Sudakov suppression of asymmetries involving TMDs

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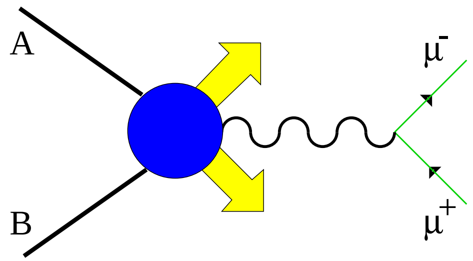
Outline

- Collinear factorization and resummation
- Angular asymmetries and noncollinear factorization
- Q^2 dependence of the transverse momentum dependence
  Coupled via Sudakov factors
- Effects of Sudakov factors in two explicit examples:
  - $\cos 2\phi$ asymmetry in Drell-Yan
  - Collins asymmetry in semi-inclusive DIS
- Effect of Wilson lines
- Conclusions

Drell-Yan process: $H_1 + H_2 \to \ell + \bar{\ell} + X$



In general, photon has a transverse momentum $\boldsymbol{q}_T$ w.r.t. P_A, P_B

Consider three cases (with each a different factorization):

- $\boldsymbol{q}_T$ integrated cross section

$$\frac{d\sigma}{dx_A dx_B} \sim \frac{d\sigma}{dQ^2 dy}$$

- $Q_T \equiv |\boldsymbol{q}_T|$ dependent cross section

$$\frac{d\sigma}{dQ^2 dy\, dQ_T^2}$$

- $\boldsymbol{q}_T$ dependent cross section

$$\frac{d\sigma}{dQ^2 dy\, d^2\boldsymbol{q}_T d\Omega} \sim \frac{d\sigma}{d^4q\, d\Omega}$$

Collinear factorization

Leading twist factorization theorem in Drell-Yan:

$$\frac{d\sigma}{dQ^2dy} = \sum_a \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_A;\mu) \sum_b \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_B;\mu)\, H_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q; \frac{\mu}{Q}, \alpha_s(\mu)\right)$$

$$x_A = e^y \sqrt{\frac{Q^2}{s}}, \quad x_B = e^{-y}\sqrt{\frac{Q^2}{s}}, \quad y = \frac{1}{2}\ln\frac{q\cdot P_A}{q\cdot P_B}$$

Q^2 is large, one deals with collinear factorization

A similar collinear factorization applies when Q_T is observed and large ($Q_T \sim Q$):

$$\frac{d\sigma}{dQ^2dy} \longrightarrow \frac{d\sigma}{dQ^2dy\,dQ_T^2}$$

$$H_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q; \frac{\mu}{Q}, \alpha_s(\mu)\right) \longrightarrow T_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q, Q_T; \mu, \alpha_s(\mu)\right)$$

T_{ab} is singular as $Q_T \to 0$, one needs to resum large logarithms

Collinear factorization plus resummation

$\Lambda^2 \ll Q_T^2 \ll Q^2$: Collins-Soper-Sterman (CSS) formalism

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} = \int d^2 b \, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \tilde{W}(b, Q; x_A, x_B) + Y(Q_T, Q; x_A, x_B) \qquad b = |\boldsymbol{b}|$$

$$\begin{aligned} \tilde{W}(b, Q; x_A, x_B) &= \sum_j e_j^2 \sum_a \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_A; 1/b) \sum_b \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_B; 1/b) \\ &\quad \times e^{-S(b,Q)} C_{ja}\left(\frac{x_A}{\xi_A}; \alpha_s(1/b)\right) C_{\bar{j}b}\left(\frac{x_B}{\xi_B}; \alpha_s(1/b)\right) \end{aligned}$$

Collins, Soper & Sterman, NPB 250 ('85) 199

$Y(x_1, x_2, Q, Q_T)$ becomes important only when $Q_T \sim Q$
Introduced to match to fixed order pQCD calculations at large Q_T

$e^{-S(b,Q)}$ = Sudakov form factor [exponentiation rather than cancellation of soft gluon contributions]

Sudakov factor

$$S(b,Q) = \int_{1/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A\left(\alpha_s(\mu)\right) \ln \frac{Q^2}{\mu^2} + B\left(\alpha_s(\mu)\right) \right]$$

At the leading log level $(\mathcal{O}(\alpha_s))$: $A = \alpha_s C_F/\pi + \mathcal{O}(\alpha_s^2)$

Without running of α_s:

$$S(b,Q) = -\alpha_s \frac{C_F}{\pi} \log^2\left(b^2 Q^2\right)$$

With running:

$$S(b,Q) = -\frac{C_F}{\beta_1} \left\{ \log\left(b^2 Q^2\right) + \log\left(\frac{Q^2}{\Lambda^2}\right) \log\left[1 - \frac{\log\left(b^2 Q^2\right)}{\log\left(Q^2/\Lambda^2\right)}\right] \right\}$$

Using only this expression in the factorization expression is valid for Q^2 very large, when the restriction $b^2 \ll 1/\Lambda^2$ is justified

If also $b^2 \gtrsim 1/\Lambda^2$ contributions are important $(\mu^2 \lesssim \Lambda^2)$, then one needs to include a nonperturbative Sudakov factor

Nonperturbative Sudakov factor

Rewriting $\tilde{W}(b)$ with help of $b_* = b/\sqrt{1 + b^2/b_{\max}^2} \leq b_{\max}$:

$$\tilde{W}(b) \equiv \tilde{W}(b_*)\, e^{-S_{NP}(b)}$$

$\tilde{W}(b_*)$ can be calculated within perturbation theory

Usually $b_{\max} = 0.5$ GeV^{-1}, such that $\alpha_s(1/b_*) \leq \alpha_s(2) \approx 0.3$

In general the nonperturbative Sudakov factor is of the form

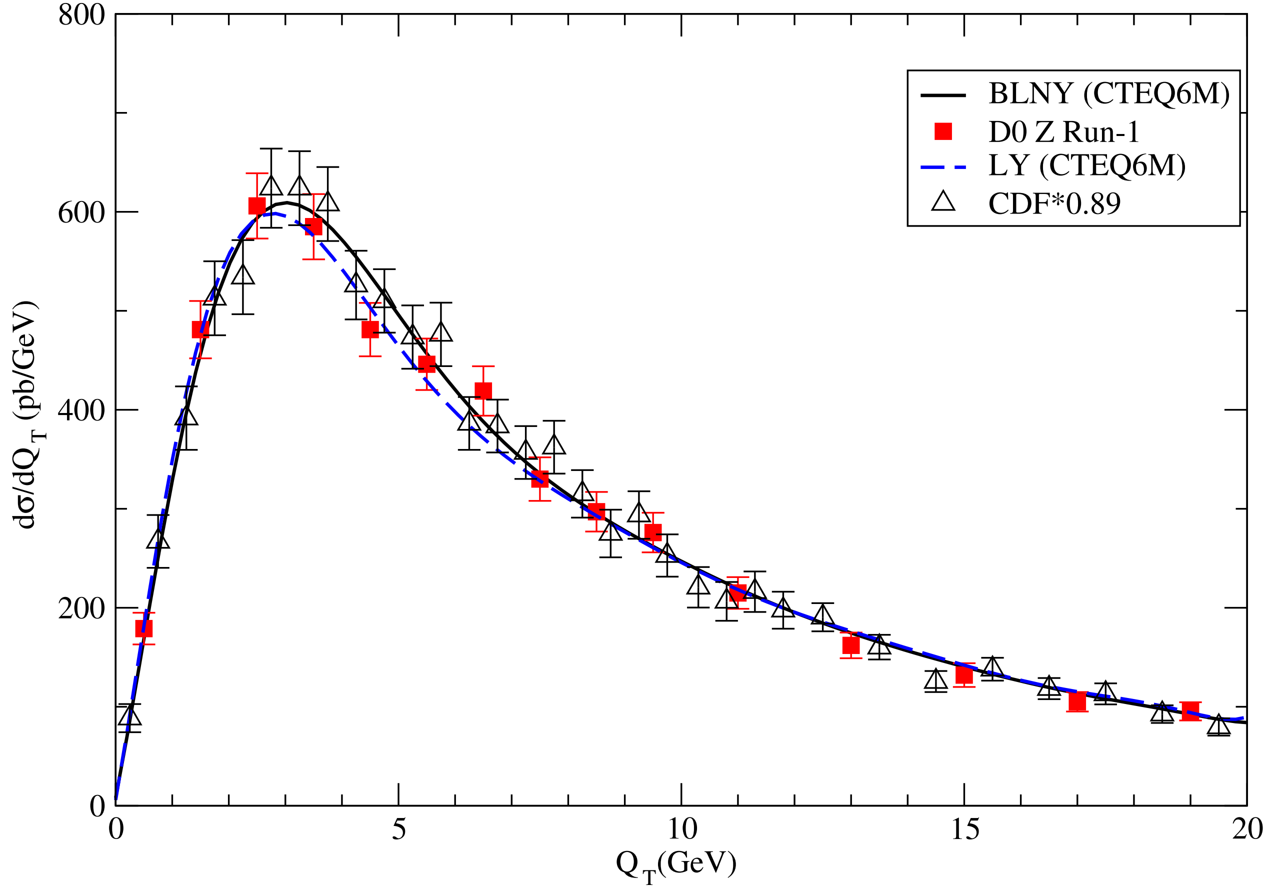
$$e^{-S_{NP}(b,Q)} = e^{-\left(\ln(Q^2/Q_0^2) g_1(b) + g_A(x_A,b) + g_B(x_B,b)\right)} \qquad Q_0 = \frac{1}{b_{\max}}$$

Collins, Soper & Sterman, NPB 250 ('85) 199

The $g_{..}$ functions are not calculable in perturbation theory and need to be fitted to experiment, in fact, they are needed to be able to describe the data

S_{NP} is Q^2 dependent (!)

Application of CSS formalism



$$\exp\left\{ - \left[g_1 + g_2 \ln \frac{Q}{2Q_0} + g_1 g_3 \ln\left(100 x_1 x_2\right) \right] b^2 \right\}$$

$$\begin{aligned} g_1 &= 0.21 \pm 0.01 \\ g_2 &= 0.68 \pm 0.02 \\ g_3 &= 0.60 \pm 0.05 \end{aligned}$$

Transverse momentum distribution of Z bosons at the Tevatron run-1 fitted using the CSS resummation formalism (includes low energy DY data in global fit)

Landry, Brock, Nadolsky, Yuan, PRD 67 ('03) 073016

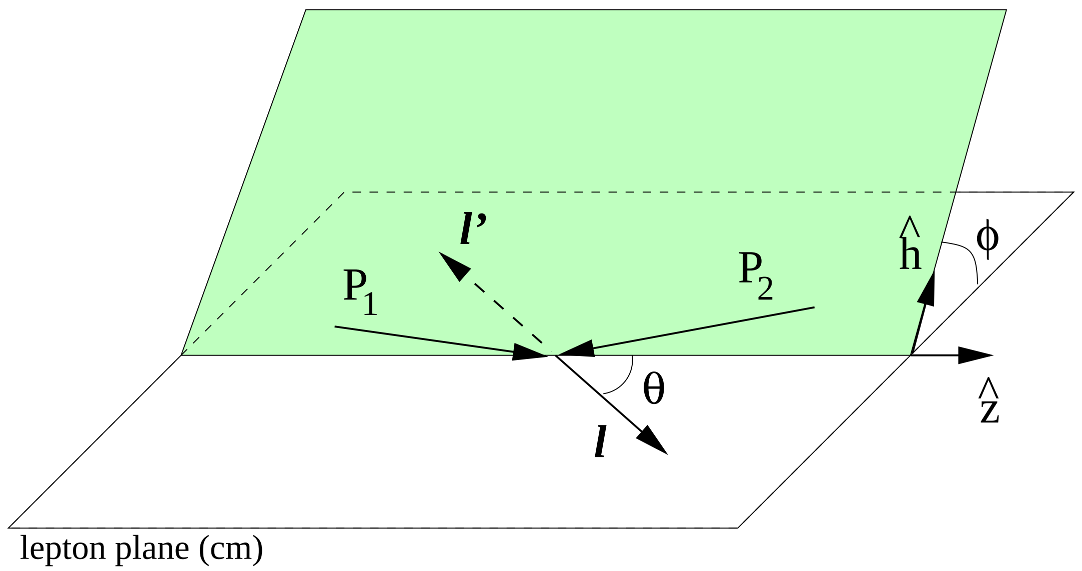
Angular asymmetries

In order to describe angular dependences of the cross section:

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} \quad \longrightarrow \quad \frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega} \sim \frac{d\sigma}{d^4 q d\Omega}$$

$d\Omega = d\cos\theta d\phi^l$, where θ and ϕ^l are the angles of one of the leptons in the lepton-pair center of mass

$d^2\boldsymbol{q}_T = d\phi^h dQ_T^2/2$ and $\phi = \phi^h - \phi^l$



Angular asymmetries

For unpolarized scattering one has the general angular dependence

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q}\right)^{-1} \frac{d\sigma}{d^4q\,d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

Fixed order perturbative calculation at $\mathcal{O}(\alpha_s)$

$$\frac{dN}{d\Omega} = \frac{3}{16\pi}\frac{1+\frac{3}{2}\rho^2}{1+\rho^2}\left[1+\frac{1-\frac{1}{2}\rho^2}{1+\frac{3}{2}\rho^2}\cos^2\theta + \frac{\rho}{(1+\frac{3}{2}\rho^2)} f\left(\frac{\xi_A}{x_A},\frac{\xi_B}{x_B}\right)\sin 2\theta\cos\phi + \frac{1}{2}\frac{\rho^2}{1+\frac{3}{2}\rho^2}\sin^2\theta\cos 2\phi\right]$$

Collins, PRL 42 ('79) 291

This expression holds (in the Collins-Soper frame) when $\rho \equiv Q_T/Q = \mathcal{O}(1)$

Beyond fixed order perturbation theory

For small Q_T one finds from fixed order (LO) perturbation theory:

$$\lambda \to 1, \quad \mu \to 0, \quad \nu \to 0$$

Not a singular limit

However, for small Q_T collinear factorization is not the right starting point

The CSS formalism applies to $d\sigma/dQ^2 dy dQ_T^2$, but it stems from a more general factorization theorem that applies to $d\sigma/dQ^2 dy d^2\boldsymbol{q}_T d\Omega$

This "CS-81" factorization theorem applied to $e^+e^- \to h_1\, h_2\, X$, not DY
Collins & Soper, NPB 193 ('81) 381

A similar factorization for SIDIS and DY was discussed recently by Ji, Ma, Yuan (PRD 71 ('05) 034005 & PLB 597 ('04) 299), but with some differences

CS-81 formalism

$$\frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \tilde{W}(\boldsymbol{b}, Q; x_A, x_B) + Y(\boldsymbol{q}_T, Q; x_A, x_B)$$

$$\begin{aligned} \tilde{W}(\boldsymbol{b}, Q; x_A, x_B) &= \sum_a \tilde{f}_{a/A}(x_A, \boldsymbol{b}; 1/b, \alpha_s(1/b)) \sum_b \tilde{f}_{b/B}(x_B, \boldsymbol{b}; 1/b, \alpha_s(1/b)) \\ &\quad \times e^{-S(b,Q)}\, H_{ab}(x_A, x_B, Q; \alpha_s(Q))\, \tilde{U}(\boldsymbol{b}; 1/b, \alpha_s(1/b)) \end{aligned}$$

Here $\tilde{f}(x, \boldsymbol{b})$ is the Fourier transform of $f(x, \boldsymbol{k}_T)$, hence one needs to deal with TMDs

Note there is no integral over momentum fractions (ξ) now

$\tilde{U}$ is a soft factor

The factorization form as discussed by Ji, Ma, Yuan (PRD 71 ('05) 034005 & PLB 597 ('04) 299) has $\mu \neq 1/b$ and the nonperturbative b region is treated differently
Idilbi, Ji, Ma, Yuan, PRD 70 ('04) 074021

Transverse momentum dependence

Collinear parton distributions are defined from

$$\Phi(x) \equiv \int \frac{d\lambda}{2\pi}\, e^{i\lambda x}\, \langle P|\overline{\psi}(0)\, \mathcal{L}[0,\lambda]\, \psi(\lambda)|P\rangle$$

For unpolarized hadrons: $\Phi(x) = \frac{1}{2} f_1(x) \not{P}$

Include parton transverse momentum $\Phi(x) \to \Phi(x, \boldsymbol{k}_T)$

$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2} \left\{ f_1(x, \boldsymbol{k}_T^2)\, \frac{\not{P}}{M} + h_1^\perp(x, \boldsymbol{k}_T^2)\, \frac{i \not{k}_T \not{P}}{M^2} \right\}$$

Without $h_1^\perp$, one can do a perturbative expansion of $\Phi(x, \boldsymbol{k}_T)$ for large k_T^2, to go from the CS-81 to the CSS formalism (this will generate the integral over ξ)

Nonperturbative k_T (or b) dependence can then be absorbed in Sudakov factor

If $h_1^\perp \neq 0$, then one *cannot* reduce to a leading twist CSS expression by expanding in k_T^2

Numerical study of Sudakov suppression

$$\frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \tilde{W}(\boldsymbol{b}, Q; x_A, x_B) + Y(\boldsymbol{q}_T, Q; x_A, x_B)$$

with $\tilde{f} \longrightarrow \tilde{\Phi}$

$$\begin{aligned} \tilde{W}(\boldsymbol{b}, Q; x_A, x_B) \quad = \quad & \sum_a \tilde{\Phi}_{a/A}(x_A, \boldsymbol{b}; 1/b, \alpha_s(1/b)) \sum_b \tilde{\Phi}_{b/B}(x_B, \boldsymbol{b}; 1/b, \alpha_s(1/b)) \\ & \times e^{-S(b,Q)}\, H_{ab}(x_A, x_B, Q; \alpha_s(Q))\, \tilde{U}(\boldsymbol{b}; 1/b, \alpha_s(1/b)) \end{aligned}$$

Scale choice is such that b dependence of soft factor $\tilde{U}$ appears only at NLLA

$$\tilde{\Phi}(x, \boldsymbol{b}) = \frac{M}{2} \left\{ \tilde{f}_1(x, \boldsymbol{b}^2)\, \frac{\not{P}}{M} + \left(\frac{\partial}{\partial b^2} \tilde{h}_1^\perp(x, \boldsymbol{b}^2) \right) \frac{2\, \not{b} \not{P}}{M^2} \right\}$$

Furthermore, assume Gaussian k_T dependence for f_1 (with Gaussian width R_u^2) and $h_1^\perp$ (with Gaussian width $R^2 > R_u^2$)

Example 1: $\cos 2\phi$ in DY

$$\frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega} \propto \left\{ 1 + \ldots + \frac{\nu(Q_T)}{2} \sin^2\theta \cos 2\phi \right\}$$

The asymmetry arising at large Q_T (associated to the Y term) to first order in α_s is

$$\nu_Y(Q_T) = \frac{Q_T^2}{Q^2 + 3Q_T^2/2}$$

The tree level expression at small Q_T due to $h_1^\perp$ is

$$\nu^{(0)}(Q_T) = \frac{h_1^\perp(x_1)\, \overline{h}_1^\perp(x_2)}{f_1(x_1)\, \overline{f}_1(x_2)} \frac{Q_T^2 R^2}{2M^2 R_u^2} \exp\left(- \left[R^2 - R_u^2 \right] \frac{Q_T^2}{2} \right)$$

Including Sudakov factors:

$$\nu(Q_T) = \frac{h_1^\perp(x_1)\, \overline{h}_1^\perp(x_2)}{f_1(x_1)\, \overline{f}_1(x_2)} \frac{\mathcal{A}(Q_T)}{2M^4 R^4}$$

Numerical estimates

$$\nu(Q_T) = \frac{h_1^\perp(x_1)\,\overline{h}_1^\perp(x_2)}{f_1(x_1)\,\overline{f}_1(x_2)}\,\frac{\mathcal{A}(Q_T)}{2M^4R^4}$$

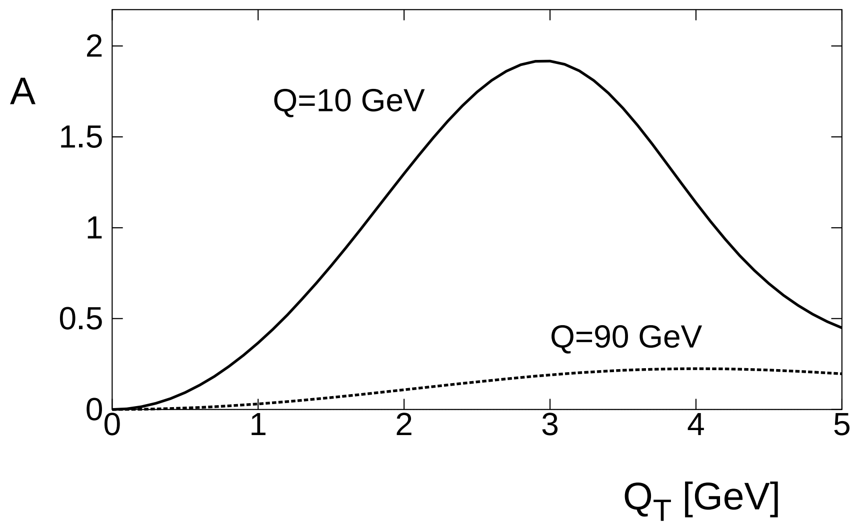
$$\mathcal{A}(Q_T) \equiv M^2\,\frac{\int_0^\infty db\, b^3\, J_2(bQ_T)\,\exp\left(-S(b_*) - S_{NP}(b)\right)}{\int_0^\infty db\, b\, J_0(bQ_T)\,\exp\left(-S(b_*) - S^u_{NP}(b)\right)}$$

Using a generic S^u_{NP}

Ladinsky & Yuan, PRD 50 ('94) R4239

Results in a considerable Sudakov suppression with increasing Q: $\sim 1/Q$

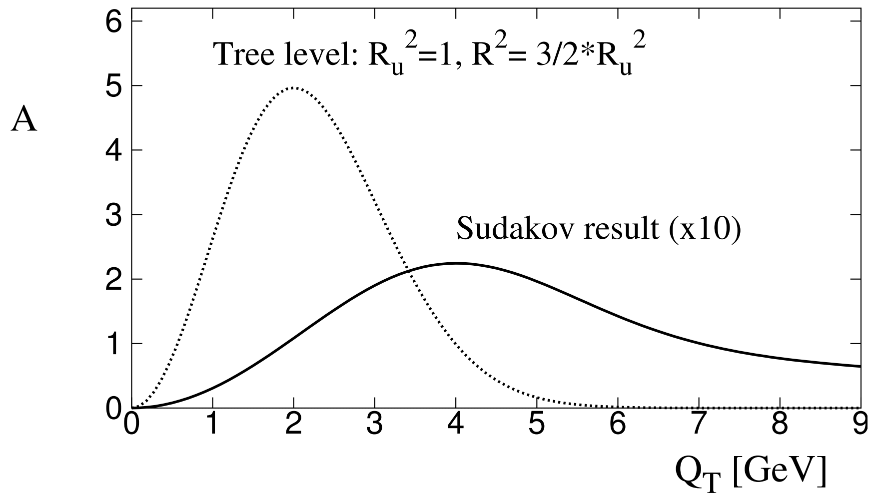
D.B., NPB 603 ('01) 195



Numerical estimates

Comparison of tree level and Sudakov results at $Q = 90$ GeV

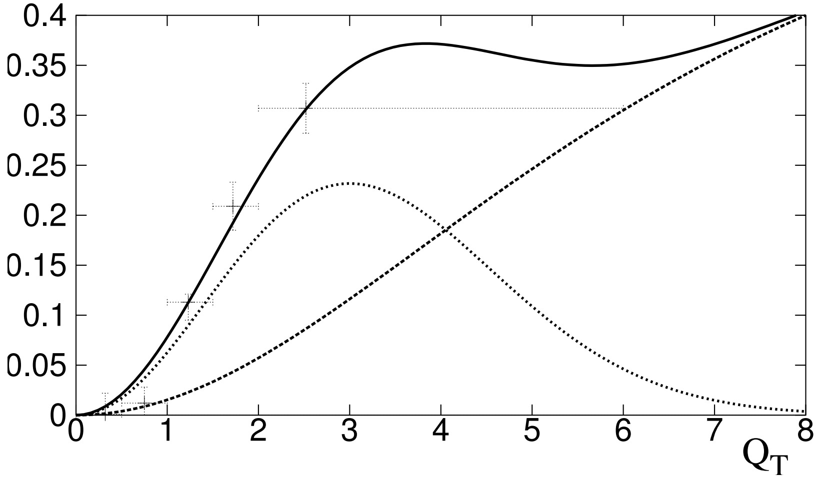
Latter is much smaller and broader



Data for $\pi^- N \to \mu^+\mu^- X$, with $N = D, W$, from NA10 Collab. ('86/'88) & E615 Collab. ('89)

with π^--beams of 140-286 GeV and lepton pair invariant mass $Q \sim 4 - 12$ GeV

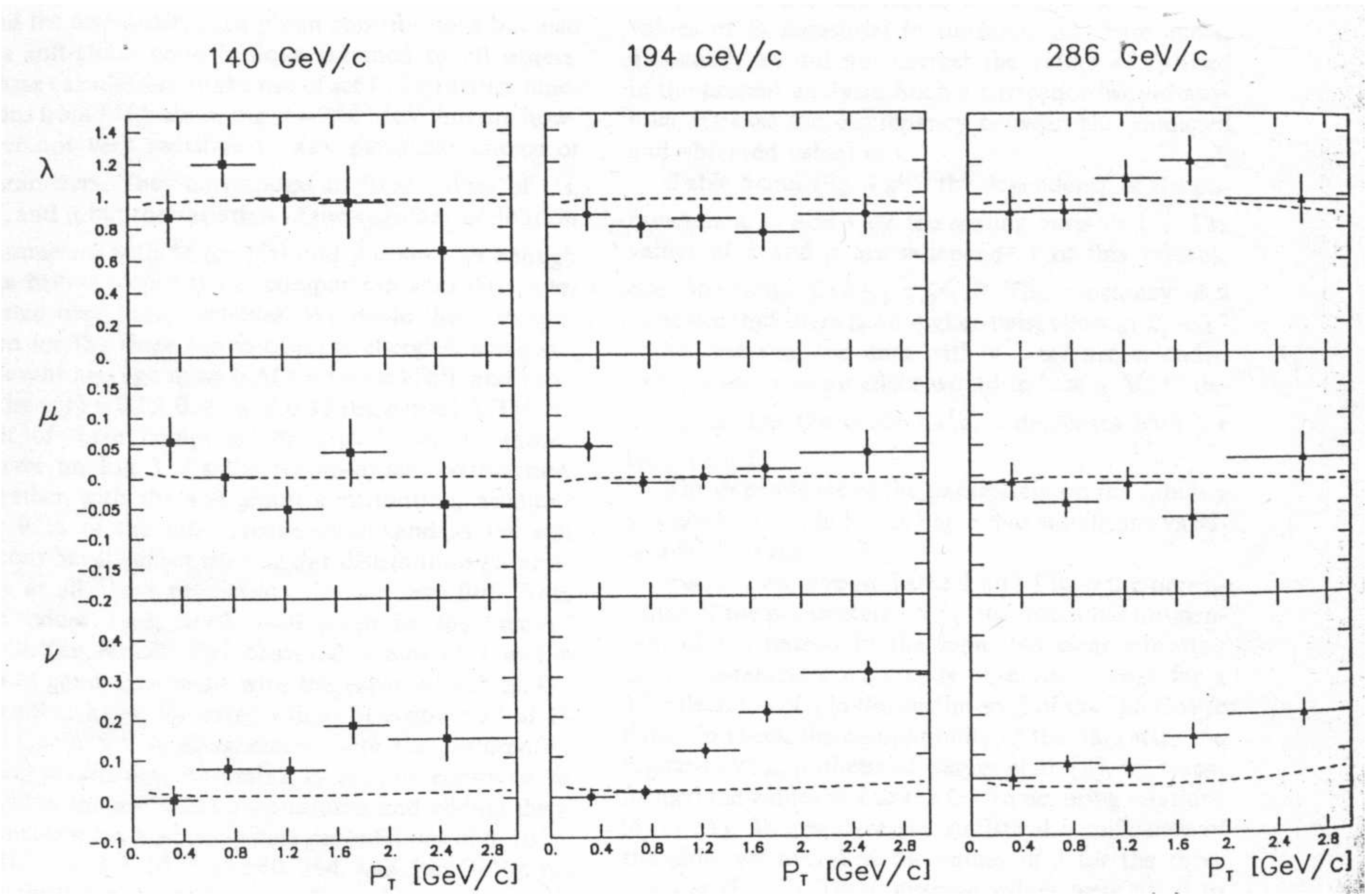
Impression of small and large Q_T contributions (at $Q = 8$ GeV) compared to DY data of NA10 ('88)



$h_1^\perp$ can be used to describe the data

D.B., PRD 60 ('99) 014012

NA10 data, ZPC 37 ('88) 545



Transverse moments

If one doesn't want to assume Gaussians, then one can consider taking Q_T moments

This leads to expressions involving $h_1^{\perp(1)}$

$$h_1^{\perp(1)}(x) \equiv \int d^2\boldsymbol{k}_T \frac{\boldsymbol{k}_T^2}{2M^2} h_1^{\perp}(x, \boldsymbol{k}_T^2)$$

It turns out that in the particular case of the $\cos 2\phi$ asymmetry

$$\int dQ_T^2 \, Q_T^2 \, \frac{d\sigma}{dQ_T^2}$$

is insensitive to the Sudakov factors

However, the Q_T^2 weight emphasizes the Y term

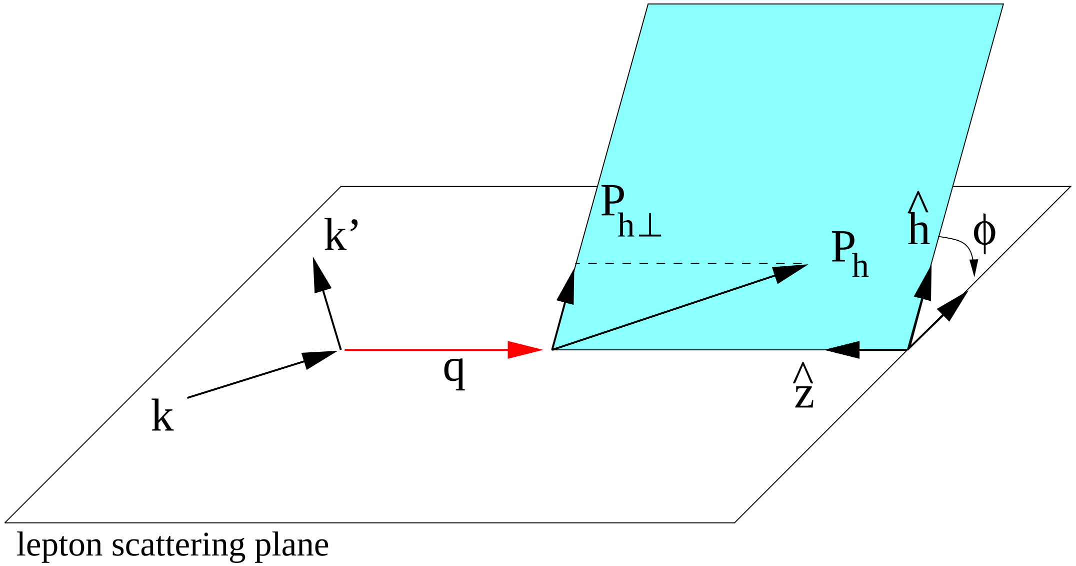
It is mostly sensitive to the high $Q_T^2 \sim Q^2$ hard gluon radiation

Solution: introduce an upper Q_T cut-off or subtract the calculable pQCD contribution

Azimuthal spin asymmetries

Semi-inclusive DIS

$e + p^{\uparrow} \to e' + \pi + X$



$$\frac{d\sigma(e\,p^{\uparrow} \to e'\pi X)}{d\Omega\; d\phi_{\pi}^{e} d|\boldsymbol{P}_{\perp}^{\pi}|^2} \propto \{1 + \ldots + |\boldsymbol{S}_T|\,\sin(\phi_{\pi}^{e} + \phi_{S}^{e})\; A_T^C + |\boldsymbol{S}_T|\,\sin(\phi_{\pi}^{e} - \phi_{S}^{e})\; A_T^S\}$$

Collins asymmetry: $A_T^C \propto h_1 H_1^{\perp}$

Sivers asymmetry: $A_T^S \propto f_{1T}^{\perp} D_1$

HERMES has measured nonzero Collins and Sivers asymmetries

Example 2: Collins asymmetry

$$\frac{d\sigma(e\,p \to e'\pi X)}{dxdzdyd\phi_e d^2\boldsymbol{q}_T} \propto \left\{1 + \ldots + |\boldsymbol{S}_T|\,\sin(\phi_C)\; A_T^C(Q_T)\right\}$$

The asymmetry's analyzing power is given by

$$A_T^C(Q_T) = \frac{\sum_a\; e_a^2\;\, B(y)\;\, h_1^a(x) H_1^{\perp a}(z)}{\sum_b\; e_b^2\;\, A(y)\;\, f_1^b(x) D_1^b(z)}\,\frac{\mathcal{A}(Q_T)}{2M_\pi^2 R^2}$$

$$\mathcal{A}(Q_T) = M_\pi\,\frac{\int db\, b^2\, J_1(bQ_T)\;\exp\left(-S(b_*) - S_{NP}(b)\right)}{\int db\, b\, J_0(bQ_T)\;\exp\left(-S(b_*) - S_{NP}^u(b)\right)}$$

$A(y) = (1 - y + \frac{1}{2}y^2)$, $B(y) = (1 - y)$

At tree level: $\mathcal{A}^{(0)}(Q_T) = \dfrac{M_\pi Q_T R^4}{R_u^2}\;\exp\left(-\left[R^2 - R_u^2\right] Q_T^2/2\right)$

The asymmetry from the Y term not yet fully investigated (Ji, Ma, Yuan, forthcoming work)

Numerical estimates

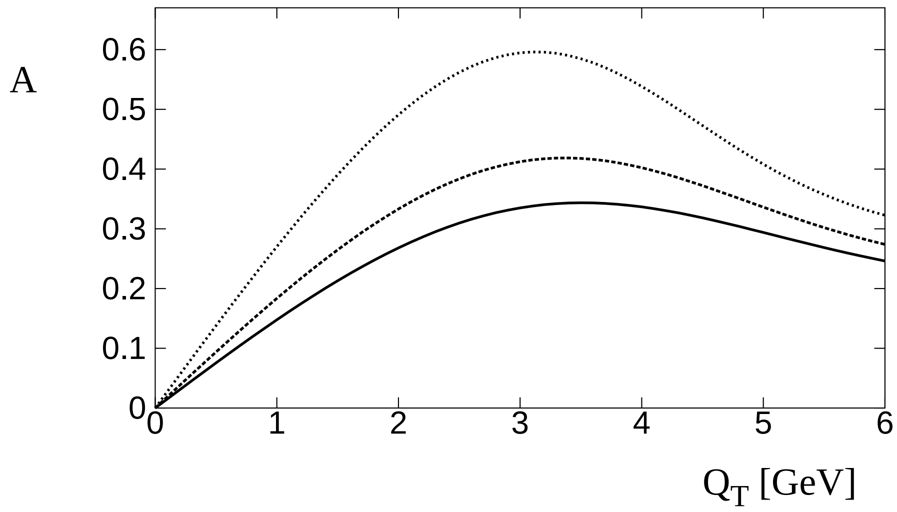
$\mathcal{A}(Q_T, Q)$

$Q = 30$ GeV (upper)

$Q = 60$ GeV (middle)

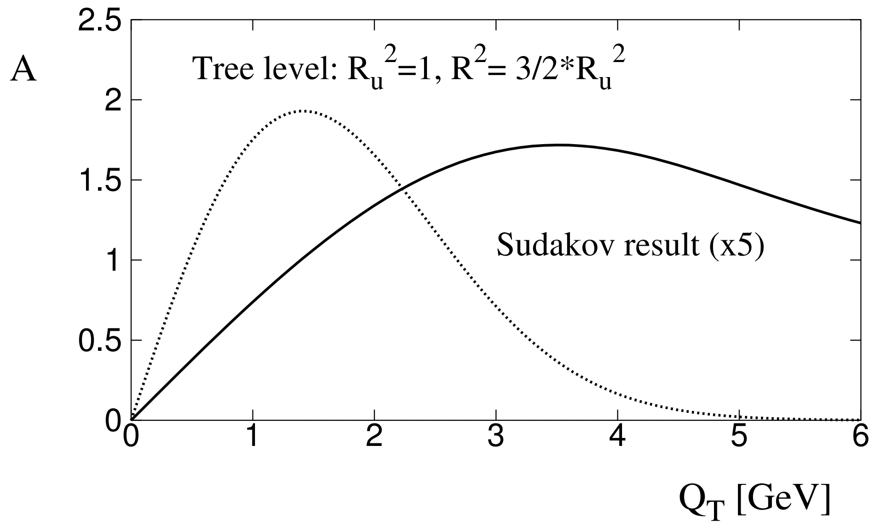
$Q = 90$ GeV (lower)

$\mathcal{A}(Q_T, Q) \sim Q^{-0.5} - Q^{-0.6}$



$\mathcal{A}(Q_T, Q = 90)$ is considerably smaller and broader than $\mathcal{A}^{(0)}$

Tree level estimates tend to overestimate transverse momentum dependent azimuthal spin asymmetries



Projecting out asymmetries

Consider the cross sections integrated, but weighted with a function of the transverse momentum of π:

$$\langle W \rangle \equiv \int d^2\boldsymbol{P}_\perp^\pi \; W \, \frac{d\sigma^{[e\,p^\uparrow \to e\,\pi\,X]}}{dx\,dy\,dz\,d\phi^e\,d\phi_\pi^e\,d|\boldsymbol{P}_\perp^\pi|^2}$$

where $W = W(|\boldsymbol{P}_\perp^\pi|, \phi_\pi^e)$ (Restrict to the case of $|\boldsymbol{P}_\perp^\pi|^2 \ll Q^2$)

If one weights with powers of the observed transverse momentum one obtains for instance the following leading order expression

$$\frac{\left\langle \sin(\phi_C)\,|\boldsymbol{P}_\perp^\pi|/M_\pi \right\rangle}{\left[4\pi\,\alpha^2\,s/Q^4\right]} = |\boldsymbol{S}_T|\,(1-y) \sum_{a,\bar{a}} e_a^2\, x\, h_1^a(x) z H_1^{\perp(1)a}(z)$$

This particular moment is insensitive to Sudakov factors and suppression

Y term asymmetry is expected to be a decreasing function of $|\boldsymbol{P}_\perp^\pi|$, so not dominant

The same conclusions apply to the Sivers asymmetry

Gauge invariant definition of TMDs

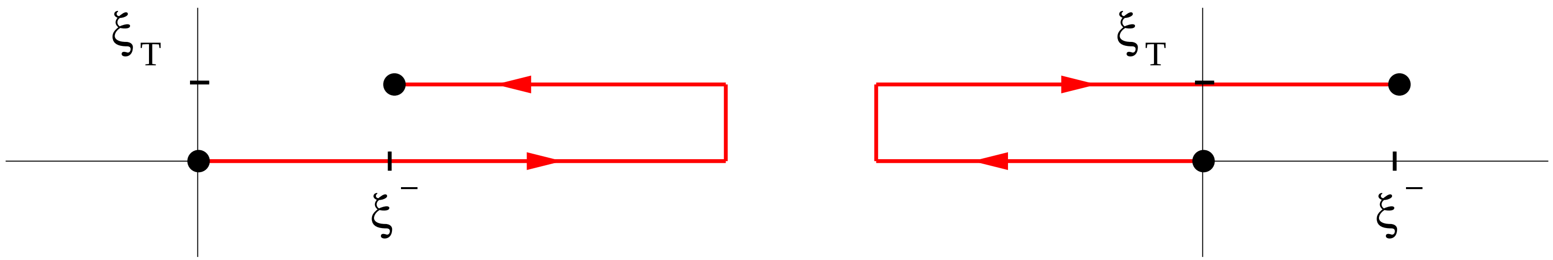
$$f_1 \propto \text{F.T.}\, \langle P|\overline{\psi}(0)\, \mathcal{L}[0,\lambda]\, \gamma^+\, \psi(\lambda n_-)|P\rangle$$

$$\mathcal{L}[0,\lambda] = \mathcal{P}\exp\left(-ig\int_0^\lambda d\eta\, A^+(\eta n_-)\right)$$

In contrast:

$$h_1^\perp\, \epsilon_T^{ij} k_{Tj} \propto \text{F.T.}\, \langle P|\overline{\psi}(0)\, \mathcal{L}[0,\xi]\, \gamma^i\gamma^+\gamma_5\, \psi(\xi)|P\rangle\Big|_{\xi=(\xi^-,0^+,\boldsymbol{\xi}_T)}$$

Proper gauge invariant definition of $h_1^\perp$ in DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing



Belitsky, Ji & Yuan, NPB 656 ('03) 165

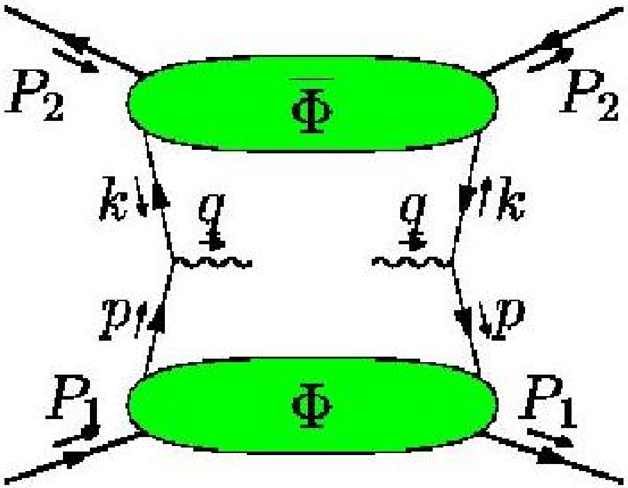
Process dependence

As a consequence (Collins, PLB 536 ('02) 43):

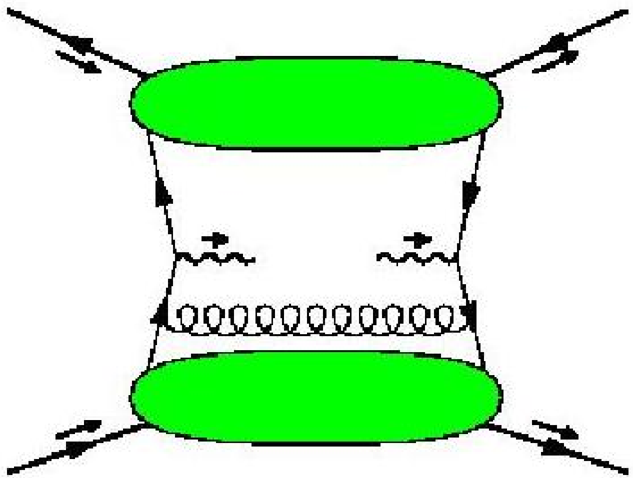
$$(h_1^\perp)_{\rm DIS} \;=\; -(h_1^\perp)_{\rm DY}$$

Derived for operator matrix elements, but the link structure itself was derived at $\mathcal{O}(\alpha_s^0)$

At $\mathcal{O}(\alpha_s)$ Bomhof, Mulders, Pijlman (PLB 596 ('04) 277) find more complicated link structures



$$\mathcal{L}^{[-]}(0,\xi)$$

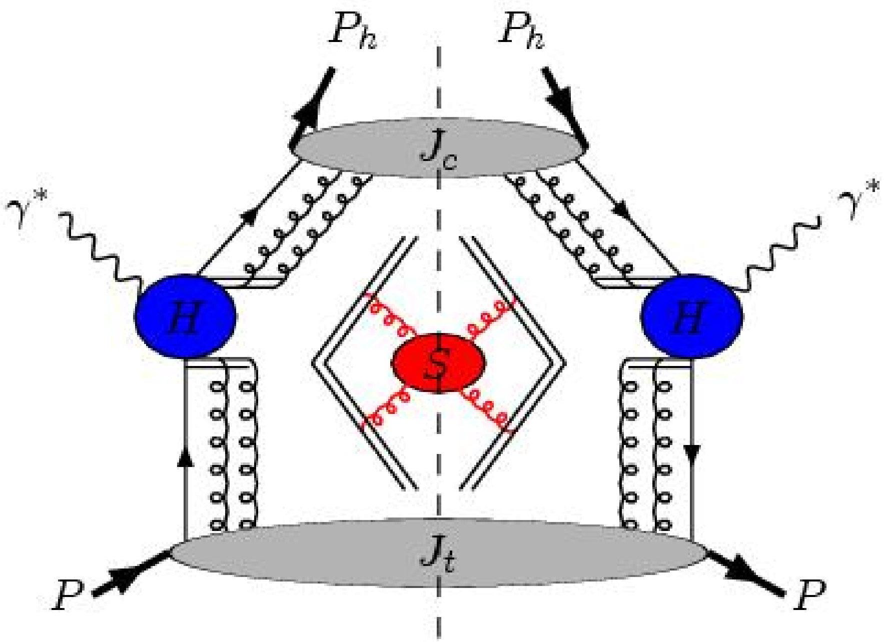


$$\frac{3}{8}\mathcal{L}^{[+]}\ \mathrm{Tr}\ \mathcal{L}^{[\Box]} - \frac{1}{8}\mathcal{L}^{[-]}$$

Process dependence

This is relevant for factorization of low Q_T DY & SIDIS

Ji, Ma, Yuan, PRD 71 ('05) 034005 & PLB 597 ('04) 299



Assuming Wilson lines in TMDs and demonstrating consistency with factorization of perturbative corrections is not the same as deriving the Wilson lines to all orders

Issue not settled yet, but probably no effect of Wilson lines in TMDs on Q^2 dependence

Conclusions

- Leading twist factorization at $Q_T^2 \ll Q^2$ requires distribution and fragmentation functions as a function of transverse momentum
- Sudakov factors need to be included – important for Q^2 evolution of azimuthal asymmetries
- In two examples this was numerically demonstrated:
  - $\cos(2\phi)$ asymmetry in Drell-Yan
  - Collins effect $\sin(\phi_C)$ asymmetry in $e\, p^\uparrow \to e'\, \pi\, X$
- Rough rule of thumb:
  - Asymmetries involving one TMD (e.g. Collins & Sivers asymmetries): $\sim 1/\sqrt{Q}$
  - Two TMDs (e.g. $\cos 2\phi$ in DY): $\sim 1/Q$
- Tree level estimates tend to overestimate transverse momentum dependent azimuthal asymmetries with increasing Q^2
- Wilson lines are not expected to affect these conclusions, but not demonstrated yet