Precision Observables in the MSSM: New Results and Applications (What is the Scale of Supersymmetry?)

Sven Heinemeyer, IFCA (CSIC–UC)

München, 12/2006

1. Introduction and motivation
2. New results for $\Delta\rho$
3. New results for M_W
4. SUSY mass scale determination from a global fit
5. Conclusions

Christmas is approaching . . .

. . . and YOU are still HERE!

⇒ some Christmas related cartoons :-)

⇒ the "Who's that Santa?" quiz

Several times a picture with a Santa Claus will show up.
The one who calls/shouts the name first gets a little present.
(Hint: it is always a physics professor)
Alles klar? :-)

"I think next year I'll go straight to my parents and cut out the middleman."

1. Introduction and motivation

Q: Which Lagrangian describes the world?

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A: Ummm . . . Let's start differently!

1. Introduction and motivation

Supersymmetry (SUSY) : Symmetry between

$$\text{Bosons} \leftrightarrow \text{Fermions}$$

$$Q\ |\text{Fermion}\rangle \rightarrow |\text{Boson}\rangle$$
$$Q\ |\text{Boson}\rangle \rightarrow |\text{Fermion}\rangle$$

Simplified examples:

$$Q\ |\text{top},\ t\rangle \rightarrow |\text{scalar top},\ \tilde{t}\rangle$$
$$Q\ |\text{gluon},\ g\rangle \rightarrow |\text{gluino},\ \tilde{g}\rangle$$

$\Rightarrow$ each SM multiplet is enlarged to its double size

Unbroken SUSY: All particles in a multiplet have the same mass

Reality: $m_e \neq m_{\tilde{e}} \Rightarrow$ SUSY is broken . . .

. . . via soft SUSY-breaking terms in the Lagrangian (added by hand)

SUSY particles are made heavy: $\boxed{M_{\text{SUSY}} = \mathcal{O}(1\ \text{TeV})}$

Supersymmetry: Motivation

The SM is in a pretty good shape.

Why MSSM? (Is it worth to double the particle spectrum?)

1.) Stability of the Higgs mass against higher-order corr.

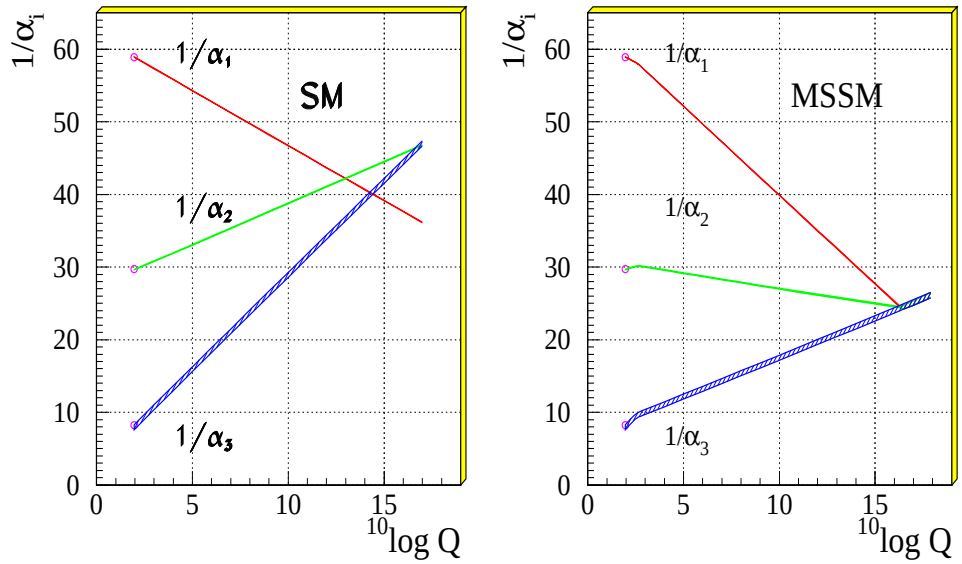
2.) Unification of gauge couplings: Not possible in the SM, but in the MSSM (although it was not designed for it.)

3.) Spontaneous symmetry breaking via Higgs mechanism is automatic in SUSY GUTs

4.) SUSY provides CDM candidate

5.) . . .

Unification of the Coupling Constants in the SM and the minimal MSSM



[*Amaldi, de Boer, Fürstenau '92*]

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin } 1 \text{ / Spin } 0$$

$$\tilde{g} \quad \tilde{\chi}^{\pm}_{1,2} \quad \tilde{\chi}^{0}_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

$\rightarrow$ CPV will mostly be neglected throughout this talk!

$\tilde{t}/\tilde{b}$ sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ($X_t = A_t - \mu^*/\tan\beta$, $X_b = A_b - \mu^* \tan\beta$):

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

soft SUSY-breaking parameters A_t, A_b also appear in ϕ-$\tilde{t}/\tilde{b}$ couplings

$$SU(2) \text{ relation} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$$

$\Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

"What do you think? Should we let him stick in the wall and have a quiet evening?"

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

Who's that Santa? (I)

Q: Which Lagrangian describes the world?

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Q': What describes the world better: SM or MSSM ?

A: Two possible ways:

- Search for new SUSY particles

new SUSY particles found
$\Updownarrow$
SM ruled out

Problem:
SUSY particles are too heavy for todays colliders, only lower limits of $\mathcal{O}(100\text{ GeV})$.

$\rightarrow$ waiting for Tevatron (2007/08. . . ?)
$\rightarrow$ waiting for LHC (2008/09. . . ?)

- Search for indirect effects of SUSY via Precision Observables

“Indeed, you look younger without the beard - and like shit.”

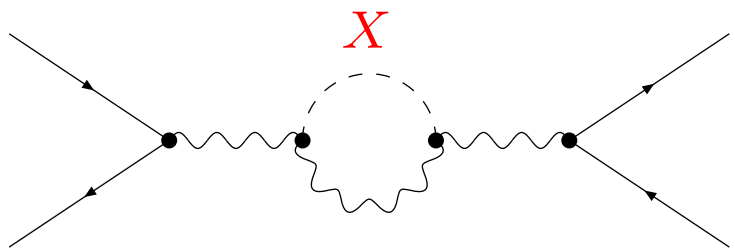
Precision Observables (POs):

Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$M_W, \sin^2\theta_{\text{eff}}, a_\mu$	$\leftrightarrow$	SM, MSSM , . . .

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections



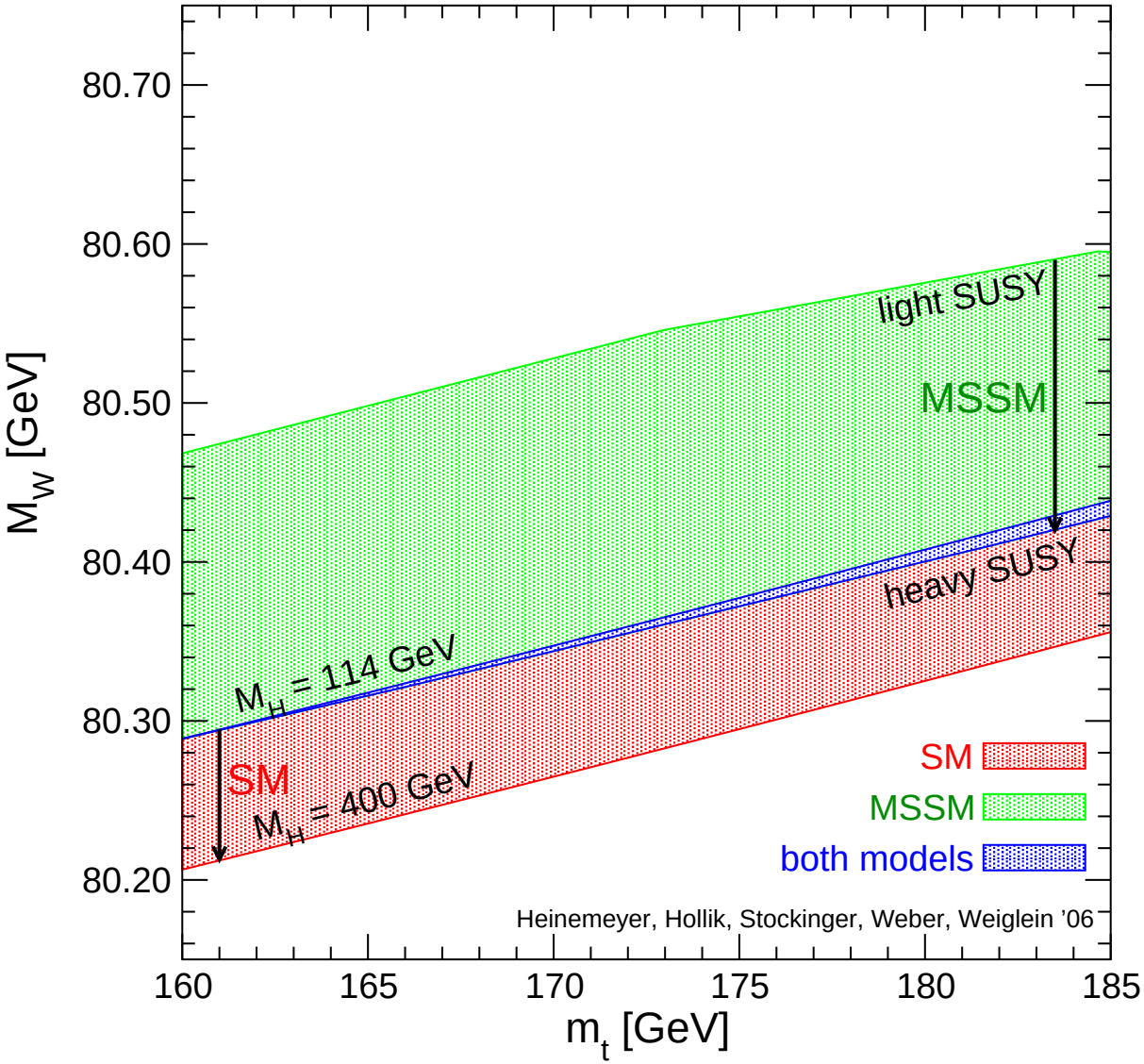
$\Downarrow$

Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

Example: Prediction for M_W in the SM and the MSSM :

[*S.H., W. Hollik, D. Stockinger, A.M. Weber, G. Weiglein '06*]



MSSM band:

scan over

SUSY masses

overlap:

SM is MSSM-like

MSSM is SM-like

SM band:

variation of M_H^{SM}

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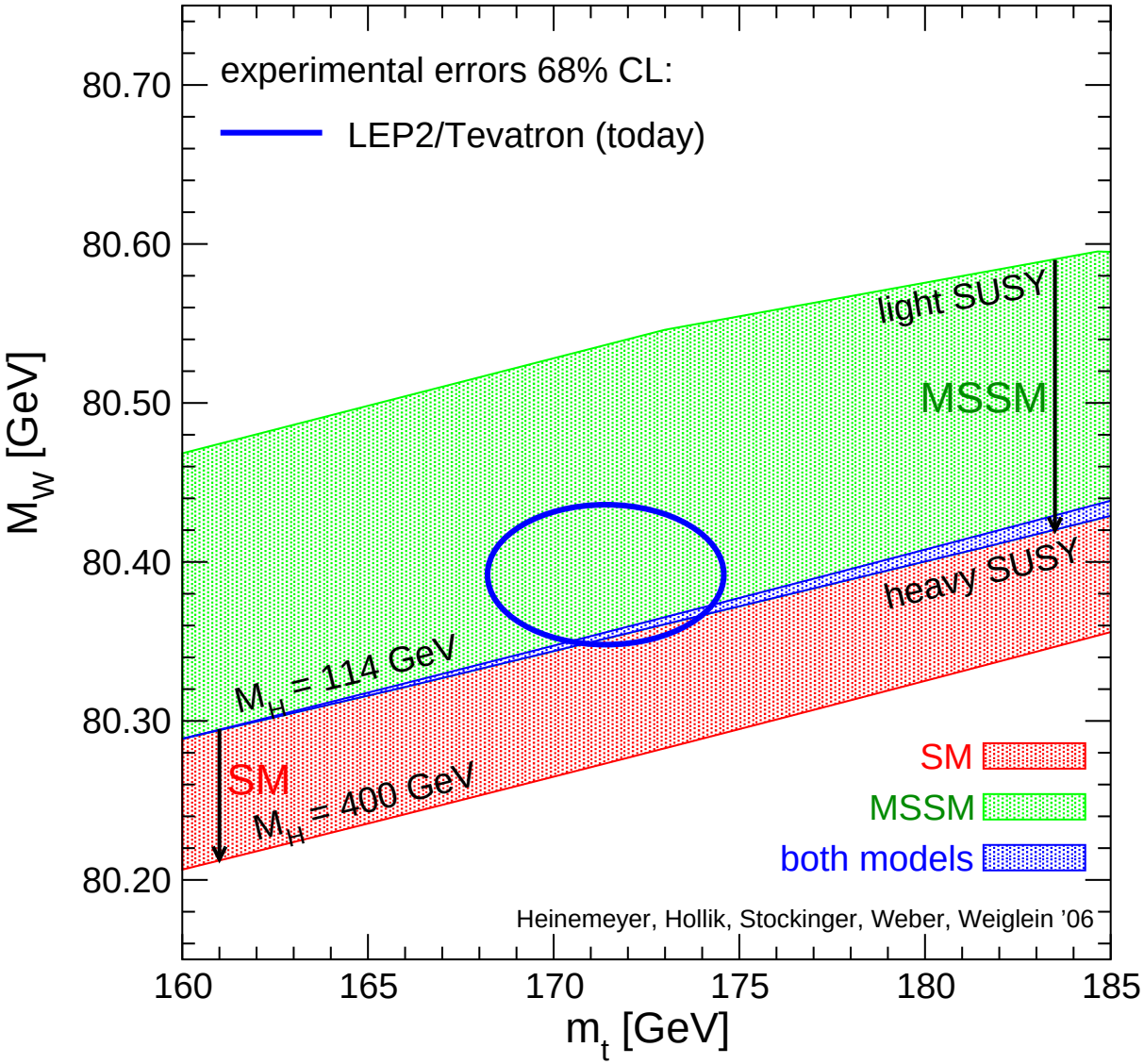

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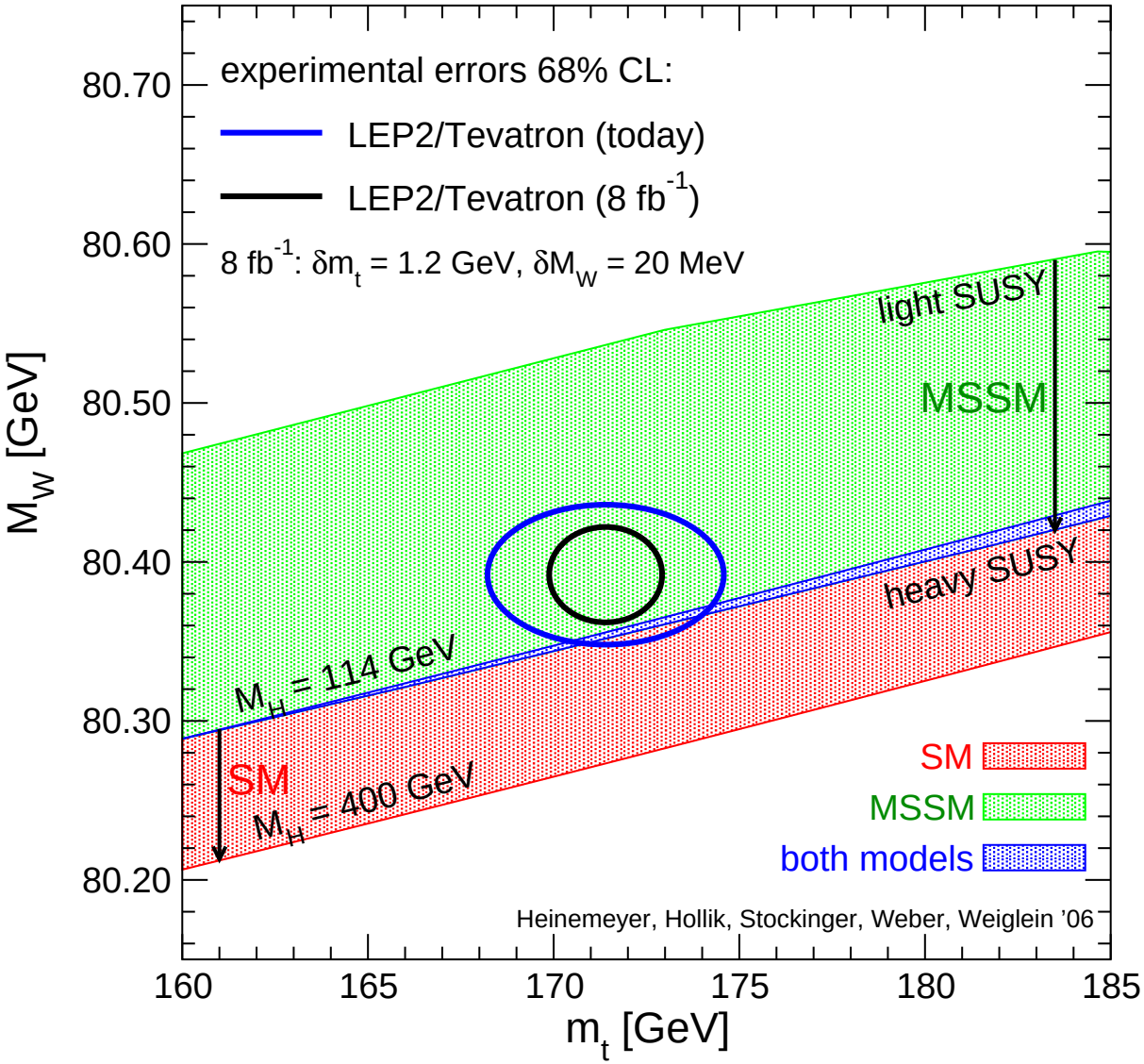

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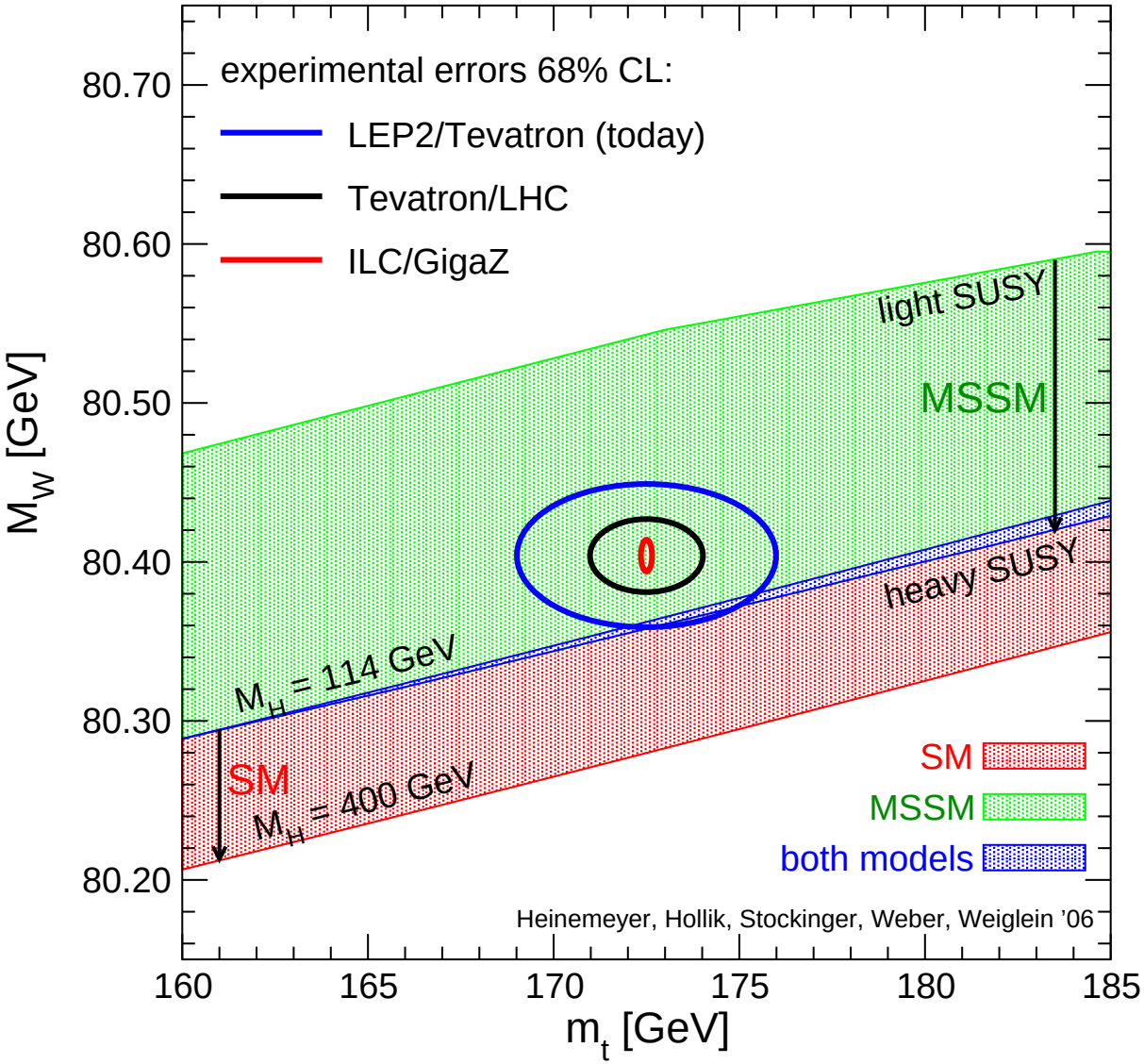

MSSM band:
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overlap:
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MSSM is SM-like

SM band:
variation of M_H^{SM}

“Again no chimney!” – “I am coming!”

2. New results for $\Delta\rho$

Precision observables: M_W, $\sin^2\theta_{\text{eff}}$, m_h, $(g-2)_\mu$, b physics, ...

A) Theoretical prediction for M_W in terms of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}(1+\Delta r)$$

$\Updownarrow$

loop corrections

B) Effective mixing angle:

$$\sin^2\theta_{\text{eff}}=\frac{1}{4\,|Q_f|}\left(1-\text{Re}\frac{g_V^f}{g_A^f}\right)$$

Higher order contributions:

$$g_V^f\to g_V^f+\Delta g_V^f,\quad g_A^f\to g_A^f+\Delta g_A^f$$

Corrections to M_W, $\sin^2\theta_{\text{eff}}$

$\rightarrow$ can be approximated with the **ρ-parameter:**

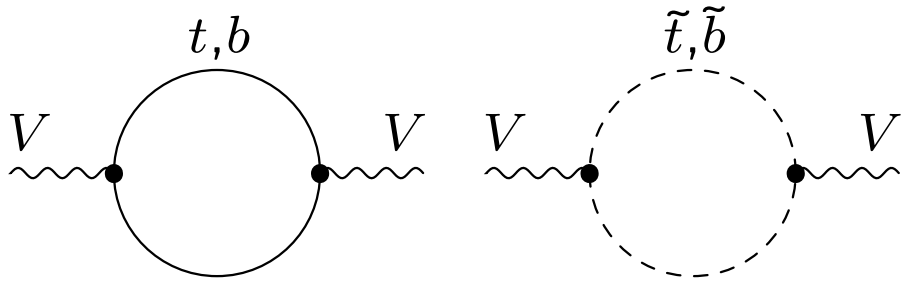
ρ measures the relative strength between
neutral current interaction and charged current interaction

$$\rho = \frac{1}{1-\Delta\rho} \qquad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

(leading, process independent terms)

$\Delta\rho$ gives the main contribution to EW observables:

$$\Delta M_W \approx \frac{M_W}{2}\frac{c_{\text{W}}^2}{c_{\text{W}}^2 - s_{\text{W}}^2}\Delta\rho, \qquad \Delta\sin^2\theta_W^{\text{eff}} \approx -\frac{c_{\text{W}}^2 s_{\text{W}}^2}{c_{\text{W}}^2 - s_{\text{W}}^2}\Delta\rho$$



Who's that Santa? (II)

Status of $\Delta\rho$ calculations (I)

- SM, $\mathcal{O}(\alpha)$: dominant one-loop
  [*M. Veltman '77*]
- SM, $\mathcal{O}(\alpha\alpha_s)$: leading two-loop corrections
  [*A. Djouadi, C. Verzegnassi '87*]
  [*A. Djouadi '88*]
- SM, $\mathcal{O}\left(\alpha_t^2\right)$: limit of $M_H \to 0$
  [*J. Van der Bij, F. Hoogeveen '87*]
- SM, $\mathcal{O}\left(\alpha_t^2\right)$: corrections with $M_H \neq 0$
  [*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '93*]
  [*J. Fleischer, F. Jegerlehner, O. Tarasov '93*]
- SM, $\mathcal{O}\left(\alpha\alpha_s^2\right)$: leading three-loop contributions
  [*K. Chetyrkin, J Kühn, M. Steinhauser '95*]
  [*L. Avdeev et al. '95*]

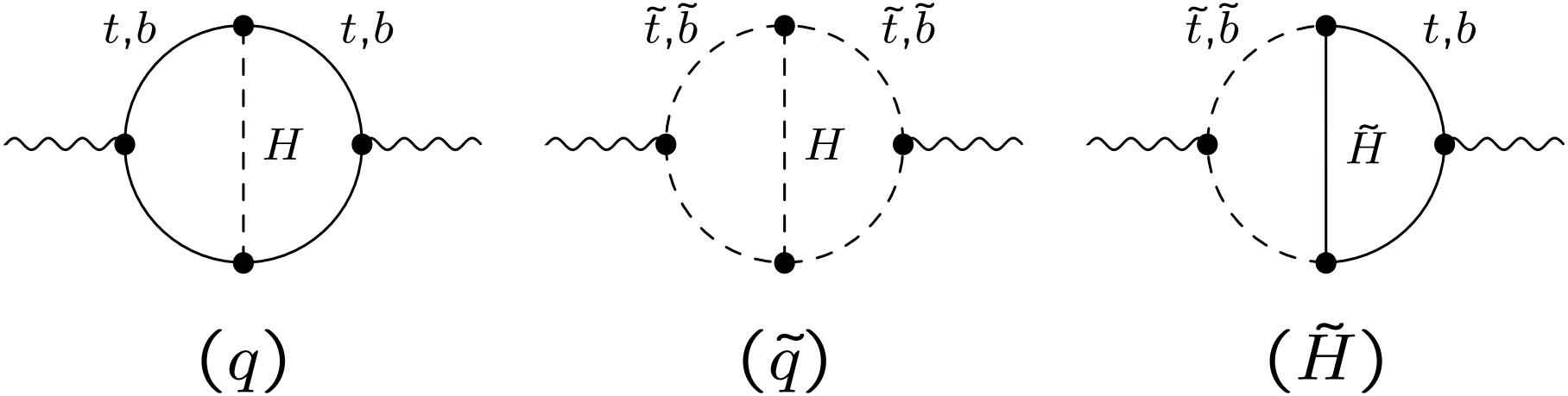
Status of $\Delta\rho$ calculations (II)

- SM, $\mathcal{O}\left(\alpha^2\alpha_s\right)$, $\mathcal{O}\left(\alpha^3\right)$: limit of $M_H \to 0$
  [*J. Van der Bij, K. Chetyrkin, M. Faisst, G. Jikia, T. Seidensticker '01*]
- SM, $\mathcal{O}\left(\alpha^2\alpha_s\right)$, $\mathcal{O}\left(\alpha^3\right)$: limits with $M_H \neq 0$
  [*M. Faisst, J. Kühn, T. Seidensticker, O. Veretin '03*]
- SM, $\mathcal{O}\left(\alpha\alpha_s^3\right)$: various four-loop contributions
  [*Y. Schröder, M. Steinhauser '05*]
  [*K. Chetyrkin, M. Faisst, J. Kühn, P. Maierhoefer, C. Sturm '06*]
  [*R. Boughezal, M. Czakon '06*]
- MSSM, $\mathcal{O}\left(\alpha\alpha_s\right)$: leading MSSM two-loop corrections
  [*A. Djouadi, P. Gambino, S.H., W. Hollik, C. Jünger, G. Weiglein '97*]
- MSSM, $\mathcal{O}\left(\alpha_t^2, \alpha_t\alpha_b, \alpha_b^2\right)$: limit of $M_{\text{SUSY}} \to \infty$
  [*S.H., G. Weiglein '01, '03*]
- New: MSSM, $\mathcal{O}\left(\alpha_t^2, \alpha_t\alpha_b, \alpha_b^2\right)$: limit of $M_{\text{SUSY}} \neq \infty$
  [*J. Haestier, S.H., D. Stöckinger, G. Weiglein '05*]

New results: $\mathcal{O}\left(\alpha_{t,b}^2\right)$ corrections to $\Delta\rho$

[*J. Haestier, S.H., D. Stöckinger, G. Weiglein '05*]

Three classes:



(q) Fermions and Higgs bosons (has SM counter part)

$(\tilde{q})$ Scalar fermions and Higgs bosons

$(\tilde{H})$ Higgsinos and fermions/sfermions

"Because of time constraints this year I send only a short greeting. Yours, S."

Calculation:

- Diagram computation in principle straightforward, but very large number of diagrams ($\mathcal{O}\left(10^3\right)$), many different scales

  $\Rightarrow$ algebra very involved

  used `TwoCalc` [*G. Weiglein '92*] , [*G. Weiglein, R. Scharf, M. Böhm '94*]

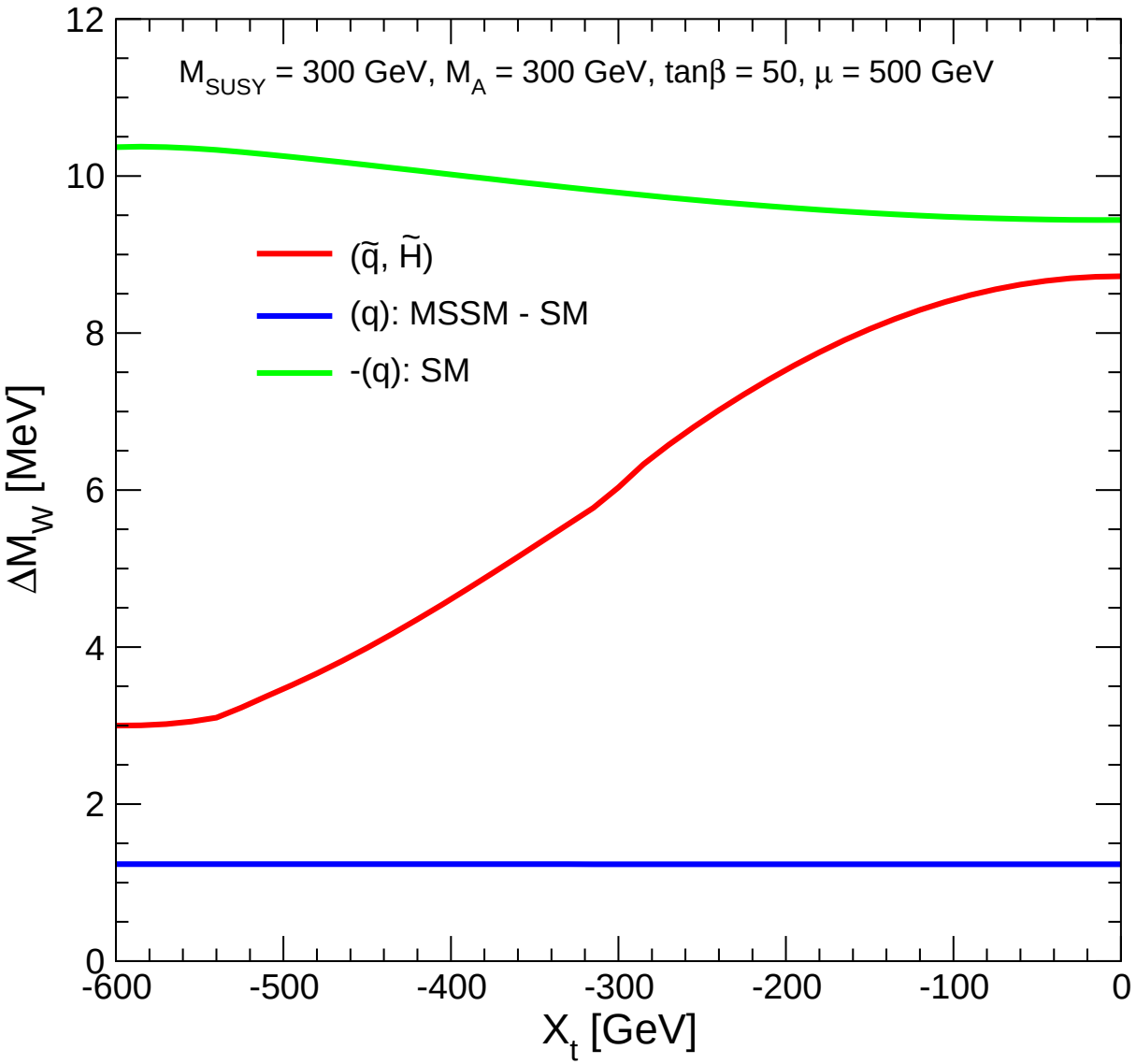
- Important to take Higgs-mass dependence into account
  [*J. van der Bij, F. Hoogeveen '87*] , [*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '92*] , [*J. Fleischer, O. Tarasov, F. Jegerlehner '93*]

  more difficult in the MSSM: M_h is not a free parameter,
  need to preserve SUSY relations

- Renormalization: SUSY + gauge invariance imply relation between the parameters in the scalar quark sector

  $\Rightarrow$ needs to be preserved by renormalization procedure

Numerical results for M_W:

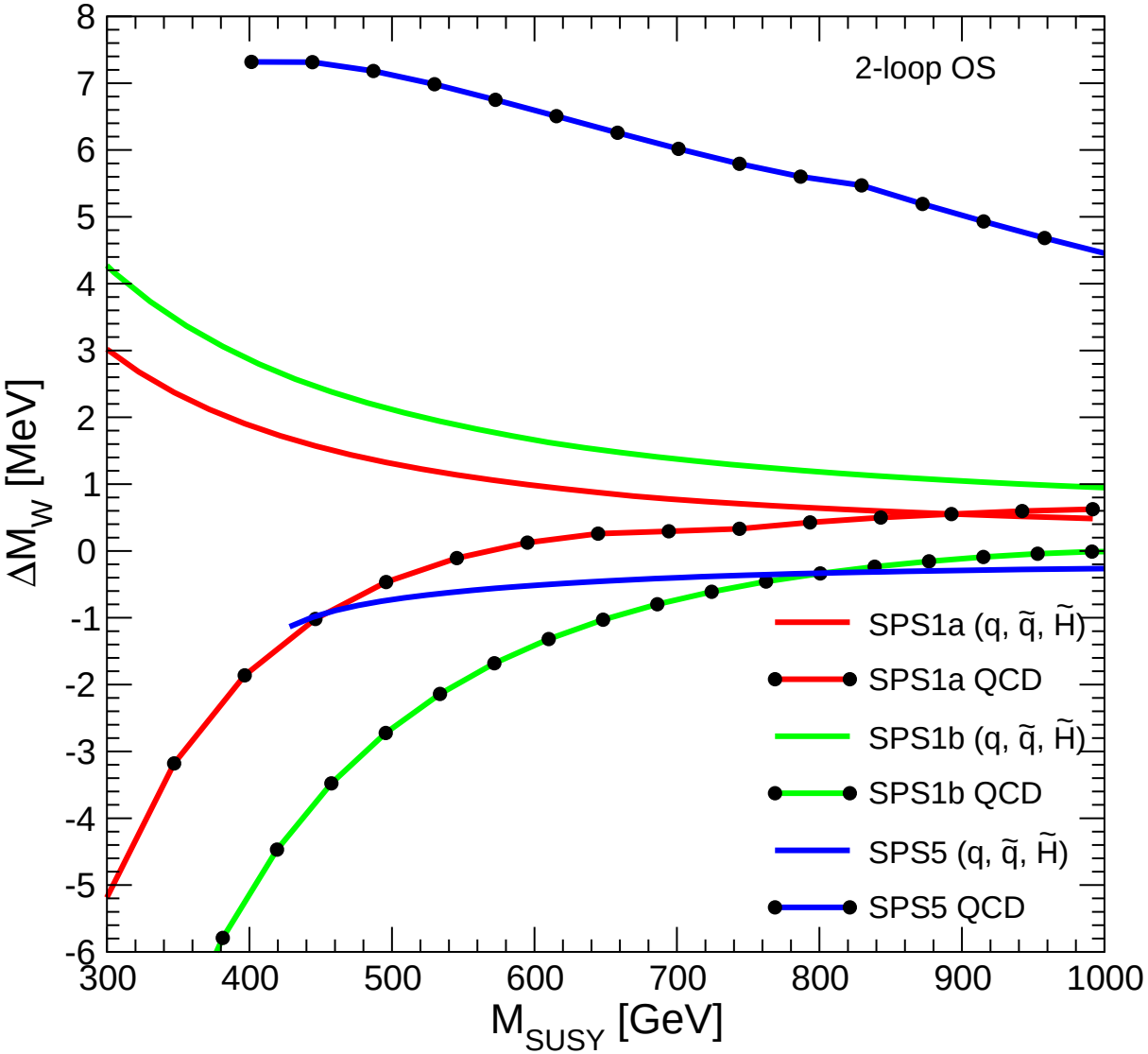


(q) : SM large

(q) : MSSM - SM small,
$\mathcal{O}$ (1 MeV)

$(\tilde{q}, \tilde{H})$: $\mathcal{O}$ (6 MeV)
non-negligible,
larger than current
SM uncertainty

Numerical results for M_W in benchmark scenarios (SPS):



2-loop QCD up to 7 MeV

$(\tilde{q}, \tilde{H})$ of similar size
non-negligible

Who's that Santa? (III)

3. New results for M_W

Theoretical prediction for M_W in terms of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}(1 + \Delta r)$$

$\Updownarrow$

loop corrections

Evaluate Δr from μ decay $\Rightarrow M_W$

One-loop result for M_W in the SM:

[*A. Sirlin '80*] , [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\text{rem}}(M_H)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

Combination of SM and MSSM result

→ use best available SM result

→ add all available MSSM corrections

→ subtract double counting

⇒ decoupling limit ok ($M_{\text{SUSY}} \to \infty$)

. . . but some corrections included only via their SM part

⇒ best solution

Status of SM calculation

- Most recently full electroweak two-loop result available
  [*Awramik, Czakon, Freitas, Hollik, Onishenko, Veretin, Walter, Weiglein '03, '04*]
- Most recent electroweak three-loop corrections via $\Delta\rho$
  [*Faisst, Kühn, Seidensticker, Veretin '04*]
- Compact parametric formula used
  [*Awramik, Czakon, Freitas, Weiglein '04*]

Status of MSSM calculations of M_W

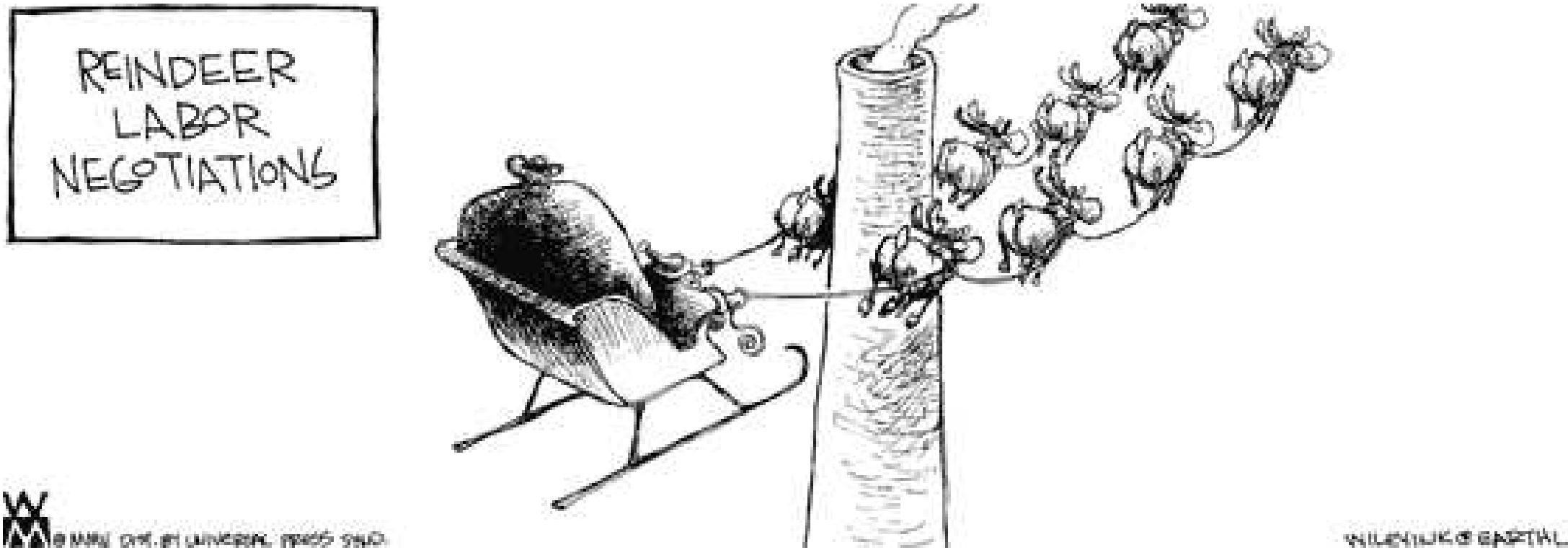
- MSSM, Δr: full one-loop corrections
  [*P. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski, J. Rosiek '94*]
  [*D. Garcia, J. Solà '94*]
- MSSM, $\Delta\rho$: leading $\mathcal{O}(\alpha\alpha_s)$ corrections
  [*A. Djouadi, P. Gambino, S.H., W. Hollik, C. Jünger, G. Weiglein '97*]
- MSSM, Δr: leading gluonic $\mathcal{O}(\alpha\alpha_s)$ corr.
  [*S.H. '98*] [*S.H., W. Hollik, G. Weiglein '04*]
- MSSM, $\Delta\rho$: leading $\mathcal{O}\left(\alpha_t^2, \alpha_t\alpha_b, \alpha_b^2\right)$ corrections
  [*S.H., G. Weiglein '01, '03*]
  [*J. Haestier, S.H., D. Stöckinger, G. Weiglein '05*]

Most recent:

- MSSM, Δr: one-loop with complex parameters
  [*S.H., W. Hollik, D. Stöckinger, A.M. Weber, G. Weiglein '06*]

Most dominant:

- $\Delta\rho$: one- or two-loop, contribution from $\tilde{t}/\tilde{b}$ sector


REINDEER
LABOR
NEGOTIATIONS

More details about our calculation:

- Complex phases in the squark sector enter only via shift in squark masses (explicit dependence drops out)

$$
\begin{aligned}
|X_t|^2 &= |A_t|^2 + |\mu \cot\beta|^2 - 2|A_t| \cdot |\mu| \cot\beta \cos(\phi_{A_t} + \phi_\mu) \\
|X_b|^2 &= |A_b|^2 + |\mu \tan\beta|^2 - 2|A_b| \cdot |\mu| \tan\beta \cos(\phi_{A_b} + \phi_\mu)
\end{aligned}
$$

  Only some phase combinations are physical, other phases can be rotated away.
  Examples for physical combinations:

$$
\begin{aligned}
&\phi_{A_t} + \phi_\mu \\
&\phi_{A_b} + \phi_\mu
\end{aligned}
$$

- Higgs mass dependence of the two-loop contributions is know to be very strong
  $\Rightarrow$ we use *FeynHiggs* (`www.feynhiggs.de`)

- All one-loop calculations have been performed with *FeynArts* and *FormCalc*
  [*T. Hahn et al '00 - '05*]

Treatment of the phase dependence beyond one-loop order

Phase dependence at the two-loop level approximated by a simple interpolation based on:

full phase dependence at the one-loop level, $M_W^{1\mathsf{L}}(\phi)$,

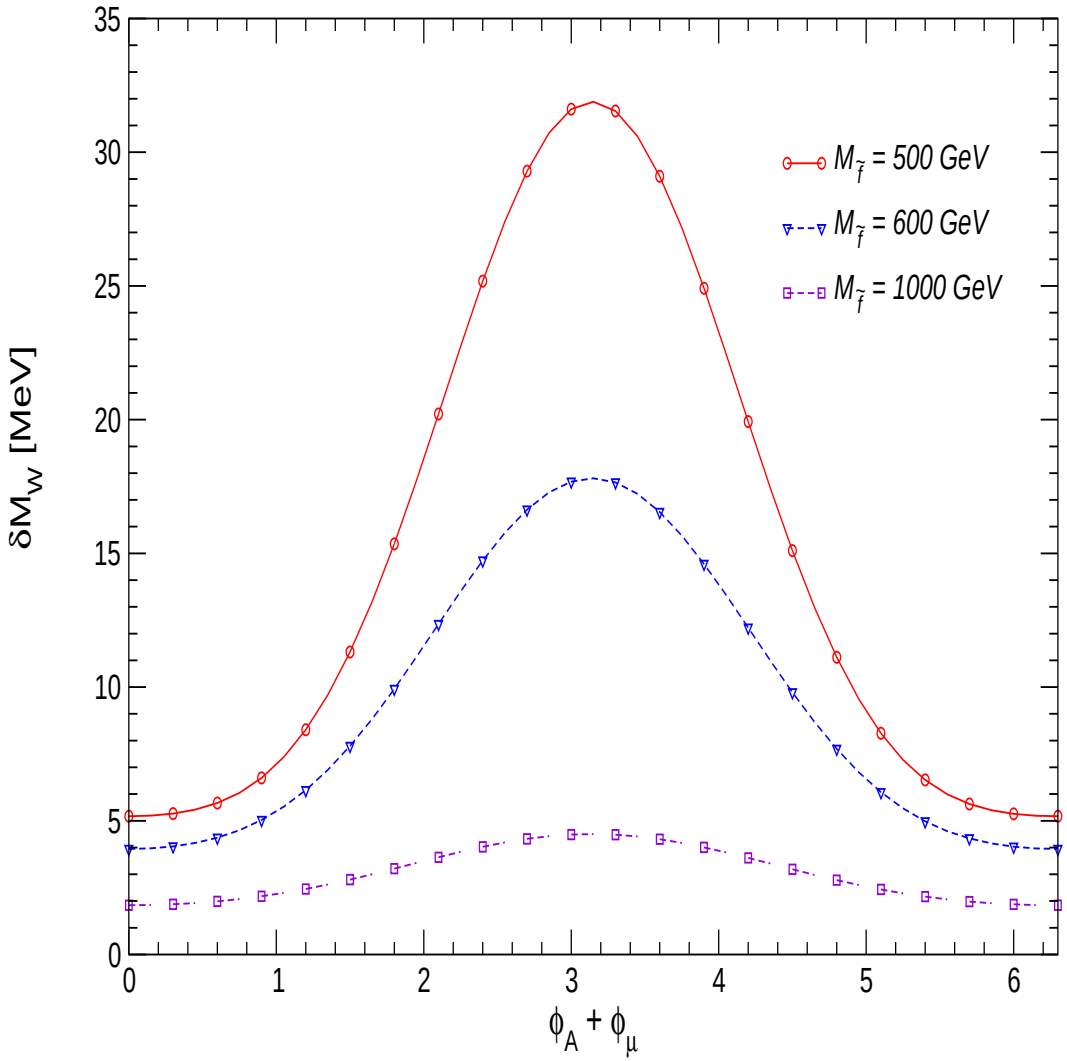
two-loop results for real parameters, $M_W^{\mathsf{full}}(0)$, $M_W^{\mathsf{full}}(\pi)$

$\Rightarrow$ Two-loop result for complex phase ϕ:

$$M_W^{\mathsf{full}}(\phi) = M_W^{1\mathsf{L}}(\phi) + \left[M_W^{\mathsf{full}}(0) - M_W^{1\mathsf{L}}(0)\right] \times \frac{1+\cos\phi}{2} + \left[M_W^{\mathsf{full}}(\pi) - M_W^{1\mathsf{L}}(\pi)\right] \times \frac{1-\cos\phi}{2}$$

Dependence of M_W on phases in stop/sbottom sector:

[*S.H., W. Hollik, D. Stöckinger, A.M. Weber, G. Weiglein '06*]

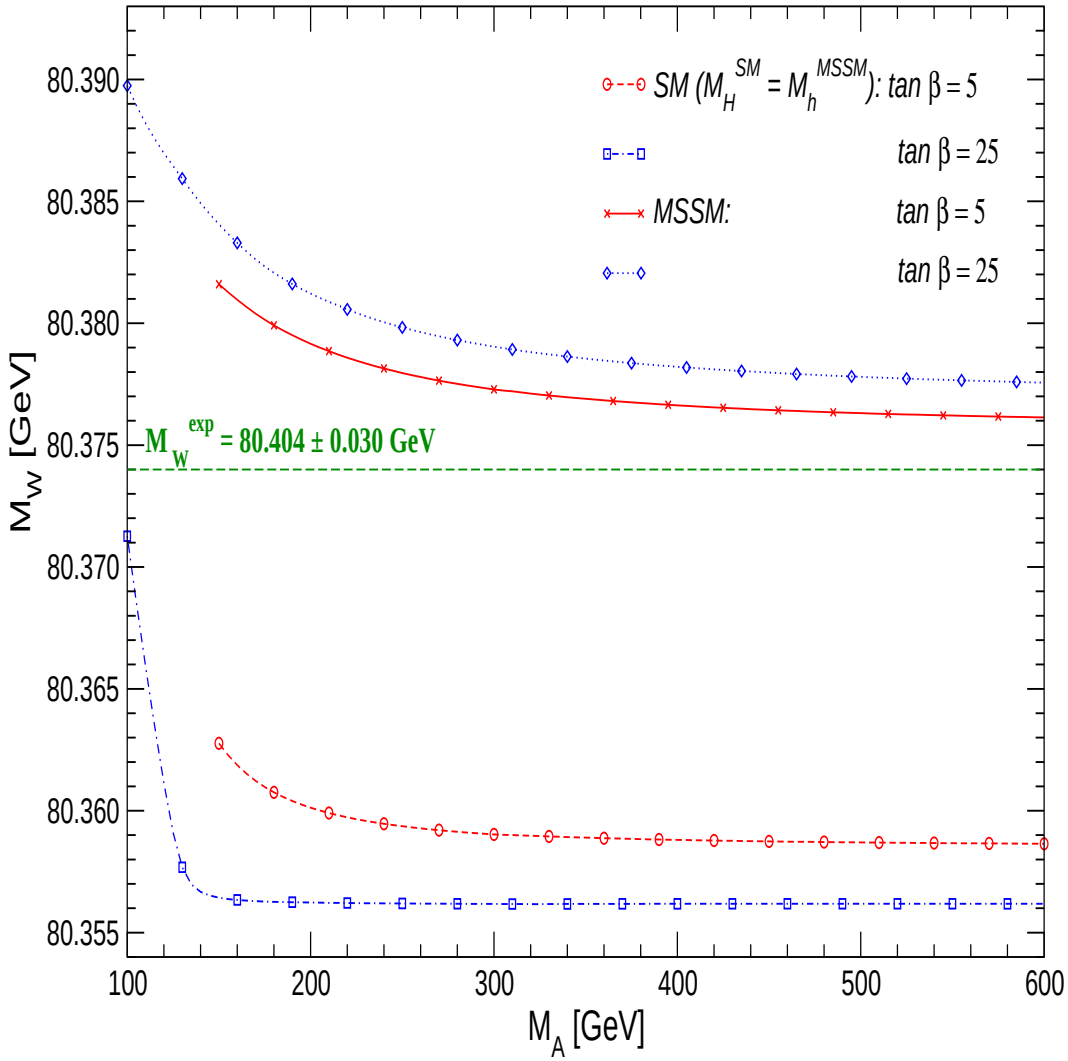


variation of $\phi_A + \phi_\mu$
$(\phi_A \equiv \phi_{A_t} = \phi_{A_b})$
other parameters:
$\tan\beta = 5$
$M_{\text{SUSY}} = 600$ GeV
$A_{t,b} = 2M_{\text{SUSY}}$
$|\mu| = 900$ GeV,

$\Rightarrow$ variation of $\phi_A + \phi_\mu$ yields shift of more than 20 MeV

Prediction of M_W, comparison with SM:

[S.H., W. Hollik, D. Stöckinger, A.M. Weber, G. Weiglein '06]



other parameters:

$M_{\text{SUSY}} = 600$ GeV

$A_{t,b} = 1200$ GeV

$\mu = M_2 = m_{\tilde{g}} = 300$ GeV,

$\phi_x = 0$

$M_W^{\text{SUSY}} - M_W^{\text{SM}} \gtrsim 20$ MeV

(typical difference)

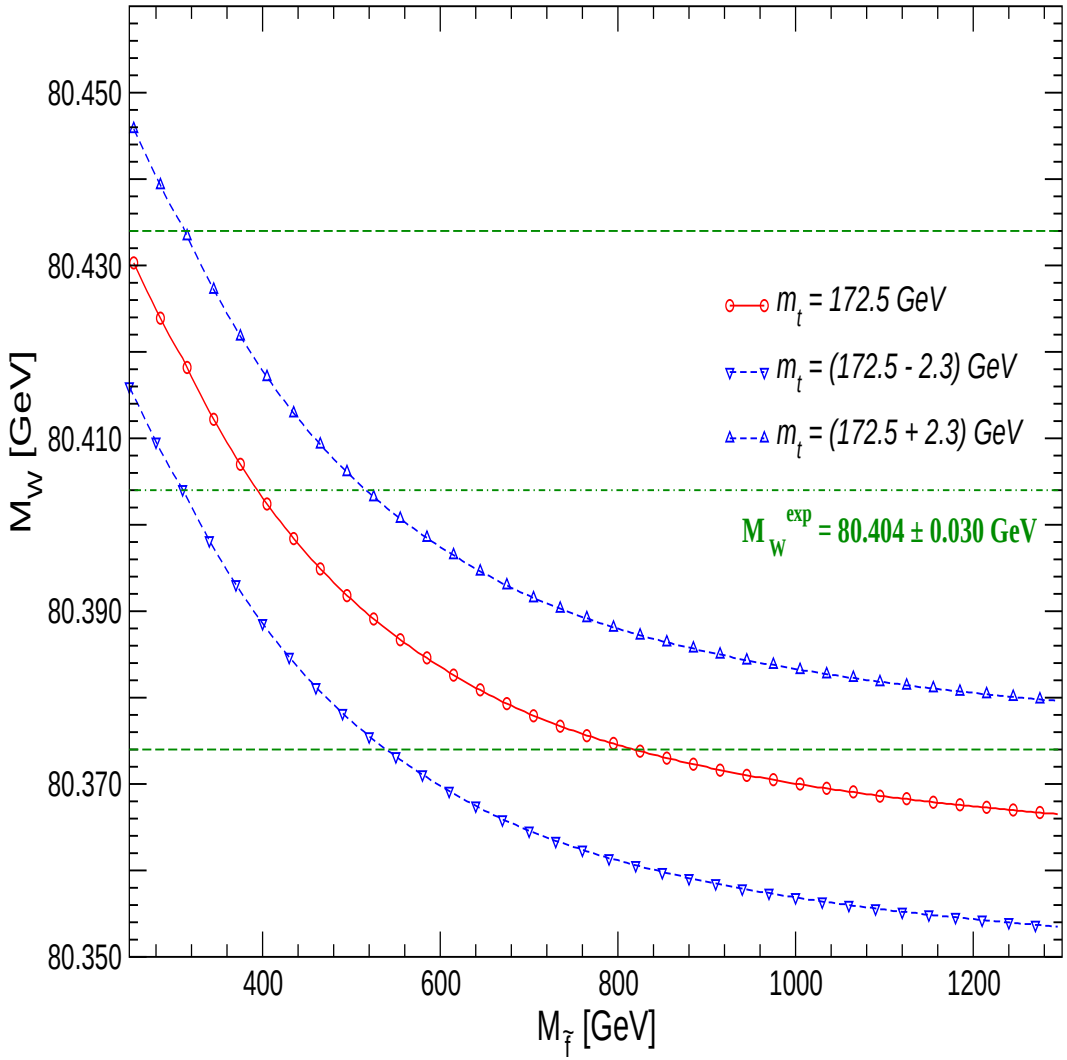
Comparison with M_W^{exp}:

MSSM agrees at 1σ

SM agrees at 2σ

Prediction of M_W: variation of m_t

[S.H., W. Hollik, D. Stöckinger, A.M. Weber, G. Weiglein '06]



other parameters:

$A_{t,b} = 2\, M_{\text{SUSY}}$

$\mu = M_2 = m_{\tilde{g}} = 300$ GeV,

$\phi_x = 0$

$\tan\beta = 10$

$\delta m_t = \pm 2.3$ GeV

$\Rightarrow \delta M_W \approx \pm 14$ MeV

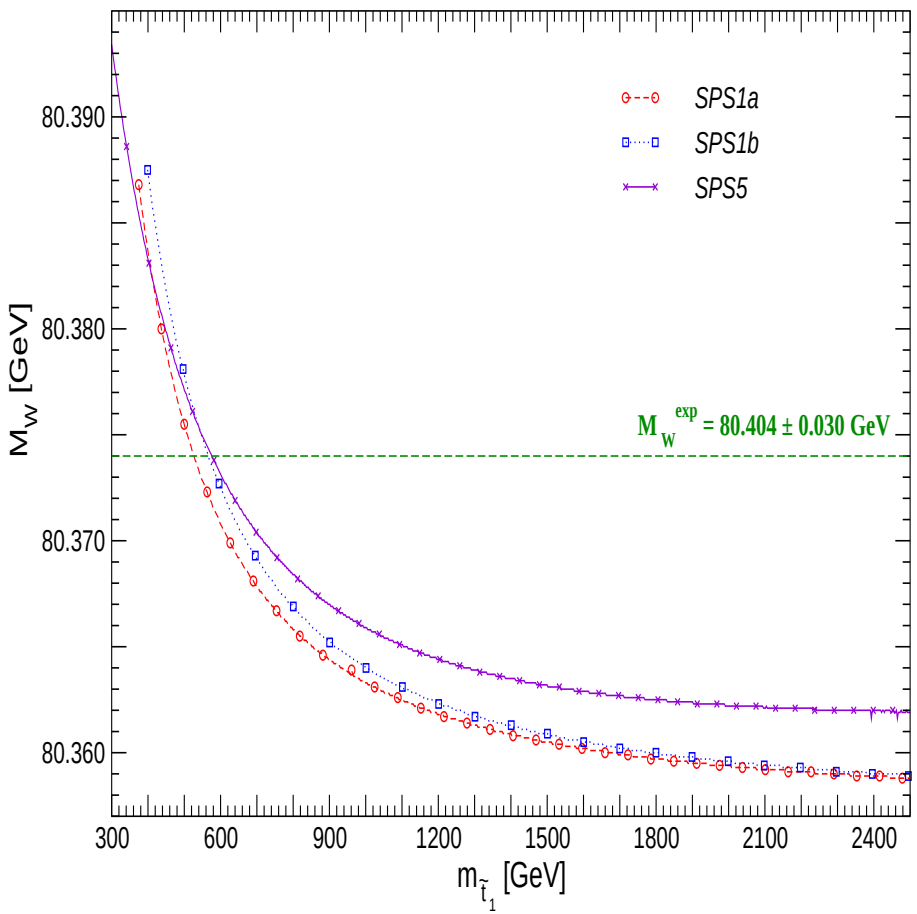
lower m_t disfavored

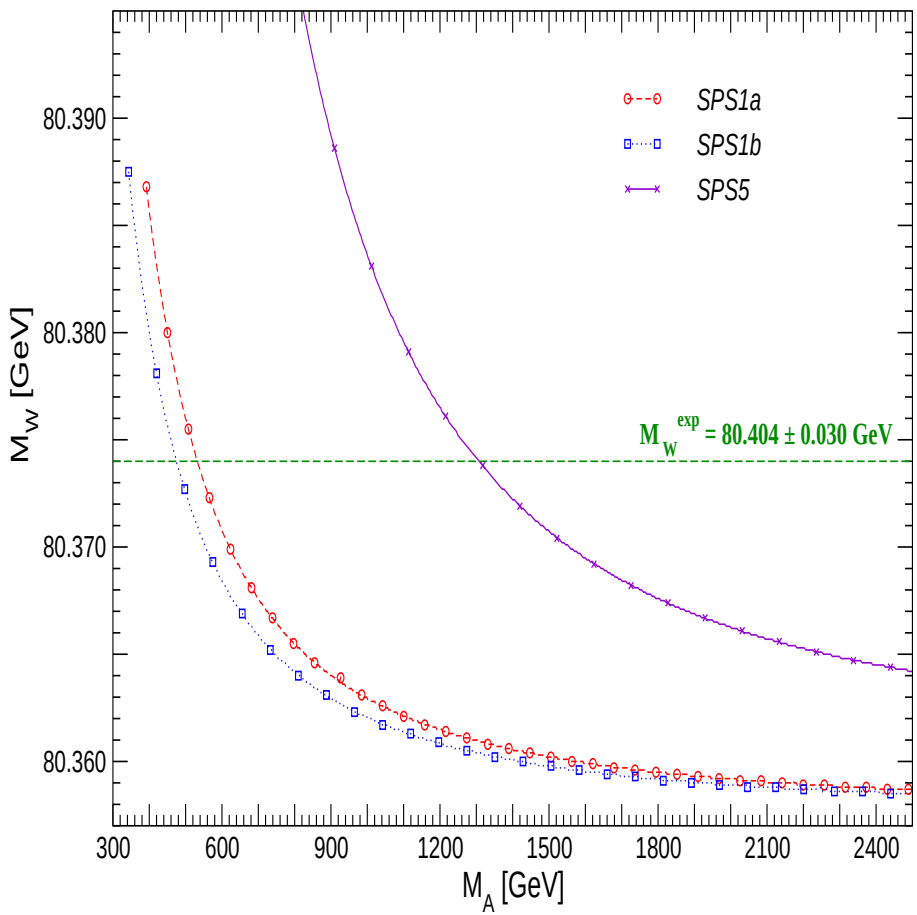
$\Rightarrow$ largest parametric uncertainty

"I have the feeling we will arrive late this year . . ."

Prediction of M_W: typical scenarios: SPS 1a, 1b, 5

[*S.H., W. Hollik, D. Stöckinger, A.M. Weber, G. Weiglein '06*]

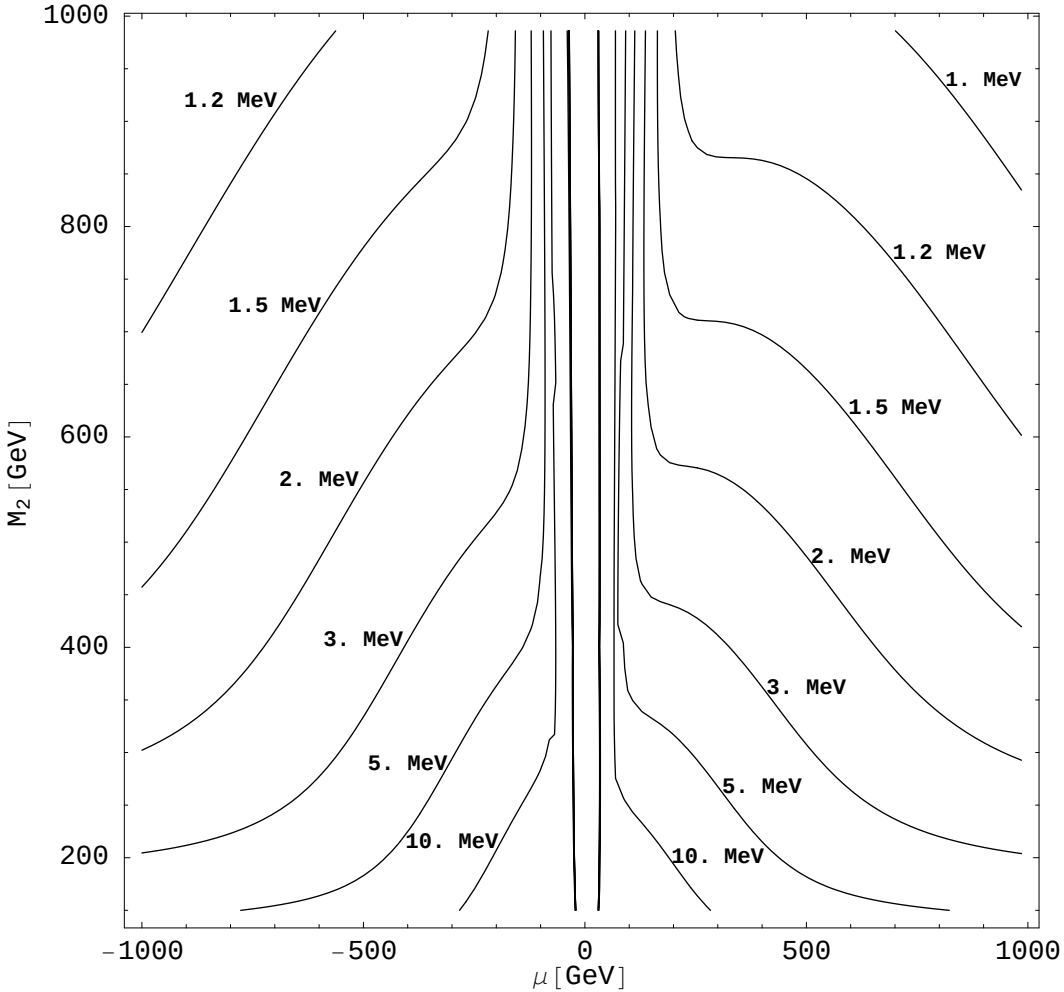




$\Rightarrow$ only "lower" masses show agreement at 1σ

Prediction of M_W: Split SUSY

[N. Arkani-Hamed, S. Dimopoulos '04] [G. Giudice, A. Romanino '04]



[S.H., W. Hollik, D. Stöckinger, A.M. Weber, G. Weiglein '06]

Difference to M_W^{SM} with $M_H^{\mathrm{SM}} = M_h$

⇒ no deviation for Tevatron, LHC, ILC precision

⇒ GigaZ can confirm only very light masses

(as expected . . .)

Split SUSY disagrees with experiment at the 2σ level

Who's that Santa? (IV) (Santa on vacation . . .)

4. SUSY mass scale determination from a global fit

What do we know about the SUSY mass scale?

1. Coupling constant unification $\Rightarrow M_{\text{SUSY}} \approx 1$ TeV
2. LSP should be cold dark matter $\Rightarrow M_{\text{SUSY}} \lesssim 1$ TeV
3. Indirect hints from existing data?
   - Focus on CMSSM, NUHM, VCMSSM and GDM
     small number of free parameters
   - hard constraint: LSP gives right amount of cold dark matter
     CMSSM: only thin strips allowed in the $m_{1/2}$–m_0 plane
     VCMSSM: even $\tan\beta$ determined
     NUHM, GDM: also strong constraints
   - Use existing data of M_W, $\sin^2\theta_{\text{eff}}$, $\text{BR}(b \to s\gamma)$, $(g-2)_\mu$, M_h
     $\Rightarrow \chi^2$ fit with these observables
     $\Rightarrow$ best fit values for masses, couplings, ...

The models (I):

CMSSM (or mSUGRA):

$\Rightarrow$ Scenario characterized by

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \mathrm{sign}\,\mu$$

m_0 : universal scalar mass parameter

$m_{1/2}$: universal gaugino mass parameter

A_0 : universal trilinear coupling

} at the GUT scale

$\tan\beta$: ratio of Higgs vacuum expectation values

$\mathrm{sign}(\mu)$: sign of supersymmetric Higgs parameter

$\Rightarrow$ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is the lightest neutralino

The models (I):

CMSSM (or mSUGRA

⇒ Scenario charac

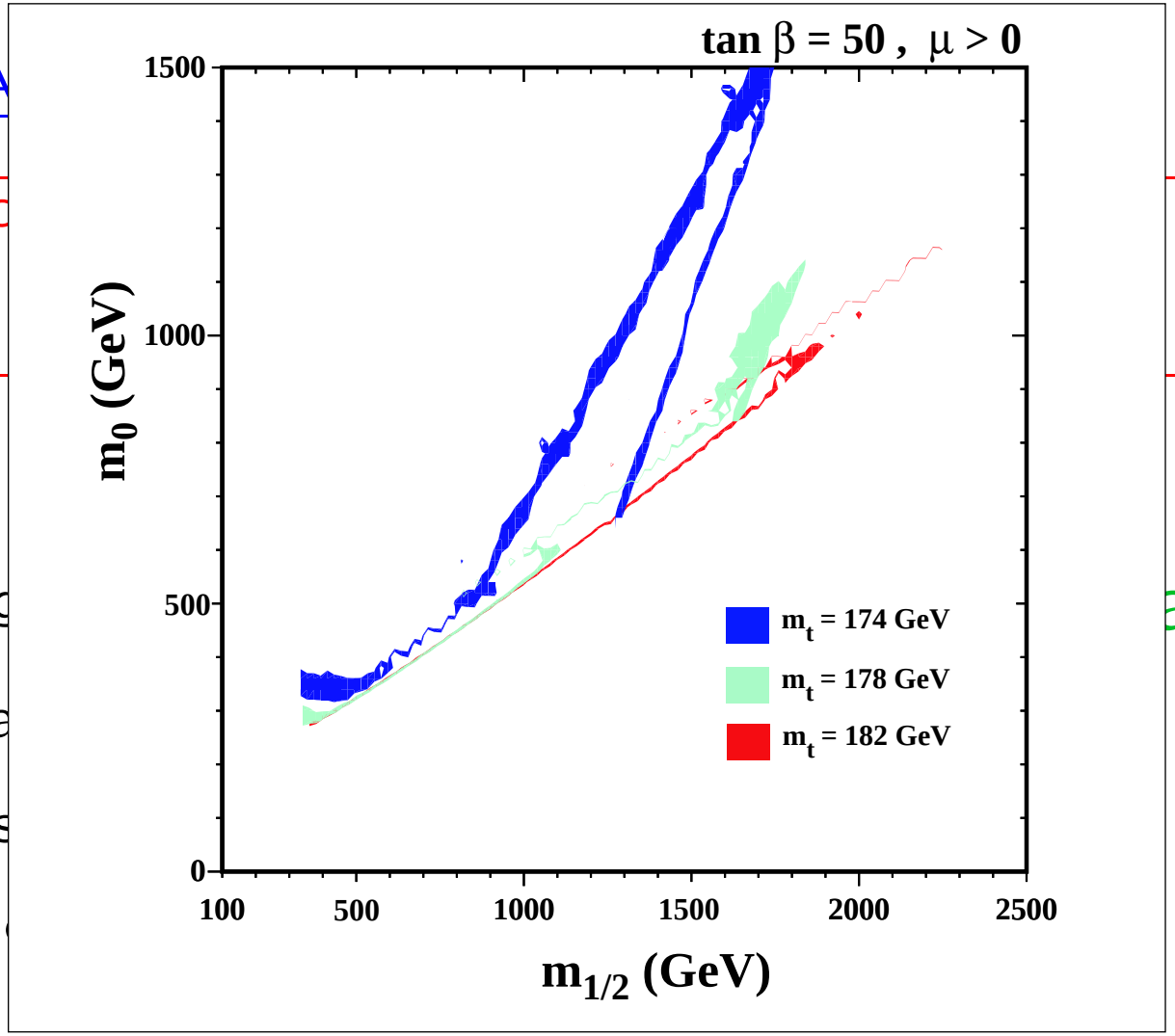
m_0 : universal scalar

$m_{1/2}$: universal gaug

A_0 : universal trilinea

$\tan\beta$: ratio of Higgs

sign(μ) : sign of sup



ale

⇒ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is the lightest neutralino

“And who of you had the cheese on the wish-list?”

The models (II):

NUHM: (Non-universal Higgs mass model)

$\Rightarrow$ besides the CMSSM parameters

$$M_A \text{ and } \mu$$

Assumption:

no unification of scalar fermion and scalar Higgs parameters at the GUT scale

$\Rightarrow$ effectively M_A and μ free parameters at the EW scale

$\Rightarrow$ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is the lightest neutralino

The models (III):

VCMSSM: (Very Constrained MSSM)

$\Rightarrow$ In addition to CMSSM: assume relation between A_0 and m_0:

$$A_0/m_0 = 0,\ 3/4,\ 3-\sqrt{3},\ 2$$

Additional constraint also fixes $\tan\beta$

Free parameters: $m_{1/2}$, A_0/m_0

m_0 and $\tan\beta$ fixed via CDM constraint

Lightest SUSY particle (LSP) is the lightest neutralino

GDM (mSUGRA): (Gravitino DM in mSUGRA)

$\Rightarrow$ In addition to CMSSM: assume relation between A_0 and m_0:

$$A_0/m_0 = 0,\ 3/4,\ 3-\sqrt{3},\ 2$$

mSUGRA: $m_{\text{gravitino}} = m_0 \Rightarrow$ gravitino can be the LSP

Free parameters: $m_{1/2}$, A_0/m_0

Lightest SUSY particle (LSP) is the gravitino

The models (III):

VCMSSM: (Very Constrained MSSM)

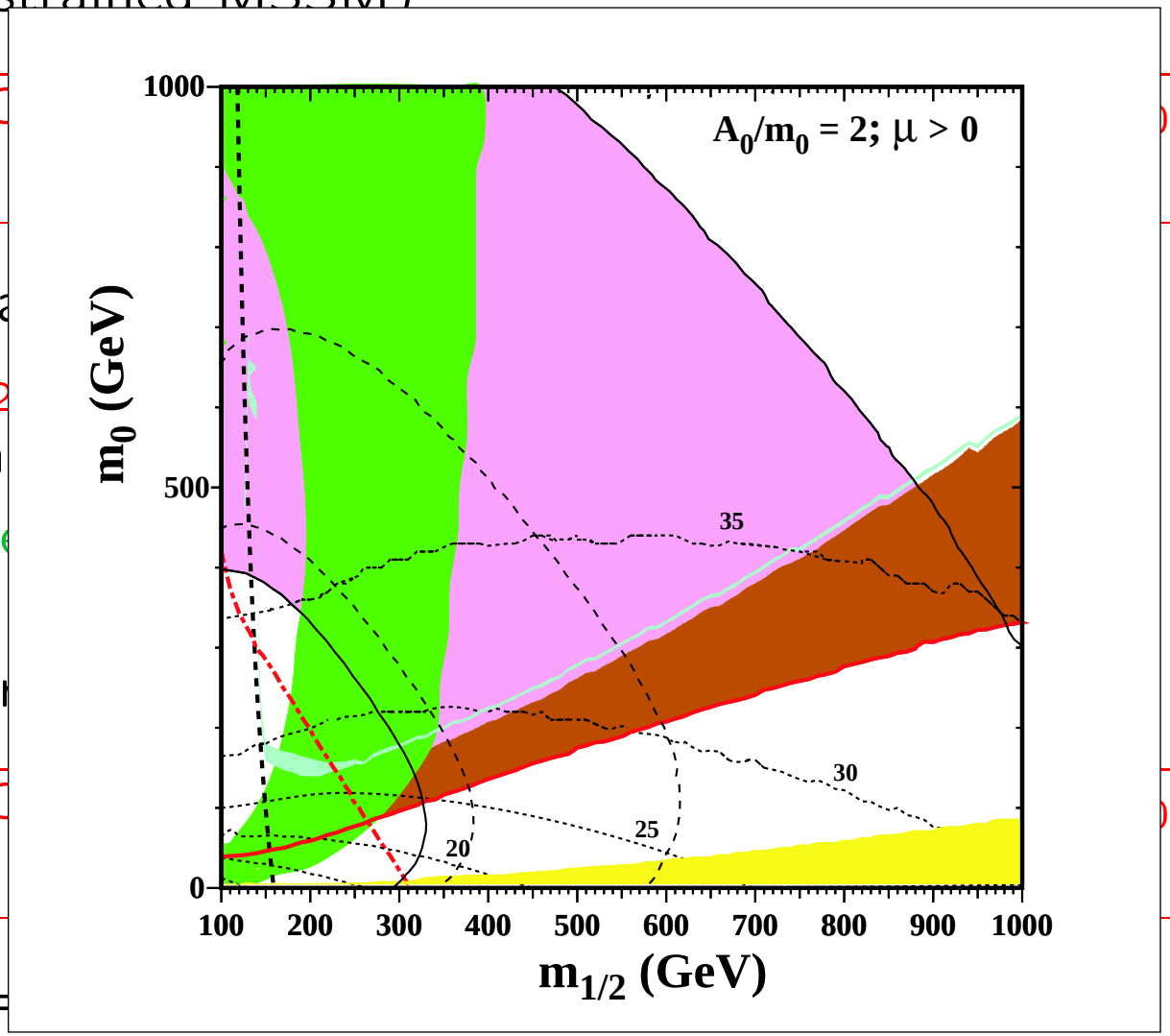
⇒ In addition to C):

Additional constraint a

Free parameters: $m_{1/2}$

m_0 and $\tan\beta$ fixed via

Lightest SUSY particle



GDM (mSUGRA): (Gr

⇒ In addition to C):

mSUGRA: $m_{\text{gravitino}} =$

Free parameters: $m_{1/2}$, A_0/m_0

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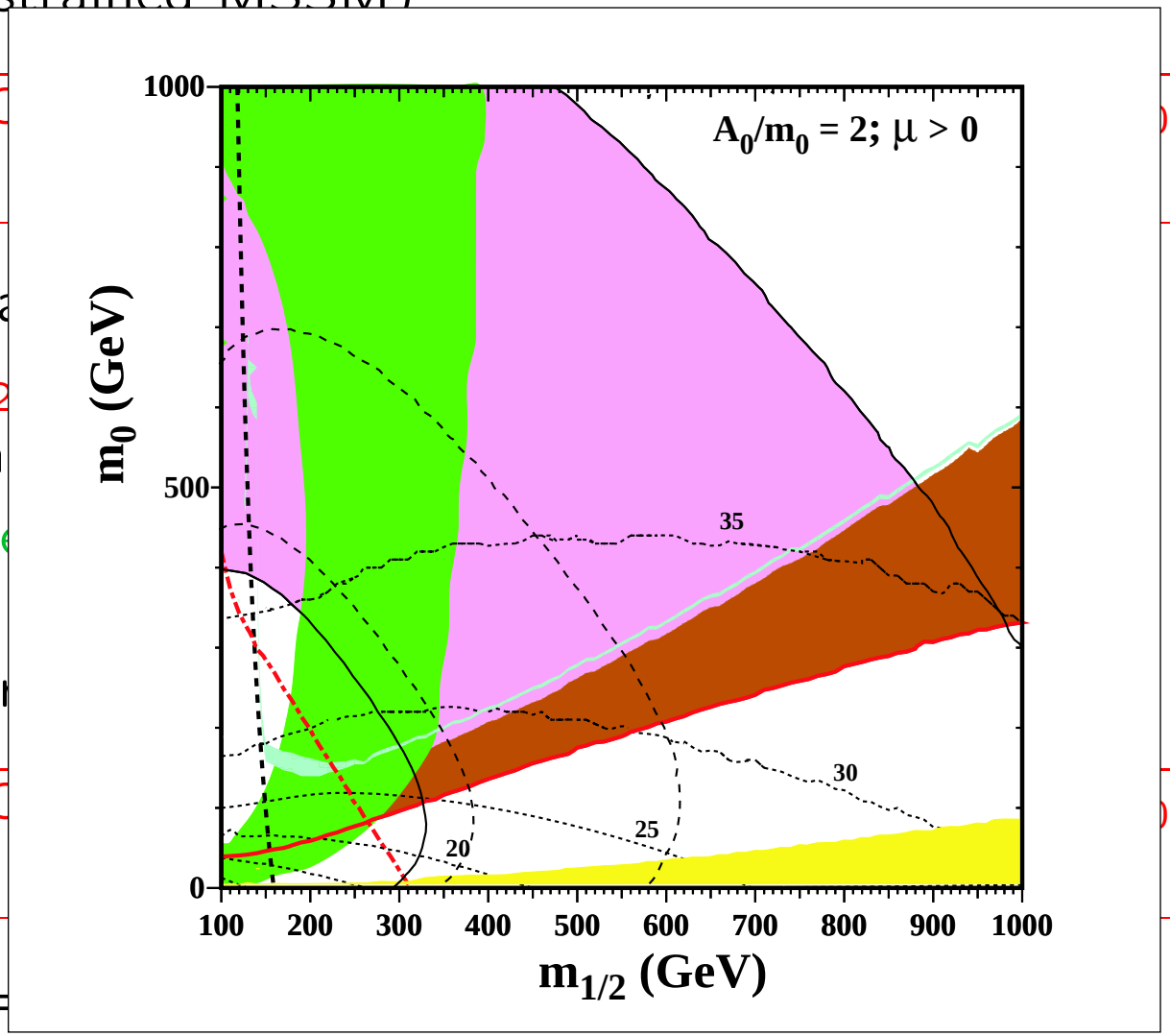
Lightest SUSY particle



GDM (mSUGRA): (Gr

⇒ In addition to C):

mSUGRA: $m_{\text{gravitino}} =$

Free parameters: $m_{1/2}$, A_0/m_0

Lightest SUSY particle (LSP) is the gravitino

The observables (I): M_W, $\sin^2\theta_{\text{eff}}$:

For χ^2 fit:

$$\chi^2_x = \left(\frac{R_x^{\text{exp}} - R_x^{\text{theo}}}{\sigma_x}\right)^2 \qquad x = M_W, \sin^2\theta_{\text{eff}}$$

R_x^{exp}: experimental value

R_x^{theo}: theory prediction

σ_x^2: (exp. error)^2 + (param. error)^2 + (intr. error)^2

experimental error

parametric error: from uncertainty in input parameters

intrinsic error: from unknown higher-order corrections

$\Rightarrow$ use most up to date calculations and error estimates

[*S.H., W. Hollik, G. Weiglein '04*]

[*J. Haestier, S.H., D. Stöckinger, G. Weiglein '05*]

[*LEPEWWG '05*]

(really most up to date used for a current re-evaluation

[*J. Ellis, S.H., K. Olive, A.M. Weber, G. Weiglein '07*])

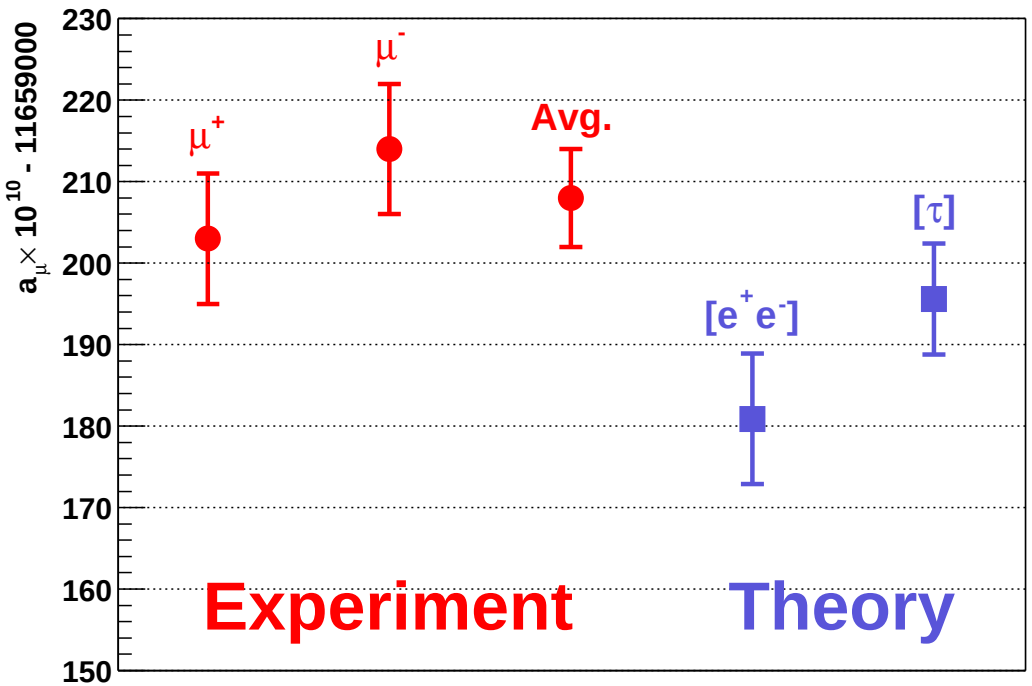
The observables (II): anomalous magnetic moment of the muon: $(g-2)_\mu$

Coupling of muon to magnetic field : $\mu - \mu - \gamma$ coupling

$$\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu}\sigma^{\mu\nu}q_\nu F_2(q^2)\right]u(p)A_\mu \qquad F_2(0) = (g-2)_\mu$$

Overview about the current experimental and SM (theory) result:

[*g-2 Collaboration, hep-ex/0401008*]



Overview of the SM theory evaluation:

Source	contr. to $a_\mu[10^{-10}]$	
LO hadr.	$\sim 695 \pm 7$ (e^+e^-)	[*Davier et al, Hagiwara et al. '03*]
		[*Ghozzi, Jegerlehner '03*]
	711.0 ± 6 (τ)	[*Davier, Eidelman, Höcker, Zhang '03*]
LBL	8 ± 4	[*Knecht, Nyffeler '02*]
	13.6 ± 2.5 tbc	[*Melnikov, Vainshtein '03*]
EW 1L	19	
EW 2L	-4	[*Czarnecki, Krause, Marciano '98*]
exp. res.	6	[*BNL E821 '04*]

$\rightarrow$ "Isospin breaking effects" in τ data problematic [*Ghozzi, Jegerlehner '03*]

e^+e^- data: good agreement between new SND, CMD2, KLOE data

$\Rightarrow$ general agreement at ICHEP'06 Moscow: discard τ data

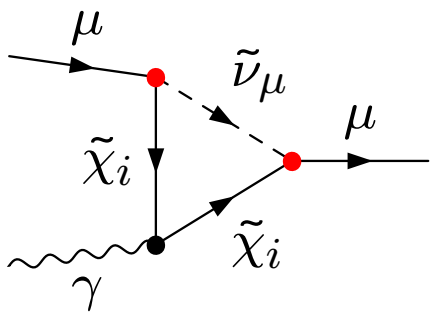
$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} \approx (27.5 \pm 8.4) \times 10^{-10} \quad (\text{before: } (25.2 \pm 9.2) \times 10^{-10})$$

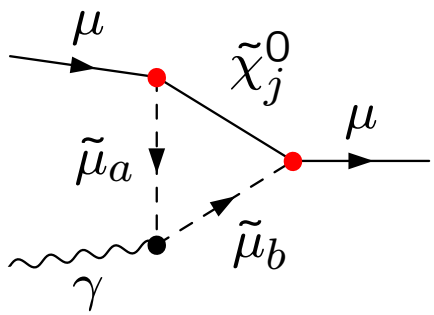
MSSM contributions to $(g-2)_\mu$:

Coupling of muon to magnetic field : $\mu - \mu - \gamma$ coupling

$$\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu}\sigma^{\mu\nu}q_\nu F_2(q^2)\right]u(p)A_\mu \qquad F_2(0) = (g-2)_\mu$$

Feynman diagrams for MSSM 1L corrections:





Enhancement factor as compared to SM:

$$\mu - \tilde{\chi}_i^\pm - \tilde{\nu}_\mu \;:\; \sim m_\mu \tan\beta$$
$$\mu - \tilde{\chi}_j^0 - \tilde{\mu}_a \;:\; \sim m_\mu \tan\beta$$

For χ^2 fit:

$$\chi_x^2 = \left(\frac{R_x^{\text{exp}} - R_x^{\text{theo}}}{\sigma_x} \right)^2 \qquad x = (g-2)_\mu$$

R_x^{exp}: experimental value $= (a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}})$

R_x^{theo}: theory prediction $= a_\mu^{\text{theo,SUSY}}$

σ_x^2: (exp. error)^2 + (param. error)^2 + (intr. error)^2

experimental error

parametric error: from uncertainty in input parameters

intrinsic error: from unknown higher-order corrections

$\Rightarrow$ use most up to date calculations and error estimates

[*S.H., W. Hollik, G. Weiglein '04*]

[*S.H., D. Stöckinger, G. Weiglein '03,'04*]

[*g-2 Collaboration, hep-ex/0401008*]

The observables (III): $b \to s\gamma$

For χ^2 fit:

$$\chi_x^2 = \left(\frac{R_x^{\text{exp}} - R_x^{\text{theo}}}{\sigma_x} \right)^2 \qquad x = \text{BR}(b \to s\gamma)$$

R_x^{exp}: experimental value

R_x^{theo}: theory prediction

σ_x^2: (exp. error)^2 + (param. error)^2 + (intr. error)^2

experimental error

parametric error: from uncertainty in input parameters

intrinsic error: from unknown higher-order corrections

$\Rightarrow$ use up to date calculations and error estimates

[*Asatrian, Hovhannisyan, Greub, Hurth, Poghosyan '05*]

[*BaBar, Belle '02, '04*]

[*HFAG '06*]

(really most up to date not yet included, . . . [*M. Misiak et al. '06*])

The observables (IV): the lightest MSSM Higgs boson mass: M_h

Contrary to the SM: M_h is not a free parameter

MSSM tree-level bound: $M_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:

$$\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector) f

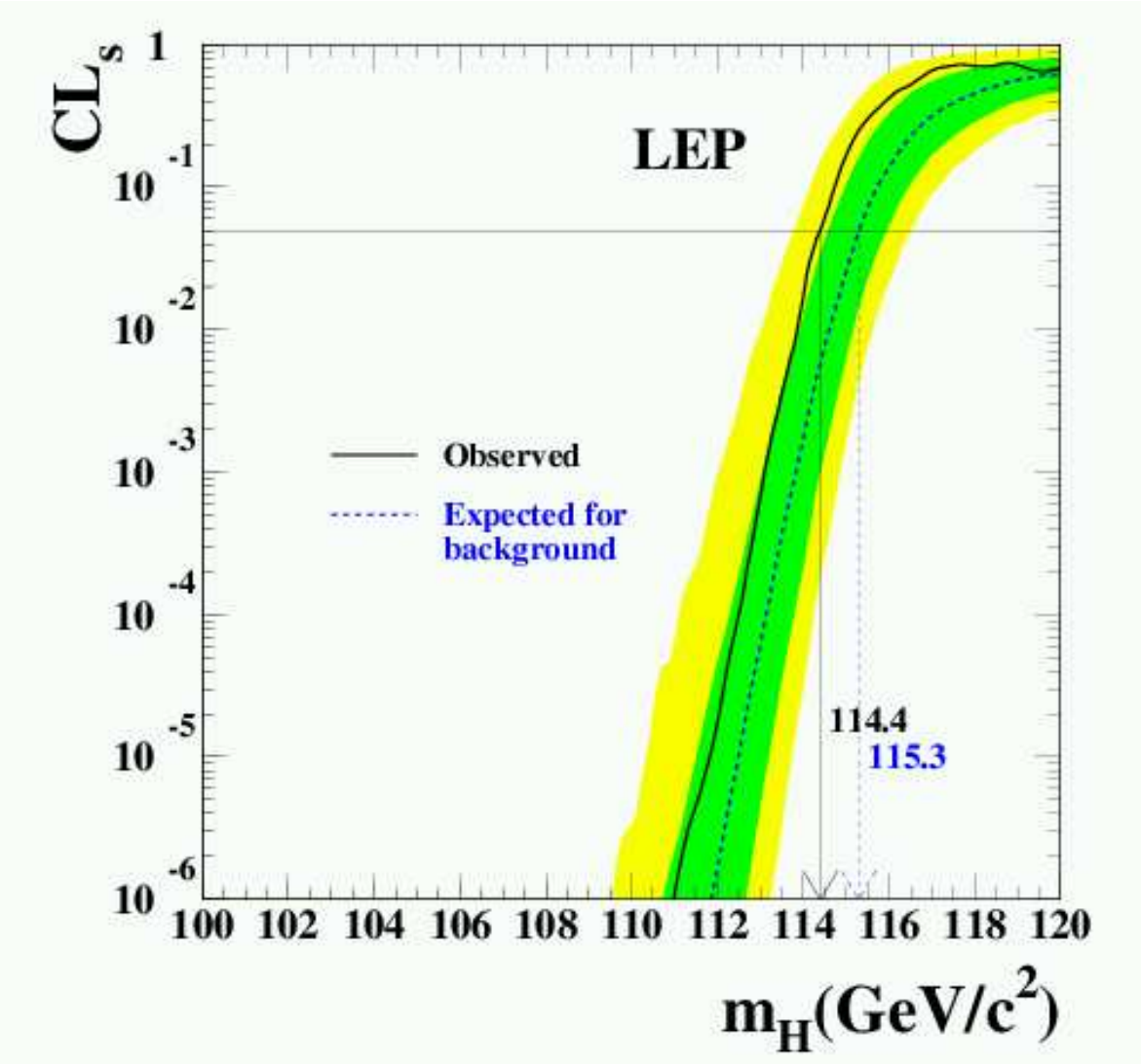
Measurement of M_h, Higgs couplings $\Rightarrow$ test of the theory

LHC: $\Delta M_h \approx 0.2$ GeV
ILC: $\Delta M_h \approx 0.05$ GeV

$\Rightarrow$ M_h will be (the best?) electroweak precision observable

In CMSSM, NUHM, VCMSSM, GDM: SM bound of M_H search can be used

[*LEP Higgs Working Group '03*]



CL_s can be used/transformed into χ^2 values

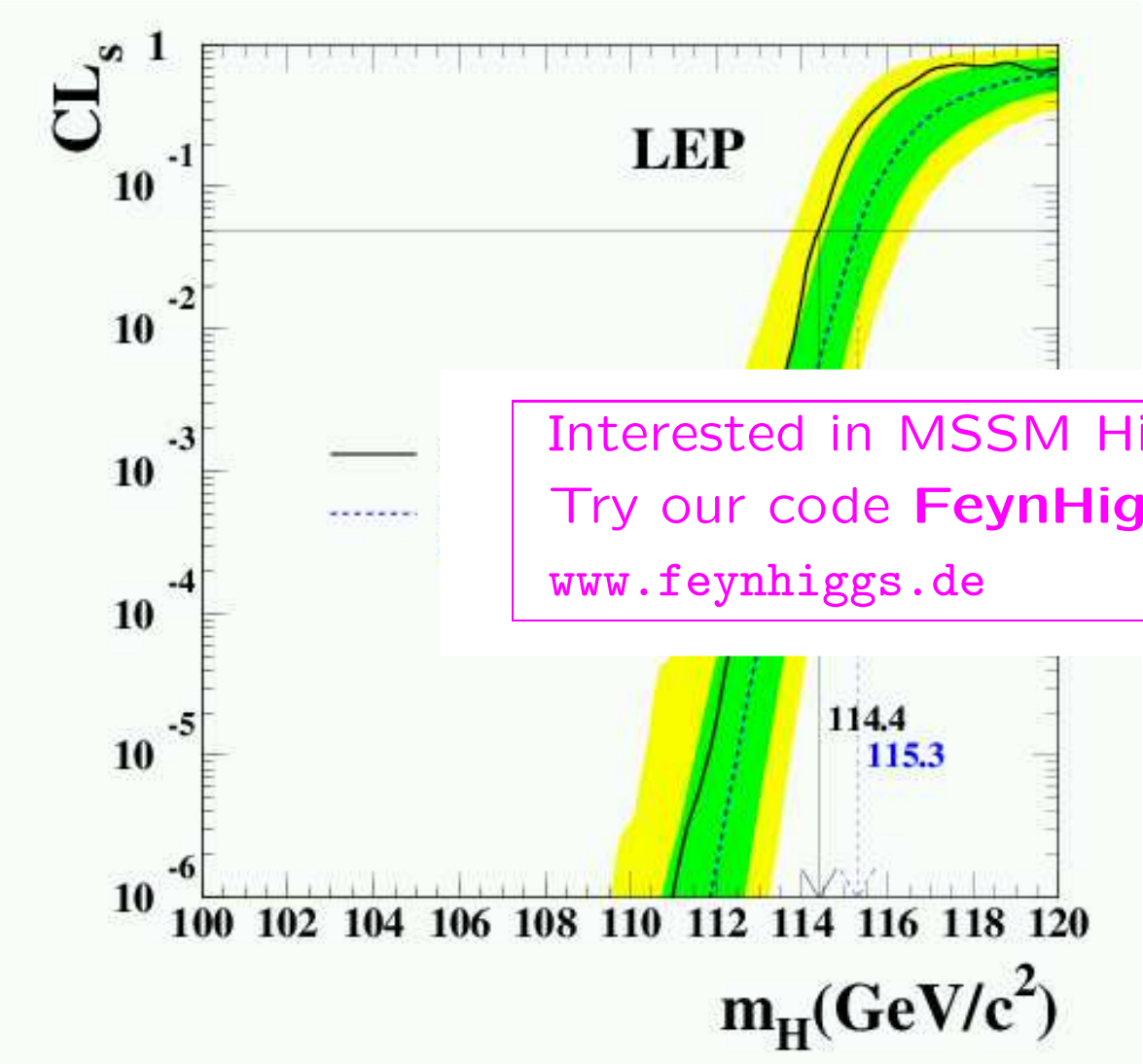
$\Rightarrow$ additional (unobserved) parameter

$\delta M_h^{\text{intr.}} \approx 3$ GeV

We use *FeynHiggs*

In CMSSM, NUHM, VCMSSM, GDM: SM bound of M_H search can be used

[*LEP Higgs Working Group '03*]



CL_s can be used/transformed into χ^2 values

observed)

Interested in MSSM Higgs physics?
Try our code **FeynHiggs**
`www.feynhiggs.de`

$\delta M_h^{\text{intr.}} \approx 3$ GeV

We use *FeynHiggs*

Fit results and collider reach

Procedure:

1. Scan over parameter space:
   - CMSSM: for fixed $\tan\beta = 10, 50$
   - NUHM: certain parameter planes,
     corresponding to CMSSM best fit points
   - VCMSSM: full parameter space ($A_0/m_0 = 0, 3/4, 3-\sqrt{3}, 2$)
   - GDM (mSUGRA): full parameter space ($A_0/m_0 = 0, 3/4, 3-\sqrt{3}, 2$)

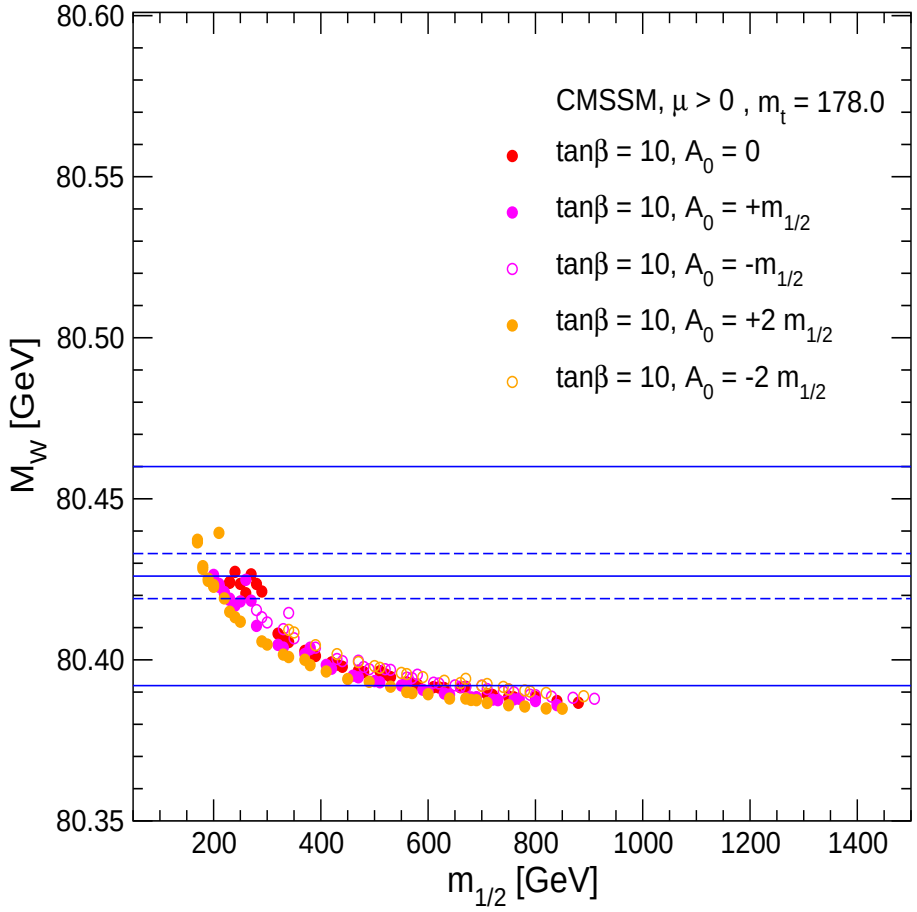
2. Perform χ^2 fit

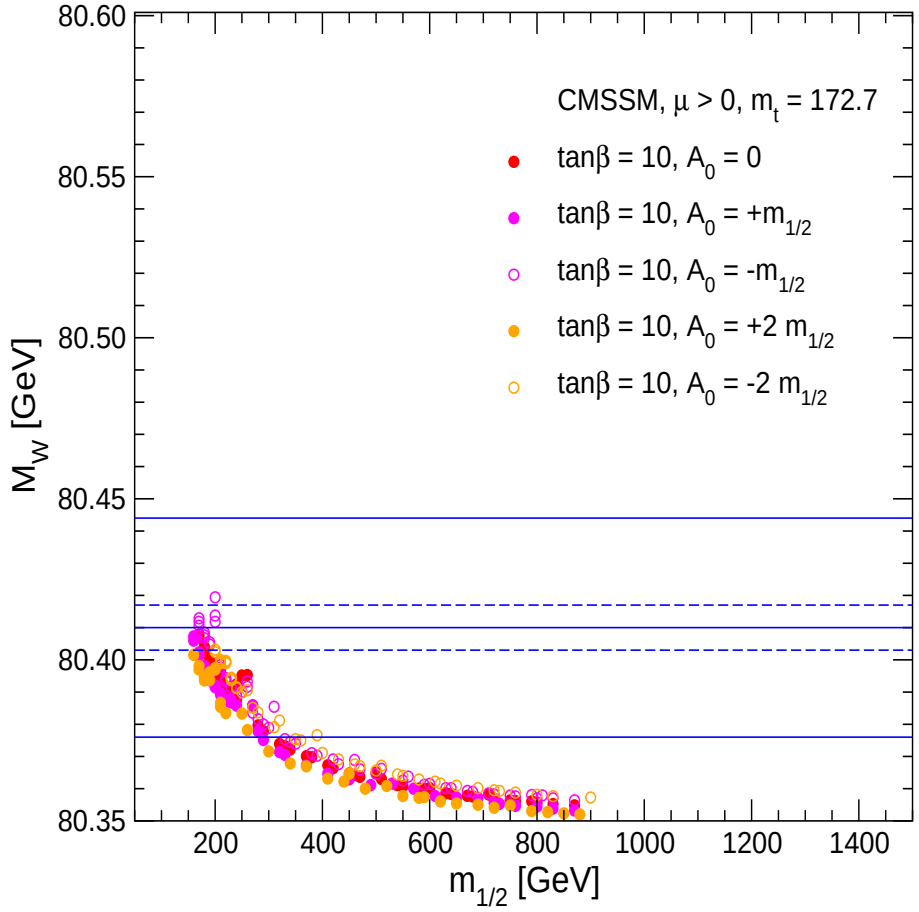
3. Find preferred values for masses
   $\Rightarrow$ collider reach

$\Rightarrow$ focus on CMSSM
all models show the same qualitative behavior

Results: CMSSM:

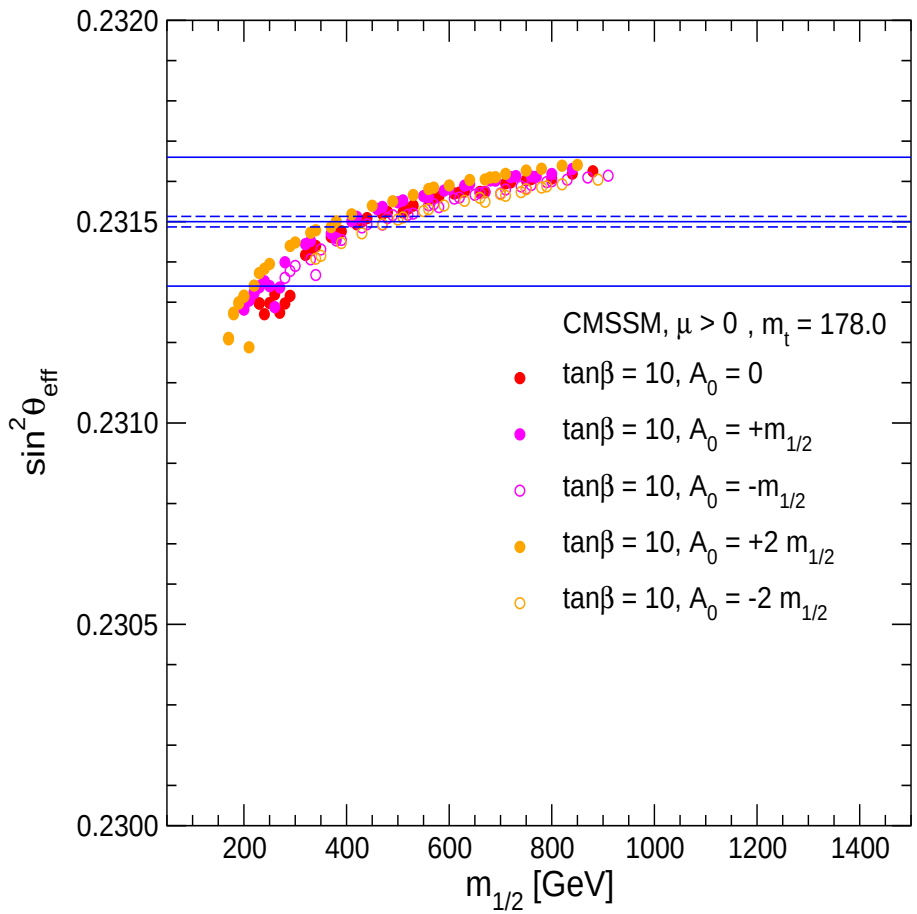
Compare $m_t = 178$ GeV with $m_t = 172.7$ GeV: M_W for $\tan\beta = 10$:

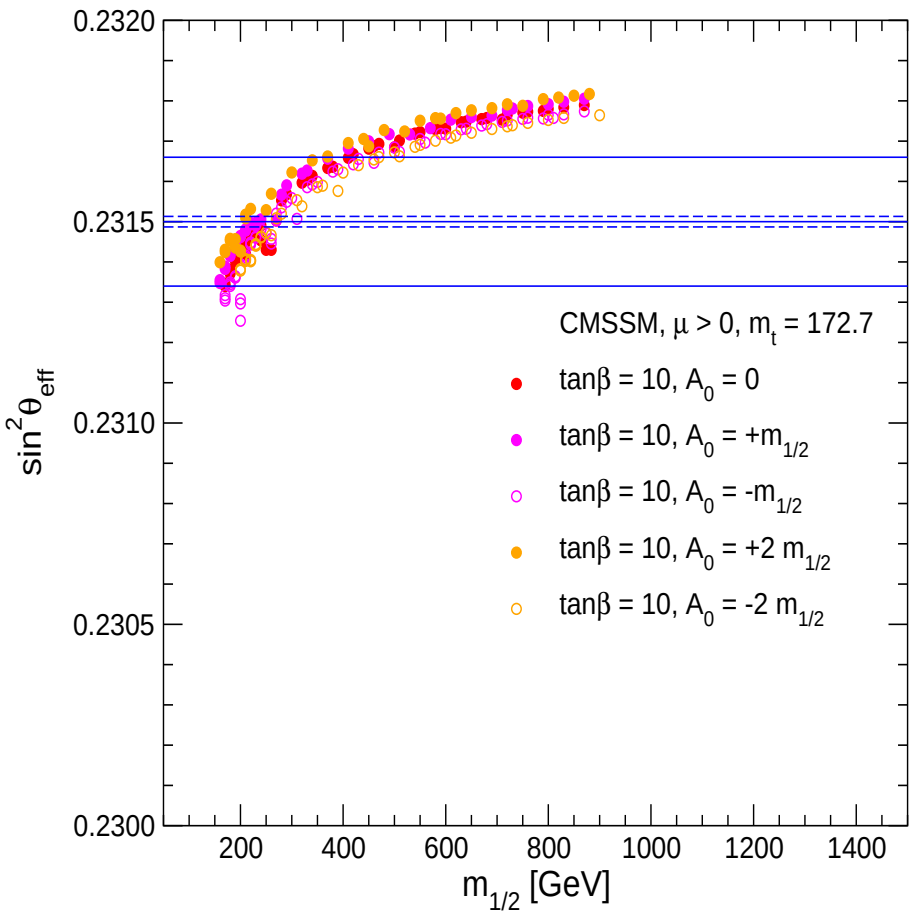




$m_t = 178 \rightarrow 172.7 \Rightarrow m_{1/2}$ lowered

Compare $m_t = 178$ GeV with $m_t = 172.7$ GeV: $\sin^2\theta_{\rm eff}$ for $\tan\beta = 10$:



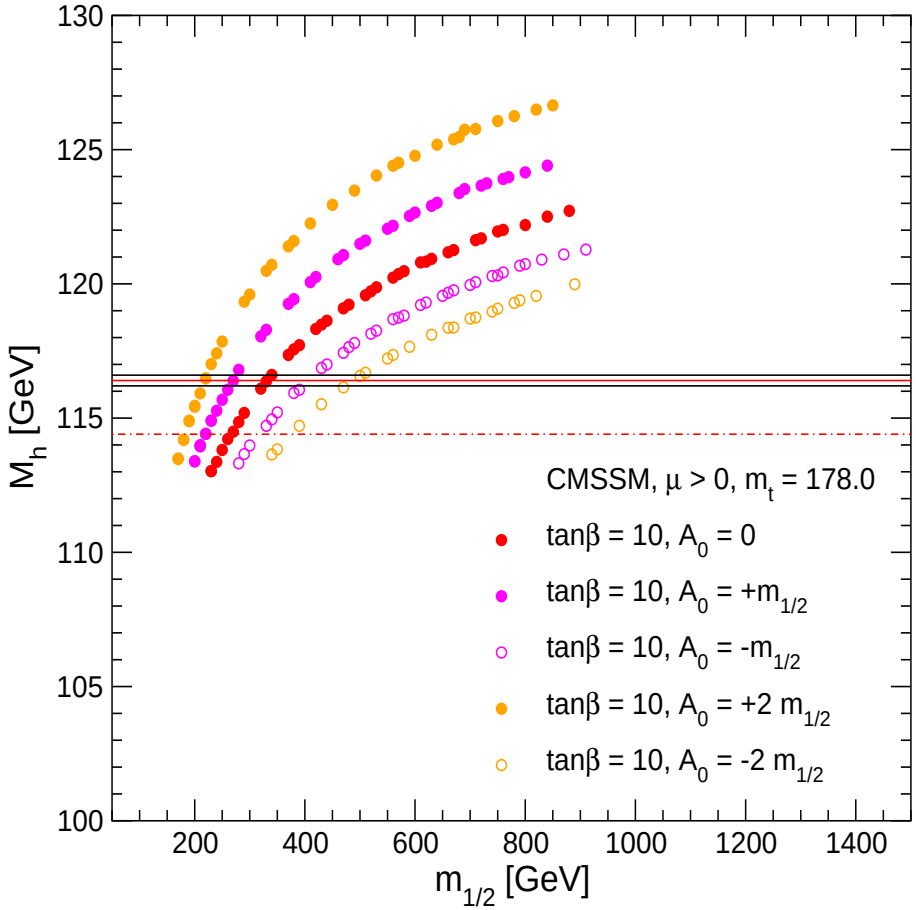


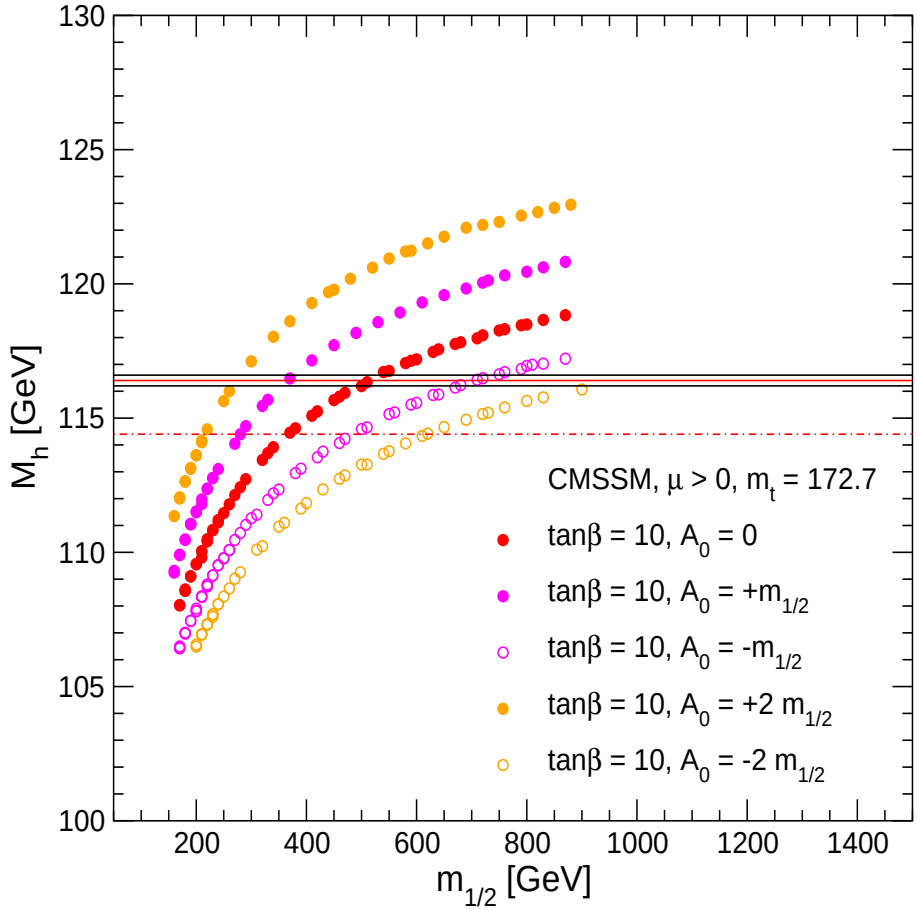
$m_t = 178 \to 172.7 \Rightarrow m_{1/2}$ lowered

Note: most recent evaluation of $\sin^2\theta_{\rm eff}$ gives results $\sim 1.5\sigma^{\rm exp}$ lower

[S.H., W. Hollik, A.M. Weber, G. Weiglein '07]

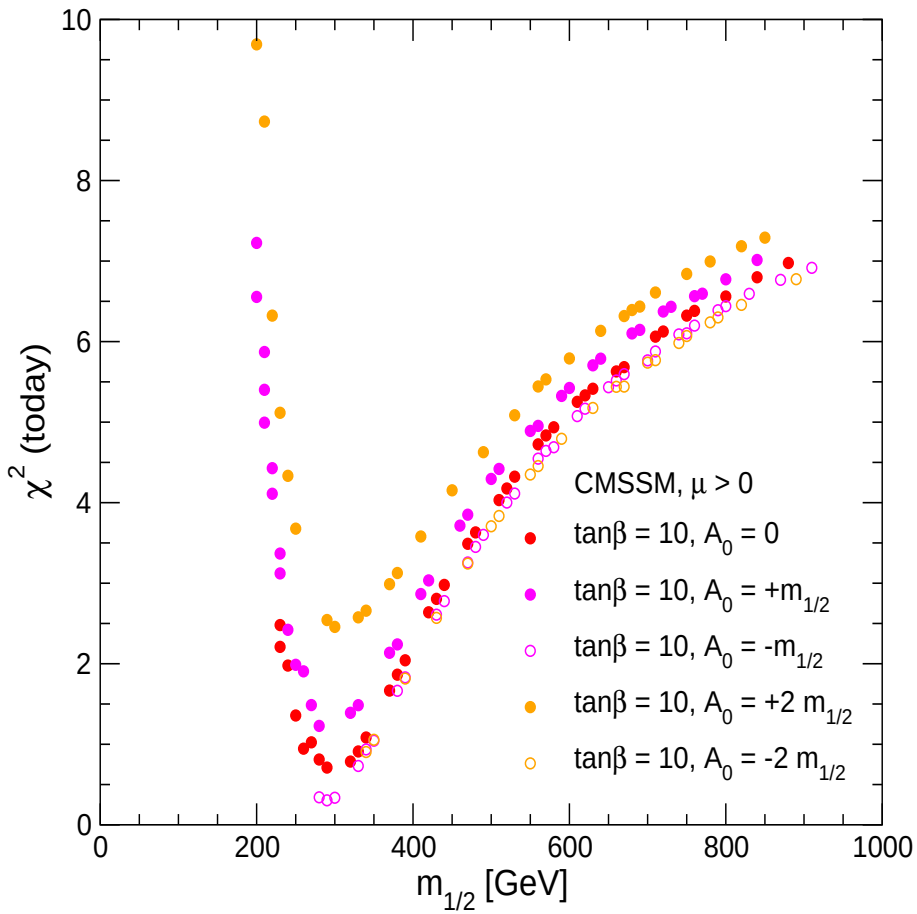
Compare $m_t = 178$ GeV with $m_t = 172.7$ GeV: M_h for $\tan\beta = 10$:

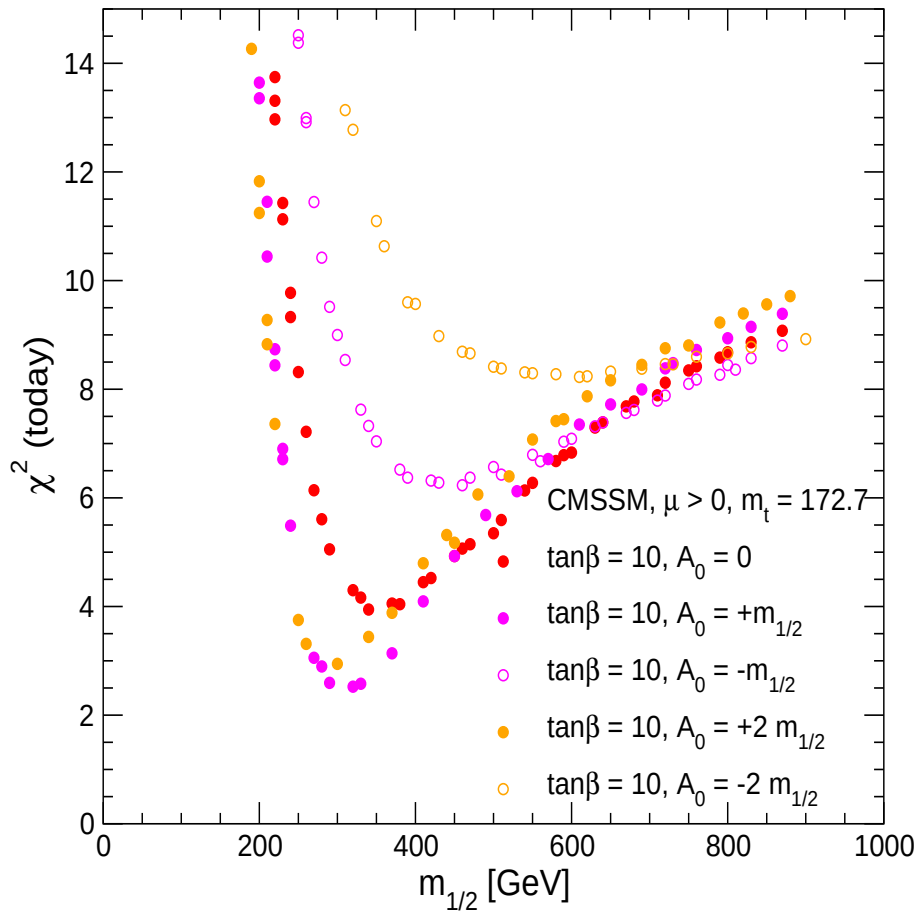




$m_t = 178 \rightarrow 172.7 \Rightarrow m_{1/2}$ increased

Effect of $m_t = 178$ GeV with $m_t = 172.7$ GeV: χ^2 fit for $\tan\beta = 10$:

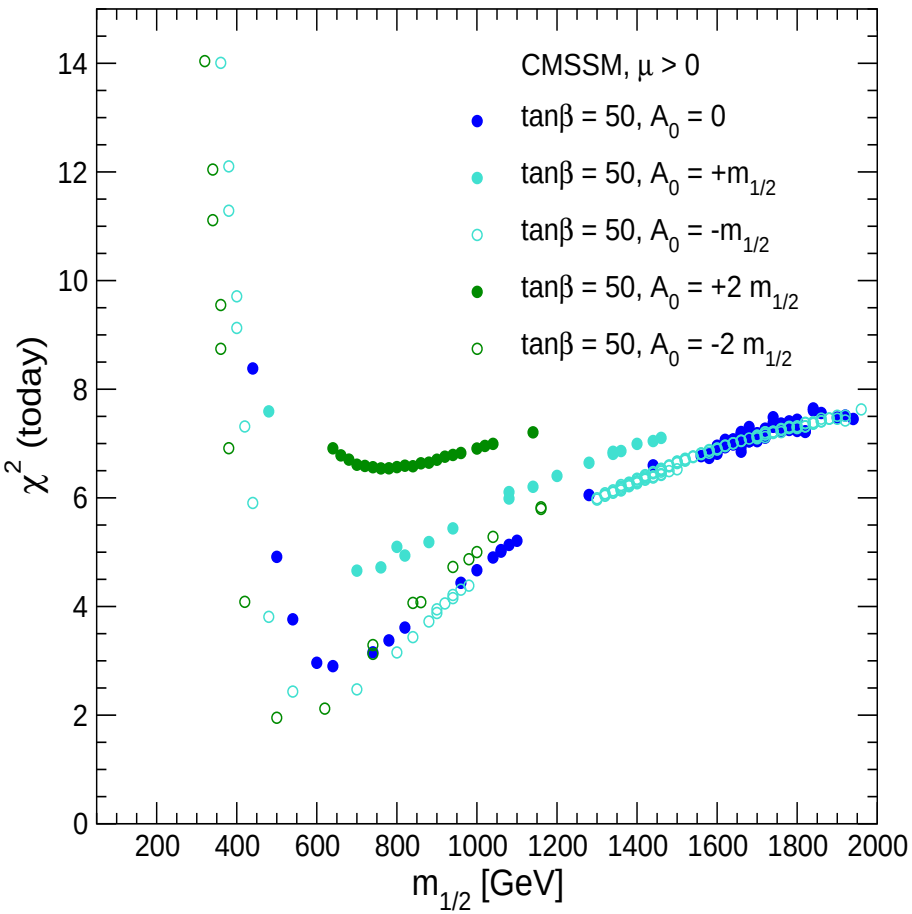


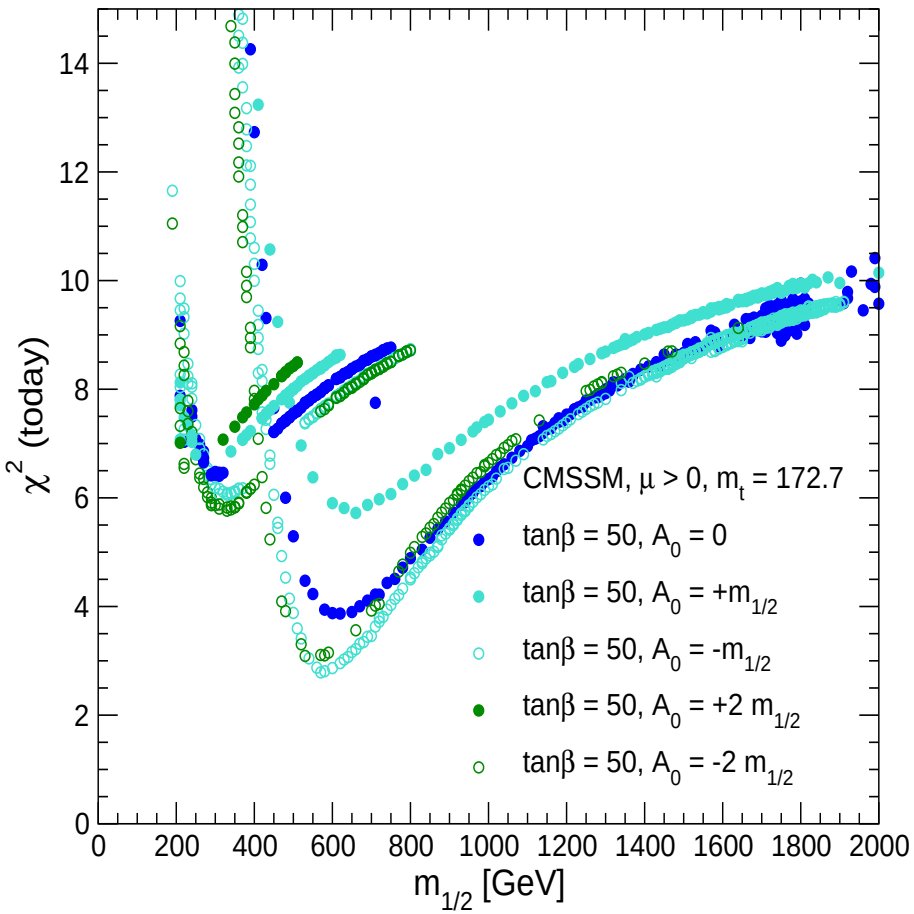


$m_t = 178 \to 172.7$ slightly higher χ^2, $A_0 > 0$ favored

$\Rightarrow M_W$ and $\sin^2\theta_{\text{eff}}$ more important, $(g-2)_\mu$ less important for low $m_{1/2}$

Effect of $m_t = 178$ GeV with $m_t = 172.7$ GeV: χ^2 fit for $\tan\beta = 50$:

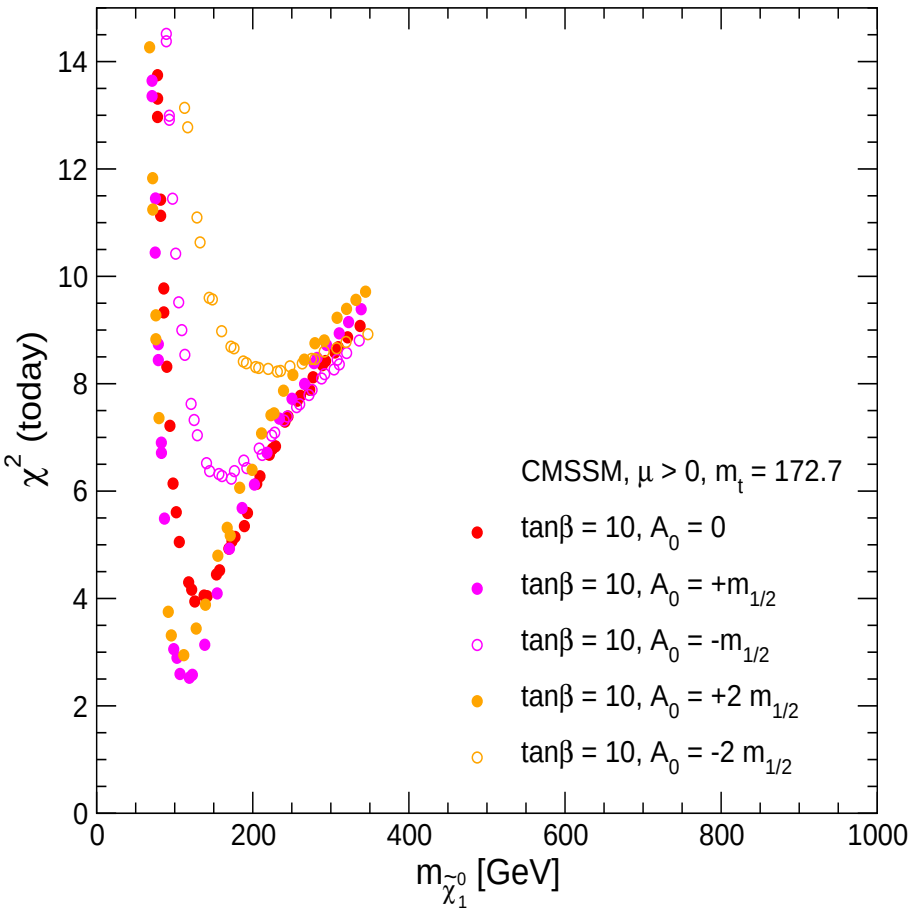


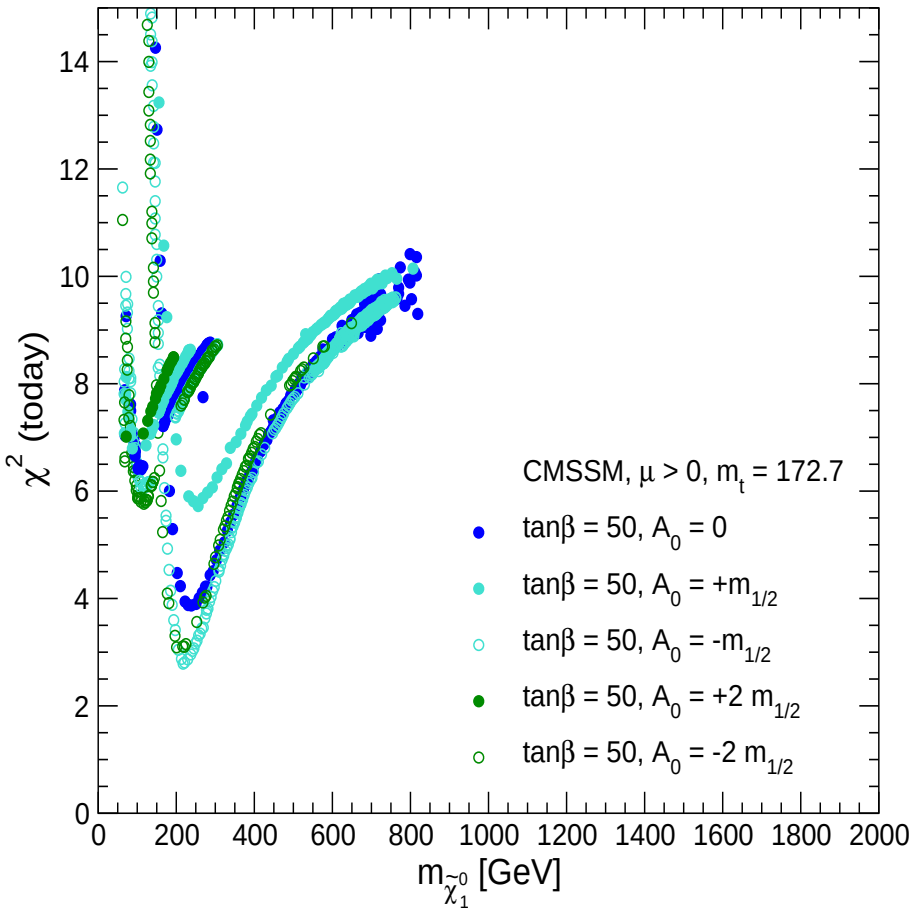


$m_t = 178 \rightarrow 172.7$ effective change minor

$\Rightarrow$ (re)appearance of focus point region at $m_{1/2} \approx 200$ GeV

CMSSM: LSP mass for $\tan\beta = 10, 50$

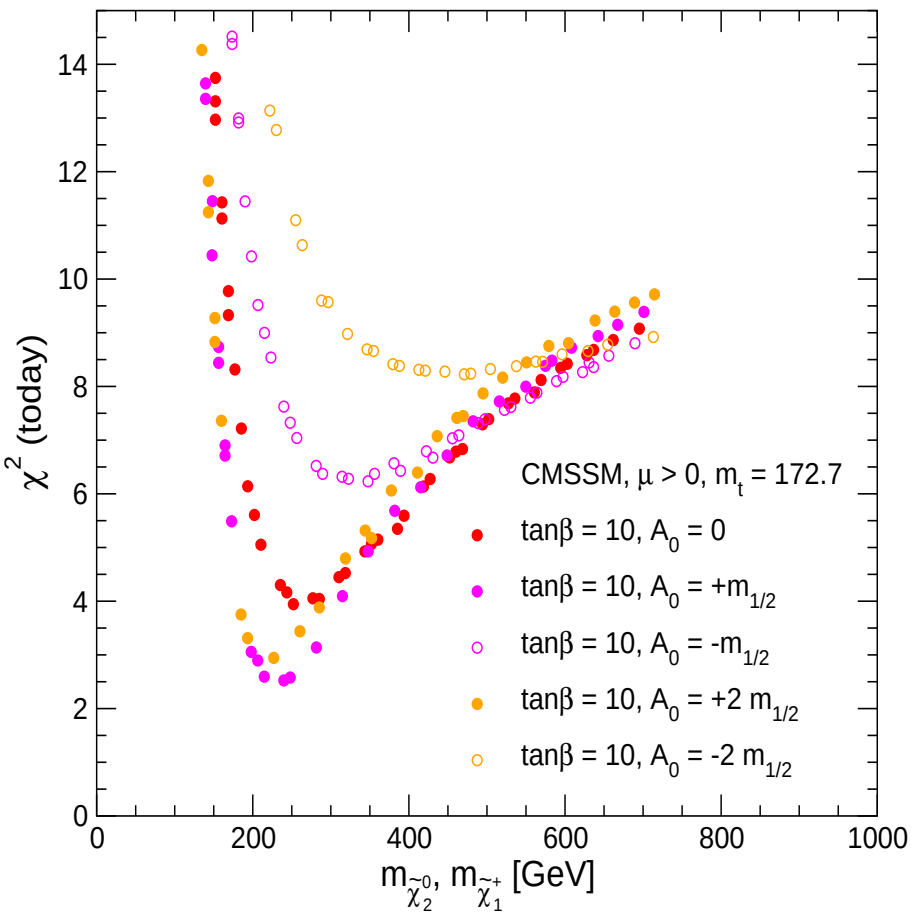


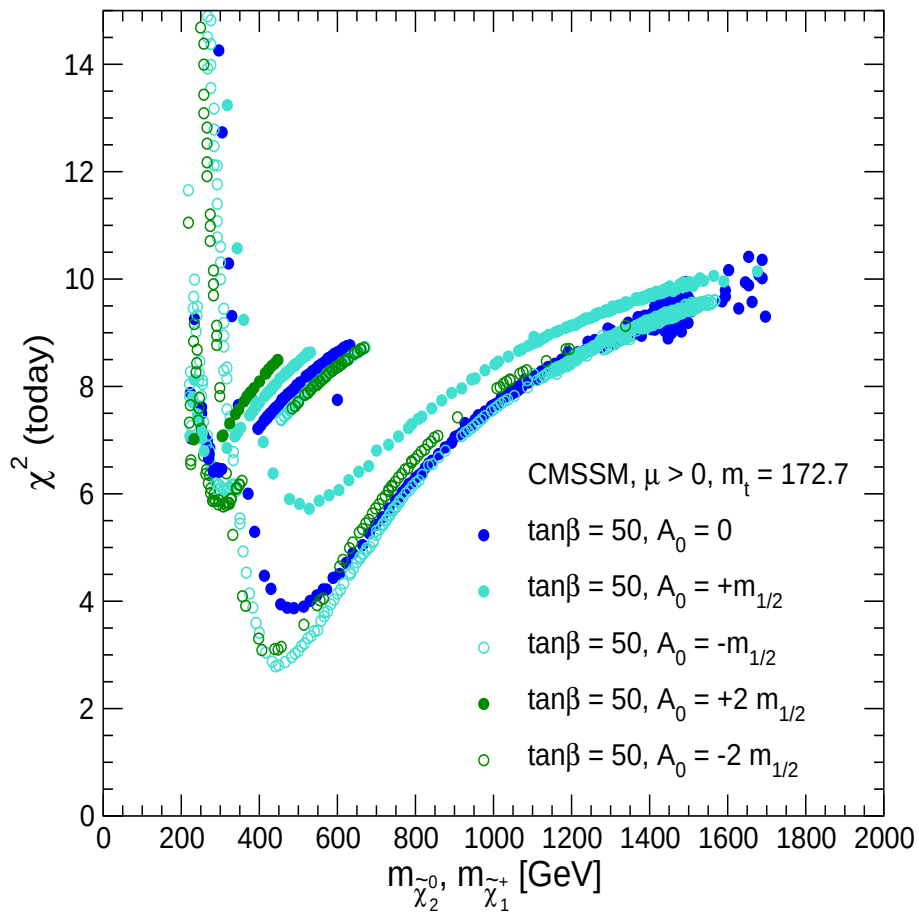


$\tan\beta = 10 \Rightarrow$ minimum at/below 200 GeV

$\tan\beta = 50 \Rightarrow$ similar

CMSSM: light neutralino/chargino masses for $\tan\beta = 10, 50$



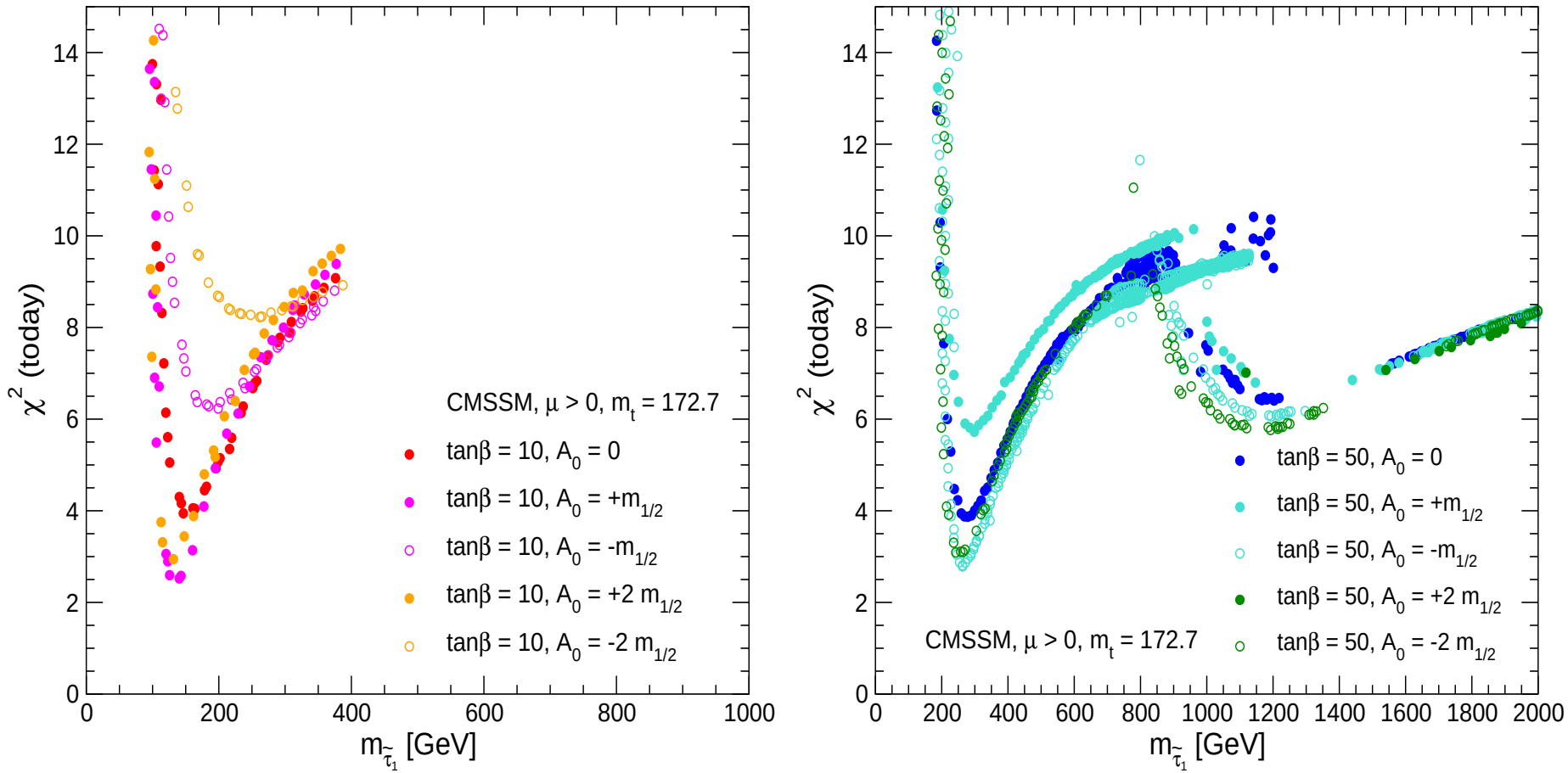


$\tan\beta = 10 \Rightarrow$ very good prospects for the LHC/ILC(1000)

$\Rightarrow$ even chances for the Tevatron

$\tan\beta = 50 \Rightarrow$ good prospects (only) for $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production

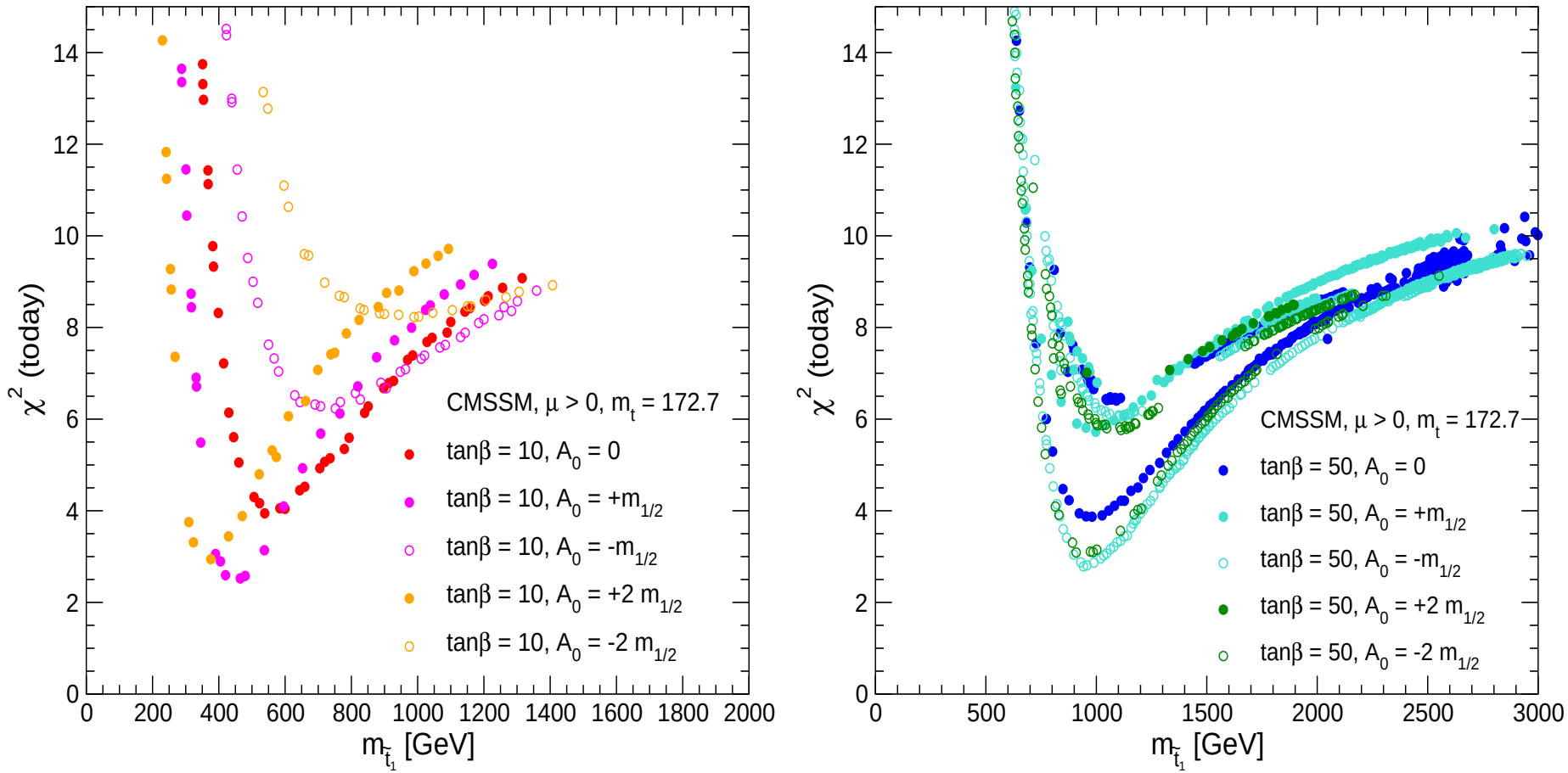
CMSSM: lightest stau mass for $\tan\beta = 10, 50$



$\tan\beta = 10 \Rightarrow$ very good prospects for all colliders

$\tan\beta = 50 \Rightarrow$ still quite good (except for Focus Point)

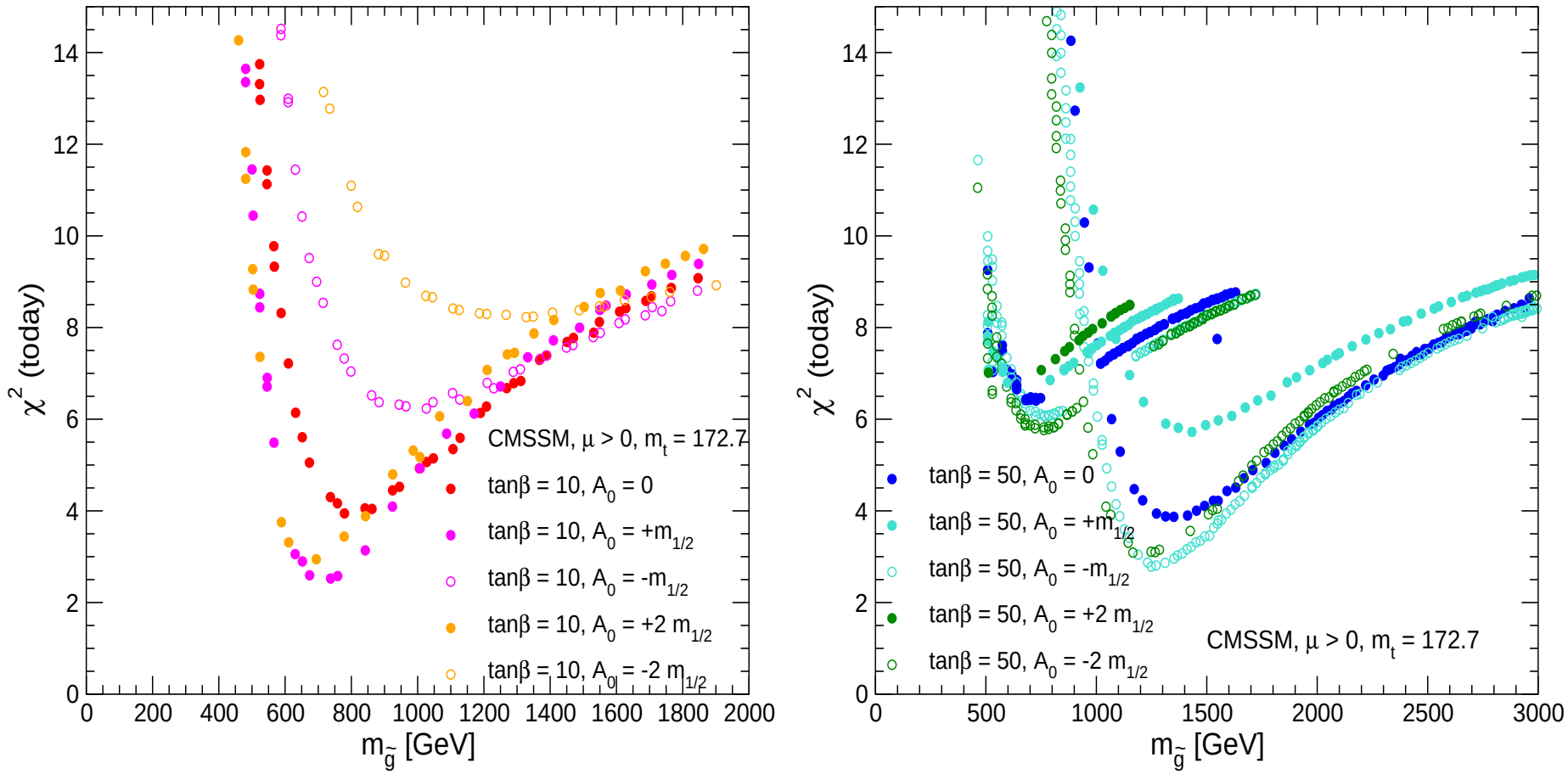
CMSSM: lightest stop mass for $\tan\beta = 10, 50$



$\tan\beta = 10 \Rightarrow$ good chances for LHC, moderate prospects for the ILC

$\tan\beta = 50 \Rightarrow$ still good for LHC, but outside the ILC(1000) reach

CMSSM: gluino mass for $\tan\beta = 10, 50$



$\tan\beta = 10 \Rightarrow$ no problem for LHC, but outside the ILC(1000) reach

$\tan\beta = 50 \Rightarrow$ no problem for LHC, but outside the ILC(1000) reach

"So he did not behave well? No presents? Just beat him up?
Ok, I understand! – Well, that message arrived in the last second!"

5. Conclusinos

- Precision observables
  - can give valuable information about the "true" Lagrangian
  - can provide bounds on SUSY parameter space

- Two-loop Yukawa correction to $\Delta\rho$:
  corrections up to $\Delta M_W = 8$ MeV, $\Delta \sin^2\theta_{\text{eff}} = -4 \times 10^{-5}$

- New result for M_W in the MSSM:
  complete one-loop with complex parameters
  + all existing MSSM higher-order corrections
  + full SM result

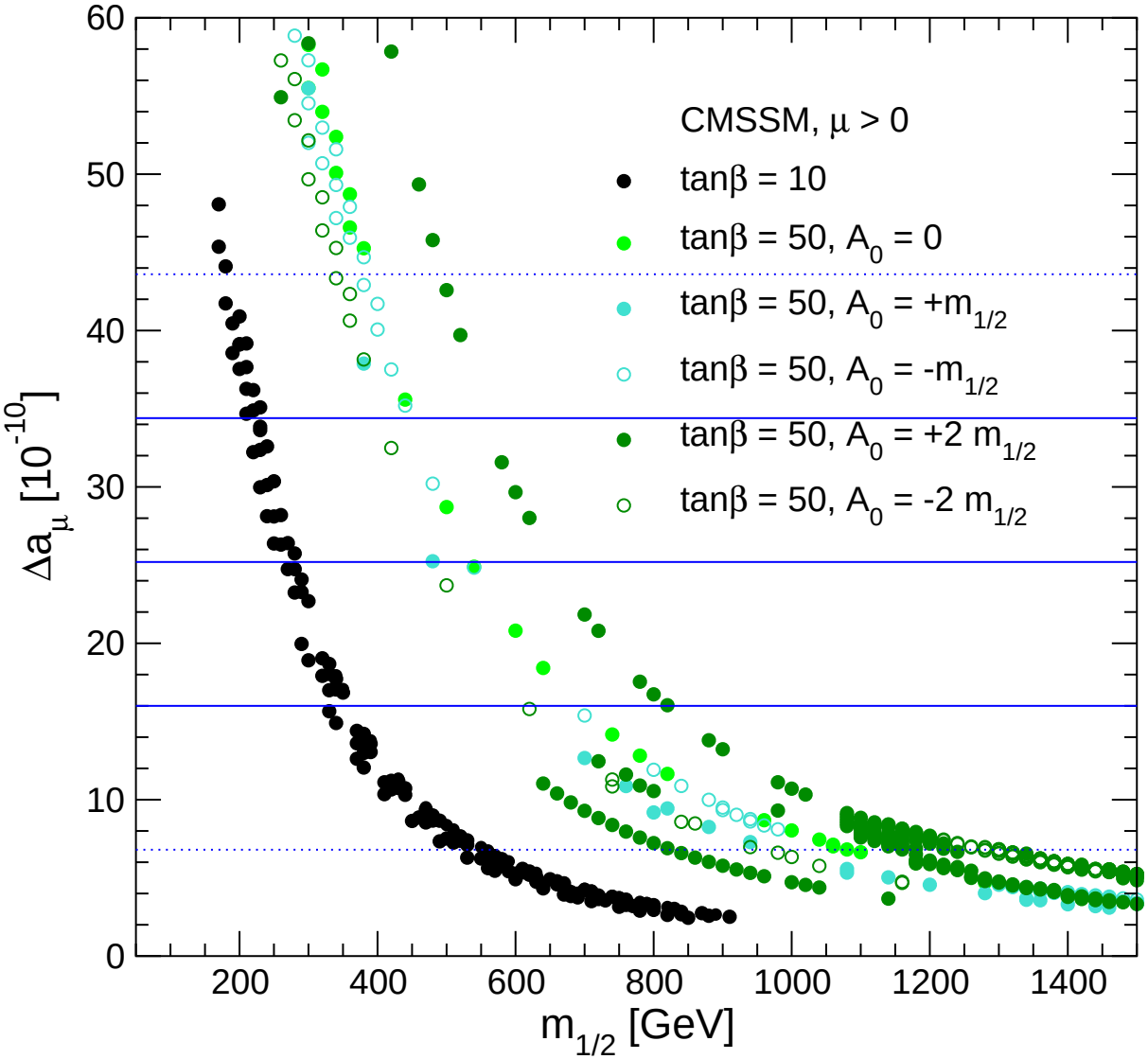
- χ^2 fit using M_W, $\sin^2\theta_{\text{eff}}$, $(g-2)_\mu$, BR$(b \to s\gamma)$, M_h
  investigated models: CMSSM, NUHM, VCMSSM, GDM (mSUGRA)

  χ^2 fit: low values, $\mathcal{O}(2)$ reached

  relatively low SUSY mass scales favored
  $\Rightarrow$ good prospects for LHC and ILC (and possibly even Tevatron)

Back-up

Example: Investigation of mSUGRA with cold dark matter constraint



Scan over $m_{1/2}$, m_0, A_0

$\tan\beta = 10, 50$

selected points give correct amount of cold dark matter

[*Ellis, S.H., Olive, Weiglein '04*]

Severe bounds on e.g. $m_{1/2}$

Remaining uncertainties:

1. SUSY 1L diagrams with a mixed $f/\tilde{f}$ loop

$\rightarrow$ same enhancement factors as
SM 1L diagrams with a closed $f/\tilde{f}$ loop attached

$\Rightarrow$ possibly of similar order

2. THDM corrections to SUSY 1L diagrams

Already known:

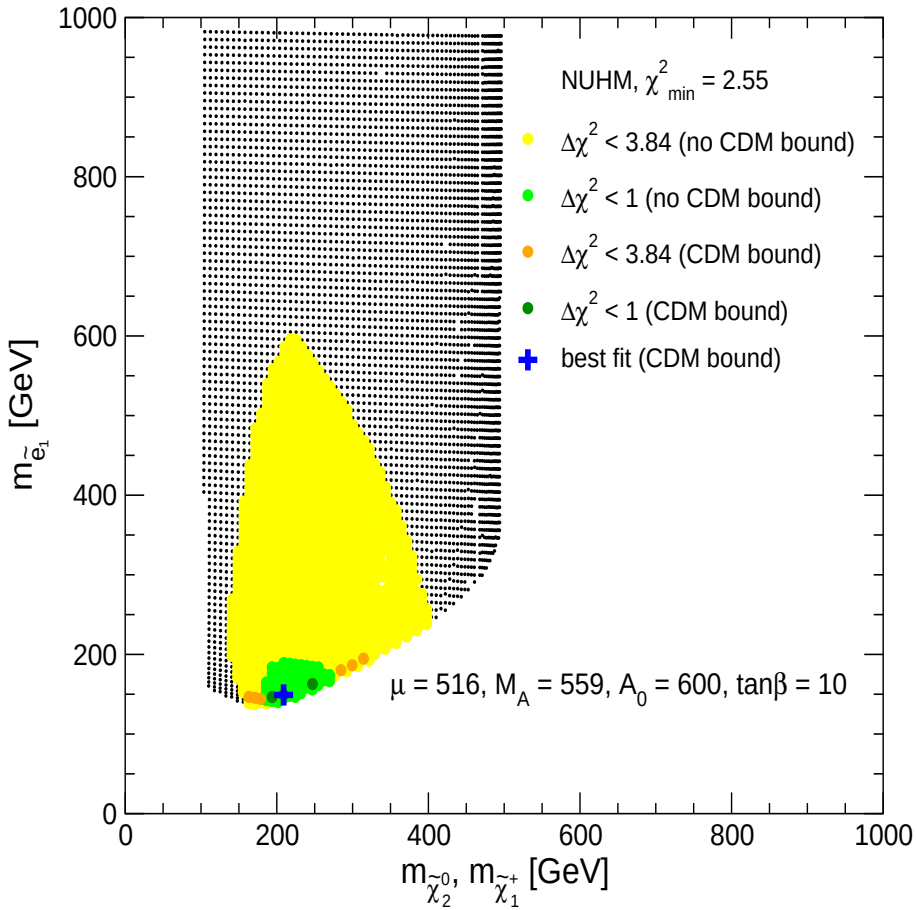
QED corrections to $a_\mu^{\text{SUSY}}(1\text{L})$: $\sim -8\%$ for $M_{\text{SUSY}} = 500$ GeV
[*G. Degrassi, G. Giudice '98*]

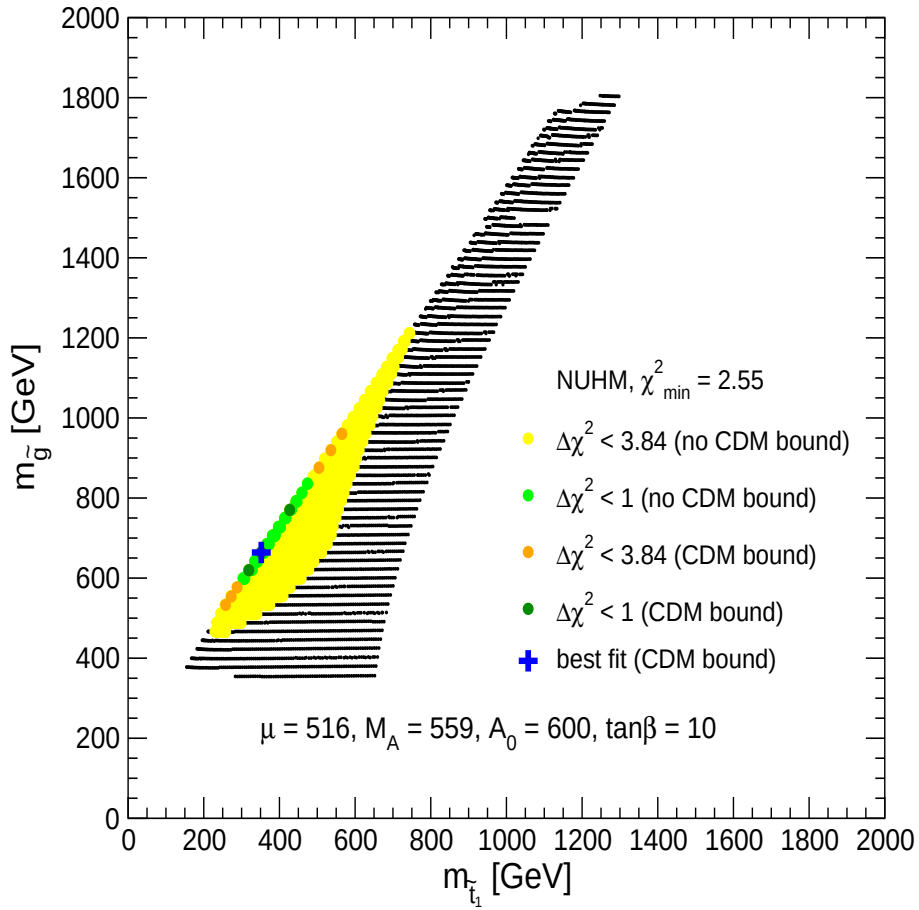
$\rightarrow$ only evaluated for a common SUSY mass scale
$\Rightarrow$ non-negligible corrections possible

$\Rightarrow$ Remaining uncertainties estimated to $\sim 6 \times 10^{-10}$
[*S.H., D. Stöckinger, G. Weiglein '04*]

Results (II): NUHM

NUHM: vary $m_{1/2}$ and m_0 around best CMSSM fit point $\tan\beta = 10$:

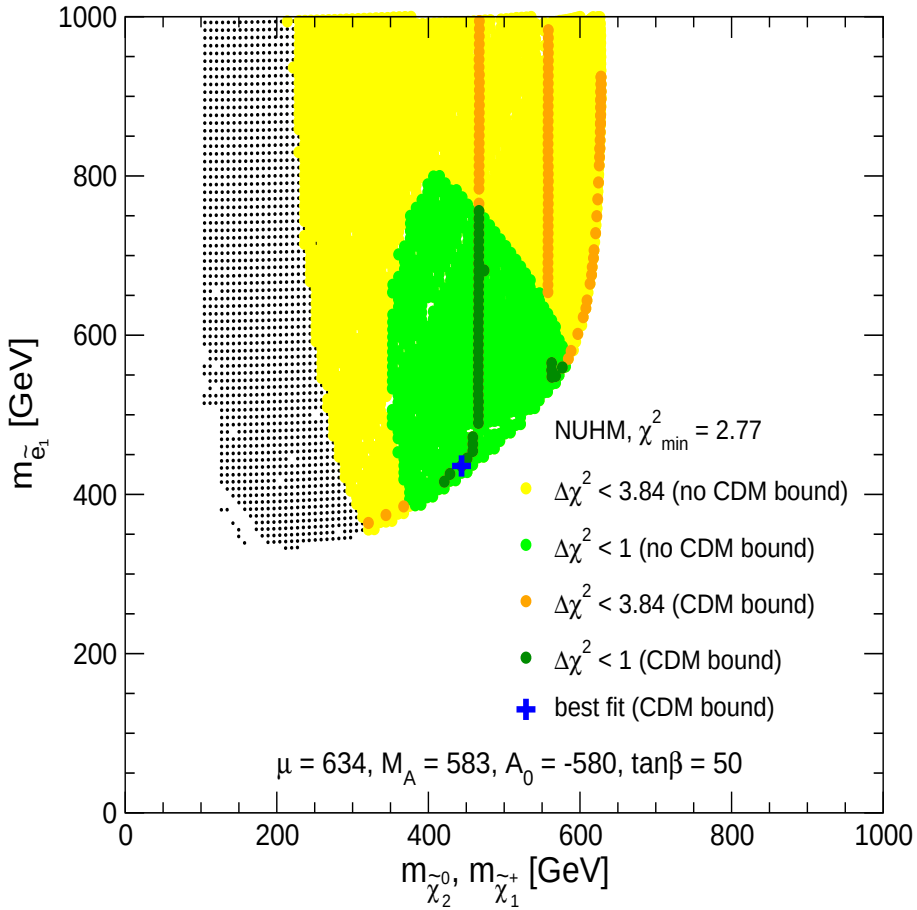


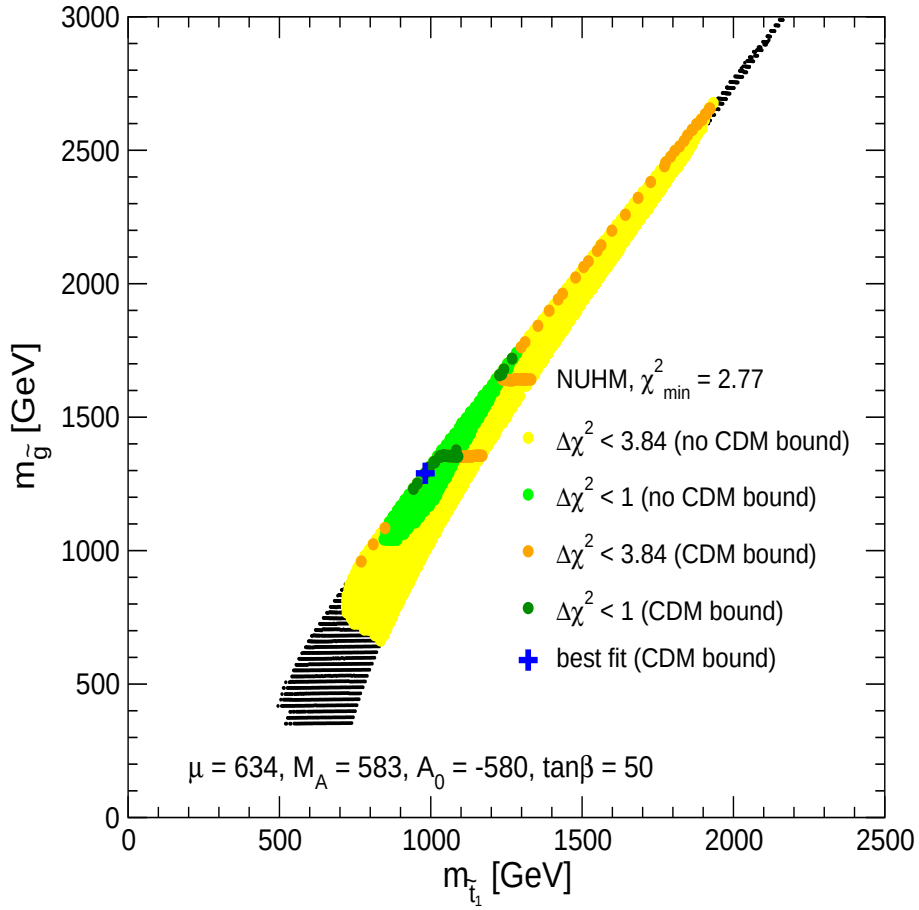


⇒ sleptons, charginos, neutralinos in reach

stops and gluinos only in reach for LHC

NUHM: vary $m_{1/2}$ and m_0 around best CMSSM fit point $\tan\beta = 50$:



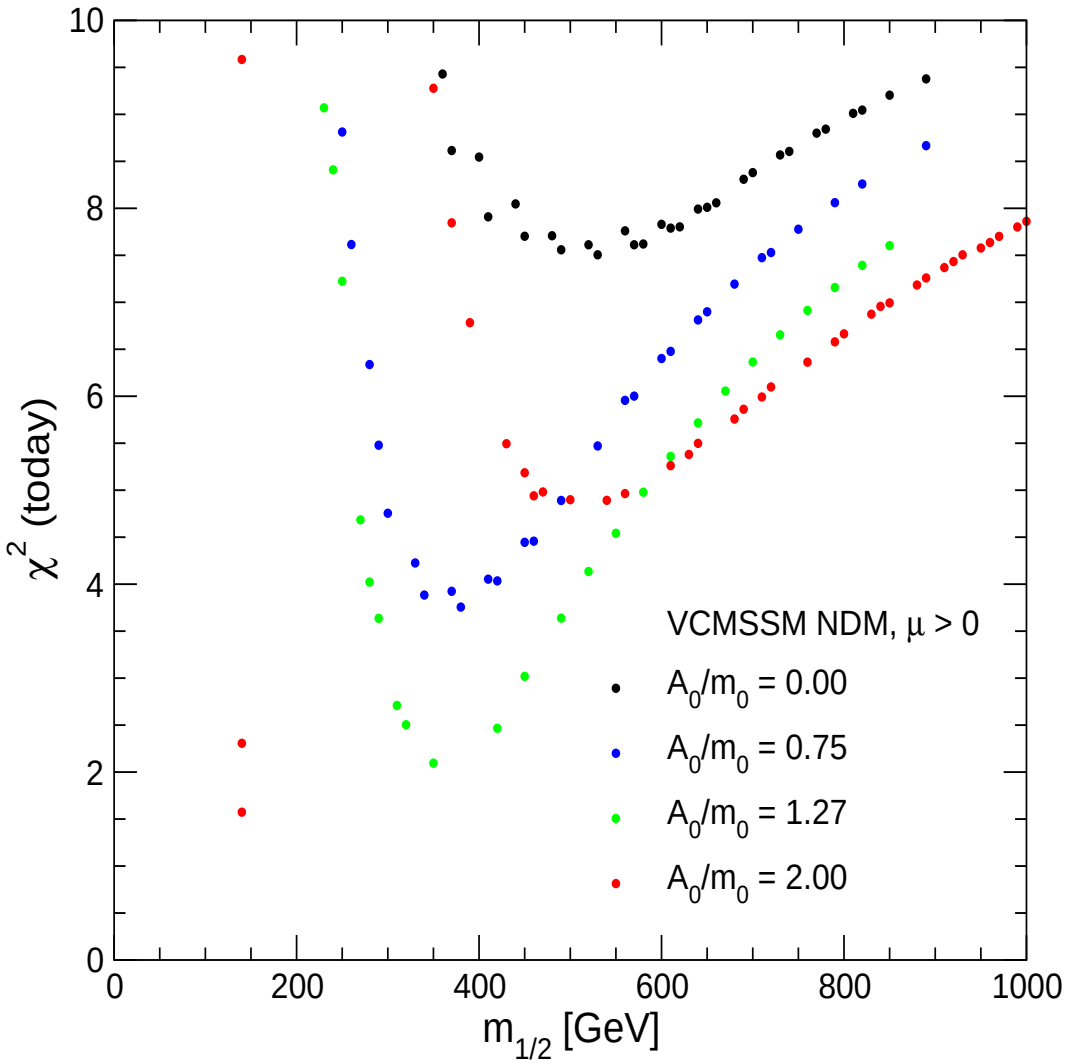


$\Rightarrow$ sleptons, charginos, neutralinos partially in reach for ILC(1000)

stops and gluinos only in reach for LHC

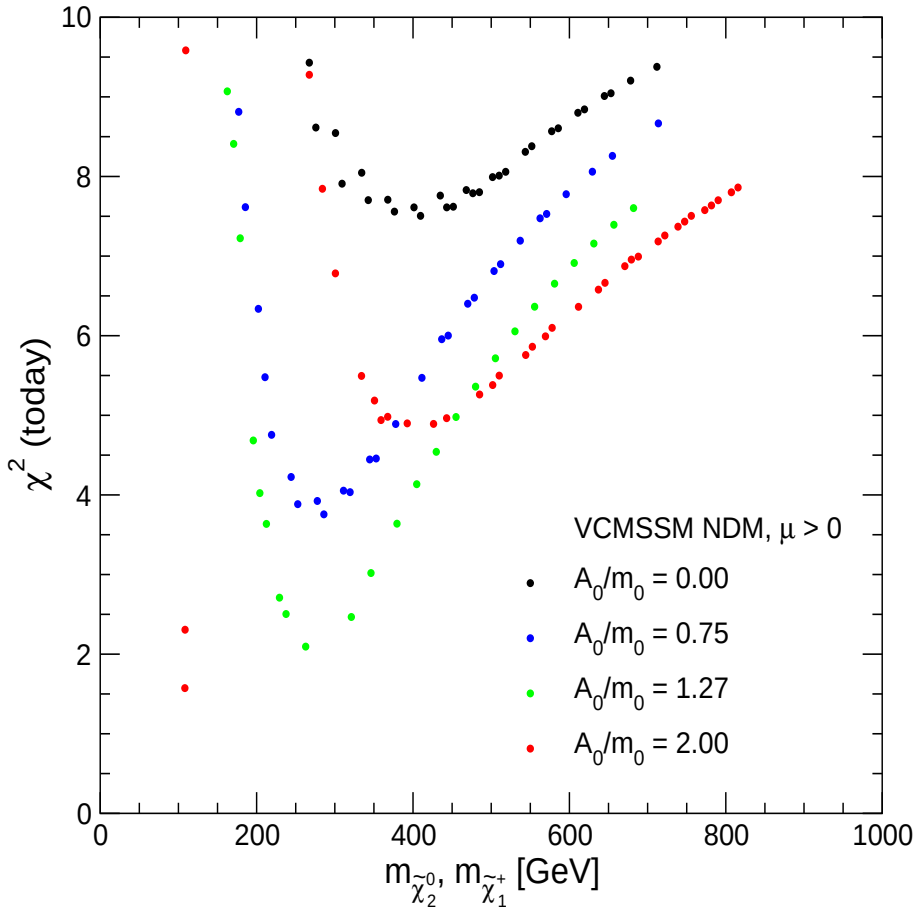
Results (III): VCMSSM

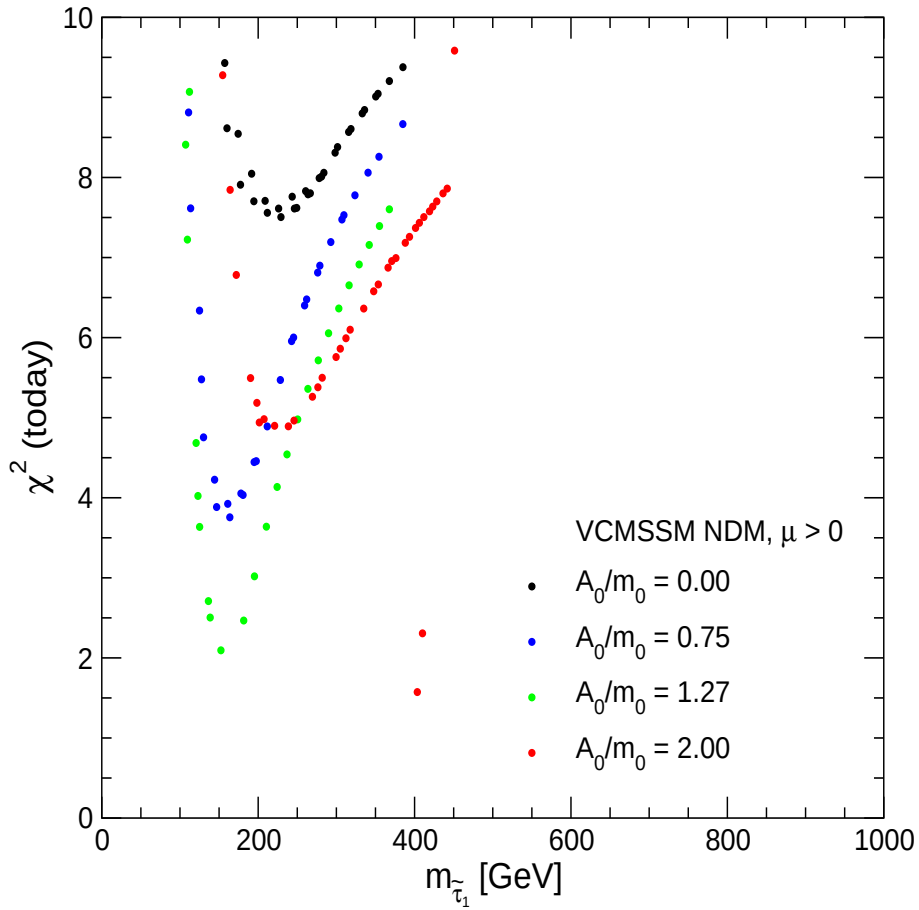
VCMSSM: scan over full parameter space



$\Rightarrow A_0/m_0 = 3/4, 3 - \sqrt{3}$ and Higgs pole favored

VCMSSM: scan over full parameter space



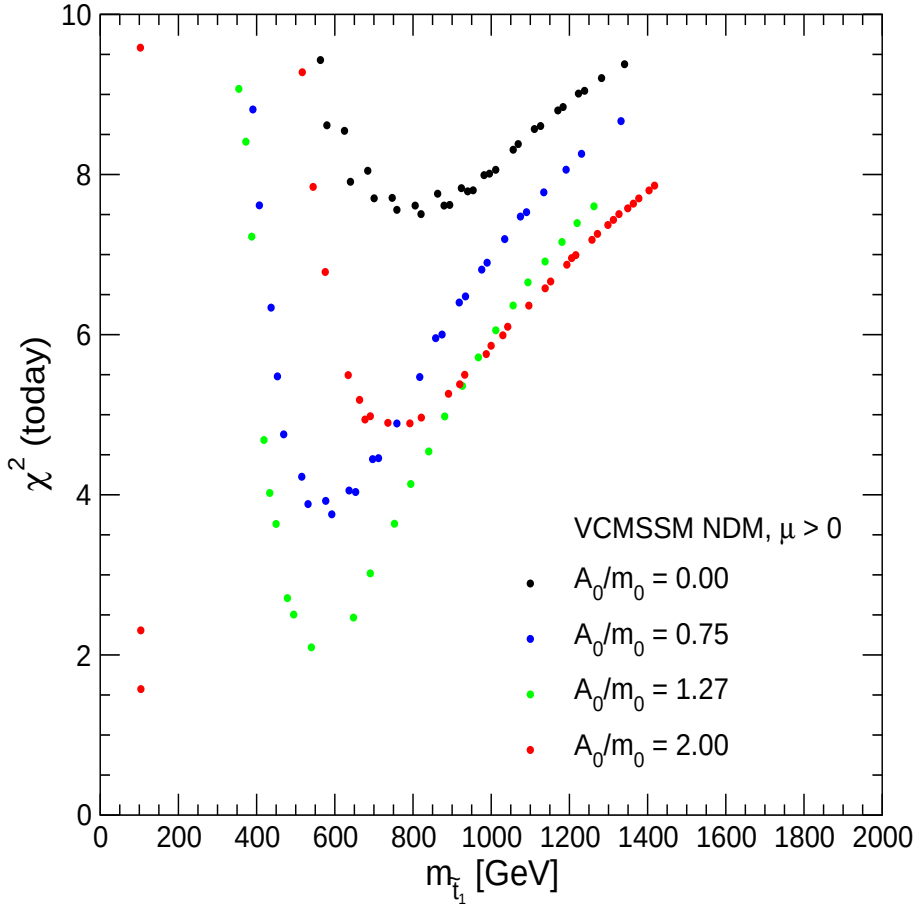


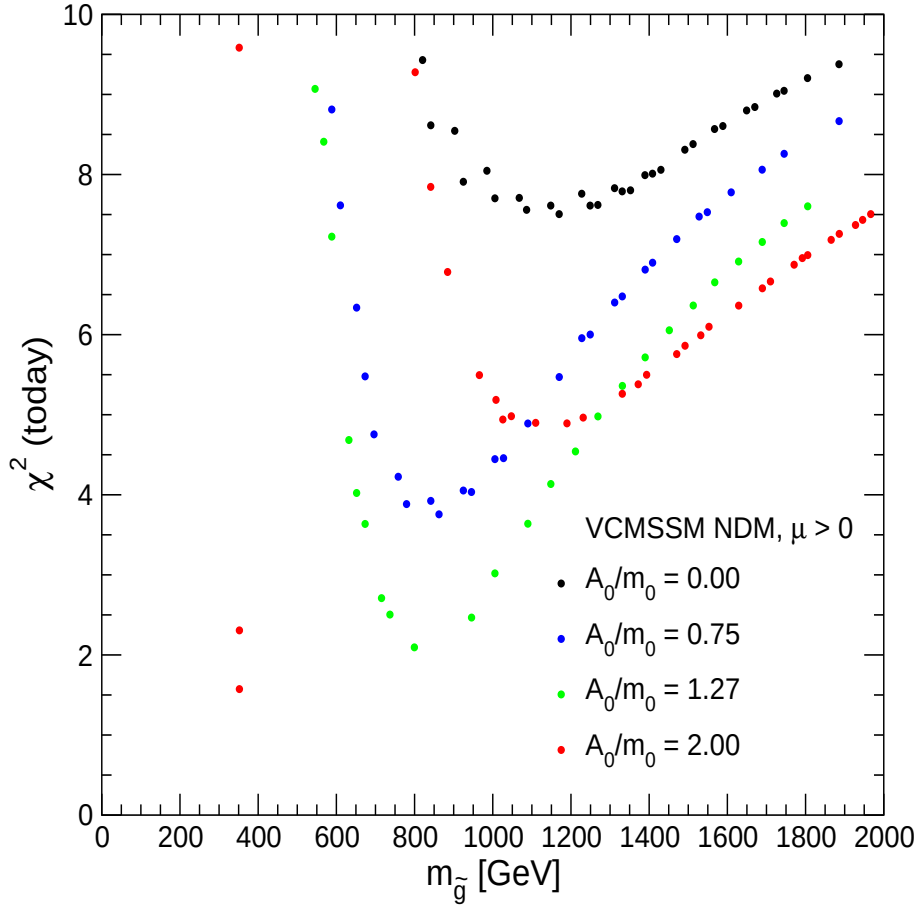
⇒ $A_0/m_0 = 3/4, 3 - \sqrt{3}$ and Higgs pole favored

⇒ sleptons, charginos, neutralinos (partially) in reach for ILC(1000)

⇒ even chances for the Tevatron and good prospects for LHC

VCMSSM: scan over full parameter space





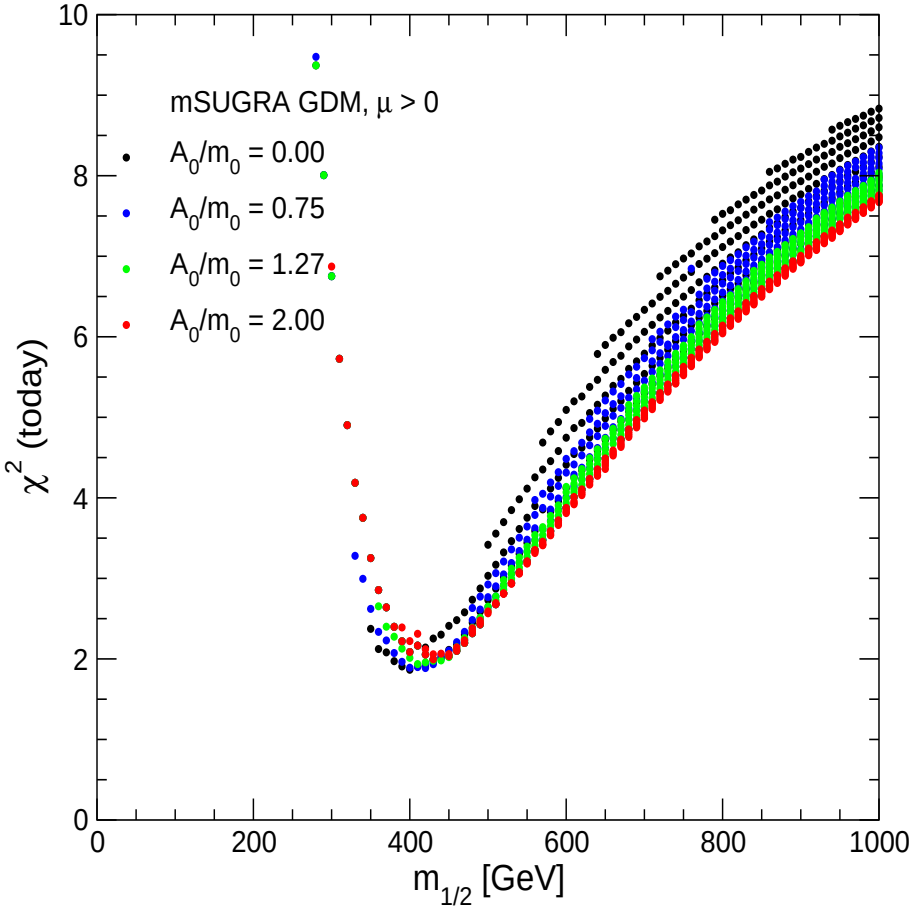
$\Rightarrow A_0/m_0 = 3/4, 3 - \sqrt{3}$ and Higgs pole favored

$\Rightarrow$ stops and gluinos out of reach for ILC, except for Higgs pole

$\Rightarrow$ LHC can cover the whole parameter space

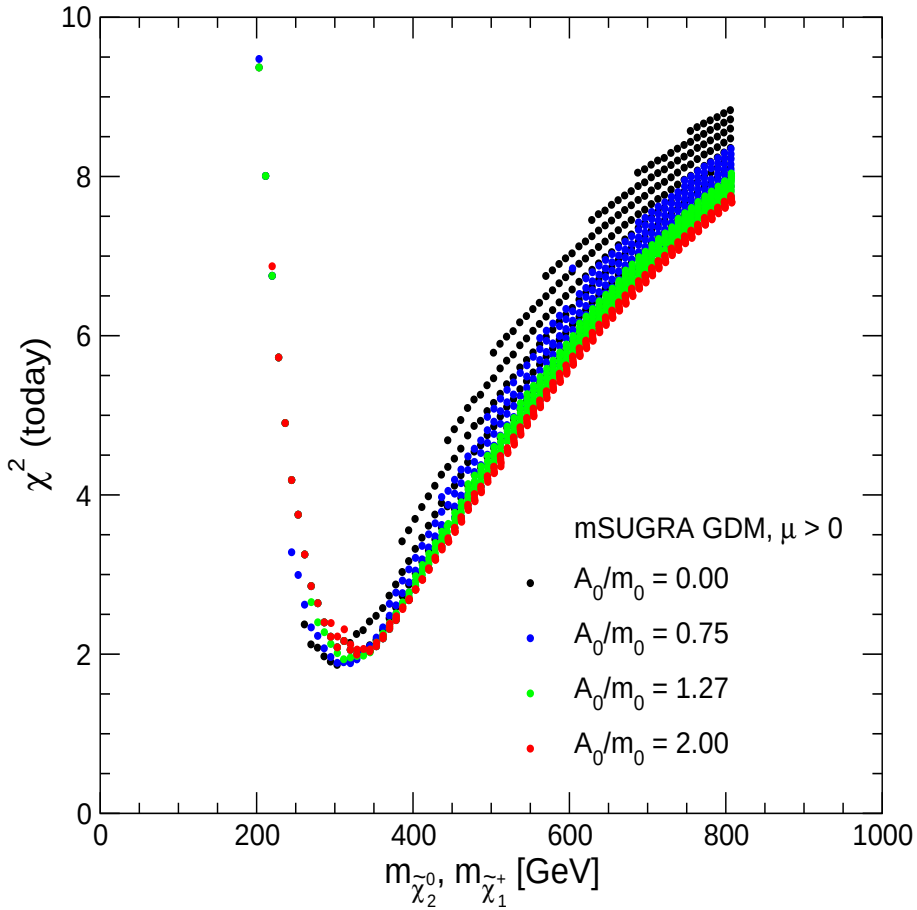
Results (IV): GDM (mSUGRA)

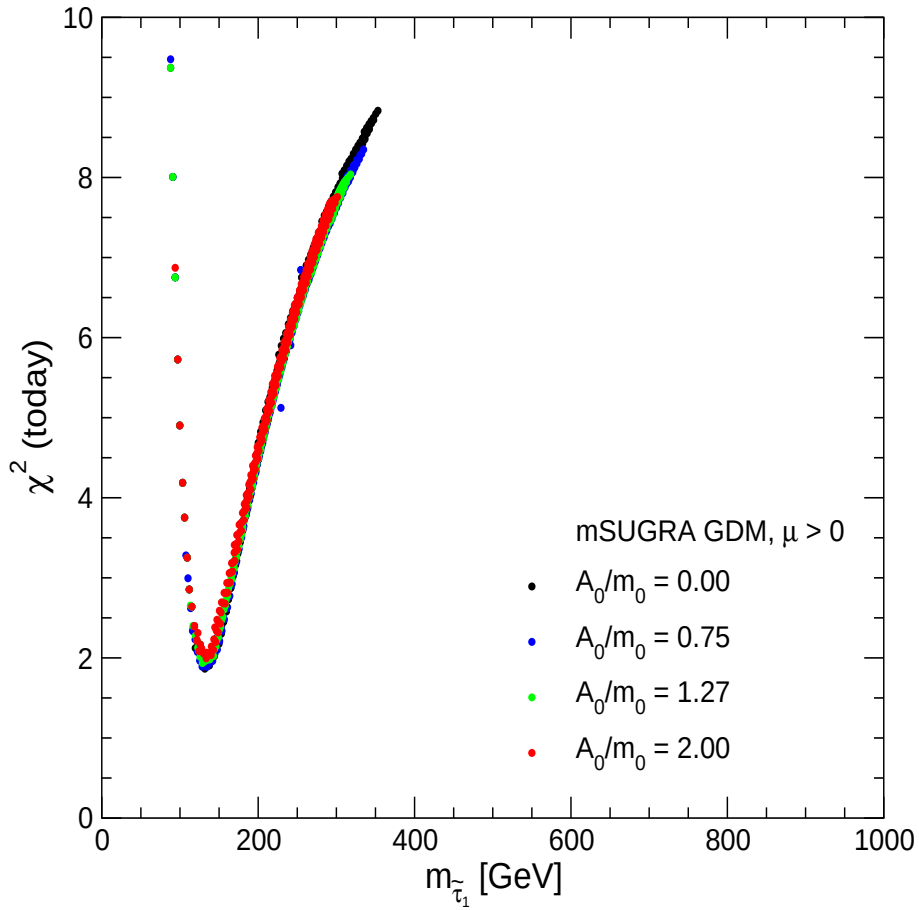
GDM (mSUGRA): scan over full parameter space



$\Rightarrow$ all A_0/m_0 values similarly good

GDM (mSUGRA): scan over full parameter space



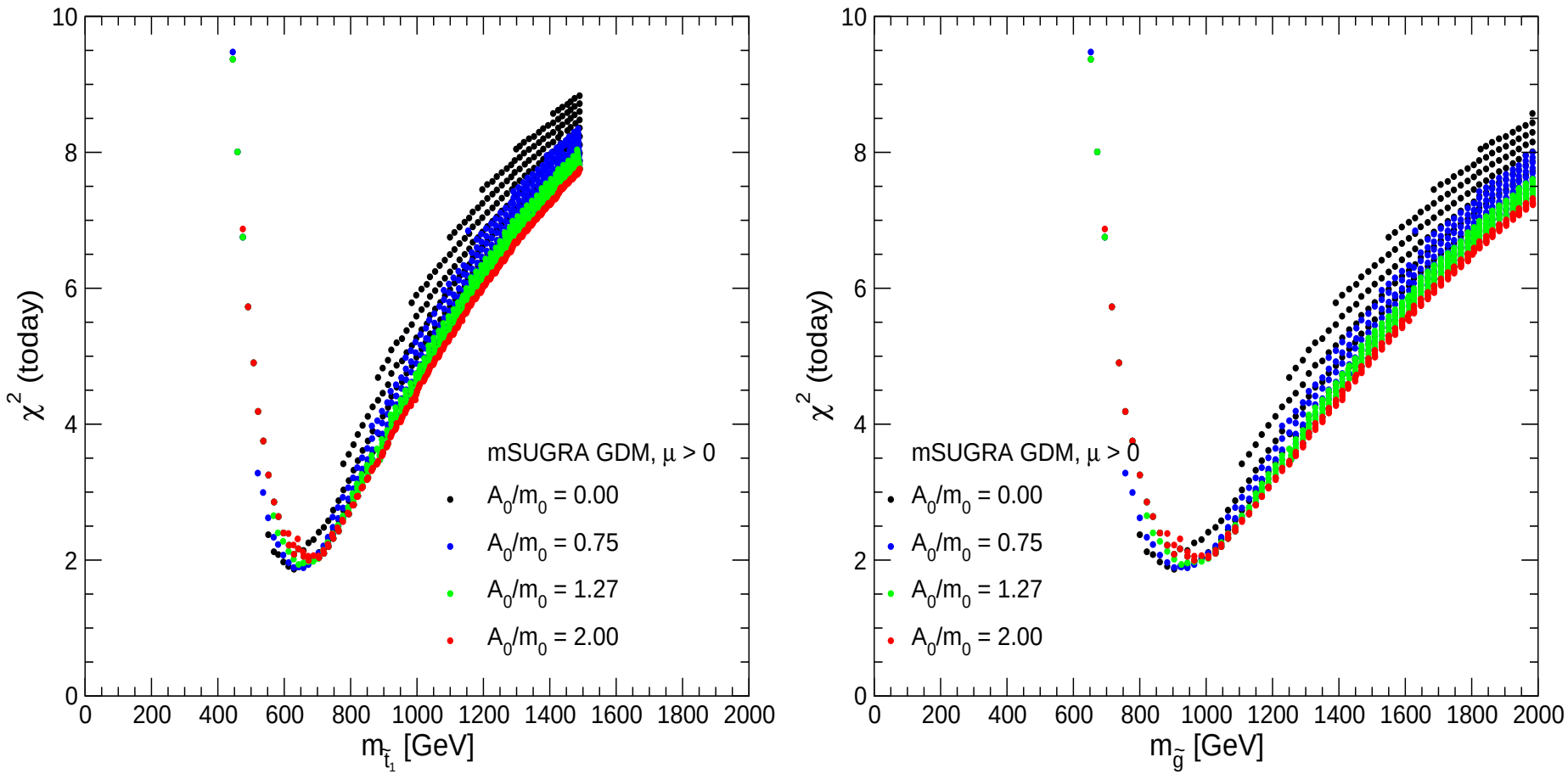


⇒ all A_0/m_0 values similarly good

⇒ sleptons, charginos, neutralinos in reach for ILC(1000) and LHC

⇒ even chances for the Tevatron

GDM (mSUGRA): scan over full parameter space



⇒ all A_0/m_0 values similarly good

⇒ stops and gluinos only in reach for the LHC