Cumulants of Conserved Charges and QCD Phase Structure

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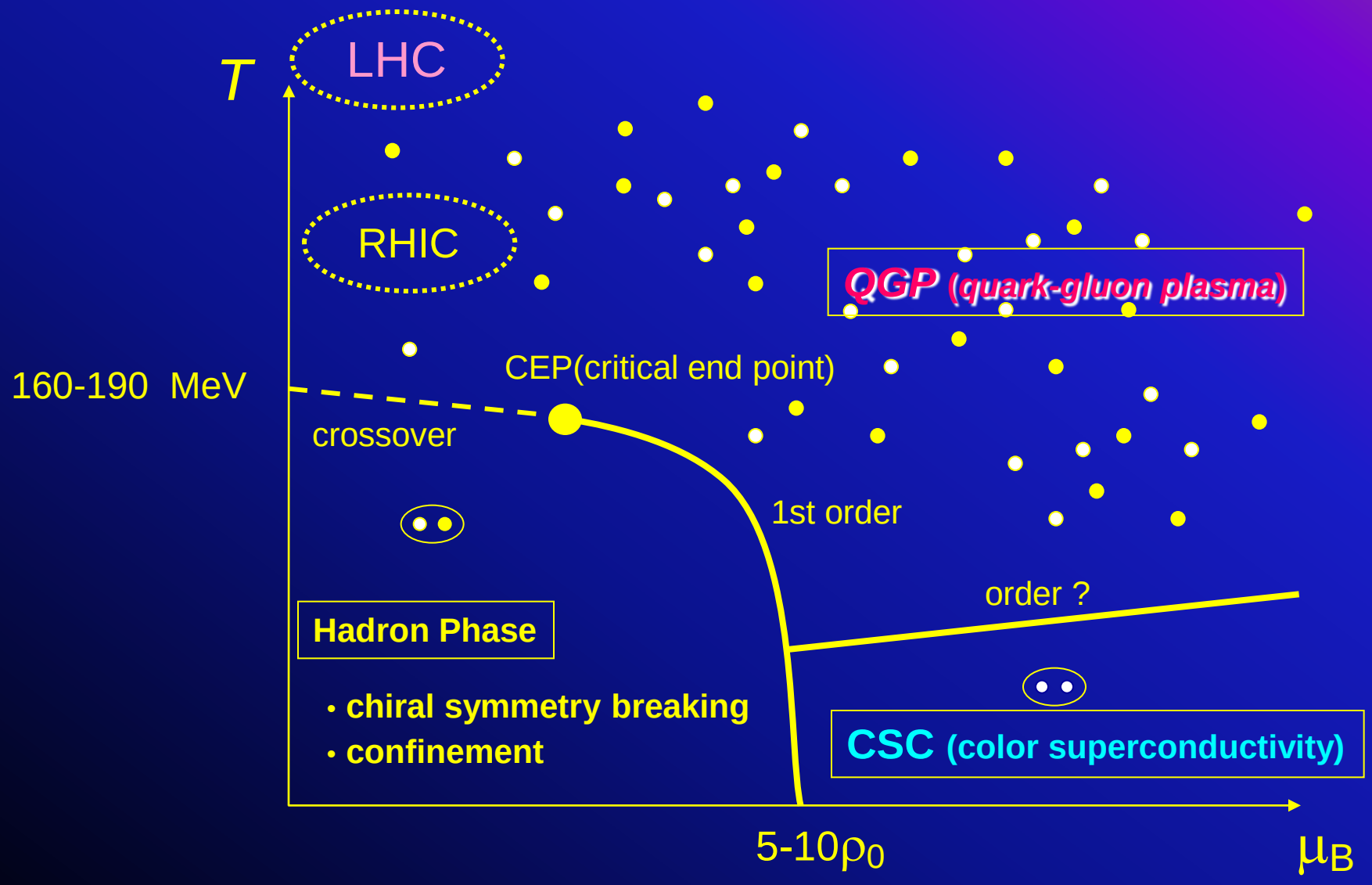
S. Ejiri, M. Kitazawa, and M. A., PRL 103 (2009) 262301
M. Kitazawa and M.A., (2011) to be published

@ Fluctuations, Correlations and RHIC Low Energy Runs

October 2011

QCD Phase Diagram



Two Meanings of Fluctuation Observables

- Distinction of Phases (conserved charge fluctuation)

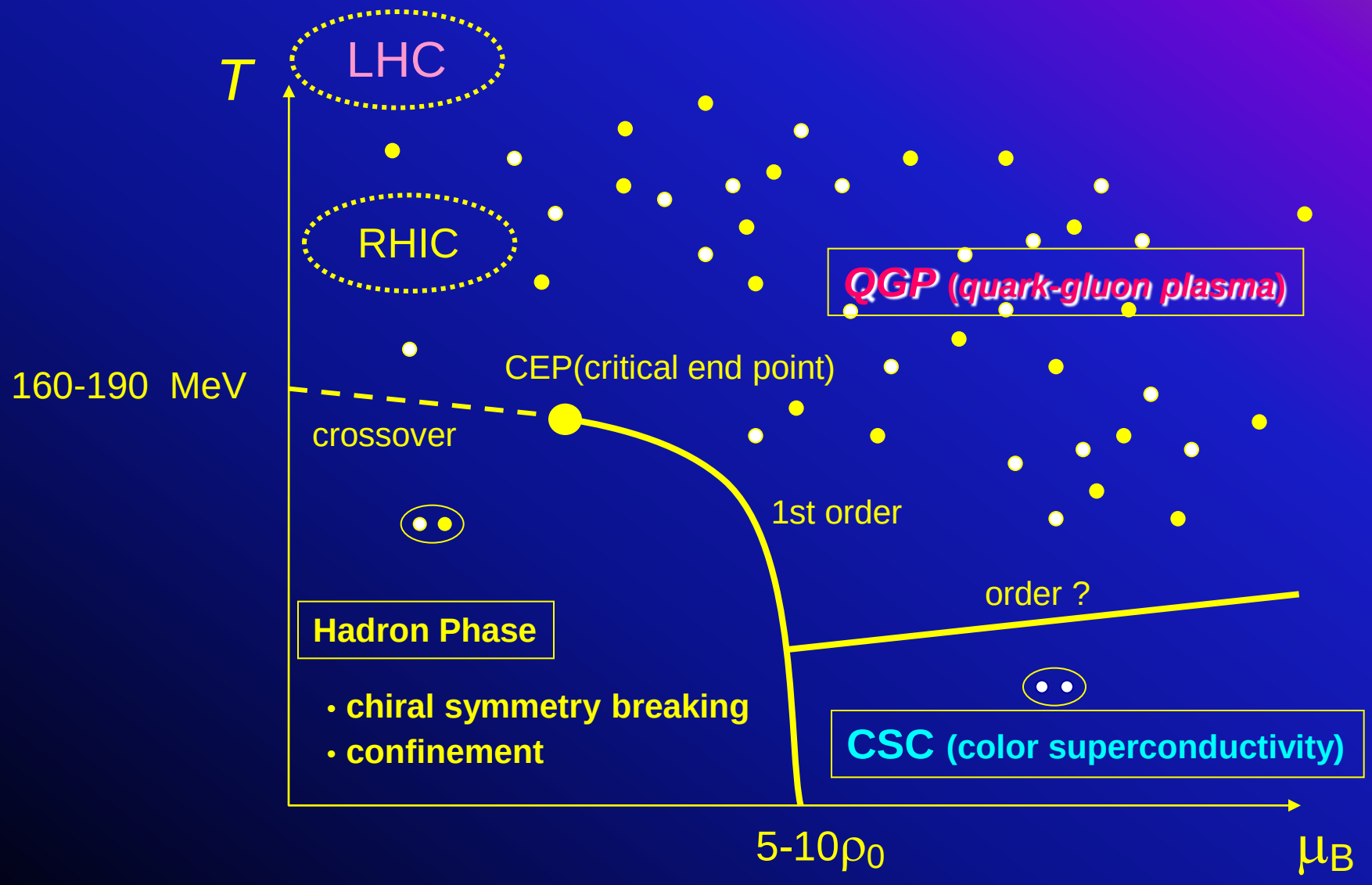
Asakawa, Heinz, Müller, Jeon, Koch, Ejiri, Kitazawa, …

- Detection of Increase of Fluctuation (or Correlation Length) at 2nd order Phase Transition

Critical End Point = 2nd order phase transition point

⇨ Phase structure of QCD

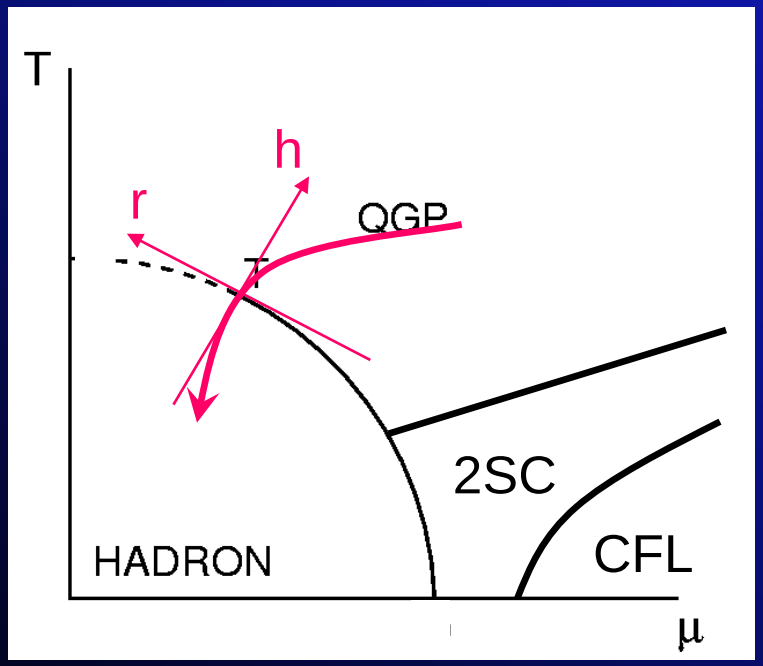
QCD Phase Diagram



CEP = 2nd order phase transition, but...

CEP =
2nd Order Phase Transition Point

Divergence of Fluctuation
Correlation Length
Specific Heat ?
Critical Opalescence?



even if the system goes right through the critical end point…

Hadrons: subject to final state interactions

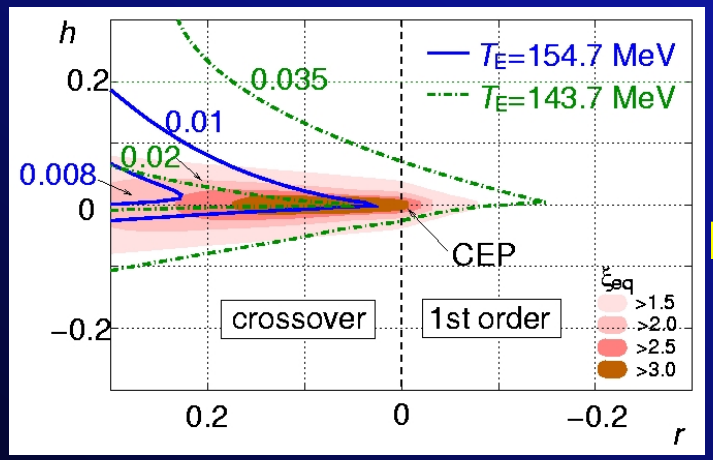
Critical slowing down should be taken into account as well

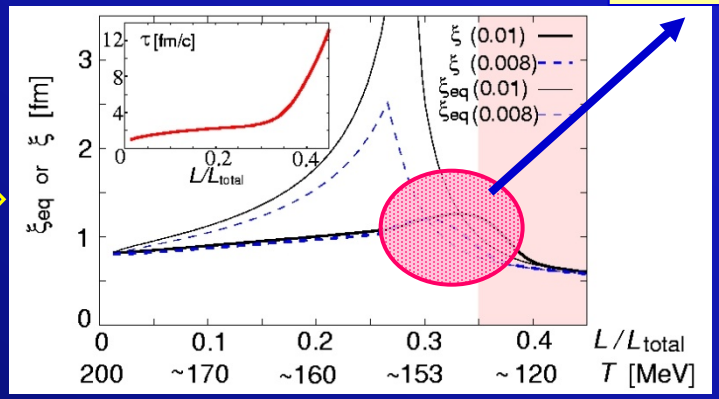
HI collision proceeds in finite time

Critical Slowing Down and Final State Int.

Furthermore,
critical slowing down limits the size of fluctuation, correlation length !

Time Evolution along given isentropic trajectories (n_B/s : fixed)





ξ and ξ_{eq} little difference

1-dim Bjorken Flow

Nonaka and M.A. (2004)

$$\frac{d}{d\tau} m_\sigma(\tau) = -\Gamma[m_\sigma(\tau)]\left(m_\sigma(\tau) - \frac{1}{\xi_{eq}(\tau)} \right)$$

$$\Gamma[m_\sigma(\tau)] = \frac{A}{\xi_0}(m_\sigma(\tau)\xi_0)^z, \; m_\sigma(\tau) = \frac{1}{\xi(\tau)}$$

$z \approx 3$ Model H (Hohenberg and Halperin RMP49(77)435)

Berdnikov and Rajagopal, 2000

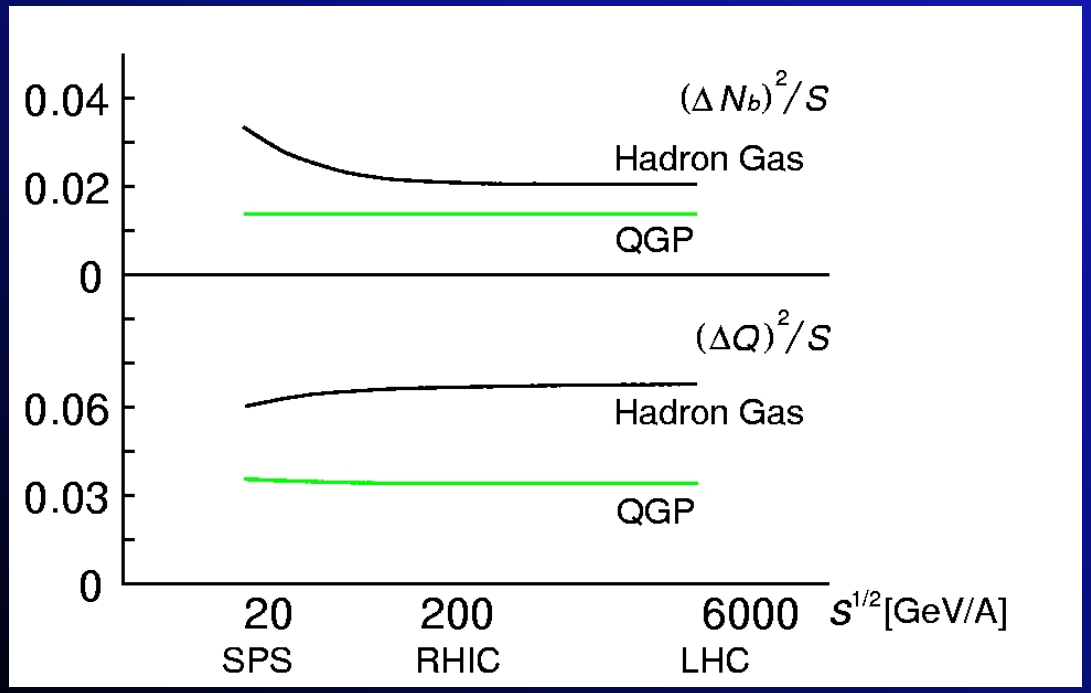
In addition, because of final state interaction correlation length at freezeout get smaller

Locally Conserved Charges

- Fluctuation of Conserved Charges: not subject to final state interactions

Conserved Charges

i. Net Baryon Number
ii. Net Electric Charge
iii. Net Strangeness
iv. Energy ...



- Net Electric Charge

Hadron Gas Phase

- ~2/3 of hadrons carry electric charge ± 1

QGP Phase

- only ~ 1/2 of d.o.f., i.e. quarks and anti-quarks carry electric charge $\pm\frac{1}{3}, \pm\frac{2}{3}$

Heinz, Müller, and M.A., PRL (2000)
Also, Jeon and Koch, PRL (2000)

Charge Fluctuation at RHIC

- D-measure

$$D = 4\frac{\langle (\delta Q)^2 \rangle}{N_{ch}}$$

Predictions

QGP phase

$D \sim 1$

Hadron phase

$D \sim 4$

($D \sim 3$: hadron resonance gas)

Experimental Value

$D = 2.8 \pm 0.05$ (STAR)

$D \sim 4$ (PHENIX)

Well-Explained by Quark Recombination

Białas, PLB(2002)
Nonaka, Müller, Bass, M.A., PRC (2005)

Odd Power Fluctuation Moments

- Fluctuation of Conserved Charges: not subject to final state interactions
  - Usually even power fluctuations such as $\left\langle (\delta Q)^2 \right\rangle$ have been considered

One of exceptions: Stephanov (2008) in conjunction with CEP

  - Usual Fluctuations such as $\left\langle (\delta Q)^2 \right\rangle$: positive definite

⇨ Absolute values carry information of states (D-measure)

Asakawa, Heinz, Müller, Jeon, Koch

On the other hand,

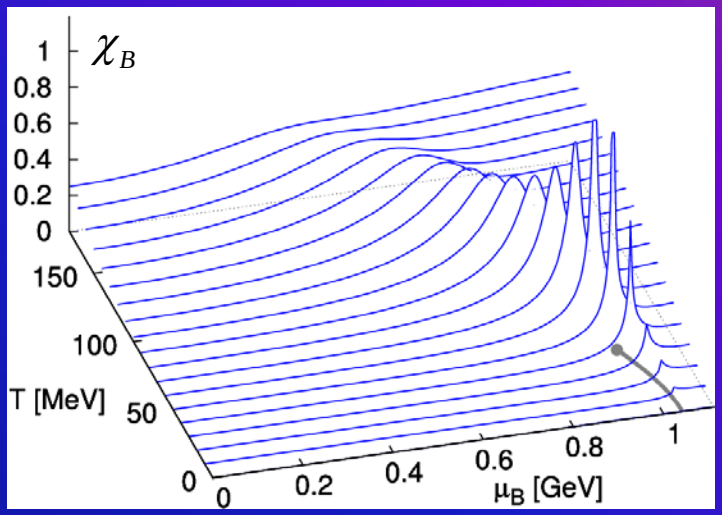
✓ Odd power fluctuations : NOT positive definite

- In general, do not vanish (exception, $\langle \delta A \rangle$)
- *Sign also carry information of states*

Physical Meaning of 3rd Fluc. Moment

χ_B : Baryon number susceptibility

in general, has a peak along phase transition



$\Rightarrow$ $\dfrac{\partial \chi_B}{\partial \mu_B}$ changes the sign at QCD phase boundary !

- In the Language of fluctuation moments:

$$\chi_B = -\frac{1}{V}\frac{\partial^2 \Omega}{\partial \mu_B^2} = \frac{\left\langle (\delta N_B)^2 \right\rangle}{VT}$$

$$\frac{\partial \chi_B}{\partial \mu_B} = -\frac{1}{V}\frac{\partial^3 \Omega}{\partial \mu_B^3} = \frac{\left\langle (\delta N_B)^3 \right\rangle}{VT^2} \equiv m_3(\mathrm{BBB})$$

more information than usual fluctuation

(Hopefully) More Easily Measured Moments

- Third Moment of Electric Charge Fluctuation

$$m_3(\mathrm{QQQ}) \equiv \frac{\left\langle (\delta N_Q)^3 \right\rangle}{VT^2} = -\frac{1}{V}\frac{\partial^3 \Omega}{\partial \mu_Q^3}$$

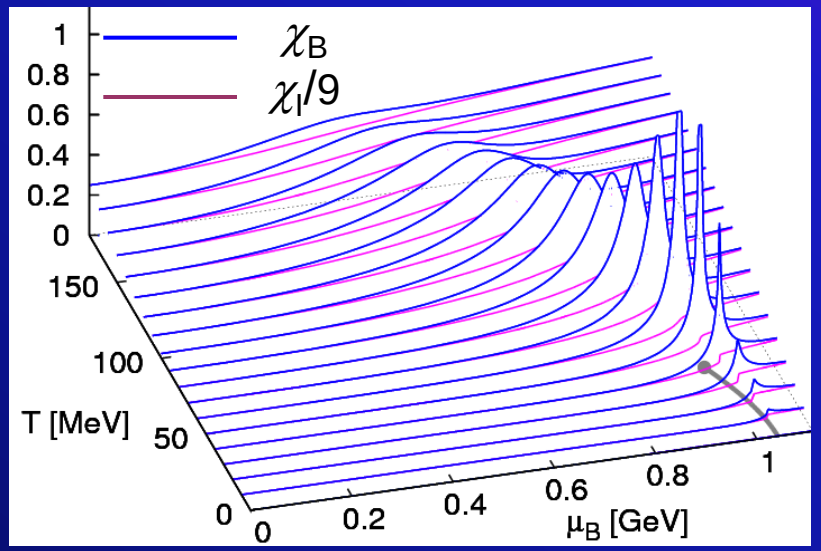
$$N_Q = \frac{2}{3}N_u - \frac{1}{3}N_d$$
$$= \frac{1}{2}N_B + \frac{1}{2}N_I$$

$$m_3(\mathrm{QQQ}) \equiv \frac{1}{8}\frac{\partial}{\partial \mu_B}\left(\frac{1}{27}\chi_B + \chi_I\right)$$

singular @CEP

iso-vector susceptibility
nonsingular when Isospin-symm.

Hatta and Stephanov 2002



Mixed Moments

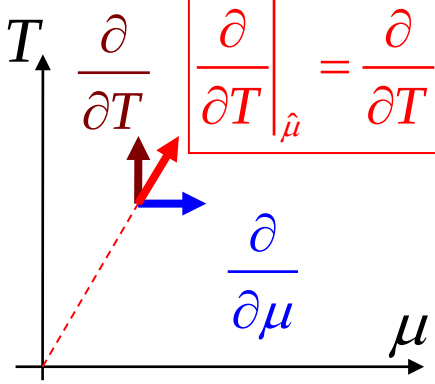
- Energy is also a conserved charge and mesurable !

$$m_3(\mathrm{BBE}) \equiv \frac{\left\langle (\delta N_B)^2 \delta E \right\rangle}{VT^3} = \frac{1}{T}\frac{\partial (T\chi_B)}{\partial T}\bigg|_{\hat{\mu}}$$

$$m_3(\mathrm{QQE}) \equiv \frac{\left\langle (\delta N_Q)^2 \delta E \right\rangle}{VT^3} = \frac{1}{T}\frac{\partial (T\chi_Q)}{\partial T}\bigg|_{\hat{\mu}}$$

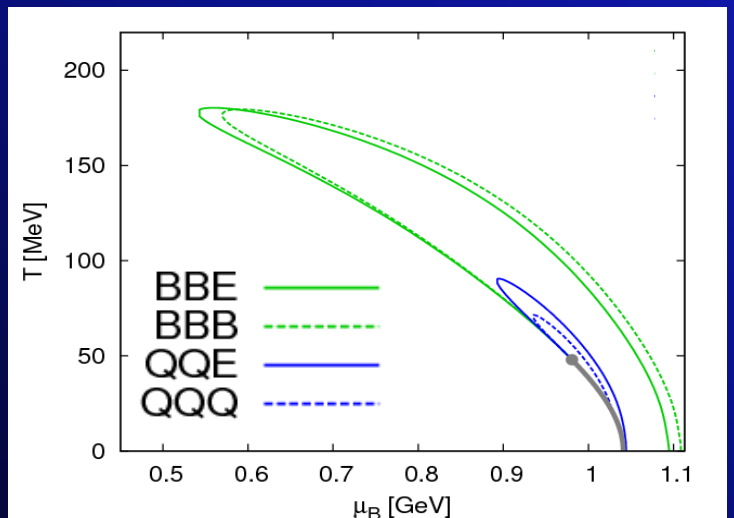
E : total energy in a subvolume

$$\hat{\mu} \equiv \mu / T$$



$$\frac{\partial}{\partial T}\bigg|_{\hat{\mu}} = \frac{\partial}{\partial T} + \frac{\mu}{T}\frac{\partial}{\partial \mu}$$

Regions with *Negative* fluctuation moments



- Result with the standard NJL parameters (N_f=2) (only as an example)

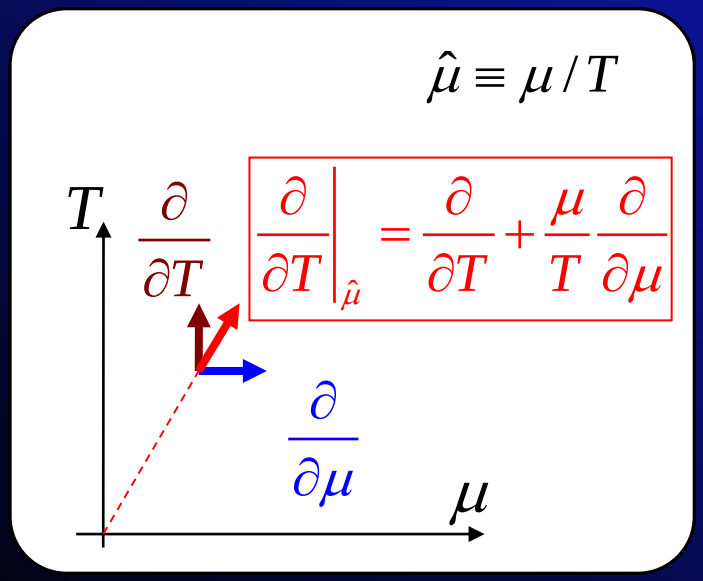
More and More 3rd Moments

$$m_3(\mathrm{EEE}) \equiv \frac{\left\langle (\delta E)^3 \right\rangle}{VT^5} = \frac{1}{T^3} \frac{\partial (T^2 C_{\hat{\mu}})}{\partial T}\bigg|_{\hat{\mu}}$$

$$m_3(\mathrm{BEE}) \equiv \frac{\left\langle \delta N_B (\delta E)^2 \right\rangle}{VT^4} \equiv \frac{1}{T} \frac{\partial C_{\hat{\mu}}}{\partial \mu_B}$$

$$m_3(\mathrm{QEE}) \equiv \frac{\left\langle \delta N_Q (\delta E)^2 \right\rangle}{VT^4} \equiv \frac{1}{2T} \frac{\partial C_{\hat{\mu}}}{\partial \mu_Q}$$

These quantities are expressed concisely as derivatives of "specific heat" at constant $\hat{\mu}$

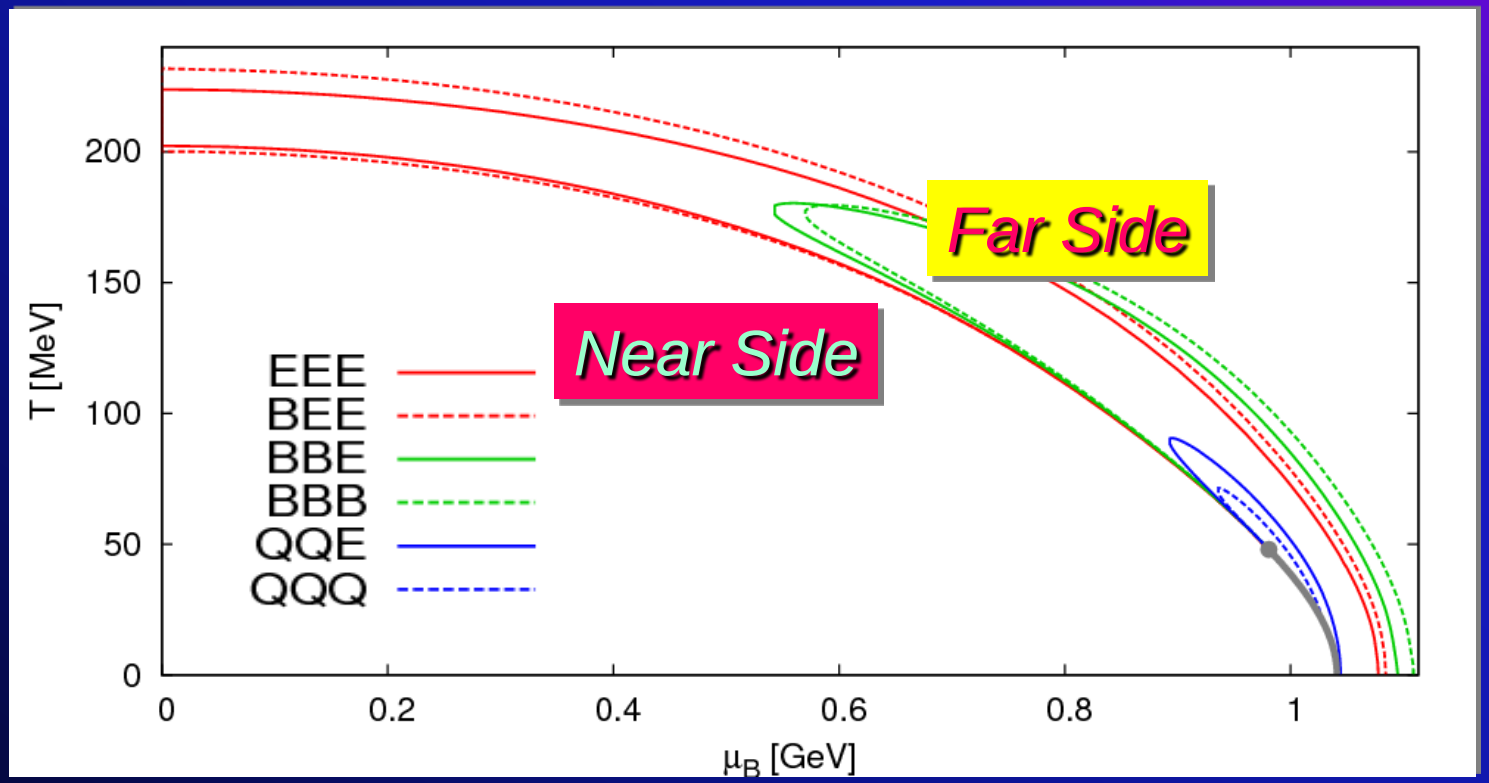


"specific heat" at constant $\hat{\mu}$

$$C_{\hat{\mu}} \equiv -T \left.\frac{\partial^2 \omega}{\partial T^2}\right|_{\hat{\mu}} = \frac{\left\langle (\delta E)^2 \right\rangle}{VT^2}$$

- diverges at critical end point
- peaks on phase transition line

Comparison of Various Moments



2-flavor NJL with standard parameters

G=5.5GeV^{-2}
m_q=5.5MeV
Λ=631MeV

- Different moments have different regions with negative moments

By comparing the signs of various moments, possible to pin down the origin of moments

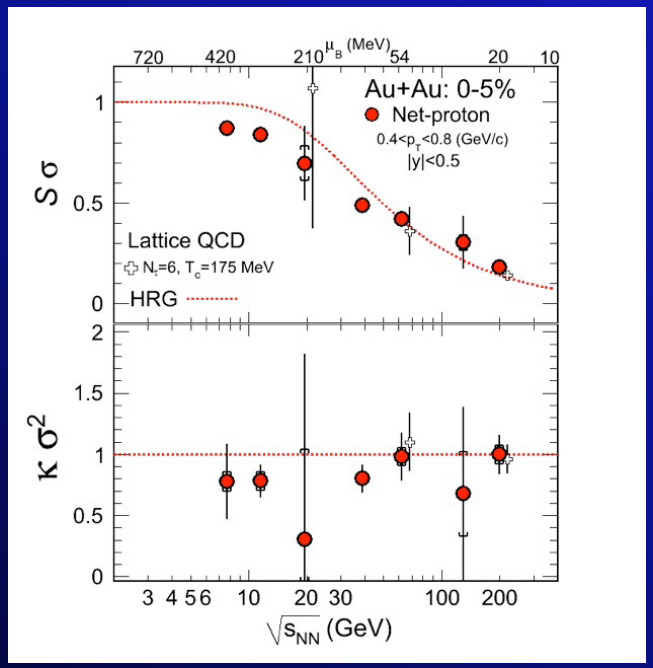
- Negative m_3(EEE) region extends to T-axis (in this particular model)
- Sign of m_3(EEE) may be used to estimate heat conductivity

Recent Progress: Proton Cumulants

- Proton Fluctuation has been attracting a lot of interest because it can be observed experimentally
- Proton Fluctuation diverges at CEP

Hatta and Stephanov, 2003

- Comparisons of experimental results and lattice predictions have been done (e.g. Gupta et al., Science 2011)



$$\chi_B^{(n)}\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)=\frac{1}{T^n}\frac{\partial^n}{\partial(\mu_B/T)^n}P\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)\Bigg|_{T/T_c}$$

$$S\sigma=\frac{T\chi_B^{(3)}}{\chi_B^{(2)}}$$

$$\kappa\sigma^2=\frac{T^2\chi_B^{(4)}}{\chi_B^{(2)}}$$

STAR, QM2011

Experiment: Net Proton
Theory: Net Baryon

Is this legitimate?

Kitazawa's Talk

Summary

- Final State Interaction and Critical Slowing Down

  ➡ Usual observables
  such as large fluctuation, critical opalescence, ...etc. do not work

- Conserved Charges and Higher Moments:

  i) Third Fluctuation Moments of Conserved Charges
  take negative values in regions on the FAR SIDE of Phase Transition
  (more information!)

  ii) Different Moments have different regions with negative moments

  By comparing different moments, possible to pin down the origin
  of hot matter, get an idea about heat conductivity...etc.

- Proton Number Cumulants and Baryon Number Ones:

  will be discussed in Kitazawa's talk