Single spin asymmetries in hadron-hadron collisions

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We study weighted azimuthal single spin asymmetries at leading order in hadron-hadron scattering using the diagrammatic approach at tree level and assuming factorization. The effects of the intrinsic transverse momenta of the partons are taken into account. We show that the way in which T-odd functions, such as the Sivers function, appear in these processes does not merely involve a sign flip when compared with semi-inclusive deep inelastic scattering, such as in the case of the Drell-Yan process. Expressions for the weighted scattering cross sections in terms of distribution and fragmentation functions folded with hard cross sections are obtained by introducing modified hard cross sections, referred to as gluonic pole cross sections.

I. INTRODUCTION

Accessing the effects arising from the transverse momentum of quarks in hadrons requires hard processes involving at least two hadrons (or hadronic jets) and a hard scale to separate them. This is most cleanly achieved in electroweak processes in which the gauge boson provides the hard scale separating the two hadronic regions. The transition between the hadronic regions and the hard subprocess is described by soft quark and gluon correlation functions, implying approximate collinearity between the quarks, gluons and hadrons involved. Without effects of quark intrinsic transverse momentum these are bilocal, lightlike separated, matrix elements where collinear gluons provide the gauge-link. Transverse momentum dependent correlation functions involve bilocal matrix elements off the lightcone [1]. Here the issue of color gauge-invariance is slightly more complex, involving gauge fields at lightcone infinity [2, 3, 4, 5, 6]. The gauge-link structure may depend on the hard subprocess and leads to observable consequences.

Absorbing the soft physics in the correlation functions requires coupling of, essentially, collinear quarks (and gluons) to the hadronic region. These partons themselves are soft. In the absence of a hard scale from an electroweak boson, as in strong interaction processes, the simplest hard subprocess (large momentum transfer) involving soft quarks and gluons is a two-to-two process.

In this paper we discuss hard hadron-hadron scattering processes using the diagrammatic approach rather than the commonly used helicity approach [7, 8, 9, 10, 11, 12]. This has the advantage that we can directly connect to the matrix elements of quark and gluon fields, without having to go through the step of rewriting them into parton distributions with specific helicities. It allows us to include the effects of collinear gluons, determining the gauge-link structure and to compare this with semi-inclusive deep inelastic scattering (SIDIS) and the Drell-Yan process (DY). We note that in this paper we use the diagrammatic approach and assume the validity of factorization.

We will consider the possibilities to measure transverse moments which are obtained from transverse momentum dependent (TMD) distribution and fragmentation functions upon integration over intrinsic transverse momentum (k_T) including a k_T-weighing. In the transverse moments the effects of the gauge-link structure remains visible. In particular, this is reflected in the time-reversal properties. Time-reversal even or odd behavior can experimentally be distinguished. In single-spin asymmetries at least one (in general an odd number of) T-odd function appears, while in unpolarized processes or double-spin asymmetries an even number of T-odd functions must appear. The importance of considering transverse momentum dependence comes from the fact that for spin 0 and spin $\frac{1}{2}$ hadrons the simple transverse momentum integrated distribution and fragmentation functions, relevant at leading order, are all T-even.

The specific hadronic process that we will consider is the 2-particle inclusive process $H_1+H_2 \to h_1+h_2+X$, which in order to separate the hadronic regions requires minimally a two-to-two hard subprocess. Also included are inclusive

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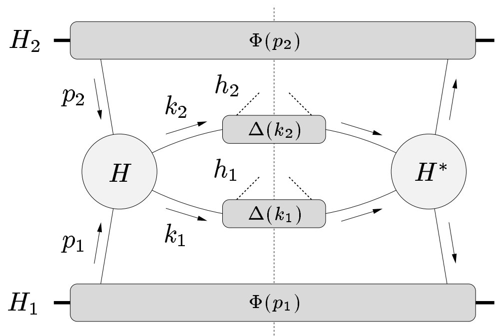


FIG. 1: The leading order contribution to the cross section of $H_1+H_2 \to h_1+h_2+X$.

hadron-jet and jet-jet production in hadron-hadron scattering. The 1-particle inclusive process $p^\uparrow+p \to \pi+X$ involving a transversely polarized proton is known to show a large single-spin asymmetry [13, 14, 15, 16, 17]. Some of the mechanisms [18, 19, 20] to explain these asymmetries involve T-odd functions, such as the Sivers distribution function or the Collins fragmentation function. These functions are expected to appear in a cleaner way in 2-particle inclusive processes [10]. Here we only consider single-hadron fragmentation functions, in which case the 2-particle inclusive production requires h_1 and h_2 to belong to different (in lowest order opposite) jets.

In this paper we limit ourselves to the (anti)quark contributions with as main goal to show the relevant gauge-link structure for the T-odd Sivers distribution functions $f_{1T}^{\perp(1)}$ and the Collins fragmentation functions $H_1^{\perp(1)}$ entering these processes. This is important for the study of universality of these functions. The paper is structured as follows. In section II we consider the kinematics particular to 2→2 particle scattering. In section III we discuss our approach and several (weighted) scattering cross section are written down for hadronic pion production and hadronic jet production in section IV. Details about the gauge-links and their consequences for distribution and fragmentation functions are dealt with in the appendices.

II. KINEMATICS

The hard scale in the process $H_1(P_1)+H_2(P_2) \to h_1(K_1)+h_2(K_2)+X$ is set by the center-of-mass energy $\sqrt{s} = E^{\rm cm}$. The leading order contribution to the scattering cross section is shown in Fig. 1. In a hard scattering process it is important to get as much information about the partonic momenta as possible, in our case including, in particular, their transverse momenta. The partonic momenta, for which $p_1{\cdot}P_1 \sim p_1^2 \sim P_1^2 = M_1^2$ are of hadronic scale, are expanded as follows

$$p_1 = x_1\, P_1 + \sigma_1\, n_1 + p_{1T}\ , \tag{1a}$$
$$p_2 = x_2\, P_2 + \sigma_2\, n_2 + p_{2T}\ , \tag{1b}$$
$$k_1 = z_1^{-1}\, K_1 + \sigma_1'\, n_1' + k_{1T}\ , \tag{1c}$$
$$k_2 = z_2^{-1}\, K_2 + \sigma_2'\, n_2' + k_{2T}\ , \tag{1d}$$

where the n_i (n_i') are lightlike vectors chosen such that $P_1\cdot n_1 \propto \mathcal{O}(s^{1/2})$ and similarly for the other partonic momenta. The fractions $x_i = p_i{\cdot}n_i/P_i{\cdot}n_i$ and $z_i^{-1} = k_i{\cdot}n_i'/K_i{\cdot}n_i'$ are lightcone momentum fractions. The quantity multiplying the vector n_i is the lightcone component conjugate to $p_i{\cdot}n_i$ and is given by

$$\sigma_i = \frac{p_i{\cdot}P_i - x_i\, M_i^2}{P_i{\cdot}n_i}\ , \tag{2}$$

(and similar expressions for σ_i'), quantities which are of order $s^{-1/2}$. If any of the 'parton' momenta is actually an external momentum (for leptons or when describing jets) the momentum fractions become unity and the transverse momenta and σ_i vanish.

Integration over parton momenta is written as

$$d^4p_1 = dx_1\, d^2p_{1T}\, d(p_1{\cdot}P_1)\ , \tag{3}$$

with $d(p_1{\cdot}P_1) = (P_1{\cdot}n_1)\, d\sigma_1$ and similar expressions for d^4p_2, d^4k_1 and d^4k_2. The integrations over the parton momentum components $(p_i \cdot P_i)$ and $(k_i \cdot K_i)$ will be included in the definitions of the TMD distribution and fragmentation

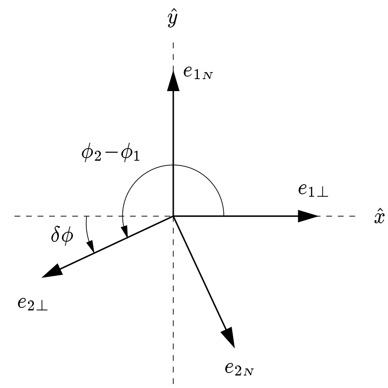


FIG. 2: Plane perpendicular to the incoming hadronic momenta.

functions. Note that the subscripts T have a different meaning in each of the above decompositions, i.e., p_{1T} is transverse to P_1 and n_1, while p_{2T} is transverse to P_2 and n_2, etc. Momentum conservation relates the partonic momenta:

$$p_1 + p_2 - k_1 - k_2 = 0 \ . \tag{4}$$

This relation, however, does not imply that the sum of the intrinsic transverse momenta, $q_T \equiv p_{1T}+p_{2T}-k_{1T}-k_{2T} \approx z_1^{-1}K_1+z_2^{-1}K_2-x_1P_1-x_2P_2$ vanishes.

We use the incoming momenta P_1 and P_2 to define perpendicular momenta $K_{i\perp}$ orthogonal to the incoming hadronic momenta, $K_{i\perp}\cdot P_1 = K_{i\perp}\cdot P_2 = 0$. For the perpendicular momenta it is convenient to scale the variables using $x_{i\perp} = 2|K_{i\perp}|/\sqrt{s}$. For the outgoing hadrons we use the pseudo-rapidities η_i defined by $\eta_i = -\ln\tan(\frac{1}{2}\theta_i)$ where the θ_i are the polar angles of these hadrons in the center-of-mass frame. All invariants involving the external momenta can be expressed in terms of these variables:

$$P_1 \cdot K_1 = \tfrac{1}{4}s\, x_{1\perp}\, e^{-\eta_1} \ , \qquad P_2 \cdot K_1 = \tfrac{1}{4}s\, x_{1\perp}\, e^{+\eta_1} \ , \tag{5a}$$

$$P_1 \cdot K_2 = \tfrac{1}{4}s\, x_{2\perp}\, e^{-\eta_2} \ , \qquad P_2 \cdot K_2 = \tfrac{1}{4}s\, x_{2\perp}\, e^{+\eta_2} \ , \tag{5b}$$

and $P_1\cdot P_2 = \frac{1}{2}s$. These identities are valid up to subleading order in $\sqrt{s}$. The remaining invariant $K_1\cdot K_2$ is not independent of the others. *To leading order*, one has

$$K_1 \cdot K_2 = \tfrac{1}{2}s\, x_{1\perp}x_{2\perp}\, \cosh^2\left[\tfrac{1}{2}(\eta_1-\eta_2)\right] \ . \tag{5c}$$

To subleading order the outgoing hadronic momenta can now be written as

$$K_1 = \frac{(K_1\cdot P_2)\, P_1 + (K_1\cdot P_1)\, P_2}{P_1\cdot P_2} + K_{1\perp} = \tfrac{1}{2}x_{1\perp}\left(e^{+\eta_1}P_1 + e^{-\eta_1}P_2\right) + K_{1\perp} \ , \tag{6a}$$

$$K_2 = \frac{(K_2\cdot P_2)\, P_1 + (K_2\cdot P_1)\, P_2}{P_1\cdot P_2} + K_{2\perp} = \tfrac{1}{2}x_{2\perp}\left(e^{+\eta_2}P_1 + e^{-\eta_2}P_2\right) + K_{2\perp} \ . \tag{6b}$$

The two perpendicular vectors $K_{1\perp}$ and $K_{2\perp}$ are approximately back-to-back (see Fig. 2). Sometimes the Feynman variables $x_{iF} = K_{iz}^{\rm cm}/K_{iz}^{\rm cm(max)} = x_{i\perp}\sinh\eta_i$ are used in the literature. Another useful variable in writing down cross sections is the quantity y, which is defined via the Mandelstam variables of the partonic subprocess, and can be related to the pseudo-rapidities,

$$y = -\frac{\hat{t}}{\hat{s}} = \sqrt{\frac{(P_1\cdot K_1)(P_2\cdot K_2)}{(P_1\cdot P_2)(K_1\cdot K_2)}} = \frac{1}{e^{(\eta_1-\eta_2)}+1} \ . \tag{7}$$

Dividing $K_{1\perp}$ and $K_{2\perp}$ by the momentum fractions one immediately sees from the decompositions of the partonic momenta that the vector

$$r_\perp = \frac{K_{1\perp}}{z_1} + \frac{K_{2\perp}}{z_2} \ , \tag{8}$$

only involves transverse momenta of partons. It is just the small projection of the transverse momentum in the perpendicular plane, $r_\perp \approx q_{T\perp}$. The vectors $K_{i\perp}$ themselves are not 'small' vectors. They are spacelike vectors with invariant length of $\mathcal{O}(\sqrt{s})$. In the analysis of the kinematics in the transverse plane the momentum fractions are not direct observables. In particular, experimentally it is more convenient to work with the directions of the vectors $K_{i\perp}$ and the corresponding orthogonal directions (see Fig. 2)

$$e_{1\perp}^\mu = \frac{K_{1\perp}^\mu}{|K_{1\perp}|} \ , \qquad e_{1N}^\mu = -\frac{2}{s}\,\frac{\epsilon^{P_1 P_2 K_1\,\mu}}{|K_{1\perp}|} = \epsilon_\perp^{\mu\nu}\, e_{1\perp\,\nu} \ , \tag{9}$$

and similarly for $K_{2\perp}$. As illustrated in Fig. 2, the direction $e_{2\perp}$ is, up to a (small)angle $\delta\phi \equiv \phi_2{-}\phi_1{-}\pi \propto \mathcal{O}(|p_T|/\sqrt{s})$, opposite to $e_{1\perp}$.

The momentum conservation relation in Eq. (4) is enforced by a delta function in the scattering cross section. The delta function can be decomposed using the basis constructed in the previous paragraph. For $R = p_1{+}p_2{-}k_1{-}k_2$ this decomposition reads

$$\delta^4(R) = \tfrac{1}{2}s\,\delta(R{\cdot}P_1)\,\delta(R{\cdot}P_2)\,\delta^2(R_\perp) = \tfrac{1}{2}s\,\delta(R{\cdot}P_1)\,\delta(R{\cdot}P_2)\,\delta^2(q_{T\perp}{-}r_\perp) \ . \tag{10}$$

The arguments of the first two delta functions involve large momenta and can be used to relate the momentum fractions x_1 and x_2 to kinematical observables. For the latter two delta functions the treatment depends on the situation. Using the orthogonal vectors $e_{1\perp}$ and e_{1N} we get, up to $\mathcal{O}(1/\sqrt{s})$,

$$R{\cdot}e_{1\perp} = e_{1\perp}{\cdot}q_T + \left(\frac{x_{1\perp}}{z_1} - \frac{x_{2\perp}}{z_2}\right)\frac{\sqrt{s}}{2} \ , \tag{11a}$$

$$R{\cdot}e_{1N} = e_{1N}{\cdot}q_T - \frac{x_{2\perp}}{z_2}\,\frac{\sqrt{s}}{2}\,\sin(\delta\phi) \ . \tag{11b}$$

In the case of two-hadron or hadron-jet production ($z_2 = 1$), the first delta function implies that at leading order $x_{1\perp}/z_1 \approx x_{2\perp}/z_2 \equiv x_\perp$, which is interpreted as the scaled parton perpendicular momentum, $x_\perp = 2\,|k_{2\perp}|/\sqrt{s}$. Using the variable $x_\perp$ as an integration variable we can write

$$\begin{aligned}\delta^4(p_1{+}p_2{-}k_1{-}k_2) = \frac{4}{s^2}\,\frac{1}{x_{1\perp}\,x_{2\perp}}\int dx_\perp\ &\delta\big(\,x_1 - \tfrac{1}{2}x_\perp(e^{\eta_1}{+}e^{\eta_2})\,\big)\,\delta\big(\,x_2 - \tfrac{1}{2}x_\perp(e^{-\eta_1}{+}e^{-\eta_2})\,\big)\\ &\times\ \delta\Big(\,z_1^{-1} - \frac{x_\perp}{x_{1\perp}}\,\Big)\,\delta\Big(\,z_2^{-1} - \frac{x_\perp}{x_{2\perp}}\,\Big)\,\delta\Big(\,\frac{e_{1N}{\cdot}q_T}{\sqrt{s}} - \frac{x_\perp}{2}\,\sin(\delta\phi)\,\Big) \ ,\end{aligned} \tag{12}$$

which shows that in one- or two-particle inclusive processes we are always left with a convolution of distribution and fragmentation functions over one momentum fraction or, equivalently, over the parton perpendicular momentum variable $x_\perp$. The last delta function shows explicitly that $\sin(\delta\phi) \propto 1/\sqrt{s}$ and that it can be used to construct cross sections weighted with one component of the intrinsic transverse momentum, i.e. $e_{1N}{\cdot}q_T$.

In the case that $K_1 = k_1$ and $K_2 = k_2$ (i.e. $z_i = 1$, $k_{iT} = 0$), such as in production of a lepton pair in Drell-Yan scattering or the (idealized) production of two jets, the delta function $\delta(R{\cdot}e_{1\perp})$ also relates intrinsic transverse momenta to observed momenta and, therefore, one can construct azimuthal asymmetries involving two components of q_T. In fact, the product of delta functions $\delta^2(R_\perp) = \delta(R{\cdot}e_{1\perp})\,\delta(R{\cdot}e_{1N})$ can be used to weigh with the transverse momenta $p_{1T}{+}p_{2T}$, as they relate $q_T = p_{1T}{+}p_{2T}$ to $q \equiv k_1{+}k_2$ in the orthogonal plane. With the natural choice of the n-vectors in the case that only two hadrons are involved such that $P_{1T} = P_{2T} = 0$, one obtains the familiar relation $p_{1T}{+}p_{2T} = q_T = q{-}x_1P_1{-}x_2P_2$, leading to

$$\delta^4(p_1{+}p_2{-}k_1{-}k_2) = \frac{2}{s}\,\delta\Big(x_1 - \frac{P_2{\cdot}q}{P_1{\cdot}P_2}\Big)\,\delta\Big(x_2 - \frac{P_1{\cdot}q}{P_1{\cdot}P_2}\Big)\,\delta^2\Big(p_{1T}{+}p_{2T}{-}q_T(q,P_1,P_2)\Big) \ . \tag{13}$$

III. CROSS SECTIONS

The scattering cross section for $p_1p_2 \to h_1h_2X$ (see Fig. 1) at tree-level is written as

$$d\sigma = \frac{1}{2s}\,|\mathcal{M}|^2\,\frac{d^3K_1}{(2\pi)^3\,2E_{K_1}}\,\frac{d^3K_2}{(2\pi)^3\,2E_{K_2}} \ , \tag{14}$$

where the matrix element is expressed in terms of hard amplitudes and correlation functions (see appendix A). It is given by

$$
\begin{aligned}
|\mathcal{M}|^2 = \int & dx_1 d^2p_{1T}\, dx_2 d^2p_{2T}\, dz_1^{-1} d^2k_{1T}\, dz_2^{-1} d^2k_{2T}\ (2\pi)^4\delta^4(p_1{+}p_2{-}k_1{-}k_2) \\
& \times\ \mathrm{Tr}\left\{\, \Phi(x_1,p_{1T})\Phi(x_2,p_{2T})\Delta(z_1,k_{1T})\Delta(z_2,k_{2T})H(p_1,p_2,k_1,k_2)H^*(p_1,p_2,k_1,k_2)\,\right\}\ .
\end{aligned}
\tag{15}
$$

The trace involves the appropriate contraction of Dirac indices in soft and hard scattering parts. A summation over color and quark flavors is understood. The phase-space elements are given by

$$
\frac{d^3K_i}{(2\pi)^3\, 2E_{K_i}} = \frac{x_{i\perp}\, s}{8\,(2\pi)^2}\, dx_{i\perp}\, d\eta_i\, \frac{d\phi_i}{2\pi}\ . \tag{16}
$$

Combining the phase space integration and the delta functions coming from partonic energy-momentum conservation, one has for back-to-back hadron-hadron production

$$
\begin{aligned}
d\sigma[h_1h_2] = \frac{1}{32\,s}\, dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\,\frac{d\phi_2}{2\pi} \int dx_\perp \int & d^2p_{1T}\, d^2p_{2T}\, d^2k_{1T}\, d^2k_{2T}\ \delta\Big(\frac{e_{1N}{\cdot}q_T}{\sqrt{s}} - \frac{x_\perp}{2}\,\sin(\delta\phi)\Big) \\
\times\ \mathrm{Tr}\left\{\Phi(x_1,p_{1T})\Phi(x_2,p_{2T})\Delta(z_1,k_{1T})\Delta(z_2,k_{2T})\right. & \left. H(p_1,p_2,k_1,k_2)H^*(p_1,p_2,k_1,k_2)\right\}\ .
\end{aligned}
\tag{17}
$$

In this expression the momentum fractions are fixed by the arguments of the delta functions in Eq. (12), i.e. one has $x_1(x_\perp,\eta_1,\eta_2)$, $x_2(x_\perp,\eta_1,\eta_2)$, $z_1(x_\perp,x_{1\perp})$, and $z_2(x_\perp,x_{2\perp})$. Since $x_{1\perp}\le x_\perp$ and $x_{2\perp}\le x_\perp$, the integration over $x_\perp$ is bounded from below.

In the hadron-jet inclusive process one has $\Delta(z_2,k_{2T}) = \delta(z_2{-}1)\,\delta^2(k_{2T})\,\not{k}_2 = x_\perp\,\delta(x_{2\perp}{-}x_\perp)\,\delta^2(k_{2T})\,\not{k}_2$, which implies that $z_2=1$ and $x_\perp = x_{2\perp}$. The cross section becomes

$$
\begin{aligned}
d\sigma[h_1j_2] = \frac{x_{2\perp}}{32\,s}\, dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\,\frac{d\phi_2}{2\pi} \int & d^2p_{1T}\, d^2p_{2T}\, d^2k_{1T}\ \delta\Big(\frac{e_{1N}{\cdot}q_T}{\sqrt{s}} - \frac{x_{2\perp}}{2}\,\sin(\delta\phi)\Big) \\
& \times\ \mathrm{Tr}\left\{\Phi(x_1,p_{1T})\Phi(x_2,p_{2T})\Delta(z_1,k_{1T})\, H(p_1,p_2,k_1,k_2)H^*(p_1,p_2,k_1,k_2)\right\}\ .
\end{aligned}
\tag{18}
$$

As stated in the previous section, in back-to-back jet production both delta functions in the perpendicular plane relate observed kinematical variables to intrinsic transverse momenta. Therefore, in jet-jet production we use the expression in Eq. (13) rather than Eq. (12) for the momentum conserving delta function, leading to

$$
\begin{aligned}
d\sigma[j_1j_2] = \frac{x_{1\perp}\,x_{2\perp}}{64}\, dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\,\frac{d\phi_2}{2\pi} \int & d^2p_{1T}\, d^2p_{2T}\ \delta^2(p_{1T}{+}p_{2T}{-}q_T) \\
& \times\ \mathrm{Tr}\left\{\Phi(x_1,p_{1T})\Phi(x_2,p_{2T})\, H(p_1,p_2,k_1,k_2)H^*(p_1,p_2,k_1,k_2)\right\}\ ,
\end{aligned}
\tag{19}
$$

where $z_1 = z_2 = 1$, $x_{1\perp} = x_{2\perp} = x_\perp$ and $q_T = q{-}x_1P_1{-}x_2P_2$.

In averaged and weighted cross sections we will encounter contractions of hard and soft pieces like

$$
\begin{aligned}
\Sigma(x_1,x_2,z_1,z_2,y) &= \int d^2p_{1T}\, d^2p_{2T}\, d^2k_{1T}\, d^2k_{2T}\ \ \mathrm{Tr}\left\{\Phi(x_1,p_{1T})\Phi(x_2,p_{2T})\Delta(z_1,k_{1T})\Delta(z_2,k_{2T})\, H\, H^*\right\} \\
&= \mathrm{Tr}\left\{\, \Phi(x_1)\Phi(x_2)\Delta(z_1)\Delta(z_2)\, H\, H^*\,\right\}\ ,
\end{aligned}
\tag{20}
$$

and

$$
\begin{aligned}
\Sigma_\partial^\alpha(x_1,x_2,z_1,z_2,y) &= \int d^2p_{1T}\, d^2p_{2T}\, d^2k_{1T}\, d^2k_{2T}\ q_T^\alpha\ \mathrm{Tr}\left\{\Phi(x_1,p_{1T})\Phi(x_2,p_{2T})\Delta(z_1,k_{1T})\Delta(z_2,k_{2T})\, H\, H^*\right\} \\
&= \mathrm{Tr}\Big\{\, \big[\, \Phi_\partial^\alpha(x_1)\Phi(x_2)\Delta(z_1)\Delta(z_2) + \Phi(x_1)\Phi_\partial^\alpha(x_2)\Delta(z_1)\Delta(z_2) \\
&\qquad - \Phi(x_1)\Phi(x_2)\Delta_\partial^\alpha(z_1)\Delta(z_2) - \Phi(x_1)\Phi(x_2)\Delta(z_1)\Delta_\partial^\alpha(z_2)\big]\, H\, H^*\,\Big\}\ .
\end{aligned}
\tag{21}
$$

These expressions for hadron-hadron scattering (and similar ones for hadron-jet and jet-jet scattering) are schematic in the sense that the tracing depends on the particular term in the sum of squared amplitudes, including both direct and interference diagrams when the hard amplitude contains more than one contribution.

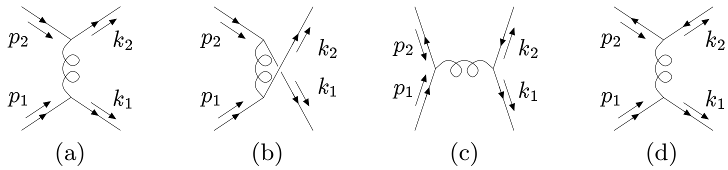


FIG. 3: Hard scattering amplitudes for quark-quark scattering: (a) t-diagram, (b) u-diagram; quark-antiquark scattering: (c) s-diagram, (d) t-diagram.

In the case of hadron-hadron or hadron-jet cross sections one finds averaged cross sections like

$$\begin{aligned}\langle\, d\sigma[h_1h_2]\,\rangle &= \int d\phi_2\;\frac{d\sigma[h_1h_2]}{d\phi_2}\\ &= \frac{dx_{1\perp}\,dx_{2\perp}\,d\eta_1\,d\eta_2}{32\,\pi\,s}\;\frac{d\phi_1}{2\pi}\int\frac{dx_\perp}{x_\perp}\;\Sigma(x_1,x_2,z_1,z_2,y)\;, \qquad (22a)\\ \langle\,\tfrac{1}{2}\sin(\delta\phi)\,d\sigma[h_1h_2]\,\rangle &= \int d\phi_2\;\tfrac{1}{2}\sin(\delta\phi)\,\frac{d\sigma[h_1h_2]}{d\phi_2}\\ &= \frac{dx_{1\perp}\,dx_{2\perp}\,d\eta_1\,d\eta_2}{32\,\pi\,s^{3/2}}\;\frac{d\phi_1}{2\pi}\int\frac{dx_\perp}{x_\perp^2}\;e_{1N}\!\cdot\!\Sigma_\partial(x_1,x_2,z_1,z_2,y)\;. \qquad (22b)\end{aligned}$$

We would like to note that in Eq. (22b), one is weighing with a dimensionless quantity which leads to a suppression with $1/\sqrt{s}$. For jet-jet cross sections one has, in principle, the possibility to access both perpendicular directions of Σ_∂^α, assuming that $q = k_1{+}k_2$ is known accurately. One could, then, weigh with q_T^α, in analogy to the Drell-Yan process [21]. In that case one weighs with dimensionful quantities, even if these are small momenta, and one does not get additional suppression involving the hard scale.

These equations will be the starting point in the calculation of cross sections. One needs to calculate the quantities in Eqs. (20) and (21). These expressions involve hard scattering amplitudes and soft correlators Φ and Δ, which are obtained as Fourier transforms of matrix elements of nonlocal combinations of quark and gluon fields. They are parametrized in terms of distribution and fragmentation functions as presented in appendix B and C. In order to render the correlators color gauge invariant a gauge-link connecting the fields is needed. In the diagrammatic calculation, gauge-links are explicitly found by taking into account, for each of the hadrons, the interactions of collinear gluons (polarizations along hadron momentum) between the soft and hard parts. These give the well-known straight-line gauge-links along the lightcone for transverse momentum integrated correlators [22], but they lead to nontrivial gauge-link paths for the TMD correlators [3, 6, 23]. The integration paths in the gauge-links $\mathcal{U}$ are process dependent, depending in particular on the hard partonic subprocesses. We indicate this dependence by a superscript $\Phi^{[\mathcal{U}]}(x,p_T)$.

The transverse momentum integrated correlator is a lightcone correlator with a unique gauge-link, in which the path dependence disappears:

$$\Phi^{[\mathcal{U}]}(x) = \int d^2p_T\;\Phi^{[\mathcal{U}]}(x,p_T) = \Phi(x)\;. \qquad (23)$$

For the transverse moments of the correlators obtained after p_T-weighing, of which we only consider the simplest one, one finds two types of lightcone correlators, a quark-quark matrix element Φ_∂ and a gluonic pole matrix element Φ_G, where the latter is multiplied by a factor that depends on the gauge-link

$$\Phi_\partial^{[\mathcal{U}]\alpha}(x) = \int d^2p_T\;p_T^\alpha\,\Phi^{[\mathcal{U}]}(x,p_T) = \Phi_\partial^\alpha(x) + C_G^{[\mathcal{U}]}\,\pi\Phi_G^\alpha(x,x)\;. \qquad (24)$$

This gluonic pole matrix element, which contains the T-odd distribution functions, was suggested in a slightly different context by Qiu and Sterman [24, 25] as the origin of single spin asymmetries. In processes like SIDIS with underlying hard process $\ell{+}q \to \ell{+}q$ and the DY process with underlying hard process $q{+}\bar{q} \to \ell{+}\bar{\ell}$, different gauge-link paths $\mathcal{U}^{[+]}$ and $\mathcal{U}^{[-]}$ appear. In these processes the corresponding factors in Eq. (24) are simply $C_G^{[\mathcal{U}^{[\pm]}]} = \pm 1$.

As was shown in Ref. [23], more complex paths enter when other subprocesses are involved, such as the two-to-two (anti)quark subprocesses in this paper. Moreover, in general several diagrams enter in the calculation. For instance, for quark-antiquark scattering both t- and s-channel amplitudes (see Fig. 3) can contribute, $H = H_{q\bar{q}}^t{+}H_{q\bar{q}}^s$, whereas for quark-quark scattering we get t- and u-channel amplitudes, $H_{qq} = H_{qq}^t{+}H_{qq}^u$. One must consider the additional diagrams that produce the gauge-links needed to render the correlation functions color gauge invariant. In these cases the gauge-links, in general, also differ for the various terms appearing in the squared amplitude $HH^\dagger$ for a given partonic subprocess, for instance for the terms $H^tH^{t\dagger}$, $H^uH^{u\dagger}$, $H^tH^{u\dagger}$ and $H^uH^{t\dagger}$ in scattering of two identical quarks. Details are explained in appendix A.

The results of the diagrammatic calculation for transverse momentum integrated cross sections involving soft and hard parts can, in leading order, be recast in the form of a folding of the quark distribution and fragmentation functions appearing in the transverse momentum integrated correlators $\Phi(x)$ and $\Delta(z)$ with hard partonic cross sections. For SIDIS one has a folding with the cross section for the hard process $\ell{+}q \to \ell{+}q$ and in the DY process a folding with the cross section for $q{+}\bar{q} \to \ell{+}\bar{\ell}$. In hadron-hadron scattering one has several hard processes. An example is qq scattering with a cross section, in the case of identical quark flavors, of the form

$$\frac{d\hat{\sigma}_{qq\to qq}}{d\hat{t}} = \sum_D \frac{d\hat{\sigma}^{[D]}_{qq\to qq}}{d\hat{t}} \ , \tag{25}$$

where the summation is over the different direct and interference contributions involving the t- and u-channel amplitudes.

A folding of distribution and fragmentation functions is also possible for weighted cross sections. The cross sections involving the link-independent parts of the transverse moments (i.e. $\Phi_\partial(x)$ and $\Delta_\partial(x)$) also lead to a folding with the normal partonic cross sections, just as for the integrated correlators $\Phi(x)$ and $\Delta(z)$. For the contractions with the gluonic pole matrix elements $\pi\Phi_G(x,x)$ and $\pi\Delta_G(x,x)$, however, the gauge-link dependence in the decomposition in Eq. (24) has important ramifications. Expressing the asymmetries as a folding of universal, one argument functions and a hard part requires a modification of the hard cross section by including the gauge-link dependent factors $C_G^{[\mathcal{U}]}$ in the various terms in these cross sections. This is a convenient way of doing since the value of these factors depends on these terms. For instance, in the example of unpolarized qq scattering for identical flavors, the functions in the gluonic pole matrix elements are folded with the *gluonic pole cross section*

$$\frac{d\hat{\sigma}_{\widehat{gq}q\to qq}}{d\hat{t}} = \sum_D C_G^{[\mathcal{U}(D)]}\,\frac{d\hat{\sigma}^{[D]}_{qq\to qq}}{d\hat{t}} \ . \tag{26}$$

The notation $\widehat{gq}$ emphasizes which quark field (in this case the first one), accompanied by a zero momentum gluon field, enters in the correlator $\pi\Phi_G(x,x)$. The parametrization of this correlator involves one-argument distribution functions, which will appear folded with the gluonic pole cross sections. At tree level often only one diagram enters. In that case the gluonic pole cross section is simply proportional to the normal partonic cross section. For instance, the sign difference between SIDIS and DY for the Sivers distribution function, a uniquely defined function appearing in the parametrization of $\pi\Phi_G(x,x)$, comes from the factors $C_G^{[\mathcal{U}^{[\pm]}]} = \pm 1$ discussed above. Instead of the folding with partonic cross sections, the Sivers function is folded with the gluonic pole cross sections

$$\frac{d\hat{\sigma}_{\ell\widehat{gq}\to\ell q}}{d\hat{t}} = +\frac{d\hat{\sigma}_{\ell q\to\ell q}}{d\hat{t}} \ , \tag{27a}$$

$$\frac{d\hat{\sigma}_{\widehat{gq}\bar{q}\to\ell\bar{\ell}}}{d\hat{t}} = -\frac{d\hat{\sigma}_{q\bar{q}\to\ell\bar{\ell}}}{d\hat{t}} \ . \tag{27b}$$

Although the gluonic pole cross sections should not be interpreted as true partonic cross sections, their concept is convenient in order to get a simple folding expression for the one-argument functions appearing in the gluonic pole matrix element $\pi\Phi_G(x,x)$ and they are easily obtained from the terms in the hard partonic cross section without soft gluons.

In the following sections the formalism described above is applied to single spin asymmetries in inclusive two-hadron production, hadron-jet and jet-jet production in $p^\uparrow p$ scattering, for which the gluonic pole cross sections for polarized (anti)quark scattering are also needed.

IV. SINGLE-SPIN ASYMMETRIES IN INCLUSIVE HADRON-HADRON SCATTERING

As a reference we first consider the cross section for 2-particle inclusive hadron-hadron scattering. The explicit expression for the cross section in terms of the distribution and fragmentation functions can be obtained by inserting the parametrizations of the correlators, Eqs. (B7) and (C8) in appendices B and C into Eq. (22a) and performing the required traces. In this paper we restrict ourselves to the quark and antiquark scattering contributions. Since the short-distance scattering subprocesses remain unobserved, all partonic subprocesses that could contribute have to be taken into account. This includes, for a realistic description, besides the (anti)quark contributions $qq{\to}qq$, $q\bar{q}{\to}q\bar{q}$ and $\bar{q}\bar{q}{\to}\bar{q}\bar{q}$, also contributions involving gluons, $qg{\to}qg$, $\bar{q}g{\to}\bar{q}g$, $gg{\to}gg$, $q\bar{q}{\to}gg$ and $gg{\to}q\bar{q}$ including their polarizations [9, 10, 11, 12, 26, 27, 28]. However, the (anti)quark contributions suffice to illustrate how the inclusion of gauge-links leads to altered strengths of specific distribution or fragmentation functions. Contributions involving gluons can simply be added incoherently to the results presented here. For the (anti)quark contributions to the averaged cross section one obtains

$$
\begin{aligned}
\langle\, d\sigma[h_1h_2]\,\rangle = dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\int\frac{dx_\perp}{x_\perp} \\
\times\ \frac{2\pi\,\alpha_s^2}{9\,\hat{s}}\Big\{\ &\big((1-y)^2+y^2\big)\sum_{q,q'} f_1^q(x_1)\bar{f}_1^q(x_2)D_1^{q'}(z_1)\bar{D}_1^{q'}(z_2) &\text{(28a)}\\
&+\frac{(1-y)^2+1}{y^2}\sum_{q,q'} f_1^q(x_1)f_1^{q'}(x_2)D_1^q(z_1)D_1^{q'}(z_2) &\text{(28b)}\\
&+\frac{(1-y)^2+1}{y^2}\sum_{q,q'} f_1^q(x_1)\bar{f}_1^{q'}(x_2)D_1^q(z_1)\bar{D}_1^{q'}(z_2) &\text{(28c)}\\
&+\frac{2}{3}\frac{(1-y)^2}{y}\sum_{q} f_1^q(x_1)\bar{f}_1^q(x_2)D_1^q(z_1)\bar{D}_1^q(z_2) &\text{(28d)}\\
&-\frac{1}{3}\frac{1}{y(1-y)}\sum_{q} f_1^q(x_1)f_1^q(x_2)D_1^q(z_1)D_1^q(z_2) &\text{(28e)}\\
&+\big(\text{quark PDFs/FFs}\leftrightarrow\text{antiquark PDFs/FFs}\big)\ \Big\}+\big(K_1\leftrightarrow K_2\big)
\end{aligned}
$$

where the summation is over all quark flavors, including the case that $q = q'$. In this expression y is given by Eq. (7) and $\hat{s}$ is $\hat{s} = x_\perp^2\, s\cosh^2[\frac{1}{2}(\eta_1{-}\eta_2)] = x_\perp^2\, s/4y(1{-}y)$. This result can be recast into a folding of the distribution and fragmentation functions appearing in $\Phi(x)$ and $\Delta(z)$ and the elementary (anti)quark cross sections given in appendix E. That is, expression (28) can be rewritten to

$$
\langle\, d\sigma[h_1h_2]\,\rangle = dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\int\frac{dx_\perp}{x_\perp}\sum_{q_1q_2q_3q_4} f_1^{q_1}(x_1)f_1^{q_2}(x_2)\,\frac{\hat{s}}{2}\,\frac{d\hat{\sigma}_{q_1q_2\to q_3q_4}}{d\hat{t}}\,D_1^{q_3}(z_1)D_1^{q_4}(z_2)\ , \tag{29}
$$

where the summation is over all quark and antiquark flavors. In the expressions above the momentum fractions are fixed by $x_{1/2} = \frac{1}{2}x_\perp\big(e^{\pm\eta_1}{+}e^{\pm\eta_2}\big)$ and $z_i = x_{i\perp}/x_\perp$.

A. Hadron-hadron production in $p^\uparrow p$ scattering: $p^\uparrow{+}p\to\pi{+}\pi{+}X$

With only one of the hadrons polarized, any nonzero spin asymmetry must involve at least one T-odd function. Restricting ourselves to hadrons with spin 0 and $\frac{1}{2}$, such functions do not show up in the transverse momentum integrated correlators $\Phi(x)$ and $\Delta(z)$. They do appear in the parametrization of the matrix elements involved in the decomposition of the transverse moments of the correlators. T-odd distribution functions only appear in the gluonic pole matrix element $\pi\Phi_G$, while T-odd fragmentation functions can appear in both the matrix elements Δ_∂ and $\pi\Delta_G$ [6]. Using the parametrizations for these functions, one can calculate $e_{1_N}{\cdot}\Sigma_\partial(x_1,x_2,z_1,z_2,y)$ and find the expression for the weighted cross section using Eq. (22b). Considering only the (anti)quark contributions in $p^\uparrow{+}p\to\pi{+}\pi{+}X$ the resulting cross section is explicitly given in appendix D, including in each term explicitly the factor $C_G^{[\mathcal{U}]}$ between braces $\{\cdot\}$.

The results can most conveniently be expressed as a folding of distribution and fragmentation functions, now including always one T-odd function and a *gluonic pole cross section*,

$$
\begin{aligned}
&\langle\, \tfrac{1}{2}\sin(\delta\phi)\, d\sigma[h_1 h_2]\,\rangle \\
&\qquad = dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\ \cos(\phi_1^S) \int \frac{dx_\perp}{x_\perp} \\
&\qquad\quad \times \Bigg\{ \frac{M_1}{x_\perp\sqrt{s}} \sum_{q_1q_2q_3q_4} f^{q_1}{}_{1T}^{\perp(1)}(x_1) f_1^{q_2}(x_2)\, \frac{\hat{s}}{2}\, \frac{d\hat{\sigma}_{\widehat{gq}_1 q_2\to q_3q_4}}{d\hat{t}}\, D_1^{q_3}(z_1) D_1^{q_4}(z_2) && \text{(30a)}\\
&\qquad\qquad + \frac{M_2}{x_\perp\sqrt{s}} \sum_{q_1q_2q_3q_4} h_1^{q_1}(x_1) h^{q_2}{}_1^{\perp(1)}(x_2)\, \frac{\hat{s}}{2}\, \frac{d\Delta\hat{\sigma}_{q_1^\uparrow \widehat{gq}_2^\uparrow\to q_3q_4}}{d\hat{t}}\, D_1^{q_3}(z_1) D_1^{q_4}(z_2) && \text{(30b)}\\
&\qquad\qquad - \frac{M_{h_1}}{x_\perp\sqrt{s}} \sum_{q_1q_2q_3q_4} h_1^{q_1}(x_1) f_1^{q_2}(x_2)\, \frac{\hat{s}}{2}\, \frac{d\Delta\hat{\sigma}_{q_1^\uparrow q_2\to q_3^\uparrow q_4}}{d\hat{t}}\, H^{q_3}{}_1^{\perp(1)}(z_1) D_1^{q_4}(z_2)\ + \big(K_1\leftrightarrow K_2\big) && \text{(30c)}\\
&\qquad\qquad - \frac{M_{h_1}}{x_\perp\sqrt{s}} \sum_{q_1q_2q_3q_4} h_1^{q_1}(x_1) f_1^{q_2}(x_2)\, \frac{\hat{s}}{2}\, \frac{d\Delta\hat{\sigma}_{q_1^\uparrow q_2\to \widehat{gq}_3^\uparrow q_4}}{d\hat{t}}\, \widetilde{H}^{q_3}{}_1^{\perp(1)}(z_1) D_1^{q_4}(z_2)\ + \big(K_1\leftrightarrow K_2\big) \Bigg\} && \text{(30d)}
\end{aligned}
$$

where the summations run over all quark and antiquark flavors and the angle ϕ_1^S is defined by $\phi_1^S = \phi_1 - \phi_S$. All non-vanishing partonic scattering cross sections and gluonic pole cross sections are functions of y or, equivalently, $\eta_1 - \eta_2$ and those that contribute to hadronic pion production are listed in appendix E.

We note the occurrence of *one* T-odd function in each of the terms in Eq. (30), the functions $f_{1T}^{\perp(1)}(x)$ and $h_1^{\perp(1)}(x)$ coming from the gluonic pole matrix element $\pi\Phi_G(x,x)$, the function $H_1^{\perp(1)}(z)$ coming from the link-independent correlator Δ_∂ and the function $\widetilde{H}_1^{\perp(1)}(z)$ coming from the gluonic pole matrix element $\pi\Delta_G$. We would, once more, like to emphasize that for fragmentation both Δ_∂ and $\pi\Delta_G$ contain T-odd functions contributing to the Collins effect. In Ref. [29, 30] it is argued that the function $H_1^{\perp(1)}$ is universal, confirmed in several model calculations [31]. This situation would occur if gluonic pole matrix elements in the case of fragmentation into final state hadrons vanish, in which case the function $\widetilde{H}_1^{\perp(1)}$ vanishes and all T-odd effects come from the 'universal' function $H_1^{\perp(1)}$. This latter function appears folded with ordinary partonic cross sections. In this paper, however, we will allow for the gluonic pole matrix element and a nonvanishing function $\widetilde{H}_1^{\perp(1)}$ for fragmentation.

B. Hadron-jet production in $p^\uparrow p$ scattering: $p^\uparrow + p \to \pi + \mathrm{Jet} + X$

We only take into account (anti)quark scattering processes in the weighted scattering cross section for $p^\uparrow + p \to \pi + \mathrm{Jet} + X$ with the pion and the jet approximately back-to-back in the perpendicular plane. This cross section can be obtained from the more involved two-particle inclusive scattering cross section (D1) by taking $D_1^q(z_2) = \delta(z_2 - 1)\delta^{j_2 q} = x_\perp\, \delta(x_{2\perp} - x_\perp)\delta^{j_2 q}$ and by letting all other fragmentation functions vanish. Here $\delta^{j_2 q}$ is a delta function in flavor space, indicating that the jet j_2 is produced by quark q. The explicit expression using the diagrammatic approach is given in appendix D. Recast into distribution and fragmentation functions folded with gluonic pole cross sections, we obtain

$$
\begin{aligned}
&\langle\, \tfrac{1}{2}\sin(\delta\phi)\, d\sigma[h_1 j_2]\,\rangle \\
&\qquad = dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\ \cos(\phi_1^S) \\
&\qquad\quad \times \Bigg\{ \frac{M_1}{x_{2\perp}\sqrt{s}} \sum_{q_1q_2q_3q_4} f^{q_1}{}_{1T}^{\perp(1)}(x_1) f_1^{q_2}(x_2)\, \frac{\hat{s}}{2}\, \frac{d\hat{\sigma}_{\widehat{gq}_1 q_2\to q_3q_4}}{d\hat{t}}\, D_1^{q_3}(z_1) && \text{(31a)}\\
&\qquad\qquad + \frac{M_2}{x_{2\perp}\sqrt{s}} \sum_{q_1q_2q_3q_4} h_1^{q_1}(x_1) h^{q_2}{}_1^{\perp(1)}(x_2)\, \frac{\hat{s}}{2}\, \frac{d\Delta\hat{\sigma}_{q_1^\uparrow \widehat{gq}_2^\uparrow\to q_3q_4}}{d\hat{t}}\, D_1^{q_3}(z_1) && \text{(31b)}\\
&\qquad\qquad - \frac{M_{h_1}}{x_{2\perp}\sqrt{s}} \sum_{q_1q_2q_3q_4} h_1^{q_1}(x_1) f_1^{q_2}(x_2)\, \frac{\hat{s}}{2}\, \frac{d\Delta\hat{\sigma}_{q_1^\uparrow q_2\to q_3^\uparrow q_4}}{d\hat{t}}\, H^{q_3}{}_1^{\perp(1)}(z_1) && \text{(31c)}\\
&\qquad\qquad - \frac{M_{h_1}}{x_{2\perp}\sqrt{s}} \sum_{q_1q_2q_3q_4} h_1^{q_1}(x_1) f_1^{q_2}(x_2)\, \frac{\hat{s}}{2}\, \frac{d\Delta\hat{\sigma}_{q_1^\uparrow q_2\to \widehat{gq}_3^\uparrow q_4}}{d\hat{t}}\, \widetilde{H}^{q_3}{}_1^{\perp(1)}(z_1) \Bigg\} && \text{(31d)}
\end{aligned}
$$

C. Jet-jet production in $p^{\uparrow}p$ scattering: $p^{\uparrow}+p \to$ Jet+Jet+X

We only take into account (anti)quark scattering processes in the weighted scattering cross section for $p^{\uparrow}+p \to$ Jet+Jet+X with approximately back-to-back jets in the perpendicular plane. As argued, in principle one can construct azimuthal asymmetries that give access to $\Sigma_{\partial}^{\alpha}(x_1, x_2, y)$, by weighing with the small momentum q_T^{α}. This requires, however, accurate determination of the jet momenta k_1 and k_2. Here we only present the cross section obtained by weighing with $\sin\delta\phi$, which can be obtained from the more involved two-particle inclusive process (D1) by taking $D_1^q(z_i) = \delta(z_i{-}1)\delta^{j_i q} = x_\perp\,\delta(x_{i\perp}{-}x_\perp)\delta^{j_i q}$ and by letting all other fragmentation functions vanish. Casting the result from the diagrammatic approach (given explicitly in appendix D) in the form of a folding, one obtains

$$\langle\, \tfrac{1}{2}\sin(\delta\phi)\, d\sigma[j_1 j_2]\,\rangle$$
$$= dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\, \cos(\phi_1^S)\; \delta(x_{1\perp}{-}x_{2\perp})$$
$$\times \Bigg\{ \frac{M_1}{\sqrt{s}} \sum_{q_1 q_2 q_3 q_4} f^{q_1}{}_{1T}^{\perp(1)}(x_1) f_1^{q_2}(x_2)\, \frac{\hat{s}}{2}\, \frac{d\hat{\sigma}_{\widehat{g}q_1 q_2 \to q_3 q_4}}{d\hat{t}} \tag{32a}$$
$$+ \frac{M_2}{\sqrt{s}} \sum_{q_1 q_2 q_3 q_4} h_1^{q_1}(x_1) h^{q_2}{}_{1}^{\perp(1)}(x_2)\, \frac{\hat{s}}{2}\, \frac{d\Delta\hat{\sigma}_{q_1^{\uparrow}\widehat{g}q_2^{\uparrow} \to q_3 q_4}}{d\hat{t}} \Bigg\} \tag{32b}$$

V. SUMMARY AND CONCLUSIONS

In this paper we have used the diagrammatic approach at tree-level to derive expressions for single transverse-spin asymmetries in 2-particle inclusive hadron-hadron collisions. The final states considered are hadron-hadron, hadron-jet and jet-jet which are approximately back-to-back in the plane perpendicular to the incoming hadrons. The single spin asymmetries require the inclusion of transverse momentum dependence for the partons. We have assumed factorization to hold in this treatment of TMD effects although it is, at present, certainly not clear whether such a factorization holds for hadron-hadron scattering processes, in particular not if one looks at explicitly TMD correlators. We have limited ourselves to the first transverse moments obtained by weighing linearly with the transverse momentum. These transverse moments show up in azimuthal asymmetries.

While single-spin asymmetries generated by fragmentation processes, in which one can have T-odd fragmentation functions, are well-known, the single-spin asymmetries connected with initial state hadrons are more subtle. Within the diagrammatic approach T-odd effects for tranverse momentum dependent distribution functions are attributed to the path structure of the gauge-link. The path depends on the specific hard process in which the correlator is used, explaining for instance the appearance of the Sivers function $f_{1T}^{\perp(1)}$ with opposite signs in SIDIS and DY [4, 32, 33]. In the transverse moments of quark and antiquark correlators the effect of the gauge-link appears via the gluonic pole matrix element, which in the case of distributions is a T-odd matrix element which gives rise to single spin asymmetries [24, 25]. In this paper we show how the effects of the gauge-link appear as factors $C_G^{[\mathcal{U}]}$, which determine the strengths with which the gluonic pole matrix elements occur. This is a generalisation of the factors ± 1 appearing in SIDIS and DY. The fact that these strengths are determined by the hard parts makes it convenient to absorb them in so-called gluonic pole cross sections. Just as the transverse momentum averaged cross sections can, in leading order, be written as a folding of universal distribution and fragmentation functions and a hard partonic cross section, the single spin asymmetries can be written as a folding of universal distribution and fragmentation functions and a gluonic pole cross section with one T-odd function.

In the case of fragmentation, we allow in our approach two possible mechanisms, implying that in the two matrix elements, the link-independent part Δ_∂ and the gluonic pole matrix element $\pi\,\Delta_G$, in which the transverse moments can be decomposed, one has both T-even and T-odd effects. For the Collins effect in fragmentation, it leads to two independent functions $H_1^{\perp(1)}$ and $\widetilde{H}_1^{\perp(1)}$, the latter appearing in the gluonic pole matrix element. Having different linear combinations of these functions in SIDIS and electron-positron annihilation spoils the comparison in that case. In hadron-hadron collisions we find other linear combinations of the two functions. If fragmentation functions are universal, as is argued in Refs. [29, 30], the tilde function $\widetilde{H}_1^{\perp(1)}$ (and the gluonic pole matrix element for fragmentation) vanishes. In that case only the contributions from $H_1^{\perp(1)}$ remain.

Our results, including the strengths of the gluonic pole matrix elements differ from those of earlier calculations, in which the effects of the gauge-links have been omitted. The effects, however, can easily be incorporated by using the gluonic pole cross sections instead of the normal hard partonic cross sections.

We have restricted ourselves to particular single spin asymmetry in which the azimuthal asymmetry arises from the deviation of the back-to-back appearance of hadron-hadron, hadron-jet and jet-jet in the perpendicular plane in hadron-hadron scattering. Without the effects of gauge-links this situation was discussed in Ref. [10]. Although experimentally more challenging, the 2-particle inclusive case is easier to analyze than the 1-particle inclusive case, where large single spin asymmetries are observed but where also subleading transverse momentum averaged T-odd fragmentation functions [34] will contribute. In principle the diagrammatic approach allows also for inclusion of these contributions. Furthermore, the methods used in this paper to include the T-odd, transverse momentum dependent effects in (anti)quark contributions, which are crucial to treat single spin asymmetries in hadron-hadron scattering, will be extended to include the gluonic contributions as well as to treat various T-even double spin asymmetries.

APPENDIX A: QUARK CORRELATORS AND GAUGE-LINKS

The starting point for the structure of the hadron→quark transition is the quark correlator $\Phi(p) \equiv \Phi(p;P,S)$ [35, 36]

$$\Phi_{ij}(p;P,S) = \int \frac{d^4\xi}{(2\pi)^4}\; e^{ip\cdot\xi} \langle P,S|\,\overline{\psi}_j(0)\psi_i(\xi)\,|P,S\rangle\ . \tag{A1}$$

Similarly, one has for the quark→hadron transition the fragmentation correlator $\Delta(k) \equiv \Delta(k;K,S)$,

$$\Delta_{ij}(k;K,S_h) = \int \frac{d^4\xi}{(2\pi)^4}\; e^{ik\cdot\xi} \langle 0|\,\psi_i(\xi)\,a_h^\dagger a_h\,\overline{\psi}_j(0)\,|0\rangle\ , \tag{A2}$$

with

$$a_h^\dagger a_h = \sum\!\!\!\!\!\!\!\int_X \frac{d^3P_X}{(2\pi)^3 2E_X}\; |P_X;K,S_h\rangle\langle P_X;K,S_h|\ . \tag{A3}$$

In the description of hard scattering processes we need the quark correlator and the fragmentation correlator integrated over, at least, the partonic momentum component $p{\cdot}P$. This leaves the TMD correlator

$$\Phi(x,p_T) = \int d(p{\cdot}P)\ \Phi(p)\ . \tag{A4}$$

Integrating the TMD correlator over or weighing it with the transverse momentum p_T, we obtain

$$\Phi(x) = \int d^2p_T\ \Phi(x,p_T)\ , \tag{A5a}$$

$$\Phi_\partial^\alpha(x) = \int d^2p_T\ p_T^\alpha\ \Phi(x,p_T)\ . \tag{A5b}$$

One finds similar expressions for the fragmentation correlator. Analogous to the above one can write down the anti-quark correlator $\overline{\Phi}$ describing the hadron→quark transition and the antiquark fragmentation correlator $\overline{\Delta}$ describing the antiquark→hadron transition.

To obtain properly gauge invariant correlators, gauge-links connecting the parton fields in the matrix elements are needed. The general structure of the gauge-links is $\mathcal{U}^{[C]}(0,\xi){=}\mathcal{P}\exp[\,-ig\int_C A(z){\cdot}dz\,]$, where the integration path C runs from 0 to ξ. Here A is the gauge field and $\mathcal{P}$ is the path-ordering operator. The integration paths in the gauge-links can be calculated by resumming all collinear gluon interactions between the soft and hard parts. Consequently, for the TMD correlators they depend on the process in which they occur. The gauge-links appearing in the quark correlator in a two-fermion hard scattering process with uncharged boson exchange, such as in QED, are readily calculated by considering the flow of the fermion lines [23]. The results from that reference are given in Fig. 4. Explicitly, we encounter the link structures

$$\mathcal{U}^{[\pm]} = U^-_{[(0^-,\mathbf{0}_T),(\pm\infty^-,\mathbf{0}_T)]}U^T_{[(\pm\infty^-,\mathbf{0}_T),(\pm\infty^-,\boldsymbol{\infty}_T)]}U^T_{[(\pm\infty^-,\boldsymbol{\infty}_T),(\pm\infty^-,\boldsymbol{\xi}_T)]}U^-_{[(\pm\infty^-,\boldsymbol{\xi}_T),(\xi^-,\boldsymbol{\xi}_T)]}\ , \tag{A6}$$

$$\mathcal{U}^{[\Box]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger}\ , \tag{A7}$$

which are build up from the gauge-links along straight lines

$$U^-_{[a,b]} = \mathcal{P}\exp\Big[-ig\int_a^b dz\ n\cdot A(z)\Big]\ , \qquad \text{and} \qquad U^T_{[a,b]} = \mathcal{P}\exp\Big[-ig\int_a^b dz_T\cdot A_T(z)\Big]\ . \tag{A8}$$

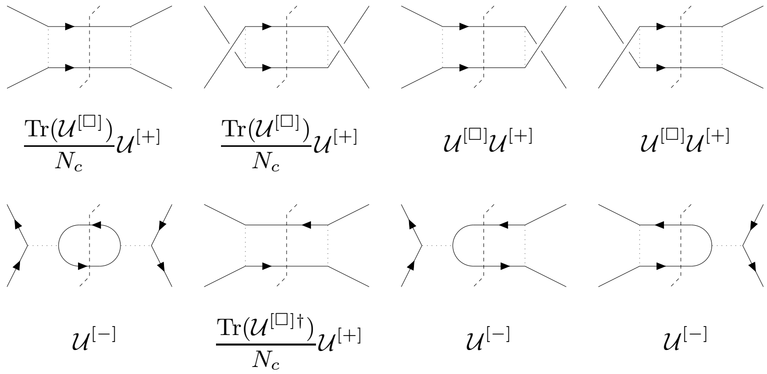
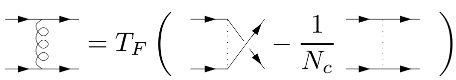
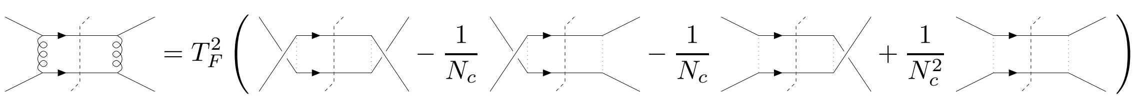


FIG. 4: Gauge-links entering in the correlator for the lower-left incoming quark for a hard two-fermion scattering process without exchange of charge. Top: quark-quark scattering; bottom: quark-antiquark scattering.

The gauge-links in the scattering of two colored fermions in QCD can be obtained from those in Fig. 4 by accounting for the flow of color charge, using well-known QCD rules for color flow such as


(A9)

For example, the tt^*-channel of quark-quark scattering can be decomposed in this way giving


(A10)

The gauge-link of this diagram can be obtained by replacing each diagram on the r.h.s. with the corresponding QED gauge-link as given by Fig. 4 and factoring out the overall color factor of the QCD diagram. The overall color factor of the gluon exchange diagram on the l.h.s. is $(\mathrm{Tr}[t^a t^b])^2 = T_F^2(N_c^2-1)$, which can also be obtained by tracing the color flow in all diagrams on the r.h.s. This color factor does not enter in the gauge-link, but in the evaluation of the diagram itself and is included in the hard amplitudes that will be used in the calculations in appendix D. Accounting for the additional factors $\mathrm{Tr}(\mathbb{1}) = N_c$ that are obtained for color loops, one obtains the gauge-link

$$T_F^2(N_c^2-1)\times\mathcal{U}_{qq}^{[tt^*]} = T_F^2\Big\{\,N_c^2\times\frac{\mathrm{Tr}(\mathcal{U}^{[\square]})}{N_c}\mathcal{U}^{[+]} - \mathcal{U}^{[\square]}\mathcal{U}^{[+]} - \mathcal{U}^{[\square]}\mathcal{U}^{[+]} + \frac{\mathrm{Tr}(\mathcal{U}^{[\square]})}{N_c}\mathcal{U}^{[+]}\Big\}\ .$$

The other diagrams can be calculated analogously. For quark-quark scattering we obtain:

$$\mathcal{U}_{qq}^{[tt^*]} = \mathcal{U}_{qq}^{[uu^*]} = \frac{1}{N_c^2-1}\Big\{\,(N_c^2+1)\,\frac{\mathrm{Tr}(\mathcal{U}^{[\square]})}{N_c}\mathcal{U}^{[+]} - 2\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\Big\}\ , \tag{A11a}$$

$$\mathcal{U}_{qq}^{[tu^*]} = \mathcal{U}_{qq}^{[ut^*]} = \frac{N_c}{N_c^2-1}\Big\{\,2N_c\,\frac{\mathrm{Tr}(\mathcal{U}^{[\square]})}{N_c}\mathcal{U}^{[+]} - \frac{N_c^2+1}{N_c}\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\Big\}\ , \tag{A11b}$$

and for quark-antiquark scattering:

$$\mathcal{U}_{q\overline{q}}^{[ss^*]} = \frac{1}{N_c^2-1}\Big\{\,N_c^2\,\frac{\mathrm{Tr}(\mathcal{U}^{[\square]\dagger})}{N_c}\mathcal{U}^{[+]} - \mathcal{U}^{[-]}\Big\}\ , \tag{A12a}$$

$$\mathcal{U}_{q\overline{q}}^{[tt^*]} = \frac{1}{N_c^2-1}\Big\{\,\frac{\mathrm{Tr}(\mathcal{U}^{[\square]\dagger})}{N_c}\mathcal{U}^{[+]} + (N_c^2-2)\,\mathcal{U}^{[-]}\Big\}\ , \tag{A12b}$$

$$\mathcal{U}_{q\overline{q}}^{[st^*]} = \mathcal{U}_{q\overline{q}}^{[ts^*]} = \frac{N_c}{N_c^2-1}\Big\{\,N_c\,\frac{\mathrm{Tr}(\mathcal{U}^{[\square]\dagger})}{N_c}\mathcal{U}^{[+]} - \frac{1}{N_c}\,\mathcal{U}^{[-]}\Big\}\ . \tag{A12c}$$

These are the gauge-link operators that enter between the quark fields in the correlator of the incoming quark:

$$\Phi^{[\mathcal{U}]}(x,p_{\scriptscriptstyle T};P,S) = \int\frac{d(\xi{\cdot}P)}{2\pi}\,\frac{d^2\xi_{\scriptscriptstyle T}}{(2\pi)^2}\;e^{ip\cdot\xi}\langle P,S|\,\overline{\psi}(0)\,\mathcal{U}(0,\xi)\,\psi(\xi)\,|P,S\rangle\ . \tag{A13}$$

The gauge-links that enter in the quark-fragmentation correlators are the time-reversed ones as compared to those in the quark-correlators. That is, a $\mathcal{U}^{[+]}$ in the quark-correlator corresponds to a $\mathcal{U}^{[-]}$ in the fragmentation correlator and a $\mathcal{U}^{[\Box]}$ to a $\mathcal{U}^{[\Box]\dagger}$, etc. The gauge-links that enter in the antiquark-correlators $\overline{\Phi}$ and $\overline{\Delta}$ are the hermitian conjugates of the gauge-links in the quark-correlators Φ and Δ of the corresponding diagrams.

We note that for the fragmentation correlators the gauge-links are all split up in parts, parts connecting to the field at ξ and others to the field at 0. Taking as an example the quark fragmentation correlators with the gauge-links $\mathcal{U}^{[\pm]}$, one has (compare with Eq. (A6))

$$\begin{aligned}\Delta^{[\pm]}(z,k_T;K,S) = \int \frac{d(\xi{\cdot}K)}{2\pi}\frac{d^2\xi_T}{(2\pi)^2}\ e^{ik\cdot\xi}\langle 0|\, U^T_{[(\pm\infty^-,\boldsymbol{\infty}_T),(\pm\infty^-,\boldsymbol{\xi}_T)]} U^-_{[(\pm\infty^-,\boldsymbol{\xi}_T),(\xi^-,\boldsymbol{\xi}_T)]}\,\psi(\xi) \\ \times\, a_h^\dagger a_h\,\overline{\psi}(0)\, U^-_{[(0^-,\mathbf{0}_T),(\pm\infty^-,\mathbf{0}_T)]} U^T_{[(\pm\infty^-,\mathbf{0}_T),(\pm\infty^-,\boldsymbol{\infty}_T)]}\,|0\rangle\ .\end{aligned} \tag{A14}$$

APPENDIX B: CONSEQUENCES OF GAUGE-LINKS FOR DISTRIBUTION FUNCTIONS

The gauge-link has important consequences for the parametrizations of the correlator due to its behavior under the time-reversal transformation. We start with the link structures enumerated in Fig. 4. The TMD correlators are link dependent. We write $\Phi^{[\pm]}$ for the correlators with gauge-links $\mathcal{U}^{[\pm]}$, $\Phi^{[\Box+]}$ for $\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}$, $\Phi^{[(\Box)+]}$ for $\frac{1}{N_c}\,\mathrm{Tr}(\mathcal{U}^{[\Box]})\mathcal{U}^{[+]}$ and $\Phi^{[(\Box^\dagger)+]}$ for $\frac{1}{N_c}\,\mathrm{Tr}(\mathcal{U}^{[\Box]\dagger})\mathcal{U}^{[+]}$. We will also need the transverse momentum integrated correlators $\Phi_D^\alpha(x)$ and $\Phi_G^\alpha(x,x{-}x')$,

$$\Phi_D^\alpha(x) = \int \frac{d(\xi{\cdot}P)}{2\pi}\ e^{ix(\xi\cdot P)}\langle P,S|\,\overline{\psi}(0)\,U^-_{[0,\xi]}\,iD^\alpha(\xi)\,\psi(\xi)\,|P,S\rangle\,\big\rfloor_{\rm LC}\ , \tag{B1}$$

$$\Phi_G^\alpha(x,x{-}x') = \int \frac{d(\xi{\cdot}P)}{2\pi}\frac{d(\eta{\cdot}P)}{2\pi}\ e^{i(x-x')(\xi\cdot P)}e^{ix'(\eta\cdot P)}\langle P,S|\,\overline{\psi}(0)\,U^-_{[0,\eta]}\,gG^{n\alpha}(\eta)\,U^-_{[\eta,\xi]}\,\psi(\xi)\,|P,S\rangle\,\big\rfloor_{\rm LC}\ , \tag{B2}$$

which are set on the lightcone (LC) where $\xi\cdot n = \xi_T = 0$ and $\eta\cdot n = \eta_T = 0$. We have also used the shorthand notation $G^{n\alpha} = g_{\mu\nu}G^{\mu\alpha}n^\nu$ for the field strength tensor. In terms of these the weighted correlators can be written as

$$\Phi_\partial^{[\pm]\alpha}(x) = \Phi_D^\alpha(x) - \int dx'\ \frac{i}{x'\mp i\epsilon}\,\Phi_G^\alpha(x,x{-}x') = \Phi_\partial^\alpha(x) \pm \pi\Phi_G^\alpha(x,x)\ , \tag{B3a}$$

$$\Phi_\partial^{[\Box+]\alpha}(x) = \Phi_D^\alpha(x) - \int dx'\ \Big\{\frac{i}{x'-i\epsilon} - 2\pi\delta(x')\Big\}\,\Phi_G^\alpha(x,x{-}x') = \Phi_\partial^\alpha(x) + 3\pi\Phi_G^\alpha(x,x)\ , \tag{B3b}$$

$$\Phi_\partial^{[(\Box)+]\alpha}(x) = \Phi_\partial^{[(\Box^\dagger)+]\alpha}(x) = \Phi_D^\alpha(x) - \int dx'\ \frac{i}{x'-i\epsilon}\,\Phi_G^\alpha(x,x{-}x') = \Phi_\partial^\alpha(x) + \pi\Phi_G^\alpha(x,x)\ , \tag{B3c}$$

where Φ_∂ without link index refers to

$$\Phi_\partial^\alpha(x) = \Phi_D^\alpha(x) - \int dx'\ P\frac{i}{x'}\ \Phi_G^\alpha(x,x{-}x')\ . \tag{B4}$$

The decompositions of the weighted correlators in terms of Φ_∂ and Φ_G are particularly useful because the former is T-even, while the latter is T-odd. They can be used as the basic matrix elements to be parametrized.

The correlators encountered in $p^\uparrow p \to \pi\pi X$ are readily obtained from the results above and can also be decomposed in terms of Φ_∂ and Φ_G. For instance, for the tt^*-channel in qq scattering we get from Eq. (A11a)

$$\begin{aligned}\Phi_\partial^{[tt^*]\alpha}(x) &= \frac{1}{N_c^2-1}\Big\{\,(N_c^2+1)\Phi_\partial^{[(\Box)+]\alpha}(x) - 2\,\Phi_\partial^{[\Box+]\alpha}(x)\,\Big\} \\ &= \frac{1}{N_c^2-1}\Big\{\,(N_c^2+1)-2\,\Big\}\,\Phi_\partial^\alpha(x) + \frac{1}{N_c^2-1}\Big\{\,(N_c^2+1)-6\,\Big\}\,\pi\Phi_G^\alpha(x,x)\ .\end{aligned}$$

The other correlators can be calculated analogously. For qq scattering we obtain:

$$\Phi_\partial^{[tt^*]\alpha}(x) = \Phi_\partial^{[uu^*]\alpha}(x) = \Phi_\partial^\alpha(x) + \frac{N_c^2-5}{N_c^2-1}\,\pi\Phi_G^\alpha(x,x)\ , \tag{B5a}$$

$$\Phi_\partial^{[tu^*]\alpha}(x) = \Phi_\partial^{[ut^*]\alpha}(x) = \Phi_\partial^\alpha(x) - \frac{N_c^2+3}{N_c^2-1}\,\pi\Phi_G^\alpha(x,x)\ , \tag{B5b}$$

$\mathcal{U}$	$\mathcal{U}^{[\pm]}$	$\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}$	$\frac{1}{N_c}\,\mathrm{Tr}(\mathcal{U}^{[\Box]})\mathcal{U}^{[+]}$	$\frac{1}{N_c}\,\mathrm{Tr}(\mathcal{U}^{[\Box]\dagger})\mathcal{U}^{[+]}$
$\Phi^{[\mathcal{U}]}$	$\Phi^{[\pm]}$	$\Phi^{[\Box+]}$	$\Phi^{[(\Box)+]}$	$\Phi^{[(\Box^\dagger)+]}$
$C_G^{[\mathcal{U}]}$	± 1	3	1	1

$\mathcal{U}$	$\mathcal{U}_{qq}^{[tt^*]}$	$\mathcal{U}_{qq}^{[uu^*]}$	$\mathcal{U}_{qq}^{[tu^*]}$	$\mathcal{U}_{qq}^{[ut^*]}$	$\mathcal{U}_{q\bar{q}}^{[tt^*]}$	$\mathcal{U}_{q\bar{q}}^{[ss^*]}$	$\mathcal{U}_{q\bar{q}}^{[st^*]}$	$\mathcal{U}_{q\bar{q}}^{[ts^*]}$
$\Phi^{[\mathcal{U}]}$	$\Phi^{[tt^*]}$	$\Phi^{[uu^*]}$	$\Phi^{[tu^*]}$	$\Phi^{[ut^*]}$	$\Phi^{[tt^*]}$	$\Phi^{[ss^*]}$	$\Phi^{[st^*]}$	$\Phi^{[ts^*]}$
$C_G^{[\mathcal{U}]}$	$\frac{N_c^2-5}{N_c^2-1}$		$-\frac{N_c^2+3}{N_c^2-1}$		$-\frac{N_c^2-3}{N_c^2-1}$	$\frac{N_c^2+1}{N_c^2-1}$		

TABLE I: The basic gauge-links (upper table) and the gauge-links in specific hard scattering (qq and $q\bar{q}$) diagrams (lower table), the notations used for the correlators and the strengths C_G of the gluonic pole contribution $\pi\Phi_G$.

and from Eq. (A12) we get for $q\bar{q}$ scattering

$$\Phi_\partial^{[ss^*]\alpha}(x) = \Phi_\partial^{[st^*]\alpha}(x) = \Phi_\partial^{[ts^*]\alpha}(x) = \Phi_\partial^\alpha(x) + \frac{N_c^2+1}{N_c^2-1}\,\pi\Phi_G^\alpha(x,x)\ , \tag{B6a}$$

$$\Phi_\partial^{[tt^*]\alpha}(x) = \Phi_\partial^\alpha(x) - \frac{N_c^2-3}{N_c^2-1}\,\pi\Phi_G^\alpha(x,x)\ . \tag{B6b}$$

The integrated quark correlator $\Phi(x)$ is parametrized in terms of quark distribution functions as follows [21, 37, 38]

$$\Phi_U(x;P) = \tfrac{1}{2}\,f_1(x)\not{P}\ , \tag{B7a}$$

$$\Phi_L(x;P) = \tfrac{1}{2}S_L\,g_1(x)\gamma_5\not{P}\ , \tag{B7b}$$

$$\Phi_T(x;P) = \tfrac{1}{2}\,h_1(x)\gamma_5\tfrac{1}{2}[\not{S}_T,\not{P}]\ , \tag{B7c}$$

where

$$\epsilon_T^{\mu\nu} = \frac{1}{P{\cdot}n}\,\epsilon^{Pn\mu\nu}\ , \qquad \text{and} \qquad S = S_L\frac{1}{M}\,P - S_L\frac{M}{2\,P{\cdot}n}\,n + S_T\ , \tag{B8}$$

with $S_L^2{+}S_T^2 = -1$. The indices U, L and T refer to unpolarized, longitudinally and transversely polarized hadrons, respectively. For the T-even transverse momentum weighted correlator $\Phi_\partial(x)$ and the T-odd gluonic pole $\pi\Phi_G(x,x)$ one has the parametrizations

$$\big(\Phi_\partial^\alpha\big)_U(x;P) = 0\ , \qquad \big(\pi\Phi_G^\alpha\big)_U(x;P) = \tfrac{1}{2}M\,ih_1^{\perp(1)}(x)\tfrac{1}{2}[\not{P},\gamma^\alpha]\ , \tag{B9a}$$

$$\big(\Phi_\partial^\alpha\big)_L(x;P) = \tfrac{1}{2}S_L\,M\,h_{1L}^{\perp(1)}(x)\gamma_5\tfrac{1}{2}[\not{P},\gamma^\alpha]\ , \qquad \big(\pi\Phi_G^\alpha\big)_L(x;P) = 0\ , \tag{B9b}$$

$$\big(\Phi_\partial^\alpha\big)_T(x;P) = \tfrac{1}{2}M\,S_T^\alpha g_{1T}^{(1)}(x)\gamma_5\not{P}\ , \qquad \big(\pi\Phi_G^\alpha\big)_T(x;P) = \tfrac{1}{2}M\,\epsilon_T^{\alpha S_T} f_{1T}^{\perp(1)}(x)\not{P}\ . \tag{B9c}$$

From the parametrizations given above and using the decomposition in Eq. (B5a), we find that the T-odd distribution functions $f_{1T}^{\perp(1)}$ and $h_1^{\perp(1)}$ appear with a multiplicative prefactor $C_G^{[tt^*]}{=}(N_c^2{-}5)/(N_c^2{-}1)$ in the contribution corresponding to the tt^*-channel in qq-scattering. This is the appropriate generalization of the factors $C_G^{[\mathcal{U}^{[\pm]}]} = \pm 1$ occurring in SIDIS and Drell-Yan scattering (as explained in section III). Similarly, the prefactors of the T-odd distribution functions appearing in the other scattering channels can be read of from Eq. (B5) for qq scattering and from Eq. (B6) for $q\bar{q}$ scattering. These prefactors are summarized in Table I. From the Eqs. (B5) and (B6) we also see that all the T-even distribution functions occur in hadron-hadron scattering in the same way as they do in SIDIS, i.e. with a prefactor +1. For antiquark distribution functions, which can be related to quark distributions in the negative x region, the same results as above apply. The antiquark distribution functions will be distinguished from their quark counterparts by an overline, e.g. $\bar{f}_1(x)$, etc.

APPENDIX C: CONSEQUENCES OF GAUGE-LINKS FOR FRAGMENTATION FUNCTIONS

The discussion on the consequences of the gauge-links for fragmentation functions is a little bit more involved than for distribution functions, due to the presence of the hadronic states $|K,X\rangle$ in the definition of the correlators, which is an *out*-state, preventing the use of time-reversal to constrain the parametrization.

All collinear interactions between the soft and hard parts result in the quark-fragmentation correlator $\Delta^{[-]}(k)$ in SIDIS and the correlator $\Delta^{[+]}(k)$ in electron-positron annihilation (see equation (A14)). The transverse-momentum integrated fragmentation correlators in these two processes are

$$\Delta^{[\pm]}(z) = \int \frac{d(\xi{\cdot}K)}{2\pi}\; e^{iz^{-1}(\xi\cdot K)} \langle 0|\, U^-_{[\pm\infty,\xi]}\, \psi(\xi)\, a_h^\dagger a_h\, \overline{\psi}(0)\, U^-_{[0,\pm\infty]}\, |0\rangle \,\big\rfloor_{\rm LC} \;. \tag{C1}$$

Although not immediately evident, it is not hard to see that, since there are only gauge-links along the n_h-direction, the two correlators are identical $\Delta^{[+]}(z) = \Delta^{[-]}(z) \equiv \Delta(z)$.

In analogy to the previous appendix we define a correlator Δ_D^α and a gluonic-pole matrix element Δ_G^α

$$\Delta_D^\alpha(z) = \int \frac{d(\xi{\cdot}K)}{2\pi}\; e^{i(\xi\cdot K)/z}\, \langle 0|\, U^-_{[\zeta,\xi]}\, iD^\alpha(\xi)\, \psi(\xi)\, a_h^\dagger a_h\, \overline{\psi}(0)\, U^-_{[0,\zeta]}\, |0\rangle \,\big\rfloor_{\rm LC} \;, \tag{C2}$$

$$\begin{aligned} \Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}{-}\tfrac{1}{z'}) = \int \frac{d(\xi{\cdot}K)}{2\pi}\frac{d(\eta{\cdot}K)}{2\pi}\; & e^{i(\xi\cdot K)/z} e^{i[(\eta\cdot K)-(\xi\cdot K)]/z'} \\ & \times \langle 0|\, U^-_{[\zeta,\eta]}\, gG^{n_h\alpha}(\eta)\, U^-_{[\eta,\xi]}\, \psi(\xi)\, a_h^\dagger a_h\, \overline{\psi}(0)\, U^-_{[0,\zeta]}\, |0\rangle \,\big\rfloor_{\rm LC} \;, \end{aligned} \tag{C3}$$

with $G^{n_h\alpha} = g_{\mu\nu}G^{\mu\alpha}n_h^\nu$ (and ζ an arbitrary point). It can be shown that in terms of these the weighted correlators can be written as

$$\Delta_\partial^{[\pm]\alpha}(z) = \Delta_D^\alpha(z) - \int d(\tfrac{1}{z'})\; \frac{i}{\frac{1}{z'} \mp i\epsilon}\; \Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}{-}\tfrac{1}{z'}) = \Delta_\partial^\alpha(z) \pm \pi\Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}) \;, \tag{C4a}$$

$$\Delta_\partial^{[-\Box^\dagger]\alpha}(z) = \Delta_D^\alpha(z) - \int d(\tfrac{1}{z'})\; \Big\{ \frac{i}{\frac{1}{z'} + i\epsilon} + 2\pi\delta\big(\tfrac{1}{z'}\big) \Big\}\, \Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}{-}\tfrac{1}{z'}) = \Delta_\partial^\alpha(z) - 3\pi\Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}) \;, \tag{C4b}$$

$$\Delta_\partial^{[-(\Box)]\alpha}(z) = \Delta_\partial^{[-(\Box^\dagger)]\alpha}(z) = \Delta_D^\alpha(z) - \int d(\tfrac{1}{z'})\; \frac{i}{\frac{1}{z'} + i\epsilon}\; \Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}{-}\tfrac{1}{z'}) = \Delta_\partial^\alpha(z) - \pi\Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}) \;, \tag{C4c}$$

where Δ_∂ without link index refers to

$$\Delta_\partial^\alpha(z) = \Delta_D^\alpha(z) - \int dz'^{-1}\; P\frac{i}{z'^{-1}}\; \Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}{-}\tfrac{1}{z'}) \;. \tag{C5}$$

As stated at the end of appendix A, the gauge-links in the fragmentation correlators in $p^\uparrow p \to \pi\pi X$ are obtained from (A11) and (A12) by time-reversal. We, then, find the following quark-fragmentation correlator for the tt^*-channel in quark-quark scattering (cf. (A11a)):

$$\begin{aligned} \Delta_\partial^{[tt^*]\alpha}(z) &= \frac{1}{N_c^2-1}\Big\{\, (N_c^2+1)\, \Delta_\partial^{[-(\Box^\dagger)]\alpha}(z) - 2\, \Delta_\partial^{[-\Box^\dagger]}(z) \,\Big\} \\ &= \frac{1}{N_c^2-1}\big\{\, (N_c^2+1) - 2 \,\big\}\, \Delta_\partial^\alpha(z) - \frac{1}{N_c^2-1}\big\{\, (N_c^2+1) - 6 \,\big\}\, \pi\Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}) \;. \end{aligned}$$

The other quark-fragmentation correlators can be calculated analogously. For quark-quark scattering we obtain:

$$\Delta_\partial^{[tt^*]\alpha}(z) = \Delta_\partial^{[uu^*]\alpha}(z) = \Delta_\partial^\alpha(z) - \frac{N_c^2-5}{N_c^2-1}\, \pi\Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}) \;, \tag{C6a}$$

$$\Delta_\partial^{[tu^*]\alpha}(z) = \Delta_\partial^{[ut^*]\alpha}(z) = \Delta_\partial^\alpha(z) + \frac{N_c^2+3}{N_c^2-1}\, \pi\Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}) \;, \tag{C6b}$$

and for quark-antiquark scattering

$$\Delta_\partial^{[ss^*]\alpha}(z) = \Delta_\partial^{[st^*]\alpha}(z) = \Delta_\partial^{[ts^*]\alpha}(z) = \Delta_\partial^\alpha(z) - \frac{N_c^2+1}{N_c^2-1}\, \pi\Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}) \;, \tag{C7a}$$

$$\Delta_\partial^{[tt^*]\alpha}(z) = \Delta_\partial^\alpha(z) + \frac{N_c^2-3}{N_c^2-1}\, \pi\Delta_G^\alpha(\tfrac{1}{z},\tfrac{1}{z}) \;. \tag{C7b}$$

The integrated fragmentation correlator $\Delta(z)$ is parametrized as follows [39]

$$z\,\Delta_U(z;K) = D_1(z)\not{K} \;, \tag{C8a}$$

$$z\,\Delta_L(z;K) = S_L\, G_1(z)\gamma_5\not{K} \;, \tag{C8b}$$

$$z\,\Delta_T(z;K) = H_1(z)\gamma_5\tfrac{1}{2}[\not{S}_T,\not{K}] \;, \tag{C8c}$$

$\mathcal{U}$	$\mathcal{U}^{[\pm]}$	$\mathcal{U}^{[-]}\mathcal{U}^{[\Box]\dagger}$	$\frac{1}{N_c}\mathcal{U}^{[-]}\,\mathrm{Tr}(\mathcal{U}^{[\Box]})$	$\frac{1}{N_c}\mathcal{U}^{[-]}\,\mathrm{Tr}(\mathcal{U}^{[\Box]\dagger})$
$\Delta^{[\mathcal{U}]}$	$\Delta^{[\pm]}$	$\Delta^{[-\Box^\dagger]}$	$\Delta^{[-(\Box)]}$	$\Delta^{[-(\Box^\dagger)]}$
$C_G^{[\mathcal{U}]}$	±1	-3	-1	-1

$\mathcal{U}$	$\mathcal{U}_{qq}^{[tt^*]}$	$\mathcal{U}_{qq}^{[uu^*]}$	$\mathcal{U}_{qq}^{[tu^*]}$	$\mathcal{U}_{qq}^{[ut^*]}$	$\mathcal{U}_{q\bar{q}}^{[tt^*]}$	$\mathcal{U}_{q\bar{q}}^{[ss^*]}$	$\mathcal{U}_{q\bar{q}}^{[st^*]}$	$\mathcal{U}_{q\bar{q}}^{[ts^*]}$
$\Delta^{[\mathcal{U}]}$	$\Delta^{[tt^*]}$	$\Delta^{[uu^*]}$	$\Delta^{[tu^*]}$	$\Delta^{[ut^*]}$	$\Delta^{[tt^*]}$	$\Delta^{[ss^*]}$	$\Delta^{[st^*]}$	$\Delta^{[ts^*]}$
$C_G^{[\mathcal{U}]}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{4}$	$-\frac{5}{4}$	$-\frac{5}{4}$	$-\frac{5}{4}$

TABLE II: The basic gauge-links (upper table) and the gauge-links in specific hard scattering (qq and $q\bar{q}$) diagrams (lower table), the notations used for the correlators and the strengths C_G of the gluonic pole contribution $\pi\Delta_G$ with N_c=3.

with

$$\epsilon_T^{\mu\nu} = \frac{1}{K{\cdot}n_h}\,\epsilon^{K n_h \mu\nu}\ , \qquad \text{and} \qquad S = S_L\frac{1}{M_h}\,K - S_L\frac{M_h}{2\,K{\cdot}n_h}\,n_h + S_T\ . \tag{C9}$$

The functions in these expansions are called quark fragmentation functions. Due to the internal soft interactions in the final-state hadron the correlators Δ_∂ and $\pi\Delta_G$ both contain T-even *and* T-odd parts [6]. Correspondingly, they have very similar parametrizations in terms of fragmentation functions. We will distinguish the fragmentation functions in these two correlators by adding a tilde to the fragmentation functions appearing in the parametrization of the gluonic pole. That is, parametrizing the correlator Δ_∂ as follows

$$z\left(\Delta_\partial^\alpha\right)_U(z;K) = M_h\, iH_1^{\perp(1)}(z)\tfrac{1}{2}[\not{K},\gamma^\alpha]\ , \tag{C10a}$$

$$z\left(\Delta_\partial^\alpha\right)_L(z;K) = S_L\, M_h\, H_{1L}^{\perp(1)}(z)\gamma_5\tfrac{1}{2}[\not{K},\gamma^\alpha]\ , \tag{C10b}$$

$$z\left(\Delta_\partial^\alpha\right)_T(z;K) = M_h\left\{S_T^\alpha G_{1T}^{(1)}(z)\gamma_5\not{K} - \epsilon_T^{\alpha S_T} D_{1T}^{\perp(1)}(z)\not{K}\right\}\ , \tag{C10c}$$

the parametrization of the gluonic pole is written as

$$z\left(\pi\Delta_G^\alpha\right)_U(\tfrac{1}{z},\tfrac{1}{z};K) = M_h\, i\widetilde{H}_1^{\perp(1)}(z)\tfrac{1}{2}[\not{K},\gamma^\alpha]\ , \tag{C11a}$$

$$z\left(\pi\Delta_G^\alpha\right)_L(\tfrac{1}{z},\tfrac{1}{z};K) = S_L\, M_h\, \widetilde{H}_{1L}^{\perp(1)}(z)\gamma_5\tfrac{1}{2}[\not{K},\gamma^\alpha]\ , \tag{C11b}$$

$$z\left(\pi\Delta_G^\alpha\right)_T(\tfrac{1}{z},\tfrac{1}{z};K) = M_h\left\{S_T^\alpha \widetilde{G}_{1T}^{(1)}(z)\gamma_5\not{K} - \epsilon_T^{\alpha S_T} \widetilde{D}_{1T}^{\perp(1)}(z)\not{K}\right\}\ . \tag{C11c}$$

The fragmentation functions appearing in these parametrizations contribute to azimuthal asymmetries in special combinations. For instance, using the decomposition in Eq. (C6a) we find that the Collins effect contributed by the tt^*-channel for qq scattering is $H_1^{\perp(1)}(z) - \frac{N_c^2-5}{N_c^2-1}\,\widetilde{H}_1^{\perp(1)}$. Similarly, the other partonic channels contribute particular combinations of fragmentation functions. The particular combination of fragmentation functions that one should take for a certain process can be read of directly from the decompositions in Eq. (C6) and (C7). That is, if we let $\mathrm{FF}(z)$ denote a generic fragmentation function appearing in the parametrizations in Eq. (C10) and (C11), then this fragmentation function will appear in the expressions for azimuthal asymmetries in the combination $\mathrm{FF}(z) - C_G^{[\mathcal{U}]}\,\widetilde{\mathrm{FF}}(z)$. In particular, we see that the tilde fragmentation functions always appear with the (process dependent) prefactors $C_G^{[\mathcal{U}]}$ summarized in Table II, while the fragmentation functions without a tilde always occur with a simple prefactor $+1$. If the gluonic pole matrix elements $\pi\Delta_G$ vanish, then so do all the tilde functions. In that case fragmentation is completely described by the universal functions appearing in the parametrization of Δ_∂. Notably, the Collins effect is always given by the term $H_1^{\perp(1)}(z)$.

For antiquark-fragmentation functions, which can be related to the quark-fragmentation functions in the negative z region, the same results as above apply.

APPENDIX D: RESULTS IN THE DIAGRAMMATIC APPROACH

In the expressions given below y is given by Eq. (7) and $\hat{s}$ is $\hat{s} = x_\perp^2\,s\,\cosh^2[\frac{1}{2}(\eta_1{-}\eta_2)] = x_\perp^2\,s/4y(1{-}y)$. The summations run over all quark flavors, including the case that $q' = q$ (where applicable). Similarly, the $\delta^{j_i q}$ are delta functions in flavor space, indicating that the jet j_i is produced by quark q. We have written the factors $C_G^{[\mathcal{U}]}$ for the T-odd distribution functions between braces $\{\cdot\}$. For the Collins functions we have written the combinations $H_1^{\perp(1)}{-}C_G^{[\mathcal{U}]}\,\widetilde{H}_1^{\perp(1)}$ between braces. The factors $C_G^{[\mathcal{U}]}$ are taken from Table I for the distribution functions and from Table II for the fragmentation functions.

$$p^{\uparrow}{+}p \to \pi{+}\pi{+}X$$

Considering only the (anti)quark contributions in $p^{\uparrow}{+}p \to \pi{+}\pi{+}X$ the resulting cross section is given by

$$\begin{aligned}
&\langle\, \tfrac{1}{2}\sin(\delta\phi)\, d\sigma[h_1h_2]\,\rangle \\
&\quad = dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\cos(\phi_1{-}\phi_S)\int \frac{dx_\perp}{x_\perp}\,\frac{\hat{s}}{2x_\perp\sqrt{s}} \\
&\qquad \times \frac{4\pi\,\alpha_S^2}{9\,\hat{s}^2}\Big\{\; M_1\,\frac{(1-y)^2+1}{y^2}\sum_{q,q'}\{\tfrac{1}{2}\}\, f_{1T}^{q\perp(1)}(x_1) f_1^{q'}(x_2) D_1^q(z_1) D_1^{q'}(z_2) && \text{(D1a)}\\
&\qquad\qquad - M_1\,\frac{1}{3}\,\frac{1}{y(1-y)}\sum_q\{-\tfrac{3}{2}\}\, f_{1T}^{q\perp(1)}(x_1) f_1^q(x_2) D_1^q(z_1) D_1^q(z_2) && \text{(D1b)}\\
&\qquad\qquad + M_1\,\big((1-y)^2+y^2\big)\sum_{q,q'}\{\tfrac{5}{4}\}\, f_{1T}^{q\perp(1)}(x_1)\bar{f}_1^q(x_2) D_1^{q'}(z_1)\bar{D}_1^{q'}(z_2) && \text{(D1c)}\\
&\qquad\qquad + M_1\,\frac{(1-y)^2+1}{y^2}\sum_{q,q'}\{-\tfrac{3}{4}\}\, f_{1T}^{q\perp(1)}(x_1)\bar{f}_1^{q'}(x_2) D_1^q(z_1)\bar{D}_1^{q'}(z_2) && \text{(D1d)}\\
&\qquad\qquad + M_1\,\frac{2}{3}\,\frac{(1-y)^2}{y}\sum_q\{\tfrac{5}{4}\}\, f_{1T}^{q\perp(1)}(x_1)\bar{f}_1^q(x_2) D_1^q(z_1)\bar{D}_1^q(z_2) && \text{(D1e)}\\
&\qquad\qquad - M_2\, 2y(1-y)\sum_{q,q'}\{\tfrac{5}{4}\}\, h_1^q(x_1)\bar{h}_1^{q\perp(1)}(x_2) D_1^{q'}(z_1)\bar{D}_1^{q'}(z_2) && \text{(D1f)}\\
&\qquad\qquad - M_2\,\frac{2}{3}(1-y)\sum_q\{\tfrac{5}{4}\}\, h_1^q(x_1)\bar{h}_1^{q\perp(1)}(x_2) D_1^q(z_1)\bar{D}_1^q(z_2) && \text{(D1g)}\\
&\qquad\qquad - M_2\,\frac{1}{3}\sum_q\{-\tfrac{3}{2}\}\, h_1^q(x_1) h_1^{q\perp(1)}(x_2) D_1^q(z_1) D_1^q(z_2) && \text{(D1h)}\\
&\qquad\qquad - M_{h_1}\, 2\frac{1-y}{y^2}\sum_{q,q'} h_1^q(x_1) f_1^{q'}(x_2)\,\big\{H_1^{q\perp(1)}(z_1){-}\tfrac{1}{2}\widetilde{H}_1^{q\perp(1)}(z_1)\,\big\}\, D_1^{q'}(z_2) && \text{(D1i)}\\
&\qquad\qquad + M_{h_1}\,\frac{2}{3}\,\frac{1}{y}\sum_q h_1^q(x_1) f_1^q(x_2)\,\big\{H_1^{q\perp(1)}(z_1){+}\tfrac{3}{2}\widetilde{H}_1^{q\perp(1)}(z_1)\,\big\}\, D_1^q(z_2) && \text{(D1j)}\\
&\qquad\qquad - M_{h_1}\, 2\frac{1-y}{y^2}\sum_{q,q'} h_1^q(x_1)\bar{f}_1^{q'}(x_2)\,\big\{H_1^{q\perp(1)}(z_1){+}\tfrac{3}{4}\widetilde{H}_1^{q\perp(1)}(z_1)\,\big\}\,\bar{D}_1^{q'}(z_2) && \text{(D1k)}\\
&\qquad\qquad - M_{h_1}\,\frac{2}{3}\,\frac{1-y}{y}\sum_q h_1^q(x_1)\bar{f}_1^q(x_2)\,\big\{H_1^{q\perp(1)}(z_1){-}\tfrac{5}{4}\widetilde{H}_1^{q\perp(1)}(z_1)\,\big\}\,\bar{D}_1^q(z_2) && \text{(D1l)}\\
&\qquad\qquad + M_{h_1}\,\frac{2}{3}\sum_q h_1^q(x_1)\bar{f}_1^q(x_2)\,\big\{\bar{H}_1^{q\perp(1)}(z_1){-}\tfrac{5}{4}\widetilde{\bar{H}}_1^{q\perp(1)}(z_1)\,\big\}\, D_1^q(z_2) && \text{(D1m)}\\
&\qquad\qquad\qquad + \big(\,\text{quarks} \leftrightarrow \text{antiquarks}\,\big)\;\Big\} + \big(\,K_1 \leftrightarrow K_2\,\big)
\end{aligned}$$

$$p + p \to \pi + \textbf{Jet} + X$$

Considering only the (anti)quark contributions in $p^{\uparrow}+p \to \pi+\text{Jet}+X$ the resulting cross section is given by

$$
\begin{aligned}
&\langle\, \tfrac{1}{2}\sin(\delta\phi)\, d\sigma[h_1 j_2]\,\rangle \\
&\quad = dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\cos(\phi_1{-}\phi_S)\,\frac{\hat{s}}{2x_{2\perp}\sqrt{s}} \\
&\qquad \times \frac{4\pi\,\alpha_S^2}{9\,\hat{s}^2}\Bigg\{\; M_1\,\frac{(1-y)^2+1}{y^2}\sum_{q,q'}\{\tfrac{1}{2}\}\, f_{1T}^{q\perp(1)}(x_1) f_1^{q'}(x_2) D_1^{q}(z_1)\,\delta^{j_2 q'} && \text{(D2a)}\\
&\qquad\qquad +M_1\,\frac{y^2+1}{(1-y)^2}\sum_{q,q'}\{\tfrac{1}{2}\}\, f_{1T}^{q\perp(1)}(x_1) f_1^{q'}(x_2) D_1^{q'}(z_1)\,\delta^{j_2 q} && \text{(D2b)}\\
&\qquad\qquad -M_1\,\frac{2}{3}\frac{1}{y(1-y)}\sum_{q}\{-\tfrac{3}{2}\}\, f_{1T}^{q\perp(1)}(x_1) f_1^{q}(x_2) D_1^{q}(z_1)\,\delta^{j_2 q} && \text{(D2c)}\\
&\qquad\qquad +M_1\,\big((1-y)^2+y^2\big)\sum_{q,q'}\{\tfrac{5}{4}\}\, f_{1T}^{q\perp(1)}(x_1) \bar{f}_1^{q}(x_2) D_1^{q'}(z_1)\,\delta^{j_2 \bar{q}'} && \text{(D2d)}\\
&\qquad\qquad +M_1\,\big((1-y)^2+y^2\big)\sum_{q,q'}\{\tfrac{5}{4}\}\, f_{1T}^{q\perp(1)}(x_1) \bar{f}_1^{q}(x_2) \bar{D}_1^{q'}(z_1)\,\delta^{j_2 q'} && \text{(D2e)}\\
&\qquad\qquad +M_1\,\frac{(1-y)^2+1}{y^2}\sum_{q,q'}\{-\tfrac{3}{4}\}\, f_{1T}^{q\perp(1)}(x_1) \bar{f}_1^{q'}(x_2) D_1^{q}(z_1)\,\delta^{j_2 \bar{q}'} && \text{(D2f)}\\
&\qquad\qquad +M_1\,\frac{y^2+1}{(1-y)^2}\sum_{q,q'}\{-\tfrac{3}{4}\}\, f_{1T}^{q\perp(1)}(x_1) \bar{f}_1^{q'}(x_2) \bar{D}_1^{q'}(z_1)\,\delta^{j_2 q} && \text{(D2g)}\\
&\qquad\qquad +M_1\,\frac{2}{3}\frac{(1-y)^2}{y}\sum_{q}\{\tfrac{5}{4}\}\, f_{1T}^{q\perp(1)}(x_1) \bar{f}_1^{q}(x_2) D_1^{q}(z_1)\,\delta^{j_2 \bar{q}} && \text{(D2h)}\\
&\qquad\qquad +M_1\frac{2}{3}\frac{y^2}{1-y}\sum_{q}\{\tfrac{5}{4}\}\, f_{1T}^{q\perp(1)}(x_1) \bar{f}_1^{q}(x_2) \bar{D}_1^{q}(z_1)\,\delta^{j_2 q} && \text{(D2i)}\\
&\qquad\qquad -M_2\,2y(1-y)\sum_{q,q'}\{\tfrac{5}{4}\}\, h_1^{q}(x_1) \bar{h}_1^{q\perp(1)}(x_2) D_1^{q'}(z_1)\,\delta^{j_2 \bar{q}'} && \text{(D2j)}\\
&\qquad\qquad -M_2\,2y(1-y)\sum_{q,q'}\{\tfrac{5}{4}\}\, h_1^{q}(x_1) \bar{h}_1^{q\perp(1)}(x_2) \bar{D}_1^{q'}(z_1)\,\delta^{j_2 q'} && \text{(D2k)}\\
&\qquad\qquad -M_2\,\frac{2}{3}(1-y)\sum_{q}\{\tfrac{5}{4}\}\, h_1^{q}(x_1) \bar{h}_1^{q\perp(1)}(x_2) D_1^{q}(z_1)\,\delta^{j_2 \bar{q}} && \text{(D2l)}\\
&\qquad\qquad -M_2\frac{2}{3}y\sum_{q}\{\tfrac{5}{4}\}\, h_1^{q}(x_1) \bar{h}_1^{q\perp(1)}(x_2) \bar{D}_1^{q}(z_1)\,\delta^{j_2 q} && \text{(D2m)}\\
&\qquad\qquad -M_2\,\frac{2}{3}\sum_{q}\{-\tfrac{3}{2}\}\, h_1^{q}(x_1) h_1^{q\perp(1)}(x_2) D_1^{q}(z_1)\,\delta^{j_2 q} && \text{(D2n)}\\
&\qquad\qquad -M_{h_1}\,2\frac{1-y}{y^2}\sum_{q,q'} h_1^{q}(x_1) f_1^{q'}(x_2)\,\big\{H_1^{q\perp(1)}(z_1){-}\tfrac{1}{2}\widetilde{H}_1^{q\perp(1)}(z_1)\,\big\}\,\delta^{j_2 q'} && \text{(D2o)}\\
&\qquad\qquad +M_{h_1}\,\frac{2}{3}\frac{1}{y}\sum_{q} h_1^{q}(x_1) f_1^{q}(x_2)\,\big\{H_1^{q\perp(1)}(z_1){+}\tfrac{3}{2}\widetilde{H}_1^{q\perp(1)}(z_1)\,\big\}\,\delta^{j_2 q} && \text{(D2p)}\\
&\qquad\qquad -M_{h_1}\,2\frac{1-y}{y^2}\sum_{q,q'} h_1^{q}(x_1) \bar{f}_1^{q'}(x_2)\,\big\{H_1^{q\perp(1)}(z_1){+}\tfrac{3}{4}\widetilde{H}_1^{q\perp(1)}(z_1)\,\big\}\,\delta^{j_2 \bar{q}'} && \text{(D2q)}
\end{aligned}
$$

$$-M_{h_1}\,\frac{2}{3}\,\frac{1-y}{y}\sum_q h_1^q(x_1)\bar{f}_1^q(x_2)\,\big\{\,H^{q\,\perp(1)}_{\;1}(z_1) - \tfrac{5}{4}\widetilde{H}^{q\,\perp(1)}_{\;1}(z_1)\,\big\}\,\delta^{j_2\bar{q}} \tag{D2r}$$

$$+M_{h_1}\,\frac{2}{3}\sum_q h_1^q(x_1)\bar{f}_1^q(x_2)\,\big\{\,\bar{H}^{q\,\perp(1)}_{\;1}(z_1) - \tfrac{5}{4}\widetilde{\bar{H}}^{q\,\perp(1)}_{\;1}(z_1)\,\big\}\,\delta^{j_2 q} \tag{D2s}$$

$$+\big(\,\text{quarks} \leftrightarrow \text{antiquarks}\,\big)\;\Big\}$$

$$p + p \rightarrow \mathbf{Jet} + \mathbf{Jet} + X$$

Considering only the (anti)quark contributions in $p^{\uparrow}{+}p \rightarrow \text{Jet}{+}\text{Jet}{+}X$ the resulting cross section is given by

$$\begin{aligned}
&\langle\, \tfrac{1}{2}\sin(\delta\phi)\, d\sigma[j_1 j_2]\,\rangle \\
&\quad = dx_{1\perp}\, dx_{2\perp}\, d\eta_1\, d\eta_2\, \frac{d\phi_1}{2\pi}\cos(\phi_1 - \phi_S)\,\delta(x_{1\perp} - x_{2\perp})\,\frac{\hat{s}}{2\sqrt{s}} \\
&\qquad \times \frac{4\pi\,\alpha_S^2}{9\,\hat{s}^2}\,\Big\{\; M_1\,\frac{(1-y)^2+1}{y^2}\sum_{q,q'}\{\tfrac{1}{2}\}\, f^{q\,\perp(1)}_{\;1T}(x_1) f_1^{q'}(x_2)\,\delta^{j_1 q}\delta^{j_2 q'}
\end{aligned} \tag{D3a}$$

$$-M_1\,\frac{1}{3}\,\frac{1}{y(1-y)}\sum_q \{-\tfrac{3}{2}\}\, f^{q\,\perp(1)}_{\;1T}(x_1) f_1^q(x_2)\,\delta^{j_1 q}\delta^{j_2 q} \tag{D3b}$$

$$+M_1\,\big((1-y)^2+y^2\big)\sum_{q,q'}\{\tfrac{5}{4}\}\, f^{q\,\perp(1)}_{\;1T}(x_1)\bar{f}_1^q(x_2)\,\delta^{j_1 q'}\delta^{j_2 \bar{q}'} \tag{D3c}$$

$$+M_1\,\frac{(1-y)^2+1}{y^2}\sum_{q,q'}\{-\tfrac{3}{4}\}\, f^{q\,\perp(1)}_{\;1T}(x_1)\bar{f}_1^{q'}(x_2)\,\delta^{j_1 q}\delta^{j_2 \bar{q}'} \tag{D3d}$$

$$+M_1\,\frac{2}{3}\,\frac{(1-y)^2}{y}\sum_q \{\tfrac{5}{4}\}\, f^{q\,\perp(1)}_{\;1T}(x_1)\bar{f}_1^q(x_2)\,\delta^{j_1 q}\delta^{j_2 \bar{q}} \tag{D3e}$$

$$-M_2\, 2y(1-y)\sum_{q,q'}\{\tfrac{5}{4}\}\, h_1^q(x_1)\bar{h}^{q\,\perp(1)}_{\;1}(x_2)\,\delta^{j_1 q'}\delta^{j_2 \bar{q}'} \tag{D3f}$$

$$-M_2\,\frac{2}{3}(1-y)\sum_q \{\tfrac{5}{4}\}\, h_1^q(x_1)\bar{h}^{q\,\perp(1)}_{\;1}(x_2)\,\delta^{j_1 q}\delta^{j_2 \bar{q}} \tag{D3g}$$

$$-M_2\,\frac{1}{3}\sum_q \{-\tfrac{3}{2}\}\, h_1^q(x_1) h^{q\,\perp(1)}_{\;1}(x_2)\,\delta^{j_1 q}\delta^{j_2 q} \tag{D3h}$$

$$+\big(\,\text{quarks} \leftrightarrow \text{antiquarks}\,\big)\;\Big\} + \big(\,\text{Jet}_1 \leftrightarrow \text{Jet}_2\,\big)$$

APPENDIX E: PARTONIC CROSS SECTIONS

In this appendix we enumerate all the (anti)quark scattering cross sections (taken from [40]) that are needed in this paper.

Quark-quark scattering

The unpolarized quark-quark scattering cross sections are given by

$$\frac{d\hat{\sigma}_{qq'\to qq'}}{d\hat{t}} = \frac{4\pi\alpha_S^2}{9\,\hat{s}^2}\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\ , \tag{E1a}$$

$$\frac{d\hat{\sigma}_{qq'\to q'q}}{d\hat{t}} = \frac{4\pi\alpha_S^2}{9\,\hat{s}^2}\,\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\ , \tag{E1b}$$

$$\frac{d\hat{\sigma}_{qq\to qq}}{d\hat{t}} = \frac{d\hat{\sigma}_{qq'\to qq'}}{d\hat{t}} + \frac{d\hat{\sigma}_{qq'\to q'q}}{d\hat{t}} - 2\,\frac{d\hat{\sigma}^{\rm I}_{qq\to qq}}{d\hat{t}}\ , \tag{E1c}$$

where $d\hat{\sigma}^{\rm I}$ represents the interference terms

$$\frac{d\hat{\sigma}^{\rm I}_{qq\to qq}}{d\hat{t}} = \frac{4\pi\alpha_S^2}{27\,\hat{s}^2}\,\frac{\hat{s}^2}{\hat{t}\hat{u}}\ . \tag{E2}$$

The polarized quark-quark scattering cross sections are

$$\frac{d\Delta\hat{\sigma}_{q^\uparrow q^\uparrow\to qq}}{d\hat{t}} = -\frac{8\pi\alpha_S^2}{27\,\hat{s}^2}\ , \tag{E3a}$$

$$\frac{d\Delta\hat{\sigma}_{q^\uparrow q'\to q^\uparrow q'}}{d\hat{t}} = -\frac{8\pi\alpha_S^2}{9\,\hat{s}^2}\,\frac{\hat{u}\hat{s}}{\hat{t}^2}\ , \tag{E3b}$$

$$\frac{d\Delta\hat{\sigma}_{q^\uparrow q\to q^\uparrow q}}{d\hat{t}} = \frac{d\Delta\hat{\sigma}_{q^\uparrow q'\to q^\uparrow q'}}{d\hat{t}} - \frac{d\Delta\hat{\sigma}^{\rm I}_{q^\uparrow q\to q^\uparrow q}}{d\hat{t}}\ , \tag{E3c}$$

with the interference term

$$\frac{d\Delta\hat{\sigma}^{\rm I}_{q^\uparrow q\to q^\uparrow q}}{d\hat{t}} = -\frac{8\pi\alpha_S^2}{27\,\hat{s}^2}\,\frac{\hat{s}}{\hat{t}}\ . \tag{E4}$$

The modified cross sections are

$$\frac{d\hat{\sigma}_{\widehat{gq}q'\to qq'}}{d\hat{t}} = \frac{N_c^2-5}{N_c^2-1}\,\frac{d\hat{\sigma}_{qq'\to qq'}}{d\hat{t}}\ , \tag{E5a}$$

$$\frac{d\hat{\sigma}_{\widehat{gq}q'\to q'q}}{d\hat{t}} = \frac{N_c^2-5}{N_c^2-1}\,\frac{d\hat{\sigma}_{qq'\to q'q}}{d\hat{t}}\ , \tag{E5b}$$

$$\frac{d\hat{\sigma}_{\widehat{gq}q\to qq}}{d\hat{t}} = \frac{N_c^2-5}{N_c^2-1}\left[\frac{d\hat{\sigma}_{qq'\to qq'}}{d\hat{t}} + \frac{d\hat{\sigma}_{qq'\to q'q}}{d\hat{t}}\right] + 2\,\frac{N_c^2+3}{N_c^2-1}\,\frac{d\hat{\sigma}^{\rm I}_{qq\to qq}}{d\hat{t}}\ , \tag{E5c}$$

$$\frac{d\Delta\hat{\sigma}_{q^\uparrow\widehat{gq}^\uparrow\to qq}}{d\hat{t}} = -\frac{N_c^2+3}{N_c^2-1}\,\frac{d\Delta\hat{\sigma}_{q^\uparrow q^\uparrow\to qq}}{d\hat{t}}\ , \tag{E5d}$$

$$\frac{d\Delta\hat{\sigma}_{q^\uparrow q'\to\widehat{gq}^\uparrow q'}}{d\hat{t}} = -\frac{N_c^2-5}{N_c^2-1}\,\frac{d\Delta\hat{\sigma}_{q^\uparrow q'\to q^\uparrow q'}}{d\hat{t}}\ , \tag{E5e}$$

$$\frac{d\Delta\hat{\sigma}_{q^\uparrow q\to\widehat{gq}^\uparrow q}}{d\hat{t}} = -\frac{N_c^2-5}{N_c^2-1}\,\frac{d\Delta\hat{\sigma}_{q^\uparrow q'\to q^\uparrow q'}}{d\hat{t}} - \frac{N_c^2+3}{N_c^2-1}\,\frac{d\Delta\hat{\sigma}^{\rm I}_{q^\uparrow q\to q^\uparrow q}}{d\hat{t}}\ . \tag{E5f}$$

The partonic cross sections can be regarded as functions of the variable y defined in (7) through

$$\frac{\hat{t}}{\hat{s}} = -y\ , \qquad \frac{\hat{u}}{\hat{s}} = -(1-y)\ , \qquad \hat{s} = \frac{x_\perp^2\,s}{4y(1-y)}\ . \tag{E6}$$

All other non-vanishing quark-quark and antiquark-antiquark scattering cross sections that contribute to hadronic pion production can be obtained from the above from symmetry considerations.

Quark-antiquark scattering

The unpolarized quark-antiquark scattering cross sections are given by

$$\frac{d\hat\sigma_{q\bar q'\to q\bar q'}}{d\hat t} = \frac{4\pi\alpha_S^2}{9\,\hat s^2}\,\frac{\hat s^2+\hat u^2}{\hat t^2}\ , \tag{E7a}$$

$$\frac{d\hat\sigma_{q\bar q\to q'\bar q'}}{d\hat t} = \frac{4\pi\alpha_S^2}{9\,\hat s^2}\,\frac{\hat t^2+\hat u^2}{\hat s^2}\ , \tag{E7b}$$

$$\frac{d\hat\sigma_{q\bar q\to q\bar q}}{d\hat t} = \frac{d\hat\sigma_{q\bar q'\to q\bar q'}}{d\hat t} + \frac{d\hat\sigma_{q\bar q\to q'\bar q'}}{d\hat t} - 2\,\frac{d\hat\sigma^{\rm I}_{q\bar q\to q\bar q}}{d\hat t}\ , \tag{E7c}$$

with the interference term

$$\frac{d\hat\sigma^{\rm I}_{q\bar q\to q\bar q}}{d\hat t} \equiv \frac{4\pi\alpha_S^2}{27\,\hat s^2}\,\frac{\hat u^2}{\hat t\hat s}\ . \tag{E8}$$

The polarized quark-antiquark scattering cross sections are

$$\frac{d\Delta\hat\sigma_{q^\uparrow\bar q^\uparrow\to q'\bar q'}}{d\hat t} = -\frac{8\pi\alpha_S^2}{9\,\hat s^2}\,\frac{\hat t\hat u}{\hat s^2}\ , \tag{E9a}$$

$$\frac{d\Delta\hat\sigma_{q^\uparrow\bar q^\uparrow\to q\bar q}}{d\hat t} = \frac{d\Delta\hat\sigma_{q^\uparrow\bar q^\uparrow\to q'\bar q'}}{d\hat t} - \frac{d\Delta\hat\sigma^{\rm I}_{q^\uparrow\bar q^\uparrow\to q\bar q}}{d\hat t}\ , \tag{E9b}$$

$$\frac{d\Delta\hat\sigma_{q^\uparrow\bar q'\to q^\uparrow\bar q'}}{d\hat t} = -\frac{8\pi\alpha_S^2}{9\,\hat s^2}\,\frac{\hat u\hat s}{\hat t^2}\ , \tag{E9c}$$

$$\frac{d\Delta\hat\sigma_{q^\uparrow\bar q\to q^\uparrow\bar q}}{d\hat t} = \frac{d\Delta\hat\sigma_{q^\uparrow\bar q'\to q^\uparrow\bar q'}}{d\hat t} - \frac{d\Delta\hat\sigma^{\rm I}_{q^\uparrow\bar q\to q^\uparrow\bar q}}{d\hat t}\ , \tag{E9d}$$

$$\frac{d\Delta\hat\sigma_{q^\uparrow\bar q\to\bar q^\uparrow q}}{d\hat t} = -\frac{8\pi\alpha_S^2}{27\hat s^2}\ , \tag{E9e}$$

with the interference terms

$$\frac{d\Delta\hat\sigma^{\rm I}_{q^\uparrow\bar q^\uparrow\to q\bar q}}{d\hat t} = -\frac{8\pi\alpha_S^2}{27\,\hat s^2}\,\frac{\hat u}{\hat s}\ , \qquad \frac{d\Delta\hat\sigma^{\rm I}_{q^\uparrow\bar q\to q^\uparrow\bar q}}{d\hat t} = -\frac{8\pi\alpha_S^2}{27\hat s^2}\,\frac{\hat u}{\hat t}\ . \tag{E10}$$

The modified cross sections are

$$\frac{d\hat\sigma_{\widehat{q}\bar q'\to q\bar q'}}{d\hat t} = -\frac{N_c^2-3}{N_c^2-1}\,\frac{d\hat\sigma_{q\bar q'\to q\bar q'}}{d\hat t}\ , \tag{E11a}$$

$$\frac{d\hat\sigma_{\widehat{q}\bar q\to q'\bar q'}}{d\hat t} = \frac{N_c^2+1}{N_c^2-1}\,\frac{d\hat\sigma_{q\bar q\to q'\bar q'}}{d\hat t}\ , \tag{E11b}$$

$$\frac{d\hat\sigma_{\widehat{q}\bar q\to q\bar q}}{d\hat t} = -\frac{N_c^2-3}{N_c^2-1}\,\frac{d\hat\sigma_{q\bar q'\to q\bar q'}}{d\hat t} + \frac{N_c^2+1}{N_c^2-1}\left[\frac{d\hat\sigma_{q\bar q\to q'\bar q'}}{d\hat t} - 2\,\frac{d\hat\sigma^{\rm I}_{q\bar q\to q\bar q}}{d\hat t}\right], \tag{E11c}$$

$$\frac{d\Delta\hat\sigma_{q^\uparrow\widehat{q}^\uparrow\to q'\bar q'}}{d\hat t} = \frac{N_c^2+1}{N_c^2-1}\,\frac{d\Delta\hat\sigma_{q^\uparrow\bar q^\uparrow\to q'\bar q'}}{d\hat t}\ , \tag{E11d}$$

$$\frac{d\Delta\hat\sigma_{q^\uparrow\widehat{q}^\uparrow\to q\bar q}}{d\hat t} = \frac{N_c^2+1}{N_c^2-1}\,\frac{d\Delta\hat\sigma_{q^\uparrow\bar q^\uparrow\to q\bar q}}{d\hat t}\ , \tag{E11e}$$

$$\frac{d\Delta\hat\sigma_{q^\uparrow\bar q'\to\widehat{q}^\uparrow\bar q'}}{d\hat t} = \frac{N_c^2-3}{N_c^2-1}\,\frac{d\Delta\hat\sigma_{q^\uparrow\bar q'\to q^\uparrow\bar q'}}{d\hat t}\ , \tag{E11f}$$

$$\frac{d\Delta\hat\sigma_{q^\uparrow\bar q\to\widehat{q}^\uparrow\bar q}}{d\hat t} = \frac{N_c^2-3}{N_c^2-1}\,\frac{d\Delta\hat\sigma_{q^\uparrow\bar q'\to q^\uparrow\bar q'}}{d\hat t} + \frac{N_c^2+1}{N_c^2-1}\,\frac{d\Delta\hat\sigma^{\rm I}_{q^\uparrow\bar q\to q^\uparrow\bar q}}{d\hat t}\ , \tag{E11g}$$

$$\frac{d\Delta\hat\sigma_{q^\uparrow\bar q\to\widehat{\bar q}^\uparrow q}}{d\hat t} = -\frac{N_c^2+1}{N_c^2-1}\,\frac{d\Delta\hat\sigma_{q^\uparrow\bar q\to\bar q^\uparrow q}}{d\hat t}\ . \tag{E11h}$$

All other non-vanishing quark-antiquark scattering cross sections that contribute to hadronic pion production can be obtained from the above from symmetry considerations.

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