SUSY/Higgs lectures: overview

Sven Heinemeyer, IFCA (CSIC–UC)

Gausdal, 01/2007

1. Introduction to SUSY
2. The general MSSM and "simplified versions"
3. The Higgs Bosons in the MSSM
4. SUSY Higgs Phenomenology at the LHC

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SUSY/Higgs lectures (I): Introduction to SUSY

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1. Motivation for SUSY
2. Symmetry and Algebra
3. Superfields and Superspace
4. Supersymmetric Lagrangians
5. Soft SUSY-breaking

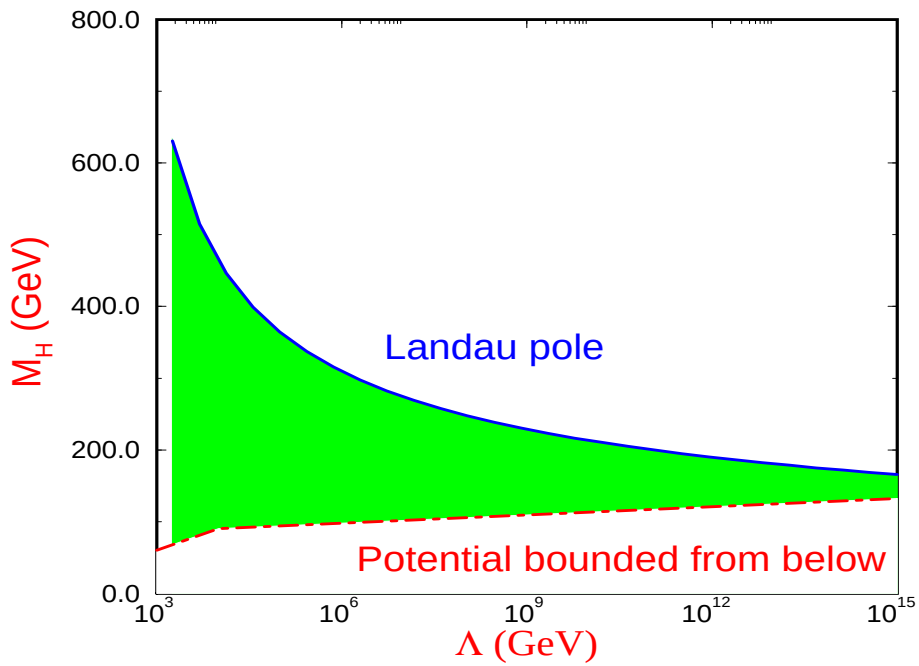
1. Motivation for SUSY

The Standard Model (SM) cannot be the ultimate theory

- The SM does not contain gravity
- Further problems: Hierarchy problem

Up to which energy scale Λ can the SM be valid?

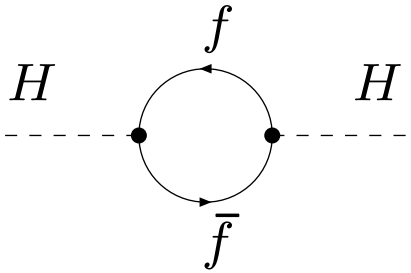
- $\Lambda < M_{\rm Pl}$: inclusion of gravity effects necessary
- stability of Higgs potential: $\Rightarrow$
- Hierarchy problem :
  Higgs mass unstable
  w.r.t. quantum corrections
  $\delta M_H^2 \sim \Lambda^2$
  (but what does this mean?)



Mass is what determines the properties of the free propagation of a particle

Free propagation:

inverse propagator: $i(p^2 - M_H^2)$

Loop corrections: 

inverse propagator: $i(p^2 - M_H^2 + \Sigma_H^f)$

QM: integration over all possible loop momenta k

dimensional analysis:

$$\Sigma_H^f \sim N_f \, \lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

for $\Lambda \to \infty$: $$\Sigma_H^f \sim N_f \, \lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim \Lambda^2} + 2m_f^2 \underbrace{\int \frac{dk}{k}}_{\sim \ln \Lambda} \right)$$

$\Rightarrow$ quadratically divergent!

For $\Lambda = M_{\mathsf{Pl}}$:

$$\Sigma_H^f \approx \delta M_H^2 \sim M_{\mathsf{Pl}}^2 \quad \Rightarrow \quad \delta M_H^2 \approx 10^{30}\, M_H^2$$

(for $M_H \lesssim 1$ TeV)

– no additional symmetry for $M_H = 0$

– no protection against large corrections

$\Rightarrow$ Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale

E.g.: Grand Unified Theory (GUT): $\delta M_H^2 \approx M_{\mathsf{GUT}}^2$

Note however: there is another fine-tuning problem in nature, for which we have no clue so far – cosmological constant

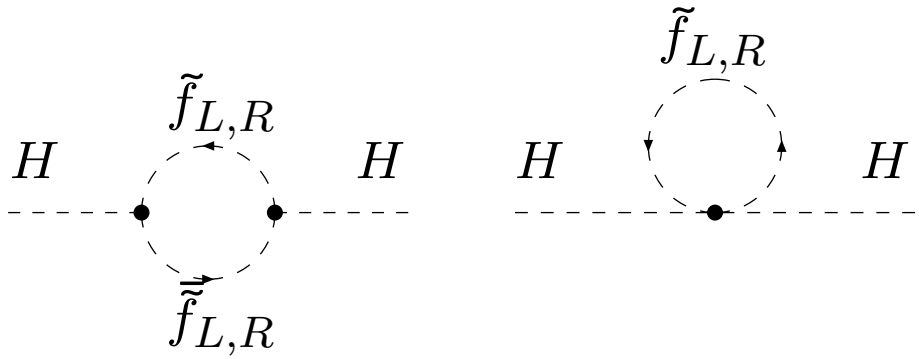
Supersymmetry:

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Effectively: SM particles have SUSY partners (e.g. $f_{L,R} \to \tilde{f}_{L,R}$)

SUSY: additional contributions from scalar fields:



$$\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2 \int d^4k \left(\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) + \text{ terms without quadratic div.}$$

for $\Lambda \to \infty$: $\quad \Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2\, \Lambda^2$

$\Rightarrow$ quadratic divergences cancel for

$$N_{\tilde{f}_L} = N_{\tilde{f}_R} = N_f$$
$$\lambda^2_{\tilde{f}} = \lambda^2_f$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

Soft SUSY breaking: $m^2_{\tilde{f}} = m^2_f + \Delta^2, \quad \lambda^2_{\tilde{f}} = \lambda^2_f$

$$\Rightarrow \Sigma_H^{f+\tilde{f}} \sim N_f \, \lambda^2_f \, \Delta^2 + \ldots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1 \text{ TeV}$$

$\Rightarrow$ SUSY at TeV scale provides attractive solution of hierarchy problem

2. Symmetry and Algebra

Symmetry: a group of transformations that leaves the Lagrangian invariant

Generators of the group fulfill certain algebra

Examples:

0. Angular rotation: $\Psi \to \Psi e^{i\theta^a L_a}$
   theory is invariant under rotation
   generators: L_a, algebra: $[L_a, L_b] = i\varepsilon_{abc} L^c$
   Quantum numbers: (max. spin)^2, spin $[l(l+1), m = +l \ldots -l]$

1. Internal symmetry: $SU(3) \times SU(2) \times U(1)$
   gauge symmetry for the description of the strong and electroweak force
   generators: T_a, algebra: $[T_a, T_b] = i f_{abc} T^c$
   Quantum numbers: color, weak isospin, hyper charge

2. Poincaré symmetry (includes rotation)
   space–time symmetries:
   Lorentz transformations: $\Lambda^{\mu\nu}$, translations: P^ρ
   Quantum numbers: mass, spin

Lorentz group: Representations of Lorentz group are labeled by two 'spins', $j_1, j_2 = 0$, where $j_1, j_2 = 0, \frac{1}{2}, 1, \ldots$

Basic representations $M_\alpha{}^\beta$ act on:

$(\frac{1}{2}, 0)$: LEFT-handed 2-component Weyl spinor, ψ_α

$(0, \frac{1}{2})$: RIGHT-handed 2-component Weyl spinor, $\bar{\psi}^{\dot{\alpha}}$

The two component Weyl spinors ψ_α (left-handed) and $\bar{\psi}^{\dot{\alpha}}$ (right-handed) transform under Lorentz transformations as follows:

$$\begin{aligned} \psi'_\alpha &= M_\alpha{}^\beta \psi_\beta; & \bar{\psi}'_{\dot{\alpha}} &= (M^*)_{\dot{\alpha}}{}^{\dot{\beta}} \bar{\psi}_{\dot{\beta}} \\ \psi'^\alpha &= (M^{-1})_\beta{}^\alpha \psi^\beta; & \bar{\psi}'^{\dot{\alpha}} &= (M^{*-1})_{\dot{\beta}}{}^{\dot{\alpha}} \bar{\psi}^{\dot{\beta}} \end{aligned}$$

where $M = \exp(i\frac{\vec{\sigma}}{2}(\vec{\vartheta} - i\vec{\varphi}))$ and $\vec{\vartheta}$ and $\vec{\varphi}$ are the three rotation angles and boost parameters, respectively

$\Rightarrow$ spinors with undotted indices (first two components of Dirac spinor) transform according to $(\frac{1}{2}, 0)$-representation of Lorentz group,

spinors with dotted indices (last two components of Dirac spinor) transform according to $(0, \frac{1}{2})$-representation

Our world (the SM) is described by:

– internal symmetry: T_a

– Poincaré symmetry: $\Lambda^{\mu\nu}$, P^ρ

internal symmetry is a trivial extension of the Poincaré symmetry:

$$[\Lambda^{\mu\nu}, T^a] = 0, \qquad [P^\rho, T^a] = 0$$

$\Rightarrow$ direct product: (Poincaré group) $\otimes$ (internal symmetry group)

Particle states characterized by maximal set of commuting observables:

$$|\underbrace{m, s; \vec{p}, s_3}_{\text{space–time}}; \underbrace{Q, I, I_3, Y, \ldots}_{\text{internal}}\rangle$$

quantum numbers

Wanted: extension of the SM

Theorem # 1: No-go theorem [*Coleman, Mandula '67*]

Any Lie-group containing Poincaré group P and internal symmetry group $\tilde{G}$ must be direct product $P \otimes \tilde{G}$

$$|\underbrace{m, s; \vec{p}, s_3}_{\text{space–time}}; \underbrace{\tilde{g}, \ldots}_{\text{internal}}\rangle$$

quantum numbers

New group $\tilde{G}$ with generators Q^α and

$$[\Lambda^{\mu\nu}, Q^\alpha] \neq 0, \quad [P^\rho, Q^\alpha] \neq 0$$

impossible

Direct product $\Rightarrow$ no irreducible multiplets can contain particles with different mass or different spin

$\Rightarrow$ new symmetry must predict new particles with the same mass and spin as in the SM

$\Rightarrow$ experimentally excluded, no such symmetry possible :-(

Theorem # 2: How-To-Avoid-the-No-go theorem

[*Gol'fand, Likhtman '71*] [*Volkov, Akulov '72*] [*Wess, Zumino '73*]

No go theorem can be evaded if instead of Lie-group (generators fulfill commutator relations):

$$[\ldots,\ldots] \rightarrow \{\ldots,\ldots\}$$

Anticommutator: $\{A, B\} = AB + BA$

$\Rightarrow$ Generator Q^α is fermionic (i.e. it has spin $\frac{1}{2}$)

$\Rightarrow$ Particles with different spin in one multiplet possible

$$Q|\text{boson}\rangle = |\text{fermion}\rangle, \quad Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Q changes spin (behavior under spatial rotations) by $\frac{1}{2}$

E.g.:

spin 2	$\rightarrow$	spin $\frac{3}{2}$	$\rightarrow$	spin 1
graviton		gravitino		photon

Simplest case: only one fermionic generator Q_α (and conjugate $\bar{Q}_{\dot\beta}$)

$\Rightarrow$ $N = 1$ SUSY algebra:

$$
\begin{aligned}
[Q_\alpha, P_\mu] = \left[\bar{Q}_{\dot\beta}, P_\mu\right] &= 0 \\
[Q_\alpha, M^{\mu\nu}] &= i(\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta \\
\left\{Q_\alpha, Q_\beta\right\} = \left\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\right\} &= 0 \\
\left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} &= 2(\sigma^\mu)_{\alpha\dot\alpha} P_\mu
\end{aligned}
$$

Energy $= H = P_0$, $\Rightarrow$ $[Q_\alpha, H] = 0$ $\Rightarrow$ conserved charge

$\Rightarrow$ SUSY: symmetry that relates bosons to fermions

unique extension of Poincaré group of symmetries of $D = 4$ relativistic QFT

Can SUSY be an exact symmetry?

Consider fermionic state $|f\rangle$ with mass m
$\Rightarrow$ there is a bosonic state $|b\rangle = Q_\alpha|f\rangle$

$P^2|f\rangle = m^2|f\rangle$

$$\Rightarrow \quad P^2|b\rangle = P^2 Q_\alpha|f\rangle = Q_\alpha P^2|f\rangle = Q_\alpha m^2|f\rangle = m^2|b\rangle$$

$\Rightarrow$ for each fermionic state there is a bosonic state with the same mass

$\Rightarrow$ states are paired bosonic $\leftrightarrow$ fermionic

$\Rightarrow$ still experimentally excluded

$\Rightarrow$ SUSY must be broken

More on spinors:

The components of the spinors are Grassmann variables ,
i.e. anticommuting c-numbers

Raising and lowering of indices through the totally antisymmetric ϵ-tensor:

$$\epsilon^{\alpha\beta} = \epsilon^{\dot\alpha\dot\beta} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \;\; \epsilon_{\alpha\beta} = \epsilon_{\dot\alpha\dot\beta} \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta; \quad \psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta; \quad \bar\psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\psi^{\dot\beta}; \quad \bar\psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta};$$

In particular

$$\epsilon^\alpha{}_\beta = \epsilon^{\alpha\gamma}\epsilon_{\gamma\beta} = \delta^\alpha{}_\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The product of two Grassmann spinors is defined through

$$\theta\zeta \equiv \theta^\alpha\zeta_\alpha = \theta^\alpha\epsilon_{\alpha\beta}\zeta^\beta = -\epsilon_{\alpha\beta}\zeta^\beta\theta^\alpha = \zeta^\beta\epsilon_{\beta\alpha}\theta^\alpha = \zeta^\beta\theta_\beta = \zeta\theta$$

$$\bar\theta\bar\zeta \equiv \bar\theta_{\dot\alpha}\bar\zeta^{\dot\alpha} = -\bar\zeta^{\dot\alpha}\bar\theta_{\dot\alpha} = \bar\zeta\bar\theta$$

The γ-matrices are defined by

$$\gamma^\mu \equiv \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot\alpha} \\ (\bar\sigma^\mu)^{\dot\alpha\alpha} & 0 \end{pmatrix}; \quad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

where $(\sigma^\mu)_{\alpha\dot\alpha} \equiv \{1, \sigma^1, \sigma^2, \sigma^3\}_{\alpha\dot\alpha}$; $(\bar\sigma^\mu)_{\alpha\dot\alpha} \equiv \{1, -\sigma^1, -\sigma^2, -\sigma^3\}_{\alpha\dot\alpha}$;

$(\sigma_\mu)_{\alpha\dot\alpha} = g_{\mu\nu}(\sigma^\nu)_{\alpha\dot\alpha}$; $(\bar\sigma_\mu)_{\alpha\dot\alpha} = g_{\mu\nu}(\bar\sigma^\nu)_{\alpha\dot\alpha}$.

The convention for the metric is: $g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}\{1, -1, -1, -1\}$.

The γ-matrices have the usual commutation relation, $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

This follows from

$$(\sigma^\mu)_{\alpha\dot\alpha}(\bar\sigma^\nu)^{\dot\alpha\alpha} = \mathrm{Tr}(\sigma^\mu\bar\sigma^\nu) = 2g^{\mu\nu}.$$

Furthermore,

$$\begin{aligned} \sigma^{\mu\nu} &= \frac{1}{4}\left(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu\right), \\ \bar\sigma^{\mu\nu} &= \frac{1}{4}\left(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu\right). \end{aligned}$$

The Lorentz covariant expressions involving 4-component Dirac spinors can be written in two-component notation using

$$\Phi = \begin{pmatrix} \lambda_\alpha \\ \bar{\phi}^{\dot{\alpha}} \end{pmatrix}; \quad \overline{\Psi} = \begin{pmatrix} \chi^\alpha & \bar{\psi}_{\dot{\alpha}} \end{pmatrix}$$

$$\begin{aligned}
\overline{\Psi}\Phi &= \chi\lambda + \bar{\psi}\bar{\phi} = \chi^\alpha\lambda_\alpha + \bar{\psi}_{\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^5\Phi &= \bar{\psi}\bar{\phi} - \chi\lambda = \bar{\psi}_{\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} - \chi^\alpha\lambda_\alpha \\
\overline{\Psi}\gamma^\mu\Phi &= \chi\sigma^\mu\bar{\phi} - \lambda\sigma^\mu\bar{\psi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} - \lambda^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^\mu\gamma^5\Phi &= \chi\sigma^\mu\bar{\phi} + \lambda\sigma^\mu\bar{\psi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} + \lambda^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^\mu\gamma^\nu\Phi &= \chi\sigma^\mu\bar{\sigma}^\nu\lambda + \bar{\psi}\bar{\sigma}^\mu\sigma^\nu\bar{\phi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}(\bar{\sigma})^{\dot{\alpha}\beta}\lambda_\beta + \bar{\psi}_{\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\beta}(\sigma^\nu)_{\beta\dot{\beta}}\bar{\phi}^{\dot{\beta}}
\end{aligned}$$

$$P_L = \frac{1}{2}(1+\gamma_5) = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}, \quad P_R = \frac{1}{2}(1-\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1} \end{pmatrix}.$$

$$\begin{aligned}
\Rightarrow P_L\Phi &= \lambda_\alpha \quad \text{left-handed Weyl spinor (LHWS)} \\
\Rightarrow P_R\Phi &= \bar{\phi}^{\dot{\alpha}} \quad \text{right-handed Weyl spinor (RHWS)}
\end{aligned}$$

Further consequences of the SUSY algebra

$$\left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

$$\Rightarrow \left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} \bar\sigma_\nu^{\dot\beta\alpha} = 2\,\underbrace{\sigma^\mu_{\alpha\dot\beta}\bar\sigma_\nu^{\dot\beta\alpha}}_{2g^\mu{}_\nu} P_\mu = 4P_\nu$$

$$\nu = 0 \Rightarrow H = P_0 = \tfrac{1}{4}\left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\}\bar\sigma_0^{\dot\beta\alpha} = \tfrac{1}{4}\left(\left\{Q_1, Q_1^\dagger\right\} + \left\{Q_2, Q_2^\dagger\right\}\right)$$

where $\bar{Q}_{\dot\alpha} = (Q_\alpha)^\dagger$

$\left\{Q_i, Q_i^\dagger\right\} = Q_i Q_i^\dagger + Q_i^\dagger Q_i$: hermitian operator, eigenvalues ≥ 0

$\Rightarrow$ for any state $|\alpha\rangle$: $\langle\alpha|H|\alpha\rangle \geq 0$

spectrum of H is bounded from below, ≥ 0

$\Rightarrow$ no negative eigenvalues

State with lowest energy: vacuum state $|0\rangle$

if vacuum state is symmetric, i.e. $Q|0\rangle = 0$, $Q^\dagger|0\rangle = 0$ for all Q

$\Rightarrow$ vacuum has zero energy, $\langle 0|H|0\rangle = E_{\text{vac}} = 0$

For spontaneous symmetry breaking: vacuum state is not invariant

$\Rightarrow$ If (global) SUSY is spontaneously broken, i.e. $Q_\alpha|0\rangle \neq 0$,

then $\langle 0|H|0\rangle = E_{\text{vac}} > 0$

$\Rightarrow$ non-vanishing vacuum energy

Further consequences for SUSY multiplets:

$\left\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\right\} = 0 \Rightarrow \bar{Q}^2_{\dot{\alpha}} = 0$ (and $Q^2_{\alpha} = 0$)

Consider multiplet, start with state of lowest helicity λ_0

application of $\bar{Q}_{\dot{\alpha}} \Rightarrow$ one additional state with helicity $\lambda_0 + \frac{1}{2}$

further application of $\bar{Q}_{\dot{\alpha}} \Rightarrow 0$, no further state

$\Rightarrow$ one fermionic + one bosonic state

(N SUSY generators $\Rightarrow 2^{N-1}$ bosonic and 2^{N-1} fermionic states)

SUSY multiplet contains equal number of bosonic and fermionic state

Most relevant multiplets (possess also CPT conjugate 'mirrors'):

- chiral supermultiplet: $-\frac{1}{2}, 0$

  Weyl fermion (quark, lepton, ...) + complex scalar (squark, slepton)

- vector supermultiplet: $-1, -\frac{1}{2}$

  Gauge boson (massless vector) + Weyl fermion (gaugino)

- graviton supermultiplet: $-2, -\frac{3}{2}$

  graviton + gravitino

3. Superfields and Superspace

Translation transformation: P_μ , parameter: x^μ

SUSY transformation: $Q_\alpha, \bar{Q}_{\dot{\alpha}}$, parameter: $\theta, \bar{\theta} \rightarrow$ anticommuting c-numbers
("Grassmann variables")

$\Rightarrow$ Extension of 4-dim. space–time by coordinates θ^α, $\bar{\theta}^{\dot{\alpha}}$: superspace

Point in superspace: $X = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$,	Superfield: $\phi(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$

Grassman variables:

SUSY transformation: $Q_\alpha, \bar{Q}_{\dot\alpha}$, parameter: $\theta, \bar{\theta} \rightarrow$ anticommuting c-numbers
("Grassmann variables")

Without spinor index:

$$\{\theta, \theta\} = 0 \;\Rightarrow\; \theta\theta = 0$$

With two-component spinor index (as it is our case):

$$\theta\theta \equiv \theta^\alpha\theta_\alpha = \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta \;\Rightarrow\; \theta\theta \neq 0,\; (\theta^1\theta_2 \neq 0)$$

Taylor expansion in Grassmann variable: $\theta^\alpha\theta^\beta\theta^\gamma = 0$ $(\alpha, \beta, \gamma = 1, 2)$

$\Rightarrow$ Taylor expansion terminates after second term, i.e. $\phi(\theta) = a + \theta\psi + \theta\theta\, f$

Integration: $\int d\theta = 0$, $\int d\theta\, \theta = 1$

$d^2\theta = -\frac{1}{4}\epsilon_{\alpha\beta} d\theta^\alpha d\theta^\beta$

$$\Rightarrow \quad \int d^2\theta\, \phi(\theta) = \int d^2\theta (a + \theta\psi + \theta\theta\, f) = f$$

With Grassmann variables:

SUSY algebra can be written in terms of commutators only

$$
\begin{aligned}
[\theta Q, \bar{\theta}\bar{Q}] &= 2\theta\sigma^\mu\bar{\theta}P_\mu \\
[\theta Q, \theta Q] = [\bar{\theta}\bar{Q}, \bar{\theta}\bar{Q}] &= 0 \\
[P^\mu, \theta Q] = [P^\mu, \bar{\theta}\bar{Q}] &= 0
\end{aligned}
$$

$\Rightarrow$ can be treated like Lie-group with anticommuting parameters

SUSY transformations: (Lagrangian should be invariant!)

Group element of finite SUSY transformation:

$$S(y,\xi,\bar{\xi}) = \exp i\left(\xi Q + \bar{\xi}\bar{Q} - y^\mu P_\mu\right)$$

in analogy to group elements for Lie-groups

ξ, $\bar{\xi}$ are independent of y^μ: global SUSY transformation

Transformation of superfield: $S(y,\xi,\bar{\xi})\phi(x,\theta,\bar{\theta})$

Use Hausdorff's formula ($e^A e^B = \ldots$) and SUSY algebra

$$\Rightarrow \quad S(y,\xi,\bar{\xi})\phi(x,\theta,\bar{\theta}) = \phi(x^\mu + y^\mu - i\xi\sigma^\mu\bar{\theta} + i\theta\sigma^\mu\bar{\xi}, \xi+\theta, \bar{\xi}+\bar{\theta})$$

representations of generators obtained from infinitesimal transformation of superfield

$$\Rightarrow \quad P_\mu = i\partial_\mu, \quad Q_\alpha = -i\,\partial_\alpha + (\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = i\,\partial_{\dot{\alpha}} - (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

with $\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}$, $\bar{\partial}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$

Definition of covariant derivatives:

$$D_\alpha = -i\,\partial_\alpha - (\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar D_{\dot\alpha} = i\,\bar\partial_{\dot\alpha} + (\theta\sigma^\mu)_{\dot\alpha}\partial_\mu$$

EXTRA ——————————————- 5 ——————————————- EXTRA

Anticommutation relations of Q_α, D_α:

$$\begin{aligned}
&\{Q_\alpha, D_\beta\} = \{\bar Q_{\dot\alpha}, \bar D_{\dot\beta}\} = \{Q_\alpha, \bar D_{\dot\beta}\} = 0\\
&\{Q_\alpha, Q_\beta\} = \{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = 0\\
&\{D_\alpha, D_\beta\} = \{\bar D_{\dot\alpha}, \bar D_{\dot\beta}\} = 0\\
&\{Q_\alpha, \bar Q_{\dot\alpha}\} = 2i(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu = 2(\sigma^\mu)_{\alpha\dot\alpha}P_\mu\\
&\{D_\alpha, \bar D_{\dot\alpha}\} = -2i(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu = -2(\sigma^\mu)_{\alpha\dot\alpha}P_\mu
\end{aligned}$$

$\Rightarrow$ D_α, $\bar D_{\dot\alpha}$ anticommute with SUSY generators

$\Rightarrow$ are invariant under SUSY transformations

$$[(\xi Q + \bar\xi\bar Q), D_\alpha] = 0, \quad [(\xi Q + \bar\xi\bar Q), \bar D_{\dot\alpha}] = 0$$

General superfield in component form

Most general form of field depending on x, θ, $\bar{\theta}$:

$$\Phi(x,\theta,\bar{\theta}) = \varphi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta F(x) + \bar{\theta}\bar{\theta}H(x) + \theta\sigma^{\mu}\bar{\theta}A_{\mu}(x) + (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\xi(x) + (\theta\theta)(\bar{\theta}\bar{\theta})D(x)$$

Further terms vanish because of $\theta\theta\theta = \bar{\theta}\bar{\theta}\bar{\theta} = 0$

Components (can be complex):

φ, F, H, D: scalar fields

A_{μ}: vector field

ψ, $\bar{\chi}$, $\bar{\lambda}$, ξ: Weyl-spinor fields

$\Rightarrow$ Too many components in 4-dim. for irreducible representation of SUSY with spin ≤ 1 (chiral or vector multiplet)

$\Rightarrow$ representation is reducible
(not all component fields mix with each other under SUSY transf.)

$\Rightarrow$ Irreducible superfields (smallest building blocks) from imposing conditions on general superfield
conditions need to be invariant under SUSY transformations:

$\bar{D}_{\dot{\alpha}}\Phi = 0$: left-handed chiral superfield (LHχSF)

$D_\alpha\Phi = 0$: right-handed chiral superfield (RHχSF)

$\Phi = \Phi^\dagger$: vector superfield

$\Rightarrow$ chiral superfields describe left- or right-handed component of SM fermion
$+$ scalar partner

LHχSF in components:

$$\begin{aligned}\phi(x,\theta,\bar{\theta}) &= \varphi(x) + \sqrt{2}\theta\psi(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\psi(x)\sigma^\mu\bar{\theta}) \\ &\quad - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\mu\partial_\mu\varphi(x) - (\theta\theta)F(x)\end{aligned}$$

φ, F: scalar fields , ψ: Weyl-spinor field

LHχSF: Transf. of component fields with infinitesimal SUSY param. $\xi, \bar{\xi}$:

$$\delta\phi(x,\theta,\bar{\theta}) = i(\xi Q + \bar{\xi}\bar{Q})\phi(x,\theta,\bar{\theta})$$

Comparison with

$$\begin{aligned}\delta\phi(x,\theta,\bar{\theta}) &= \delta\varphi(x) + \sqrt{2}\theta\delta\psi(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\delta\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\delta\psi(x)\sigma^\mu\bar{\theta}) \\ &\quad - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\mu\partial_\mu\delta\varphi(x) - (\theta\theta)\delta F(x)\end{aligned}$$

$\Rightarrow$ determination of $\delta\varphi$, $\delta\psi$, δF:

$$\delta\varphi = \sqrt{2}\xi\psi \qquad \text{boson} \rightarrow \text{fermion}$$

$$\delta\psi_\alpha = -\sqrt{2}F\xi_\alpha - i\sqrt{2}(\sigma^\mu\bar{\xi})_\alpha\partial_\mu\varphi \qquad \text{fermion} \rightarrow \text{boson}$$

$$\delta F = \partial_\mu(-i\sqrt{2}\psi\sigma^\mu\bar{\xi}\,) \qquad F \rightarrow \text{total derivative}$$

RHχSF: analogously

Vector superfield in components:

$$
\begin{aligned}
V(x,\theta,\bar\theta) &= c(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \theta\sigma^\mu\bar\theta v_\mu(x) \\
&+ \frac{i}{2}(\theta\theta)(M(x)+iN(x)) - \frac{i}{2}(\bar\theta\bar\theta)(M(x)-iN(x)) \\
&+ i(\theta\theta)\bar\theta\left(\bar\lambda(x) + \frac{i}{2}\partial_\mu\chi(x)\sigma^\mu\right) - i(\bar\theta\bar\theta)\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right) \\
&+ \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\left(D(x) - \frac{1}{2}\partial^\mu\partial_\mu c(x)\right)
\end{aligned}
$$

Number of components can be reduced by SUSY gauge transformation:

Wess-Zumino gauge: $\chi(x) = c(x) = M(x) = N(x) \equiv 0$

Vector SF: $V(x,\theta,\bar\theta) = \ldots + i(\theta\theta)\bar\theta\bar\lambda(x) - i(\bar\theta\bar\theta)\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D(x) + \ldots$

$\delta D = -\xi\sigma^\mu\partial_\mu\bar\lambda(x) - \partial_\mu\lambda(x)\sigma^\mu\bar\xi$ $\qquad D \to$ total derivative

4. Supersymmetric Lagrangians

Aim: construct an action that is invariant under SUSY transformations:

$$\delta \int d^4x \mathcal{L}(x) = 0$$

Satisfied if $\mathcal{L} \longrightarrow \mathcal{L} +$ total derivative

F and D terms (the terms with the largest number of θ and $\bar{\theta}$ factors) of chiral and vector superfields transform into a total derivative under SUSY transformations

$\Rightarrow$ Use F-terms (LHχSF, RHχSF) and D-terms (Vector SF) to construct an invariant action:

$$S = \int d^4x \left(\int d^2\theta \mathcal{L}_F + \int d^2\theta d^2\bar{\theta} \mathcal{L}_D \right)$$

If Φ is a LHχSF $\Rightarrow$ Φ^n is also a LHχSF (since $\bar{D}_{\dot{\alpha}}\Phi^n = 0$ for $\bar{D}_{\dot{\alpha}}\Phi = 0$)

$\Rightarrow$ products of chiral superfields are chiral superfields, products of vector superfields are vector superfields

F-term Lagrangian:

$$\mathcal{L}_F = \int d^2\theta \sum_{ijk} \left(a_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \lambda_{ijk} \Phi_i \Phi_j \Phi_k \right) + \text{ h.c.}$$

Terms of higher order in Φ_i lead to non-renormalizable Lagrangians

$\Rightarrow$ F-term Lagrangian contains mass terms, scalar–fermion interactions ($\rightarrow$ superpotential), but no kinetic terms

D-term Lagrangian:

$$\mathcal{L}_D = \int d^2\theta d^2\bar{\theta} V$$

$\Rightarrow$ D-term Lagrangian contains kinetic terms

Example: the Wess–Zumino Lagrangian

Construction of Lagrangian from chiral superfields Φ_i

$\Rightarrow$ Φ_i, $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$

$\Phi_i^\dagger\Phi_i$: vector superfield, $(\Phi_i^\dagger\Phi_i)^\dagger = \Phi_i^\dagger\Phi_i$

$$\left[\Phi_i^\dagger\Phi_i\right]_{\theta\theta\bar\theta\bar\theta} = F^\dagger F + (\partial_\mu\varphi^*)(\partial^\mu\varphi) + \frac{i}{2}(\psi\sigma^\mu\partial_\mu\bar\psi - \partial_\mu\psi\sigma^\mu\bar\psi) + \partial_\mu(\ldots)$$

Auxiliary field F can be eliminated via equations of motion

$$\begin{aligned}\Rightarrow \quad \mathcal{L}_D \;=\; & \frac{i}{2}(\psi_i\sigma^\mu\partial_\mu\bar\psi_i - (\partial_\mu\psi_i)\sigma^\mu\bar\psi_i) - \frac{1}{2}m_{ij}(\psi_i\psi_j + \bar\psi_i\bar\psi_j) \\ & + (\partial_\mu\varphi_i^*)(\partial^\mu\varphi_i) - \sum_i\left|a_i + \frac{1}{2}m_{ij}\varphi_j + \frac{1}{3}\lambda_{ijk}\varphi_j\varphi_k\right|^2 \\ & - \lambda_{ijk}\varphi_i\psi_j\psi_k - \lambda_{ijk}^\dagger\varphi_i^*\bar\psi_j\bar\psi_k\end{aligned}$$

Auxiliary fields are eliminated via equations of motions:

$$\begin{aligned} \text{abelian}: \quad F &= m\varphi^* + g\varphi^{*2} \\ \text{non-abelian, gauge group } G: \quad D^G &= \ldots \sum_a g_G \left(\varphi_i^\dagger (T_G)^a \varphi_i\right) \end{aligned}$$

(internal indices of T_G, φ_i suppressed)

$$\Rightarrow \quad \mathcal{L}_D = F\,F^* + \frac{1}{2}\sum_G D^G (D^G)^\dagger + \ldots$$

Lagrangian for scalar fields φ_i and spinor fields ψ_i with the same mass m_{ii} contains couplings of type $hf\bar{f}$ and $\tilde{h}\tilde{f}\bar{f}$ with the same strength

$\Rightarrow$ SUSY implies relations between masses and couplings

$\mathcal{L}$ can be rewritten as kinetic part + contribution of superpotential $\mathcal{V}$:

$$\mathcal{V}(\varphi_i) = a_i\varphi_i + \frac{1}{2}m_{ij}\varphi_i\varphi_j + \frac{1}{3}\lambda_{ijk}\varphi_i\varphi_j\varphi_k$$

$$\Rightarrow \quad \mathcal{L} = \frac{i}{2}(\psi_i\sigma^\mu\partial_\mu\bar{\psi}_i - (\partial_\mu\psi_i)\sigma^\mu\bar{\psi}_i) + (\partial_\mu\varphi_i^*)(\partial^\mu\varphi_i)$$
$$- \sum_i \left|\frac{\partial\mathcal{V}}{\partial\varphi_i}\right|^2 - \frac{1}{2}\frac{\partial^2\mathcal{V}}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\mathcal{V}^*}{\partial\varphi_i^*\partial\varphi_j^*}\bar{\psi}_i\bar{\psi}_j$$

$\mathcal{V}$ determines all interactions and mass terms

Without proof:

characteristics of $\mathcal{V}$ = characteristics of $\mathcal{L}$

Special case $a_i = 0$: Wess–Zumino model

$N > 1$ SUSY

So far: $N = 1$ SUSY, simplest case, only one fermionic generator and its hermitian adjoint: Q_α, $\bar{Q}^{\dot{\beta}}$

$\Rightarrow$ one superpartner for photon: photino

N-extended SUSY: N generators Q^A_α, $\bar{Q}^{\dot{\beta}}_B$, $A = 1, \ldots, N$

$\Rightarrow$ N superpartners for the photon, ...

Generalization of anticommutator relation:

$$\{Q^A_\alpha, Q^B_\beta\} = \epsilon_{\alpha\beta} X^{AB}$$

$X^{AB} = -X^{BA}$: "central charges"

Problem:

helicity $+\frac{1}{2}$ and helicity $-\frac{1}{2}$ fermions are in same supermultiplet
(e.g.: hypermultiplet for $N = 2$ SUSY)

$\Rightarrow$ helicity $+\frac{1}{2}$ and $-\frac{1}{2}$ fermions need to transform in the same way under gauge transformations

not possible for chiral fermions of electroweak theory

$\Rightarrow$ $N > 1$ SUSY theories are 'non-chiral'

$\Rightarrow$ $N = 1$ SUSY theories are the best candidates for a realistic low-energy theory (extension of the SM)

However: $N > 1$ SUSY have interesting properties

e.g.: $N = 4$ SUSY field theory (flat space) is finite

Seiberg–Witten solution in $N = 2$ SUSY, ...

5. Soft SUSY-breaking

Exact SUSY: $m_f = m_{\tilde{f}}, \ldots$

$\Rightarrow$ in a realistic model: SUSY must be broken

Only satisfactory way for model of SUSY breaking:

$$\boxed{\text{spontaneous SUSY breaking}}$$

Specific SUSY-breaking schemes (see below) in general yield effective Lagrangian at low energies, which is supersymmetric except for explicit soft SUSY-breaking terms

Soft SUSY-breaking terms: do not alter dimensionless couplings

(i.e. dimension of coupling constants of soft SUSY-breaking terms > 0)
otherwise: re-introduction of the hierarchy problem

$\Rightarrow$ no quadratic divergences (in all orders of perturbation theory)

scale of SUSY-breaking terms: $M_{\text{SUSY}} \lesssim 1$ TeV

Classification of possible soft breaking terms:

[*L. Girardello, M. Grisaru '82*]

- scalar mass terms: $m^2_{\phi_i} |\phi_i|^2$
- trilinear scalar interactions: $A_{ijk}\phi_i\phi_j\phi_k + \text{h.c.}$
- gaugino mass terms: $\frac{1}{2}m\bar{\lambda}\lambda$
- bilinear terms: $B_{ij}\phi_i\phi_j + \text{h.c.}$
- linear terms: $C_i\phi_i$

$\Rightarrow$ relations between dimensionless couplings unchanged

no additional mass terms for chiral fermions

A. Unconstrained models (MSSM):

agnostic about how SUSY breaking is achieved

no particular SUSY breaking mechanism assumed, parameterization of possible soft SUSY-breaking terms

$\Rightarrow$ relations between dimensionless couplings unchanged
no quadratic divergences

most general case:

$\Rightarrow$ 105 new parameters: masses, mixing angles, phases

Good phenomenological description for universal breaking terms

B. Constrained models (mSUGRA, . . .):

assumption on the scenario that achieves spontaneous SUSY breaking
$\Rightarrow$ prediction for soft SUSY-breaking terms
in terms of small set of parameters

Experimental determination of SUSY parameters

$\Rightarrow$ Patterns of SUSY breaking

Wanted: spontaneous SUSY breaking

Remember:

$$\{Q, \bar{Q}\} \sim P_\mu \quad \Rightarrow \quad Q\bar{Q} \sim P_\mu$$
$$P_0 = H \quad \Rightarrow \quad Q\bar{Q} \sim H$$

$\{Q, \bar{Q}\} \sim Q\bar{Q} + \bar{Q}Q$ (with $\bar{Q} = Q^\dagger$): hermitian Operator $\Rightarrow$ Eigen values ≥ 0

$\Rightarrow$ For any state $|\alpha\rangle$: $\langle\alpha|H|\alpha\rangle \geq 0$
spectrum bounded from below

Vacuum: $|0\rangle$

No spontaneous symmetry breaking: vacuum symm.: $Q|0\rangle = 0$, $\bar{Q}|0\rangle = 0$

$$\Rightarrow \langle 0|H|0\rangle = E_{\text{vac}} = 0$$

Spontaneous symmetry breaking: vacuum is not invariant: $Q|0\rangle \neq 0$

$$\Rightarrow \langle 0|H|0\rangle = E_{\text{vac}} > 0$$

$\Rightarrow$ non-vanishing vacuum energy

Spontaneous breaking of global SUSY

If global SUSY is spontaneously broken $\Rightarrow$ $\langle 0|H|0\rangle = E_{\text{vac}} > 0$

$\Rightarrow$ fields need vacuum expectation value

$\Rightarrow$ either $\langle F\rangle > 0$ (F-term breaking) or $\langle D\rangle > 0$ (D-term breaking)

$\Rightarrow$ requires that $F_i = 0$, $D^a = 0$ cannot be simultaneously satisfied for any values of the fields

F-term breaking:

need linear term in superpotential $\mathcal{V}(\phi_i) = a_i\phi_i + \frac{1}{2}m_{ij}\phi_i\phi_j + \frac{1}{3}\lambda_{ijk}\phi_i\phi_j\phi_k$

$\Rightarrow$ requires a chiral superfield that is a singlet under all gauge groups

$\Rightarrow$ not possible within the MSSM

D-term breaking:

Does not work in the MSSM (leads to charge and color-breaking minima)

Problems can be overcome if SUSY breaking happens in a 'hidden sector', i.e. by fields which have only very small couplings to ordinary matter

SUSY breaking in the hidden sector:
– tree-level (like F- and D-term breaking)
– dynamical breaking (similar to chiral symmetry breaking in QCD), . . .

SUSY-breaking terms in the MSSM arise radiatively via interaction that communicates SUSY breaking rather than through tree-level couplings to SUSY breaking v.e.v.s

$\Rightarrow$ phenomenology depends mainly on mechanism for communicating SUSY breaking rather than on SUSY-breaking mechanism itself

If mediating interactions are $\approx$ flavor-diagonal
$\Rightarrow$ universal soft-breaking terms

"Hidden sector":	$\longrightarrow$	Visible sector:
SUSY breaking		MSSM

"Gravity-mediated": mSUGRA
"Gauge-mediated": GMSB
"Anomaly-mediated": AMSB
"Gaugino-mediated"
...

SUGRA: mediating interactions are gravitational

GMSB: mediating interactions are ordinary electroweak and QCD gauge interactions

AMSB, Gaugino-mediation: SUSY breaking happens on a different brane in a higher-dimensional theory

(more details later)