Chapter 1

Extra Dimensions

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We begin with a discussion of a model with a *flat* extra dimension which addresses the flavor hierarchy of the Standard Model (SM) using profiles for the SM fermions in the extra dimension. We then show how flavor violation and contributions to the electroweak precision tests can be suppressed [even with O(TeV) mass scale for the new particles] in this framework by suitable modifications to the basic model. Finally, we briefly discuss a model with a *warped* extra dimension in which all the SM fields propagate and we sketch how this model "mimics" the earlier model in a flat extra dimension. In this process, we outline a "complete" model addressing the Planck-weak as well as the flavor hierarchy problems of the SM.

1.1. Introduction

Extra dimensions is a vast subject so that it is difficult to give a complete review in 5 lectures. The reader is referred to excellent lectures on this subject already available such as references [1–4] among others. Similarly, the list of references given here is incomplete and the reader is referred to the other lectures for more references.

We begin with some (no doubt this is an incomplete list) motivations for studying models with extra dimensions:

(i) Extra dimensional models can address or solve many of the prob-

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lems of the Standard Model (SM): for example, the various hierarchies unexplained in the SM – that between the Planck and electroweak scales [often called the "(big) hierarchy problem"] and also among the quark and lepton masses and mixing angles (often called the flavor hierarchy). We will show how both these problems are solved using extra dimensions in these lectures.
Extra dimensional models can also provide particle physics candidates for the dark matter of the universe (such a particle is absent in the SM). We will *not* address this point in these lectures.

(ii) Extra dimensions seem to occur in (and in fact are a necessary ingredient of) String Theory, the only known, complete theory of quantum gravity (see K. Dienes' lectures at this and earlier summer schools).

(iii) Although we will *not* refer to this point again, it turns out [5] that, under certain circumstances, extra dimensional theories can be a (weakly coupled) "dual" description of strongly coupled four-dimensional ($4D$) theories as per the correspondence between $5D$ anti-de Sitter (AdS) spaces and $4D$ conformal field theories (CFT's) [6].

The goal of these lectures is a discussion of the theory and phenomenology of some types of extra dimensional models, especially their applications to solving some of the problems of the SM of particle physics. The main concept to be gleaned from these lectures is that

- extra dimensions appear as a tower of particles (or modes) from the $4D$ point of view (a la the standard problem of a particle in $1D$ box studied in quantum mechanics).

The lightest mode (which is often massless and hence is called the zero-mode) is identified with the observed or the SM particles. Whereas, the heavier ones are called Kaluza-Klein (KK) modes and appear as new particles (beyond the SM). It is these particles which play a crucial role in solving problems of the SM, for example they could be candidates for dark matter of the universe or these particles can cut-off the quadratically divergent quantum corrections to the Higgs mass. These particles also give rise to a variety of signals in high-energy collider (i.e., via their on-shell or real production) and in low-energy experiments (via their off-shell or virtual effects). This is especially true if the masses of these KK modes are around the TeV scale, as would be the case if the extra dimension is relevant to explaining the Planck-weak hierarchy.

Here is a rough outline of the lectures. In lecture 1, we begin with the basics of KK decomposition in *flat* spacetime with one extra dimension compactified on a circle. We will show how obtaining chiral fermions requires an *orbifold* compactification instead of a circle. In lecture 2, we will consider a simple solution to the flavor hierarchy using the profiles of the SM fermions in the extra dimension. However, we will see that such a scenario results in too large contributions to flavor changing neutral current (FCNC) processes (which are ruled out by experimental data) if the KK scale is around the TeV scale – this is often called a flavor *problem*. Then, in lecture 3, we will consider a solution to this flavor problem based on the idea of large kinetic terms (for $5D$ fields) localized on a "brane". Another kind of measurement of properties of the SM particles (not involving flavor violation), called Electroweak Precision Tests, will be also be studied in this lecture, including the problem of large contributions to one such observable called the T (or ρ) parameter. In lecture 4, we will solve this problem of the T parameter by implementing a "custodial isospin" symmetry in the extra dimension. We will then briefly discuss some collider phenomenology of such models and some questions which are unanswered in these models. Finally, we will briefly study models based on *warped* spacetime in lecture 5, indicating how such models "mimic" the models in *flat* spacetime (with large brane kinetic terms) studied in the previous lectures. We will sketch how some of the open questions mentioned in lecture 4 can be addressed in the warped setting, resulting in a "complete" model.

1.2. Lecture 1

1.2.1. *Basics of Kaluza-Klein Decomposition*

Consider the following $5D$ action for a (real) scalar field (here and henceforth, the coordinates x^μ will denote the usual $4D$ and the coordinate y will denote the extra dimension):

$$S_{5D} = \int d^4x \int dy \Big[\left(\partial^M \Phi \right) \left(\partial_M \Phi \right) - M^2 \Phi \Phi \Big] \tag{1.1}$$

Since gravitational law falls off as $1/r^2$ and not $1/r^3$ at long distances, it is clear that we must compactify the extra dimension. Suppose we compactify the extra dimension on a circle (S^1), i.e., with y unrestricted ($-\infty < y < \infty$), but with y identified with $y + 2\pi R$[a]. We

[a]Equivalently, we can restrict the range of y: $0 \leq y \leq 2\pi R$, imposing the condition that

impose periodic boundary conditions on the fields as well, i.e., we require $\Phi(y = 2\pi R) = \Phi(y)$. Then, we can (Fourier) expand the $5D$ scalar field as follows:

$$\Phi = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{n=+\infty} \phi^{(n)}(x) e^{iny/R} \tag{1.2}$$

where the coefficient in front has been chosen for proper normalization.

Substituting this expansion into S_{5D} and using the orthonormality of profiles of the Fourier modes in the extra dimension (i.e., $e^{iny/R}$) to integrate over the extra dimension, we obtain the following $4D$ action:

$$S_{4D} = \int d^4x \sum_n \left[\left(\partial_\mu \phi^{(n)} \right) \left(\partial^\mu \phi^{(n)} \right) - \left(M^2 + \frac{n^2}{R^2} \right) \phi^{(n)} \phi^{(n)} \right] \tag{1.3}$$

This implies that from the $4D$ point of view the $5D$ scalar field appears as an (infinite) tower of $4D$ fields which are called the Kaluza-Klein (KK) modes: $\phi^{(n)}$ with mass^2, $m_n^2 = M^2 + n^2/R^2$ (note that the n^2/R^2 contribution to the KK masses arises from ∂_5 acting on the profiles) [see Fig. 1.1 (a)].

The lightest or zero-mode ($n = 0$) has mass M (strictly speaking it is massless only for $M = 0$). The non-zero KK modes start at $\sim 1/R$ (for the case $M \ll 1/R$) which is often called the *compactification scale*. We can easily generalize to the case of δ extra dimensions, each of which is compactified on a circle of same radius to obtain the spectrum: $m_n^2 = M^2 + \sum_{i=1}^{\delta} n_i^2/R^2$. However, in these lectures, *we will restrict to only one extra dimension.*

Thus, we see that the signature of an extra dimension from the $4D$ point of view is the appearance of infinite tower of KK modes: to repeat, the lightest (zero)-modes is identified with the SM particle and the heavier ones (KK modes) appear as new particles beyond the SM.

1.2.2. *Orbifold*

Mathematically speaking, a circle is a (smooth) manifold since it has no special points. We can "mod out" this smooth manifold by a discrete symmetry to obtain an "orbifold". Specifically, we impose the discrete (Z_2) identification: $y \leftrightarrow -y$ in addition to $y \equiv y + 2\pi R$. Thus, the physical or

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$y = 0$ same as $y = 2\pi R$.

fundamental domain extends only from $y = 0$ to $y = \pi R$[b] – this compactification is denoted by S^1/Z_2: see Fig. 1.2.

The endpoints of the orbifold ($y = 0, \pi R$) do *not* transform under Z_2 and hence are called *fixed* points of the orbifold. Also, note that the end points of this extra dimension are not identified with each other either by the periodicity condition $y \equiv y + 2\pi R$ (unlike the endpoints $y = 0, 2\pi R$ on S^1) or by the Z_2 symmetry.

Let us consider how the KK decomposition is modified in going from a circle to an orbifold. We can rewrite the earlier KK decomposition in terms of functions which are even and odd under $y \rightarrow -y$:

$$\Phi(x,y) = \frac{1}{\sqrt{2\pi R}}\phi^{(0)} + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \Big[\phi_+^{(n)} \cos\frac{ny}{R} + \phi_-^{(n)} \sin\frac{ny}{R} \Big] \tag{1.4}$$

with the identification $\phi_\pm^{(n>0)} \equiv 1(i)/\sqrt{2}\left(\phi^{(n)} \pm \phi^{(-n)}\right)$.

We must require the physics, i.e., S_{5D}, to be invariant under $y \rightarrow -y$. For this purpose, we assign an (intrinsic) parity transformation to Φ:

$$\Phi(x,-y) = P\Phi(x,y) \tag{1.5}$$

with $P = \pm 1$, i.e., Φ being even or odd. This assignment sets $\phi_-^{(n>0)} = 0$ for $P = +1$ and $\phi_+^{(n)} = 0$ [including $\phi^{(0)}$] for $P = -1$ see Fig. 1.1 (b).

Thus, a summary of orbifold compactification is that[c]: (i) it reduces the number of modes by a factor of 2 and (ii) it removes or projects out the *zero*-mode for the case of the $5D$ field being *odd* under the parity.

1.2.3. *Fermions on a Circle: Chirality Problem*

One possible representation of the $5D$ Clifford algebra for fermions:

$$\{\Gamma_M, \Gamma_N\} = 2\eta_{MN} \tag{1.6}$$

is provided by the usual Dirac (4×4) matrices

$$\Gamma_\mu = \gamma_\mu, \;\; \Gamma_5 = -i\gamma_5 \tag{1.7}$$

Thus, we see that the smallest (irreducible) representation for $5D$ fermions has 4 (complex) components (cf. 2-component complex or Weyl spinor

[b]Equivalently, we can still pretend that it extends from $y = 0$ to $y = 2\pi R$ as before, but with the region $y = \pi R$ to $y = 2\pi R$ *not* being independent of the region $y = 0$ to $y = \pi R$.

[c]We will see later how an orbifold is "useful" in the case of $5D$ fermion/gauge fields because of these properties.

in $4D$, where the 2×2 Pauli matrices form a representation of Clifford algebra).

Consider the following $5D$ action for fermions

$$S_{5D} = \bar{\Psi} \left(i \partial_M \Gamma^M - M \right) \Psi \tag{1.8}$$

When the extra dimension is compactified on a circle, we can plug in the usual decomposition $\Psi_{\alpha=1-4} = \sum_n \psi_\alpha^{(n)} e^{iny/R}$ to find the $4D$ action:

$$S_{4D} = \sum_n \bar{\psi^{(n)}} \left(i \gamma_\mu \partial^\mu - M - in/R \right) \psi^{(n)} \tag{1.9}$$

Thus, we obtain a tower of Dirac (4-component) spinors from the $4D$ point of view: $m_n^2 = M^2 + n^2/R^2$: see Fig. 1.3 (a).

Consider the case $M = 0$. We see that there are *non*-chiral massless (or zero) modes: explicitly, in the Weyl representation of Dirac matrices, i.e.,

$$\gamma_\mu = \begin{pmatrix} \mathbf{0} & \sigma_\mu \\ \sigma_\mu & \mathbf{0} \end{pmatrix} \tag{1.10}$$

$$\gamma_5 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix} \tag{1.11}$$

$$\sigma_\mu = \left(\sigma_{i=1..3}, \mathbf{1} \right), \tag{1.12}$$

$\psi^{(0)}_{\alpha=1-4}$ decomposes as $\sim \left[\psi_L^{(0)}(\alpha = 1,2), \psi_R^{(0)}(\alpha = 3,4) \right]$, where L (R) refers to left (right) chirality (or helicity) under the $4D$ Lorentz transformation. The problem is that if the $5D$ fermion transforms under some $5D$ gauge symmetry, then the L and R (massless) chiralities (zero-modes) transform identically under this gauge symmetry. Hence, such a scenario cannot correspond to the SM, where the fermions are known to be chiral, i.e., the left-handed (LH) and right-handed (RH) ones transform as doublets and singlets, respectively under the $SU(2)_{weak}$ gauge symmetry.

1.2.4. *Fermion Chirality from Orbifold*

We can obtain chiral fermions by compactifying the $5D$ theory on an orbifold instead of a circle as follows. Suppose we choose Ψ_L to be even under the Z_2 parity. Then, Ψ_R must be odd since the $5D$ action contains the term $\bar{\Psi}\Gamma^5 \partial_5 \Psi \ni \Psi_L^\dagger \partial_5 \Psi_R$, which must be even so that the $5D$ action is Z_2-invariant (note that ∂_5 is odd under parity).

We obtain the following decomposition:

$$\Psi_{L\ (R)} \sim \sum_n \psi^{(n)}_{L\ (R)} \cos \frac{ny}{R} \ (\sin \frac{ny}{R}) \tag{1.13}$$

Thus, (for case of the $5D$ mass, $M = 0$[d]) we get a massless zero-mode only for Ψ_L (even field): see Fig. 1.3(b). Of course, we could have chosen Ψ_R to be even instead to obtain a RH zero-mode.

1.3. Lecture 2

1.3.1. *Zero-Mode Fermion Profiles*

We see that the massless (chiral) mode on an orbifold has a *flat* profile [see Eq. (1.13)]. So, if all the SM fermions have $M = 0$, then the extra dimension does not provide any resolution of the flavor hierarchy, i.e., we need to put hierarchies in $5D$ Yukawa couplings (similar to the situation in the SM) in order to obtain hierarchies in the $4D$ Yukawa couplings.

We must then consider modifying the profiles of the fermion zero-modes in order to solve the flavor hierarchy problem using the extra dimension. We can try adding a bare mass term: $\bar{\Psi}\Psi = \Psi_L^\dagger \Psi_R + h.c.$, but such a mass term breaks the Z_2 symmetry (again since $\Psi_{L,R}$ transform oppositely under the parity) . The solution to this problem [7] is to couple the $5D$ fermion to a Z_2-odd scalar with the following $5D$ Lagrangian:

$$\begin{aligned} \mathcal{L}_{5D} = \bar{\Psi} \left(i \partial_M \Gamma^M - h \Phi \right) \Psi + \\ \left(\partial_M \Phi \right)^2 - \lambda \left(\Phi^2 - V^2 \right)^2 \end{aligned} \tag{1.14}$$

The point is that the potential $V(\Phi) = \lambda \left(\Phi^2 - V^2 \right)^2$ forces a vacuum expectation value (vev) for Φ which is a constant in y in-between the endpoints of the extra dimension (often called the "bulk"). However, such a vev tends to "clash" with $\Phi = 0$ at the endpoints (as required by the scalar being odd under the Z_2 parity). As a result, we obtain a (approximately) "kink-anti-kink" profile for the scalar vev (see references [7] for more details) as in Fig. 1.4. Such a profile for the scalar vev is equivalent to adding a Z_2-*odd* $5D$ mass for the fermion. The point is that with such a scalar vev we have a *spontaneous* breaking of the Z_2 symmetry – recall that it is this Z_2 symmetry which prevented us from writing such a mass term to begin with, i.e., a *bare* mass term would correspond to an *explicit* breaking of this symmetry.

Let us then consider how the KK decomposition is modified in the pres-

[d] We will see in the next section that only a "special" form of mass term is allowed on an orbifold.

ence of such an (odd) bulk fermion mass term. The $5D$ action is

$$S_{5D} = \bar{\Psi}\Big[i\partial_M \Gamma^M + M\epsilon(y)\Big]\Psi \tag{1.15}$$

where $\epsilon(y) = +1(-1)$ for $\pi R > y > 0(-\pi R < y < 0)$. It is easy to see that the eigenmodes are no longer *single* sin or cos, but instead are linear *combinations* of these basis functions. Hence, we have to work harder to obtain the eigenmodes.

1.3.2. *General Procedure for KK Reduction*

We will now take a slight detour to discuss the procedure to obtain the KK decomposition for a general $5D$ action and return to apply this procedure to the above $5D$ fermion case.

For simplicity, consider a $5D$ scalar field decomposed into modes as follows: $\Phi(x,y) = \sum_n \phi^{(n)}(x) f_n(y)$. Plug this expansion into the simple $5D$ action:

$$S_{5D} = \int d^4x \int dy \Big[\left(\partial^M \Phi\right)\left(\partial_M \Phi\right) - M^2 \Phi\Phi \Big] \tag{1.16}$$

We *require* that, *after* integrating over the extra dimension, we get

$$S_{4D} = \int d^4x \sum_n \left[\left(\partial_\mu \phi^{(n)}\right)\left(\partial^\mu \phi^{(n)}\right) - \left(M^2 + \frac{n^2}{R^2}\right)\phi^{(n)}\phi^{(n)} \right] \tag{1.17}$$

so that we can interpret $\phi^{(n)}$'s as particles (KK modes) from the $4D$ point of view.

This requirement gives us the following two equations: matching kinetic terms in S_{4D} of Eq. (1.17) to the ∂_μ (or $4D$) part of the kinetic term obtained from S_{5D} gives us the following:

(i) orthonormality condition

$$\int dy f_n^*(y) f_n(y) = 1 \tag{1.18}$$

whereas matching the mass terms in S_{4D} of Eq. (1.17) to the $5D$ mass term (M) and the action of ∂_5 on the profiles in S_{5D} gives us the

(ii) differential equation:

$$\partial_y^2 f_n(y) - M^2 f_n^2(y) = -m_n^2 f_n^2(y) \tag{1.19}$$

Thus the KK decomposition reduces to an eigenvalue problem, solving which gives us the KK masses (eigenvalues) m_n and their profiles $f_n(y)$ (eigenfunctions). This is very reminiscent of solving the problem of Schroedinger equation for a particle in a $1D$ box in quantum mechanics.

For the above simple case of a $5D$ scalar with a bulk mass, we get the following solutions to the differential equation [i.e., Eq. (1.19)]: $f_n(y) \sim e^{\pm i\sqrt{m_n^2 - M^2}y}$ for $m_n^2 \geq M^2$. In addition, the periodicity condition, i.e., $f_n(y) = f_n(y + 2\pi R)$ requires $\sqrt{m_n^2 - M^2} = n^2/R^2$ so that $m_n^2 = M^2 + n^2/R^2$ (as before). The reader should think about the possibility $m_n^2 < M^2$ (where we get exponentially rising or decaying profiles) to show that we cannot satisfy the continuity of derivative at $y = 0, \pi R$ in this case and hence we cannot have such solutions for a scalar.

The above procedure can be generalized to more complicated $5D$ actions and for other spin fields.

1.3.3. *Solution to Flavor Puzzle*

Next, we return to the problem of the KK decomposition of a $5D$ fermion with the (odd) mass term and with $\Psi_{L\ (R)}$ being even (odd) under Z_2 parity. As outlined above, we plug $\Psi_{L,R} = \psi^{(n)}(x) f_{L,R\ n}(y)$ into S_{5D} to obtain the differential equations:

$$\left[-\partial_5 + M\epsilon(y) \right] f_{L\ n} = m_n f_R \tag{1.20}$$

$$\left[\partial_5 + M\epsilon(y) \right] f_{R\ n} = m_n f_L \tag{1.21}$$

Note that (as mentioned before) cos or sin are solutions only for $M = 0$, but not for $M \neq 0$ [On a circle, the mass term M has no $\epsilon(y)$ so that $f_{L,R\ n} \sim e^{iny/R}$ are indeed solutions.].

It is easy to solve for the zero-mode profile ($m_n = 0$) even for $M \neq 0$ (the $m_n \neq 0$ case is difficult to solve due to the two differential equations being coupled):

$$\begin{aligned} f_{L\ 0}(y) &= N e^{My} \quad (0 \leq y \leq \pi R) \\ &= N e^{-My} \quad (0 \geq y \geq -\pi R) \end{aligned} \tag{1.22}$$

(N is a normalization factor: see exercise 1 in appendix).

Note that for RH modes, $f_{R\ 0} \sim e^{\mp My}$ solves the eigenvalue equation, but it clashes with vanishing of $f_{R\ 0}(y)$ at $y = 0, \pi R$ as required by Ψ_R being odd under Z_2 parity. Thus, as expected from the parity choice, there is no RH zero-mode. Note that there is a discontinuity in the derivative of $f_{L\,0}$ at $y = 0, \pi R$ (Fig. 1.4), which precisely matches the $\epsilon(y)$ term (cf. scalar case earlier where such profiles cannot satisfy the requirement of continuity of derivative at the fixed points). The point is that $M \neq 0$ still gives a mass*less* fermion mode (unlike for a scalar).

We will now see how the flavor hierarchy can be accounted for without any large hierarchies in the $5D$ theory: see exercise 1 and Fig. 1.5. For simplicity, suppose the SM Higgs field is localized at $y = \pi R$ (each end of the extra dimension is often called a "brane", motivated by String Theory) and add the following coupling of $5D$ fermions to it:

$$S_{5D} \ni \int d^4x dy \delta(y - \pi R) H \Psi_L \Psi'_R \lambda_{5D} \tag{1.23}$$

where Ψ and Ψ' are two *different* $5D$ fermion fields which are $SU(2)_L$ doublets and singlets with M, M' being their $5D$ masses, respectively. Note that Ψ_L and Ψ'_R are chosen to be even under Z_2 so that they give the LH and RH zero-modes, respectively. Since Ψ_R and Ψ'_L vanish at the $y = \pi R$ brane, they do not couple to the Higgs as seen in Eq. (1.23). Plugging in the zero-mode profiles, we obtain the effective $4D$ Yukawa coupling, i.e., $\lambda_{4D} H \psi_L^{(0)} \psi'^{(0)}_R$:

$$\begin{aligned} \lambda_{4D} &\approx \lambda_{5D} \times f_{L\,0}(\pi R) f_{R\,0}(\pi R) \\ &\propto \lambda_{5D} e^{\left(M - M'\right)} \end{aligned} \tag{1.24}$$

Let us consider the hierarchy between the down (d) and strange (s) quark masses for example. For simplicity, we set λ_{5D} to be the same for d, s and also $M = -M'$ for each quark to obtain (up to small dependence of normalization on M's)

$$\begin{aligned} \frac{m_d}{m_s} &\sim e^{2 \Delta M \pi R} \\ &\sim 1/100 \;\; \text{which is the required, i.e., experimental value} \end{aligned} \tag{1.25}$$

so that $\Delta M \equiv M_d - M_s \sim -2$ [for example, $M_d = -3, M_s = -1$] in units of $1/(\pi R)$ suffices to obtain the hierarchy in $4D$ masses (or Yukawa couplings).

The crucial point is that we did not invoke any large hierarchies in the $5D$ or fundamental parameters (M or λ_{5D}), but we can still obtain large hierarchies in the $4D$ Yukawa couplings.

1.3.4. *Intermediate Summary: Basic Concepts*

Before moving on, let us summarize:

(i) A $5D$ field appears as a tower of KK modes from $4D$ point of view, with each mode having a profile in the extra dimension.
(ii) The profiles and the KK masses are obtained by solving an eigenvalue problem (or wave equations in $5D$ space-time).
(iii) The coupling of particles (i.e., zero and KK modes) is proportional to the overlap of their profiles in the extra dimension.

1.3.5. *Gauge Field on a Circle*

Next, we consider $5D$ gauge fields with the following $5D$ action[e]:

$$S_{5D} = \int d^4x dy \frac{1}{4} \mathcal{F}_{MN} \mathcal{F}^{MN} \tag{1.26}$$

$$= \int d^4x dy \frac{1}{4} \left(\mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} + \mathcal{F}_{\mu 5} \mathcal{F}^{\mu 5} \right) \tag{1.27}$$

with

$$\mathcal{A}_M = \mathcal{A}_\mu + \mathcal{A}_5 \tag{1.28}$$

As usual, the KK decomposition is achieved by plugging in the expansion $\mathcal{A}_{\mu,\,5} = \sum_n A^{(n)}_{\mu,\,5} f_{\mu,\,5\;n}(y)$ into S_{5D}. It is easy to see that this procedure is similar to that for a $5D$ scalar, up to the presence of Lorentz index and gauge fixing. It is straightforward to include the Lorentz index in the KK decomposition, but there are subtleties with gauge fixing – we will not go into details of the latter issue in these lectures (for a discussion of this issue, see, for example, 1st reference in [3]).

The end result is that, on a circle, both $\mathcal{A}_\mu$ and $\mathcal{A}_5$ components have zero-modes – the former is a vector, whereas the latter is a scalar from the $4D$ point of view: see Fig. 1.6(a).

Thus, we encounter a *unification of spins* in the sense that massless $4D$ scalars can be obtained from $5D$ gauge fields. If the $4D$ scalar $A_5^{(0)}$ remains massless, then it will result in an extra long range force which would be ruled out by experiments. However, this scalar does acquire a mass from loop corrections (see lecture 5) so that such a light scalar (almost zero-mode) might not be a *robust* problem (unlike the chirality problem with fermions on a circle).

[e]Once the SM fermions propagate in the extra dimension, we can show that the SM gauge fields also have to do the same to preserve gauge invariance.

1.3.6. *Gauge Field on an Orbifold*

In any case, it is possible to get rid of the $\mathcal{A}_5$ zero-mode using orbifold compactification as follows. Notice that for

$$\mathcal{F}_{\mu 5} = \partial_\mu \mathcal{A}_5 - \partial_5 \mathcal{A}_\mu \tag{1.29}$$

to have a well-defined Z_2 parity, we have two choices:

(i) $\mathcal{A}_\mu$ is even – it has a zero-mode which is identified with the SM gauge boson – which implies that $\mathcal{A}_5$ is odd and so does not have a zero-mode [see Fig. 1.6(b)] or

(ii) $\mathcal{A}_\mu$ is odd (no zero-mode gauge boson) so that $\mathcal{A}_5$ is even and has a zero-mode.

As we will see later, the A_5 zero-mode in case (ii) can play the role of SM Higgs, but for now, we will make the choice (i), i.e., $\mathcal{A}_{\mu\ (5)}$ is even (odd) so that we do have a zero-mode (i.e., SM) gauge boson.

Hence, we obtain the following KK decomposition for this gauge field on an orbifold [Fig. 1.6 (b)]:

$$f_{\mu\ 0} = \frac{1}{\sqrt{2\pi R}} \quad \text{(i.e., a flat profile)} \tag{1.30}$$

$$f_{\mu\ n}(y) = \frac{1}{\sqrt{\pi R}} \cos ny/R \tag{1.31}$$

$$f_{5\ n}(y) = \frac{1}{\sqrt{\pi R}} \sin ny/R \tag{1.32}$$

We have normalized the modes over $-\pi R \leq y \leq +\pi R$, even though the physical domain is from $y = 0$ to $y = \pi R$. We can show that $A_\mu^{(n \neq 0)}$ "eats" $A_5^{(n)}$ to form a massive spin-1 gauge boson from the following mass terms

$$\mathcal{F}_{\mu 5}^2 \ni \partial_\mu \mathcal{A}_5 \partial_5 \mathcal{A}^\mu \tag{1.33}$$

$$\sim \sum_n A_\mu^{(n)} \partial^\mu A_5^{(n)} \partial_y f_{\mu\ n}(y) \tag{1.34}$$

These mass terms mixing $A_\mu^{(n)}$ and $A_5^{(n)}$ are similar to the ones in the SM: $W_\mu \partial^\mu H \langle H \rangle$ (which indicate that the longitudinal polarization of W is the unphysical component of Higgs, i.e., the equivalence theorem).

1.3.7. *Couplings of Gauge Modes*

We now calculate the couplings of the various gauge modes to the matter particles (in this case fermions) based on their profiles. We can show that the coupling of zero-mode is the same to all fermion modes (whether zero or KK):

$$\int d^4xdy\bar{\Psi}\Gamma^M\left(\partial_M+g_5\mathcal{A}_M\right)\Psi \ni \sum_n \bar{\psi_L^{(n)}}\gamma^\mu\psi_L^{(n)}\times\int dy f_{L\,n}^2(y) \left(\partial_\mu+A_\mu^{(0)}\frac{g_5}{\sqrt{2\pi R}}\right) \tag{1.35}$$

$$= \bar{\psi_L^{(n)}}\gamma^\mu\psi_L^{(n)}\left(\partial_\mu+g_4A_\mu^{(0)}\right) \quad \text{(for all } n\text{)} \tag{1.36}$$

with

$$g_4\,(\text{or } g_{SM})=\frac{g_5}{\sqrt{2\pi R}} \tag{1.37}$$

The point is that the profile of the gauge zero-mode is flat so that the overlap integrals appearing in the kinetic term for fermion mode and in the coupling to gauge zero-mode are identical. This universality of the zero-mode gauge coupling is actually guaranteed by $4D$ gauge invariance.

However, the couplings of zero-mode fermions to gauge KK modes (coming from the overlap of profiles) are *non*-universal, i.e., these couplings depend on the $5D$ fermion mass (see Fig. 1.7):

$$g(n,M)=g_5\int dy\left(Ne^{-My}\right)^2\times f_{\mu\,n}(y) \tag{1.38}$$

$$\equiv g_4\times a(n,M) \tag{1.39}$$

where a is an $O(1)$ quantity (see exercise 1). The reason is that the gauge KK profile is not flat (unlike for zero-mode) or equivalently there is no analog of $4D$ gauge invariance for the massive (KK) gauge modes.

1.3.8. *Flavor Problem from Gauge KK Modes*

Such non-universal couplings of gauge KK modes to fermion zero-modes results in flavor violation as follows [8]. The point is that the couplings of the gauge KK modes to zero-mode fermions are flavor *diagonal*, but non-universal in the interaction (or weak) basis:

$$g_4\left(\bar{d_L}_{\text{weak}}\;\bar{s_L}_{\text{weak}}\right)\begin{pmatrix}a_d & 0\\ 0 & a_s\end{pmatrix}\gamma^\mu A_\mu^{(n)}\begin{pmatrix}d_{L\,\text{weak}}\\ s_{L\,\text{weak}}\end{pmatrix} \tag{1.40}$$

which results in the appearance of flavor violating couplings *after* a unitary rotation to the mass basis:

$$...g_4 D_L^\dagger \text{diag}\left(a_d, a_s\right) D_L ... \to g_4 \left(a_s - a_d\right) \left(D_L\right)_{12} \times \\ \bar{d}_L \text{mass} \gamma^\mu A_\mu^{(n)} s_L \text{ mass} \tag{1.41}$$

where D_L is the unitary transformation to go from the interaction (or weak) basis to the mass basis (for left-handed down-type quarks).

Hence, we obtain a contribution to, for example, $K - \bar{K}$ mixing amplitude:

$$\mathcal{M}_{KK} \sim \frac{g_4^2}{M_{KK}^2} \left(a_s - a_d\right)^2 \left(D_L\right)_{12}^2 \tag{1.42}$$

The SM contribution to $K - \bar{K}$ mixing amplitude has a suppression mechanism (see below):

$$\mathcal{M}_{SM} \sim \frac{g_4^4}{16\pi^2} \frac{m_c^2}{M_W^4} \left(V_{us} V_{ud}\right)^2 \tag{1.43}$$

where $V_{us,\, ud}$ are the Cabibbo-Kobayashi-Maskawa (CKM) mixing angles. Since the data agrees with the SM prediction, we must require the KK contribution to be smaller than the SM one and hence we can set a bound on the KK mass. Using

$$\left(a_s - a_d\right) \sim O(1/10) \tag{1.44}$$

(see exercise 1), i.e., the fact that the couplings of gauge KK modes to down and strange quarks are $O(1)$ different, we get

$$M_{KK} \gtrsim 20 \text{ TeV} \tag{1.45}$$

assuming that the the D_L mixing angles are of order the CKM mixing angles. Such a large KK mass scale could result in a tension with a solution to the Planck-weak hierarchy problem: we would like the KK scale to be $\sim$ TeV for this purpose (we will see later how the KK mass scale is related to the EW scale).

For completeness, we briefly review FCNC's in the SM below. We begin with the transformation of quarks from weak to mass basis. The Yukawa couplings of the SM fermions to the Higgs (or the mass terms) are 3×3 complex matrices (denoted by M_d in the down quark sector) in the generation space. Such matrices can be diagonalized by *bi*-unitary transformations, $D_{L,R}$. For simplicity, consider the 2 generation case (this analysis can be easily generalized to the case of 3 generations), where this transformation can be explicitly written as

$$
\begin{aligned}
& \left(\bar{d_L}_{\text{weak}}\; \bar{s_L}_{\text{weak}}\right) (M_d)_{2\times 2} \begin{pmatrix} d_{R\,\text{weak}} \\ s_{R\,\text{weak}} \end{pmatrix} = \\
& \left(\bar{d_L}_{\text{mass}}\; \bar{s_L}_{\text{mass}}\right) M_d^{\text{diag.}} \begin{pmatrix} d_{R\,\text{mass}} \\ s_{R\,\text{mass}} \end{pmatrix}
\end{aligned}
\tag{1.46}
$$

where

$$
\begin{pmatrix} d_{L,R\,\text{weak}} \\ s_{L,R\,\text{weak}} \end{pmatrix} = D_{L,R} \begin{pmatrix} d_{L,R\,\text{mass}} \\ s_{L,R\,\text{mass}} \end{pmatrix} \tag{1.47}
$$

$$
\begin{aligned}
M_d^{\text{diag.}} & \equiv D_L^{\dagger} M_d D_R \\
& = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}
\end{aligned}
\tag{1.48}
$$

There are no tree-level FCNC in the SM since the gluon, γ and Z vertices preserve flavor in spite of the above transformations. Of course, the reason is that the couplings of gluon, γ and Z in the weak (or interaction) basis are universal. Explicitly,

$$
\begin{aligned}
& g_Z \left(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W\right) \left(\bar{d_L}_{\text{weak}}\; \bar{s_L}_{\text{weak}}\right) Z_\mu \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} d_{L\,\text{weak}} \\ s_{L\,\text{weak}} \end{pmatrix} \\
= \quad & \ldots \left(\bar{d_L}_{\text{mass}}\; \bar{s_L}_{\text{mass}}\right) Z_\mu \gamma^\mu D_L^{\dagger} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D_L \begin{pmatrix} d_{L\,\text{mass}} \\ s_{L\,\text{mass}} \end{pmatrix} \\
= \quad & \ldots \textstyle\sum_{i=d,s} \bar{d}^i_{L\,\text{mass}} Z_\mu \gamma^\mu d^i_{L\,\text{mass}}
\end{aligned}
\tag{1.49}
$$

as compared to Eqs. (1.40) and (1.41).

However, the charged current (W) couplings *are* non-diagonal in the mass basis:

$$
\begin{aligned}
& \tfrac{g}{\sqrt{2}} \left(\bar{u_L}_{\text{weak}}\; \bar{c_L}_{\text{weak}}\right) W_\mu \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} d_{L\,\text{weak}} \\ s_{L\,\text{weak}} \end{pmatrix} \\
= \; & \ldots \left(\bar{u_L}_{\text{mass}}\; \bar{c_L}_{\text{mass}}\right) W_\mu \gamma^\mu U_L^{\dagger} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D_L \begin{pmatrix} d_{L\,\text{mass}} \\ s_{L\,\text{mass}} \end{pmatrix} \\
= \; & \ldots \textstyle\sum_{i=u,c\; j=d,s} \bar{u}^i_{L\,\text{mass}} W^\mu \gamma_\mu V_{CKM\,ij} d^j_{L\,\text{mass}}
\end{aligned}
\tag{1.50}
$$

where the CKM matrix

$$
\begin{aligned}
V_{CKM} & \equiv U_L^{\dagger} D_L \\
& \neq 1
\end{aligned}
\tag{1.51}
$$

since the transformations in the up and down sectors are, in general, not related. Hence, the charged currents do convert up-type quark of one generation to a down-type quark of a *different* generation. So, we can use the charged current interactions more than once, i.e., in loop diagrams, to change one down-type quark to another down-type quark, for example, to obtain a $\Delta S = 2$ process via a box diagram.

Naively, we can estimate the size of this box diagram

$$\begin{aligned}\mathcal{M}_{SM} &\sim g_2^4 \int \frac{d^4k}{(2\pi)^4} V^*_{CKM\ is} V^*_{CKM\ js} V_{CKM\ id} V_{CKM\ jd} \frac{1}{\not{k} - m_i} \frac{1}{\not{k} - m_j} \frac{1}{k^2 - M_W^2} \\ &\sim g_2^4 \left(V_{us} V_{cd} \right)^2 \frac{1}{16\pi^2 M_W^2} \end{aligned} \tag{1.52}$$

(neglecting $m_{i,j}$ in the up quark propagators: more on this assumption below) which turns out to be too large compared to the experimental value!

However, this is where the Glashow-Iliopoulos-Maiani (or GIM) mechanism comes in. Using the unitarity of the CKM matrix,

$$\sum_i V^\dagger_{si} V_{id} = 0, \tag{1.53}$$

we find that $\mathcal{M}_{SM}$ vanishes if $m_i = m_j$, in particular if we neglect the quark masses as we did above. Hence, the amplitude must be proportional to the non-degeneracy of the up-type quark masses, i.e., for the two generation case we find that

$$\mathcal{M}_{SM} \sim \frac{g_2^4}{16\pi^2} \left(V_{us} V_{cd} \right)^2 \frac{m_c^2 - m_u^2}{M_W^4} \tag{1.54}$$

which was used earlier in Eq. (1.43). The point is that we get an extra suppression of $\sim m_c^2 / M_W^2 \sim 10^{-4}$ compared to the naive estimate in 2nd line of Eq. (1.52).

1.4. Lecture 3

As we saw in the previous lecture, the extra dimensional model which addresses the flavor hierarchy does *not* have analog of the GIM suppression in the gauge KK contribution to flavor violation. The reason is that the couplings of the strange and down quarks to the gauge KK modes, denoted by $a_{s,d}$ (in units of g_4), are $O(1)$, and different.

In order to solve this problem, we would like to modify the gauge KK profile, for example, a more favorable picture would be as in Fig. 1.8, where gauge KK modes are localized near the $y = \pi R$ brane whereas light fermions are localized near the $y = 0$ brane as usual. The point is that in this case couplings of fermions to the gauge KK modes (even though still non-universal) are $\ll 1$ (in units of g_4) so that the FCNC's are suppressed. So, the question is how to modify KK decomposition in general and, in particular, how to obtain the profiles as in Fig. 1.8.

1.4.1. *Brane Kinetic Terms*

We consider a modification to the extra dimensional model by adding interactions for the $5D$ gauge fields which are localized at the fixed points (branes). The point is that such interactions are allowed for an orbifold, but not on a circle, where there are no such "special" points in the extra dimension. In fact, consistency of the model at the quantum level requires the presence of such terms since such terms are generated by loops even if they are absent at tree-level [9].

Specifically, we study the Lagrangian:

$$\begin{aligned}\mathcal{L}_{5D} = &-\frac{1}{4}\Big[\mathcal{F}_{MN}\mathcal{F}^{MN} + \delta(y) r \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\Big] + \\ &\bar{\Psi}\left(\partial_M + g_5 \mathcal{A}_M\right)\Gamma^M \Psi \end{aligned} \tag{1.55}$$

Simple dimensional analysis gives $\Big[\mathcal{A}_M\Big] = 3/2$, $\big[\Psi\big] = 2$, $\Big[g_5\Big] = -1/2$ (here $\Big[...\Big]$ denotes mass dimension) so that the brane kinetic term has mass dimension -1 (i.e., it has dimension of a length) and is therefore denoted by r.

It is sometimes convenient to use a different normalization for $\mathcal{A}_M$: $\mathcal{A}_M \rightarrow \hat{\mathcal{A}}_M / g_5$ in terms of which the action is:

$$\begin{aligned}\mathcal{L}_{5D} = &-\frac{1}{4}\Big[\frac{1}{g_5^2}\hat{\mathcal{F}}_{MN}\hat{\mathcal{F}}^{MN} + \delta(y)\frac{r}{g_5^2}\hat{\mathcal{F}}_{\mu\nu}\hat{\mathcal{F}}^{\mu\nu}\Big] + \\ &\bar{\Psi}\left(\partial_M + \hat{\mathcal{A}}_M\right)\Gamma^M \Psi \end{aligned} \tag{1.56}$$

With this normalization, we have $\Big[\hat{\mathcal{A}}_M\Big] = 1$ (as in $4D$) so that the brane kinetic term is dimensionless: we can then define a brane-localized "coupling" as $1/g^2_{\text{brane}} \equiv r/g_5^2$.

We will now study how the KK decomposition is modified in the presence of these brane kinetic terms. Consider the case of a scalar field for simplicity (the gauge case which we are really interested in is similar). Here, we will only give a summary: for details, see exercise 2 and reference [10] for example.

Following the procedure outlined in lecture 2, we find that the orthonormality condition is modified (relative to the case of no brane terms):

$$\int dy f_n^*(y) f_m(y) \Big[1 + r\delta(y) \Big] = \delta_{mn} \tag{1.57}$$

and the profiles and mass eigenvalues are given by solving the differential equation:

$$\Big[\partial_y^2 + m_n^2 + r\delta(y) m_n^2 \Big] f_n(y) = 0 \tag{1.58}$$

The solutions $f_n(y)$ of this equation are linear combination of sin and cos, in particular, a *different* one for $y = 0$ to $y = \pi R$ and $y = -\pi R$ to $y = 0$.

In addition, in order to solve for the coefficients of sin, cos in these linear combinations, we must impose conditions such as continuity of $f_n(y)$ at $y = 0$, periodicity of $f_n(y)$ and matching the discontinuity in derivative of $f_n(y)$ to $\delta(y)$ in Eq. (1.57).

1.4.2. *Couplings of gauge modes*

It turns out that the zero-mode of the gauge field continues to have a flat profile: only its normalization affected by brane term such that

$$g_4 = \frac{g_5}{\sqrt{r + 2\pi R}} \tag{1.59}$$

For large brane kinetic terms,

$$g_4 \approx \frac{g_5}{\sqrt{r}} \tag{1.60}$$

Let us now consider couplings of gauge *KK* modes to particles localized on the branes in the limit of large brane terms. We find that

(i) the coupling of gauge KK mode to a particle (say light SM fermion) localized at $y = 0$ is *suppressed* (compared to zero-mode): $g_5 \times f_n(0) \sim g_4 / \sqrt{r/R}$.
(ii) Whereas, the coupling to particles (such as the Higgs) localized at $y = \pi R$ is *enhanced* compared to the zero-mode (or SM) gauge coupling : $g_5 \times f_n(\pi R) \sim g_4 \times \sqrt{r/R}$

The intuitive understanding is that large brane kinetic terms "repel" gauge KK mode from that brane (see Fig. 1.8).

1.4.3. *Solution to Flavor* Problem

In reality, the light SM fermions are not exactly localized at the $y = 0$ brane, but we find a similar suppression in their coupling to gauge KK mode for the actual profiles of the light fermions which are exponentials *peaked* at $y = 0$. Hence, based on the rough size of the coupling mentioned in point (i) above, we can show that FCNC's from exchange of gauge KK modes are suppressed by a factor of r/R relative to the case of without brane kinetic terms, i.e., large brane kinetic terms provide an *analog* of GIM suppression in the SM.

One might wonder if we are introducing a new hierarchy since we need $r/R \gg 1$. However, that's not really the case since a mild hierarchy of $O(10)$ is enough. In fact, we will see in lecture 5 how we can *effectively* obtain the same effect as that of such large brane kinetic terms in a *warped* extra dimension without introducing *any* brane terms and therefore any hierarchy in the $5D$ theory at all.

1.4.4. *Electroweak Precision Tests*

Having seen how to suppress contributions of the gauge KK modes to FCNC's, we will now consider their contributions to flavor-*preserving* observables called electroweak precision tests (EWPT). There are 3 such effects which we discuss in turn.

1.4.4.1. 4-*fermion operators*

Tree-level exchange of gauge KK modes also generates flavor-preserving 4-fermion operators, Fig. 1.9. We can compare these effects to SM (i.e., zero-mode) Z exchange which has coefficient $\sim g_Z^2/m_Z^2$ and use the fact that the experimental data on these operators agrees with the SM prediction at the $\sim 0.1\%$ level. For $r = 0$ (no brane term), we found that gauge KK coupling $\approx \sqrt{2} g_4$ for fermions localized at $y = 0$ (recall that light fermions are localized near $y = 0$) so that we obtain a limit of $m_{KK} \gtrsim$ a few TeV. However, for large brane kinetic terms, the gauge KK couplings and hence the coefficients of these operators are further suppressed by a factor of $\sim r/R$ so that $m_{KK} \sim$ TeV is *easily* allowed by the data.

The other 2 effects originate from the mixing of zero and KK modes for W, Z via the Higgs vev which we now discuss. The gauge group in the bulk is $SU(2)_L \times U(1)_Y$. We first perform the KK decomposition (i.e., obtain zero and KK modes) for $W_{i=1,2,3}$ and B (hypercharge) setting $v = 0$. At this level, there is no kinetic or mass mixing between these modes.

Next, we turn on the Higgs vev. For $v \neq 0$, we obtain masses for zero-modes of B and W_i and mass *mixing* between W_3 and B zero-modes (as in the SM). We define photon and Z zero-modes, $Z_\mu^{(0)}$ and $A_\mu^{(0)}$, to be combinations of $W_3^{(0)}$ and $B^{(0)}$ such that the *zero*-mode mass mixing is diagonalized (as in the SM). We first define the zero-mode gauge couplings (we neglect the brane terms for simplicity here, but it is straightforward to include them): $g_{W^{(0)}} = g_{5\ 2}/\sqrt{2\pi R}$, $g_{Z^{(0)}} = g_{5\ Z}/\sqrt{2\pi R}$, where $(g_{5\ Z}^2 = g_{5\ 2}^2 + g_5^{\prime\,2})$. The weak mixing angle between $W_3^{(0)}$ and $B^{(0)}$, i.e., $\sin^2\theta_W$ is the ratio of these zero-mode gauge couplings.

It turns out to be convenient to define the KK modes, $Z^{(n)}$ and $A^{(n)}$ ($n \neq 0$), using *same* (0-mode) mixing angles. The reason is that with this definition, the KK photon modes $A_\mu^{(n)}$ do not couple to Higgs (just like zero-mode) and hence decouple from the other modes.

However, the crucial point is that the $W^\pm$ zero mode mixes with the *KK* modes of $W^\pm$ via mass terms coming from the Higgs vev localized at $y = \pi R$ (similarly for Z). Therefore, the mass eigenstates, i.e., *SM* $W^\pm$ and Z, are *admixtures* of zero and KK modes. To understand this effect, we can diagonalize the 2×2 mass matrix (for zero and 1st KK mode) for simplicity (see exercise 3).

1.4.4.2. *Shift in coupling of SM fermions to Z*

The above zero-KK mode mixing for W, Z induced by Higgs vev results in a shift in the coupling of SM W, Z to a fermion localized at $y = 0$ from the pure zero-mode coupling, i.e., SM Z has a (small) KK Z component so that $g_Z = g_{Z^{(0)}} + \delta g_Z$. We can estimate this effect via mass insertion diagrams as in Fig. 1.10 which are valid for $v \times$ couplings $\ll m_{KK}$ to find $\delta g_Z / g_{Z^{(0)}} \sim g_{Z^{(0)}}^2 v^2 / m_{KK}^2$: see exercise 3 for a more accurate calculation. Note that there is *no* enhancement in δg_Z for large brane kinetic terms ($r/R \gg 1$). The point is that the enhancement in the coupling (relative to the zero-mode coupling) at the Higgs-KK Z vertex cancels the suppression in the coupling at the fermion-KK Z vertex (cf. the effect on the W, Z masses below). Just like the case of 4-fermion operators, the measured couplings of SM fermions to Z agree with the SM prediction at the $\sim 0.1\%$ level so that we obtain a limit of $m_{KK} \gtrsim$ a few TeV.

1.4.4.3. *Shift in ratio of W and Z masses or ρ parameter*

The mixing of zero and KK W modes induced by the Higgs vev also results in a shift in SM W mass from the pure zero-mode mass (a similar effect also happens for SM Z) as in Fig. 1.11:

$$M_W^2 = M_{W^{(0)}}^2 + \delta M_W^2, \text{ where} \tag{1.61}$$

$$M_{W^{(0)}}^2 = \frac{1}{4} g_{W^{(0)}}^2 v^2 \tag{1.62}$$

$$\delta M_W^2 \sim g_{W^{(0)}}^4 \frac{v^4}{m_{KK}^2} \frac{r}{R} \tag{1.63}$$

This effect, in turn, shifts the ρ parameter defined as

$$\rho = \frac{M_W^2}{M_Z^2} \times \frac{g_Z^2}{g_2^2} \tag{1.64}$$

The point is that $\rho = 1$ in the SM (at the tree-level) and $\Delta\rho_{expt.} \equiv \rho_{expt.} - 1 \sim 10^{-3}$. Actually, there is a subtlety in this definition for the $5D$ model due to the fact that the couplings of the Z boson to the SM fermions are also modified from the pure zero-mode Z coupling: $g_Z = g_{Z^{(0)}} + \delta g_Z$. However, as we discussed earlier, $\delta g_{Z,\,W}$ are not enhanced by $r/R \gg 1$ so that we can set $g_Z \approx g_{Z^{(0)}}$ in $\Delta\rho$ to find

$$\delta\rho \equiv \rho - 1 \sim \left(g_{Z^{(0)}}^2 - g_{W^{(0)}}^2 \right) \frac{v^2}{m_{KK}^2} \times \frac{r}{R} \tag{1.65}$$

The crucial point is that $\Delta\rho$ is *enhanced* by the presence of large brane kinetic terms such that we must require $m_{KK} \gtrsim 10$ TeV for $r/R \sim 10$ (as needed to solve the flavor problem).

1.5. Lecture 4

In this lecture, we will show how to solve the problem of large corrections to the ρ parameter discussed in lecture 3. For this purpose, we have to introduce a "custodial isospin" symmetry in the extra dimension. We will then discuss some signals of this extra dimensional scenario.

1.5.1. *Custodial Isospin in SM*

We will first review why $\rho = 1$ in the SM at the tree-level. The starting point is that the Higgs potential, $V(|H|)$ in the SM with the *complex* doublet Higgs written as

$$H = (h_1, h_2, h_3, h_4) \tag{1.66}$$

has a global $SO(4)$ symmetry (corresponding to rotations among the 4 *real* fields, h_i). Moreover, $SO(4)$ is isomorphic to $SU(2) \times SU(2)$ – one of these $SU(2)$'s in fact corresponds to the usual gauged $SU(2)_L$ group and the other one is usually denoted by $SU(2)_R$. The crucial point is that the global symmetry of the Higgs potential is enhanced compared to the gauged $SU(2)_L$ symmetry. The Higgs vev:

$$\langle H \rangle = (0, 0, 0, v) \tag{1.67}$$

breaks the global $SO(4)$ symmetry of the Higgs sector (in isolation) to $SO(3)$ – the gauged $SU(2)_L$ symmetry is broken in this process so that the W_i^L gauge bosons acquire masses. The unbroken $SO(3)$ symmetry (which is global) is isomorphic to an $SU(2)$ – clearly this unbroken $SU(2)$ is the *diagonal* subgroup of the 2 original $SU(2)$'s and is often called *custodial isospin*. It is this remnant symmetry which enforces equal masses for $W^L_{i=1,2,3}$.

Of course, W_3^L only mixes with B (there is no mixing for $W_L^{\pm}$). This mixing results in the neutral mass, $M_Z^2 = 1/4 \; v^2 \left(g_2^2 + g'^{\,2} \right)$, not being equal to the charged mass, $M_W^2 = 1/4 \, v^2 g_2^2$. That is the reason why there is a factor of g_Z^2/g_2^2 in the definition $\rho = M_W^2/M_Z^2 \; g_Z^2/g_2^2$: this factor takes the "violation of custodial symmetry" due to the gauging of hypercharge into account.

1.5.2. *Custodial Isospin* Violation *in* $5D$

Based on the above discussion, the sizable $\Delta\rho$ in the $5D$ model signals violation of custodial isospin symmetry somewhere in the $5D$ theory. First we begin with identifying the precise origin of custodial isospin *violation* and then we will come up with a solution to this problem. As we saw in lecture 3, $\Delta\rho$ from gauge KK modes $\propto \left(g^2_{Z^{(0)}} - g^2_{W^{(0)}} \right) \sim g^2_{B^{(0)}}$ just as in the SM. So, the origin of large $\Delta\rho$ or custodial isospin violation seems to be similar to that in the SM, i.e., it is due to gauging of hypercharge and the resulting mixing of W_3 with B. However, the point is that there are *additional* mixing effects (compared to the SM) in the $5D$ model due to the presence of KK modes (the mixing of zero-modes amongst each other is same as in the SM). In particular, $W^{(0)}_{L\,3} - B^{(n)}$ mixing occurs only in neutral sector and has no charged counterpart, whereas $W_L^{(0)} - W_L^{(n)}$ mixing is symmetric between charged and neutral sectors.

The origin of this dichotomy between charged and neutral sectors is the fact that the symmetry gauged in $5D$ is same as in the SM, i.e., $SU(2)_L \times U(1)_Y$, so that we have KK modes only for $W_L^{3,\,\pm}$ and B: there are no no charged partners for the B KK modes. This new effect (the custodial isospin violation due to B KK modes) is not taken into account by the factor of g_Z^2/g_2^2 in the definition of ρ – the point is that this factor only accounts for the mixing only amongst *zero*-modes, i.e., the $W_{L\,3}^{(0)} - B^{(0)}$ mixing. To repeat, $W_{L\,3}^{(0)} - W_{L\,3}^{(n)}$ mixing *does* have a counterpart in the charged sector. Moreover, $W_{L\,3}^{(0)} - B^{(n)}$ mass term $\sim g_{W^{(0)}} g_5' \times f_n(\pi R) v^2 \sim g_{W^{(0)}} g_{B^{(0)}} v^2 \sqrt{r/R}$ so that this effect is *enhanced* for large brane terms!

1.5.3. *Custodial Isospin* Symmetry *in* 5*D*

It is clear that we need *extra* charged KK modes to partner $B^{(n)}$ if we wish to suppress $\Delta\rho$. We can achieve this goal by promoting the hypercharge gauge boson to be a triplet. Hence, we can restore custodial isospin symmetry in the $5D$ model by enlarging the $5D$ *gauge* symmetry to $SU(2)_L \times SU(2)_R$ [11]. It turns out that we need something like $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ to obtain the correct fermion hypercharges as follows. Hypercharge is identified with a subgroup of $U(1)_R$ and $U(1)_{B-L}$: $Y = T_{3R} + (B-L)/2$, with $T_{3R} = \pm 1/2$ for $(u,d)_R$ and $(\nu,e)_R$ and $B-L = 1/3, -1$ for q, l (it is easy to check that this reproduces the SM hypercharges). Note that we still have extra neutral KK modes from $U(1)_{B-L}$ (which have no charged counterpart), but these KK modes do not couple to Higgs since the Higgs has $B-L$ charge of zero: only KK $W_{L,R}^{3,\pm}$ couple to Higgs such that the KK exchanges which give the shifts in masses respect custodial isospin (i.e., they are the same in the charged and the neutral channels).

Of course, we must break $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$, i.e., we must require that there are no zero-modes for $W_R^\pm$ and the extra $U(1)$ which is the combination of $U(1)_R$ and $U(1)_{B-L}$ orthogonal to $U(1)_Y$. However, this breaking must (approximately) preserve degeneracy for (at least the lighter) $W_R^\pm$ and W_R^3 modes such that $\Delta\rho$ continues to be (at least approximately) protected. It is clear that for this purpose we require degeneracy in both the mass of these modes and their coupling to the Higgs. This might seem to be challenging at first, but note that, for large brane kinetic terms ($r/R \gg 1$), KK modes are localized near $y = \pi R$. Therefore, if we break custodial isospin on the $y = 0$ brane, then the degeneracy between W_R^3 and $W_R^\pm$ is not significantly affected by this breaking. Specifically, we write down a *large* mass term for $W_R^\pm$ and the extra $U(1)$ at $y = 0$ which can originate from a localized scalar vev (different from the SM Higgs). We can show that this is equivalent to requiring vanishing of these gauge fields at $y = 0$ (odd or Dirichlet boundary condition: section 3.3 of reference [2]). This illustrates the general idea that breaking a $5D$ gauge symmetry by a large mass term localized on a brane is equivalent to breaking by boundary condition.

1.5.4. *Signals*

Let us consider some of the signals of this extra-dimensional set-up. A quick glance at Fig. 1.8 tells us that the coupling of gauge KK modes to top quark is enhanced compared to the SM couplings, whereas the couplings to the light SM fermions are suppressed (all based on the profiles for these modes).

We begin with real production of gauge KK modes, for example, the KK gluon. Due to the $\sim$ TeV mass for these particles, it is clear that we have to consider such a process at the Large Hadron Collider (LHC). Based on the above couplings, we typically find a broad resonance decaying into top pairs making it a challenge to distinguish the signal from SM background. It turns out that due to a constraint from a shift in the $Z \to \bar{b}b$ coupling[f], we cannot localize b_L and hence its partner t_L too close to the Higgs brane, forcing us to localize t_R near the Higgs brane in order to obtain the large top mass. Hence the KK gluon dominantly decays to RH top quark. We can use this fact (and noting that the SM $t\bar{t}$ production is approximately same for LH and RH top quarks) for the purpose of signal versus background discrimination [12]. It is easy to distinguish this signal for the extra dimension from SUSY: there is no missing energy (at least in this process) and top quark is treated as "special" in the sense that it has a larger coupling (than the other SM fermions) to the new particles, namely KK modes, unlike in SUSY.

We can also consider *virtual* exchange of gauge KK modes.

(i) In analogy with the shift in the coupling of SM fermion to the Z that we considered earlier, we see that $\bar{t}tZ$ is shifted compared to the SM prediction (or compared to $\bar{u}uZ$ and $\bar{c}cZ$) since top quark (up quark) is localized near $y = \pi R$ ($y = 0$) brane. Such an effect can be easily measured at the International Linear Collider (ILC) [13].

(ii) From the above discussion, it is clear that the couplings of the top

[f] This shift in the coupling originates from diagrams similar to the ones we considered earlier for the shift in coupling of SM fermion to the Z: see Fig. 1.10. Such shifts are enhanced if SM fermion is localized near $y = \pi R$ brane, where gauge KK mode is peaked.

and charm quarks to the KK Z are diagonal, but not universal in the weak or interaction basis. Once we rotate to the mass basis, there is a flavor violating coupling to KK Z to the top and the charm quark. In turn, this effect induces a flavor violating coupling of the *SM* Z to the top and charm quarks (via mixing of KK and zero-mode Z), resulting in a flavor violating decay of the top quark: $t \to cZ$. Such decays can be probed at the LHC [14].

1.5.5. *Summary of Model and Unanswered Questions*

So, far we have considered a model with the SM gauge and fermion propagating in the bulk of a *flat* extra dimension, with the Higgs localized on or near one of the branes. The other SM particles (gauge bosons and fermions) are identified with zero-modes of the corresponding $5D$ fields.

We have seen that a solution to the flavor hierarchy of the SM is possible using profiles for the SM fermions (again, these are the zero-modes of the $5D$ fields) in the extra dimension; in particular, top and bottom quarks can be localized near the Higgs brane, whereas the 1st and 2nd generation (or light) fermions can be localized near the other brane. Moreover, the resulting flavor problem due to non-universal couplings of gauge KK modes to the SM fermions (for a few TeV KK scale) can be ameliorated with large brane kinetic terms for $5D$ gauge fields on *non*-Higgs brane (i.e., where the light fermions are localized).

We also studied constraints from electroweak precision tests on this set-up and found that these constraints can also be satisfied for $m_{KK} \sim$ TeV, provided there is a custodial isospin symmetry in the bulk to protect the observable related to the ratio of W/Z masses (the ρ parameter).

This set-up still leaves some questions unanswered:

(i). We have assumed so far that $m_{KK} \sim$ TeV, but why is it $\ll M_{Pl.}$?

(ii). Is there a mild hierarchy problem associated with having large brane kinetic terms? Moreover, it seems a bit arbitrary that such terms appear only at $y = 0$ brane (where light SM fermions are localized) and not at $y = \pi R$.

We will see in the next lecture that both these questions can be answered by using a *warped* geometry (instead of flat extra dimension).

Furthermore,

(iii). Why does Higgs have a negative (mass)^2 or why does electroweak symmetry breaking (EWSB) occur? What sets this mass scale?

Specifically, can the hierarchy $m_H \ll M_{Pl}$ be due to some dynamics giving $m_H \sim m_{KK}$ which, in turn, is $\sim$ TeV?

(iv). Why is the Higgs localized on or near one of the branes?

These questions will be answered by a *combination* of Higgs being A_5, i.e., the 5th component of bulk gauge field and warped geometry.

1.6. Lecture 5

In this lecture, we will be brief: for details and a more complete set of references, see the excellent set of lectures by Sundrum [3].

1.6.1. *Warped Extra Dimension (RS1)*

We begin with a review of the original Randall-Sundrum model (RS1) [15]: see Fig. 1.12. It consists of an extra-dimensional interval ($y = 0$ to πR as before), but with the gravitational action containing a bulk cosmological constant (CC) and brane tensions (localized or $4D$ CC's):

$$\begin{aligned} S_{5D} &= \int d^4x dy \sqrt{-\det G} \left(M_5^3 \mathcal{R}_5 - \Lambda \right) \\ S_{brane\ 1,\ 2} &= \int d^4x \sqrt{-\det g_{1,\ 2}} T_{1,\ 2} \end{aligned} \tag{1.68}$$

with $g_{\mu\nu\ 1}(x) = G_{\mu\nu}(x, y = 0)$ and $g_{\mu\nu\ 2}(x) = G_{\mu\nu}(x, y = \pi R)$, where $g_{\mu\nu}$'s are the induced metrics on the branes and G_{MN} is the bulk metric. Also, M_5 is the $5D$ Planck scale and $\mathcal{R}_5$ is the $5D$ Ricci scalar.

With the following two fine-tunings:

$$T_1 = -T_2 = 24 k M_5^3, \tag{1.69}$$

where the (curvature) scale, k is defined using $\Lambda = 24 k^2 M_5^3$, we obtain a flat (or Minkowski), but y-dependent $4D$ metric as a solution of the $5D$ Einstein's equations:

$$(ds)^2 = e^{-2ky} \eta_{\mu\nu} (dx)^\mu (dx)^\nu + (dy)^2 \tag{1.70}$$

Thus, the geometry is that of a slice of anti-de Sitter space in $5D$ (AdS$_5$). The y-dependent coefficient of the $4D$ metric, i.e., e^{-ky} is called the "warp factor".

$4D$ **gravity**: The $4D$ graviton (which is the zero-mode of the $5D$ gravitational) corresponds to fluctuations around the flat spacetime background, i.e., $g^{(0)}_{\mu\nu}(x) \approx \eta_{\mu\nu} + h^{(0)}_{\mu\nu}(x)$. As usual, we plug this fluctuation into the $5D$

action and integrate over the extra dimensional coordinate to find an effective $4D$ action for $g^{(0)}_{\mu\nu}(x)$:

$$S_{4D} = \frac{M_5^3}{k}\left(1 - e^{-2k\pi R}\right)\int d^4x\sqrt{-\det g^{(0)}}\mathcal{R}_4[g^{(0)}] \tag{1.71}$$

from which we can deduce the $4D$ Planck scale:

$$\begin{aligned} M_{Pl}^2 &= \frac{M_5^3}{k}\left(1 - e^{-2k\pi R}\right) \\ &\approx \frac{M_5^3}{k} \text{ for } kR \gg 1 \end{aligned} \tag{1.72}$$

We choose $k \lesssim M_5$ so that the higher curvature terms in the $5D$ action are small and hence can be neglected. Thus, we get the following order of magnitudes for the various mass scales:

$$k \lesssim M_5 \lesssim M_{Pl} \sim 10^{18} \text{ GeV} \tag{1.73}$$

It turns out that the $4D$ graviton is (automatically) localized near $y = 0$ (which is hence called the Planck or UV brane) - that is why the $4D$ Planck scale is finite even if we go to the decompactified limit of $R \to \infty$ in Eq. (1.72). Specifically, its profile is $\sim e^{-2ky}$.

1.6.2. *Solution to Planck-Weak Hierarchy*

The motivation for the RS1 model is to solve the Planck-weak hierarchy problem. Let us now see how this model achieves it. Assume that a $4D$ Higgs field is localized on the $y = \pi R$ brane which is hence called the TeV or IR brane:

$$\begin{aligned} S_{\text{Higgs}} = \int d^4x\sqrt{-\det g_2}\Big[& g^{\mu\nu}_{\text{ind.}}\ \partial_\mu H \partial_\nu H - \\ & \lambda\left(|H|^2 - v_0^2\right)^2\Big] \end{aligned} \tag{1.74}$$

where the natural size for v_0 is the $5D$ gravity or fundamental scale (M_5). Using the metric induced on the TeV brane, $g_{\mu\nu\,2} = G_{\mu\nu}(y = \pi R) = g^{(0)}_{\mu\nu}e^{-2k\pi R}$, the action for the Higgs field becomes

$$\begin{aligned} S_{\text{Higgs}} = \int d^4x\sqrt{-\det g^{(0)}}\Big[& e^{-2k\pi R}g^{(0)\ \mu\nu}\partial_\mu H \partial_\nu H - \\ & e^{-4k\pi R}\lambda\left(|H|^2 - v_0^2\right)^2\Big] \end{aligned} \tag{1.75}$$

Now comes the crucial point: we must rescale the Higgs field to go to canonical normalization, $H \equiv \hat{H} e^{k\pi R}$, which results in

$$S_{\text{Higgs}} = \int d^4x \sqrt{\det g^{(0)}} \Big[g^{(0)\;\mu\nu} \partial_\mu \hat{H} \partial_\nu \hat{H} - \lambda \left(|\hat{H}|^2 - v_0^2 e^{-2k\pi R} \right)^2 \Big] \tag{1.76}$$

Note that the Higgs mass is "warped-down" to $\sim$ TeV from the $5D$ (or the $4D$) Planck scale if we have the following *modest* hierarchy between the radius (or the proper distance) of the extra dimension and the AdS curvature scale.

$$\begin{aligned} k\pi R &\sim \log\left(M_{Pl} / \text{TeV} \right) \\ &\sim 30 \text{ or} \\ R &\sim \frac{10}{k} \end{aligned} \tag{1.77}$$

Moreover, the quartic coupling is unchanged and hence the Higgs vev (or weak scale) is also at the TeV scale, assuming $\lambda \sim O(1)$.

Note that the radius of the extra dimension is not a fundamental or $5D$ parameter, rather it is determined by the dynamics of the theory. Hence, in order to have complete solution to the hierarchy problem (without any hidden fine-tuning), we must show that the radius can be stabilized at the required size without further (large) fine-tuning of parameters of the $5D$ theory. In fact, stabilization of such a radius can be achieved using a bulk scalar (Goldberger-Wise mechanism) [16], provided we invoke a *mild* hierarchy $M^2/k^2 \sim O(1/10)$, where M is the $5D$ mass of the scalar.

Thus, we see that the Planck-weak hierarchy can be obtained from $O(10)$ hierarchy in the fundamental or $5D$ theory! In general, a large ("exponential") hierarchy for the $4D$ mass scales can be obtained from a small hierarchy in the $5D$ parameters.

The central feature of a warped extra dimension is that the *effective* $4D$ *mass scale depends on position* in the extra dimension. In order to have a more intuitive understanding of this feature, consider the position $y \sim y_0$ where the metric is:

$$(ds)^2_{y \sim y_0} \sim e^{-2ky_0} \eta_{\mu\nu} (dx)^\mu (dx)^\nu + (dy)^2 \tag{1.78}$$

In terms of the rescaled coordinate and mass scale: $\hat{x} \equiv e^{-ky_0} x$, $\hat{m}_{4D} \equiv e^{ky_0} m_{4D}$, we get

$$(ds)^2_{y \sim y_0} \sim \eta_{\mu\nu} (d\hat{x})^\mu (d\hat{x})^\nu + (dy)^2 \tag{1.79}$$

The advantage of the new coordinates $\hat{x}$ is that we have a "flat" metric in terms of it so that we expect $\hat{m}_{4D} \sim m_{5D}$ (such a relationship is valid in the absence of warping). Converting back to original mass scales, we find $m_{4D} \sim e^{-ky_0} m_{5D}$, i.e., $4D$ mass scales are warped compared to $5D$ mass scales. *An analogy with the expanding Universe is useful*: just as $3D$ space expands with time, in the warped extra dimension, the $4D$ space-*time* "expands" (or contracts) with motion along the 5^{th} dimension.

1.6.3. *Summary of RS1*

The preceding discussion leads us to the "master equation" for a warped extra dimension:

$$M_{4,\,\text{eff.}}(y) \sim M_5 \times e^{-ky}$$

relating the effective $4D$ mass scales on the left-hand side (LHS) of the above equation to the fundamental or $5D$ mass scale on the right-hand side (RHS) by the warp factor. Applying it to the case of the $4D$ graviton localized at $y \sim 0$, we get

$$M_{Pl} \sim M_5 \tag{1.80}$$

so that we must choose the $5D$ Planck scale to be

$$M_5 \sim 10^{18}\text{GeV} \tag{1.81}$$

Whereas, the Higgs sector is localized at $y \sim \pi R$ so that

$$M_{\text{weak}} \sim M_5 \times e^{-k\pi R} \tag{1.82}$$

so that

$$M_{\text{weak}} \sim \text{TeV} \tag{1.83}$$

provided we have a mild hierarchy

$$\begin{aligned} k\pi R &\sim \log\left(M_{Pl}/\text{TeV}\right) \\ &\sim 30 \end{aligned} \tag{1.84}$$

1.6.4. *Similarity with* Flat *TeV-Size Extra Dimension with Large Brane Terms*

In the original RS1, it was assumed that the entire SM, i.e., including fermion and gauge fields, is localized on the TeV brane. However, it was subsequently realized that, in oder to solve the Planck-weak hierarchy problem, only the SM Higgs boson has to be localized on or near the TeV brane

– the masses of non-Higgs fields, i.e., fermions and gauge bosons, are protected by gauge and chiral symmetries, respectively.

So, we are led to consider RS1 with the SM gauge [17] and fermion fields [18] propagating in the bulk (with the Higgs still being on or near the TeV brane). It turns out that the profiles for the SM fermions in the bulk can address the flavor hierarchy just as in the case of flat extra dimension. Moreover, solving the wave equation in curved spacetime, we can show [17–19] that all KK modes are localized near the IR brane (that too automatically, i.e., with*out* actual brane terms) and the KK masses are given by $m_{KK} \sim ke^{-k\pi R}$ and *not* $1/R$ [note that, based on Eqs. (1.73) and (1.77) $1/R$ is of the size of the $4D$ Planck scale!]. Hence, we find $m_{KK} \sim$ TeV given the choice of parameters to solve the Planck-weak hierarchy problem! A very rough intuition for localization of KK modes near the TeV brane is that the KK modes can minimize their mass by "living" near IR brane, where all mass scales are warped down. In this sense, the warped extra dimension "mimics" large brane kinetic terms of flat geometry – recall that the large brane kinetic terms in a flat extra dimension result in a similar localization of KK modes. In addition, the hierarchy $m_{KK} \ll M_{Pl.}$ is explained by the warped geometry. This addresses the 1st and 2nd questions outlined at the end of the previous lecture.

Because of this localization of KK modes near the TeV brane, we find that the solution to the flavor problem and the discussion of the electroweak precision tests (including custodial isospin) goes through (roughly) as in the case of a flat extra dimension.

1.6.5. Unification of Spins: Higgs as A_5

We now return to the other (3rd and 4th) questions asked at the end of the previous lecture, namely, what sets the scale of EWSB or Higgs mass and why is Higgs localized on the TeV brane?

We will show in this and the next subsection that obtaining the SM Higgs as the 5th component of $5D$ gauge field (or A_5) can resolve the 3rd question and then outline in the final subsection how combining the idea of Higgs as A_5 with the warped geometry answers the 4th question, resulting in a "complete" model.

As a warm-up for the idea of Higgs as A_5 (see the review [20] for references), consider an $SU(2)$ gauge theory in an extra dimension which is compactified on a circle (S^1). As we saw earlier, for $n \neq 0$, the $A_\mu^{(n)}$ modes "eat" $A_5^{(n)}$ modes to form massive spin-1 states. Moreover, there is a (massless) zero-mode A_5, which is in adjoint representation of $SU(2)$, i.e., it is charged under the $SU(2)$ gauge symmetry. We can introduce a $SU(2)$ doublet fermion in the bulk which will acquire a Yukawa coupling $\sim g$ to the A_5 zero-mode from the interaction $\bar{\Psi}_L A_5 \Psi_R$ coming from the $5D$ covariant derivative. Hence, this scenario is often called "Gauge-Yukawa unification".

Note that this scalar has no potential at the tree-level since it is part of a $5D$ gauge field. We will now discuss the potential for A_5 zero-mode induced by loop effects to find that it is *finite*. Naively, the scalar $(\text{mass})^2$ gets quadratically divergent loop corrections: $m^2_{A_5^{(0)}} \sim g_4^2 / \left(16\pi^2\right) \Lambda^2_{UV}$. However, from the $5D$ point of view, it is clear that $5D$ gauge invariance protects the A_5 scalar mass from receiving divergent loop corrections (there is no counter-term to absorb such divergences and so these must be absent). The reader is referred to the 1st reference in [3] for a detailed calculation of $m^2_{A_5^{(0)}}$ coming from a fermion loop for the simpler case of a $U(1)$ gauge field in the bulk. The summary is that loop contributions to $m^2_{A_5^{(0)}}$ are "cut-off" by R^{-1}:

$$m^2_{A_5^{(0)}} \sim \frac{g_4^2}{16\pi^2} R^{-2} \tag{1.85}$$

Intuitively, the understanding is that A_5 behaves as a "regular" scalar till $E \sim R^{-1}$: see Fig. 1.13(a). Beyond these energies, the quantum corrections "realize" that A_5 is part of a $5D$ gauge field. Therefore, the loop contributions from $E \gtrsim R^{-1}$ are highly suppressed, in particular, there is no divergence. Thinking in terms of KK modes, there is a cancellation in the loop diagram among the different modes. We can then ask: what did we gain relative to a "regular" scalar (which is not an A_5 zero-mode, but is localized on a brane or originates in a $5D$ scalar field)? To answer this question, we need to know what is Λ_{UV}, the scale which cuts of the divergence in the case of a regular scalar. The $4D$ SM (without gravity) is renormalizable so that the cut-off is the Planck scale (where quantum gravity becomes important). However, the $5D$ *gauge theory*, even without gravity, *is non-renormalizable* and therefore must be defined with a cut-off (which is not related to the Planck scale): see Fig. 1.13(b). The reason is that the $5D$ gauge coupling constant is dimensionful so that the $5D$ loop expansion grows with energy: $g_5^2 E / \left(16\pi^2\right)$. Since we cannot extrapolate the $5D$ gauge theory beyond the energy scale where the loop expansion parameter becomes ~ 1, we must introduce a cut-off at this scale:

$$\begin{aligned}\Lambda_{UV} &\sim \frac{16\pi^2}{g_5^2} \\ &\sim \frac{16\pi}{g_4^2} R^{-1}\end{aligned} \tag{1.86}$$

where we have set the brane terms to be small so that $g_4 \sim g_5/sqrtR$. Note that this cut-off is not much larger than the compactification scale since $g_4 \sim 1$ in the SM. Thus, we find that $m^2_{A_5^{(0)}}$ is suppressed relative to the mass^2 in the case of a regular scalar by $\sim (\Lambda_{UV} R)^2 \sim \left(16\pi/g_4^2\right)^2$: we *do* gain by going to A_5.

Next, we discuss how to use A_5 for *radiative symmetry breaking (often called Hosotani mechanism)* [21]. Continuing with the case of $SU(2)$ on S^1, we see that a vev for the A_5 zero-mode, $\langle A_5^{(0)} \rangle$ can break $SU(2)$ gauge symmetry to a $U(1)$ gauge symmetry. The point is that fermion loops typically give $m^2_{A_5^{(0)}} < 0$, whereas gauge loops are of opposite sign. However, the fermion contributions can win if the number of fermion degrees of freedom is larger than that of gauge bosons.

Thus, we have a "cartoon" of the SM in the following sense. We can identify the $SU(2)$ gauge group that we considered above with the SM W's. We will then get $M_{W^\pm} \sim R^{-1}$ (coming from $\langle A_5 \rangle$), whereas W_3 [corresponding to the unbroken $U(1)$ gauge symmetry] remains massless (it is the "photon"). Finally, the $\bar{\Psi}_L A_5 \Psi_R$ coupling mentioned above gives a fermion mass $M_{\psi^{(0)}} \sim R^{-1} \sim M_W$ which is roughly correct for top quark (since $m_t \sim M_W$).

Of course, this model is far from being realistic:

(i). We must require $1/R \gg 100$ GeV since we have not seen any KK modes in experiments so far which have probed energy scales up to $\sim$ TeV (either directly in the highest energy colliders or indirectly via virtual effects of new particles). To satisfy this constraint, we can fine-tune the fermion versus the gauge loop contributions to A_5 mass such that $M_{W^\pm}$ or $\langle A_5 \rangle \sim 100 \text{ GeV} \ll R^{-1}$.

(ii). More importantly, we do not have fermion chirality on a circle.

1.6.6. *Towards Realistic Higgs as A_5: Chirality and Enlarging the Gauge Group*

As we saw earlier, we can obtain chiral fermions by going to an orbifold: S^1/Z_2. However, if we require A_μ of $SU(2)$ to be even under Z_2 (such that we get a corresponding zero-mode, i.e., a massless $4D$ gauge boson), then the A_5's are necessarily odd. Thus, we lose the scalar zero-mode. In any case, the scalar was in the adjoint representation of $SU(2)$, whereas we need a doublet for EW symmetry breaking.

The trick is to enlarge the gauge group to $SU(3)$ and to break it down to $SU(2) \times U(1)$ by boundary condition as follows. Choose the following parities under Z_2 for the fundamental representation

$$\begin{pmatrix} 3 \end{pmatrix} \to P \begin{pmatrix} 3 \end{pmatrix}, \quad \text{where}$$

$$P = \begin{pmatrix} + & & \\ & + & \\ & & - \end{pmatrix} \tag{1.87}$$

Given this parity choice, can derive the transformation of any other representation under Z_2. For example, consider fields in the adjoint representation, Φ_a ($a = 1...8$), written as a 3×3 matrix, $\Phi_a T^a$, where T^a's are generators of the fundamental representation. This matrix transforms as

$$\begin{pmatrix} 8 \end{pmatrix} \to P^\dagger \begin{pmatrix} 8 \end{pmatrix} P \sim \begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix} \tag{1.88}$$

This implies that if the A_μ's belonging to $SU(2) \times U(1)$ are chosen to be even (and hence have a zero-mode), then the A_μ's of the coset group $SU(3)/\left[SU(2) \times U(1)\right]$ are odd (i.e., do not have a zero-mode). This choice of parities thus achieves the desired breaking pattern $SU(3) \to SU(2) \times U(1)$. Moreover, the A_5's of $SU(3)/\left[SU(2) \times U(1)\right]$ are even, giving us a scalar zero-mode which is a doublet of the unbroken $SU(2)$ group as desired.

Furthermore, just like in the case of the breaking $SU(2) \to U(1)$ discussed earlier, the breaking of $SU(2) \times U(1)$ can be achieved by vev of A_5 which is generated by loop corrections. Moreover, due to usage of fundamental representation for this radiative symmetry breaking, the rank of the gauge group is also broken, i.e., we have an unbroken $U(1)$ symmetry.

A $5D$ fermion which is a triplet of $SU(3)$ gives zero-modes for LH $SU(2)$ doublet *and* RH singlet:

$$\Psi_L = \begin{pmatrix} \Psi_L^D + \\ \Psi_L^S - \end{pmatrix}$$
$$\Psi_R = \begin{pmatrix} \Psi_R^D - \\ \Psi_R^S + \end{pmatrix} \tag{1.89}$$

where D and S denote $SU(2)$ doublet and singlet, respectively – recall that the parities of the RH and LH fields must be opposite. Moreover, the Yukawa coupling for the zero-mode fermions comes from the interaction $\bar{\Psi}_L^D A_5 \Psi_R^S$. Thus, we are getting closer to the SM!

1.6.7. *Realistic Higgs as A_5 in* Warped *Extra Dimension*

When we construct the previous model in a warped extra dimension, it turns out that the $A_5^{(0)}$ is automatically localized near the TeV brane [22] – recall that in order to solve the hierarchy problem, we would like the Higgs to be localized precisely there. Thus, A_5 zero-mode is an excellent candidate for SM Higgs!

As "finishing touches", we can add an extra $U(1)$ to obtain the correct hypercharges for the fermions and similarly a custodial isospin symmetry to protect the ρ parameter [23]. Also, it turns out that the Ψ_L^D and Ψ_R^S have (effectively) "opposite" sign of $5D$ mass, M (recall that this mass is not coming from $\langle A_5^{(0)} \rangle$) in the sense that if the LH zero-mode is localized near $y = 0$, then the RH zero-mode must be near $y = \pi R$ (or vice versa): see exercise 1. To relax this constraint, i.e., to obtain more freedom in localization of LH versus RH zero-modes, we can instead obtain LH and RH SM fermions as zero-modes of different bulk multiplets. However, then the question arises: since A_5 only couples fermions within the same fermionic multiplet, how do we obtain Yukawa couplings? The solution is to mix fermionic multiplets by adding mass terms localized at the endpoints of the extra dimension.

1.6.8. *Epilogue*

Due to lack of time, we have not considered other extra dimensional models with connections to the weak scale (and gravitational aspects of extra dimensional models in general). Here, we give a summary of the essential features of these other models: for details, see the references below and other lectures [1–4]. Arkani-Hamed, Dimopoulos and Dvali (ADD) proposed a scenario where only gravity propagates in extra dimensions, with all the SM fields localized on a brane [24]. The idea is that the fundamental or higher-dimensional gravity scale is $\sim$ TeV (and not the $4D$ Planck scale), while the weakness of gravity (or largeness of $4D$ or observed Planck scale) is accounted for by diluting the strength of gravity using extra dimensions which are much larger in size than the fundamental length scale, i.e., $R \gg 1/$ TeV. The crucial point is that the gravitational force law has been tested only for distances larger than $O\,(100)\,\mu m$ so that such very large extra dimensions could be consistent with current experiments. Only the graviton has KK modes in this framework, that too very light, resulting in interesting phenomenology both from real and virtual production of these KK modes. These KK modes couple with the usual $4D$ gravitational strength, but their large multiplicity can compensate for this very weak coupling.

At the other extreme is the model called Universal Extra Dimensions (UED) [25]. This scenario has a flat extra dimension(s) in which *all* the SM fields (including Higgs) propagate. The $5D$ fields have no brane localized interactions at the tree-level: of course, loops will generate small brane terms. Moreover, there are no $5D$ masses for fermions and Higgs so that profiles for all zero-modes (including all fermions, gauge fields and Higgs) are flat. Hence, we do not have a solution to the flavor hierarchy of the SM unlike in the scenario considered in these lectures. The motivation for UED is more phenomenological: there is a remnant of extra dimensional momentum or KK number conservation (dubbed KK parity) which forbids a coupling of a *single* lightest (level-1 and in general, odd level) KK mode to SM particles. Such a coupling *is* allowed for level-2 (and in general, even level) KK modes, but it is still suppressed by the small (loop-induced size) of brane kinetic terms.[g] Hence, the contributions from KK exchange to precision tests are suppressed (in particular, tree-level exchange of odd level modes is forbidden), *easily* allowing KK mass scale *below* a TeV for level-1 and even level-2 modes (cf. the lower limit of *a few* TeV in the scenario studied in these lectures). Thus, KK modes can be more easily produced at colliders (even though it is clear that the odd level KK modes have to be pair produced). Moreover, the lightest KK particle (LKP) is stable and can be a good dark matter candidate [26].

[g]This coupling does not preserve KK number conservation or extra dimensional translation invariance and hence must arise from interactions localized on the branes which violate these symmetries.

Finally, we mention the $5D$ Higgsless models [27], where EW symmetry itself is broken by boundary conditions like the breaking of $5D$ custodial isospin symmetry mentioned in lecture 4 [or the breaking $SU(3) \to SU(2) \times U(1)$ considered in lecture 5 in order to obtain Higgs as A_5]. The idea is that there is no light Higgs in the spectrum in order to unitarize WW scattering, which is instead accomplished by exchange of gauge KK modes. These KK modes then must have mass $\lesssim 1$ TeV. It turns out that due to such a low KK scale, the simplest such models are severely constrained by precision tests, but it is possible to avoid some of these constraints by suitable model-building.

Acknowledgments

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Appendix A. Excercises

A.1. Exercise 1

A.1.1. *Zero-Mode Fermion and 4D Yukawa Coupling*

Show that the normalized profile for LH zero-mode fermion (i.e., choosing Ψ_L to be even) is (lecture 2):

$$\begin{aligned} f_{L\,0}(y) &= \sqrt{\frac{M}{e^{2M\pi R}-1}} e^{My} \quad (0 \le y \le \pi R) \\ &= \sqrt{\frac{M}{e^{2M\pi R}-1}} e^{-My} \quad (0 \ge y \ge -\pi R) \end{aligned} \tag{A.1}$$

where the normalization is over $0 \le y \le 2\pi R$ (even though the physical domain is from $y = 0$ to $y = \pi R$). Similarly, if we choose Ψ_R to be even instead of Ψ_L, then the RH zero-mode profile is

$$\begin{aligned} f_{R\,0}(y) &= \sqrt{\frac{-M}{e^{-2M\pi R}-1}} e^{-My} \quad (0 \le y \le \pi R) \\ &= \sqrt{\frac{-M}{e^{-2M\pi R}-1}} e^{+My} \quad (0 \ge y \ge -\pi R) \end{aligned} \tag{A.2}$$

Note the opposite sign of M in the LH versus RH zero-mode profiles [following from Eqs. (1.20) and 1.21)]. Assuming that the SM Higgs field is localized at $y = \pi R$, we see that we need $M < 0$ (> 0) for LH (RH) fermion to obtain small fermion wavefunction at the location of the Higgs and hence small $4D$ Yukawa couplings for light fermions (1st and 2nd generations). So, we can neglect $e^{\pm M\pi R}$ compared to 1 wherever appropriate.

The zero-mode ($4D$ or SM) Yukawa coupling in terms of the $5D$ Yukawa coupling: $\int dy d^4x \delta(y) \lambda_{5D} H \Psi_L \Psi'_R$ [where Ψ_L is $SU(2)_L$ doublet and Ψ'_R is $SU(2)_L$ singlet] is:

$$\lambda_{4D} \approx \lambda_{5D} M e^{2M\pi R} \tag{A.3}$$

and the $4D$ mass of fermion is

$$m \approx \lambda_{4D} v, \tag{A.4}$$

where, for simplicity, we assume equal size of $5D$ masses, i.e., $M = -M'$, for doublet and singlet fermions.

A.1.2. *Coupling of Zero-mode Fermion to Gauge KK mode: No Brane Kinetic Terms*

The profile for n^{th} gauge KK mode ($m_n = n/R$) is:

$$f_n(y) = \frac{1}{\sqrt{\pi R}} \cos(m_n y) \tag{A.5}$$

Calculate the coupling of zero-mode fermion to gauge KK modes in terms of the coupling of zero-mode gauge field (i.e., SM gauge coupling), $g_4 \equiv g_5/\sqrt{2\pi R}$:

$$g(n, M) = g_4 a(n, M) \tag{A.6}$$

You should obtain:

$$a(n, M) \approx \sqrt{2} \frac{4M^2}{4M^2 + (n/R)^2} \tag{A.7}$$

Use $m_{d,s} = 1$ MeV, 100 MeV and the Higgs vev $v \approx 100$ GeV. Assume, for simplicity, that $\lambda_{5D} M = 1$ for both s, d – otherwise, we have to solve a transcendental equation to obtain M (given the $4D$ Yukawa coupling). Calculate the $5D$ masses $M_{s,d}$ and show that $a(1, M_s) - a(1, M_d) \approx 0.1$.

Compare $K - \bar{K}$ mixing from KK Z exchange as in lecture 2

$$\frac{g_Z^2}{m_{KK}^2} \Big[a(1, M_s) - a(1, M_d) \Big]^2 (\text{mixing angle})^2 \tag{A.8}$$

to the SM amplitude

$$\frac{g_2^4}{16\pi^2}\frac{m_c^2}{M_W^4}\,(\text{mixing angle})^2 \tag{A.9}$$

to obtain bound on m_{KK} of ≈ 20 TeV, using $g_Z \approx 0.75$ and $g_2 \approx 0.65$ for the SM Z and $SU(2)_L$ gauge couplings.

It turns out that another observable called ϵ_K (which is the imaginary or CP-violating part of the above $K-\bar{K}$ mixing amplitude) gives a stronger bound on KK mass scale of ~ 100 TeV.

A.2. Exercise 2

A.2.1. *General Brane Kinetic Terms*

The Lagrangian is

$$\mathcal{L}_{5D} \ni -\frac{1}{4}\mathcal{F}_{MN}\mathcal{F}^{MN} - \frac{1}{4}\delta(y) r \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} \tag{A.10}$$

where r has dimension of length.

Go through the derivation outlined in lecture 3, i.e., f_n satisfies the orthonormality condition:

$$\int dy f_n^*(y) f_m(y)\Big[1 + r\delta(y)\Big] = \delta_{mn} \tag{A.11}$$

and the differential equation:

$$\Big[\partial_y^2 + m_n^2\left(1 + r\delta(y)\right)\Big] f_n(y) = 0 \tag{A.12}$$

The solution is

$$\begin{aligned} f_n(y) &= a_n \cos(m_n y) + b_n \sin(m_n y) \text{ for } y \geq 0 \\ &= \tilde{a}_n \cos(m_n y) + \tilde{b}_n \sin(m_n y) \text{ for } y \leq 0 \end{aligned} \tag{A.13}$$

Use the following 4 conditions to obtain relations between coefficients a, b's and to solve for m_n: (i) continuity at $y = 0$, (ii) discontinuity in derivative matches brane term, (iii) f_n is even and (iv) periodicity of f_n. In particular, condition (iv) is satisfied by repeating (or copying) f_n between $-\pi R$ and πR to between πR and $3\pi R$ and so on. However, continuity of f_n at $y = \pi R$ has to be imposed and similarly that of derivative of f_n (assuming no brane kinetic term at $y = \pi R$).

You should find

$$\begin{aligned} a_n &= \tilde{a}_n \\ \frac{b_n}{a_n} &= -\frac{r m_n}{2} \\ b_n &= -\tilde{b}_n \\ \frac{b_n}{a_n} &= \tan\left(m_n \pi R\right) \end{aligned} \tag{A.14}$$

so that eigenvalues are solutions to

$$\tan\left(m_n \pi R\right) = -\frac{r m_n}{2} \tag{A.15}$$

Finally, calculate

$$\frac{1}{a_n^2} = \pi R \left(1 + \frac{1}{4} r^2 m_n^2 + \frac{r}{2\pi R}\right) \tag{A.16}$$

from normalization.

A.2.2. Large *Brane Kinetic Terms*

Verify approximate results shown in lecture 3 for large brane kinetic terms, $r/R \gg 1$, namely,

(i) $m_n \approx (n + 1/2)/R$,

(ii) $1/g_4^2 \approx r/g_5^2$

and for lightest KK modes (small n)

(iii) coupling of a fermion localized at $y = 0$ to gauge KK mode $\sim g_4/\sqrt{r/R}$

(iv) coupling of gauge KK mode to a fermion/Higgs field localized on $y = \pi R$ brane $\sim g_4\sqrt{r/R}$.

We can generalize these couplings of gauge KK mode to the case of a zero-mode fermion with a profile in the bulk – it's just that we have to do an overlap integral as in problem 2 of exercise 1. Calculate the new $a(1, M_s) - a(1, M_d)$. For $r/R \gg 1$, show that it is smaller than before (i.e., without brane terms) so that $K - \bar{K}$ mixing is suppressed and a lower KK mass scale is allowed.

A.3. Exercise 3

As discussed in lecture 3, the zero and KK modes of Z are defined by setting the Higgs vev to zero. However, due to non-zero Higgs vev, the zero and KK modes of Z mix via mass terms – kinetic terms are still diagonal. The $Z^{(0)}$-$Z^{(1)}$ (i.e., 1^{st} KK mode of Z) mass matrix is:

$$\mathcal{L}_{mass} \ni \left(Z_\mu^{(0)} Z_\mu^{(1)} \right) \begin{pmatrix} m^2 & \Delta m^2 \\ \Delta m^2 & M^2 \end{pmatrix} \begin{pmatrix} Z^{\mu\,(0)} \\ Z^{\mu\,(1)} \end{pmatrix} \tag{A.17}$$

where $m^2 = 1/4\; g^2_{Z^{(0)}} v^2$, mixing term $\Delta m^2 = 1/4\; g_{Z^{(0)}} g_{5\;Z} f_1\,(\pi R)\,v^2$ and $M^2 = m^2_{KK} + 1/4\; g^2_{5\;Z} f_1^2\,(\pi R)\,v^2$. Here, $f_1\,(\pi R)$ is wavefunction of $Z^{(1)}$ evaluated at the Higgs brane ($y = \pi R$). Also, $g_{Z^{(0)}} = g_{5\;Z}/\sqrt{2\pi R + r}$ denotes the coupling of $Z^{(0)}$ (where r is the brane kinetic term at $y = 0$) and $g_{5\;Z} = \sqrt{g^2_{5\;2} + g'^{\,2}_5}$ denotes the $5D$ coupling of Z, with $g_{5\;2}$ and g'_5 being the $5D$ gauge couplings of $SU(2)$ and $U(1)_Y$, respectively (assume, for simplicity, the same brane kinetic term r for all gauge fields).

Diagonalize this mass matrix, assuming $v^2/m^2_{KK} \times$gauge couplings $\ll 1$ where appropriate, i.e., determine

(i) the unitary transformation to go from $\left(Z^{(0)} Z^{(1)} \right)$ to physical basis and

(ii) the eigenvalues of the mass matrix.

There are 2 effects of this diagonalization.

A.3.1. *Shift in Coupling of a Fermion to Z*

Given couplings of a fermion to $Z^{(0)}$ and $Z^{(1)}$ (KK basis)

$$\mathcal{L}_{coupling} \ni \bar{\psi} \gamma^\mu (g, G) \begin{pmatrix} Z_\mu^{(0)} \\ Z_\mu^{(1)} \end{pmatrix} \psi \tag{A.18}$$

use the above unitary transformation to calculate the couplings to the fermion in the physical basis, denoted by Z_{light} (which is SM Z) and Z_{heavy}.

Specifically, calculate the coupling of a fermion localized at $y = 0$ to the SM Z using $g = g_{Z^{(0)}}$ and $G = g_{5\;Z} f_1\,(0)$ in the above equation, where $f_1(0)$ is wavefunction of $Z^{(1)}$ evaluated at the fermion brane ($y = 0$).

Verify that the shift in the coupling of this fermion to the SM Z from the zero-mode Z coupling (i.e., $g_{Z^{(0)}}$) is as shown in lecture 3: $\delta g_Z / g_{Z^{(0)}} \sim g^2_{Z^{(0)}} v^2/m^2_{KK}$, in particular, that there is *no* enhancement for large brane kinetic terms, $r/R \gg 1$.

A.3.2. *Shift in Z mass*

The lighter eigenvalue of mass matrix is the SM Z mass. Verify that the shift in the SM Z mass from the purely zero-mode mass, i.e., $1/4 g^2_{Z^{(0)}} v^2$, is as shown in lecture 3, in particular, that there *is* an enhancement in this shift due to $r/R \gg 1$ (when the shift is expressed in terms of $g_{Z^{(0)}}$).

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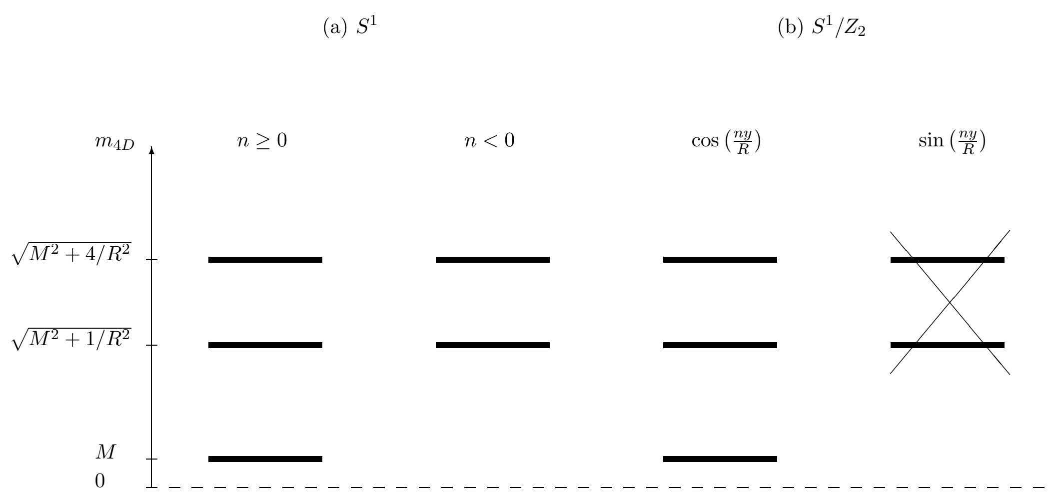

Fig. 1.1. KK decomposition of a $5D$ scalar on a circle (a) and an orbifold (b), choosing even parity.

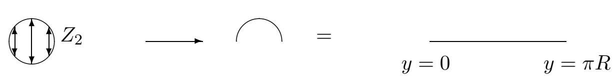

Fig. 1.2. Going from a circle to an orbifold using Z_2 symmetry

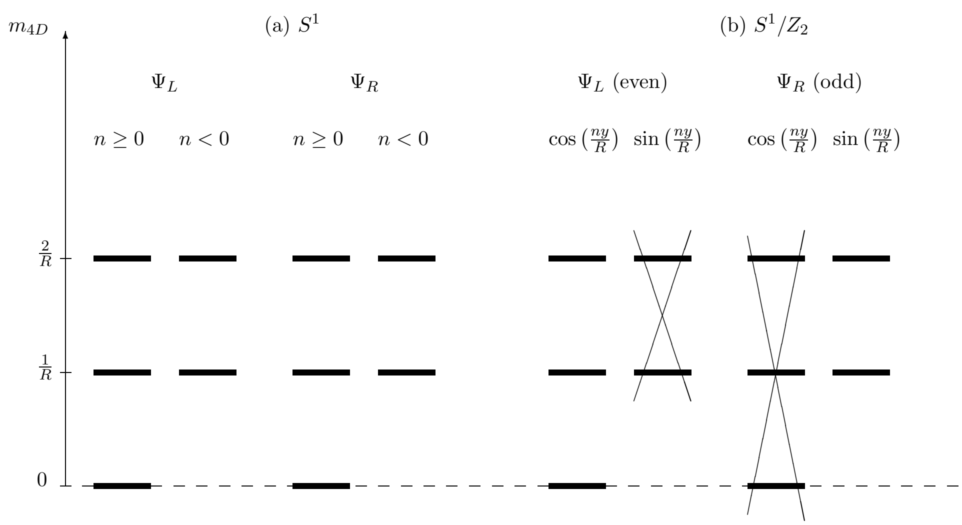

Fig. 1.3. KK decomposition for a $5D$ fermion on a circle (a) and an orbifold (b) with even parity for Ψ_L.

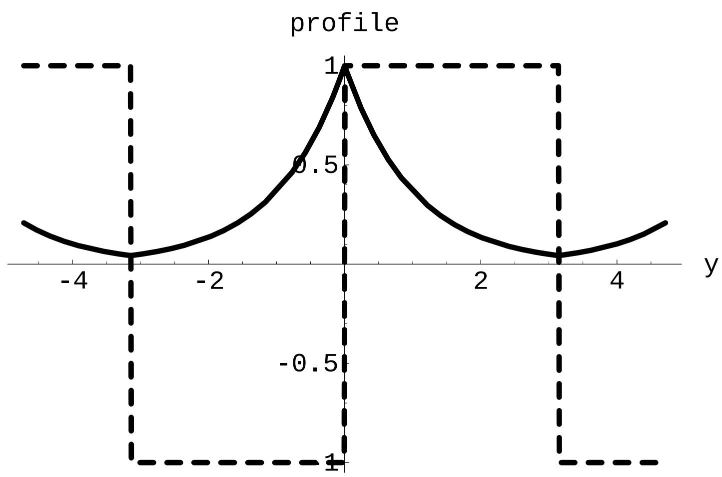

Fig. 1.4. Profile of odd mass term (dashed line) and fermion zero-mode (solid line). Here and henceforth, we set radius of extra dimension, $R = 1$ in all figures.

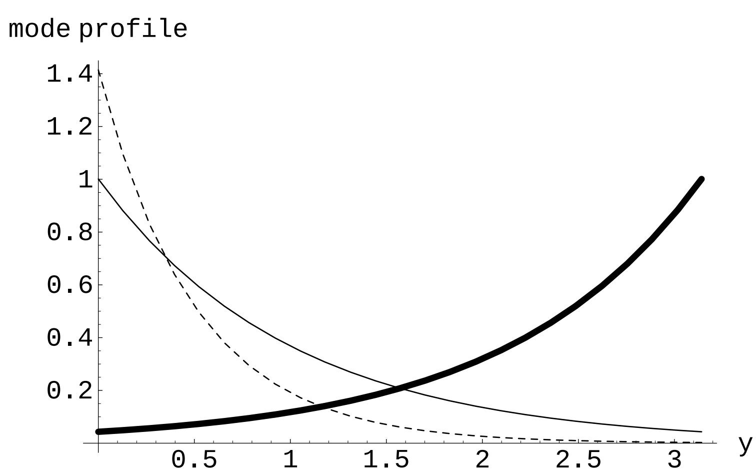

Fig. 1.5. Profiles for down (dashed line: $5D$ mass, $M = -2$), strange (thin solid line: $M = -1$) and top quarks (thick solid line: $M = +1$). The SM Higgs is localized on the $y = \pi R$ brane.

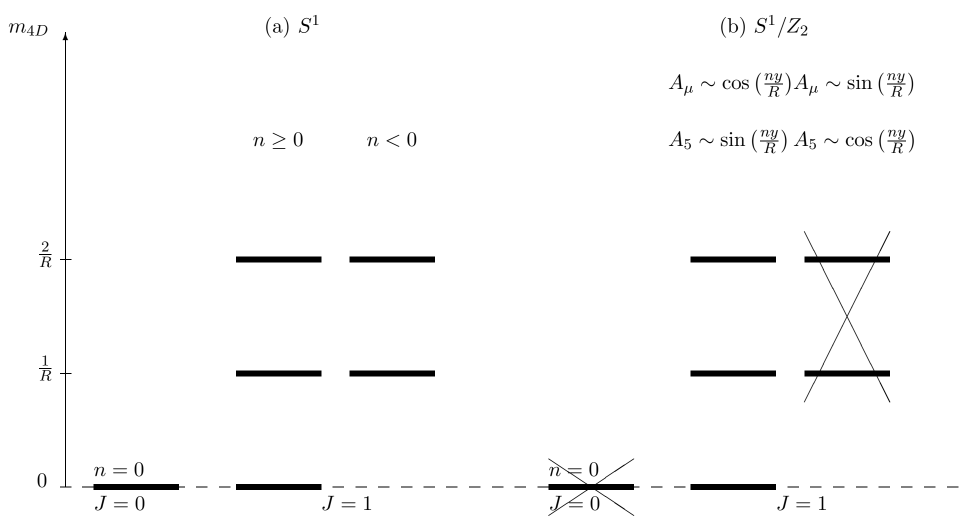

Fig. 1.6. KK decomposition for a $5D$ gauge field on a circle (a) and on a orbifold (b) with choice of even parity for $\mathcal{A}_\mu$.

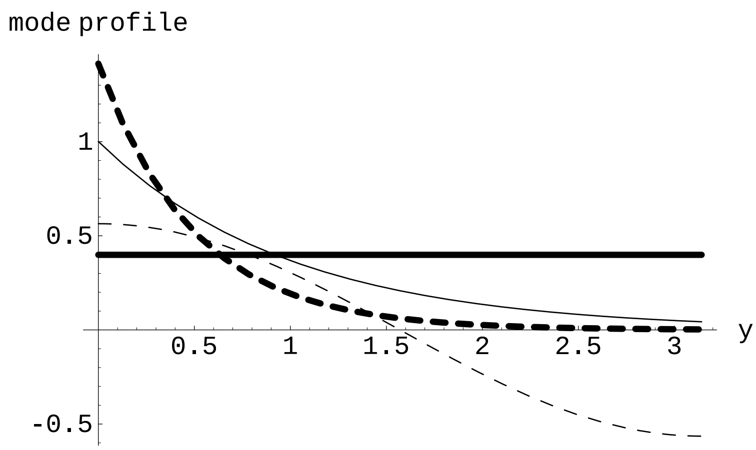

Fig. 1.7. Profiles for down (thick dashed line) and strange (thin solid line) quarks and the gauge zero-mode (thick solid line) and 1st KK mode (thin dashed line). The SM Higgs is localized on the $y = \pi R$ brane.

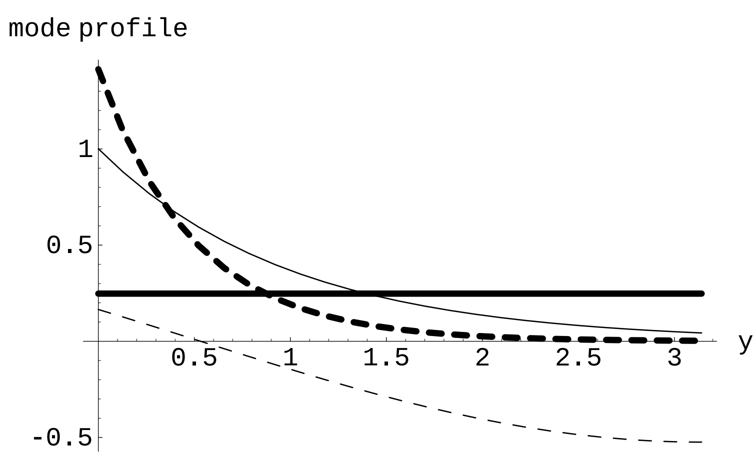

Fig. 1.8. Same as Fig. 1.7, but with brane kinetic term, $r/R = 10$, for gauge fields on $y = 0$ brane.

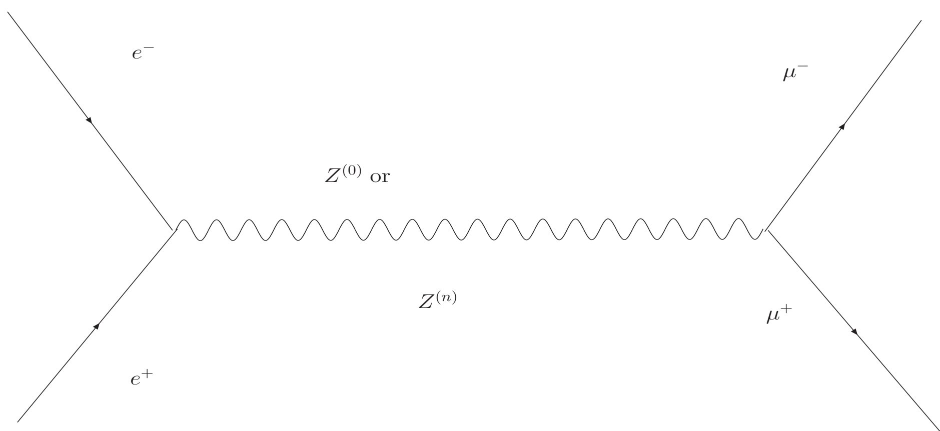

Fig. 1.9. 4-fermion operators generated by exchange of zero and KK modes of Z.

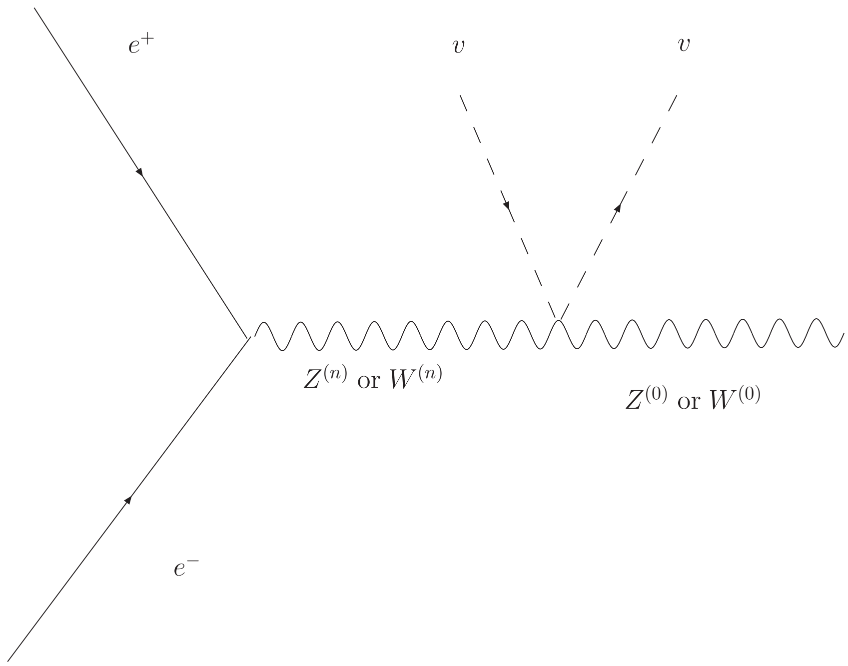

Fig. 1.10. Shift in the coupling of a SM fermion to SM Z from the zero-mode gauge coupling due to the mixing of zero and KK modes of Z.

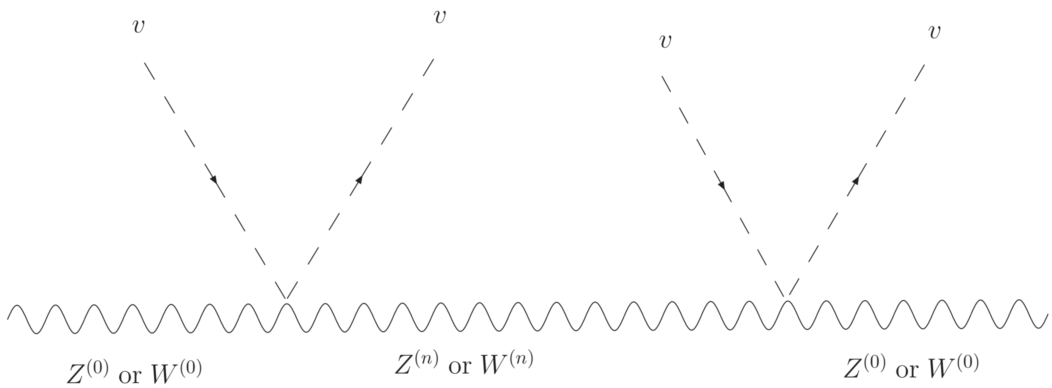

Fig. 1.11. Shift in the masses of SM W, Z from the zero-mode masses due to the mixing of zero and KK modes.

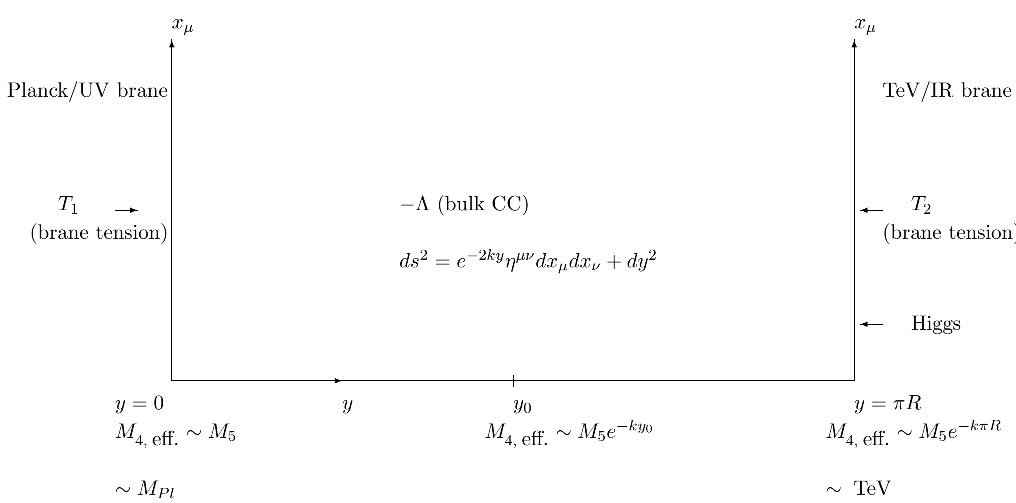

Fig. 1.12. The Randall-Sundrum (RS1) model.

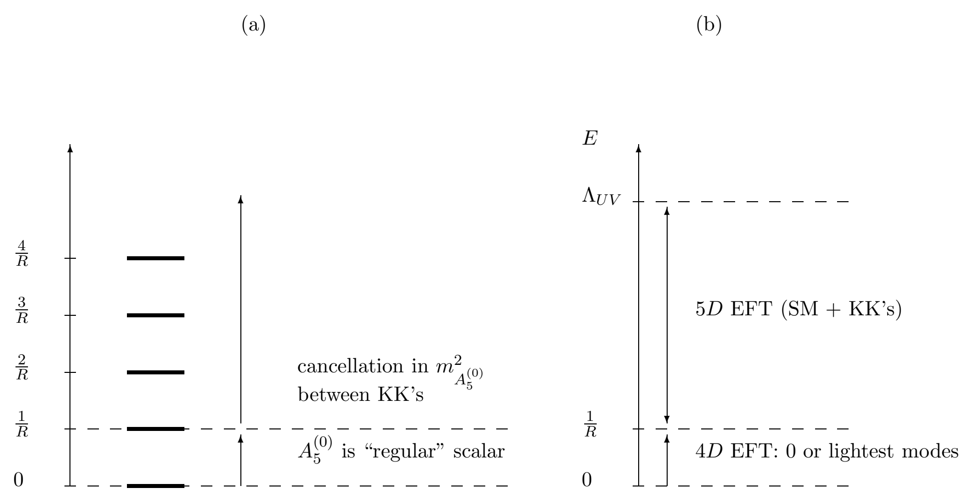

Fig. 1.13. Contributions to mass of A_5 (a) and various energy scales in the $5D$ model (b).