Neutrinoless Double Beta Decay

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Lecture # 1

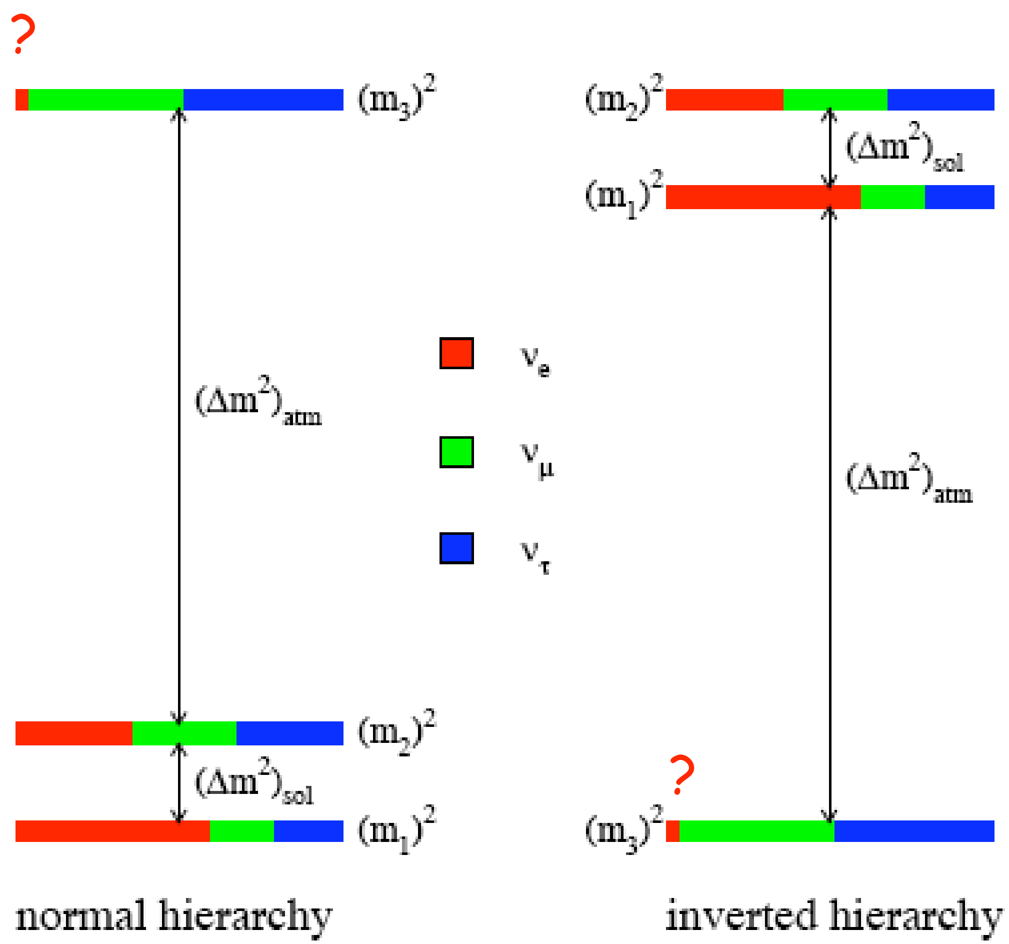
TASI-06, Boulder, June 12

Outline:

Lecture # 1: Introduction (fundamentals of $\beta\beta$ decay)
Mechanism of $0\nu\beta\beta$ decay
$\langle m_{\beta\beta} \rangle$ versus other observables

Lecture # 2: Experimental status and near term plans
Nuclear matrix elements
Neutrino magnetic moment and neutrino mass

From oscillation experiments we know that neutrinos are massive and mixed. We also deduce the composition of mass eigenstates in terms of the flavor eigenstates, and splitting (in Δm^2) of the mass eigenstates



In either case electron neutrinos ν_e are a superposition of two closely spaced states ν_1 and $\nu_{2.}$ The third state, further apart, contains very little, if any, (<5%) of ν_e.

What we do not know?

- Are neutrinos Majorana particles? ←
- What is the pattern of neutrino masses?
- What is the absolute mass scale? ←
- Is CP invariance violated in the lepton sector?
- Is there a relation between all of this and the baryon excess in the Universe?

Here, observation of the neutrinoless $\beta\beta$ decay would help to answer the questions with arrows in a unique way.

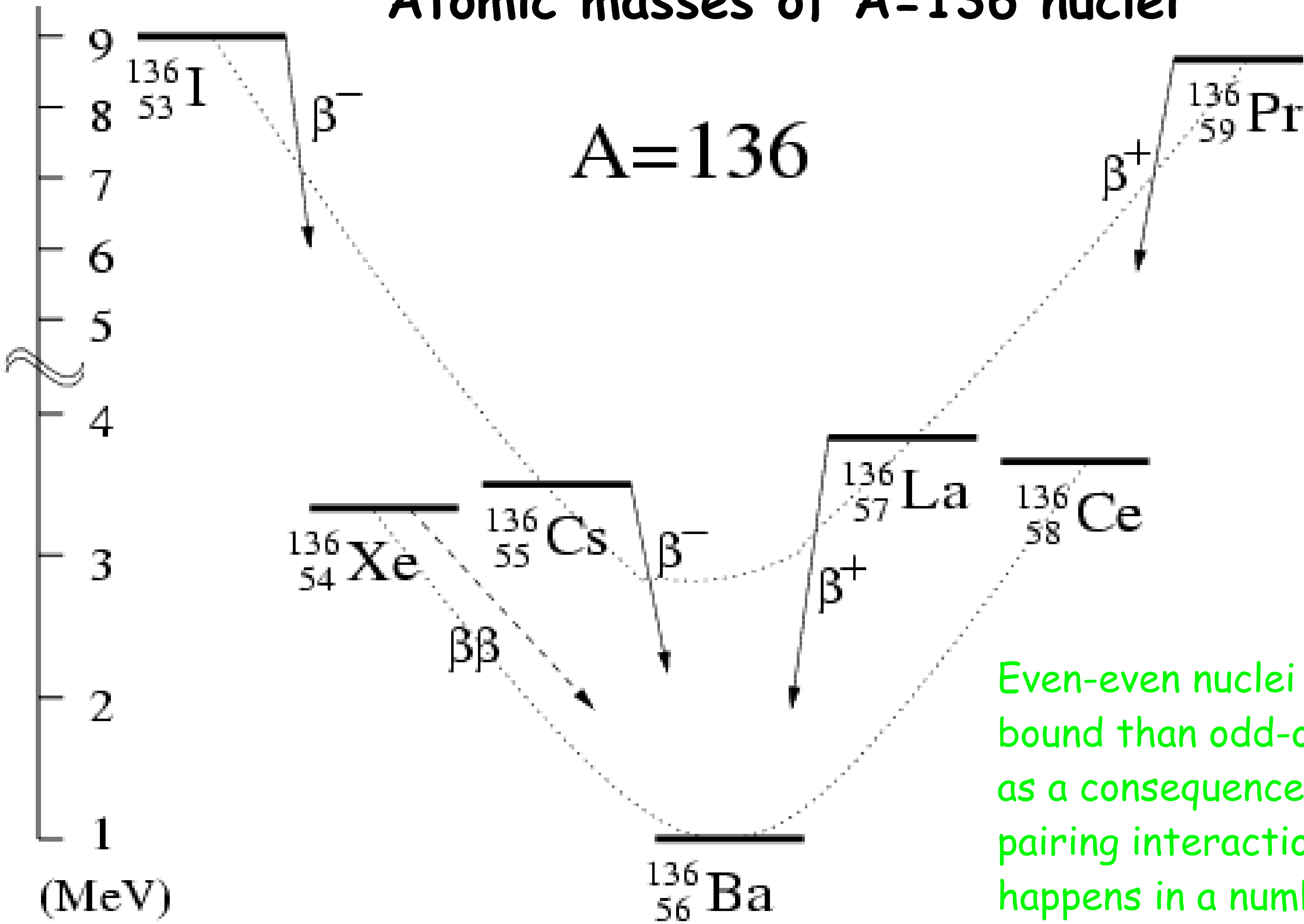
APS Joint Study on the Future of Neutrino Physics (2004)

(physics/0411216)

We recommend, as a high priority, a phased program of sensitive searches for neutrinoless double beta decay (first on the list of recommendations)

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

Atomic masses of A=136 nuclei

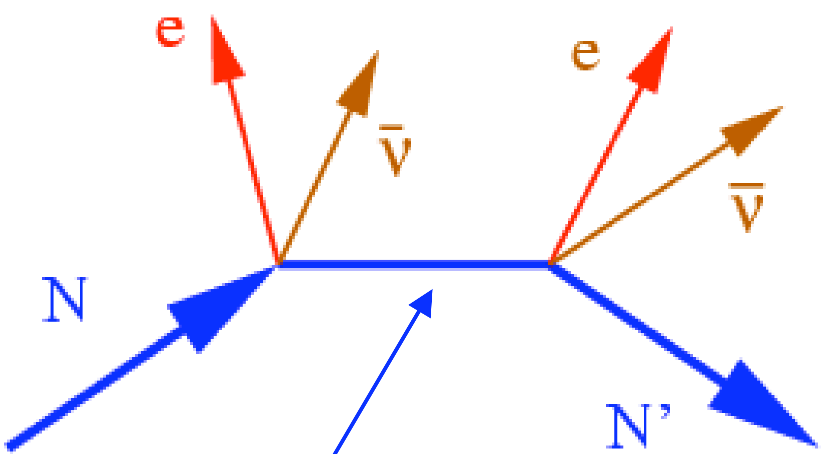


Even-even nuclei are more bound than odd-odd ones as a consequence of pairing interaction. This happens in a number of cases.

^{136}Xe and ^{136}Ce are stable against β decay, but unstable against $\beta\beta$ decay ($\beta^-\beta^-$ for ^{136}Xe and $\beta^+\beta^+$ for ^{136}Ce)

Two modes of $\beta\beta$ decay

$2\nu\ \beta\beta$ decay: a standard process in nuclear physics



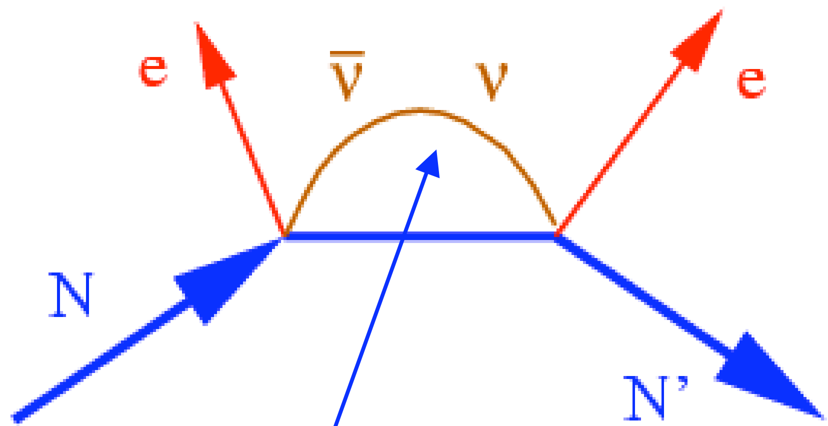
virtual state in the intermediate odd-odd nucleus

$0\nu\ \beta\beta$ decay: a hypothetical process

→ $m_\nu \neq 0$

→ $\bar{\nu} = \nu$

since helicity has to "flip"



here the exchanged neutrino is virtual as well

From G. Gratta

Common features:

i) Light particles (electrons and antineutrinos) carry essentially all energy (Q-value). The nuclear recoil is negligible since $Q/Am_n \ll 1$.

ii) The transition involves the 0^+ ground state of the initial nucleus and (in almost all cases) the 0^+ ground state of the final nucleus. In few cases the transition to excited 0^+ states is energetically possible, but suppressed by phase space. (But the 2ν decay to the excited 0^+ state was observed in few cases).

iii) Both processes are of second order of weak interaction, $\sim G_F^4$, hence inherently slow. The phase space factors alone, for $2\nu \sim Q^{11}$, for $0\nu \sim Q^5$ give preference for the 0ν mode, which is, however, forbidden by lepton number conservation.

Distinct features:

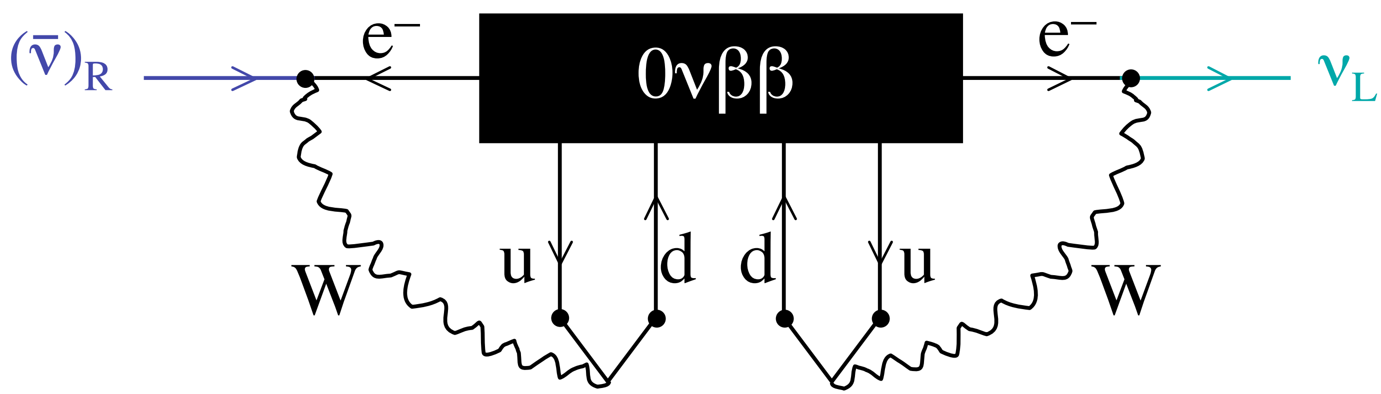
i) In $2\nu\beta\beta$ the two decaying neutrons are uncorrelated, while in the $0\nu\beta\beta$ decay they are correlated.

ii) In $2\nu\beta\beta$ decay the sum electron spectrum is continuous, with $T_1 + T_2$ peaked below Q/2. The tail for $T_1 + T_2 \to Q$ goes like $(\Delta E/Q)^6$.

iii) In $0\nu\beta\beta$ decay the $T_1 + T_2 = Q$ smeared only by recoil and energy resolution.

Whatever processes cause 0νββ, i.e. independently of the content of the dd->uuee black box below, its observation would imply the existence of a Majorana mass term:

Schechter and Valle 82



$(\bar{\nu})_R \rightarrow \nu_L$: A Majorana mass term

$0\nu\beta\beta$ decay violates the total lepton number conservation law, and thus requires `physics beyond the Standard Model'.

Other possible LNV processes are e.g.
$\mu^- + Z \rightarrow e^+ + (Z-2)$ (Br. < 10^{-12})
$K^+ \rightarrow \mu^+\mu^+\pi^-$ (Br. < 3.0×10^{-9})
$\bar{\nu}_e$ emission from the Sun (Br. < ~10^{-4})
etc.

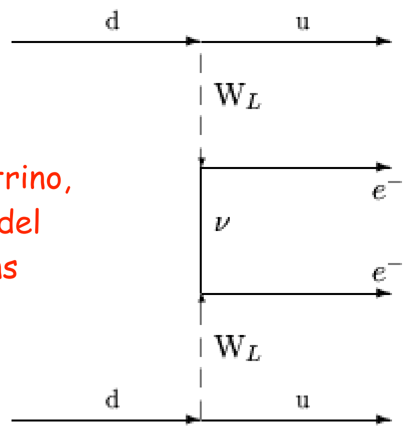
$0\nu\beta\beta$ decay is the most sensitive probe of LNV. Why?
100 kg ~ 1 kmole ~ 10^{27} nuclei,
contrast this to an accelerator beam (Fermilab produces ~10^{20}p/year)

What is in the black box?

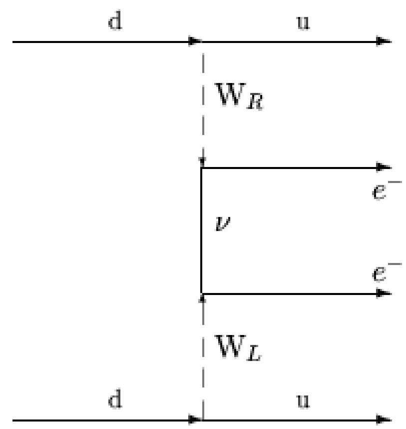
All these diagrams can contribute to the $0\nu\ \beta\beta$ decay

Light Majorana neutrino, only Standard Model weak interactions

Exchange of a **light** neutrino, only left-handed currents



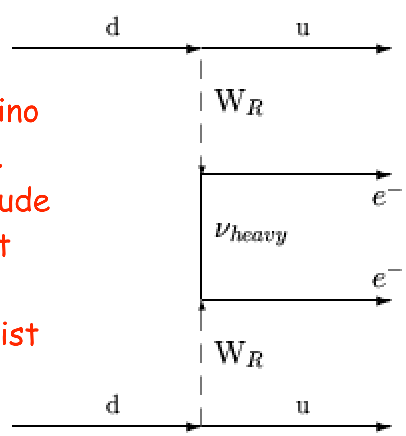
Exchange of a light or heavy neutrino and one **right-handed** $\mathbf{W}_R$



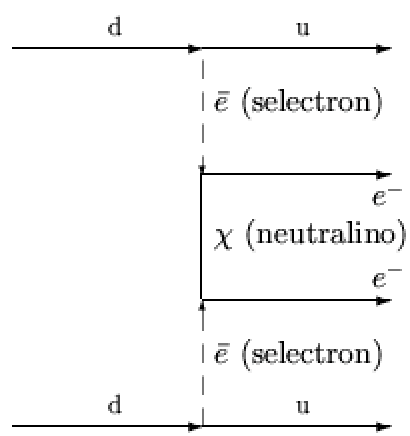
Light or heavy Majorana neutrino. Model extended to include right-handed W_R. Mixing extended between the left and right-handed neutrinos. Light Majorana neutrinos exist also.

Heavy Majorana neutrino interacting with W_R. Model extended to include right-handed current interactions. Light Majorana neutrinos exist also.

Exchange of a **heavy** neutrino, short range hadron physics at play



Exchange of **supersymmetric** particles, R-symmetry violated



Supersymmetry with R-parity violation. Many new particles invoked. Light Majorana neutrinos exist also.

It is well known that the amplitude for the light neutrino exchange scales as $\langle m_{\beta\beta} \rangle$. On the other hand, if heavy particles of scale Λ are involved the amplitude scales as $1/\Lambda^5$ (dimension 9 operator)

.

The relative size of the heavy (A_H) vs. light particle (A_L) exchange to the decay amplitude is (a crude estimate, due originaly to Mohapatra)

$A_L \sim G_F^2\, m_{\beta\beta}/\langle k^2 \rangle$, $\quad A_H \sim G_F^2\, M_W^4/\Lambda^5$,

where Λ is the heavy scale and $k \sim 50$ MeV is the virtual neutrino momentum.

For $\Lambda \sim 1$ TeV and $m_{\beta\beta} \sim 0.1 - 0.5$ eV $A_L/A_H \sim 1$, hence both mechanisms contribute equally.

$$A_L/A_H \sim m_{\beta\beta}\,\Lambda^5/\langle k^2\rangle M_W^4$$

Thus for $m_{\beta\beta}$ = 0.2 eV, $\langle k^2\rangle = 50^2$ MeV^2, and $A_L/A_H \sim 1$

$\Lambda^5 \sim 50^2 \times 10^{12} \times 80^4 \times 10^{36}/0.2$ eV $\sim 5\times10^{59}$ eV

$\Rightarrow \Lambda \sim 10^{12}$ eV = 1 TeV

Clearly, the heavy particle mechanism could compete with the light Majorana neutrino exchange only if the heavy scale Λ is between about 1 - 5 TeV. Smaller Λ are already excluded and larger ones will be unobservable due to the fast Λ^5 scale dependence.

Observing the $0\nu\beta\beta$ decay will not (in general) make it possible to draw conclusion about the `mechanism' of the process. We need additional information.

We shall discuss how the study of lepton flavor violation (LFV) can help us to decide what mechanism is responsible for the 0nbb decay if it is observed in a foreseeable future.

This is based on "Lepton number violation without supersymmetry" Phys.Rev.D 70 (2004) 075007

V. Cirigliano, A. Kurylov, M.J.Ramsey-Musolf, and P.V.

and on "Neutrinoless double beta decay and lepton flavor violation" Phys. Rev. Lett. 93 (2004) 231802

V. Cirigliano, A. Kurylov, M.J.Ramsey-Musolf, and P.V.

In the standard model lepton flavor conservation is a consequence of vanishing neutrino masses. However, the observation of neutrino oscillations shows that neutrinos are massive and that the flavor is not conserved. Hence a more general theory must contain LFV of charged leptons generated probably at some high scale.

There is a long history of searches for LFV with charged leptons, like $\mu \rightarrow e + \gamma$, muon conversion $\mu^- + (Z,A) \rightarrow e^- + (Z,A)$, or $\mu^+ \rightarrow e^+ + e^+ + e^-$.

Impressive limits for the branching ratios have been established:

$$B_{\mu\to e\gamma} = \frac{\Gamma(\mu \longrightarrow e\gamma)}{\Gamma(\mu \longrightarrow e\nu_\mu \bar{\mu}_e)} < 1.2\times10^{-11}$$

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \longrightarrow e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \longrightarrow \nu_\mu + (Z-1,A))} < 8\times10^{-13}$$

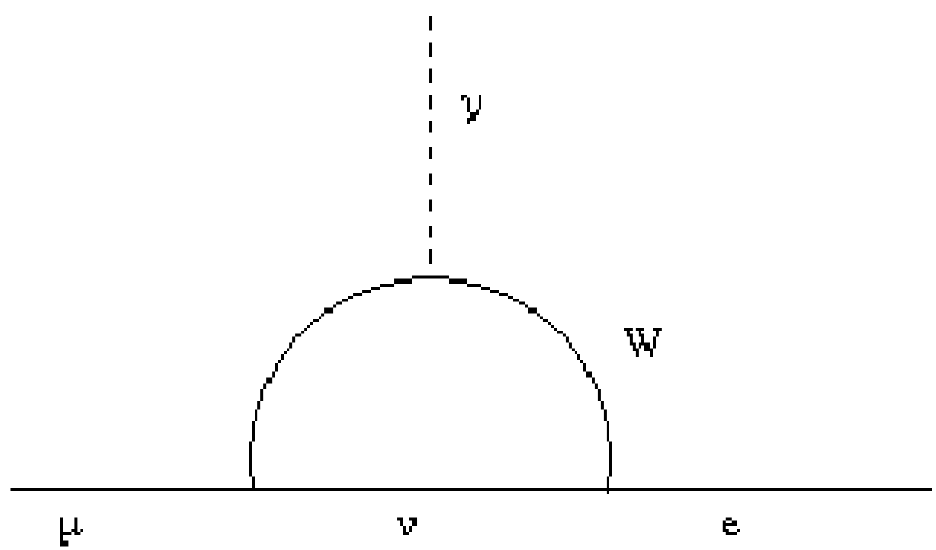
There are ambitious new proposals with much better sensitivities:

MECO (now unfortunately cancelled): $B_{\mu \to e} < 5\times10^{-17}$ on Al

MEG (now being built at PSI): $B_{\mu \to e+\gamma} < 5\times 10^{-14}$

i.e. improvement by a factor of ~ 1000 - 10000.

The direct effect of neutrino mass is "GIM suppressed" by a factor of $(\Delta m_\nu^2/M_W^2)^2 \sim 10^{-50}$ hence unobservable.



So, why are people even looking for LFV?

Because most particle physics models of `physics beyond the Standard Model' contain LFV originating at some high mass scale. Most of them also contain LNV and, naturally, all realistic models should include light and mixed neutrinos, known to exist.

If the scales of **both** LFV and LNV are well above the weak scale, then $\Gamma_{0\nu\beta\beta} \sim \langle m_{\beta\beta}\rangle^2$ and $\langle m_{\beta\beta}\rangle$ can be derived from the $0\nu\beta\beta$ decay rate. However, the `dangerous' case is when **both** LFV and LNV scales are low (~ TeV). In that case there might be an ambiguity in interpreting the results of $0\nu\beta\beta$ decay experiments.

In the most popular SUSY-GUT scenario (for SU(5) GUT) one has the branching ratios

$$B_{\mu\to e\gamma} = 2.4\times 10^{-12}\left(\frac{|V_{ts}|}{0.04}\frac{|V_{td}|}{0.01}\right)^2\left(\frac{100\ \mathrm{GeV}}{m_{\tilde\mu}}\right)^4$$

$$B^{\mathrm{Ti}}_{\mu\to e} = 5.8\times 10^{-12}\ \alpha\ \left(\frac{|V_{ts}|}{0.04}\frac{|V_{td}|}{0.01}\right)^2\left(\frac{100\ \mathrm{GeV}}{m_{\tilde\mu}}\right)^4$$

Thus a) MEG and MECO should see an effect, and

b) m -> e + g is enhanced by a factor ~1/a compared to m -> e conversion.

The feature b) is generic for theories with high scale LNV

Linking LNV to LFV Summary:

$$B_{\mu\to e\gamma} = \frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\nu_\mu\bar{\mu}_e)} \qquad B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A)\to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A)\to \nu_\mu + (Z-1,A))}$$

- SM extensions with low (~ TeV) scale LNV $\Rightarrow$** $\mathcal{R} = \dfrac{B_{\mu\to e}}{B_{\mu\to e\gamma}} >> 10^{-2}$

  ** In absence of fine-tuning or hierarchies in flavor couplings. Important caveat!

- SM extensions with high (GUT) scale LNV $\left[\,\Gamma_{0\nu\beta\beta} \sim m_{\beta\beta}^2\,\right] \Rightarrow$

$$\mathcal{R} \sim O(\alpha/\pi) \sim 10^{-3} - 10^{-2}$$

Thus the ratio *R* can be used as a \`diagnostic tool' for low scale LNV

Effective theory description

$$\mathcal{L}_{0\nu\beta\beta} = \sum_i \frac{\tilde{c}_i}{\Lambda^5} \tilde{O}_i$$

$$\mathcal{L}_{\rm LFV} = \sum_i \frac{c_i}{\Lambda^2} O_i$$

Operators (omitting L ↔ R)

$$\tilde{O}_i = \bar{q}\Gamma_1 q \;\; \bar{q}\Gamma_2 q \;\; \bar{e}\Gamma_3 e^c$$

$$O_{\sigma L} = \frac{e}{(4\pi)^2} \overline{\ell_{iL}}\, \sigma_{\mu\nu} i \not{D}\, \ell_{jL}\; F^{\mu\nu} + \mathrm{h.c.}$$

$$O_{\ell L} = \overline{\ell_{iL}}\, \ell^c_{jL}\; \overline{\ell^c_{kL}}\, \ell_{mL}$$

$$O_{\ell q} = \overline{\ell_i}\Gamma_\ell \ell_j \; \overline{q}\Gamma_q q$$

- $O_{\sigma L}$ arises at loop level
- $O_{\ell L}$, $O_{\ell q}$ may arise at tree level
- Leading pieces in c_i are nominally of order (Yukawa)^2

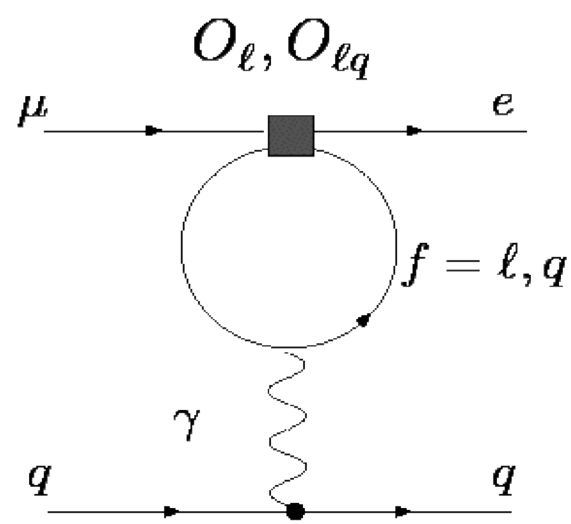
The ratio R can be expressed in terms of the constants c_i as follows

$$\mathcal{R} = \frac{\Phi}{48\pi^2} \frac{\left| e^2\,\eta_1\, c_{\sigma L} \;+\; e^2\left(\eta_2\, c_{\ell L} \;+\; \eta_3\, c_{\ell q}\right) \log \frac{\Lambda^2}{m_\mu^2} \;+\; \eta_4\,(4\pi)^2\, c_{\ell q} \;+\; \ldots \right|^2}{e^2 \left(|c_{\sigma L}|^2 + |c_{\sigma R}|^2 \right)}$$

- Phase space + overlap integrals: $\Phi = \dfrac{Z\, F_p^2(m_\mu^2)}{g_V^2 + 3 g_A^2} \sim O(1)$ for light nuclei
- η_n are coefficients of O(1)
- Origin of large logs:

  one loop operator mixing

  [Raidal-Santamaria '97]



Thus from the expression for R it follows:

$$\mathcal{R} = \frac{\Phi}{48\pi^2} \frac{\left| e^2\,\eta_1\, c_{\sigma L} \;+\; e^2\left(\eta_2\, c_{\ell L} \;+\; \eta_3\, c_{\ell q}\right) \log \frac{\Lambda^2}{m_\mu^2} \;+\; \eta_4\,(4\pi)^2\, c_{\ell q} \;+ \ldots \right|^2}{e^2 \left(|c_{\sigma L}|^2 + |c_{\sigma R}|^2 \right)}$$

(i) No tree level $c_{\ell L}$, $c_{\ell q}$ $\Rightarrow$ $\mathcal{R} \sim \frac{\Phi\,\eta_1^2\,\alpha}{12\pi} \sim 10^{-3} - 10^{-2}$

(ii) Tree level $c_{\ell L}$, $c_{\ell q}$ $\Rightarrow$ log enhancement and $\mathcal{R} \sim O(1)$

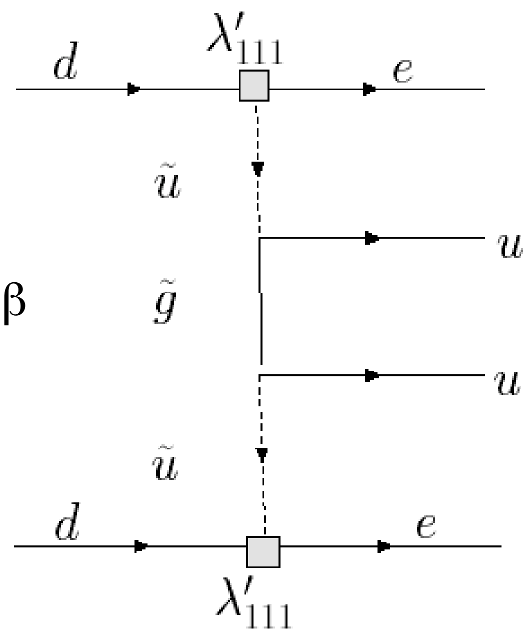
(iii) Tree level $c_{\ell q}$ $\Rightarrow$ $\mathcal{R} \gg 1$

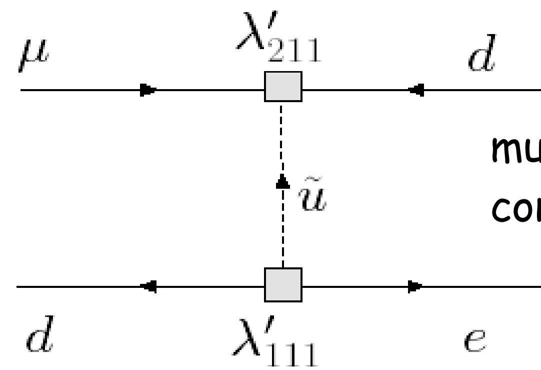
We need to show that in models with low scale LNV O_l and/or O_{lq} are generated at tree level. We offer no general proof, but two illustrations.

Illustration I: RPV SUSY [R = $(-1)^{3(B-L)+2s}$]

$$W_{RPV} = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k + \mu'_i L_i H_u$$

0νββ





muon conversion

$$\frac{\tilde{c}_i}{\Lambda^5} \sim \frac{\pi\alpha_s}{m_{\tilde{g}}} \frac{\lambda'^2_{111}}{m^4_{\tilde{f}}} \; ; \; \frac{\pi\alpha_2}{m_\chi} \frac{\lambda'^2_{111}}{m^4_{\tilde{f}}}$$

$$\frac{c_\ell}{\Lambda^2} \sim \frac{\lambda_{i11}\lambda^*_{i21}}{m^2_{\tilde{\nu}_i}}, \frac{\lambda^*_{i11}\lambda_{i12}}{m^2_{\tilde{\nu}_i}}$$

$$\frac{c_{\ell q}}{\Lambda^2} \sim \frac{\lambda'^*_{11i}\lambda'_{21i}}{m^2_{\tilde{d}_i}}, \frac{\lambda'^*_{1i1}\lambda'_{2i1}}{m^2_{\tilde{u}_i}}$$

$$\frac{c_\sigma}{\Lambda^2} \sim \frac{\lambda\lambda^*}{m^2_{\tilde{\ell}}}, \frac{\lambda'\lambda'^*}{m^2_{\tilde{q}}}$$

Clearly, the way to avoid the connection between LFV and LNV is if $l'_{111} \gg l'_{211}$, etc. That is if l' is nearly flavor diagonal. Note that empirically both l_{ijk} and l'_{ijk} are small $\ll 1$.

For the discussion of neutrino masses in the R-parity violating supersymmetric models see Y. Grossman and S. Rakshit, hep-ph/0311310

Generally, a hierarchical neutrino spectrum is predicted, but small neutrino masses require some fine tuning. Note also that R-parity violation excludes LSP as a dark matter candidate. Discovering it would exclude R-parity violation.

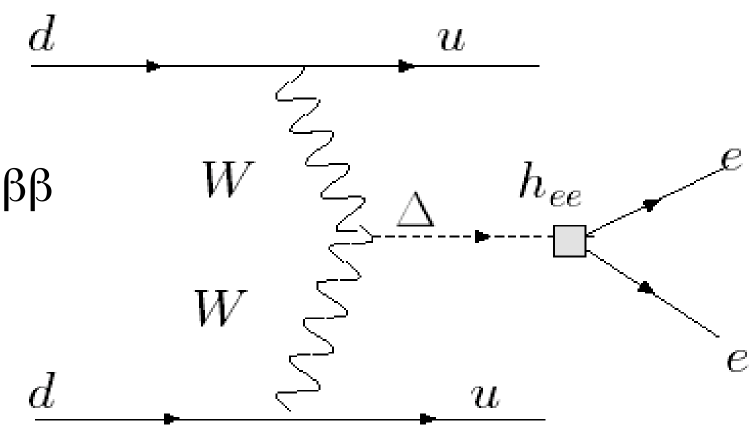
Illustration II: Left-Right Symmetric Model

$$\mathrm{SU(2)_L} \otimes \mathrm{SU(2)_R} \otimes \mathrm{U(1)_{B\text{-}L}} \;\Rightarrow\; \mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y} \;\Rightarrow\; \mathrm{U(1)_{EM}}$$

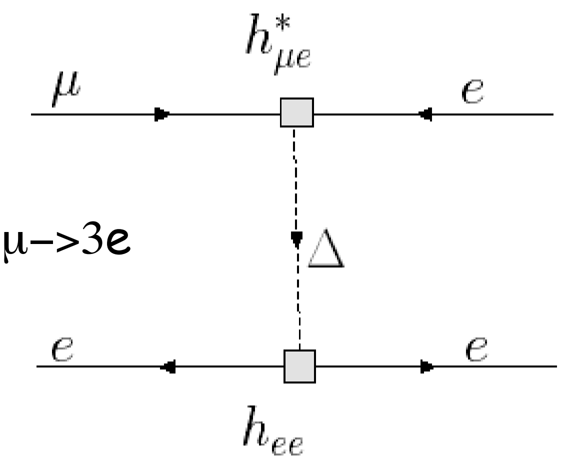
$$\mathcal{L}_Y^{\mathrm{lept}} = -\overline{L_L}^{\,i}\left(y_D^{ij}\,\Phi + \tilde{y}_D^{ij}\,\tilde{\Phi}\right) L_R^j - \overline{(L_L)^c}^{\,i}\, y_M^{ij}\,\tilde{\Delta}_L\, L_L^j - \overline{(L_R)^c}^{\,i}\, y_M^{ij}\,\tilde{\Delta}_R\, L_R^j$$

$$\Rightarrow \mathcal{L}_{\delta^{\pm\pm}_{L,R}} = \frac{g}{2}\left[\delta^{++}_{L,R}\,\overline{l^c}\,(h\,P_{L,R})\,l + \delta^{--}_{L,R}\,\overline{l}\left(h^\dagger\,P_{R,L}\right) l^c\right]$$

0νββ



μ->3e



$$\frac{\tilde{c}_i}{\Lambda^5} \sim \frac{g_2^4}{M_{W_R}^4}\frac{1}{M_{\nu_R}} \;;\; \frac{g_2^3}{M_{W_R}^3}\frac{h_{ee}}{M_\Delta^2} \qquad \frac{c_\ell}{\Lambda^2} \sim \frac{h_{\mu i}h^*_{ie}}{m_\Delta^2} \qquad \frac{c_\sigma}{\Lambda^2} \sim \frac{(h^\dagger h)_{e\mu}}{M_{W_R}^2}$$

LRSM Matter fields:

$$L_{iL} = \begin{pmatrix} \nu'_i \\ l'_i \end{pmatrix}_L : (1/2 : 0 : -1)\,,\ L_{iR} = \begin{pmatrix} \nu'_i \\ l'_i \end{pmatrix}_R : (0 : 1/2 : -1)\ ,$$

$$Q_{iL} = \begin{pmatrix} u'_i \\ d'_i \end{pmatrix}_R : (1/2 : 0 : 1/3)\,,\ Q_{iR} = \begin{pmatrix} u'_i \\ d'_i \end{pmatrix}_R : (0 : 1/2 : 1/3)\ .$$

Higgs sector

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix},\ \Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^0 & -\delta_{L,R}{}^+/\sqrt{2} \end{pmatrix}\ ,$$

$$\langle\phi\rangle = \begin{pmatrix} \kappa_1/\sqrt{2} & 0 \\ 0 & \kappa_2/\sqrt{2} \end{pmatrix},\ \langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}\ ,$$

h_{ij} are coupling constants of leptons and the doubly charged Higgs

$$\mathcal{L}_{\delta^{\pm\pm}_{L,R}} = \frac{g}{2}\left[\delta^{++}_{L,R}\,\overline{l^c}\,(h_{L,R}\,P_{L,R})\,l + \delta^{--}_{L,R}\,\bar{l}\left(h^{\dagger}_{L,R}\,P_{R,L}\right)l^c\right],$$

They are related to the mixing matrix K_R of the heavy neutrinos

$$h_{ij} = \sum_{n=\mathrm{heavy}} \left(K_R\right)_{ni}\left(K_R\right)_{nj}\sqrt{x_n}\,,$$

$$\left(h^{\dagger}h\right)_{e\mu} = \left(\tilde{h}^{\dagger}\tilde{h}\right)_{e\mu} = \sum_{n=\mathrm{heavy}} x_n\left(K_R^{\dagger}\right)_{en}\left(K_R\right)_{n\mu} \equiv g_{\mathrm{lfv}}\,,$$

$$x_n = \left(\frac{M_n}{M_{W_2}}\right)^2 ,$$

Note that g_{lfv} vanishes for degenerate heavy neutrinos, but h_{ij} need not.

Within LRSM the LFV branching ratios depend only on g_{lfv} .

$$B_{\mu\to e\gamma} \approx 10^{-7} \times |g_{\rm lfv}|^2 \left(\frac{1\ \mathrm{TeV}}{M_{W_R}}\right)^4$$

$$B^{\rm Al}_{\mu\to e} \approx 10^{-7} \times \alpha |g_{\rm lfv}|^2 \left(\frac{1\ \mathrm{TeV}}{M_{\delta_R^{++}}}\right)^4 \left(\log \frac{M^2_{\delta_R^{++}}}{m^2_\mu}\right)^2$$

Thus the present limits suggest that either the scale is >> 1 TeV, or that g_{lfv} is very small, i.e. that he heavy neutrino spectrum is degenerate or has very little mixing.

Lets assume that $0\nu\beta\beta$ decay is caused by the exchange of light Majorana neutrinos.

What is the relation of the deduced fundamental parameters and the neutrino mixing matrix? Or, in other words, what is the relation between the $0\nu\beta\beta$ decay rate and the absolute neutrino mass?

As long as the mass eigenstates ν_i are Majorana neutrinos, the $0\nu\beta\beta$ decay **will occur**, with the rate

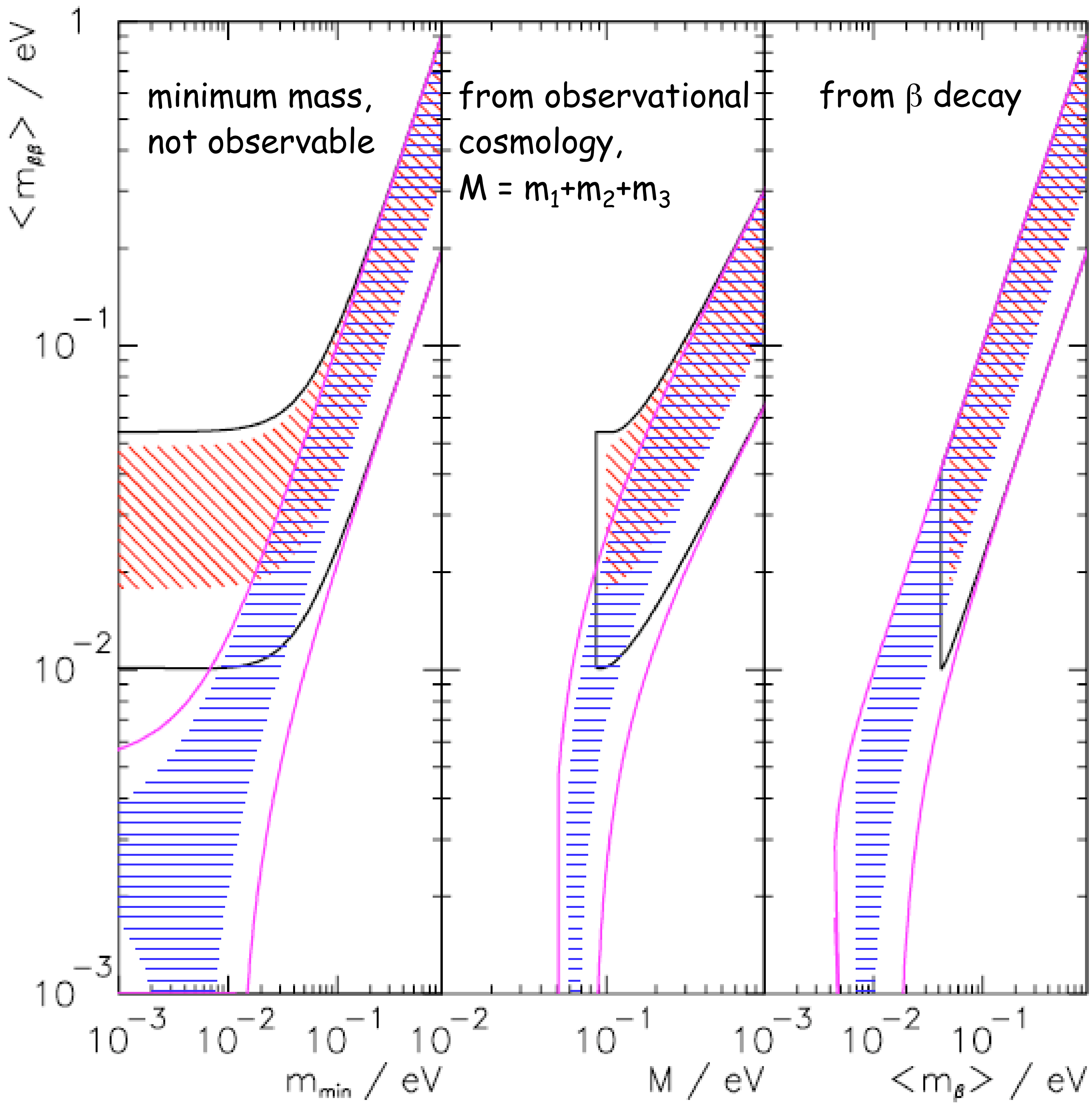
$1/T_{1/2} = G(E_{tot},Z)\,(M^{0\nu})^2 \langle m_{\beta\beta}\rangle^2$,

where $G(E_{tot},Z)$ is easily calculable phase space factor, $M^{0\nu}$ is the nuclear matrix element, calculable with difficulties (and discussed below), and

$$\langle m_{\beta\beta}\rangle = \Sigma_i\, |U_{ei}|^2 \exp(i\alpha_i)\, m_i,$$

where α_i are unknown Majorana phases (only two of them are relevant). (Actually, if $\theta_{13} \sim 0$, only one phase, one angle, and two masses matter.)

We can relate $\langle m_{\beta\beta}\rangle$ to other observables related to the absolute neutrino mass.



$\langle m_{\beta\beta} \rangle$ vs. the absolute mass scale

blue shading: normal hierarchy, $\Delta m^2_{31} > 0$.

red shading: inverted hierarchy $\Delta m^2_{31} < 0$

shading:best fit parameters, lines 95% CL errors.

Thanks to A. Piepke

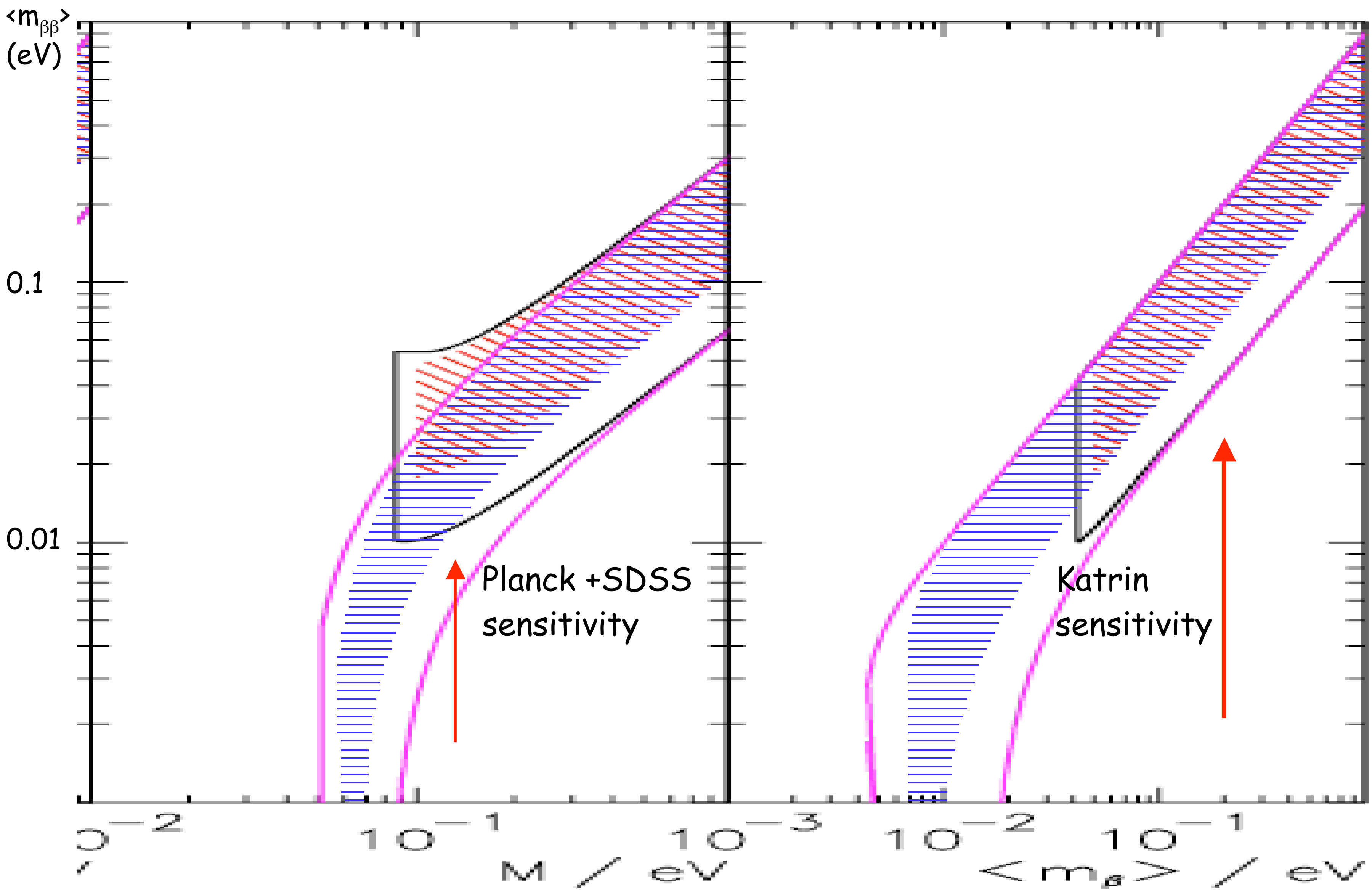
Three regions of $\langle m_{\beta\beta} \rangle$ of interest:

i) Degenerate mass region where all $m_i \gg \Delta m_{31}^2$. There $\langle m_{\beta\beta} \rangle > 0.1$ eV. $T_{1/2}$ for $0\nu\beta\beta$ decay $< 10^{26\text{-}27}$ y in this region. This region will be explored during the next 3-5 years with $0\nu\beta\beta$ decay experiments using ~100 kg sources . Moreover, most if not all of that mass region will be explored also by study of ordinary β decay and by the `observational cosmology'. These latter techniques are independent of whether neutrinos are Majorana or Dirac particles.

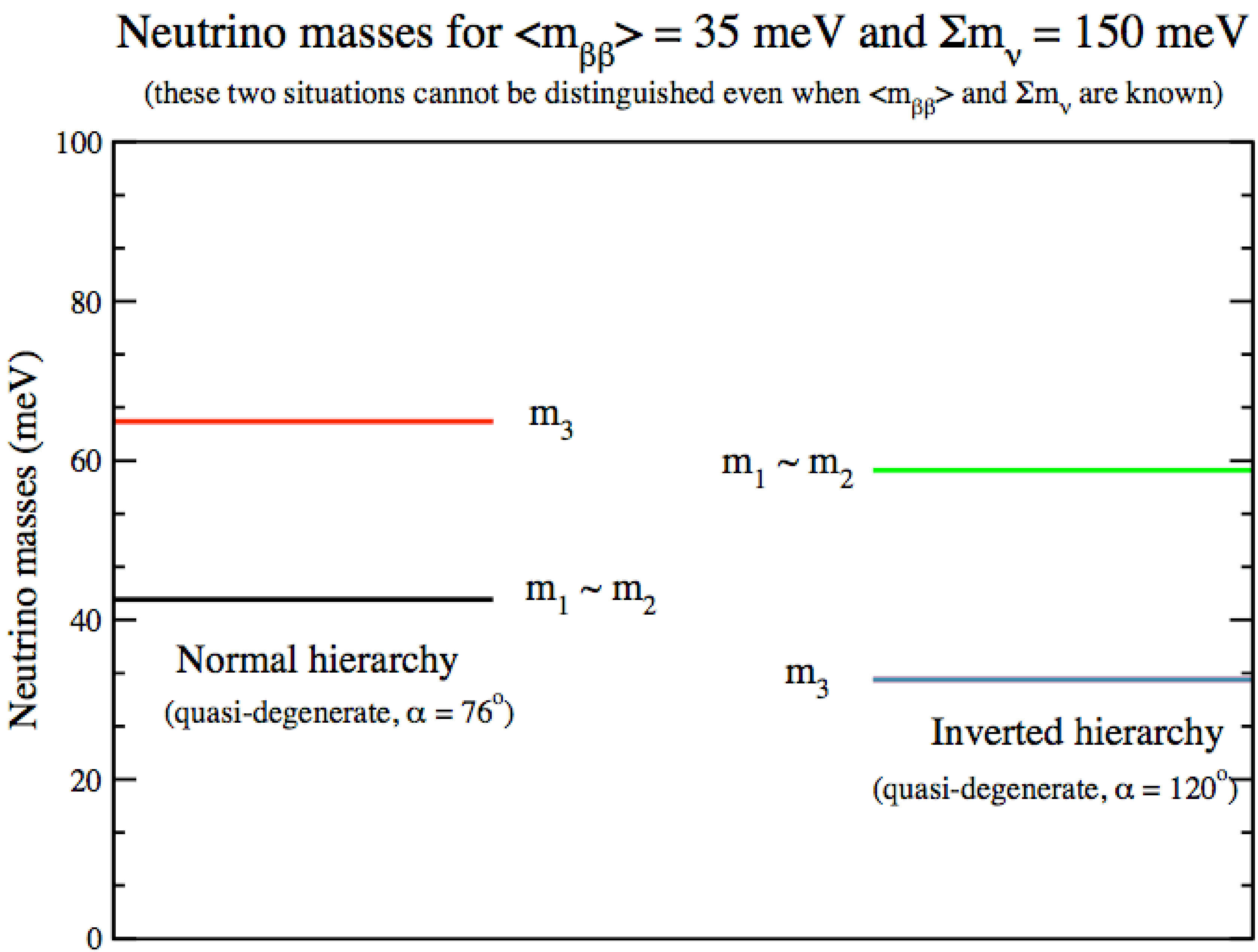
ii) Inverted hierarchy region where m_3 could be $< \Delta m_{31}^2$. However, quasidenegerate normal hierarchy is also possible for $\langle m_{\beta\beta} \rangle \sim 20\text{-}100$ meV. $T_{1/2}$ for $0\nu\beta\beta$ decay is $10^{27\text{-}28}$ years here, and could be explored with ~ton size experiments. Proposals for such experiments, with timeline ~10 years, exist.

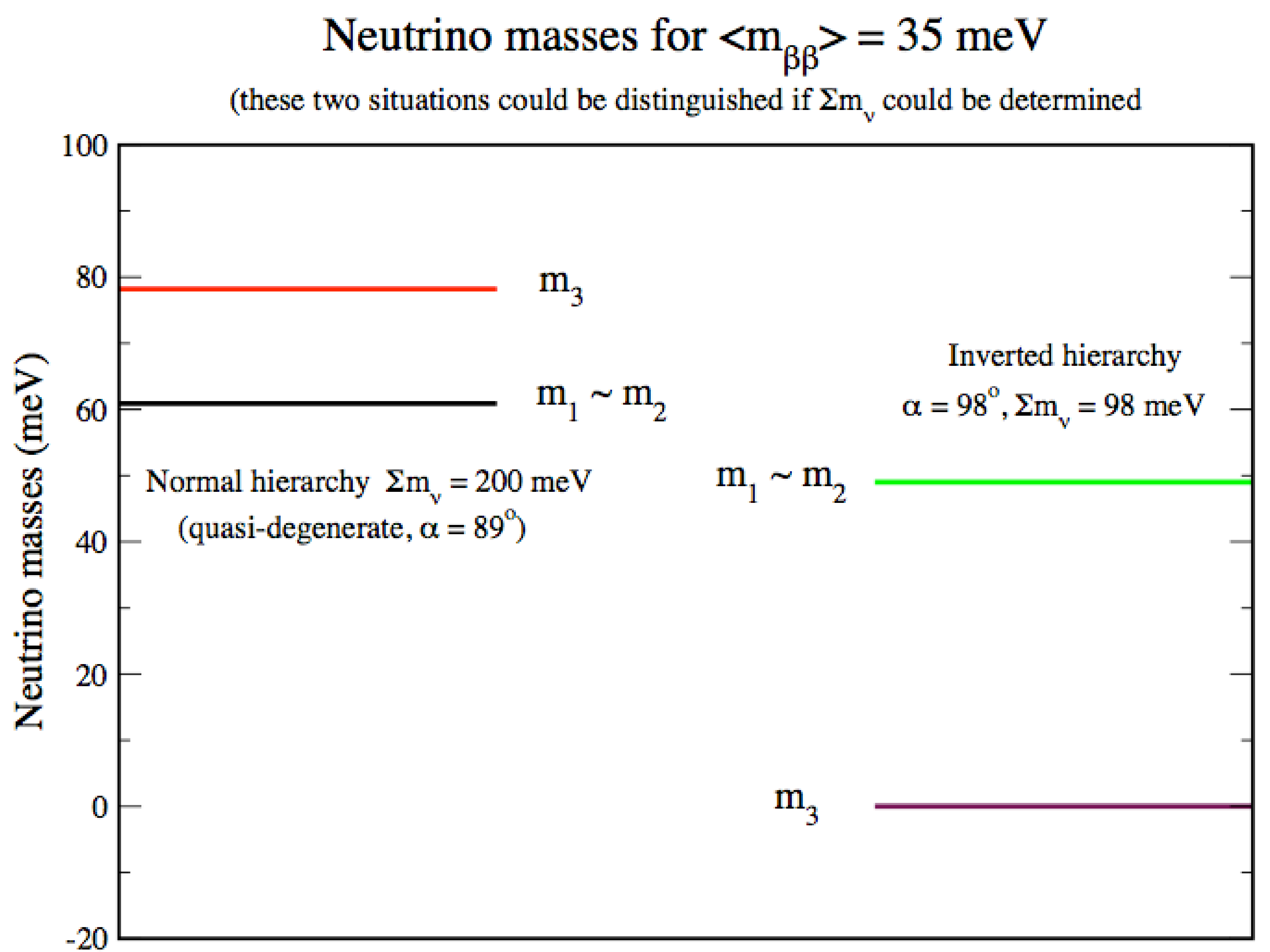
iii) Normal mass hierarchy, $\langle m_{\beta\beta} \rangle < 20$ meV. It would be necessary to use ~100 ton experiments. There are no realistic ideas how to do it.

The degenerate mass region will be explored by the next generation of $0\nu\beta\beta$ experiments and also probed by ways independent on Majorana nature of neutrinos.



However, life is not simple. Even with infinite precision …




Neutrino masses for <mββ> = 35 meV
(these two situations could be distinguished if Σmν could be determined
100
80
60
40
20
0
-20
Neutrino masses (meV)
m3
m1 ~ m2
Normal hierarchy Σmν = 200 meV
(quasi-degenerate, α = 89°)
Inverted hierarchy
α = 98°, Σmν = 98 meV
m1 ~ m2
m3