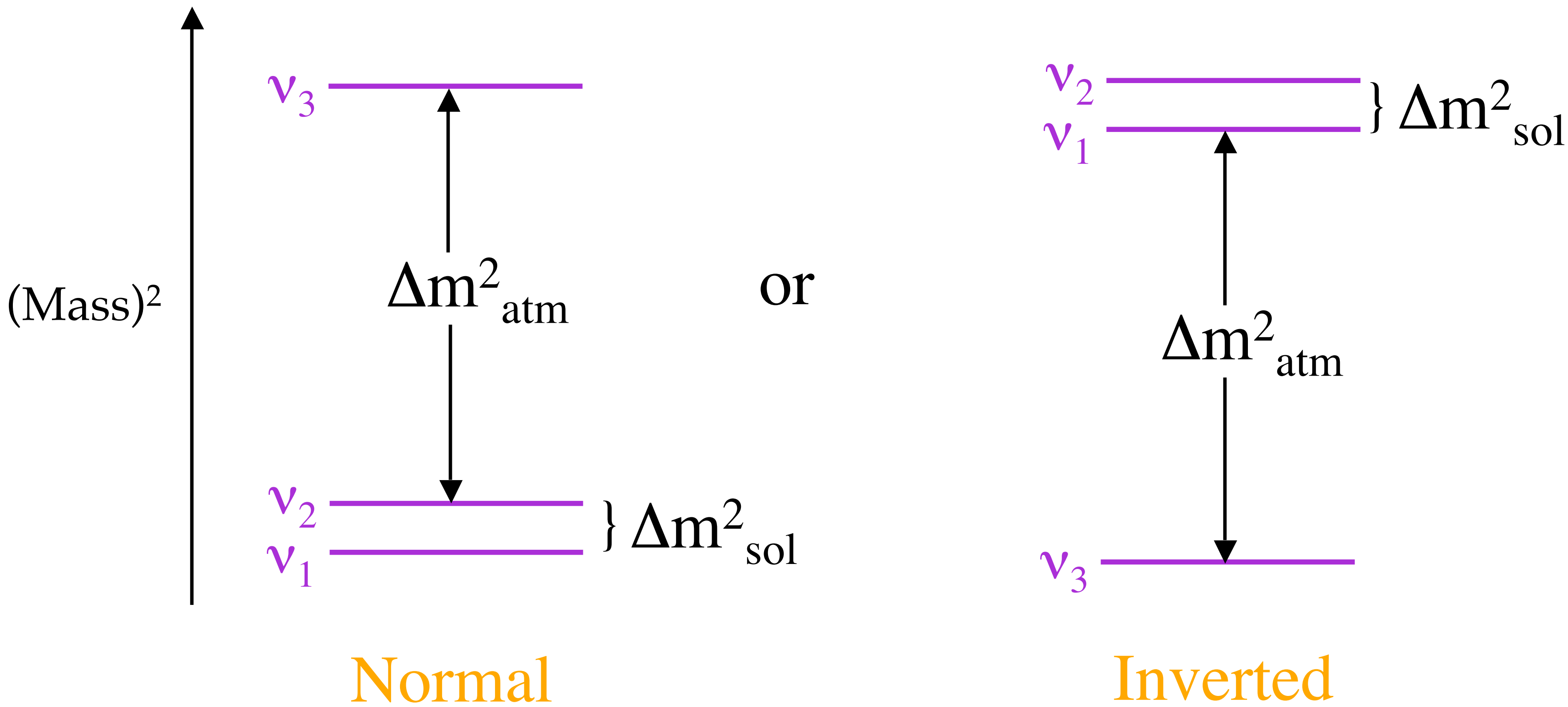
TASI 2006
June 23, 2006


What We
Have Learned

The (Mass)^2 Spectrum



$$\Delta m^2_{sol} \cong 8 \times 10^{-5}\ \mathrm{eV}^2, \quad \Delta m^2_{atm} \cong 2.7 \times 10^{-3}\ \mathrm{eV}^2$$

Are there *more* mass eigenstates, as LSND suggests?

Leptonic Mixing

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

$(W \to e\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau)$

are superpositions of the mass eigenstates:

$$|\nu_\alpha> = \sum_i U^*_{\alpha i}\, |\nu_i> \quad .$$

Neutrino of flavor
$\alpha = e, \mu,$ or τ

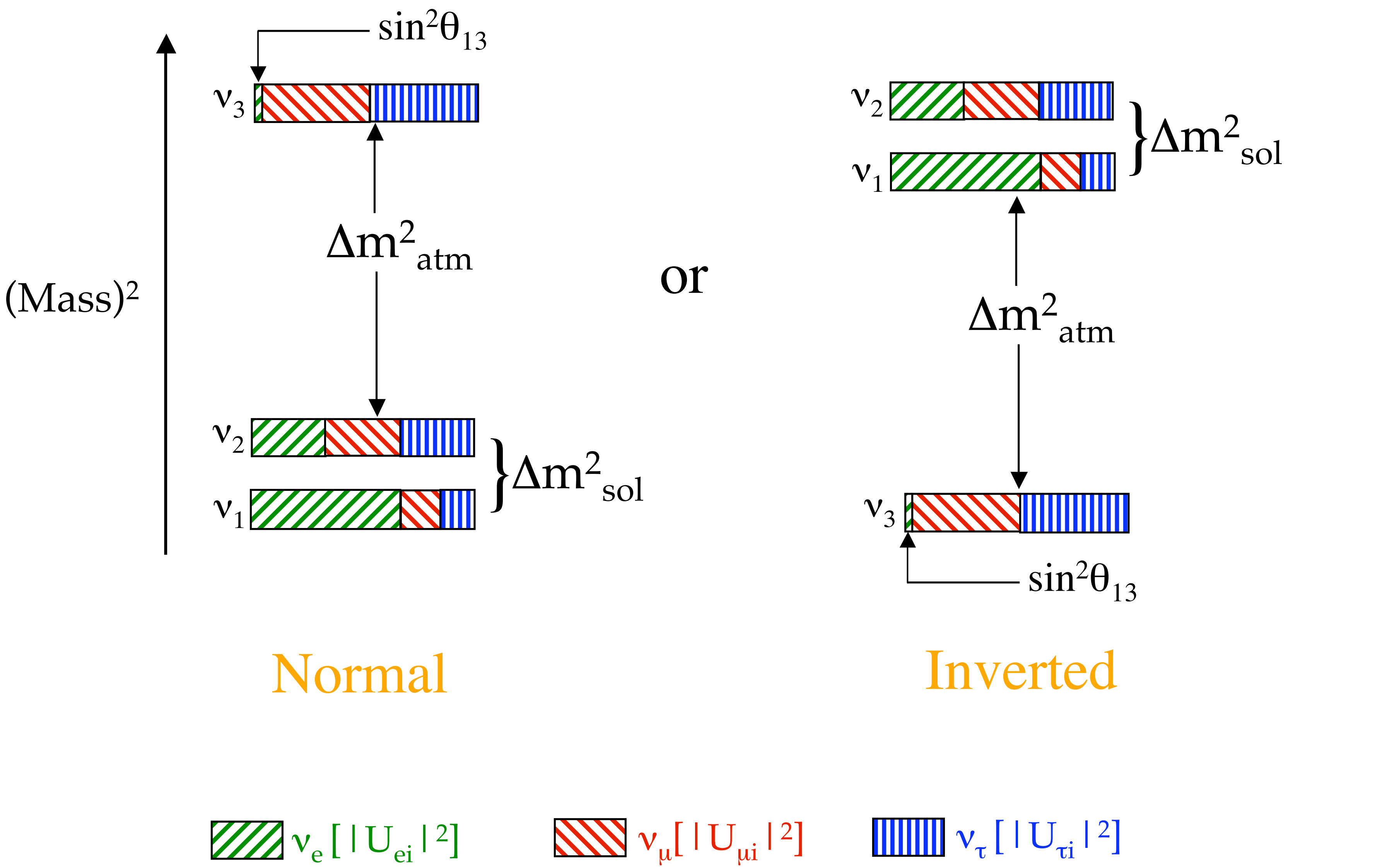
Unitary Leptonic Mixing Matrix

Neutrino of definite mass m_i

Inverting: $|\nu_i> = \sum_\alpha U_{\alpha i}\, |\nu_\alpha> .$

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

The spectrum, showing its approximate flavor content, is



The Mixing Matrix

$$U = \overset{\text{Atmospheric}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}} \times \overset{\text{Cross-Mixing}}{\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}} \times \overset{\text{Solar}}{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$c_{ij} \equiv \cos\theta_{ij}$$
$$s_{ij} \equiv \sin\theta_{ij}$$

$$\times \underset{\text{Majorana } \cancel{CP} \text{ phases}}{\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$\theta_{12} \approx \theta_{sol} \approx 34°$, $\theta_{23} \approx \theta_{atm} \approx 37\text{-}53°$, $\theta_{13} \lesssim 10°$

δ would lead to $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq P(\nu_\alpha \to \nu_\beta)$. $\cancel{CP}$

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

CP Violation From Majorana Phases

Majorana CP-Violating Phases

The 3x3 quark mixing matrix: 1 CP phase

When $\bar{\nu}_i = \nu_i$ —

The 3x3 lepton mixing matrix: 3 CP phases

The 2 extra phases, α_1 and α_2, are called Majorana phases.

Each Majorana phase is associated with a particular ν mass eigenstate ν_i:

$$U_{\rho i} = U^0_{\rho i}\, e^{i\frac{\alpha_i}{2}}\ ;\ \text{all } \rho\,. \qquad U = \begin{array}{c} \\ \\ \end{array}\begin{array}{cccc} \nu_1 & \nu_2 & \nu_3 & \\ \left[\begin{array}{c} U^0_{e1} e^{i\frac{\alpha_1}{2}} \\ U^0_{\mu 1} e^{i\frac{\alpha_1}{2}} \\ U^0_{\tau 1} e^{i\frac{\alpha_1}{2}} \end{array}\right. & \begin{array}{c} U^0_{e2} e^{i\frac{\alpha_2}{2}} \\ U^0_{\mu 2} e^{i\frac{\alpha_2}{2}} \\ U^0_{\tau 2} e^{i\frac{\alpha_2}{2}} \end{array} & \left.\begin{array}{c} U^0_{e3} \\ U^0_{\mu 3} \\ U^0_{\tau 3} \end{array}\right] & \begin{array}{c} e \\ \mu \\ \tau \end{array} \end{array}$$

Bilenky, Hosek, and Petcov; Schechter and Valle, Doi et al.

An **L-conserving** process:

$$\mathrm{Amp}[e^- W^+ \to \nu \to \mu^- W^+]$$

$$\sim \Sigma_i \underbrace{\langle \mu^- W^+ | H | \nu_i \rangle}_{U_{\mu i}} \mathrm{Propagator}(\nu_i) \underbrace{\langle \nu_i | H | e^- W^+ \rangle}_{U_{ei}^*}$$

$$\sim \Sigma_i \, U_{\mu i} \, \mathrm{Propagator}(\nu_i) \, U_{ei}^*$$

An **L-nonconserving** process:

$$\mathrm{Amp}[e^+ W^- \to \nu \to \mu^- W^+]$$

$$\sim \Sigma_i \langle \mu^- W^+ | H | \nu_i \rangle \, \mathrm{Propagator}(\nu_i) \, \langle \nu_i | H | e^+ W^- \rangle$$

$$\mathrm{CTP:}\ \langle \nu_i | H | e^+ W^- \rangle = \langle \nu_i | H | e^- W^+ \rangle^* = U_{ei}$$

So $\mathrm{Amp}[\,\cancel{L}\,] \sim \Sigma_i \, U_{\mu i} \, \mathrm{Propagator}(\nu_i) \, U_{ei}$

This is sensitive to Majorana phases.

Majorana phases have physical consequences, but only in physical processes that involve violation of L.

They do not affect ν flavor oscillation, but they do affect $0\nu\beta\beta$:

$$m_{\beta\beta} = \left| \sum_i m_i U_{ei}^2 \right|$$

clearly depends on the relative phase of U_{e1}^2 and U_{e2}^2.

But the rate for $0\nu\beta\beta$, whatever it may be, is not a CP-violating difference between a process and its CP-mirror image.

Can Majorana Phases Lead to Manifest $\cancel{CP}$?

Manifest $\cancel{CP}$:

$$\text{Rate [Process]} \neq \text{Rate [}\overline{\text{Process}}\text{]}$$

The "Dirac" $\cancel{CP}$ phase in the quark mixing matrix causes such inequalities.

Can Majorana phases cause them too?

Yes, they can cause them in heavy neutrino decay in the early universe (Leptogenesis).

(Fukugita & Yanagida)

But can they cause rate inequalities in present-day processes?

Yes, although only in processes that are extremely difficult to observe.

(de Gouvêa, BK, Mohapatra)

An example is $e^{\pm}W^{\mp} \rightarrow \nu \rightarrow \mu^{\mp}W^{\pm}$.

$$\text{Amp}\,[e^+W^- \to \nu \to \mu^-W^+] =$$

Distance

$$= S\ \Sigma_i\ \underbrace{U_{ei}U_{\mu i}}_{\text{Has Maj. phases}}\ m_i/E\ \underbrace{\exp\{-im_i^2\,(L/2E)\}}_{\nu\text{ propagator}}$$

Kinematics

Helicity suppression

$$\text{Amp}\,[e^-W^+ \to \nu \to \mu^+W^-] =$$

$$= S\ \Sigma_i\ U_{ei}^*U_{\mu i}^*\ m_i/E\ \exp\{-im_i^2\,(L/2E)\}$$

Suppose only 2 generations matter:

$$U = \begin{array}{c} \\ \nu_e \\ \nu_\mu \end{array}\!\!\begin{array}{c} \begin{array}{cc} \nu_1 & \nu_2 \end{array} \\ \begin{bmatrix} ce^{i\frac{\alpha}{2}} & s \\ -se^{i\frac{\alpha}{2}} & c \end{bmatrix} \end{array}$$

$c \equiv \cos\theta$

$s \equiv \sin\theta$

$\alpha \equiv$ a Majorana phase.

$$\Gamma\left[e^{+}W^{-} \to \nu \to \mu^{-}W^{+}\right]$$
$$= K\frac{\sin^2 2\theta}{4E^2}\left[m_1^2 + m_2^2 - 2m_1 m_2 \cos\left(\Delta m^2 \frac{L}{2E} - \alpha\right)\right]$$

(Schechter & Valle)

$$\Gamma\left[e^{-}W^{+} \to \nu \to \mu^{+}W^{-}\right]$$
$$= K\frac{\sin^2 2\theta}{4E^2}\left[m_1^2 + m_2^2 - 2m_1 m_2 \cos\left(\Delta m^2 \frac{L}{2E} + \alpha\right)\right]$$

Here,

K = irrelevant constant = $|S|^2$

$m_{1,2}$ = masses of $\nu_{1,2}$

$\Delta m^2 = m_2^2 - m_1^2$

Note the two rates are not the same.

Why Are There 3 Generations?

If the preponderance of **MATTER** over antimatter in the universe arose from $\cancel{CP}$ in quark mixing, we could argue that—

- It takes ≥ 3 generations to have $\cancel{CP}$ in quark mixing.
- It takes $\cancel{CP}$ in quark mixing to have **MATTER** >> antimatter.
- It takes **MATTER** >> antimatter to have us.

But $\cancel{CP}$ in quark mixing is completely inadequate for **MATTER** >> antimatter.

Majorana phases can produce the manifest $\cancel{CP}$

$$\Gamma[N \rightarrow \ell^+ + \text{Higgs}^-] \neq \Gamma[N \rightarrow \ell^- + \text{Higgs}^+]$$

in the early universe. *This* ***(Leptogenesis)*** may be the origin of **MATTER** >> antimatter.

It takes only 2 generations to have manifest $\cancel{CP}$ from Majorana phases.

So why are there 3??

Dirac and Majorana Masses and the See-Saw Mechanism

The See-Saw Mechanism

For a *Dirac* neutrino mass eigenstate ν of mass m, the mass term in the Lagrangian density is —

$$L_m = -m\bar{\nu}\nu$$

Then —

$$\langle \nu \text{ at rest} \left| H_m \right| \nu \text{ at rest} \rangle = \langle \nu \text{ at rest} \left| m \int d^3x \, \bar{\nu}\nu \right| \nu \text{ at rest} \rangle = m$$

Hamiltonian

For a *Majorana* neutrino mass eigenstate ν of mass m, the mass term in the Lagrangian density is —

$$L_m = -\frac{m}{2}\bar{\nu}\nu$$

with $\underbrace{\nu^c = \text{(phase factor) x } \nu}_{\text{Antineutrino = Neutrino}}$

Then —

$$\langle \nu \text{ at rest} | H_m | \nu \text{ at rest} \rangle = \left\langle \nu \text{ at rest} \left| \frac{m}{2} \int d^3x \, \bar{\nu}\nu \right| \nu \text{ at rest} \right\rangle = m$$

{The matrix element of $\bar{\nu}\nu$ is doubled in the Majorana case.}

Chiral fields:

Chirally left- and right-handed fermion fields satisfy the constraints —

$$P_L f_L \equiv \frac{(1-\gamma_5)}{2} f_L = f_L \qquad \text{and} \qquad P_R f_R \equiv \frac{(1+\gamma_5)}{2} f_R = f_R$$

For a *massless* fermion, chirality = helicity.

In the Standard Model (SM), only chirally left-handed fermion fields couple to the W boson.

Therefore, it is convenient to express the SM in terms of "underlying" chiral fields.

Expressed in terms of chiral fields, any mass term connects only fields of *opposite* chirality:

$$\bar{g}_R f_L$$

Chiral fermion fields

$$\bar{j}_L k_L = \bar{j}_R k_R = 0$$

Chiral fermion fields

For example —

$$\bar{j}_L k_L = \overline{\left(\frac{1-\gamma_5}{2}\right) j}\left(\frac{1-\gamma_5}{2}\right) k = \bar{j}\left(\frac{1+\gamma_5}{2}\right)\left(\frac{1-\gamma_5}{2}\right) k = 0$$

Note: Charge conjugating a chiral field reverses its chirality.

Dirac Mass Term

For quarks, charged leptons and *maybe* neutrinos.

Suppose $\nu_L{}^0$ and $\nu_R{}^0$ are underlying chiral fields in terms of which the SM, extended to include neutrino mass, is written.

The Dirac mass term is then —

$$L_D = -m_D \overline{\nu_R{}^0} \nu_L{}^0 + \text{ h.c. } = -m_D (\overline{\nu_R{}^0} \nu_L{}^0 + \overline{\nu_L{}^0} \nu_R{}^0)$$

In terms of $\nu \equiv \nu_L{}^0 + \nu_R{}^0$, $L_D = -m_D \bar{\nu}\nu$, since

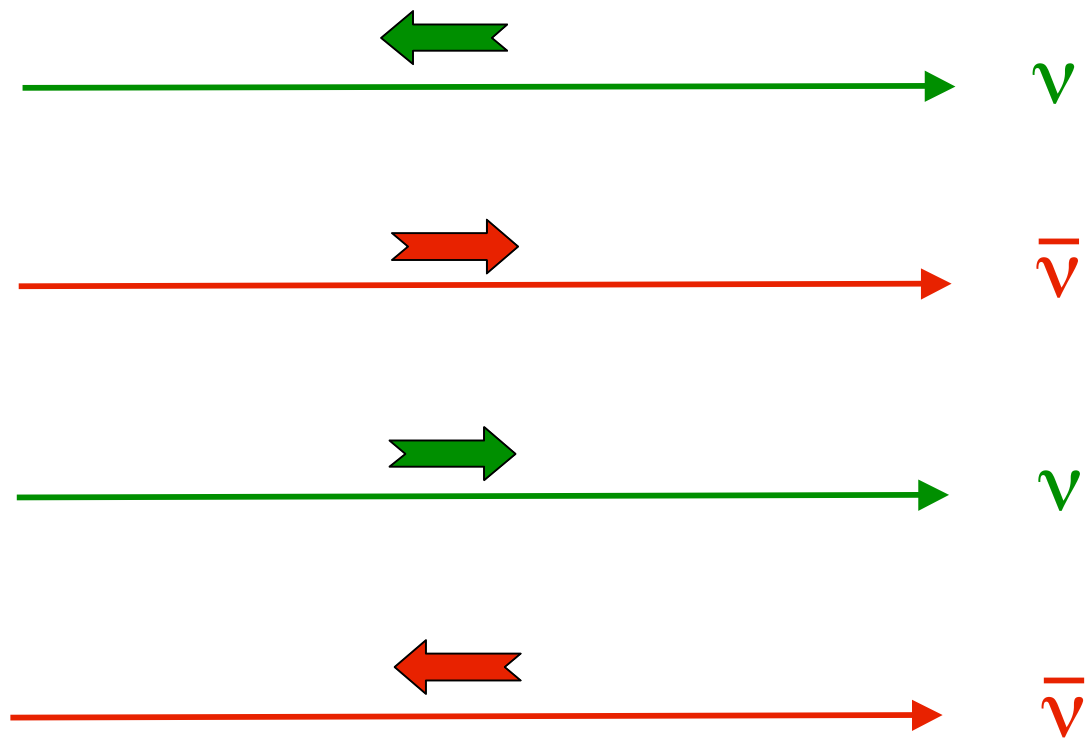
$$\bar{\nu}\nu = \overline{\left(\nu_L{}^0 + \nu_R{}^0\right)}\left(\nu_L{}^0 + \nu_R{}^0\right) = \overline{\nu_R{}^0} \nu_L{}^0 + \overline{\nu_L{}^0} \nu_R{}^0$$

ν is the mass eigenstate, and has mass m_D.

We have 4 mass-degenerate states:

L is conserved.

$\bar{\nu} \neq \nu$



This collection of 4 states is a Dirac neutrino plus its antineutrino.

Majorana Mass Term

For neutrinos only.

Suppose $\nu_R{}^0$ is an electroweak singlet chiral field.

The right-handed Majorana mass term is then —

$$L_R = -\frac{m_R}{2}\overline{\left(\nu_R{}^0\right)^c}\nu_R{}^0 + \text{ h.c. } = -\frac{m_R}{2}\left[\overline{\left(\nu_R{}^0\right)^c}\nu_R{}^0 + \overline{\nu_R{}^0}\left(\nu_R{}^0\right)^c\right]$$

In terms of $\nu \equiv \nu_R{}^0 + \left(\nu_R{}^0\right)^c$, $L_R = -\frac{m_R}{2}\bar{\nu}\nu$, since

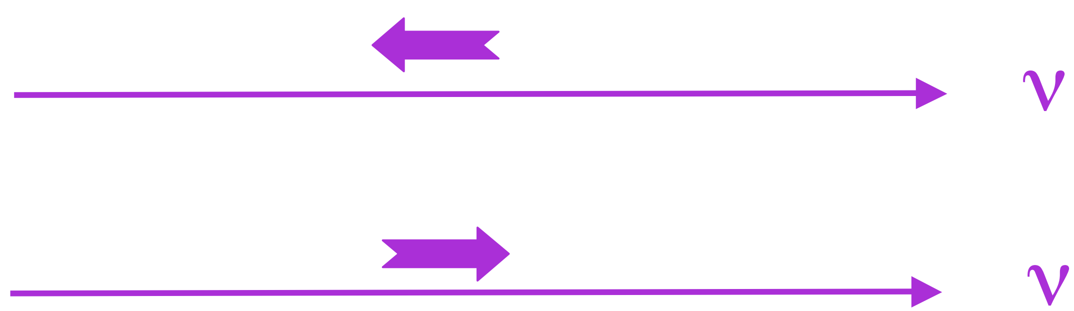
$$\bar{\nu}\nu = \overline{\left[\nu_R{}^0 + \left(\nu_R{}^0\right)^c\right]}\left[\nu_R{}^0 + \left(\nu_R{}^0\right)^c\right] = \overline{\left(\nu_R{}^0\right)^c}\nu_R{}^0 + \overline{\nu_R{}^0}\left(\nu_R{}^0\right)^c$$

ν is the mass eigenstate, and has mass m_R.

$$\nu^c = \left[\nu_R{}^0 + \left(\nu_R{}^0\right)^c\right]^c = \left(\nu_R{}^0\right)^c + \nu_R{}^0 = \nu$$

Thus, ν is its own antiparticle. It is a Majorana neutrino.

We have only 2 mass-degenerate states:



The See-Saw

We include *both* Majorana and Dirac mass terms:

$$L_m = -m_D \overline{\nu_R{}^0} \nu_L{}^0 - \frac{m_R}{2} \overline{\left(\nu_R{}^0\right)^c} \nu_R{}^0 + \text{ h.c.}$$

$$= -\frac{1}{2}\left[\overline{\left(\nu_L{}^0\right)^c}, \overline{\nu_R{}^0}\right]\begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix}\begin{bmatrix} \nu_L{}^0 \\ \left(\nu_R{}^0\right)^c \end{bmatrix} + \text{ h.c.}$$

We have used $\overline{\left(\nu_L{}^0\right)^c} m_D \left(\nu_R{}^0\right)^c = \overline{\nu_R{}^0} m_D \nu_L{}^0$.

$M_\nu = \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix}$ is called the neutrino mass matrix.

No SM principle prevents m_R from being extremely large.

But we expect m_D to be of the same order as the masses of the quarks and charged leptons.

Thus, we assume that $m_R >> m_D$.

M_ν can be diagonalized by the transformation —

$$Z^T M_\nu Z = D_\nu$$

With $\rho \equiv m_D / m_R << 1$,

$$Z \cong \begin{bmatrix} 1 & \rho \\ -\rho & 1 \end{bmatrix} \overbrace{\begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}}^{\text{Makes eigenvalues positive}}$$

and

$$D_\nu \cong \begin{bmatrix} m_D{}^2 / m_R & 0 \\ 0 & m_R \end{bmatrix}$$

Define $\begin{bmatrix} \nu_L \\ N_L \end{bmatrix} \equiv Z^{-1} \begin{bmatrix} \nu_L{}^0 \\ \left(\nu_R{}^0\right)^c \end{bmatrix}$ and $\begin{bmatrix} \nu \\ N \end{bmatrix} \equiv \begin{bmatrix} \nu_L + \left(\nu_L\right)^c \\ N_L + \left(N_L\right)^c \end{bmatrix}$.

Majorana neutrinos

Then —

$$L_m = -\frac{1}{2}\frac{m_D{}^2}{m_R}\bar{\nu}\nu - \frac{1}{2}m_R\bar{N}N$$

Mass of ν

Mass of N

(Mass of ν) x (Mass of N) = $m_D{}^2 \sim m_{\text{quark or lepton}}{}^2$

The See-Saw Relation

What Happened?

The Majorana mass term split a Dirac neutrino into two Majorana neutrinos.