The Top Quark: From Theory to Experiment

Sven Heinemeyer, IFCA (CSIC)/Zaragoza

Elba, 06/2006

1. Introduction
2. Electroweak Precision Physics
3. Higgs Physics in the MSSM
4. Renormalization Group Running and Dark Matter
5. Conclusions $\oplus$ theorist's wish list

1. Introduction

Top-quark mass is a fundamental parameter of the electroweak theory

By far the largest quark mass,
largest mass of all known fundamental particles

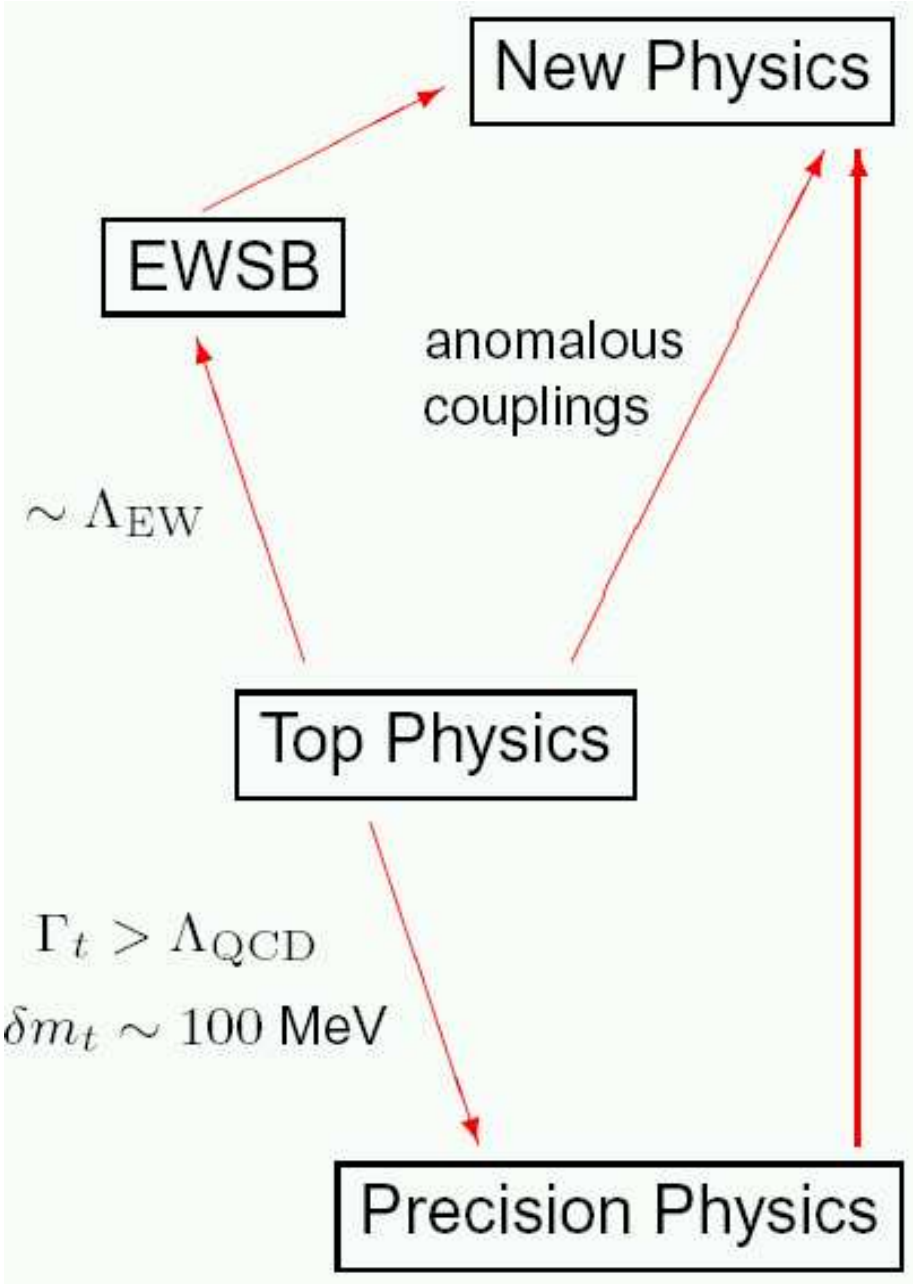
Window to new physics?

Large coupling to the Higgs boson; physics of flavor;
prediction of m_t from underlying theory?

Radiative corrections

$\Rightarrow$ non-decoupling effects proportional to powers of m_t

$\Rightarrow$ Need to know m_t very precisely in order to have sensitivity to effects of new physics



EWSB: just a heavy quark?
special role for t in EWSB?
strong constraint on any model

Precision physics:
$\delta m_t^{\rm exp}$ leading parametric uncertainty
$\rightarrow$ could obscure new physics

SUSY: m_t crucial input parameter
drives SSB/unification

Little Higgs: heavier top

Tevatron: "rough" measurements
of mass, couplings, BRs

LHC: the same (but better!?)

ILC: high precision of everything

$\Rightarrow$ need good knowledge now to prepare
for LHC/ILC

What is the top mass?

Particle masses are not observables
one can only measure cross sections, decay rates, ...

Additional problem for the top mass:

what is the mass of a colored object?

Top pole mass is not IR safe (affected by large long-distance contributions), cannot be determined to better than $\mathcal{O}(\Lambda_{\rm QCD})$

Measurement of m_t:

- At Tevatron, LHC:
  kinematic reconstruction, fit to invariant mass distribution
  $\Rightarrow$ "pole" mass

- At the ILC:
  mainly from threshold behavior $\Rightarrow$ threshold mass

Experimental accuracy of m_t:

Measurement $\Leftrightarrow$ comparison data from Monte Carlo

$\rightarrow$ you measure the mass that is implemented in your MC

$\Rightarrow$ measured mass is not strictly model independent

Situation at the Tevatron:

$$\boxed{\begin{array}{lcll} \delta m_t & = & 2.3 \text{ GeV} & (\text{Tevatron, today}) \\ \delta m_t & \approx & 1.2 \text{ GeV} & (\text{Tevatron, future?}) \end{array}}$$

Situation at the LHC:

$$\boxed{\delta m_t \approx 1 \text{ GeV} \quad (\text{LHC})}$$

Situation at the ILC:

$$\boxed{\delta m_t \lesssim 100 \text{ MeV} \quad (\text{ILC})}$$

$\Rightarrow$ show physics examples in which the top quark is crucial

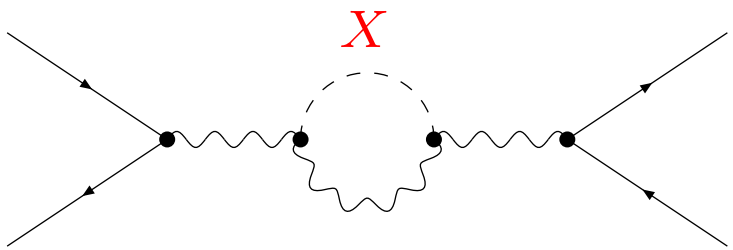
2. Electroweak Precision Observables

Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
M_W, $\sin^2\theta_{\text{eff}}, a_\mu$, ...	$\leftrightarrow$	SM, MSSM , ...

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections



$\Downarrow$

Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

Precision observables: M_W, $\sin^2\theta_{\text{eff}}$, m_h, $(g-2)_\mu$, b physics, ...

Theoretical prediction for M_W in terms of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}\left(\frac{1}{1-\Delta r}\right)$$

$\Updownarrow$

loop corrections

Theoretical prediction for the effective mixing angle:

$$\sin^2\theta_{\text{eff}}=\frac{1}{4\,|Q_f|}\left(1-\frac{\text{Re}\,g_V^f}{\text{Re}\,g_A^f}\right)$$

Higher order contributions:

$$g_V^f\to g_V^f+\Delta g_V^f,\quad g_A^f\to g_A^f+\Delta g_A^f$$

Leading m_t contributions:

One-loop result for M_W in the SM:

[*A. Sirlin '80*] , [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \underset{\sim \log \frac{M_Z}{m_f}}{\Delta\alpha} - \underset{\sim m_t^2}{\frac{c_W^2}{s_W^2}\Delta\rho} + \Delta r_{\text{rem}}(M_H)$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

Leading m_t contribution to $\sin^2\theta_{\text{eff}}$:

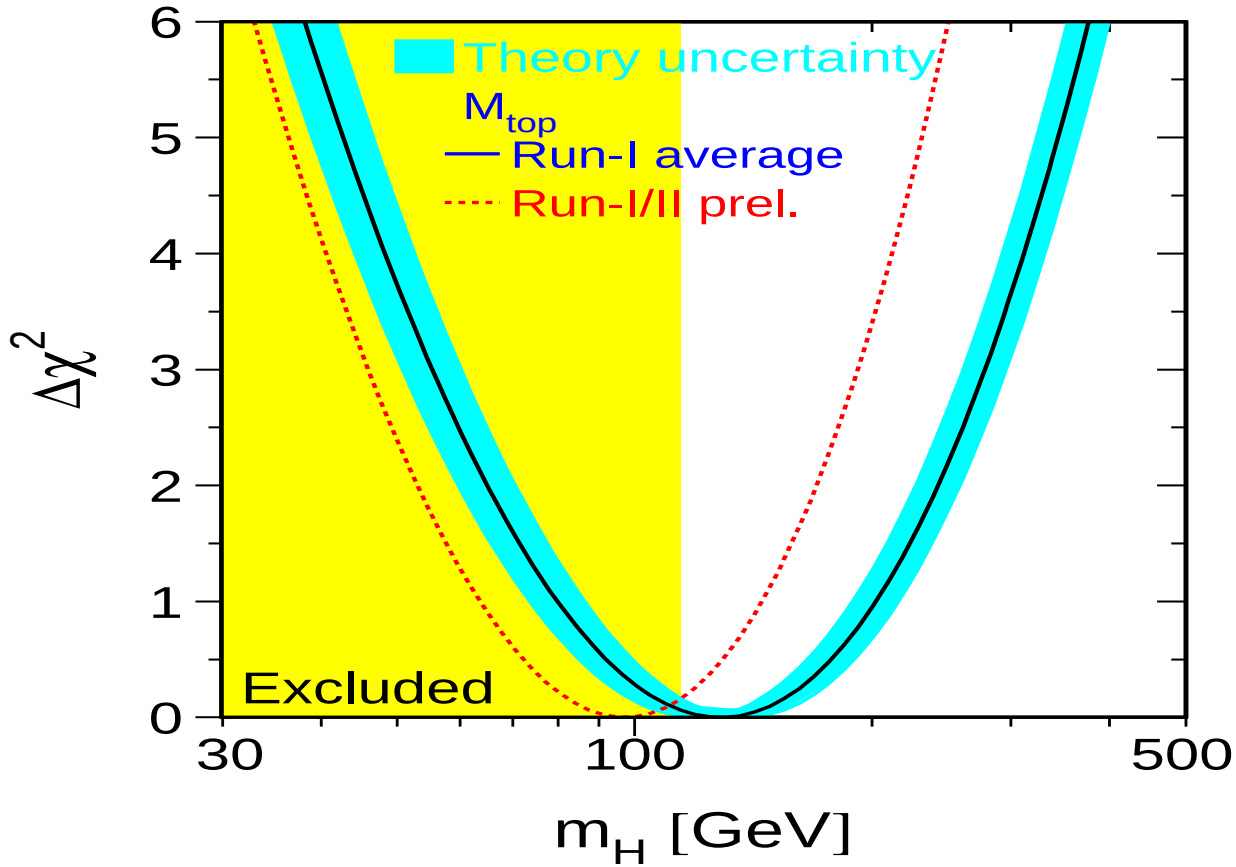
$$\Delta \sin^2\theta_{\text{eff}} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2}\Delta\rho \sim m_t^2$$

Example: effect of shift in m_t value on global fit to all EWPO data:

Old m_t value:
$m_t = 178.0 \pm 4.3$ GeV

"Newer" m_t value:
$m_t = 174.3 \pm 3.4$ GeV

[*Tevatron EWWG '05*]



[*LEPEWWG '05*]

$\Rightarrow$ best fit value of M_H shifted by more than 20 GeV

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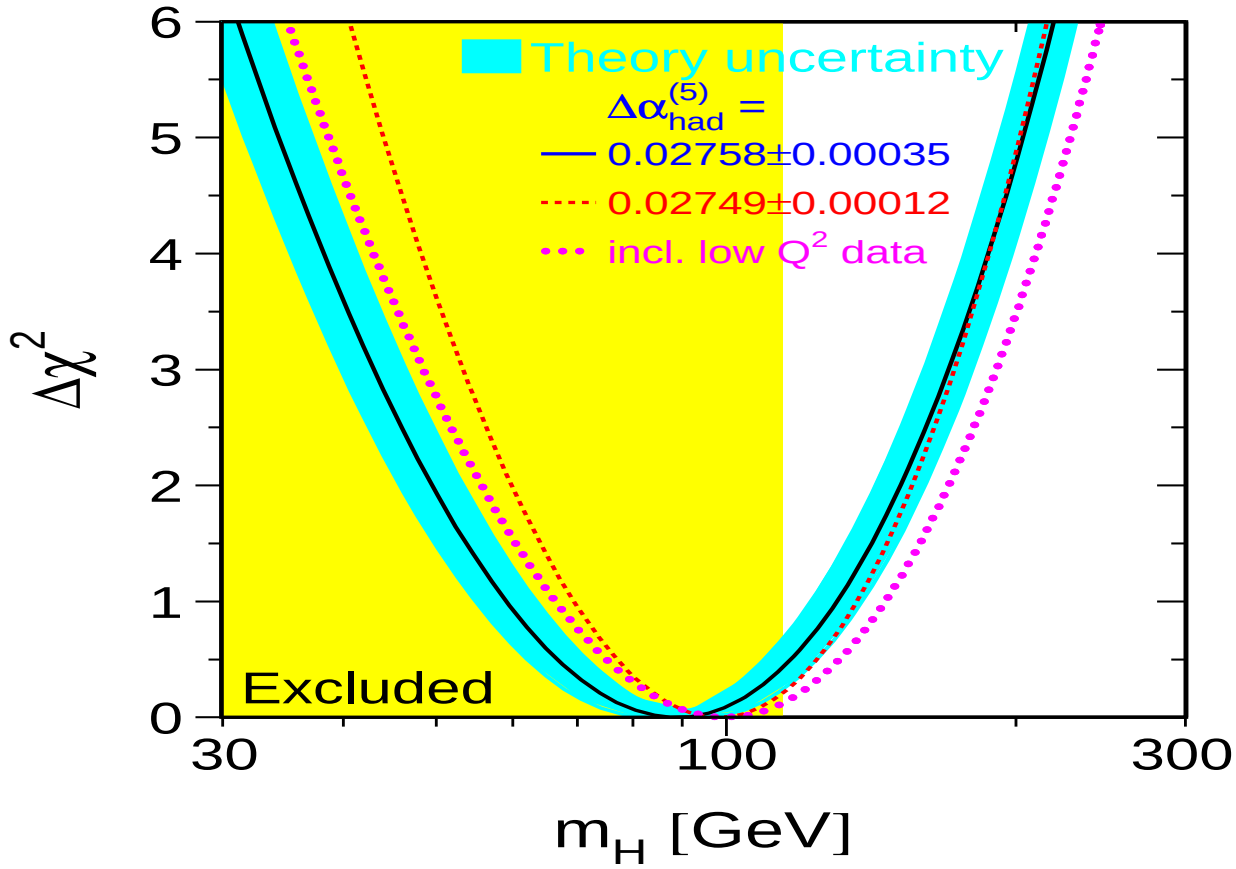
Current m_t value:
$m_t = 172.5 \pm 2.3$ GeV

[*Tevatron EWWG '06*]



[*LEPEWWG '06*]

$\Rightarrow$ current best fit value: $M_H = 89^{+42}_{-30}$ GeV, $M_H < 175$ GeV at 95% C.L.

Experimental errors:

	today	Tevatron	LHC	ILC	GigaZ
$\delta \sin^2\theta_{\text{eff}} (\times 10^5)$	17	17	17	–	1.3
δM_W [MeV]	34	20	15	10	7

Relevant SM parametric errors: $\delta(\Delta\alpha_{\text{had}}) = 5 \times 10^{-5}$, $\delta M_Z = 2.1$ MeV

	$\delta m_t = 2$	$\delta m_t = 1$	$\delta m_t = 0.1$	$\delta(\Delta\alpha_{\text{had}})$	δM_Z
$\delta \sin^2\theta_{\text{eff}}$ [10^{-5}]	6	3	0.3	1.8	1.4
ΔM_W [MeV]	12	6	1	1	2.5

To keep the parametric error induced by m_t at/below the level of other uncertainties:

$\Rightarrow$ $\delta m_t \lesssim 0.2$ GeV for M_W, $\delta m_t \lesssim 0.5$ GeV for $\sin^2\theta_{\text{eff}}$

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$\Rightarrow$ Tevatron can contribute substantially here

The most prominent example: m_t vs. M_W

Several steps:

1. Evaluate theory prediction in the SM:
   - take an m_t value
   - vary last free parameter, M_H^{SM}, in a reasonable range: 114 . . . 400 GeV
   - evaluate M_W prediction

   $\Rightarrow$ Red band in the m_t–M_W plane

2. Evaluate theory prediction in the MSSM:
   - Scan over all MSSM parameters independently
   - take an m_t value
   - evaluate the Higgs boson mass (not a free parameter)
   - evaluate M_W prediction [*hep-ph/0604147*]

   $\Rightarrow$ Green band in the m_t–M_W plane

   $\Rightarrow$ small blue overlap area of both models in m_t–M_W plane

   $\rightarrow$ MSSM behaves SM-like: all SUSY masses heavy

   $\rightarrow$ SM behaves MSSM-like: $M_H^{\text{SM}} \lesssim 135$ GeV

3. Compare with experimental values

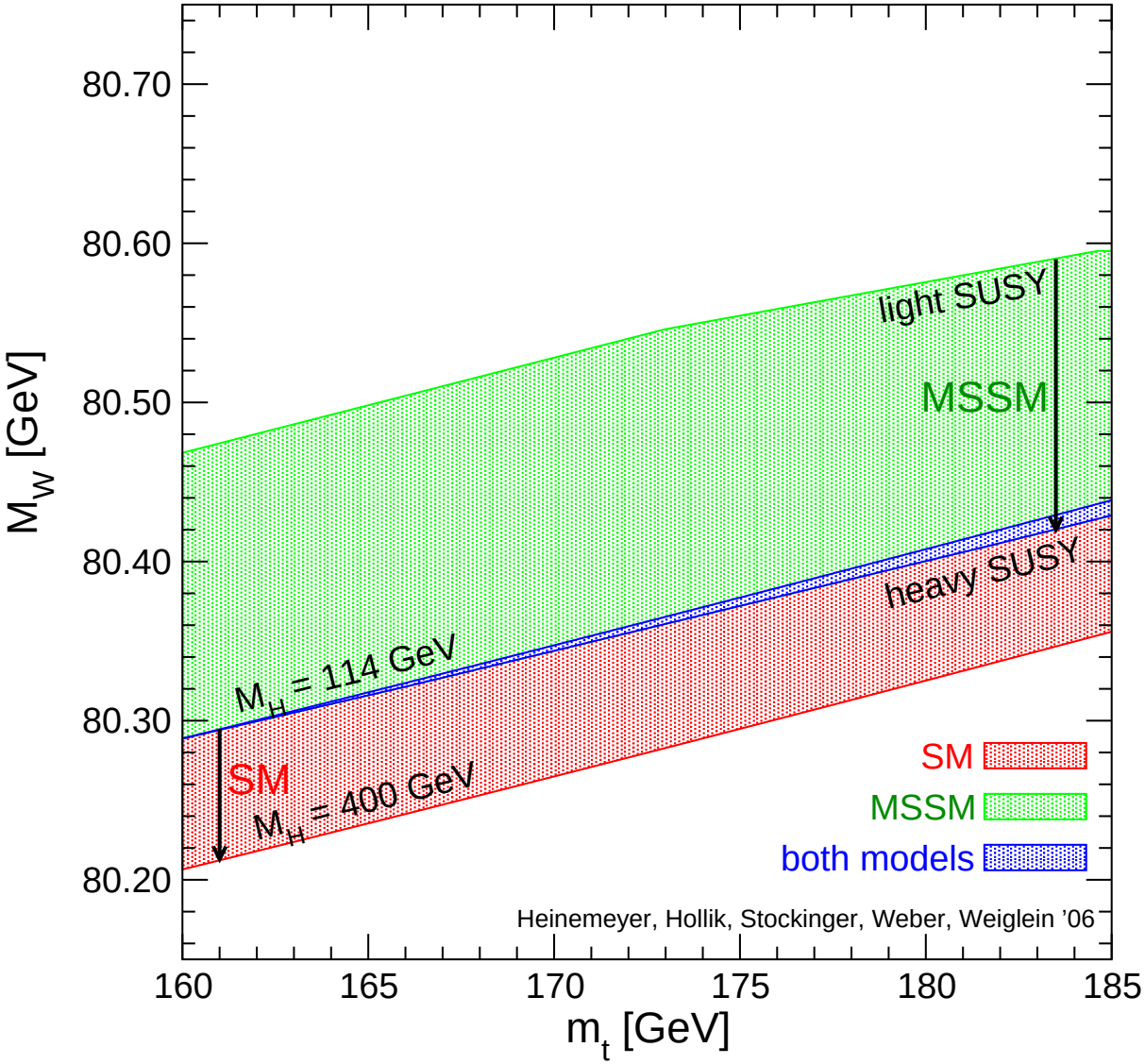
4. Read off preferred parameter space

   SM: preferred Higgs boson mass

   MSSM: preferred SUSY masses

Prediction for M_W in the SM and the MSSM :

[*S.H., W. Hollik, D. Stockinger, A.M. Weber, G. Weiglein '06*]



MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of M_H^{SM}

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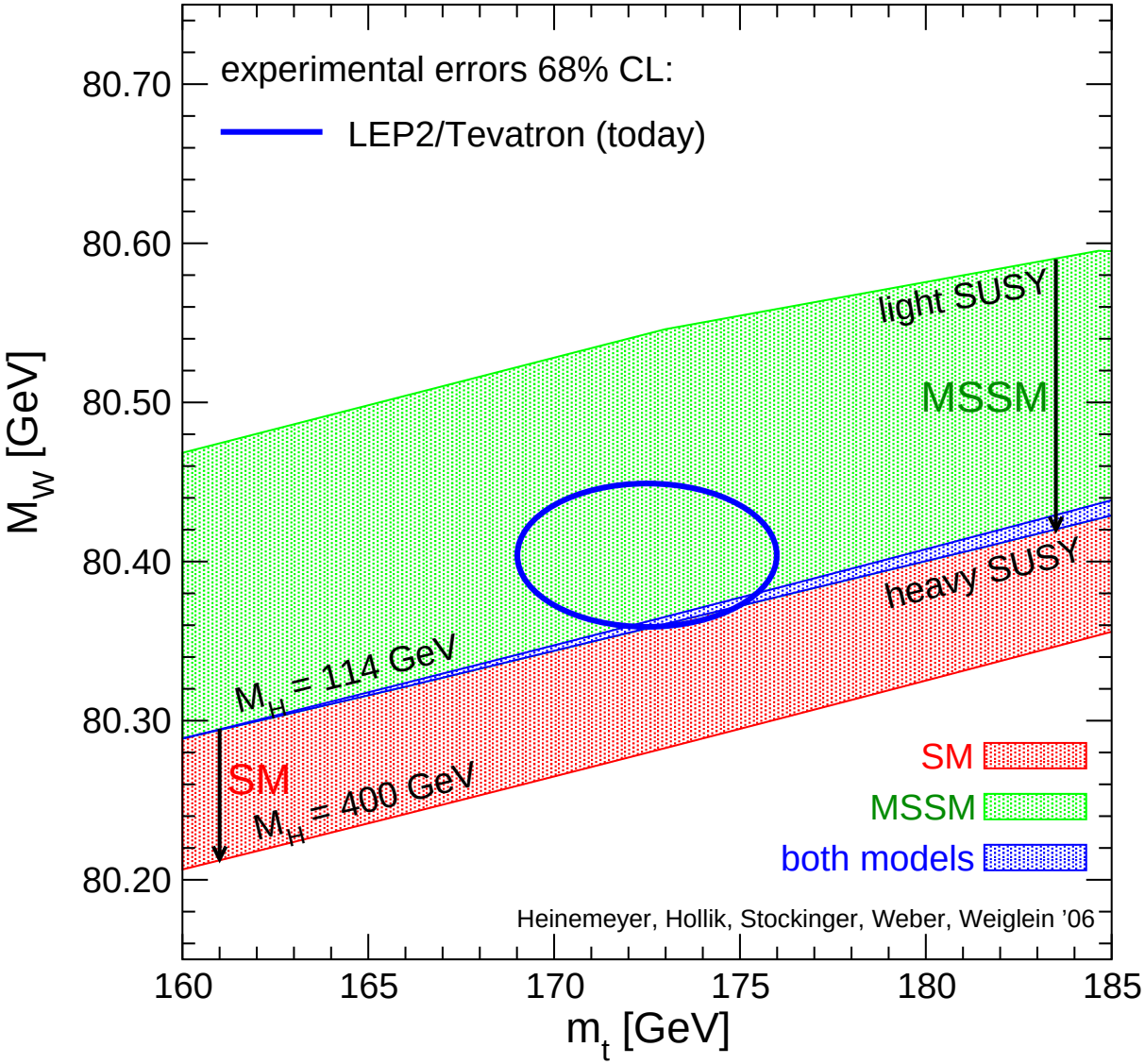

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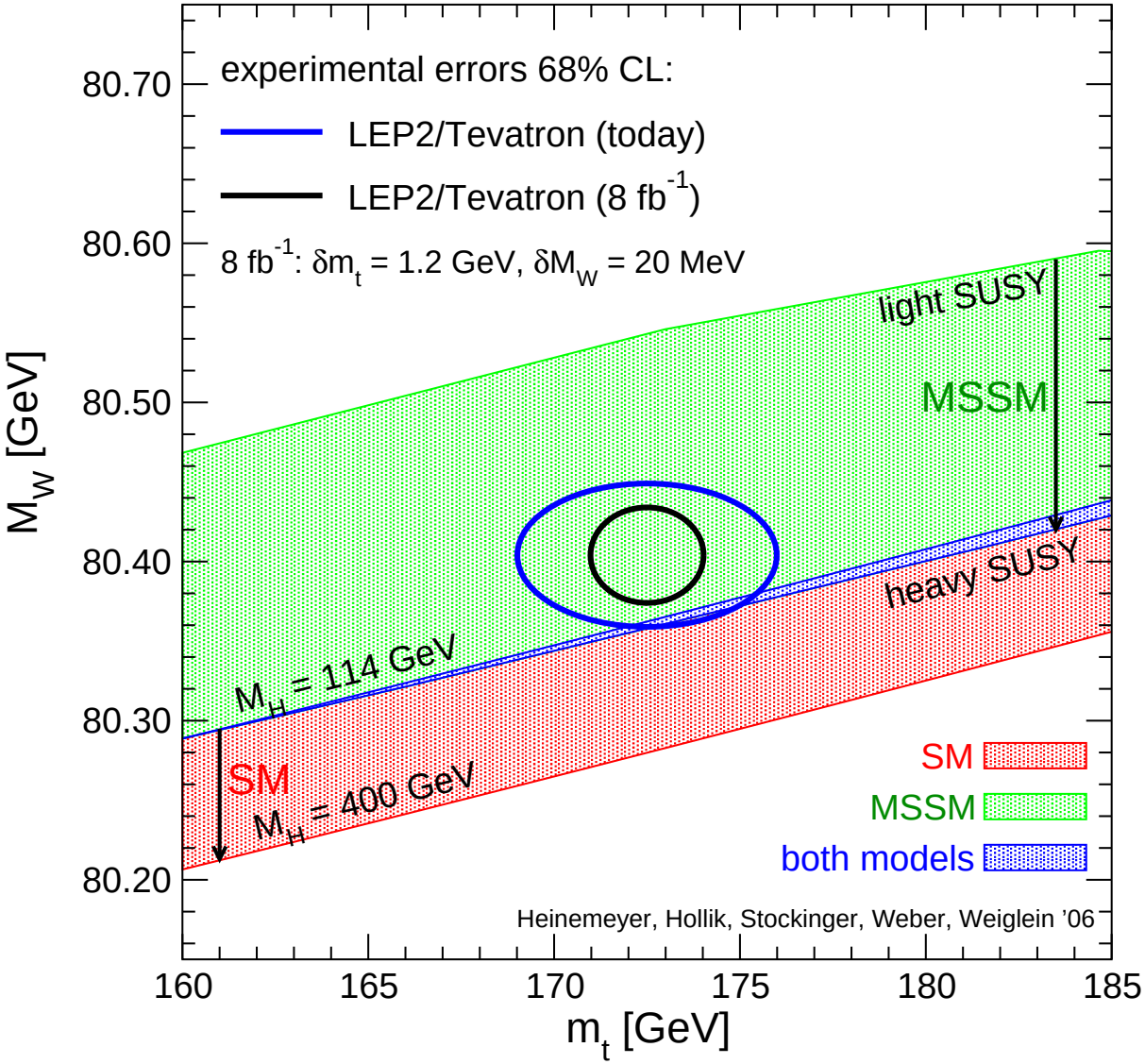

MSSM band:

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SM band:

variation of M_H^{SM}

Determination of preferred SUSY masses:

→ focus on mSUGRA/CMSSM

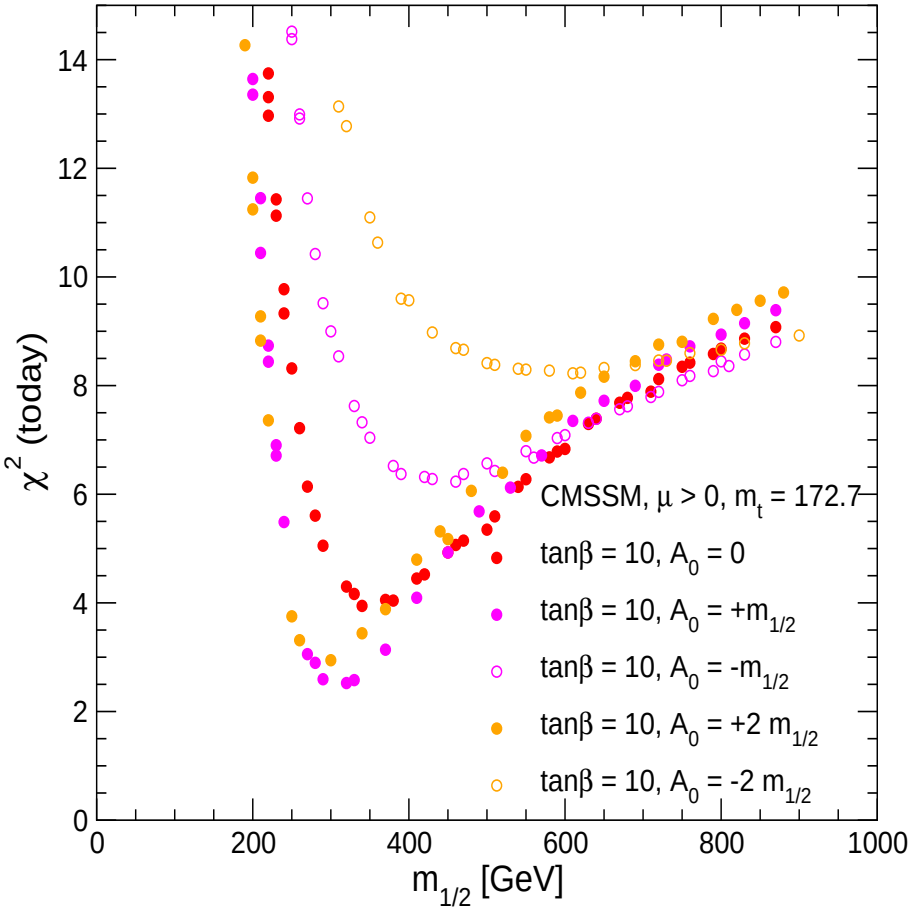
Procedure:

1. use as many as possible precision observables:
   M_W, $\sin^2\theta_{\text{eff}}$, $(g-2)_\mu$, BR$(b \to s\gamma)$, M_h, CDM
   (CDM allows only thin strips in the CMSSM parameter space)
   → interested in results including B-physics? Ask me for my back-up!
2. Scan over (remaining) parameter space:
   – fixed $\tan\beta = 10, 50$
   – fixed $A_0/m_0 = 0, \pm 1, \pm 2$
3. Perform χ^2 fit
   m_t plays an important role
4. Find preferred values for masses
   ⇒ Tevatron reach (see talk by S.H. in SUSY session :-)

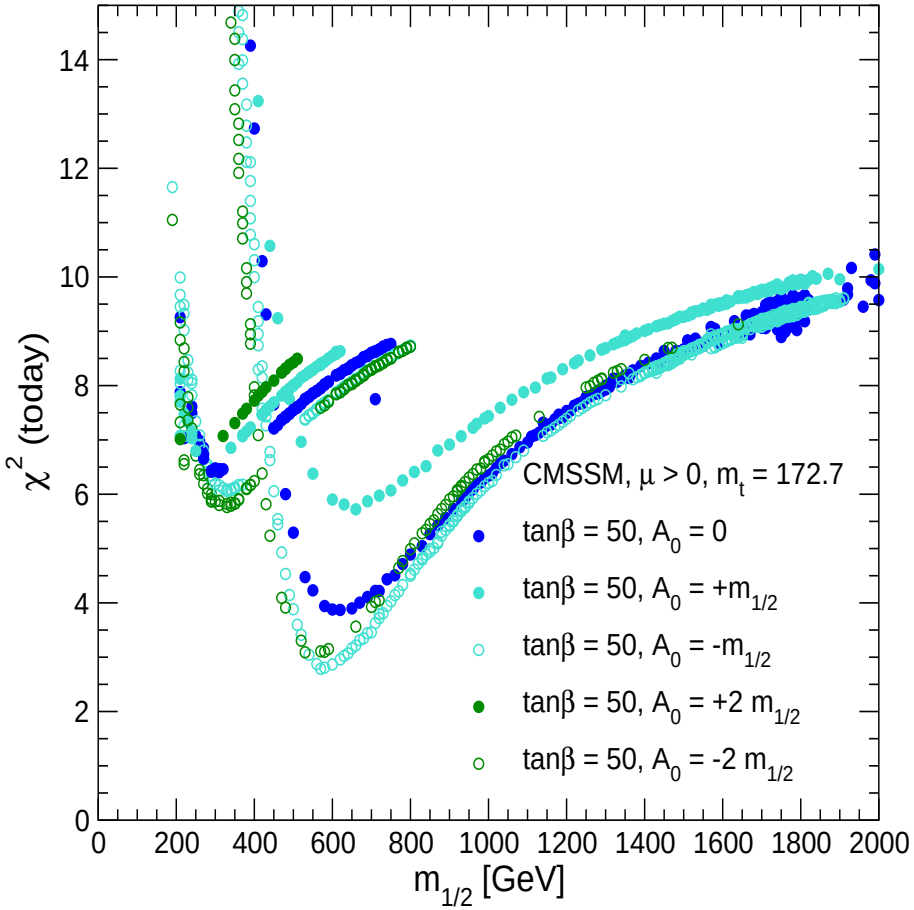
Result for total χ^2:

[*J. Ellis, S.H., K. Olive, G. Weiglein '06*]

$\tan\beta = 10$



$\tan\beta = 50$



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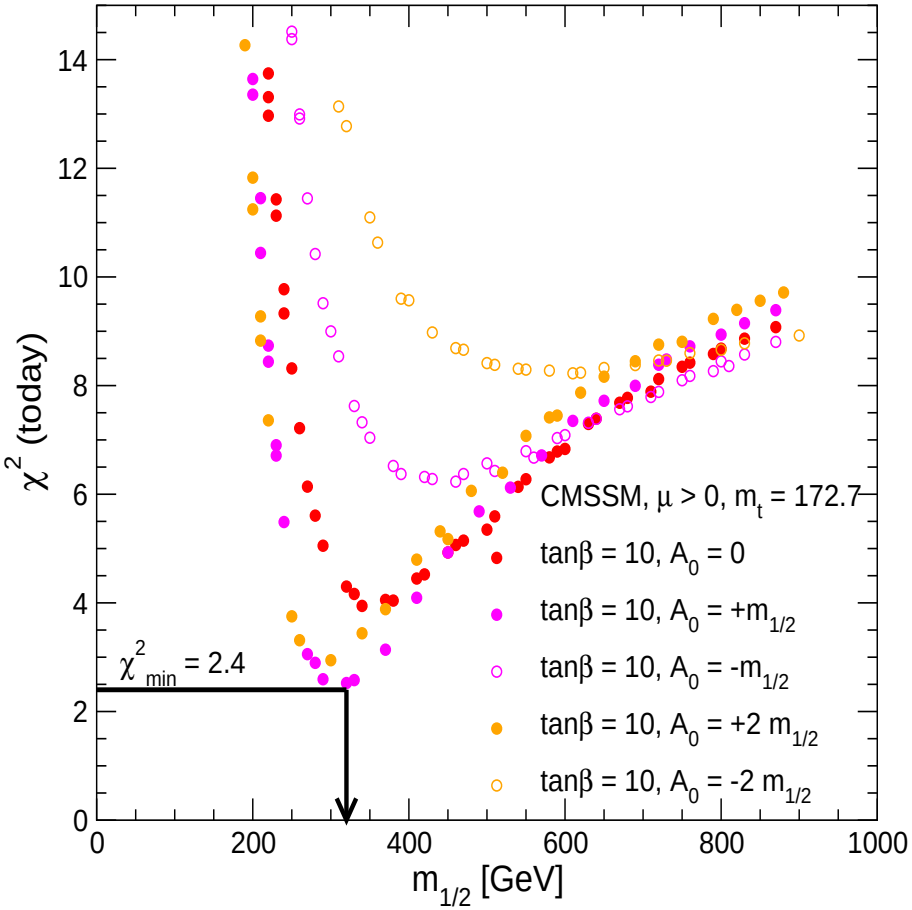

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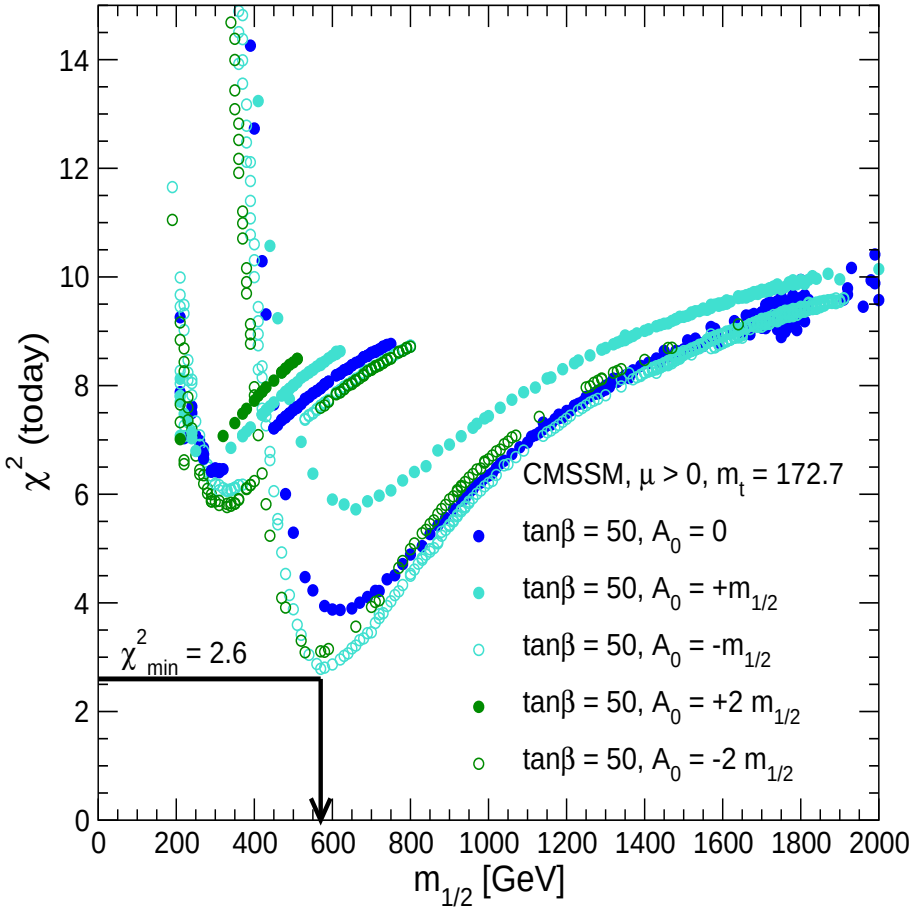

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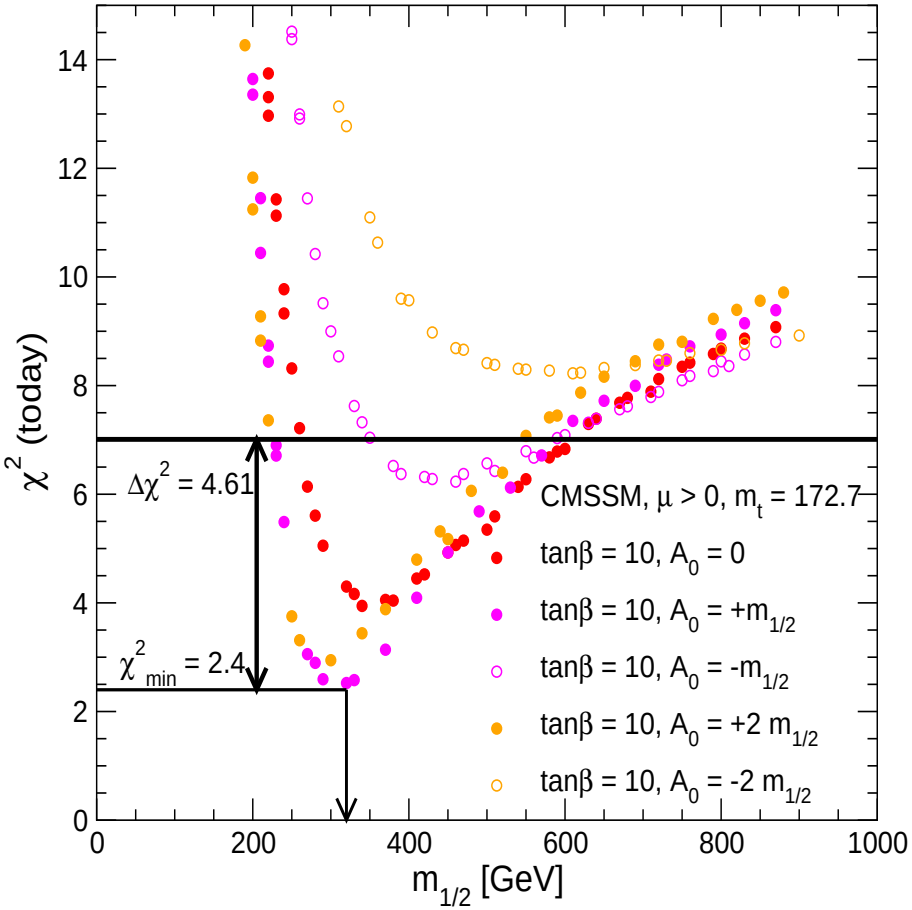

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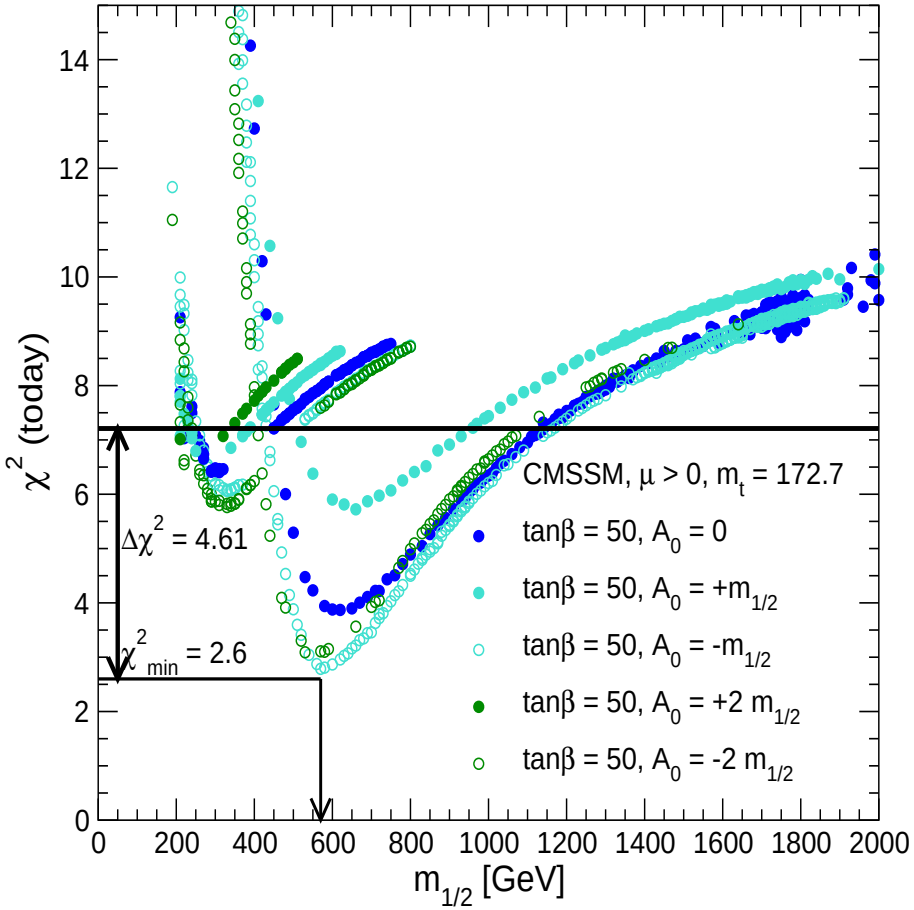

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$\Rightarrow$ upper bound on $m_{1/2}$

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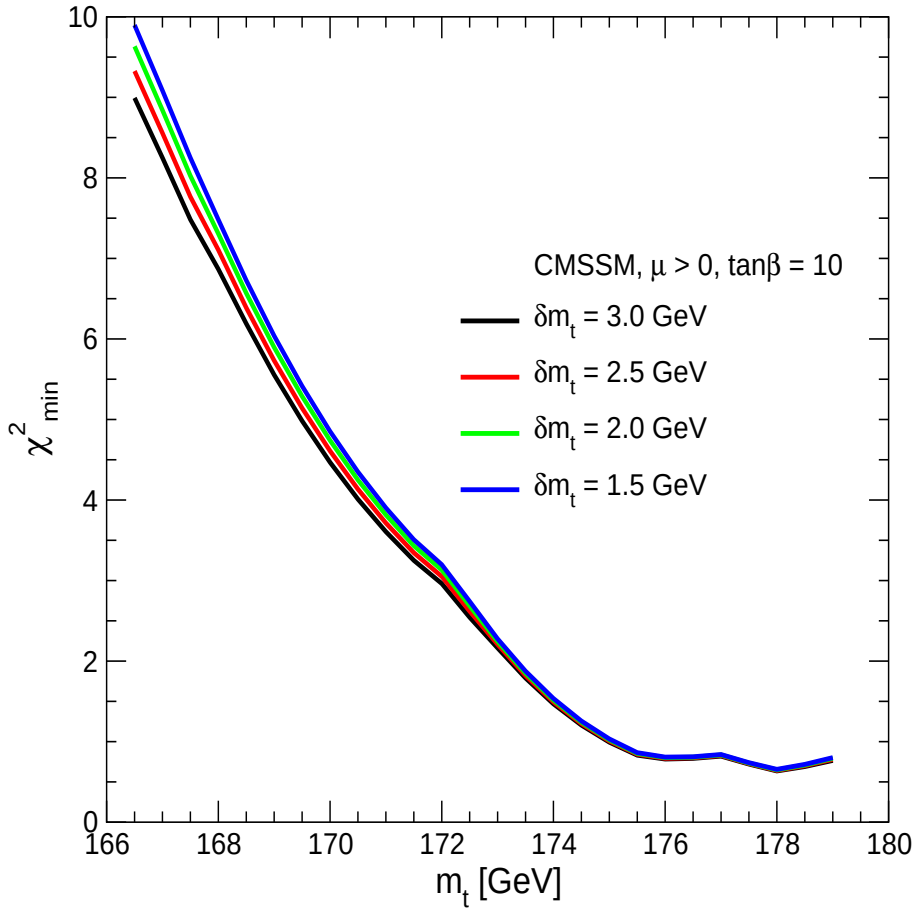
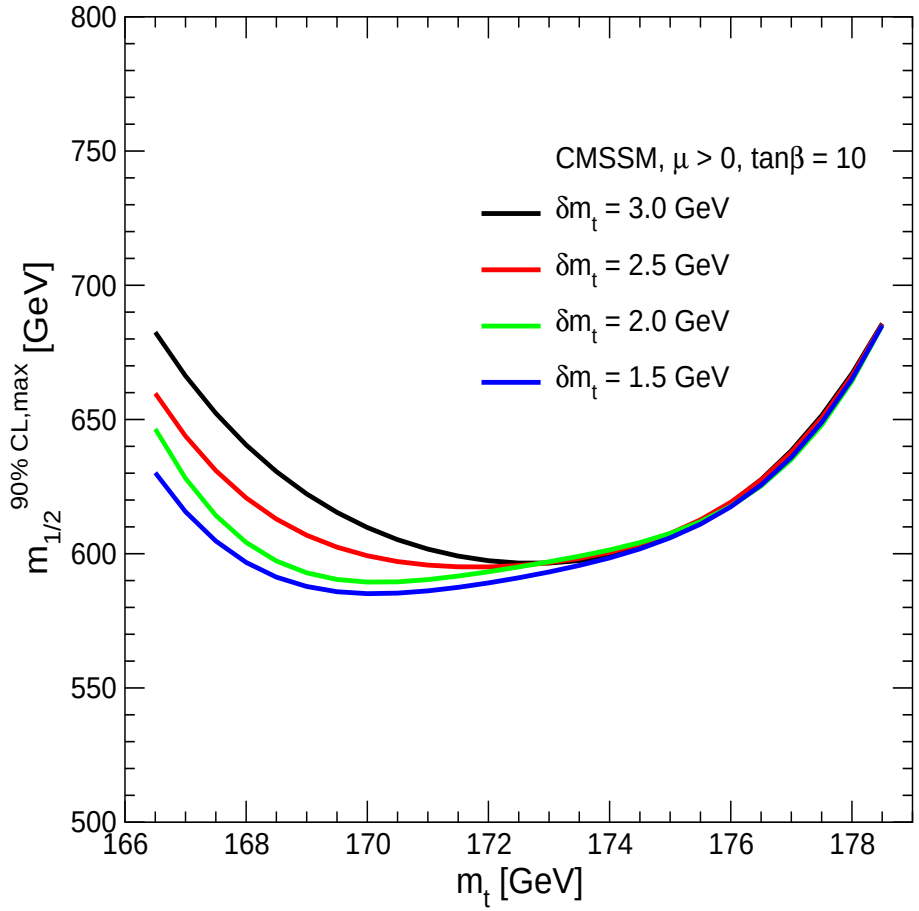
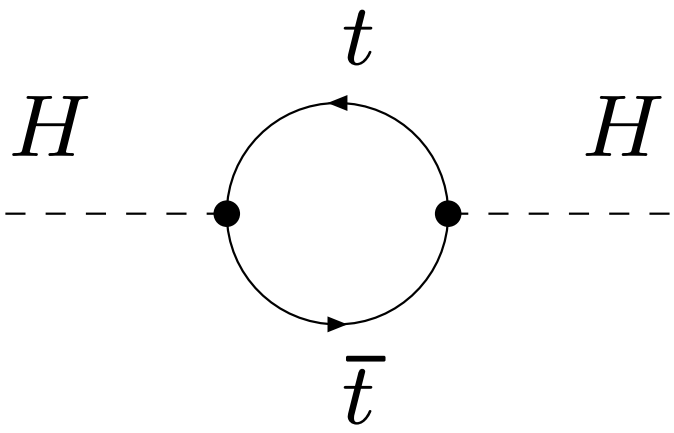
influence of m_t: [*J. Ellis, S.H., K. Olive, G. Weiglein '06*]





3. Higgs Physics in the MSSM

Nearly any model: large coupling of the Higgs to the top quark:



$\Rightarrow$ one-loop corrections $\Delta m_h^2 \sim G_\mu m_t^4$

$\Rightarrow$ M_H depends sensitively on m_t in all models where M_H can be predicted (SM: M_H is free parameter)

SUSY as an example: $\Delta m_t \approx \pm 2$ GeV $\Rightarrow$ $\Delta m_h \approx \pm 2$ GeV

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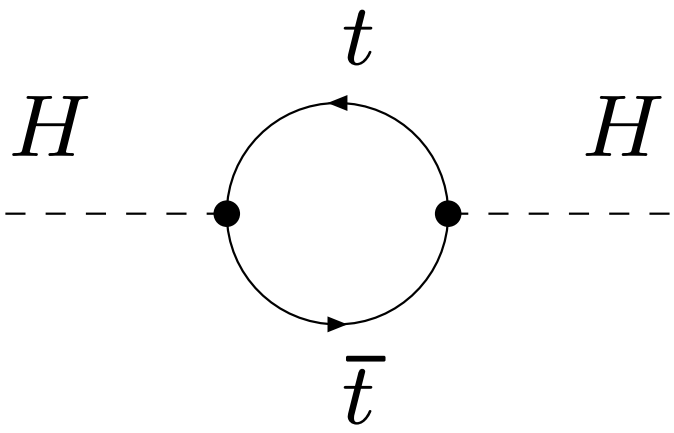

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SUSY as an example: $\Delta m_t \approx \pm 2$ GeV $\Rightarrow$ $\Delta m_h \approx \pm 2$ GeV

$\Rightarrow$ Precision Higgs physics needs precision top physics

MSSM Higgs sector:

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + \frac{\phi_1 + i\chi_1}{\sqrt{2}} \\ \phi_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{\phi_2 + i\chi_2}{\sqrt{2}} \end{pmatrix}$$

$$\begin{aligned} V &= m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ &\quad + \underbrace{\frac{g'^2 + g^2}{8}}_{} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{} |H_1 \bar{H}_2|^2 \end{aligned}$$

gauge couplings, in contrast to SM $\Rightarrow m_h^{\text{tree}} \leq M_Z$

physical states: $h^0, H^0, A^0, H^\pm$ Goldstone bosons: $G^0, G^\pm$

Input parameters: $\tan\beta = \frac{v_2}{v_1}$, $M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$

Large radiative corrections (Yukawa sector, ...):

Yukawa couplings: $\frac{e\, m_t}{2 M_W s_W}$, $\frac{e\, m_t^2}{M_W s_W}$, ...

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$, $\mathcal{O}(100\%)$!

$\tilde{t}$ sector of the MSSM: (scalar partner of the top quark)

Mass matrix for $\tilde{t}_L, \tilde{t}_R$:

$$(\tilde{t}_L, \tilde{t}_R) \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{1t} & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{2t} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

$$\Big\Downarrow \leftarrow \text{Diagonalization}, \theta_{\tilde{t}}$$

$$(\tilde{t}_1, \tilde{t}_2) \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}$$

$m_t X_t = m_t(A_t - \mu \cot\beta)$ $\Rightarrow$ large mixing possible

$\Rightarrow$ Physical parameters: $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$

$\Rightarrow$ Soft SUSY-breaking parameters: $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$, A_t

$\Rightarrow$ m_t plays an important role

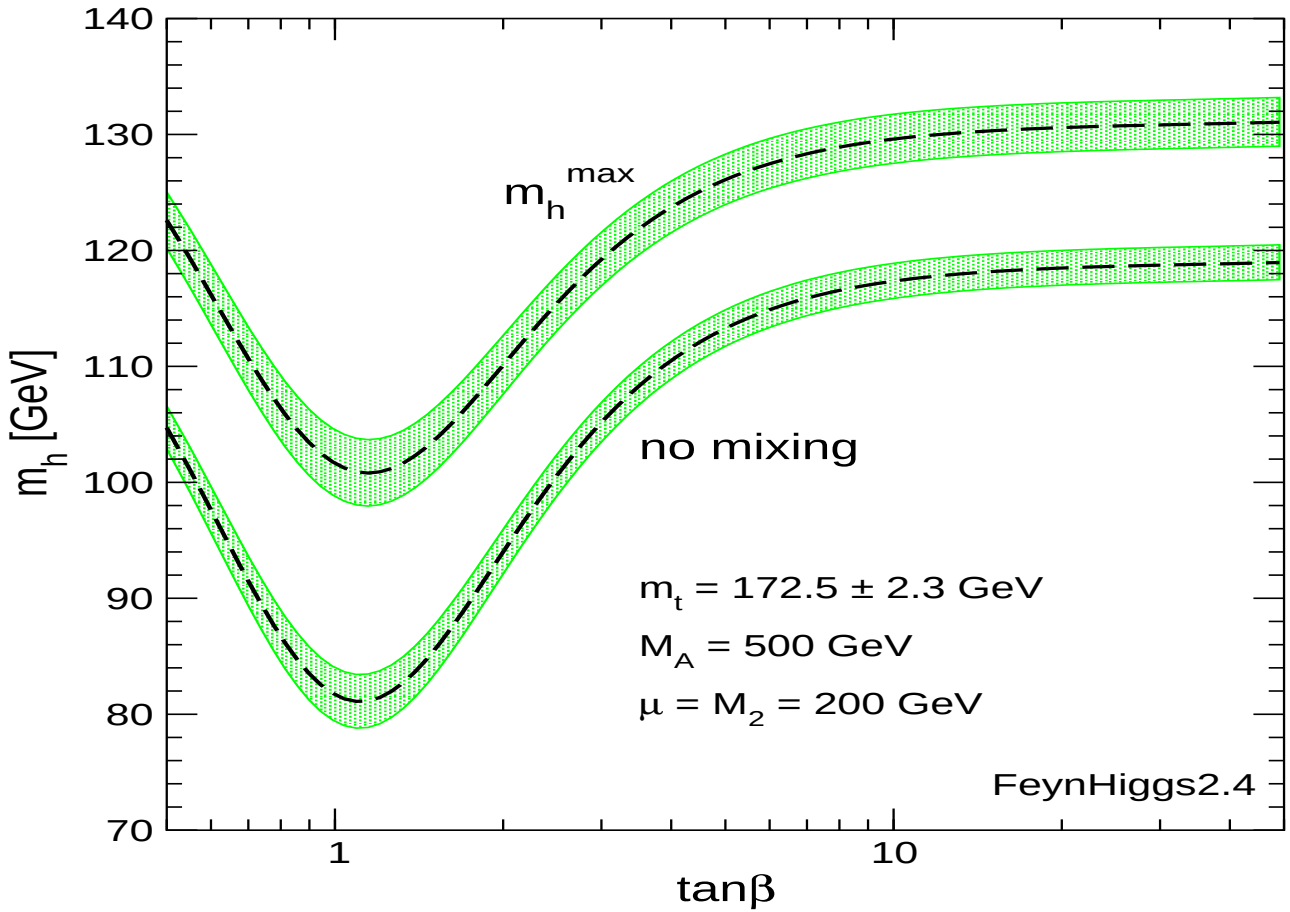
Theory uncertainties in m_h prediction:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

– From unknown higher order corrections $\Rightarrow \Delta m_h \approx 3$ GeV
(if $b/\tilde{b}$ sector is under control [*S.H., W. Hollik, H. Rzehak, G. Weiglein '04*])

– From uncertainties in input parameters
$\Delta m_t \approx \pm 2$ GeV
$\Rightarrow \Delta m_h \approx \pm 2$ GeV

Upper bound:
$m_h \lesssim 135$ GeV



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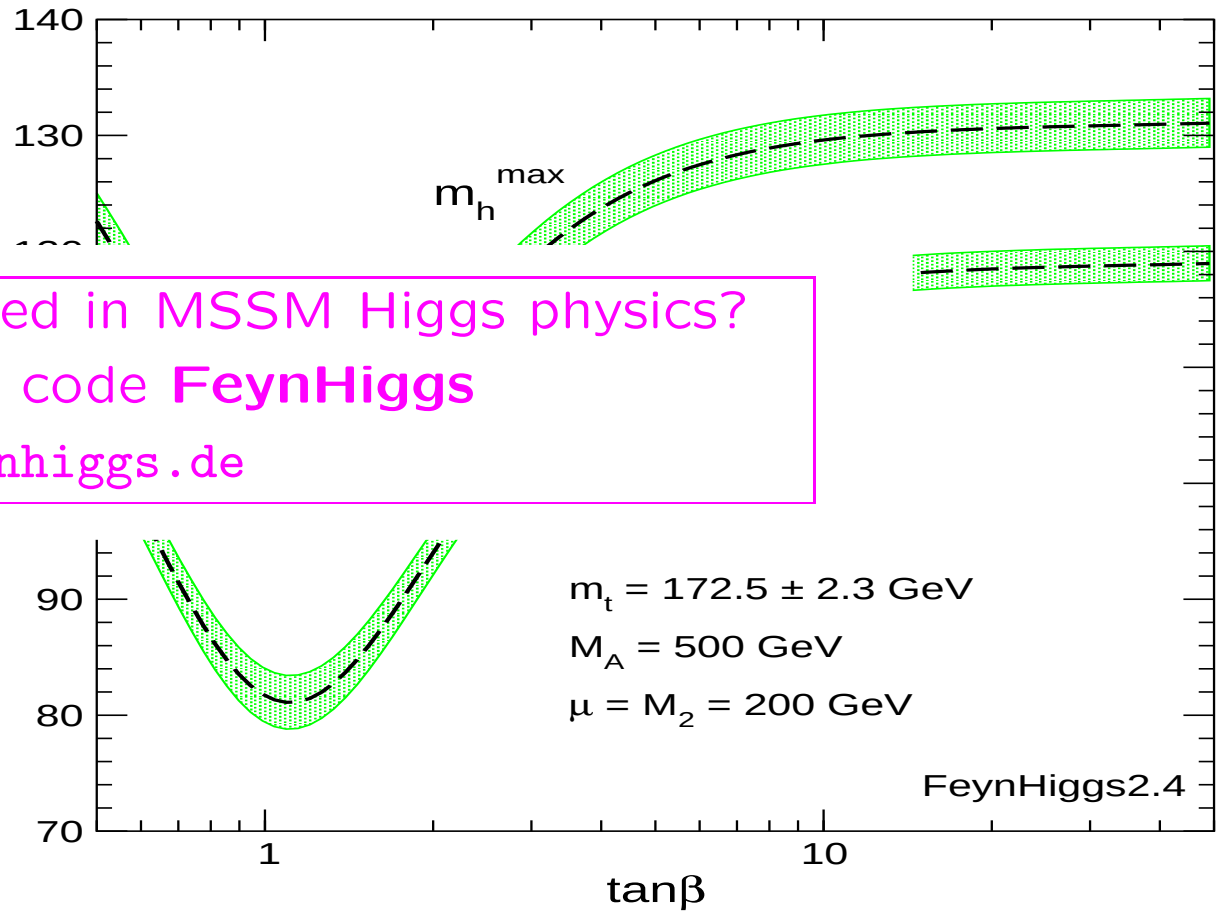
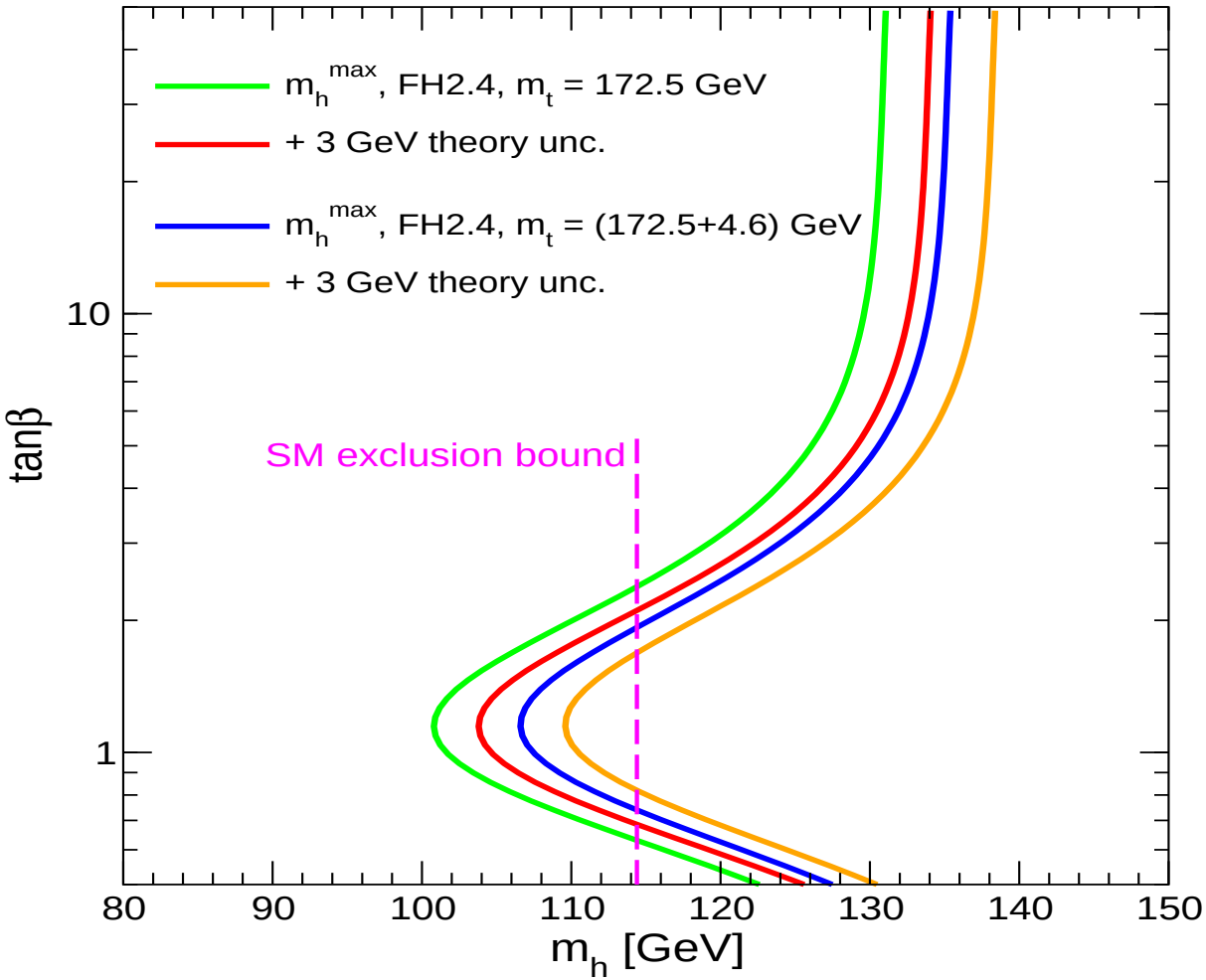


Interested in MSSM Higgs physics?
Try our code **FeynHiggs**
`www.feynhiggs.de`

Effect on $\tan\beta$ exclusion bounds



$m_h^{\max}$ scenario, *FeynHiggs*2.4, $m_t = 172.5$ GeV

$m_h^{\max}$ scenario, *FeynHiggs*2.4, + 3 GeV theory unc.

$\Rightarrow$ $\tan\beta$ bound considerably weakened by theory unc.

Exp. error in m_t:

$m_h^{\max}$ scenario, $m_t = 177.1$

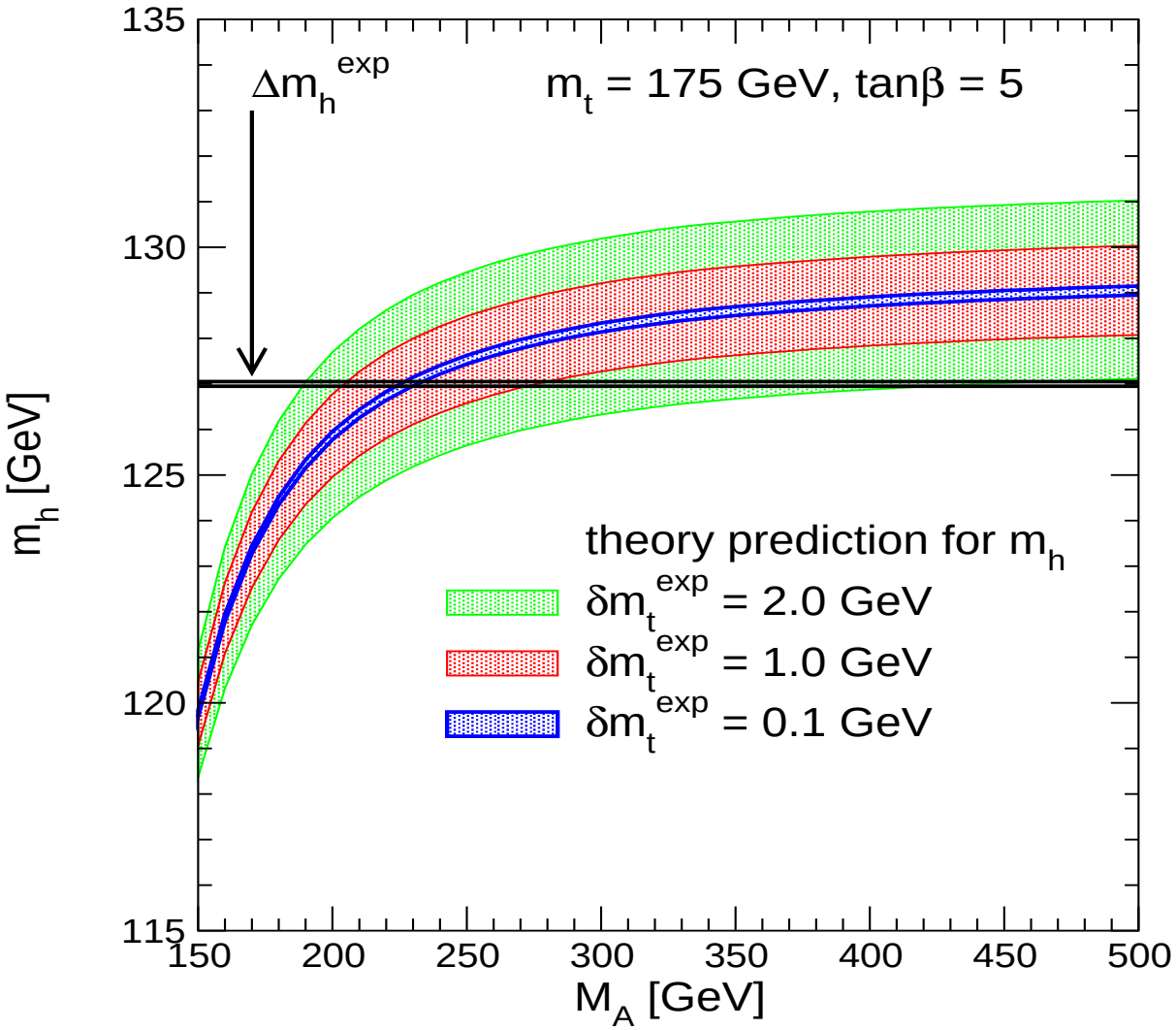
$m_h^{\max}$ scenario $m_t = 177.1$ + 3 GeV theory unc.

$\Rightarrow$ hardly any $\tan\beta$ exclusion

$\Rightarrow$ considerable improvement needed in both: δm_t^{exp} and δm_h^{theo}

Example: parametric uncertainty vs. experimental error:

[*S.H., S. Kraml, W. Porod, G. Weiglein '03*]



m_h^{max} benchmark scenario (no experimental errors on SUSY parameters):

current situation

vs.

future Tevatron measurement

vs.

ILC precision

vs.

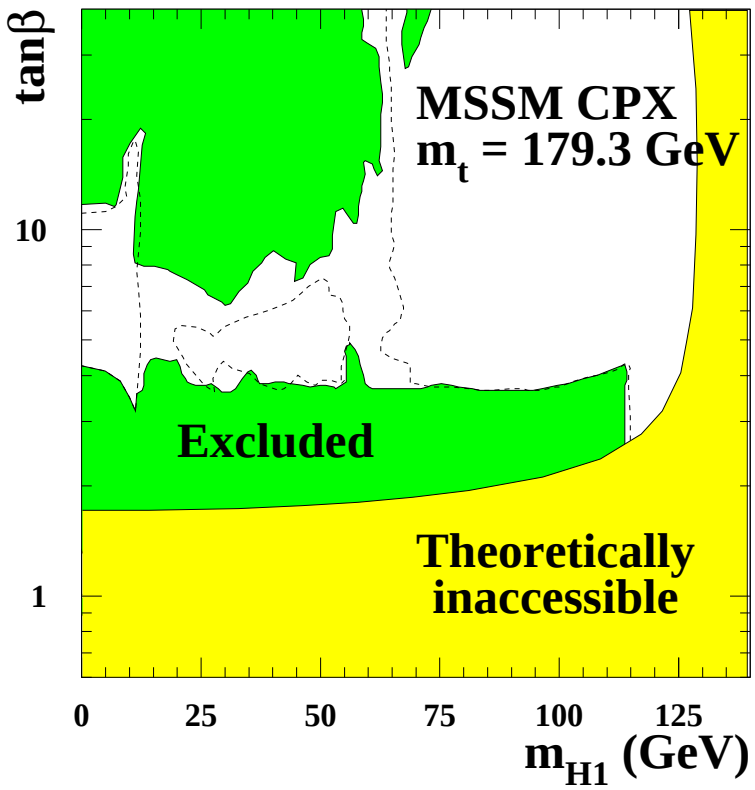
experimental resolution

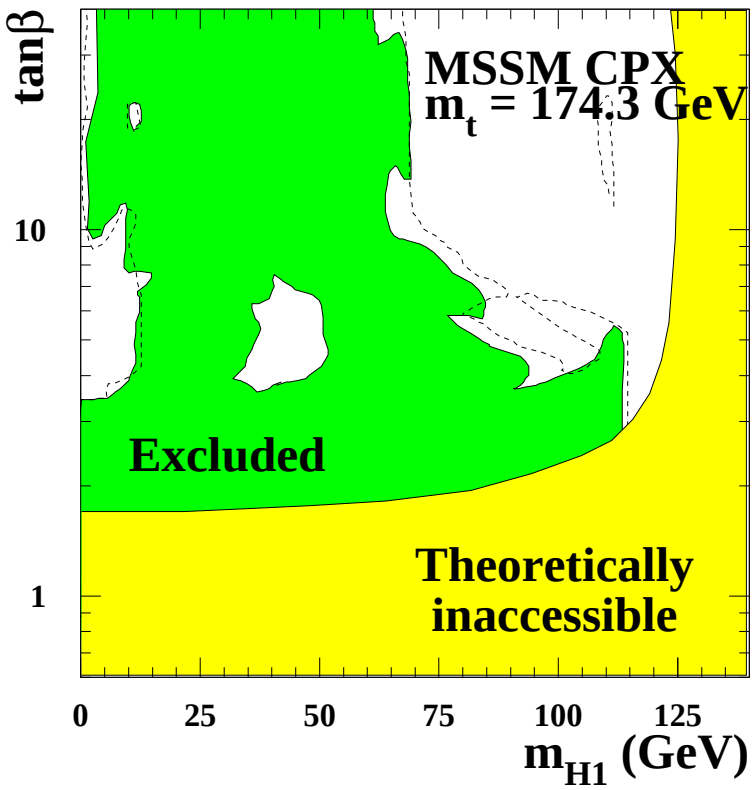
Higgs search in the MSSM with complex parameters: (CPX scenario)

[*LEP Higgs Working Group '06*]

⇒ large uncovered holes, even for very low Higgs masses

⇒ observe m_t dependence:





What the Tevatron can do to cover the holes:

⇒ measure a **small** and **accurate** m_t value

4. Renormalization group running and Dark Matter

→ focus on mSUGRA/CMSSM

mSUGRA/CMSSM:

⇒ Scenario characterized by

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \text{sign}\,\mu$$

m_0 : universal scalar mass parameter

$m_{1/2}$: universal gaugino mass parameter

A_0 : universal trilinear coupling

} at the GUT scale

$\tan\beta$: ratio of Higgs vacuum expectation values

sign(μ) : sign of supersymmetric Higgs parameter

⇒ particle spectra from renormalization group running to weak scale

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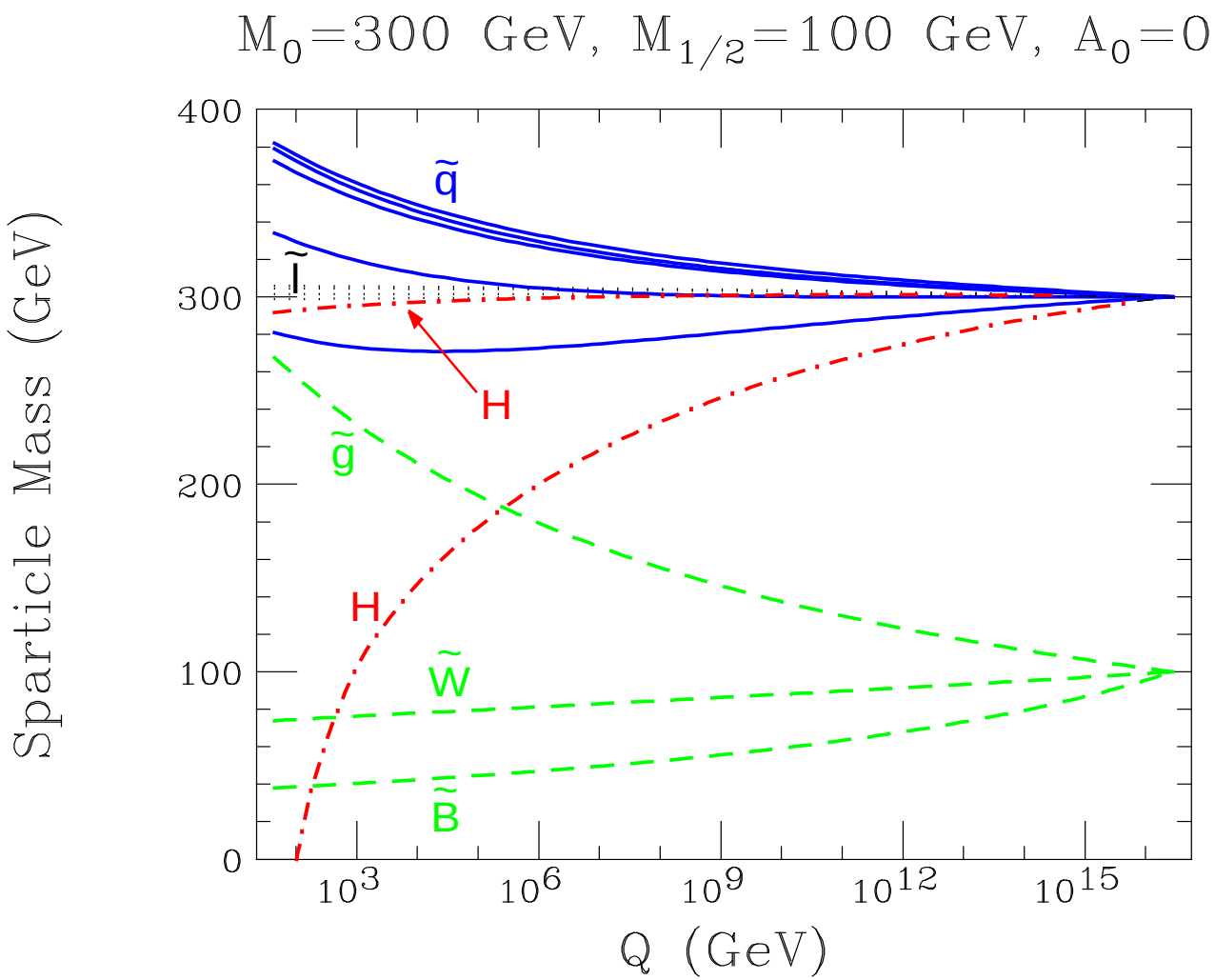


⇒ m_t plays an important role, e.g.: Higgs mechanism ("H" turns negative)

Prediction of Cold Dark Matter:

[*J. Ellis, S.H., K. Olive, G. Weiglein '04*]

CDM prediction in terms of $m_{1/2}$, m_0 and m_t

$A_0 = 0$, $\tan\beta = 10, 50$ fixed

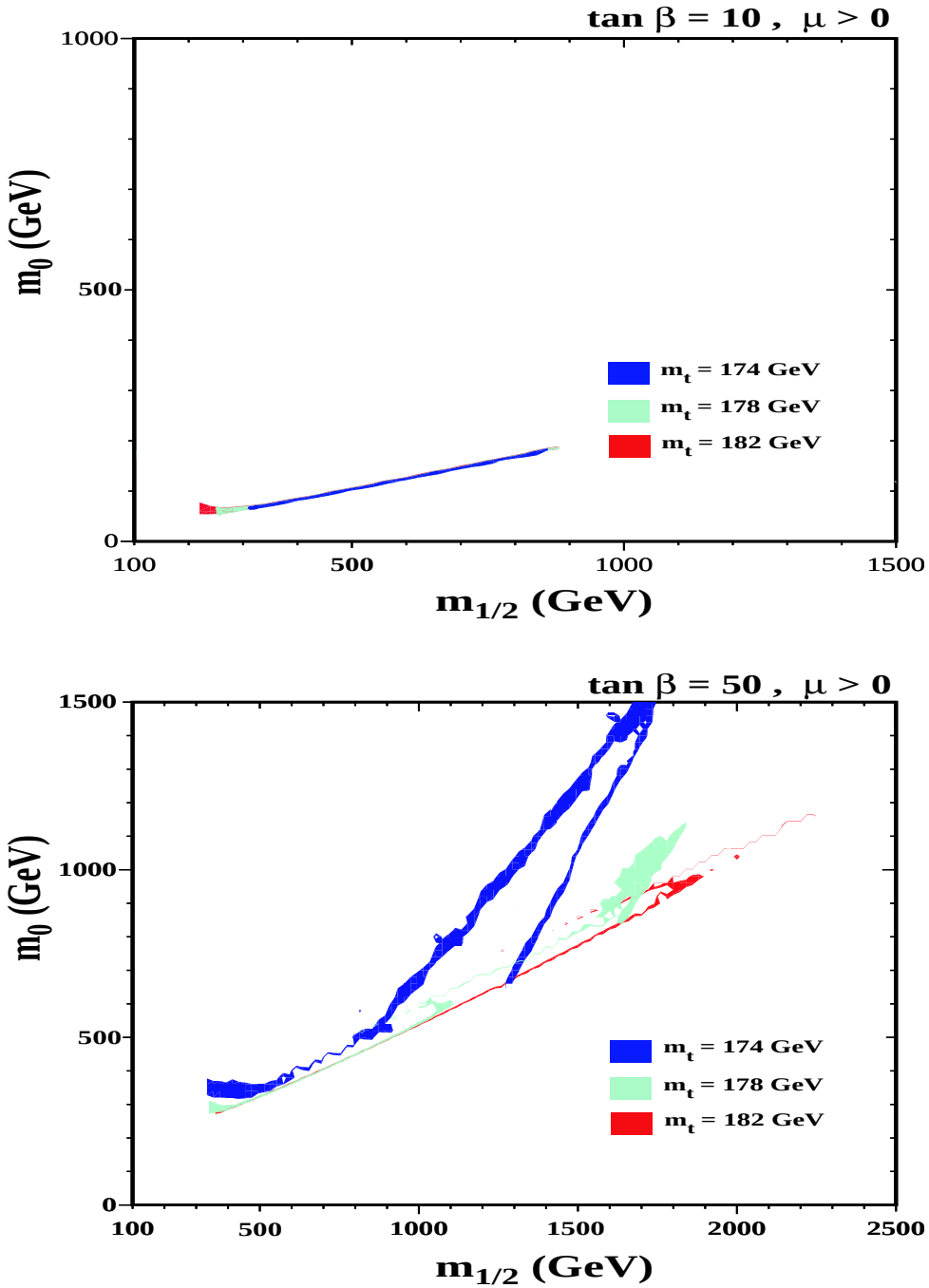
$m_t = 178.0 \pm 4.3$ GeV

Shown: parameters allowed by $0.094 \leq \Omega_\chi h^2 \leq 0.129$

[*WMAP '03*]

$\Rightarrow$ strong variation for large $\tan\beta$

$\Rightarrow$ precise m_t value needed for precise CDM predictions!



Test of unification:

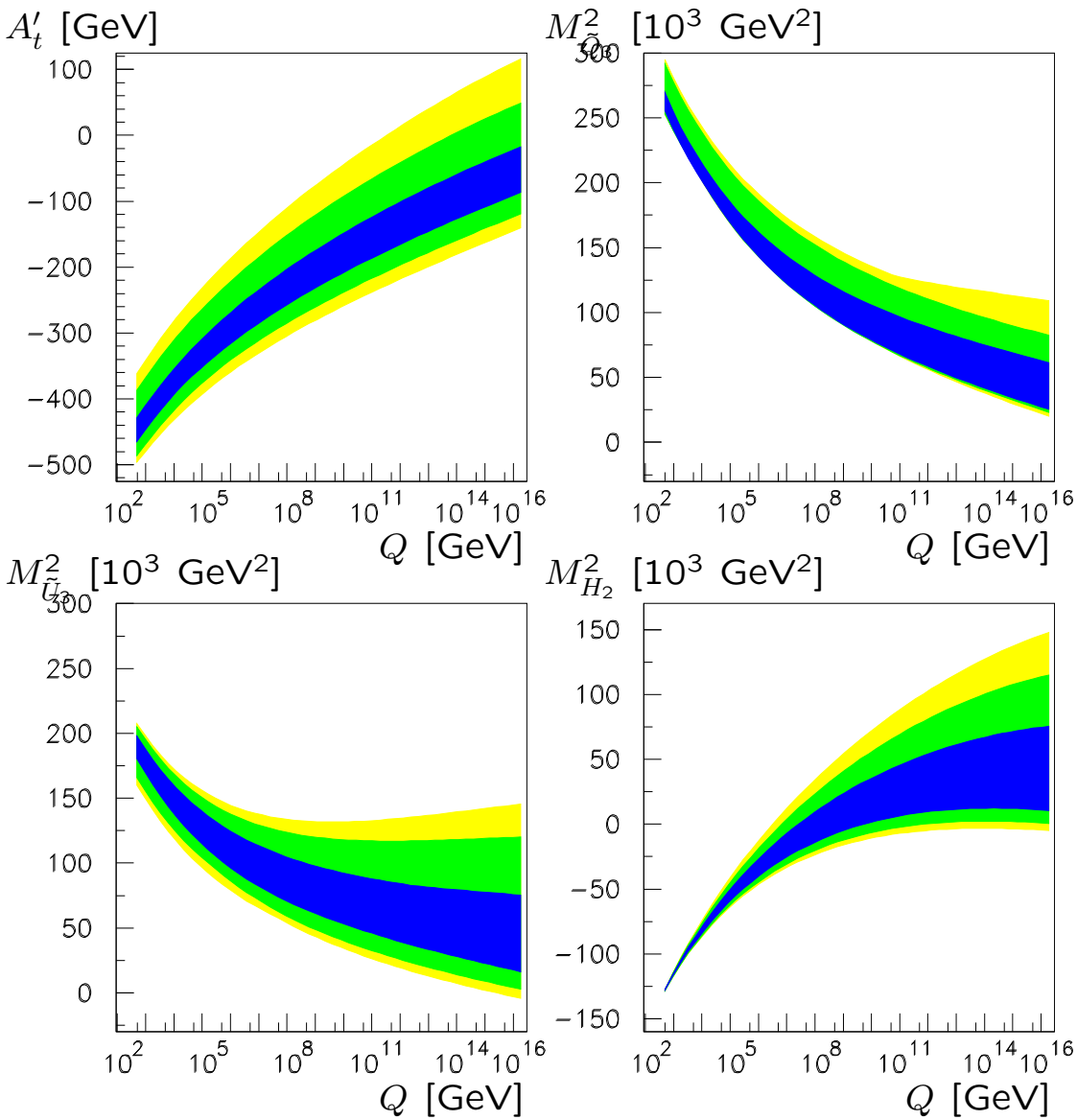
[S.H., S. Kraml, W. Porod, G. Weiglein '03]

Low-energy measurement of SUSY parameters
$\Rightarrow$ extrapolation to GUT scale

m_t enters RGEs
via Yukawa couplings,
stop masses, loop effects

$\delta m_t = 2.0$ GeV (Tev. today)
$\delta m_t = 1.0$ GeV (Tev. future)
$\delta m_t = 0.1$ GeV (ILC)

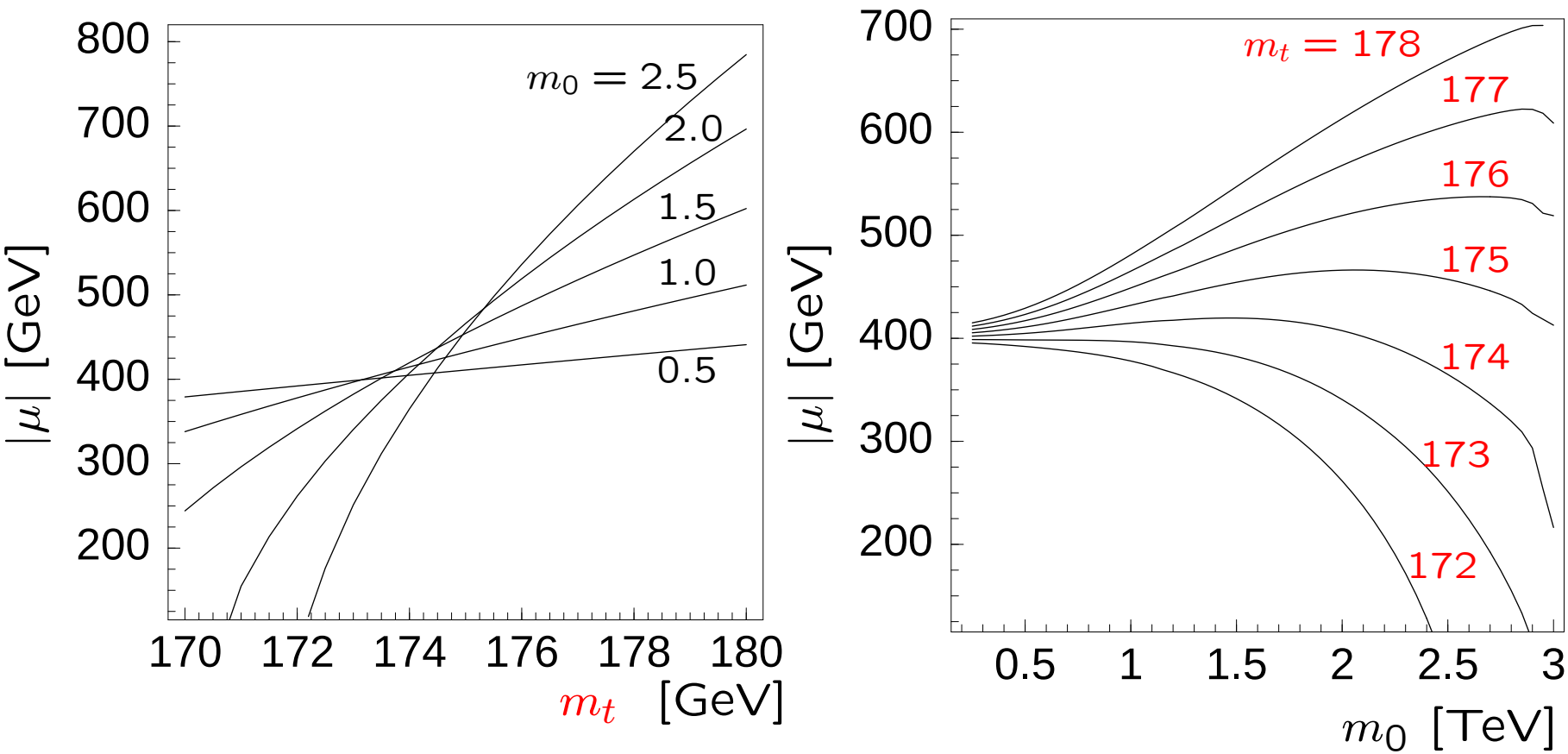
$\Rightarrow$ large improvement in precision of GUT scale parameters



Prediction of neutralino/chargino masses:

[*S.H., S. Kraml, W. Porod, G. Weiglein '03*]

Start with GUT parameters: $m_{1/2}$, m_0, A_0, $\tan\beta = 10$
derive low-energy parameters: μ, ...



$\Rightarrow$ Precise knowledge of m_t crucial (for large m_0)

5. Conclusinos $\oplus$ theorist's wish list

- The top quark is a special particle:
  - role for t in EWSB?
  - drives SSB/unification in SUSY
  - δm_t^{exp} induces uncertainty for precision physics $\Rightarrow$ can obscure NP
- Electroweak precision observables:
  - m_t is crucial for bounds on M_H^{SM}
  - Tevatron might be nearly as good as LHC ???
  - EWPO favor relatively light SUSY mass scale, again m_t is crucial
    $\Rightarrow$ good prospects for direct searches at Tevatron
- Higgs physics in the MSSM

  Precision Higgs physics needs precision top physics (in nearly any model)
  - Scalar top sector needs precise knowledge on m_t
  - reliable exclusion bounds need precise m_t ($\tan\beta$, CPX scenario, ...)
- mSUGRA/CMSSM: RGE running needs precise m_t

  Cold Dark Matter predictions / Tests of unification
- Theorist's wish list:

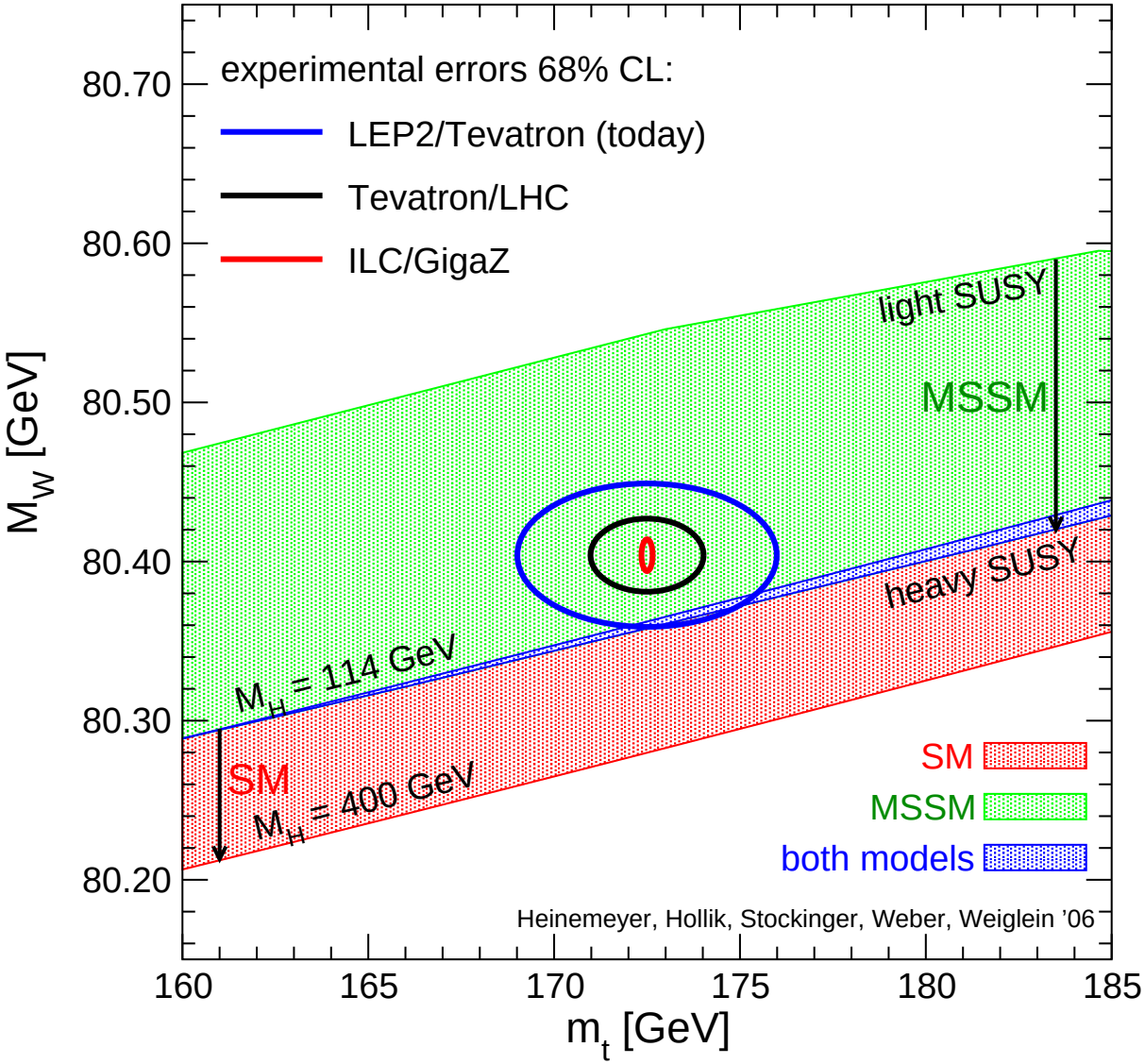
  The Tevatron has a potential to crack the SM with the t ...

  Please try as hard as you can with the t, it is definitely worth the effort!

Back-up

Prediction for M_W in the SM and the MSSM :

[*S.H., W. Hollik, D. Stockinger, A.M. Weber, G. Weiglein '06*]



MSSM band:

scan over

SUSY masses

overlap:

SM is MSSM-like

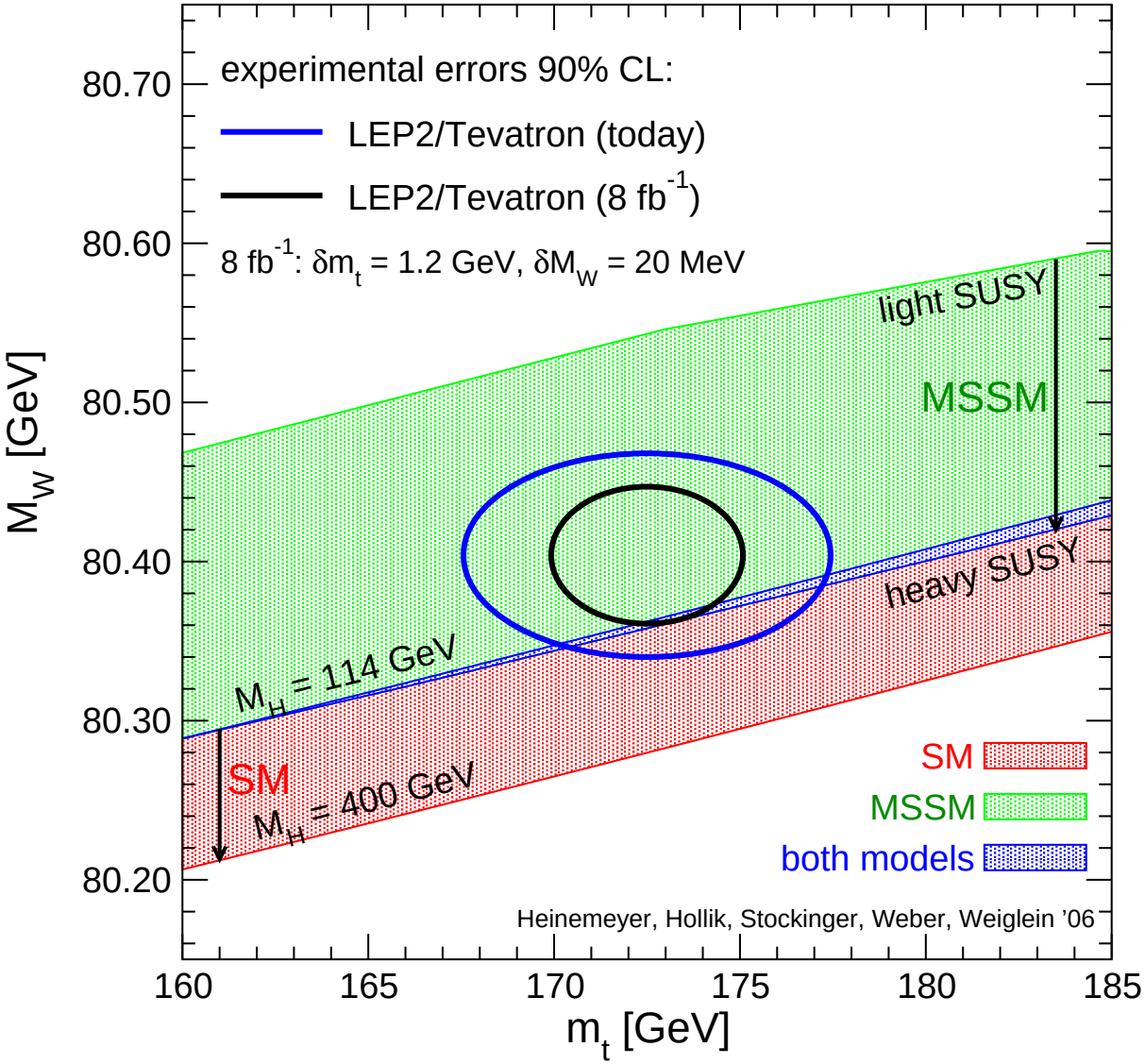
MSSM is SM-like

SM band:

variation of M_H^{SM}

Prediction for M_W in the SM and the MSSM :

[*S.H., W. Hollik, D. Stockinger, A.M. Weber, G. Weiglein '06*]



MSSM band:

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Prediction for M_W in the SM and the MSSM :

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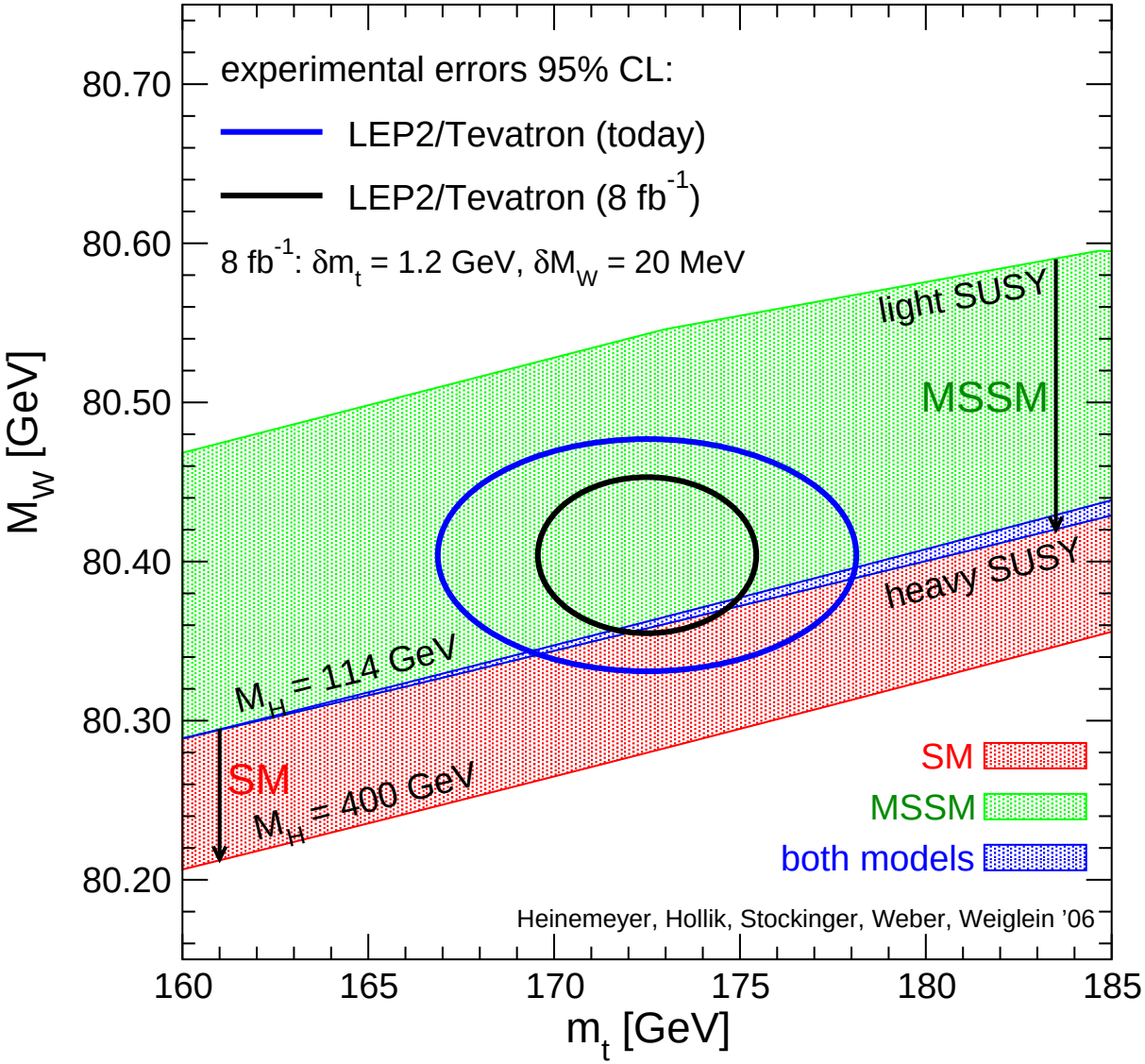


MSSM band:
scan over
SUSY masses

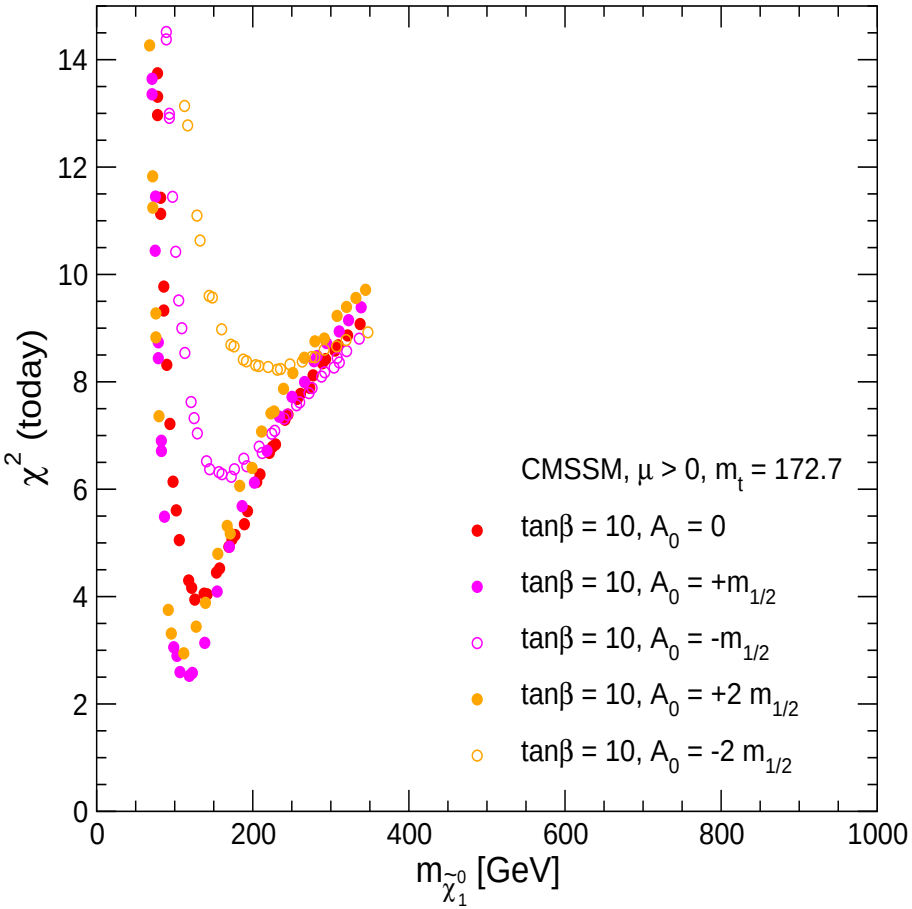
overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of M_H^{SM}

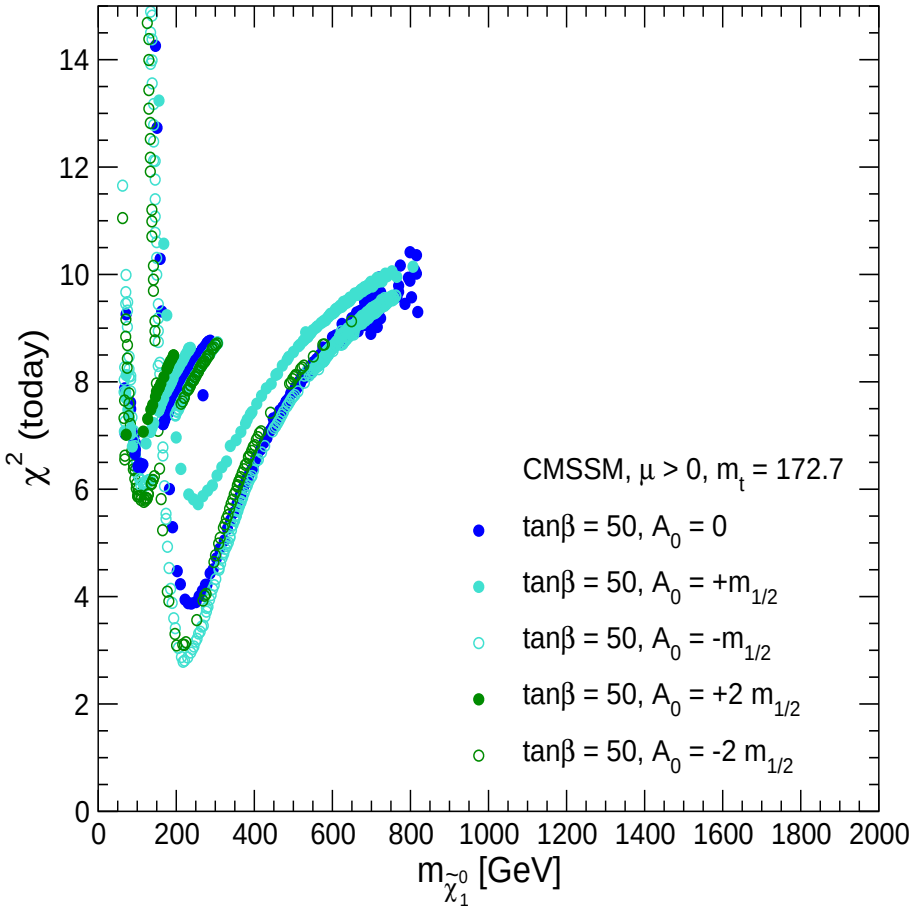
CMSSM: LSP mass for $\tan\beta = 10, 50$

[*J. Ellis, S.H., K. Olive, G. Weiglein '06*]

$\tan\beta = 10$



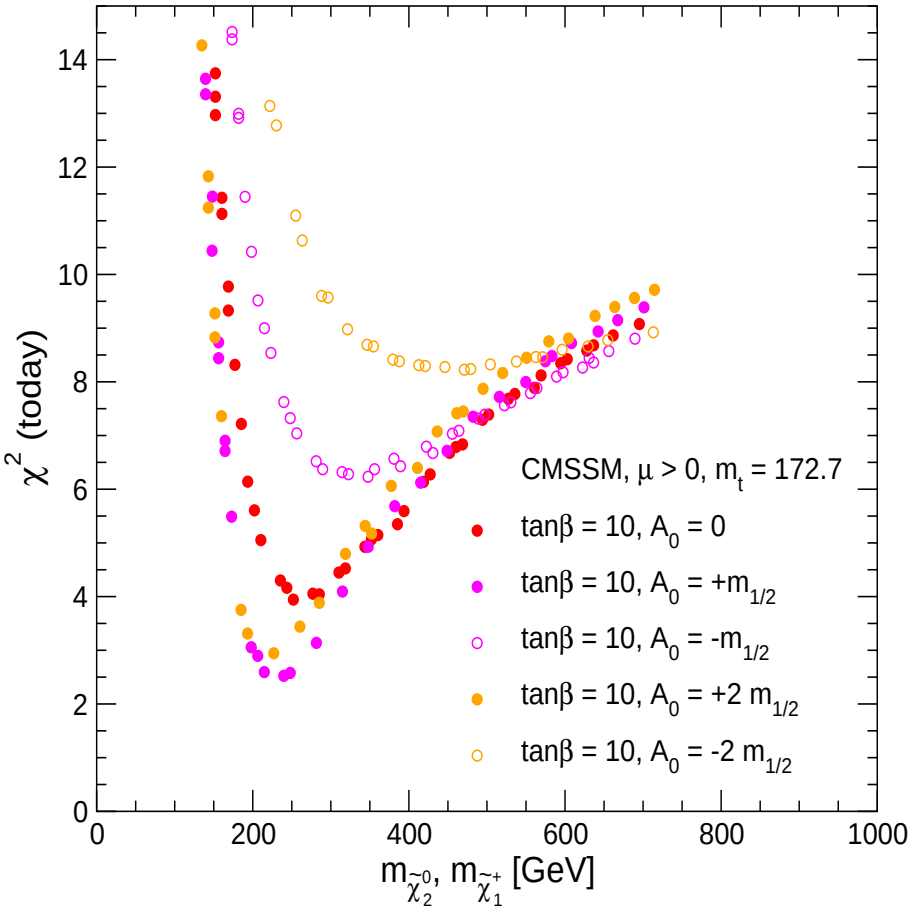
$\tan\beta = 50$



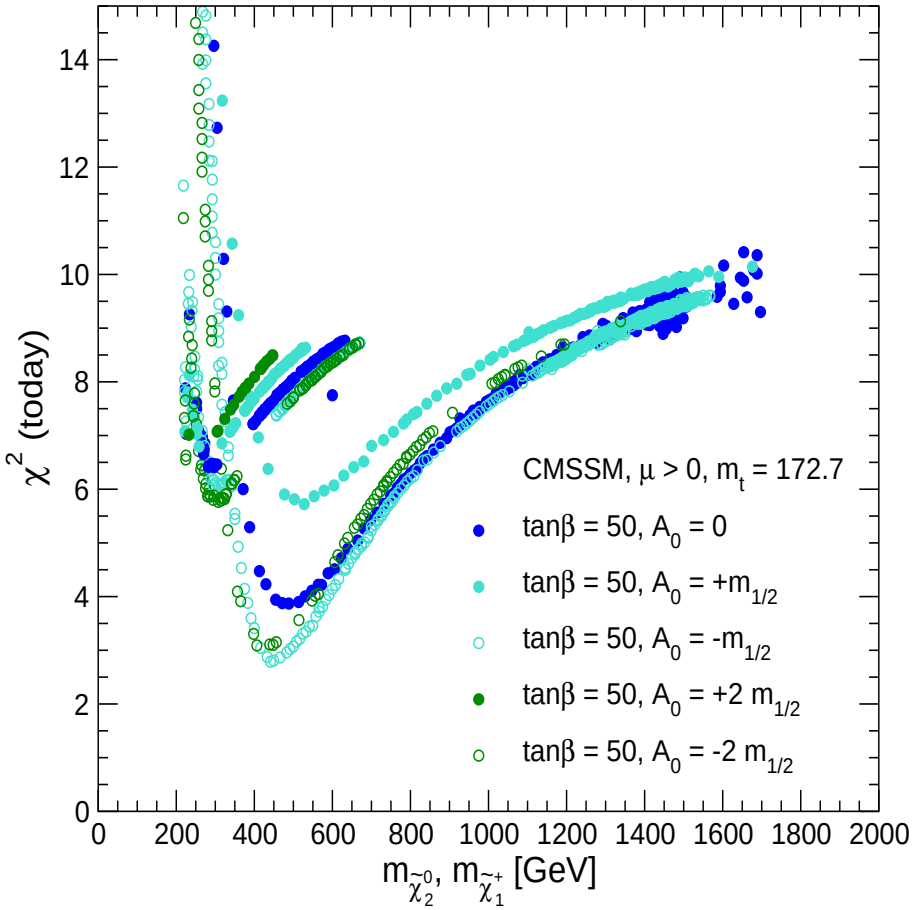
CMSSM: light neutralino/chargino masses for $\tan\beta = 10, 50$

[*J. Ellis, S.H., K. Olive, G. Weiglein '06*]

$\tan\beta = 10$

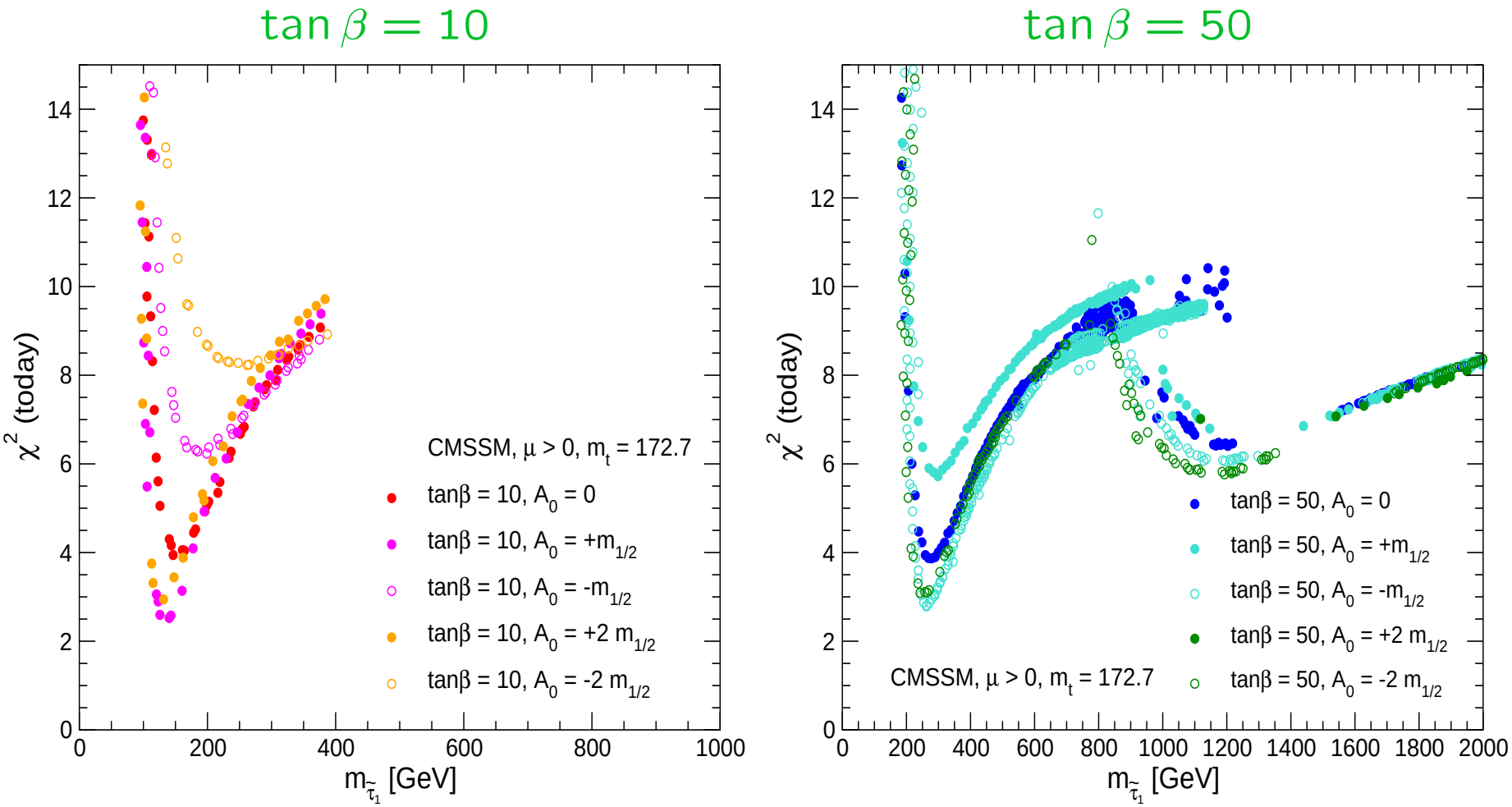


$\tan\beta = 50$



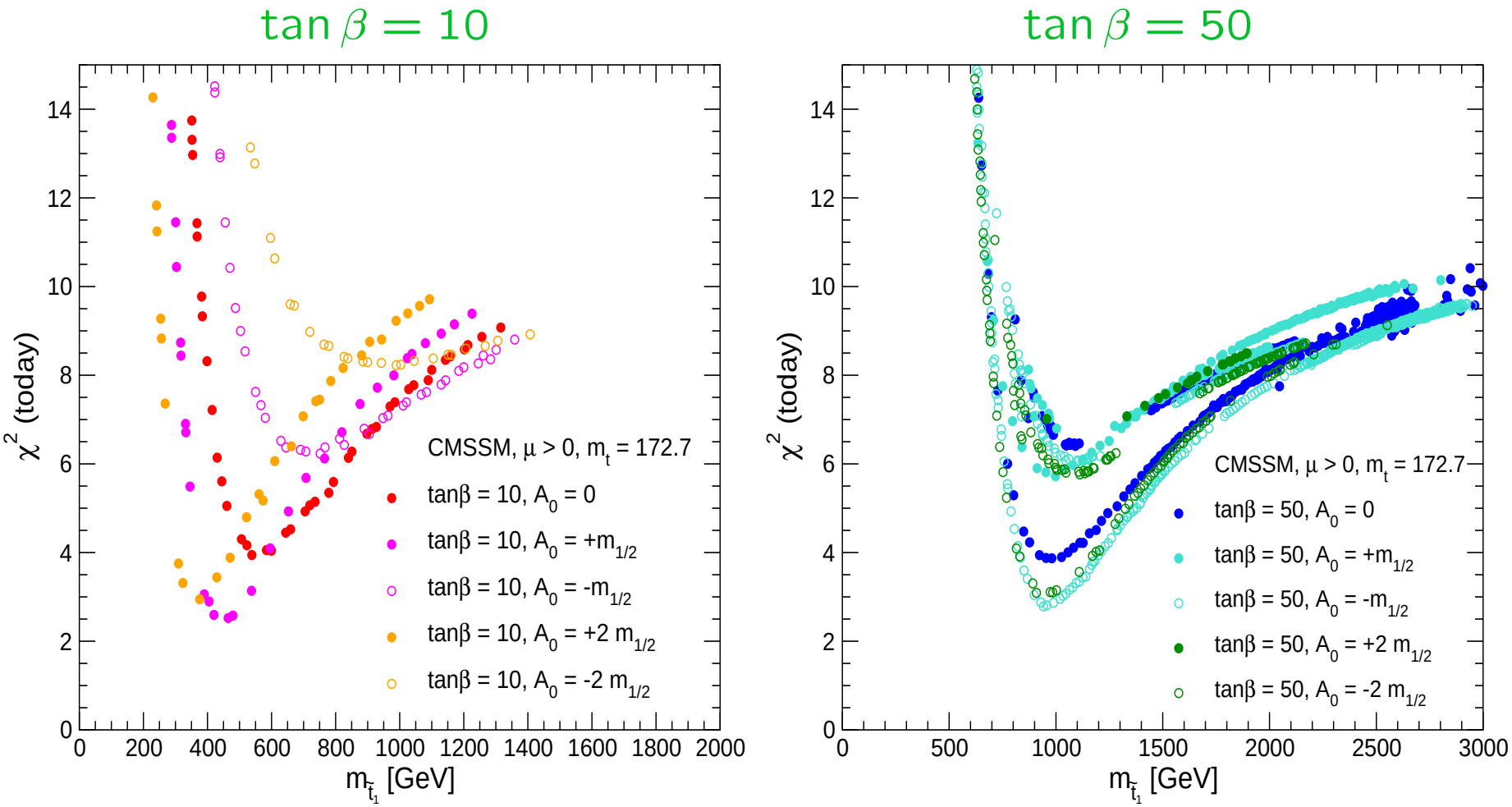
CMSSM: lightest stau mass for $\tan\beta = 10, 50$

[*J. Ellis, S.H., K. Olive, G. Weiglein '06*]



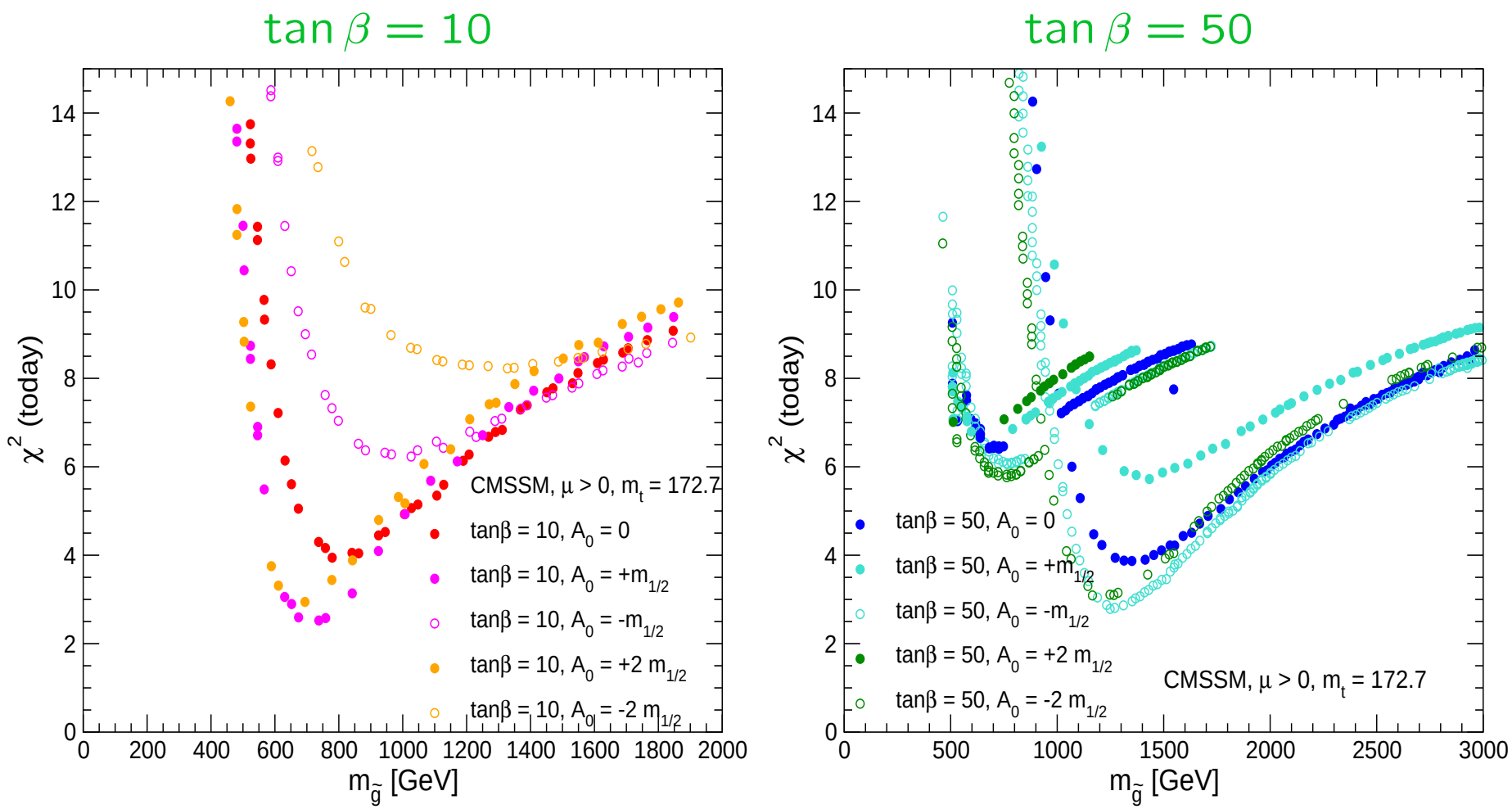
CMSSM: lightest stop mass for $\tan\beta = 10, 50$

[*J. Ellis, S.H., K. Olive, G. Weiglein '06*]



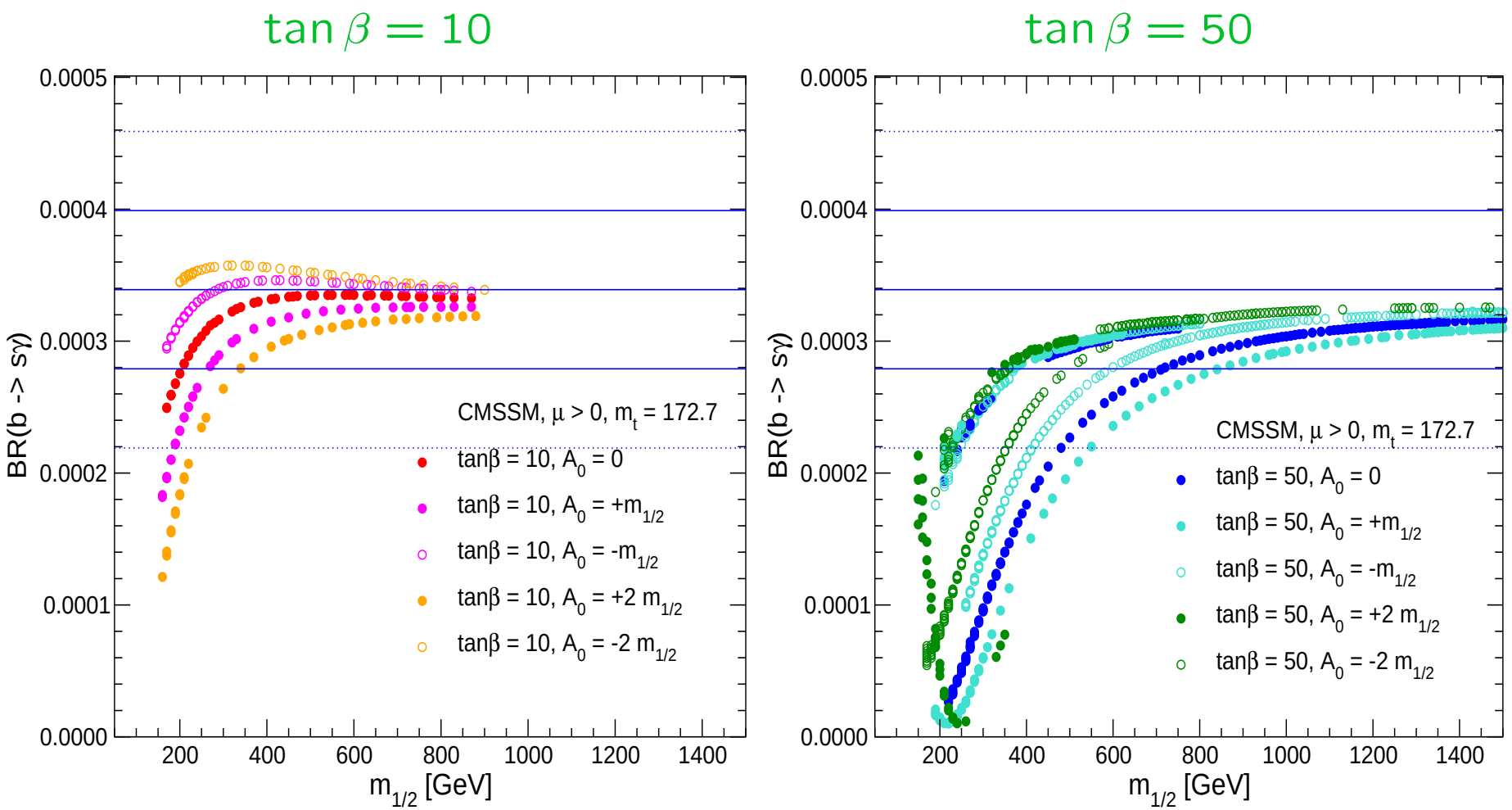
CMSSM: gluino mass for $\tan\beta = 10, 50$

[*J. Ellis, S.H., K. Olive, G. Weiglein '06*]



CMSSM result for total BR($b \to s\gamma$):

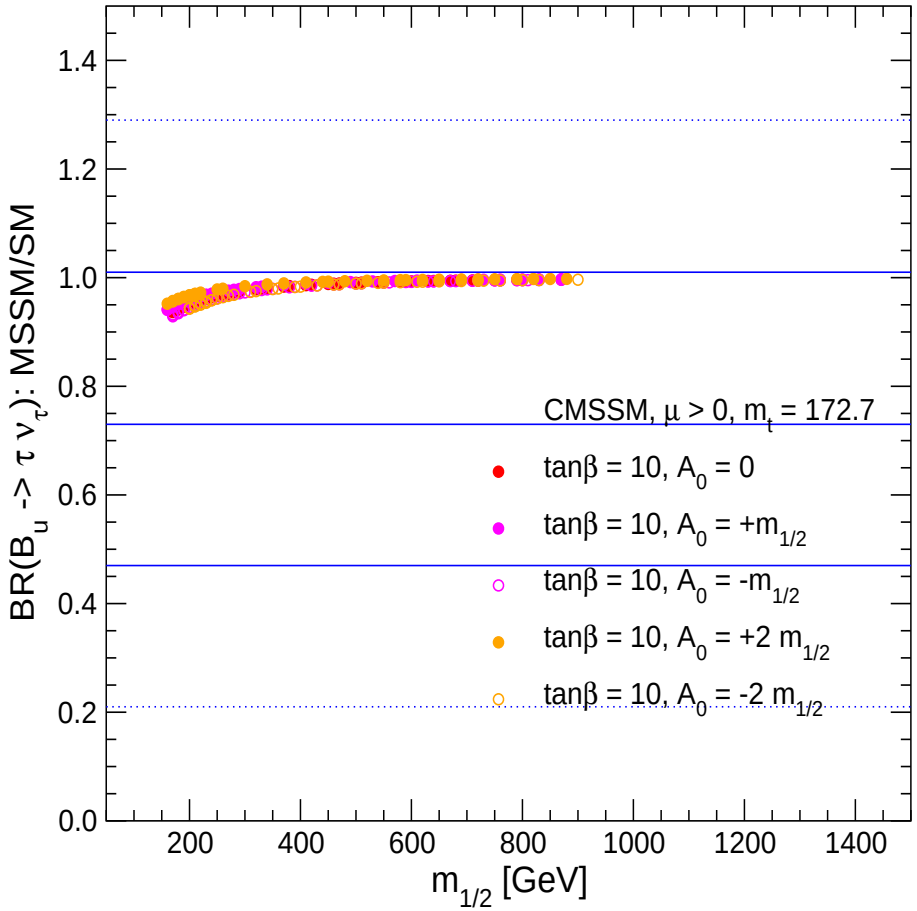
[*J. Ellis, S.H., K. Olive, G. Weiglein '06*]



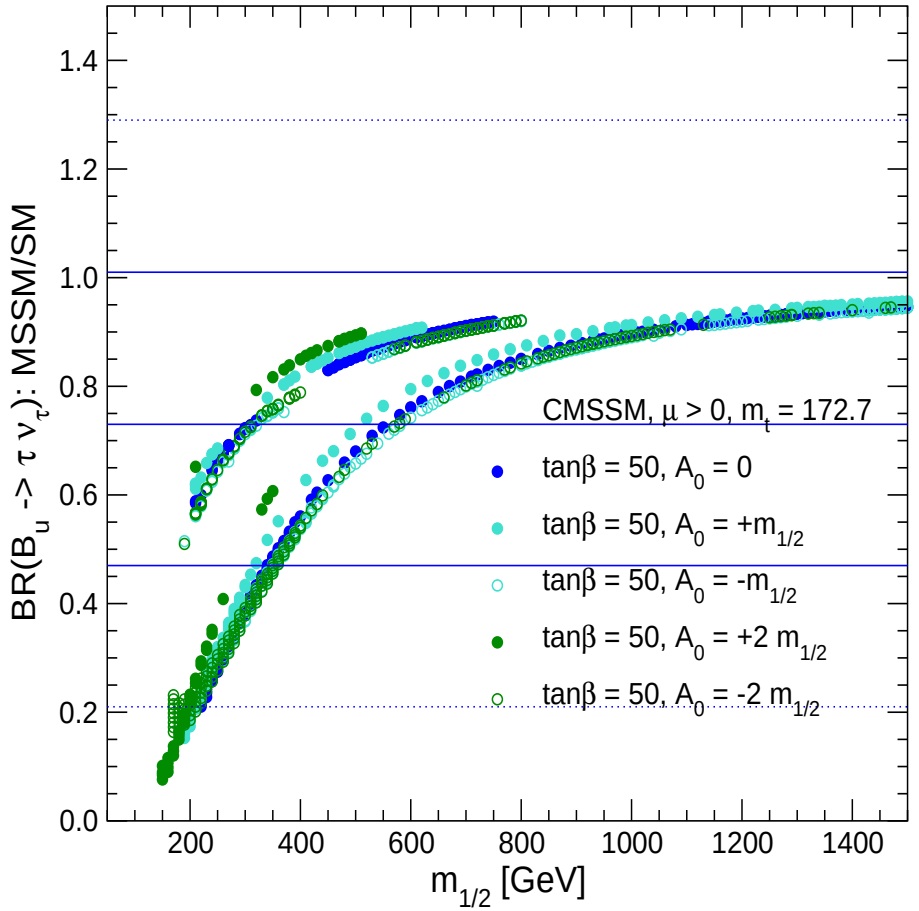
CMSSM result for total BR($B_u \to \tau\nu_\tau$) (MSSM/SM):

[*J. Ellis, S.H., K. Olive, G. Weiglein '06*]

$\tan\beta = 10$

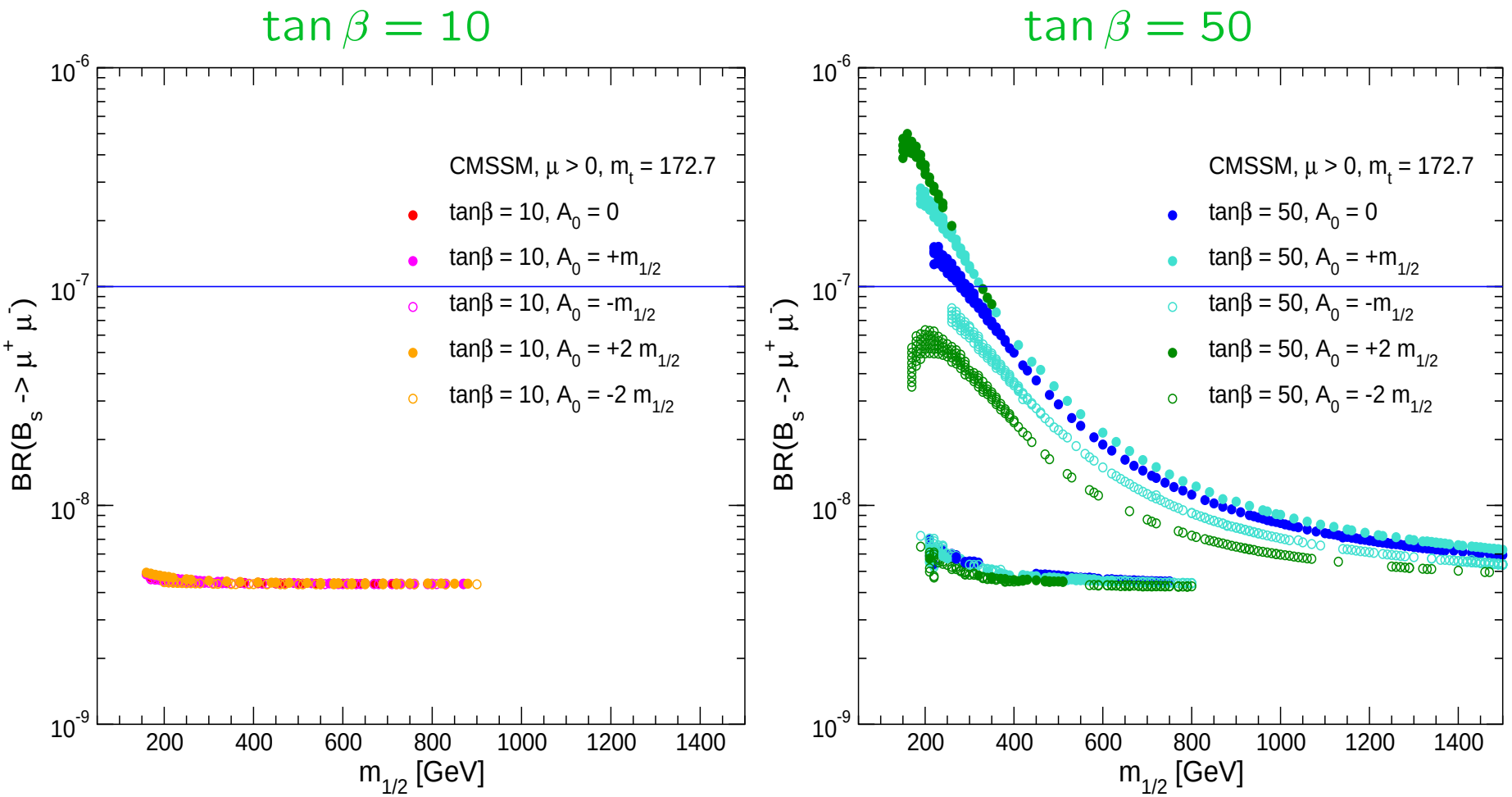


$\tan\beta = 50$



CMSSM result for total BR($B_s \to \mu^+\mu^-$):

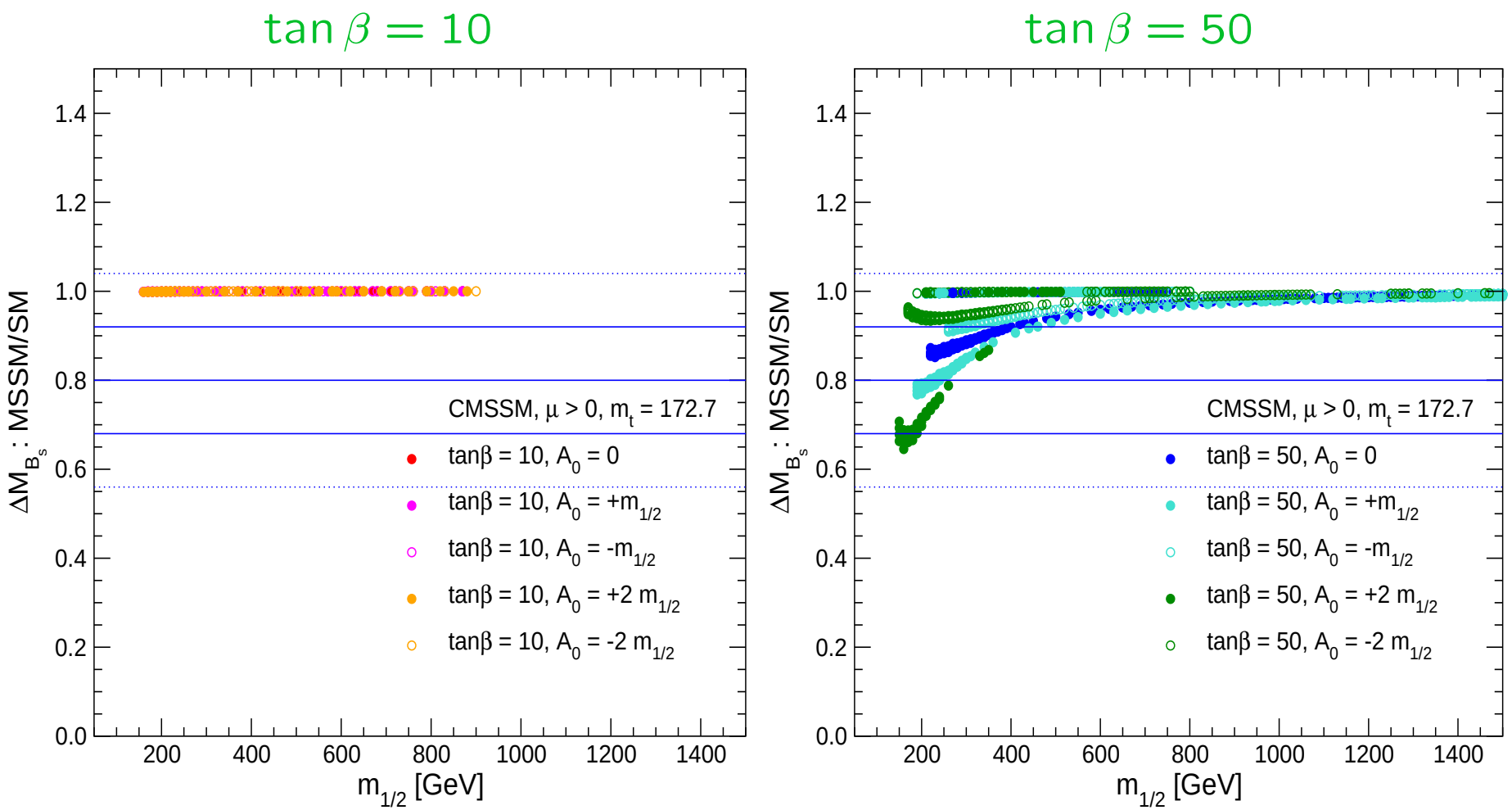
[*J. Ellis, S.H., K. Olive, G. Weiglein '06*]



→ thanks to Cheng-Ju Stephen Lin and Matthew Herndon!

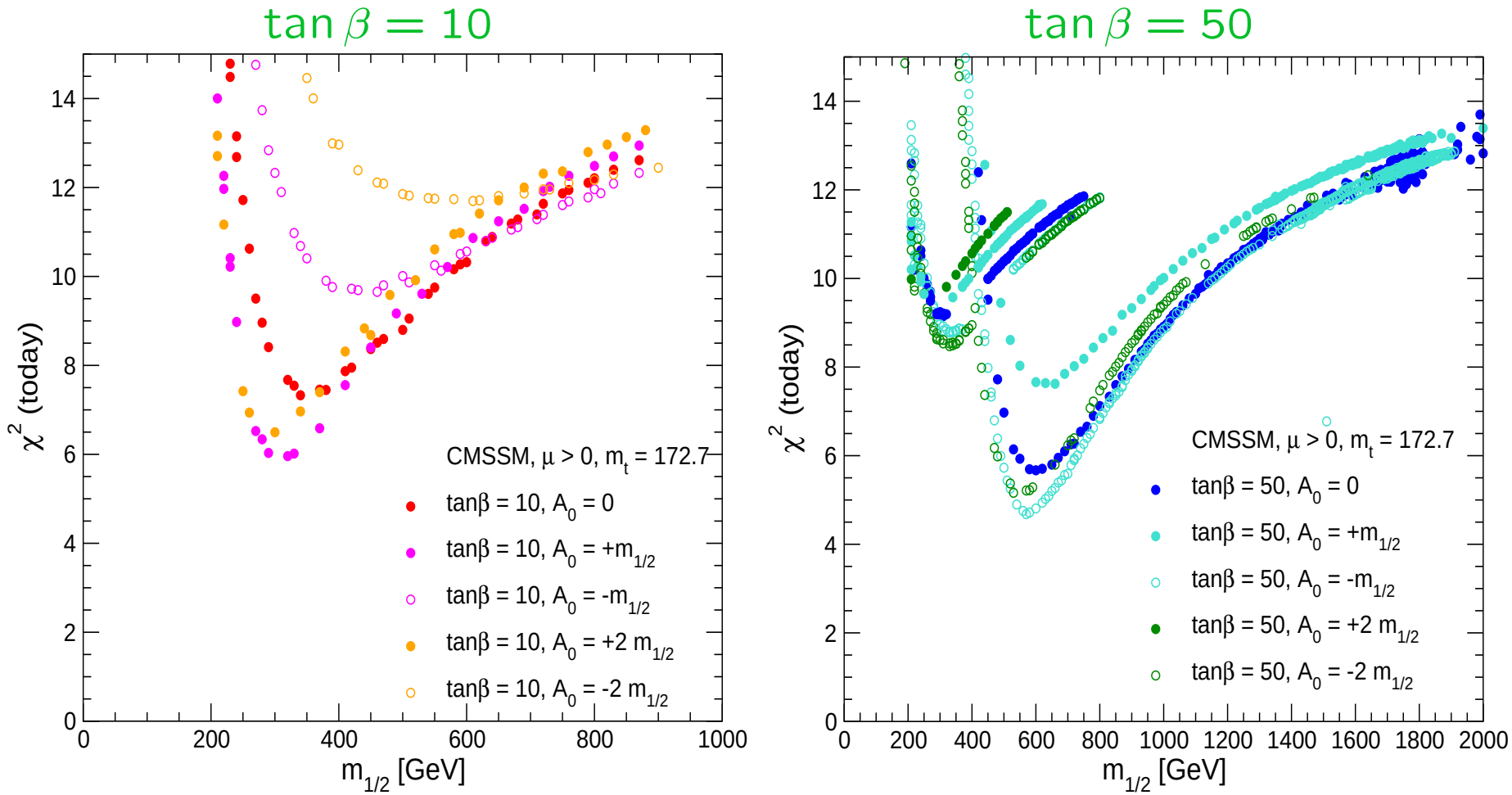
CMSSM result for total ΔM_{B_s} (MSSM/SM):

[*J. Ellis, S.H., K. Olive, G. Weiglein '06*]



Result for total χ^2 including all B-physics observables:

[*J. Ellis, S.H., K. Olive, G. Weiglein '06 – PRELIMINARY*]



⇒ overall fit still very good, $\chi^2_{\text{tot,min}} \approx 5$

⇒ preference for relatively light SUSY masses confirmed