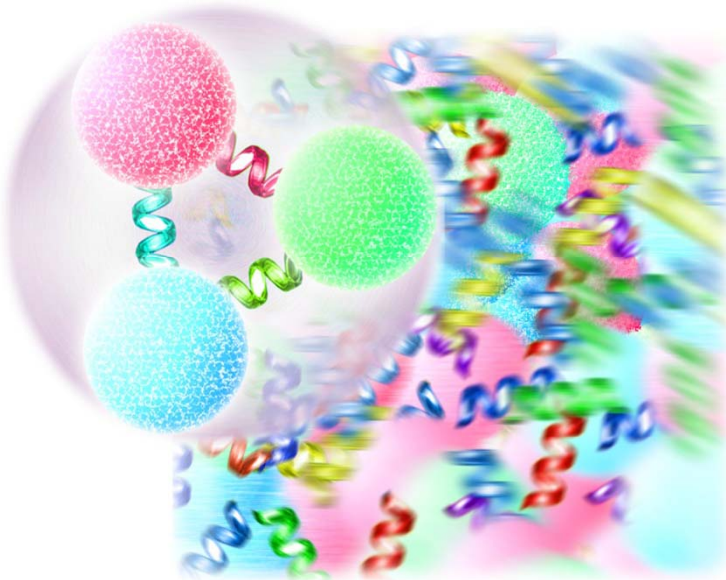
Talk @ Workshop on Fluctuations, Correlations and RHIC Low Energy Run
Brookhaven OCT.3rd, 2011

"Three Quarks for Muster Mark"
--- Lessons Learned from Susceptibilities



Jinfeng Liao

Indiana University, Physics Dept. & CEEM

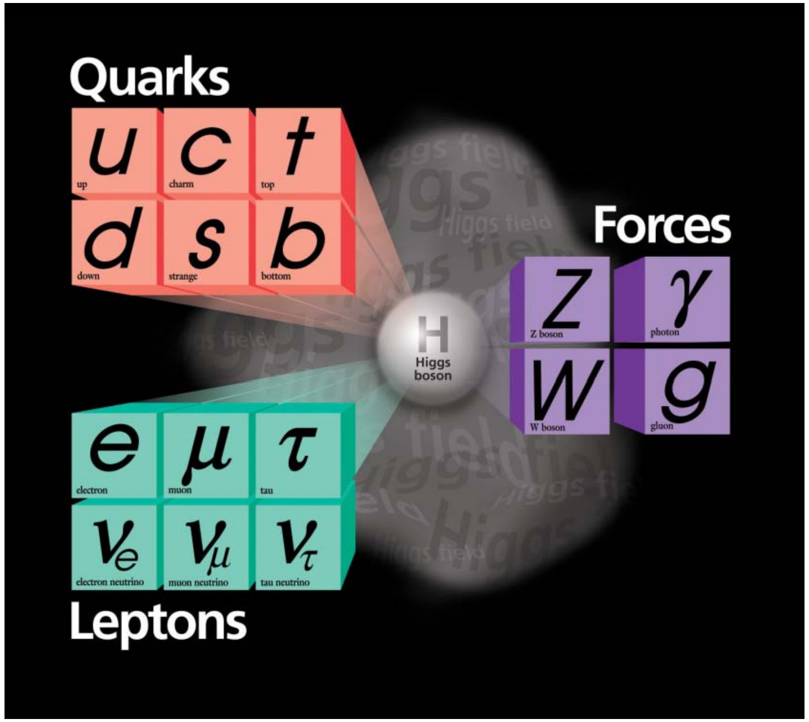
RIKEN BNL Research Center

Outline

- Susceptibilities: from hadrons to quarks
- Quasi-particles: are we there yet?
- Strongly interacting quarks: holography?
- The uneasy "color": to be bound or not to be?
- Summary

References:
JL & Shuryak, Phy. Rev. D 73(2006)014509;
Kim & JL, Nucl. Phys. B 822(2009)201.

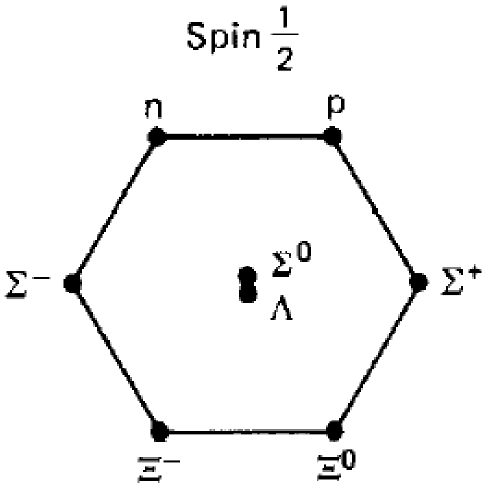
Building Blocks of the Standard Model

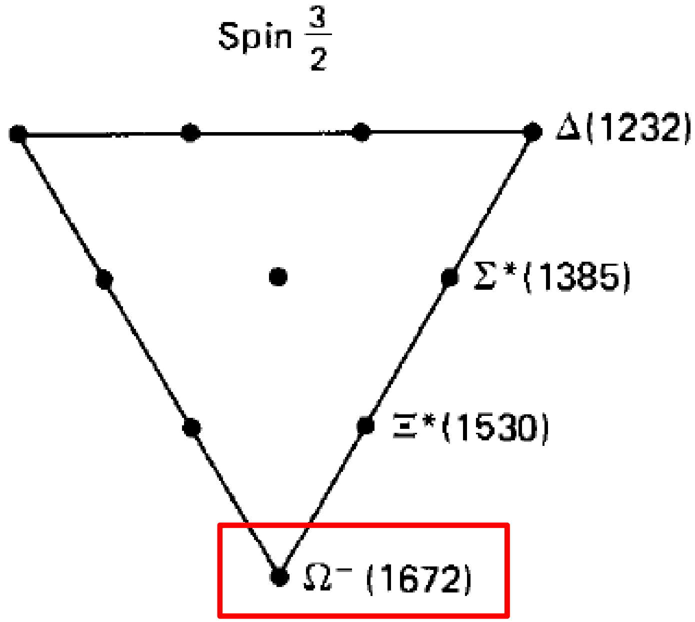


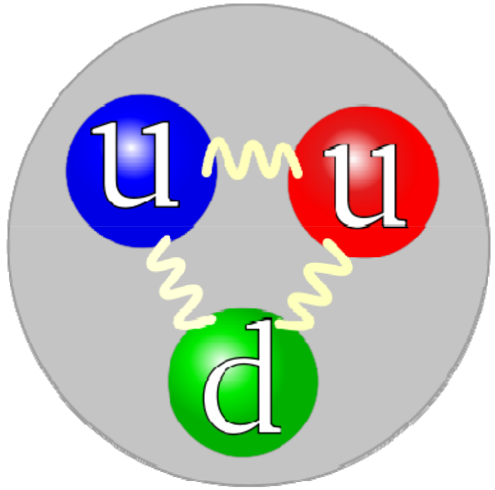
Matter → Atom → Nuclei → Hadrons(Baryons, Mesons) → Quarks

Color confinement

"Three Quarks for Muster Mark"



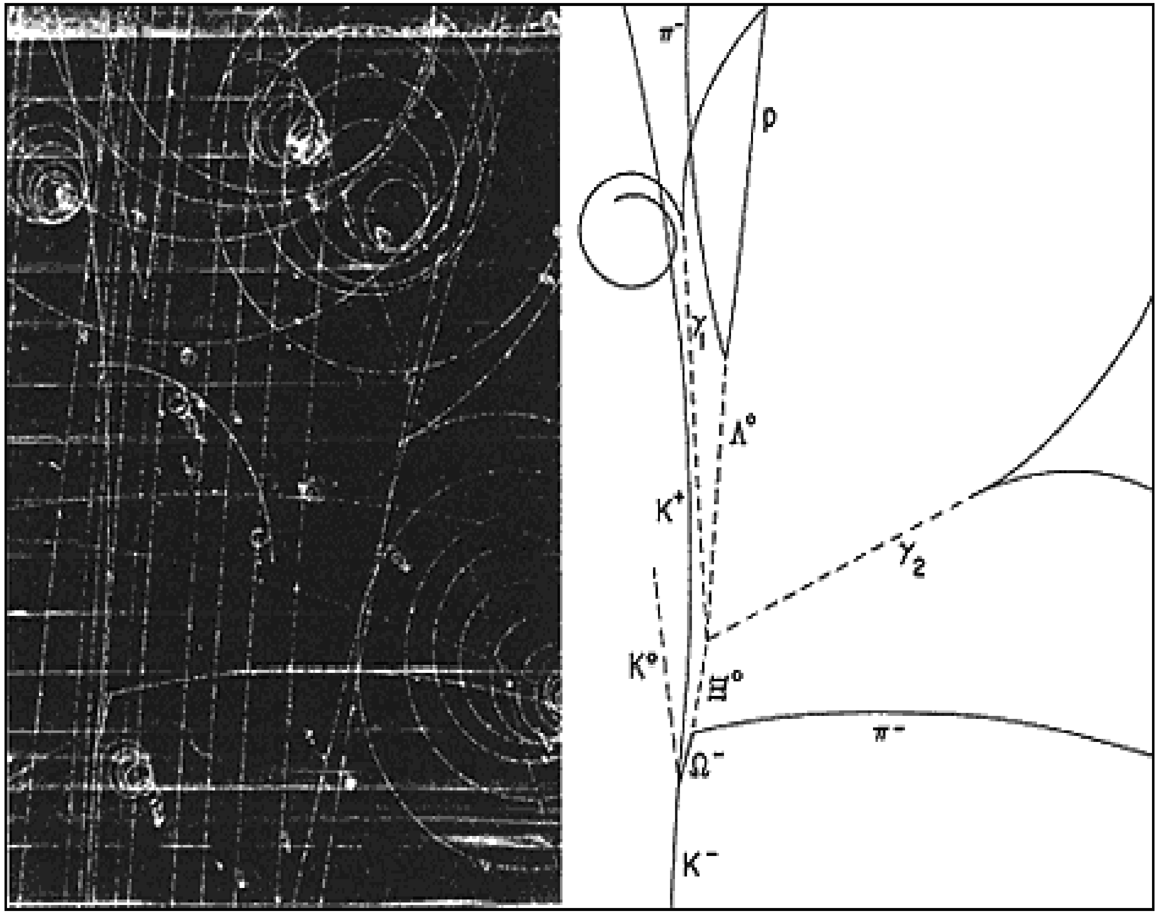




"Then, in one of my occasional perusals of Finnegans Wake, by James Joyce, I came across the word "quark" in the phrase ***"Three quarks for Muster Mark".*** ......
In any case, **the number three fitted perfectly the way quarks occur in nature."**

Today we know:
The number 3 is a "magic number" not only for 3 light favors, but more importantly for the 3 colors!

The Omega-Minus Baryon



N. Samios

$$K^- + p \to \Omega^- + K^+ + K^0$$

$$\Omega^- \to \Xi^0 + \pi^-$$

$$\Xi^0 \to \Lambda^0 + \pi^0$$

$$\pi^0 \to \gamma_1 + \gamma_2$$

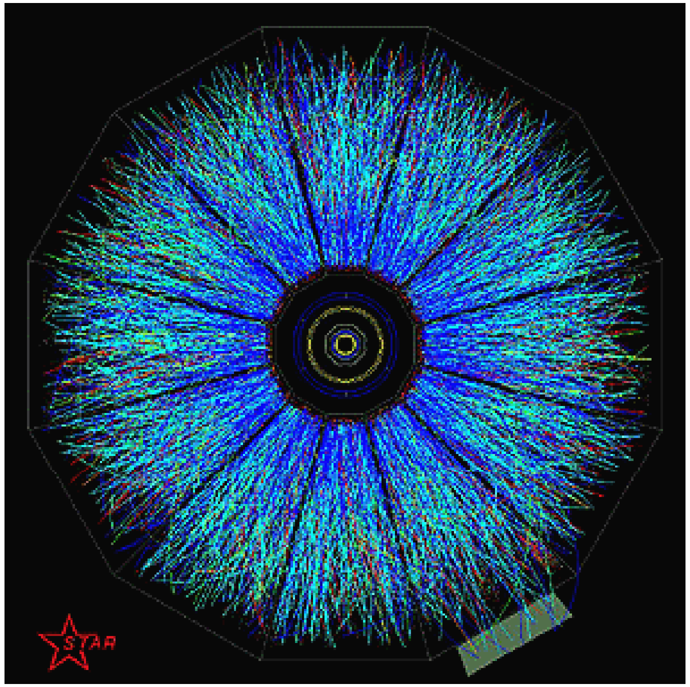
$$\gamma_2 \to e^+ + e^-$$

$$\gamma_1 \to e^+ + e^-$$

$$\Lambda^0 \to \pi^- + p.$$

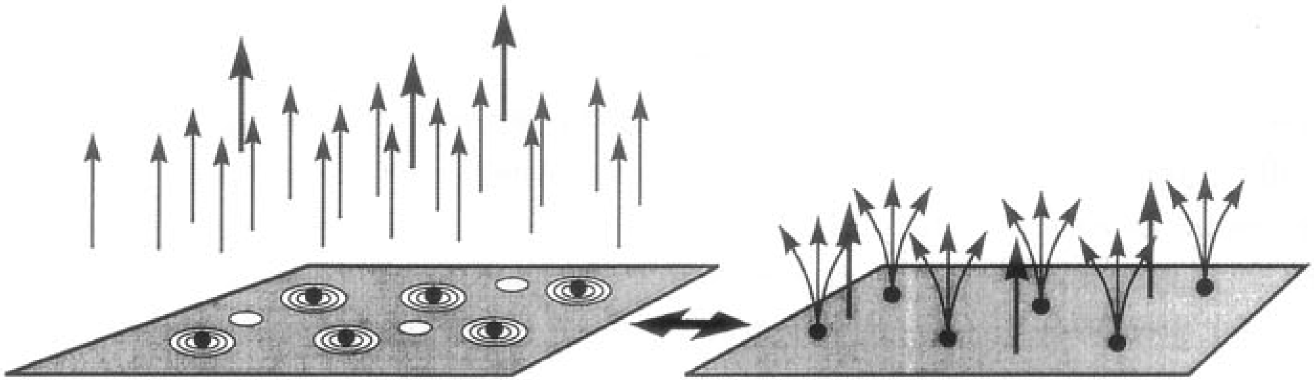
In the hadronic world:
the baryon number is always transmitted in its entire unity !

Forty Years Later @ Brookhaven

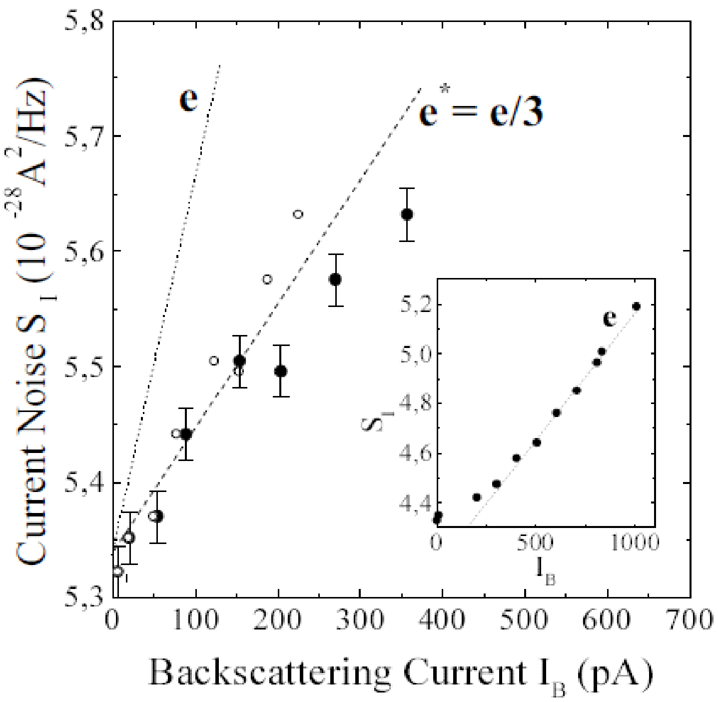


The primordial hot soup directly made of quarks and gluons, QGP,
has been created, in which *we think*
the baryon number is to be transmitted in its broken fraction, 1/3,
that would be otherwise not seen !

In Search of Fractional Charge



Fractional Quantum Hall State: quasi-particle with charge (1/3) of electron's



New state of matter

→→

New degrees of freedom

& conserved charge carrier

→→

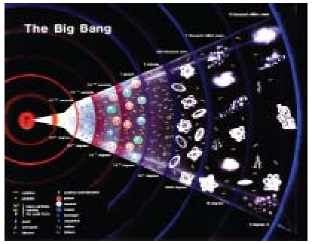
New basic UNIT for transmission

of conserved charge

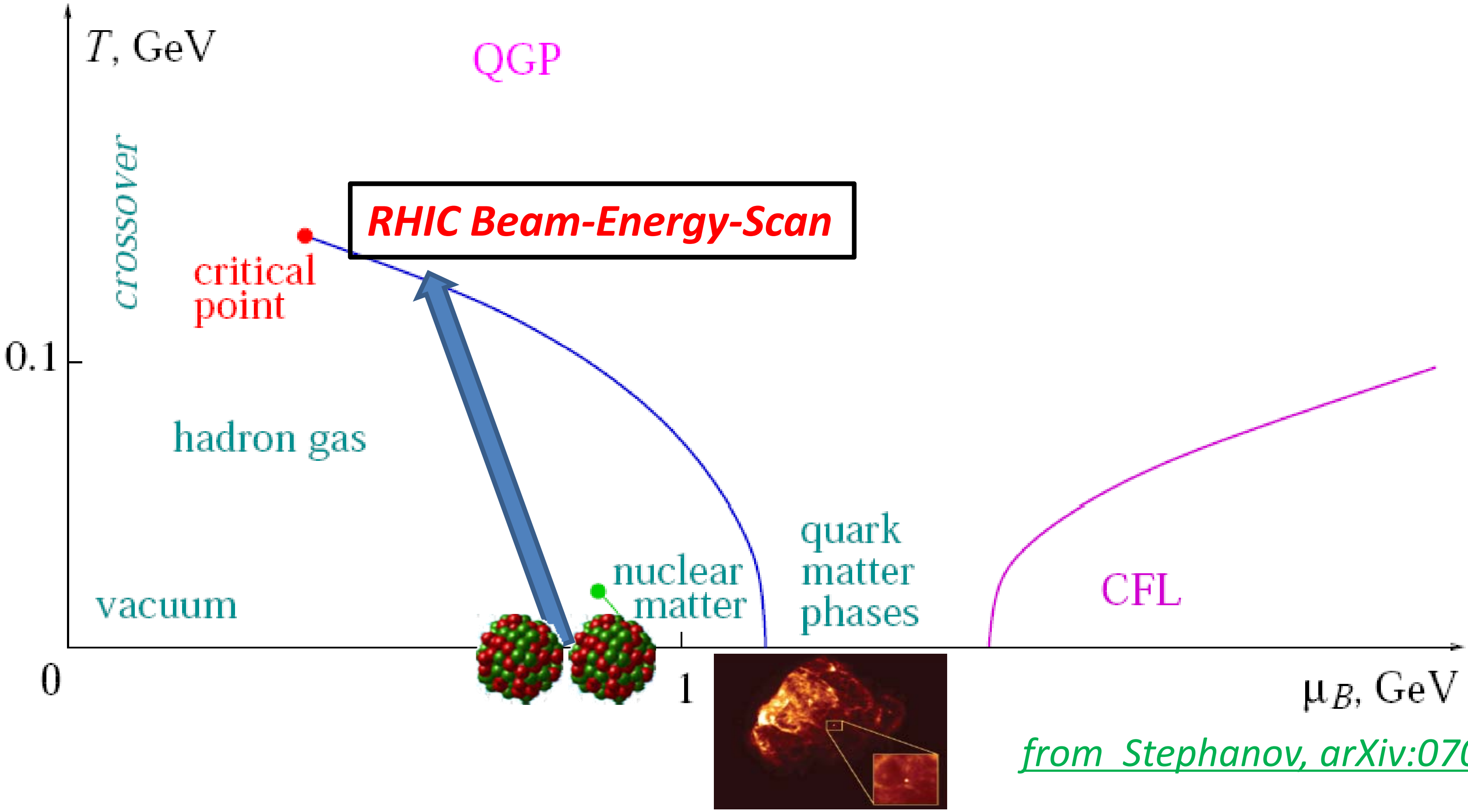
→→

New behavior of fluctuation & correlation

Phases of QCD



Looking for distinctive patterns of fluctuations & correlations
→Heavy ion collisions (HIC) in laboratory
→ Lattice QCD on super-computers

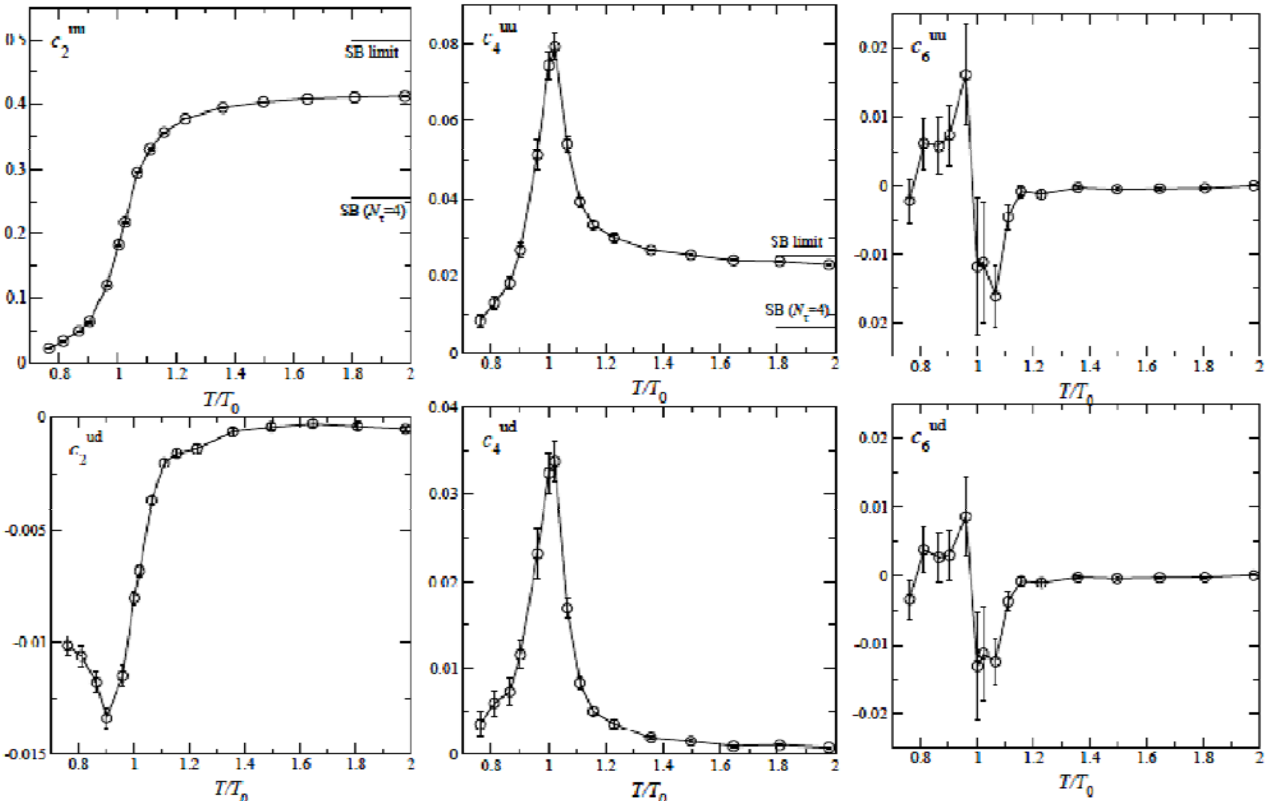


from Stephanov, arXiv:0701002

Susceptibilities

$$\boxed{\langle \delta Q_i \delta Q_j \rangle \;=\; T^2 \frac{\partial^2}{\partial \mu_i \partial \mu_j} \log(Z) \equiv V T \chi_{i,j}}$$

$$\chi^{(n_i,n_j,n_k)} \equiv \frac{1}{VT} \frac{\partial^{n_i}}{\partial(\mu_i/T)^{n_i}} \frac{\partial^{n_j}}{\partial(\mu_j/T)^{n_j}} \frac{\partial^{n_k}}{\partial(\mu_k/T)^{n_k}} \log Z.$$



Susceptibilities:
Taylor coefficients
for expanding the
pressure in terms of
chemical potentials

See reviews in e.g. V. Koch, arXiv:0701002

Susceptibilities: Benchmark I

$$P(T,\mu) = T^4 \sum_{n=0}^{\infty} \frac{d_n(T)}{n!} \left(\frac{\mu}{T}\right)^n$$

Consider a free gas of heavy particles <u>(non-relativistic, N.R.),</u> with baryon number B, and mass M>>T:

$$\boxed{d_n^{\text{free}}\Big|_{\text{NR}} = N_i \left(\frac{M}{2\pi T}\right)^{\frac{3}{2}} e^{-\frac{M}{T}} \times 2B^n \equiv \mathcal{F}\left[\frac{M}{T}\right] B^n.}$$

- Same T-dependence for all orders
- d_n is positive, proportional to B^n
- at any order the ratio d_(n+2)/d_n = B^2

Susceptibilities: Benchmark II

$$P(T,\mu) = T^4 \sum_{n=0}^{\infty} \frac{d_n(T)}{n!} \left(\frac{\mu}{T}\right)^n$$

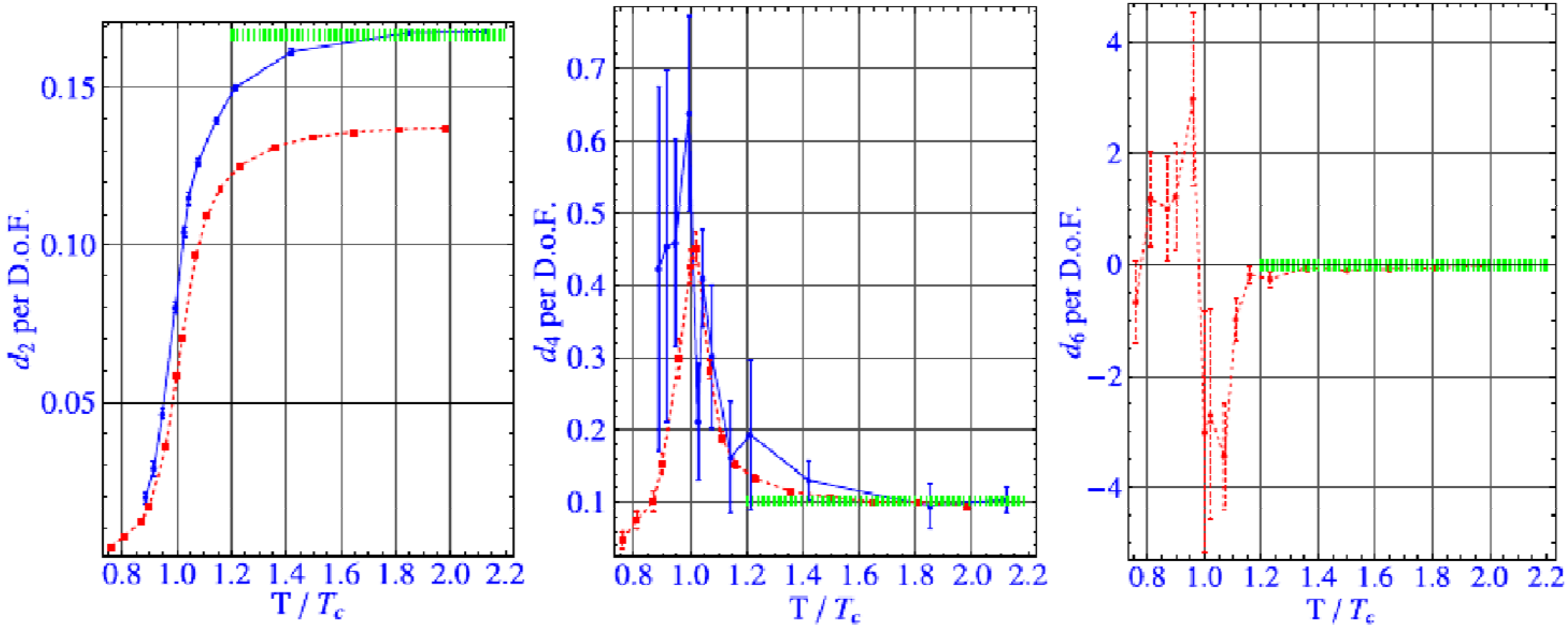
Consider a free gas of massless particles (Stefan-Boltzmann, S.B.), with baryon number B, and mass M=0:

$$\boxed{d_2^{\mathrm{free}}\big|_{\mathrm{UR}} = N_i \frac{B^2}{6}, \qquad d_4^{\mathrm{free}}\big|_{\mathrm{UR}} = N_i \frac{B^4}{\pi^2}, \qquad d_{n>4}^{\mathrm{free}}\big|_{\mathrm{UR}} = 0.}$$

- No T-dependence, up to n=4
- d_n is positive, proportional to B^n
- d_4/d_2 ~ B^2

Susceptibilities from Lattice QCD

LQCD data: Bielefeld-BNL 2005 2-flavor with heavy pion; 2009 “almost physical” pion mass



Below Tc: behavior close to the N.R.-benchmark , ratios~B=1

i.e. hadronic(baryonic) resonance gas (verified by many)

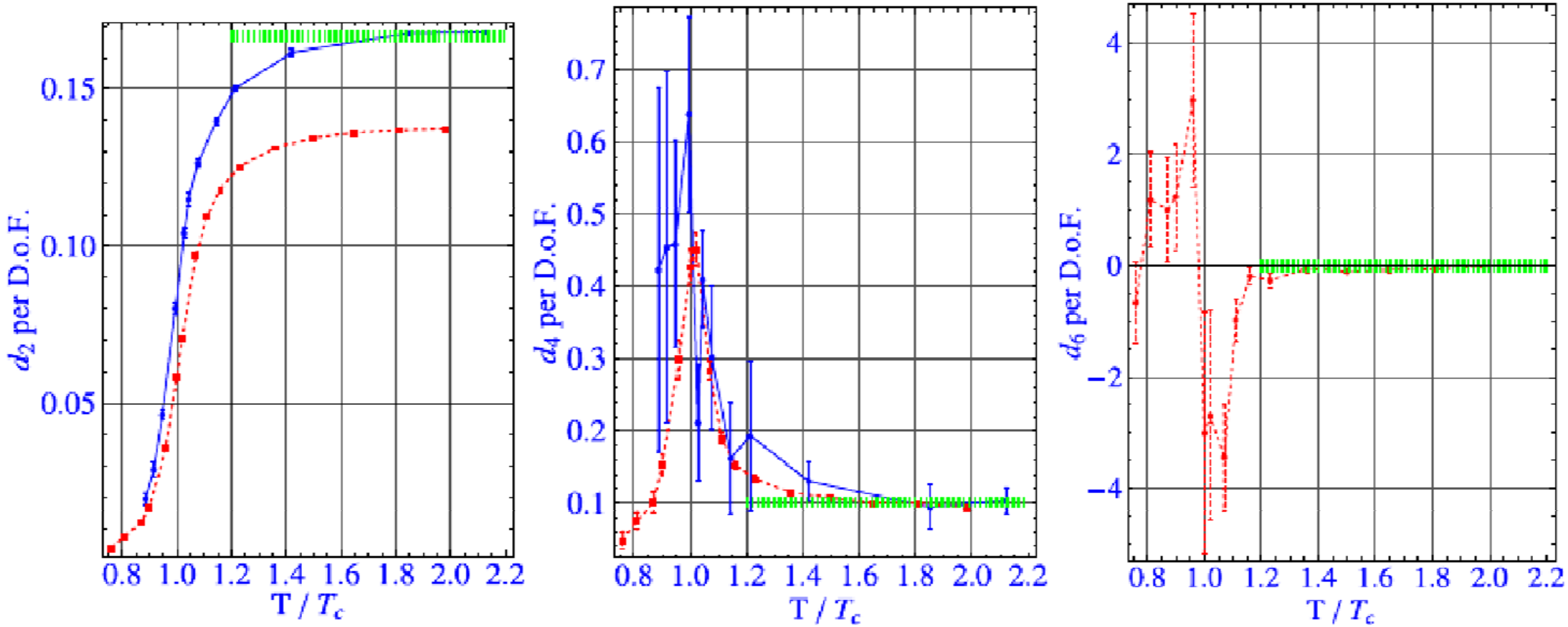
Well above Tc (at least 1.5Tc up): behavior close to S.B., ratios~B=1/3

i.e. seemingly (quasi-)quarks that are VERY light ! ?

[caution: stay tuned till conclusive and agreed data from varied groups]

Susceptibilities from Lattice QCD

LQCD data: Bielefeld-BNL 2005 2-flavor with heavy pion; 2009 “almost physical” pion mass



Yet, a few nontrivial questions to be answered:

- **what about the near-Tc plasma? The behavior there resembles neither!**
- **what are those nontrivial structures --- peaks and wiggles ---- due to?**
- **is the nearly S.B. behavior at higher T in accord with our integrated picture of QGP in this T-region?**

Different models have varied answers --- let’s examine them.

Quasi-particle Models

Free gas of quark quasi-particles with medium generated mass: M(T , \mu)

$$d_2 = \frac{\partial(n_B/T^3)}{\partial\tilde{\mu}}\Big|_{\mu=0} = -\frac{2g}{2\pi^2}\int dx x^2 n^2 F^{(1)}(\epsilon_0),$$

$$d_4 = \frac{\partial^3(n_B/T^3)}{\partial\tilde{\mu}^3}\Big|_{\mu=0}$$
$$= -\frac{2g}{2\pi^2}\int dx x^2\Big[n^4 F^{(3)}(\epsilon_0) + 3n^2 F^{(2)}(\epsilon_0)\frac{\tilde{m}_0}{\epsilon_0} \times\Big(\frac{\partial^2\tilde{m}}{\partial\tilde{\mu}^2}\Big|_{\mu=0}\Big)\Big],$$

$$d_6 = \frac{\partial^5(n_B/T^3)}{\partial\tilde{\mu}^5}\Big|_{\mu=0}$$
$$= -\frac{2g}{2\pi^2}\int dx x^2\Big[n^6 F^{(5)}(\epsilon_0) + 10n^4 F^{(4)}(\epsilon_0)\frac{\tilde{m}_0}{\epsilon_0} \times\Big(\frac{\partial^2\tilde{m}}{\partial\tilde{\mu}^2}\Big|_{\mu=0}\Big) + 15n^2 F^{(3)}(\epsilon_0)\frac{\tilde{m}_0^2}{\epsilon_0^2}\Big(\frac{\partial^2\tilde{m}}{\partial\tilde{\mu}^2}\Big|_{\mu=0}\Big)^2 + 5n^2 F^{(2)}(\epsilon_0)\Big(\frac{\tilde{m}_0}{\epsilon_0}\Big(\frac{\partial^4\tilde{m}}{\partial\tilde{\mu}^4}\Big|_{\mu=0}\Big) + \frac{3x^2}{x^2+\tilde{m}_0{}^2}\Big(\frac{\partial^2\tilde{m}}{\partial\tilde{\mu}^2}\Big|_{\mu=0}\Big)^2\Big)\Big].$$

The main message:

$$d_2\,(T) \rightarrow \{M\}_{T,\mu=0}$$

$$d_4\,(T) \rightarrow \Big\{M, \frac{d^2 M}{d\mu^2}\Big\}_{T,\mu=0}$$

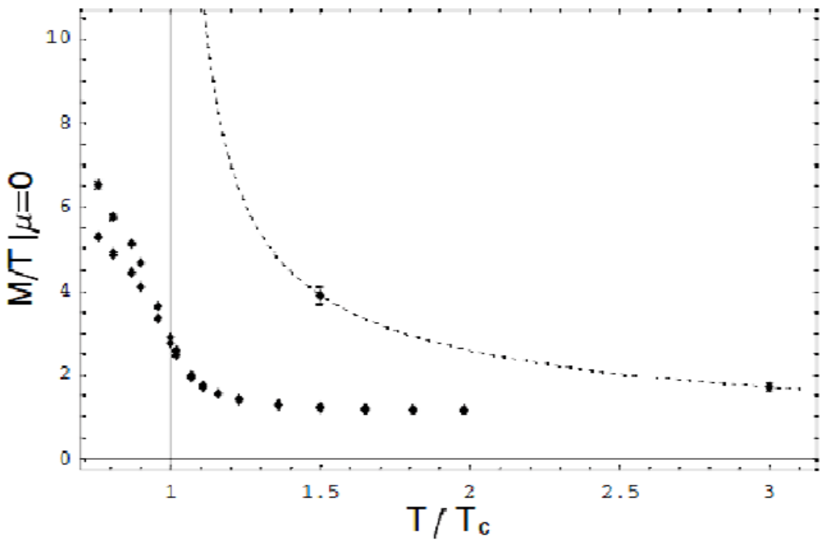
$$d_4\,(T) \rightarrow \Big\{M, \frac{d^2 M}{d\mu^2}, \frac{d^4 M}{d\mu^4}\Big\}_{T,\mu=0}$$

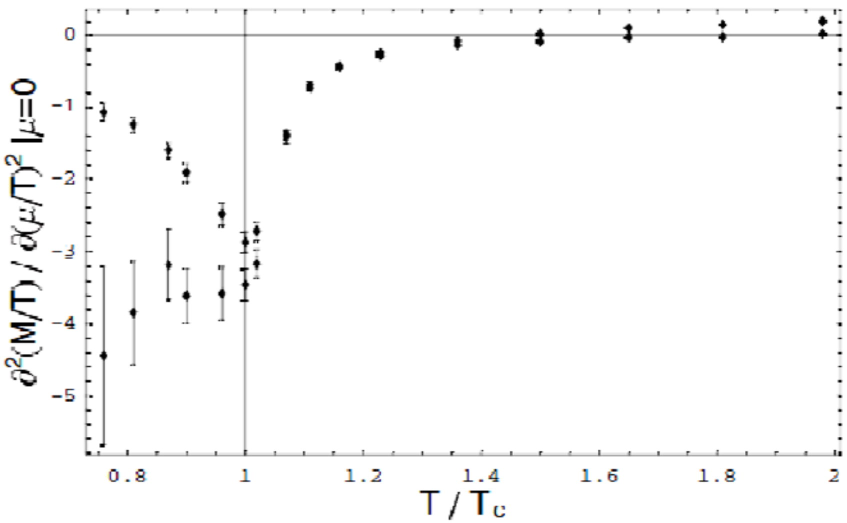
(

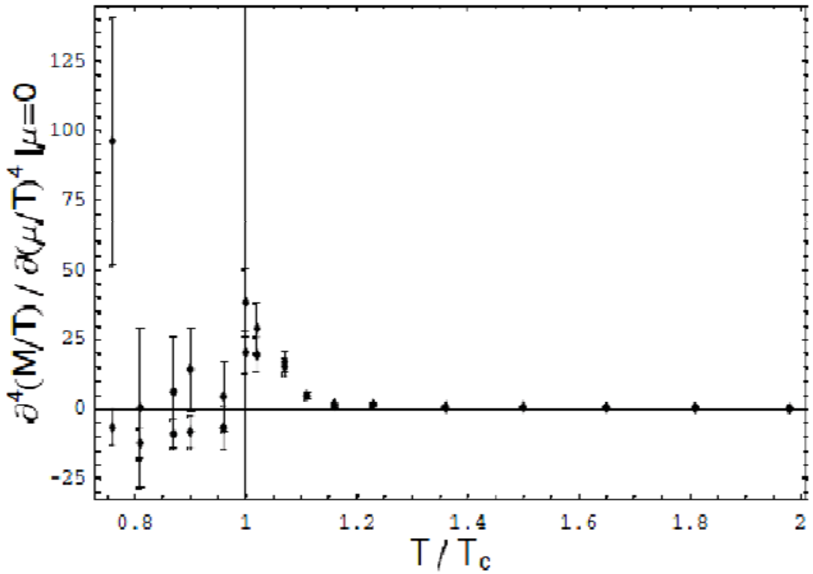
JL & Shuryak, Phy. Rev. D 73(2006)014509.

Quasi-particle Models

Free gas of quark quasi-particles with medium generated mass: M(T , \mu)





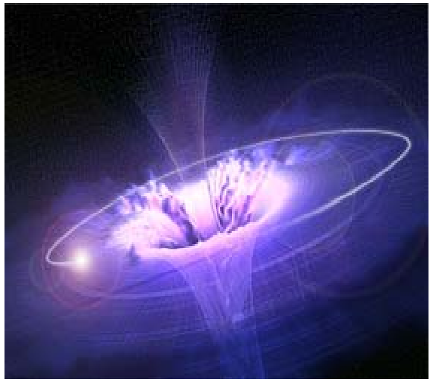


<u>Issues with quasi-particle models:</u>

- dynamical explanation of such dependence?
  e.g. HTL not generating these…
- masses compared with direct lattice results?
  e.g. M_g/q ~ 1-3 T (Petreczky…; Karsch…)
- Polyakov loop suppression ?!
- what about the “perfect fluid” and quenching?
  → susceptibilities of quarks at strong coupling?

Susceptibilities from Holography

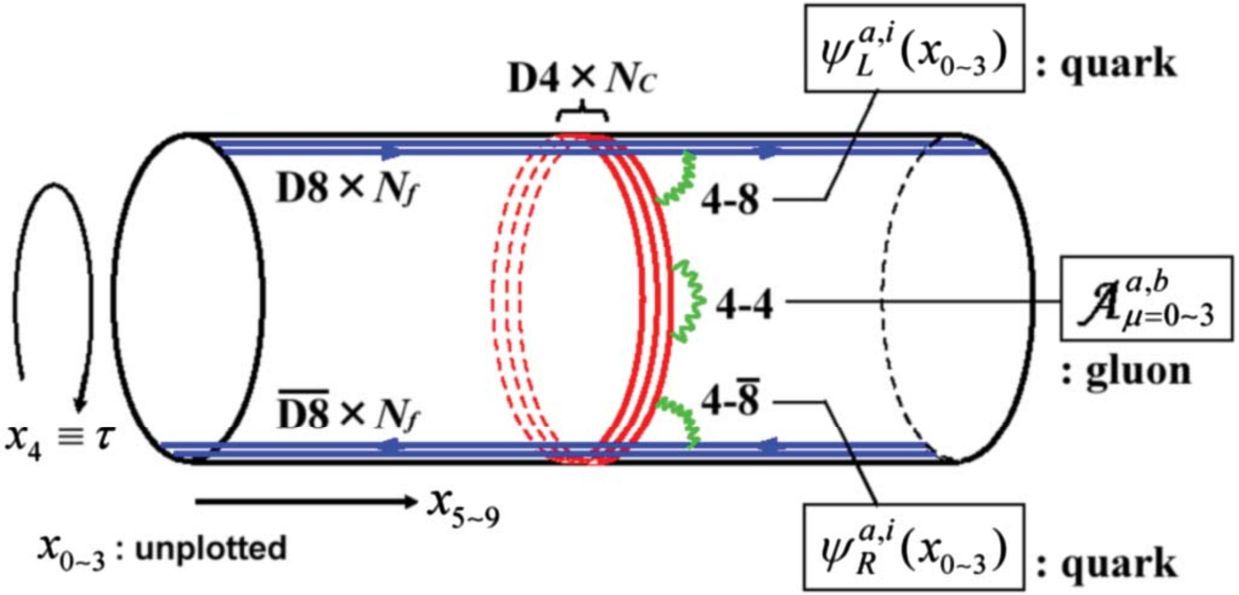
RHIC produces the sQGP: we need benchmarks of susceptibilities at strong coupling!
May holography provide a useful benchmark as in the case of e.g. shear viscosity?



$$s(\lambda \to \infty) / s(\lambda = 0) = 3/4$$

$$\eta / s \geq 1/(4\pi)$$

For baryonic susceptibilities: Sakai-Sugimoto model (D4/D8 branes)



Susceptibilities from Holography

Thermodynamics of QGP phase in Sakai-Sugimoto Model:

$$P_{\mathrm{QGP}}[T, d(T,\mu)] = \left[\frac{2}{7}\Gamma_A d^{\frac{7}{5}} + \frac{2}{7}u_T(d^2+u_T^5)^{\frac{1}{2}} - \frac{2}{7}u_T d\, {}_2\mathrm{F}_1\left(\frac{1}{5},\frac{1}{2};\frac{6}{5};-\frac{u_T^5}{d^2}\right)\right],$$

$$\Gamma_A d^{\frac{2}{5}} - u_T\, {}_2\mathrm{F}_1\left(\frac{1}{5},\frac{1}{2};\frac{6}{5};-\frac{u_T^5}{d^2}\right) - \mu = 0, \quad \Gamma_A = \frac{\Gamma(\frac{3}{10})\Gamma(\frac{6}{5})}{\sqrt{\pi}}.$$

Results for susceptibilities in QGP phase from such a holographic model of QCD:

$$d_n = \xi_n N_s N_c N_f \left(\frac{1}{\lambda\tilde{T}}\right)^{n-3}$$

$\lambda = g^2 N_c$

$\tilde{T} = T / T_c$

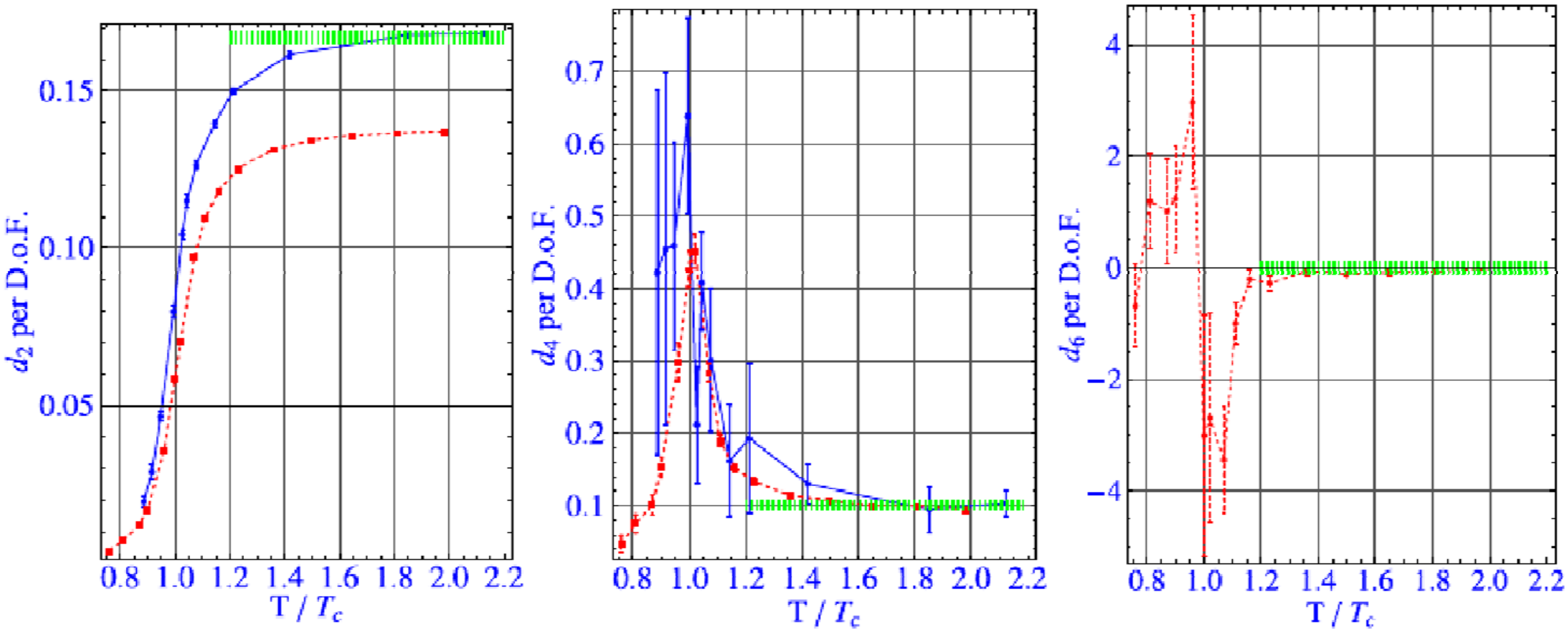
$$\boxed{\frac{d_2}{N_s N_f N_c} \approx 0.012\cdot\lambda\tilde{T}, \qquad \frac{d_4}{N_s N_f N_c} \approx \frac{0.37}{\lambda\tilde{T}}, \qquad \frac{d_6}{N_s N_f N_c} \approx -\frac{26}{\lambda^3\tilde{T}^3}.}$$

Very interesting dynamical feature:

- **alternating signs (beyond 6-th order as we checked)**
- **strong coupling suppressing higher fluctuations while enhancing leading order**
- **temperature dependence is unusual as well**

Kim & JL, Nucl. Phys. B 822(2009)201.

A Quick Comparison

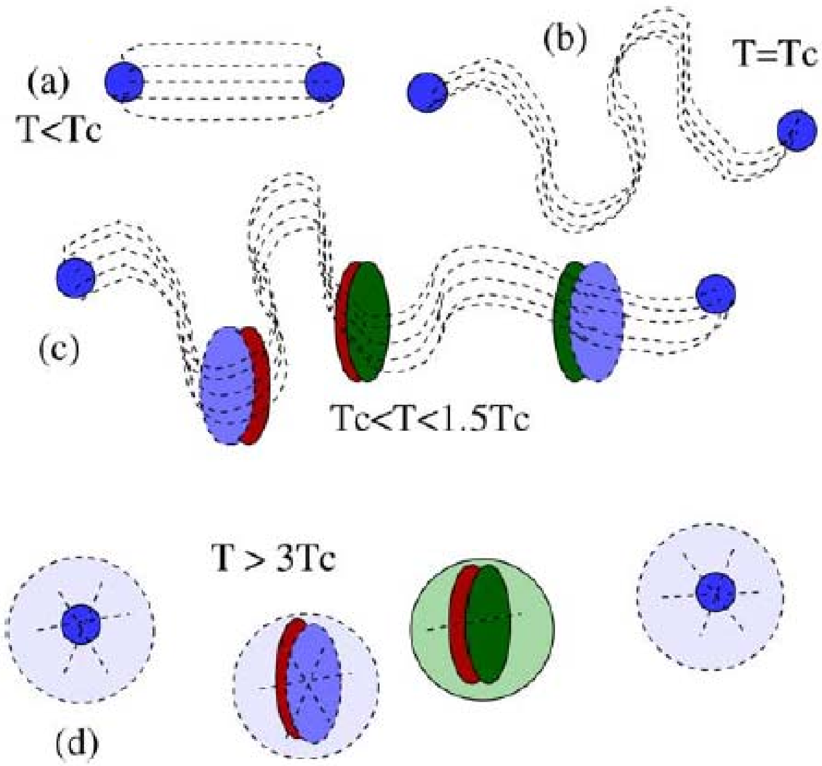


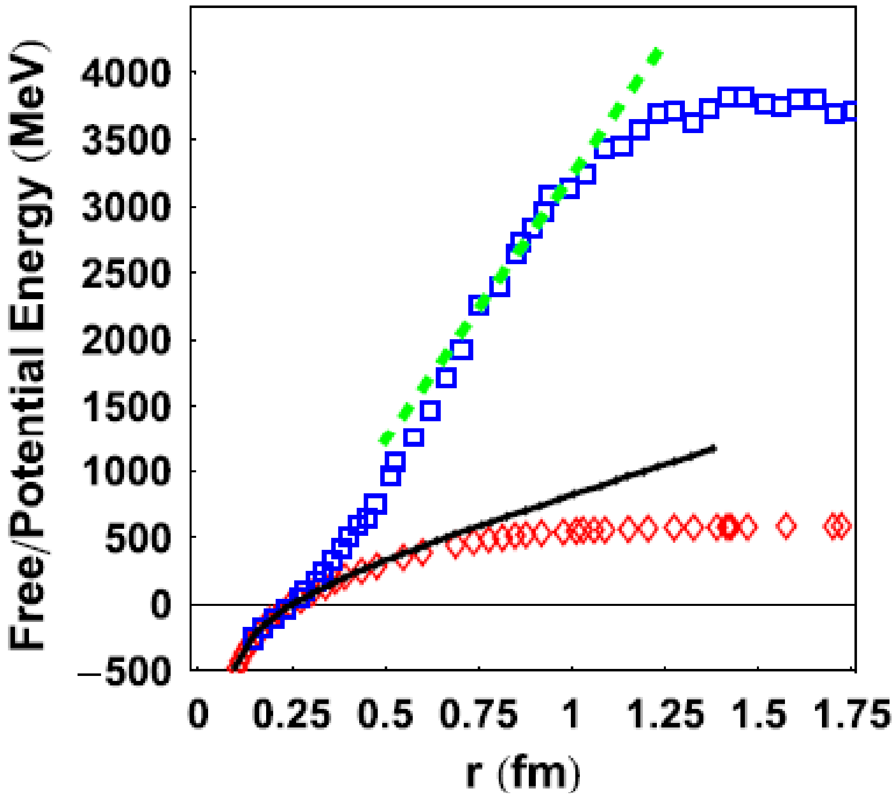
$$\boxed{\frac{d_2}{N_s N_f N_c} \approx 0.012 \cdot \lambda \tilde{T}, \qquad \frac{d_4}{N_s N_f N_c} \approx \frac{0.37}{\lambda \tilde{T}}, \qquad \frac{d_6}{N_s N_f N_c} \approx -\frac{26}{\lambda^3 \tilde{T}^3}.}$$

- **qualitatively similar near Tc, but not quantitatively, and need further understanding… (as in all holographic calculations)**
- **a useful (but only a) benchmark for how quarks might contribute to susceptibilities at strong coupling**

Back to Baryons: Dead or Alive in sQGP?

- RHIC phenomenology → collective flow, jet quenching,… → strongly coupled!
- Lattice QCD → strong screening kicks in late; Polyakov line restores late;
  VERY strong potential between color charges!





JL&Shuryak,
Nucl. Phys. A 775(2006)224.

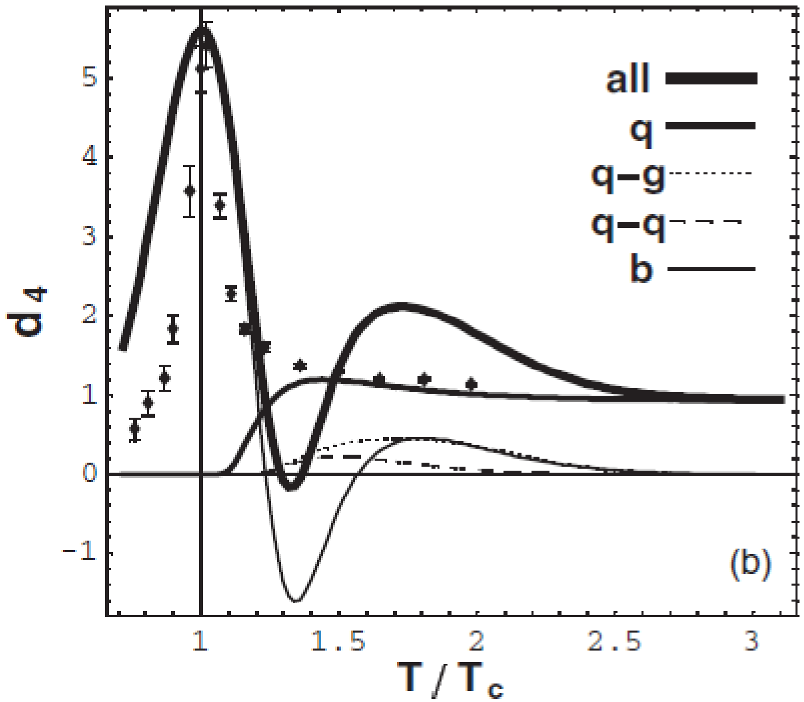
JL&Shuryak,
Phy. Rev. D 82(2010)094007.

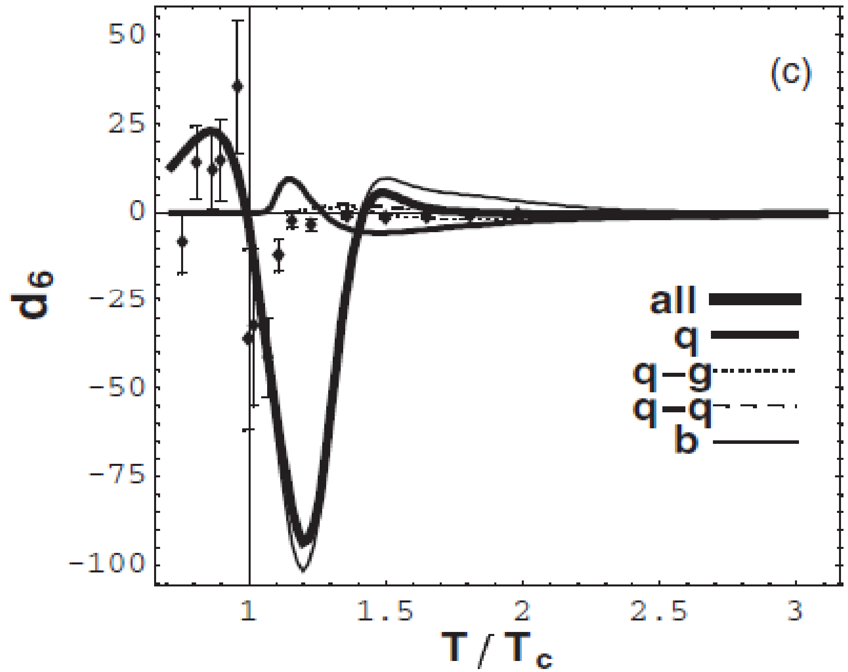
Susceptibilities: Baryons v.s. Quarks

Thermal factor : $e^{-M_q/T}$ v.s. $e^{-M_B/T}$

Charge factor : $\left(B_q = \frac{1}{3}\right)^n$ v.s. $\left(B_B = \frac{1}{3}\right)^n$

Baryons, or baryonic correlations, can contribute much more prominently in higher order susceptibilities, particularly the *4-th and 6-th orders*!





Baryons dominate the near Tc peaks and wiggles in the 4-th and 6-th order.

JL & Shuryak, Phy. Rev. D 73(2006)014509.

Cross-Flavor Susceptibilities

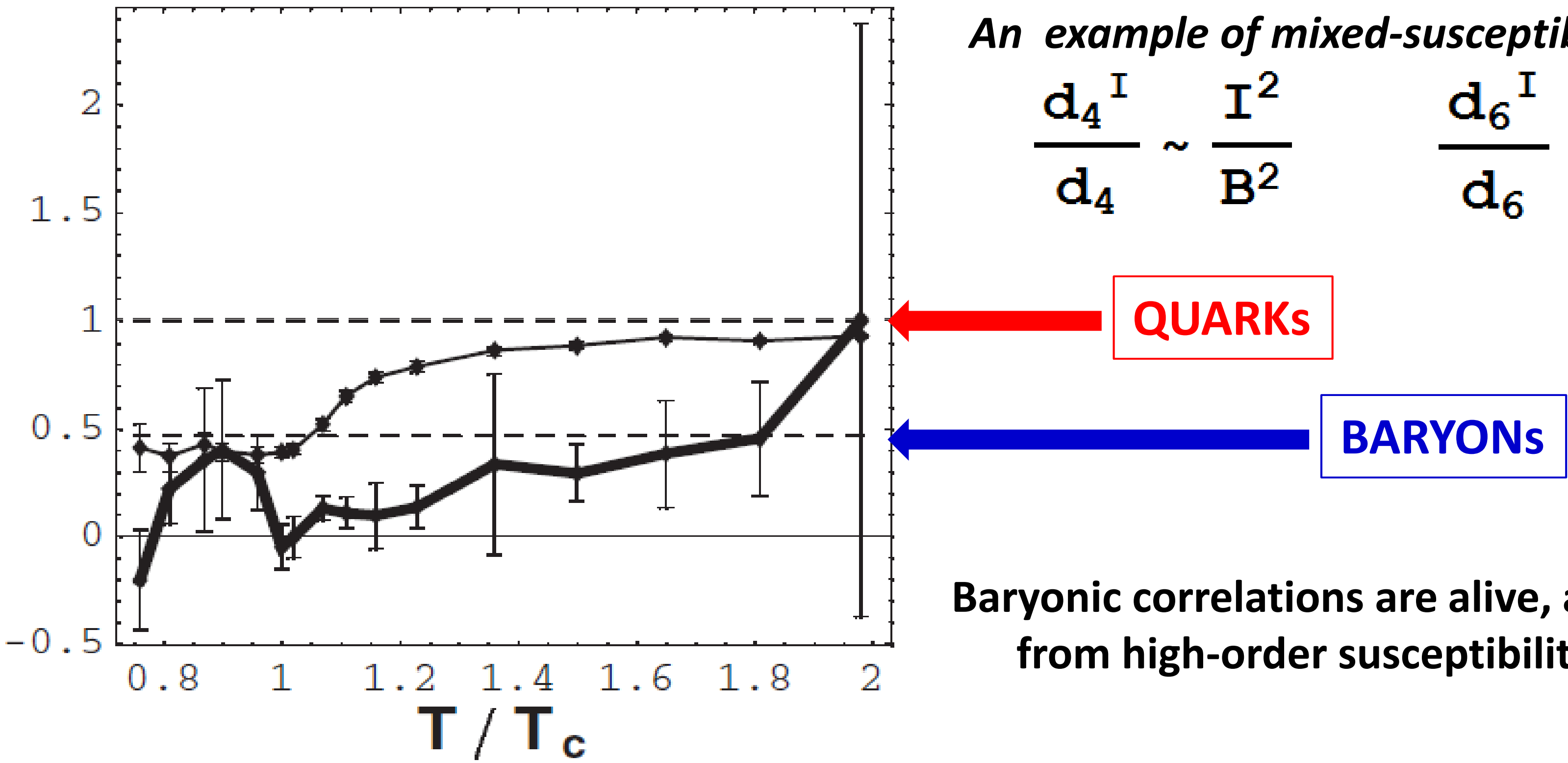
There are more conserved charges (within QCD): Baryon , Isospin , Strangeness
→ Provides additional probes of charge carriers

N : I = 1 / 2 , B = 1

Δ : I = 3 / 2 , B = 1

Q (u, d) :

I = 1 / 2, B = 1 / 3



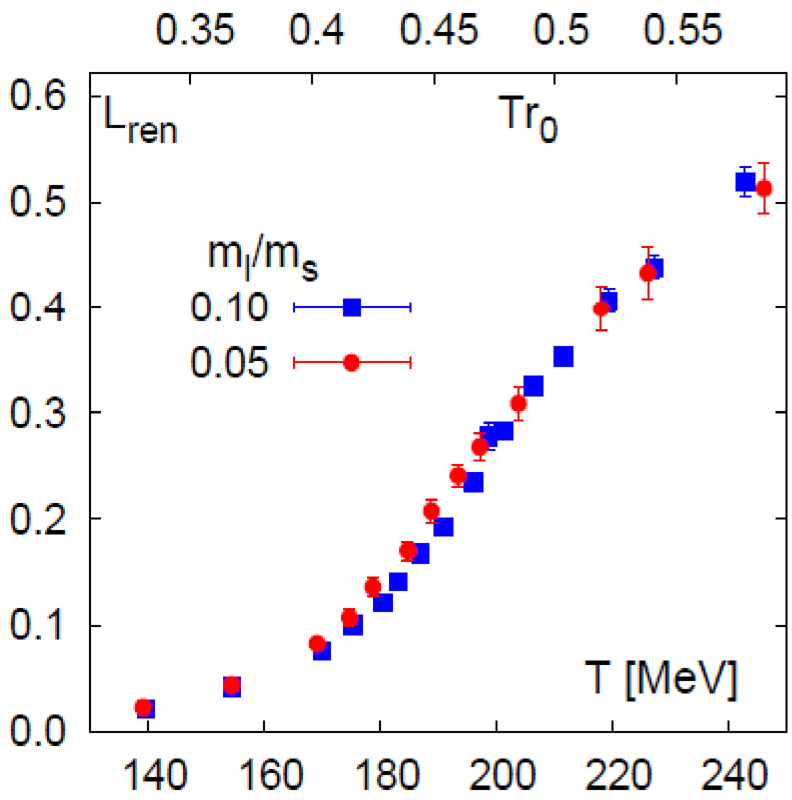
An example of mixed-susceptibilities

$$\frac{d_4{}^{I}}{d_4} \sim \frac{I^2}{B^2} \qquad \frac{d_6{}^{I}}{d_6} \sim \frac{I^2}{B^2}$$

Baryonic correlations are alive, as seen from high-order susceptibilities!

Other example: B-S correlations by Koch, et al.

And There is Polyakov Loop



From HotQCD 2009

Color is not completely liberated yet !
(semi-QGP by Pisarski, et al)

$$Q : \langle L \rangle \text{ suppression}$$

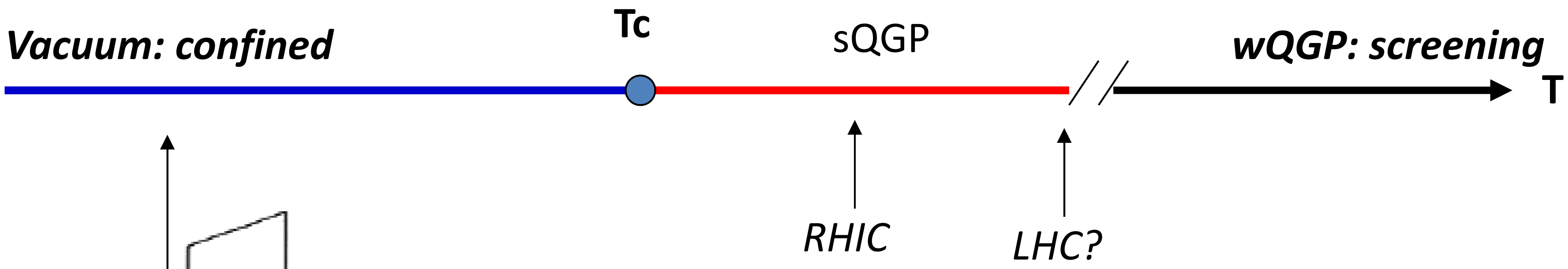
Color-less combination of three quarks however do not suffer from L !

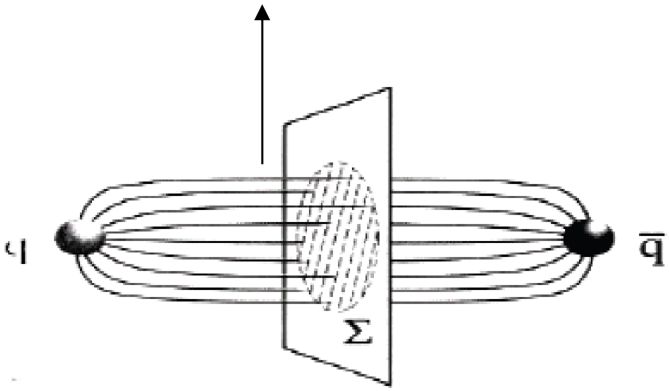
$$QQQ \sim B : \langle L_r L_g L_b \rangle = 1$$

(PNJL models by Ratti, Weise, et al)

The confining physics of QCD vacuum is NOT immediately gone in near –Tc QGP!

"Three Quarks for Muster Mark" in a New Context: Near-Tc QGP





Electric Flux Tube:
Magnetic **Condensate**

Susceptibilities from BARYONs: colorless combination of THREE (constituent) quarks

Post-confinement: Continuation of confining physics into sQGP from the magnetic component

--- the magic of THREE still at work !

--- would be great to see it gone at LHC!

Plasma of E-charges
E-screening: g T
M-screening: g^2 T

Susceptibilities from individual colorful QUARKs

JL & Shuryak, PRC2007,PRL2008

Summary

- Susceptibilities of conserved charges provide sensitive probe to the charge carriers, therefore important for ***finding new D.o.F in new phases of matter***.
- Three models are examined***: quasi-particles, holography, & bound states***.
- ***Baryonic correlations*** are particularly important for understanding the ***near-Tc structures*** in higher order susceptibilities calculated from lattice QCD.
- More generally, a ***post-confinement scenario for sQGP*** where "three-quarks for muster mark" naturally continues from vacuum into plasma.

Thank you!