What Do We Know About EWSB?

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Lecture 2

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Basics

- SM is SU(2) x U(1) theory
  - Two gauge couplings: g and g'
- Higgs potential is $V=-\mu^2\phi^2+\lambda\phi^4$
  - Two free parameters
- Four free parameters in gauge-Higgs sector

Basics, #2

- Chose parameters in gauge/Higgs sector
  - α=1/137.0359895(61)
  - G_F =1.16637(1) x 10^{-5} GeV $^{-2}$
  - M_Z=91.1875 $\pm$ 0.0021 GeV
  - M_H

  Express everything else in terms of these parameters

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{\pi\alpha}{2\left(1-\frac{M_W^2}{M_Z^2}\right)M_W^2}$$

Inadequacy of Tree Level Calculations

- Mixing angle is predicted quantity
  - On-shell definition $\cos^2\theta_W = M_W^2/M_Z^2$
  - Predict M_W

$$M_W^2 = \pi\sqrt{2}\,\frac{\alpha}{G_F}\left(1-\sqrt{1-\frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}}\right)^{-1}$$

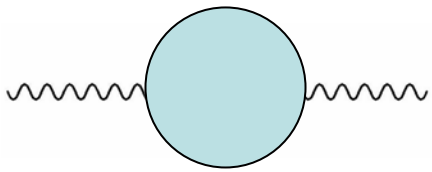
$$s_W{}^2 c_W{}^2 = \frac{\pi\alpha}{G_F M_Z^2}$$

  - Plug in numbers:
    - M_W predicted =80.939 GeV
    - M_W(exp) =80.399 $\pm$ 0.025 GeV
  - Need to calculate beyond tree level

Quantum Corrections

- Relate tree level to one-loop corrected masses

$$-i\Pi^{\mu\nu}_{XY} =$$



$$\Pi^{\mu\nu}_{XY}(k^2) = g^{\mu\nu}\Pi_{XY}(k^2) + k^\mu k^\nu B_{XY}(k^2)$$

$$M^2_{V0} = M^2_V + \Pi_{VV}(M^2_V)$$

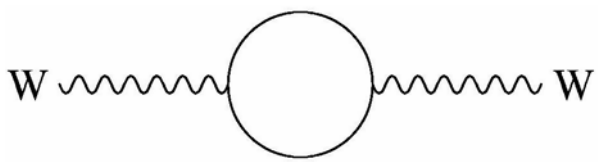
- Majority of corrections at one-loop are from 2-point functions

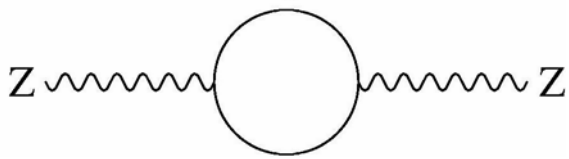
Note sign conventions for 2-point functions

Example of Quantum Corrections

- Example: $$\rho = \frac{M_W^2}{M_Z^2 \cos_W^2} = 1 + \delta\rho$$

$$\boxed{\delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}}$$





Top quark contributes to W and Z 2-point functions

Top Quark Corrections to ρ Parameter

- 2-point functions of W, Z

$$\Pi_{ZZ}(0) = \frac{g^2 N_c}{32\pi^2 c_W^2}\left(\frac{4\pi}{m_t^2}\right)^{\varepsilon} \frac{m_t^2}{\varepsilon}\left(R_t - L_t\right)^2$$

$$\Pi_{WW}(0) = \frac{g^2 N_c}{32\pi^2}\left(\frac{4\pi}{m_t^2}\right)^{\varepsilon} m_t^2\left(\frac{1}{\varepsilon} + \frac{1}{2}\right)$$

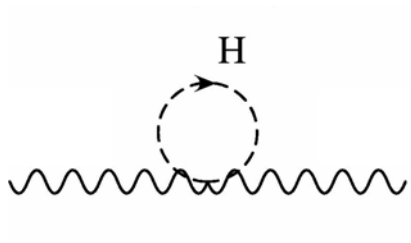
$L_t = 1 - 4s_W^2/3$

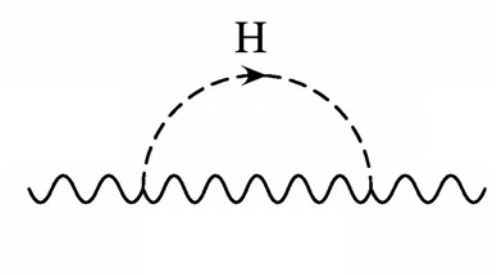
$R_t = -4s_W^2/3$

$$\delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} = \frac{G_F}{\sqrt{2}}\frac{N_c}{8\pi^2}\left(\frac{m_t^2}{M_W^2}\right)$$

(Neglecting $\log(m_t)$ pieces and using on-shell definition of $\sin\theta_W$)

Heavy Higgs Contribution to $\delta\rho$





$$\rho = \frac{-3\alpha}{16\pi c_W^2} \log\left(\frac{M_H^2}{M_W^2}\right)$$

- In on-shell scheme, Higgs contributes logarithmically to quantum corrections & top quark contributes quadratically

Modification of tree level relations

$$G_F = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2\theta_W} \frac{1}{(1-\Delta r)}$$

- Δr is a physical quantity which incorporates 1-loop corrections

- Contributions to Δr from top quark and Higgs loops

$$\Delta r^t = -\frac{3G_F m_t^2}{8\sqrt{2}\pi^2}\left(\frac{\cos^2\theta_W}{\sin^2\theta_W}\right)$$

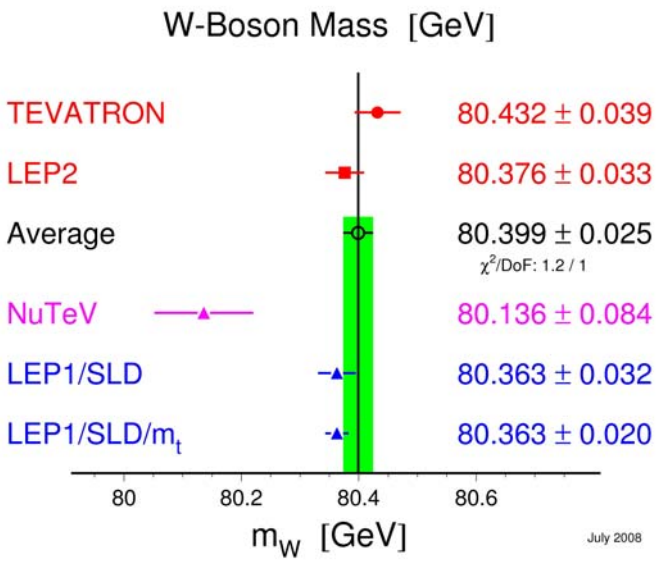
$$\Delta r^H = \frac{11G_F M_W^2}{24\sqrt{2}\pi^2}\left(\ln\frac{M_H^2}{M_W^2} - \frac{5}{6}\right)$$

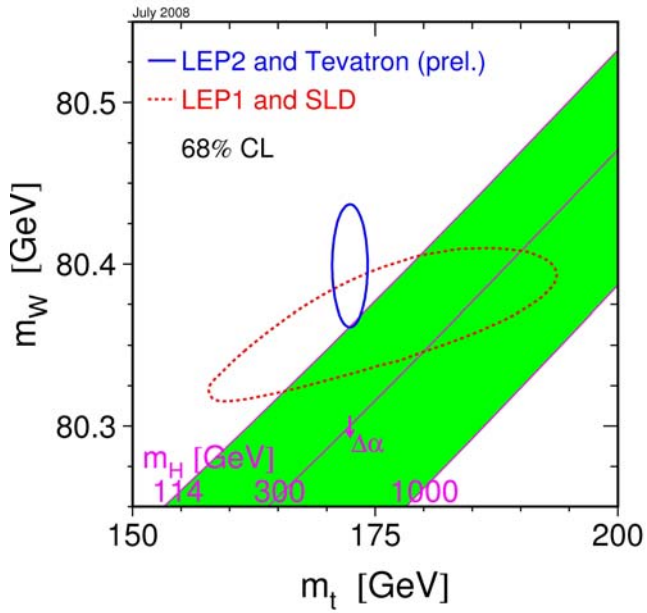
Extreme sensitivity of precision measurements to m_t

Understanding Higgs Limit

Theory: Input M_Z, G_F, α

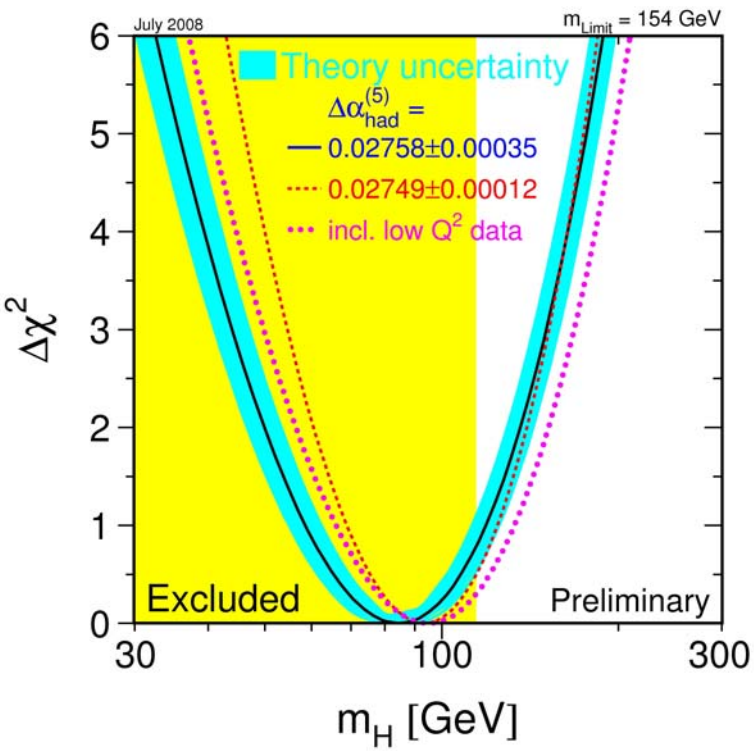
$\rightarrow$ Predict M_W





Consistency between direct and indirect measurements of M_W and m_t a strong test of theory!

Precision Measurements Limit M_H



- LEP EWWG (July, 2008):
  - m_t=172.4 ± 1.2 GeV
  - $M_H=84^{+34}_{-26}$ GeV
  - M_H < 154 GeV (one-sided 95% cl)
  - M_H < 185 GeV (Precision measurements plus direct search limit)

Best fit in region excluded from direct searches

Caveats

- Low Q^2 data not included in fit
  - Doesn't include atomic parity violation in cesium, parity violation in Moller scattering, & neutrino-nucleon scattering (NuTeV)
  - Higgs fit not hugely sensitive to low Q^2 data
- $M_H < 185$ GeV
  - Higgs limit moves around with m_t

Higgs limit assumes SM!

EW Measurements test SM

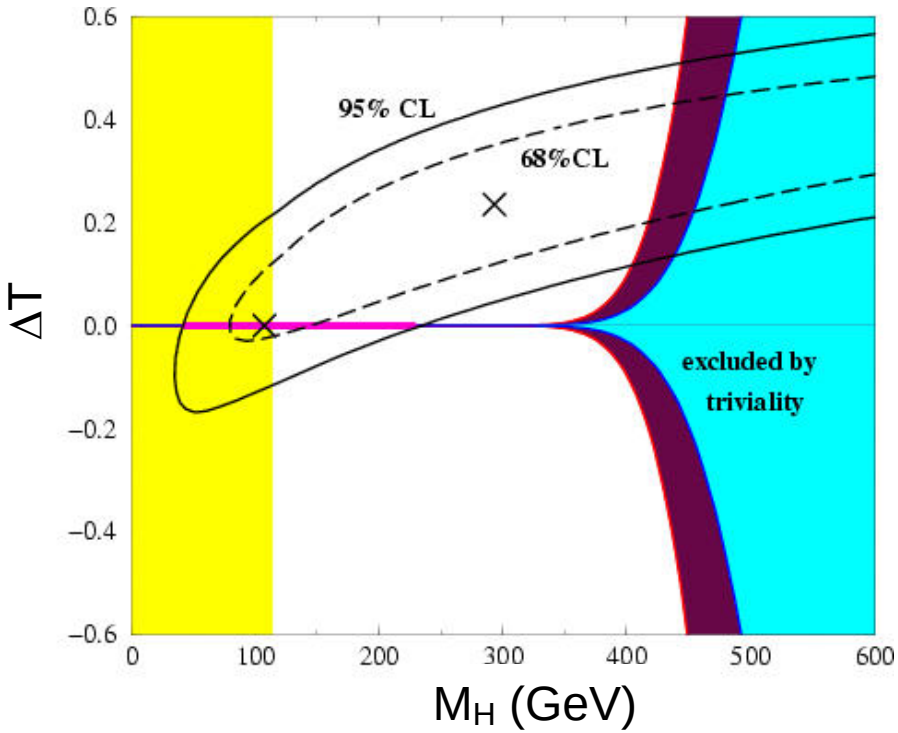
We have a model….
And it works to the 1% level

- Consistency of precision measurements at multi-loop level used to constrain models with new physics
- If a new model predicts some deviation from the SM, it has to be small

This fit ASSUMES SM

Limits on M_H ***Assume*** SM

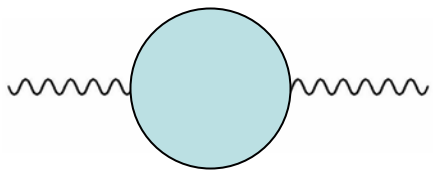
- $M_H \approx$ 450-500 GeV allowed with large isospin violation ($\alpha\Delta T=\delta\rho$) and higher dimension operators



We don't know what the model is which produces the operators which generate large $\delta\rho$

S,T,U formalism

- Suppose “new physics” contributes primarily to gauge boson 2 point functions
  - cf Δr where vertex and box corrections are small



- Also assume “new physics” is at scale $M >> M_Z$
- Two point functions for $\gamma\,\gamma$, WW, ZZ, γZ

S,T,U, (#2)

- Taylor expand 2-point functions

$$\boxed{\Pi_{ij}(q^2) = \Pi_{ij}(0) + q^2 \Pi'_{ij}(0) + \ldots}$$

- Keep first two terms
- Remember that QED Ward identity requires any amplitude involving EM current vanish at $q^2=0$

$$\boxed{\Pi_{\gamma\gamma}(0) = 0}$$

S,T,U (#3)

- To $O(q^2)$, there are 6 coefficients:

$$\Pi_{\gamma\gamma} = q^2 \Pi'_{\gamma\gamma}(0) + \ldots$$

$$\Pi_{WW} = \Pi_{WW}(0) + q^2 \Pi'_{WW}(0) + \ldots$$

$$\Pi_{\gamma Z} = q^2 \Pi'_{\gamma Z}(0) + \ldots$$

$$\Pi_{ZZ} = \Pi_{ZZ}(0) + q^2 \Pi'_{ZZ}(0) + \ldots$$

- Three combinations of parameters absorbed in α, G_F, M_Z
- In general, 3 independent coefficients which can be extracted from data

S,T,U, (#4)

$$\alpha T = \frac{\Pi_{WW}^{new}(0)}{M_W^2} - \frac{\Pi_{ZZ}^{new}(0)}{M_Z^2}$$

$$\frac{\alpha}{4 s_W^2 c_W^2} S = \frac{\Pi_{ZZ}^{new}(M_Z^2)}{M_Z^2} - \frac{\Pi_{ZZ}^{new}(0)}{M_Z^2}$$

$$\frac{\alpha}{4 s_W^2}(S+U) = \frac{\Pi_{WW}^{new}(M_W^2)}{M_W^2} - \frac{\Pi_{WW}^{new}(0)}{M_W^2}$$

SM contributions in α, G_F, and M_Z

Advantages: Easy to calculate

Valid for many models

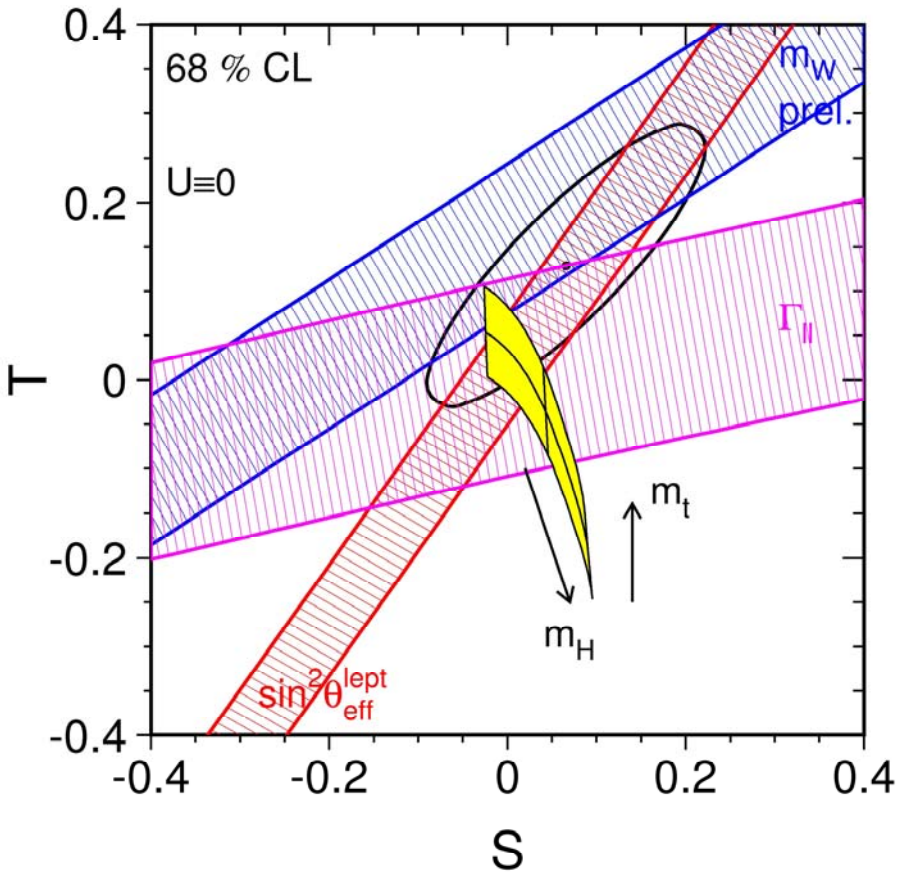
Experimentalists can give you model independent fits

Limits on S & T

- A model with a heavy Higgs requires a source of large (positive) ΔT

$$\alpha T = \delta\rho$$

- Fit assumes M_H=150 GeV
- S/T/U approach subtracts off SM contributions



Theoretical Limits on M_H

- Unitarity
  - If unitarity is violated, interactions grow with energy (cf longitudinal W's)
- Perturbativity of couplings
  - If perturbativity is violated, loop corrections may be larger than tree
- Limits tell us where minimal SM is valid
  - Unitarity and perturbativity provide strong limits on beyond the SM physics
  - These are model builders tools
- Renormalization of Higgs mass
  - What about naturalness?

Unitarity

- Consider 2 $\rightarrow$ 2 elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|A|^2$$

- Partial wave decomposition of amplitude

$$A = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l$$

- a_l are the spin *l* partial waves

Unitarity

- $P_l(\cos\theta)$ are Legendre polynomials:

$$\int_{-1}^{1} dx P_l(x) P_{l'}(x) = \frac{2\delta_{l,l'}}{2l+1}$$

$$\sigma = \frac{8\pi}{s}\sum_{l=0}^{\infty}(2l+1)\sum_{l'=0}^{\infty}(2l'+1)a_l a_{l'}^{*}\int_{-1}^{1} d\cos\theta P_l(\cos\theta)P_{l'}(\cos\theta)$$

$$= \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)\left|a_l\right|^2$$

- Sum of positive definite terms

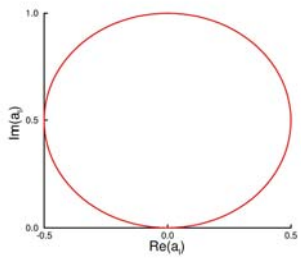
More on Unitarity

- Optical theorem $\sigma = \frac{1}{s}\mathrm{Im}\left[A(\theta=0)\right] = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)\left|a_l\right|^2$

$$\mathrm{Im}(a_l) = \left|a_l\right|^2$$

Optical theorem derived assuming only conservation of probability

- Unitarity requirement:



$$\left|\mathrm{Re}(a_l)\right| \le \frac{1}{2}$$

More on Unitarity

- Idea: Use unitarity to limit parameters of theory

Cross sections which grow with energy always violate unitarity at some energy scale

- Remember $W_L(p)$ with $\varepsilon_L \sim p/M_W$

Aside on WW Scattering

$$A(V_L^1 \ldots V_L^N \to V_L^1 \ldots V_L^{N'}) = (i)^N (-i)^{N'} A(\omega_1 \ldots \omega_N \to \omega_1 \ldots \omega_{N'}) + O\left(\frac{M_W^2}{E^2}\right)$$

This is a statement about scattering amplitudes, NOT individual Feynman diagrams

$\omega^{\pm}$, z are Goldstone bosons which are eaten by the Higgs mechanism to give the W & Z bosons their longitudinal components

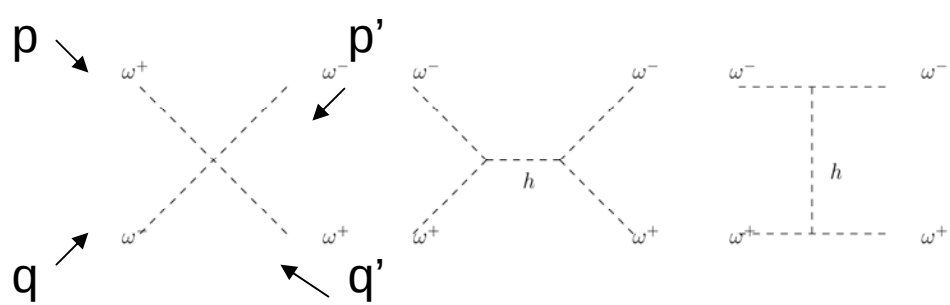
$W^+W^- \to W^+W^-$

- Recall scalar potential

$$V = \frac{M_H^2}{2} H^2 + \frac{M_H^2}{2v} H\left(H^2 + z^2 + 2\omega^+\omega^-\right) + \frac{M_H^2}{8v^2}\left(H^2 + z^2 + 2\omega^+\omega^-\right)^2$$

- $\omega^+\omega^- \to \omega^+\omega$

$$iA(\omega^+\omega \to \omega^+\omega^-) = -2i\frac{M_H^2}{v^2} + \left(-i\frac{M_H^2}{v}\right)^2 \frac{i}{t - M_H^2} + \left(-i\frac{M_H^2}{v}\right)^2 \frac{i}{s - M_H^2}$$



$$s = (p+q)^2$$

$$t = (p-p')^2$$

$$u = (q-p')^2$$

$\omega^+\omega^- \to \omega^+\omega^-$

- Two interesting limits:
  - s, t >> M_H^2

$$A(\omega^+\omega^- \to \omega^+\omega^-) \to -2\frac{M_H^2}{v^2}$$

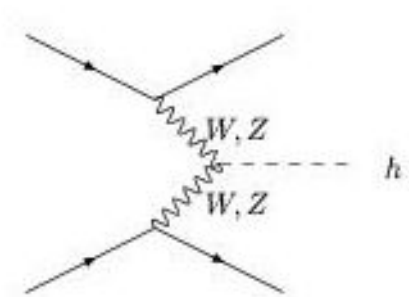
$$a_0^0 \to -\frac{M_H^2}{8\pi v^2}$$

  - s, t << M_H^2

$$A(\omega^+\omega^- \to \omega^+\omega^-) \to -\frac{u}{v^2}$$

$$a_0^0 \to -\frac{s}{32\pi v^2}$$

Remember: This is physical process



Use Unitarity to Bound Higgs

$$\left|\mathrm{Re}(a_l)\right| \leq \frac{1}{2}$$

- High energy limit:

$$a_0^0 \to -\frac{M_H^2}{8\pi v^2}$$

$M_H < 800$ GeV

- Heavy Higgs limit

$$a_0^0 \to -\frac{s}{32\pi v^2}$$

$E_c \sim 1.7$ TeV

$\to$ ***New physics at the TeV scale***

Can get more stringent bound from coupled channel analysis

Another Sort of Limit: Landau Pole

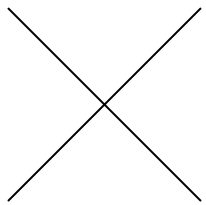
- M_H is a free parameter in the Standard Model
- Can we derive limits from consistency?
- Consider a scalar potential:

$$V = \frac{M_H^2}{2} H^2 + \frac{\lambda}{4} H^4$$

- This is potential at electroweak scale
- Parameters evolve with energy in a calculable way

Consider $HH \rightarrow HH$

- Real scattering, $s+t+u=4M_H^2$
- Consider momentum space-like and off-shell: $s=t=u=Q^2<0$
- Tree level: $iA_0=-6i\lambda$



HH→HH, #2

- One loop:

$$iA_s = (-6i\lambda)^2 \frac{1}{2}\int \frac{d^n k}{(2\pi)^2} \frac{i}{k^2 - M_H^2} \frac{i}{(k+p+q)^2 - M_H^2}$$

$$= \frac{9\lambda^2}{8\pi^2}(4\pi\mu^2)\Gamma(\varepsilon)\left(M_H^2 - Q^2 x(1-x)\right)^{-\varepsilon}$$

- $A = A_0 + A_s + A_t + A_u$

$$A = -6\lambda\left(1 + \frac{9\lambda}{16\pi^2}(4\pi\mu^2)\Gamma(\varepsilon)\left(M_H^2 - Q^2 x(1-x)\right)^{-\varepsilon} + \ldots\right)$$

HH→HH, #3

- Sum the geometric series to define running coupling

$$A = -6\lambda\left(1 + \frac{9\lambda}{16\pi^2}\log\frac{Q^2}{M_H^2}\right) + \ldots$$

$$A = \frac{6\lambda}{1 - \frac{9\lambda}{8\pi^2}\log\left(\frac{Q}{M_H}\right)} \equiv 6\lambda(Q)$$

- $\lambda(Q)$ blows up as $Q \to \infty$ (called Landau pole)

HH→HH, #4

- This is independent of starting point
- BUT…. Without $\lambda\phi^4$ interactions, theory is non-interacting
- Require quartic coupling be finite

$$\frac{1}{\lambda(Q)} > 0$$

HH→HH, #5

- Use $\lambda = M_H^2/(2v^2)$ and approximate $\log(Q/M_H) \rightarrow \log(Q/v)$
- Requirement for $1/\lambda(Q) > 0$ gives upper limit on M_H

$$M_H^2 < \frac{32\pi^2 v^2}{9\log\left(\frac{Q^2}{v^2}\right)}$$

- Assume theory is valid to 10^{16} GeV
  - Gives upper limit on $M_H < 180$ GeV
- Can add fermions, gauge bosons, etc.

We expect Higgs at electroweak scale

High Energy Behavior of λ

- Renormalization group scaling $\frac{1}{\lambda(Q)} = \frac{1}{\lambda(\mu)} + (\ldots)\log\left(\frac{Q}{\mu}\right)$

$$\boxed{16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 + 12\lambda g_t^2 - 12 g_t^4 + (gauge)}$$

$$t \equiv \log\left(\frac{Q^2}{\mu^2}\right) \qquad g_t = \frac{M_t}{v}$$

- *Large λ (Heavy Higgs):* self coupling causes λ to grow with scale
- *Small λ (Light Higgs):* coupling to top quark causes λ to become negative

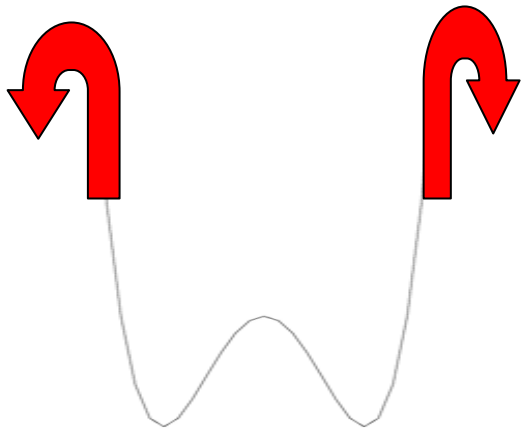
Does Spontaneous Symmetry Breaking Happen?

- SM requires spontaneous symmetry
- This requires $V(v) < V(0)$
- For small λ

$$16\pi^2 \frac{d\lambda}{dt} \approx -16 g_t^4$$

- Solve

$$\lambda(\Lambda) \approx \lambda(v) - \frac{3 g_t^4}{4\pi^2} \log\left(\frac{\Lambda^2}{v^2}\right)$$



Does Spontaneous Symmetry Breaking Happen? (#2)

- $\lambda(\Lambda) > 0$ gives lower bound on M_H

$$M_H^2 > \frac{3v^2}{2\pi^2}\log\left(\frac{\Lambda^2}{v^2}\right)$$

- If Standard Model valid to 10^{16} GeV

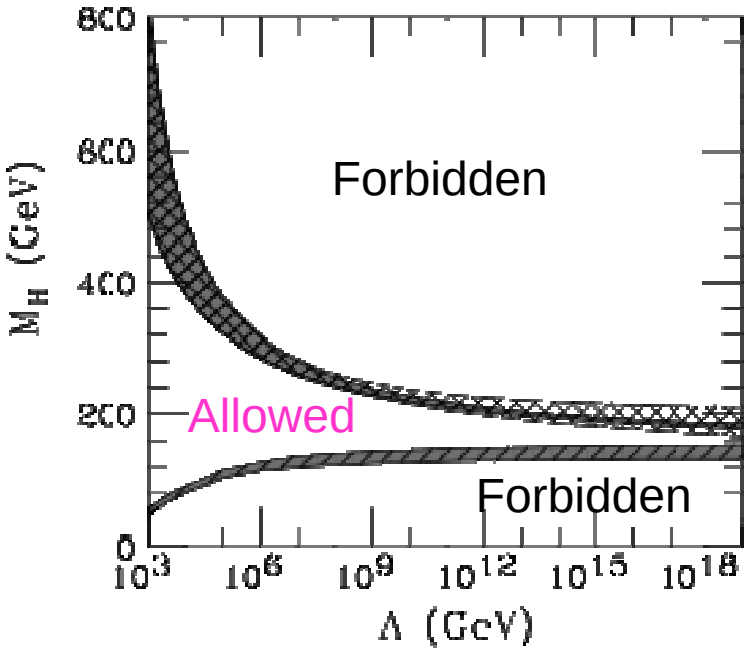
$$M_H > 130\ GeV$$

- For any given scale, Λ, there is a theoretically consistent range for M_H

Light Higgs Theoretically Attractive

➢Extrapolate Higgs potential to high scale Λ

$$V = \lambda\,(\Phi^{+}\Phi - v^{2})^{2}$$



•Standard Model is only consistent to GUT scale for small range of Higgs masses

•Heavy Higgs implies new physics at some low scale

Problems with the Higgs Mechanism

- We often say that the SM cannot be the entire story because of the quadratic divergences of the Higgs Boson mass
- Whether this is a problem or not is somewhat a matter of taste

Masses at One-Loop

- First consider a fermion coupled to a massive complex Higgs scalar

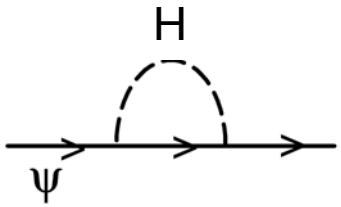
$$L = \overline{\Psi}(i\partial)\Psi + \left|\partial_\mu \phi\right|^2 - m_H \left|\phi\right|^2 - \left(\lambda_F \overline{\Psi}_L \Psi_R \phi + h.c.\right)$$

- Assume symmetry breaking as in SM:

$$\phi = \frac{(H+v)}{\sqrt{2}} \qquad\qquad m_F = \frac{\lambda_F v}{\sqrt{2}}$$

Masses at One-Loop, #2

- Calculate mass renormalization for Ψ



$$-i\Sigma_F(p) = \left(\frac{-i\lambda_F}{\sqrt{2}}\right)^2 (i)^2 \int \frac{d^4k}{(2\pi)^4} \frac{k + m_F}{[k^2 - m_F^2][(k-p)^2 - m_H^2]}$$

Renormalized Fermion Mass

$$\delta m_F = \Sigma_F(p)\Big|_{p=m_F}$$

$$= i\frac{\lambda_F^2}{32\pi^4}\int_0^1 dx \int d^4k' \frac{m_F(1+x)}{[k'^2 - m_F^2 x^2 - m_s^2(1-x)]^2}$$

- Do integral in Euclidean space

$$k_0 \to ik_4$$

$$d^4k' \to id^4k_E$$

$$k'^2 = k_0^2 - \left|\vec{k}\right|^2 \to k_4^2 - \left|\vec{k}\right|^2 = -k_E^2$$

$$\int d^4k_E\, f(k_E^2) = \pi^2 \int_0^{\Lambda^2} y\, dy\, f(y)$$

Renormalized Fermion Mass, #2

- Renormalization of fermion mass:

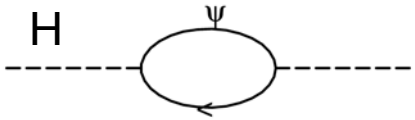
$$\delta m_F = -\frac{\lambda_F^2 m_F}{32\pi^2}\int_0^1 dx\ (1+x)\int_0^{\Lambda^2}\frac{y\,dy}{[y+m_F^2x^2+m_H^2(1-x)]^2}$$

$$= -\frac{3\lambda_F^2 m_F}{32\pi^2}\log\left(\frac{\Lambda^2}{m_F^2}\right)+\ldots..$$

Symmetry and the Fermion Mass

- $\delta m_F \approx m_F$
  - $m_F=0$, then quantum corrections vanish
  - When $m_F=0$, Lagrangian is invariant under
    - $\Psi_L \rightarrow e^{i\theta_L}\Psi_L$
    - $\Psi_R \rightarrow e^{i\theta_R}\Psi_R$
  - $m_F \rightarrow 0$ increases the symmetry of the threoy
  - Yukawa coupling (proportional to mass) breaks symmetry and so corrections $\approx m_F$

Scalars are very different



$$-i\Sigma_H(p^2) = -\left(\frac{-i\lambda_F}{\sqrt{2}}\right)^2 (i)^2 \int \frac{d^4k}{(2\pi)^4} \frac{Tr\left[(k+m_F)((k-p)+m_F)\right]}{(k^2-m_F^2)[(k-p)^2-m_F^2]}$$

$$I_1(a) = \int_0^1 dx \ \log\left(1-ax(1-x)\right)$$

$$\delta M_H^2 = \Sigma_H(m_H^2) = -\frac{\lambda_F^2\Lambda^2}{8\pi^2} + \left(m_H^2 - m_F^2\right)\log\left(\frac{\Lambda}{m_F}\right)$$

$$+(2m_F^2 - \frac{m_H^2}{2})\left(1 + I_1\left(\frac{m_H^2}{m_F^2}\right)\right) + O\left(\frac{1}{\Lambda^2}\right)$$

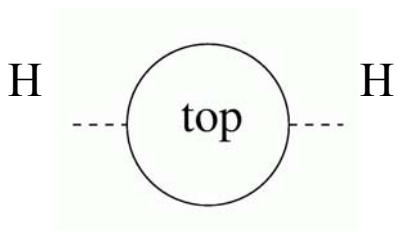
- M_H diverges quadratically!
- ***Quadratic sensitivity to high mass scales***

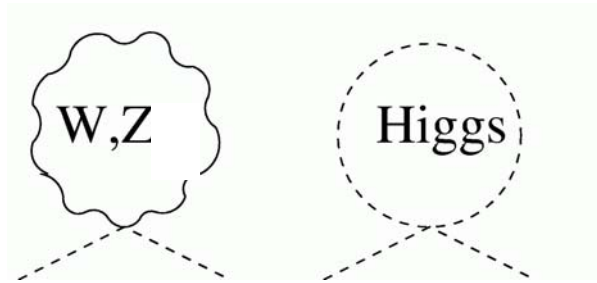
Scalars (#2)

- M_H diverges quadratically!
- Requires large cancellations (hierarchy problem)
- Can do this in Quantum Field Theory
- H does not obey decoupling theorem
  - Effects of heavy particle (H) does not decouple as $M_H \rightarrow \infty$
- $M_H \rightarrow 0$ doesn't increase symmetry of theory
  - Nothing protects Higgs mass from large corrections

Light Scalars are Unnatural

- Higgs mass grows with scale of new physics, Λ
- No additional symmetry for M_H=0, no protection from large corrections





$$\delta M_H^2 = \frac{G_F}{4\sqrt{2}\pi^2}\Lambda^2\left(6M_W^2 + 3M_Z^2 + M_H^2 - 12M_t^2\right)$$

$$= -\left(\frac{\Lambda}{0.7\ \mathrm{TeV}}\ 200\ \mathrm{GeV}\right)^2$$

$M_H \leq 200$ GeV requires large cancellations

What's the problem?

- Compute M_H in dimensional regularization and absorb infinities into definition of M_H

$$M_H^2 = M_{H0}^2 + \frac{1}{\varepsilon}(\ldots)$$

- Perfectly valid approach
- Except we know there is a high scale (associated with gravity)

Try to cancel quadratic divergences by adding new particles

- SUSY models add scalars with same quantum numbers as fermions, but different spin
- Little Higgs models cancel quadratic divergences with new particles with same spin

New particles assumed to be at TeV scale for cancellation of quadratic divergences

We expect something new at the TeV scale

- If it’s a SM Higgs then we have to think hard about what the quadratic divergences are telling us
- SM Higgs mass is highly restricted by requirements of agreement with precision electroweak data and theoretical consistency