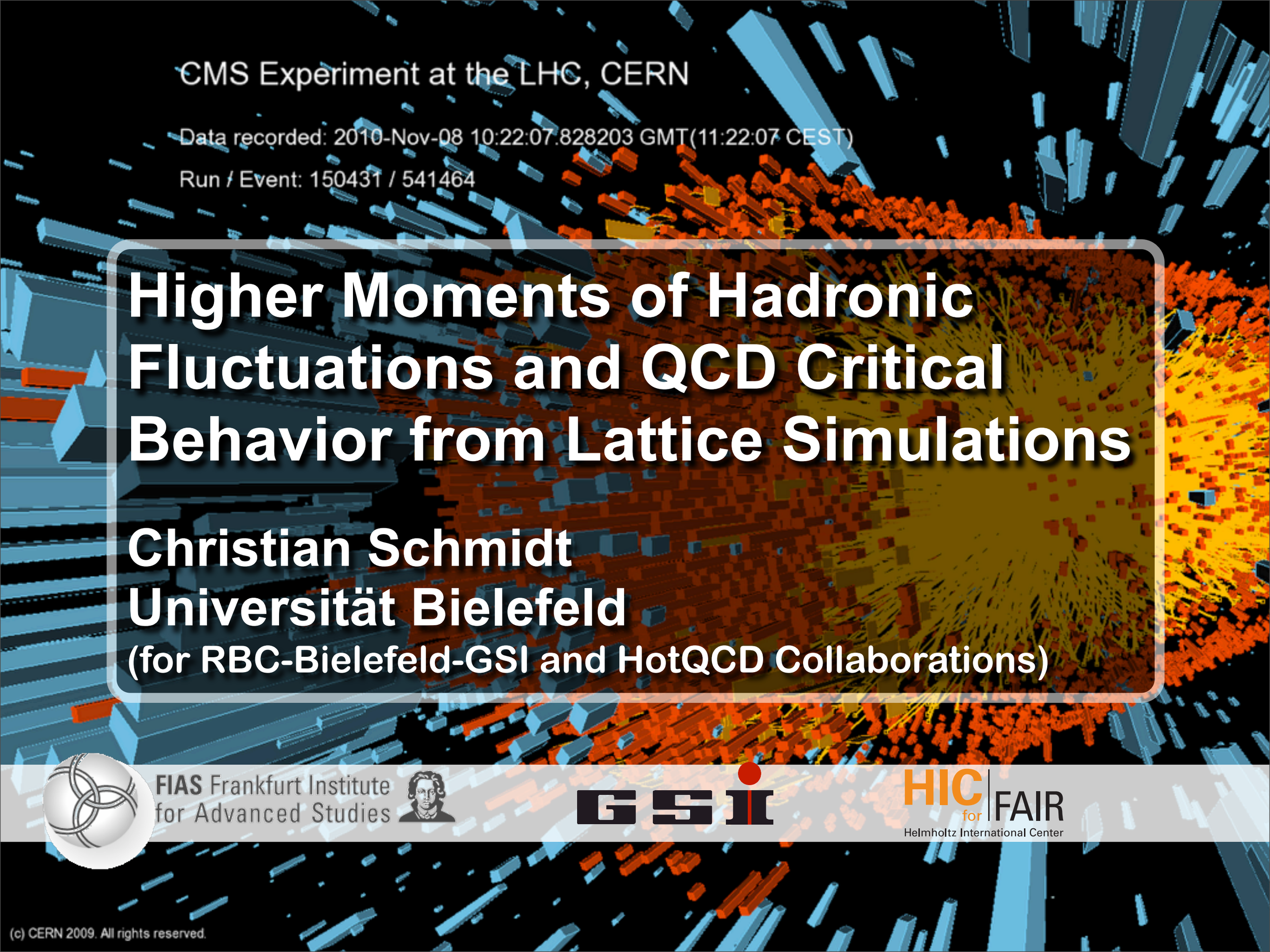

Higher Moments of Hadronic Fluctuations and QCD Critical Behavior from Lattice Simulations

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(for RBC-Bielefeld-GSI and HotQCD Collaborations)

FIAS Frankfurt Institute for Advanced Studies
GSI
HIC for FAIR Helmholtz International Center

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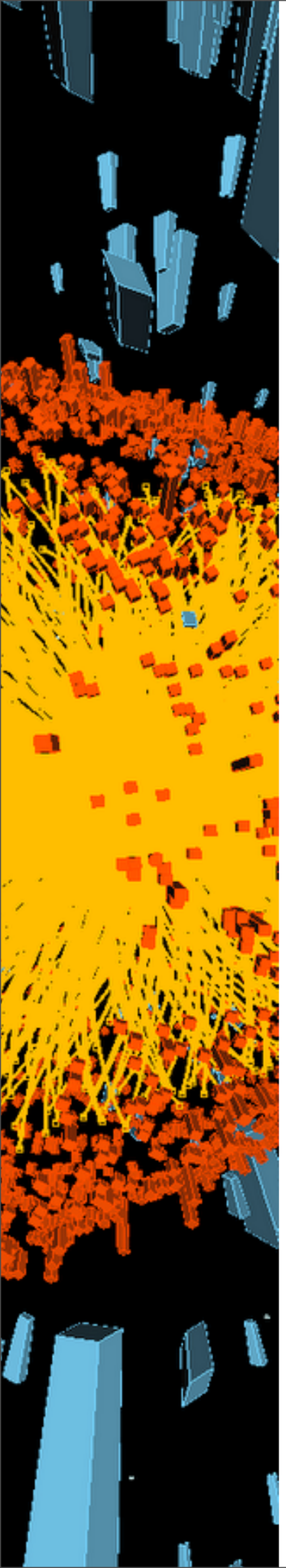


Overview

1) Introduction

- the QCD phase diagram in (μ, T)- and mass-plane
- critical behavior in the chiral limit
- lattice QCD and distortion of hadronic spectrum

2) Analyzing QCD critical behavior

- the transition temperature
- the curvature of the phase transition line
- critical behavior and hadronic fluctuations

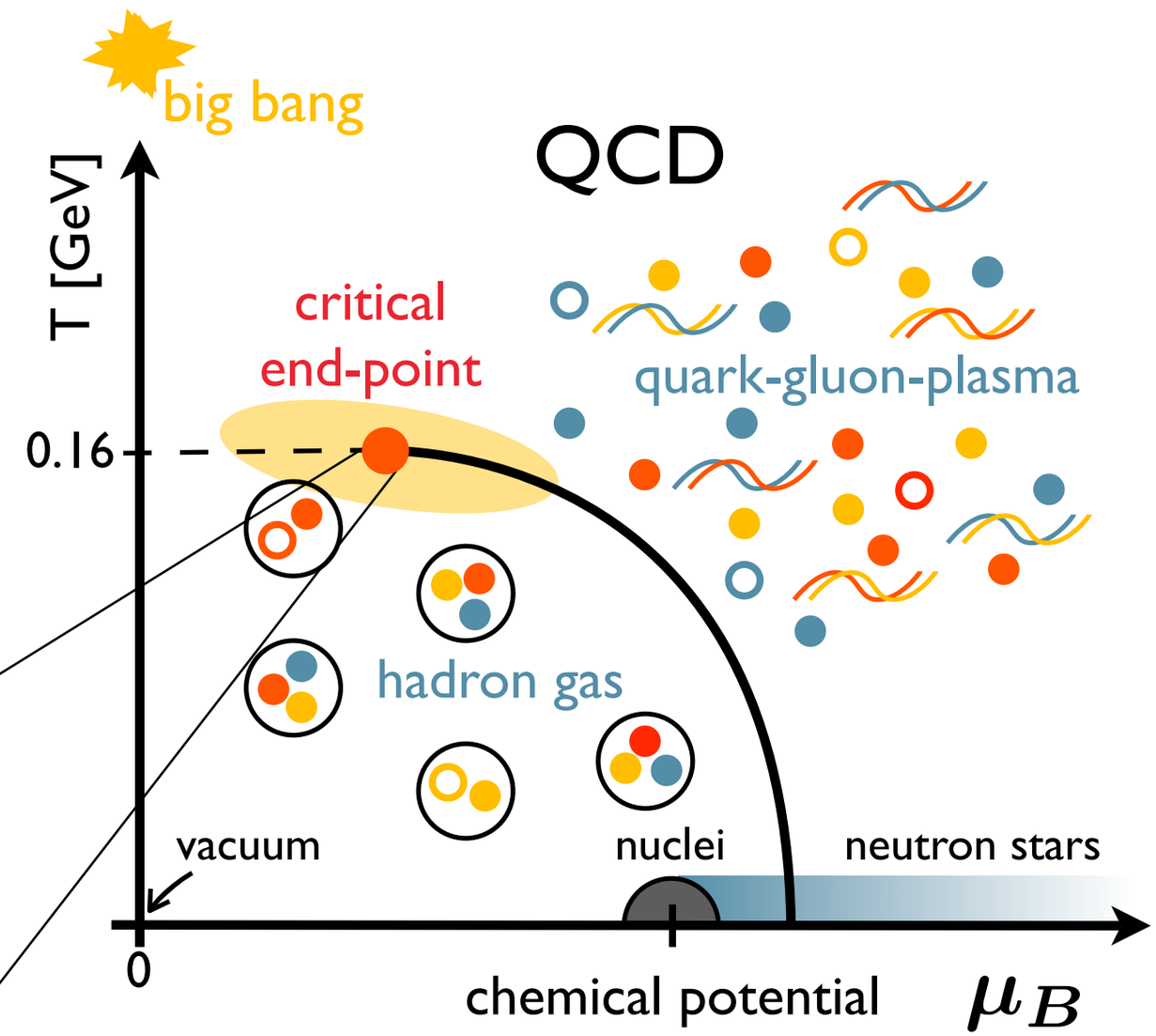
3) Fluctuations on the lattice vs. HRG and experiment

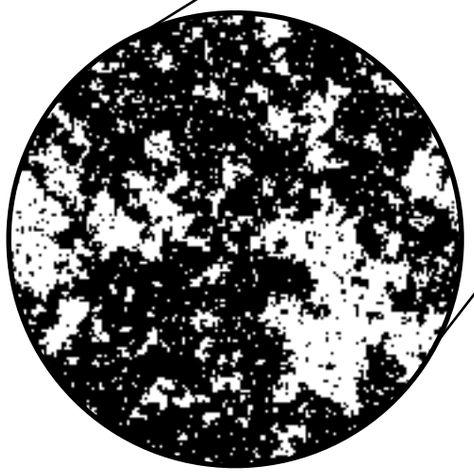
- the approach to the HRG
- deviations from experiment

4) Summary

Expected phase diagram of QCD:

- **Phases:**
  1) hadronic states at low T, low densities
  2) quasi-free quarks and gluons hat high T, high densities
- **Mechanisms**:
  1) spontaneous chiral symmetry breaking
  2) (de-)confinement
- **Critical end-point?**





$\Rightarrow$ **Universal behavior within a scaling region**

Expected phase diagram of QCD:

- **Phases:**
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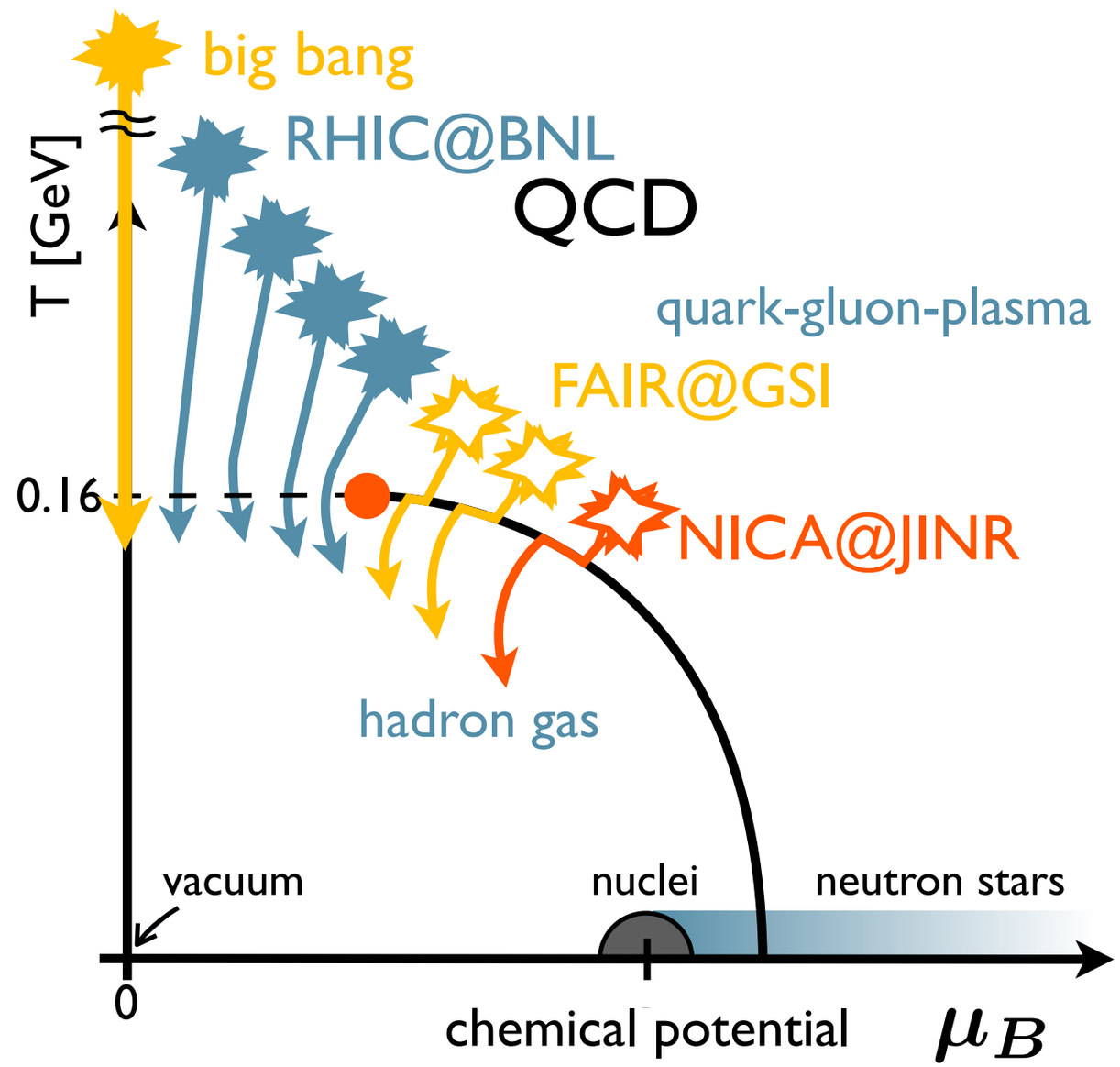
  2) quasi-free quarks and gluons hat high T, high densities
- **Mechanisms**:
  1) spontaneous chiral symmetry breaking
  2) (de-)confinement
- **Critical end-point?**
- **Relevance:**
  1) early universe
  2) hybrid stars?
  3) heavy ion collision



More critical points in QCD:

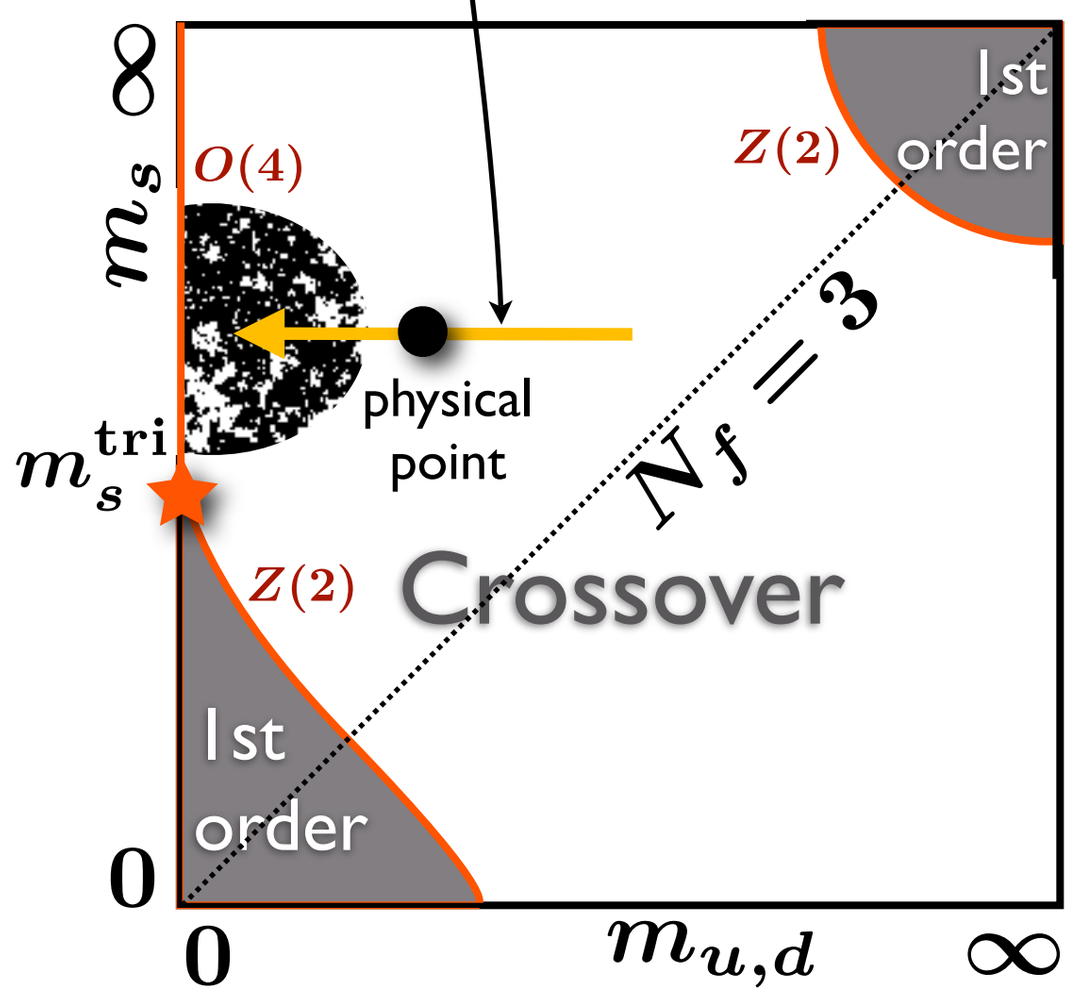
- chiral symmetry of 2-flavor QCD:
  $SU_L(2) \times SU_R(2) \simeq O(4)$
- hence, if m_s is large in (2+1)-flavor QCD:
  expect universal behavior as of 3d-$O(4)$ spins
- no clear evidence from simulations for many years
- staggered fermions preserve a flavor non-diagonal $U(1)$-part of chiral symmetry

$\Rightarrow$ **look for $O(2)$-critical behavior**

scaling studies:

p4, $N_\tau = 4$:
$m_q = (2/5 - 1/80)m_s$
Ejiri et al, PRD 80 (2009) 094505

asqtad, HISQ, $N_\tau = 6, 8, 12$:
$m_q = (1/5 - 1/20)m_s$
HotQCD in preparation



Mapping QCD to the O(N)-universal behavior:

- Thermodynamics in the vicinity of a critical point:

free energy density: $$-\frac{1}{V}\ln Z = \underbrace{f_s(t,h)}_{\text{(singular part)}} + \underbrace{f_r(T,V,H)}_{\text{(regular part)}}$$

scaling hypothesis: f_s is a generalized homogenous function

$$f_s(t,h) = b^{-d} f_s(b^{y_t} t, b^{y_h} h)$$

$\Longrightarrow$ generates scaling functions

- Critical behavior is controlled by **two** relevant **scaling fields**:

$$t = \frac{1}{t_0}\left(\left(\frac{T}{T_c}-1\right) + \kappa_q\left(\frac{\mu_q}{T}\right)^2 + \kappa_s\left(\frac{\mu_s}{T}\right)^2 + \kappa_{qs}\left(\frac{\mu_q\mu_s}{T}\right)^2\right)$$

$$h = \frac{1}{h_0}\frac{m_q}{m_s}$$

$\Longrightarrow$ **3 unique parameters zero density**

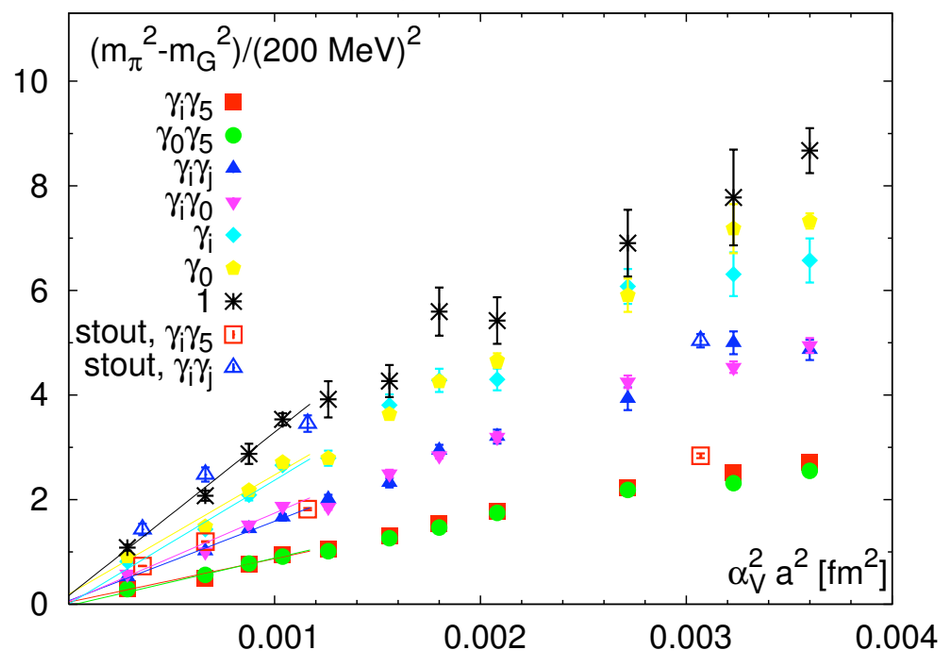
$\Longrightarrow$ **3 additional unique parameters at nonzero density**

- **Make use of known O(N)-scaling functions and fit non universal parameters**

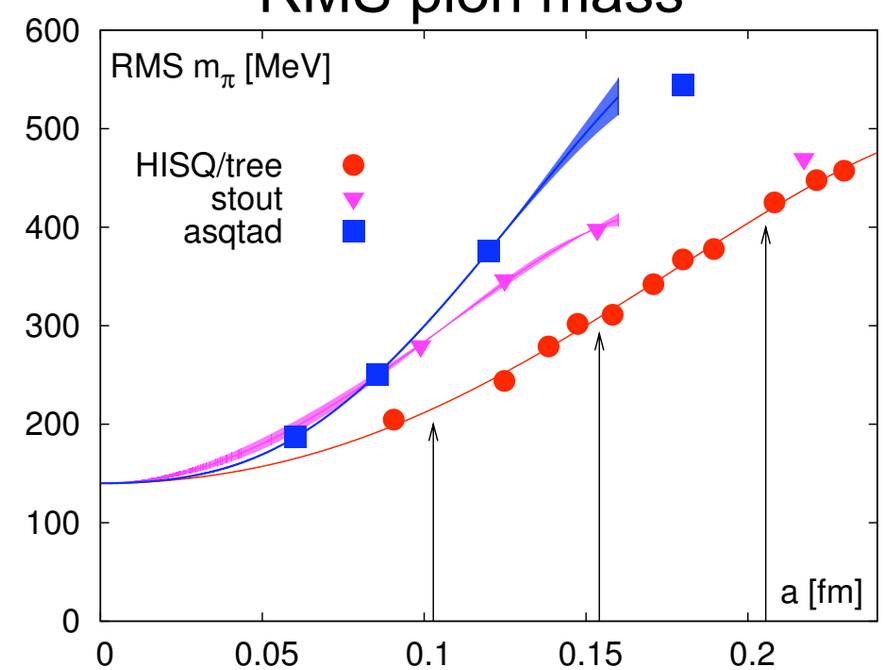
Breaking of flavor symmetry with staggered fermions:

- use smeared (asqtad,p4) and projected smeared (HISQ) links to reduce the flavor symmetry breaking:

HISQ vs. stout



RMS pion mass

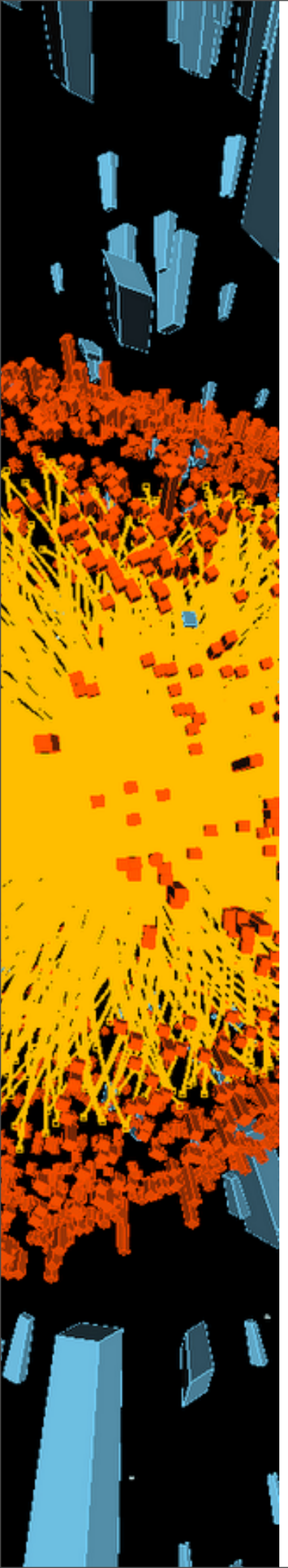


- define RMS pion mass as

$$m_\pi = \sqrt{\frac{1}{16}\left(m^2_{\gamma_5} + m^2_{\gamma_0\gamma_5} + 3m^2_{\gamma_i\gamma_5} + 3m^2_{\gamma_i\gamma_j} + 3m^2_{\gamma_i\gamma_0} + 3m^2_{\gamma_i} + m^2_{\gamma_0} + m^2_1\right)}$$

$\Longrightarrow$ **additional $\mathcal{O}(a^2)$ cutoff effects due to the distortion of the hadronic spectrum of QCD**

$\Longrightarrow$ **flavor symmetry breaking is most suppressed by the HISQ action**



1) Introduction (lattice setup)

- previous results on fluctuations have been obtained with p4-action and $m_q/m_s = 0.1\,,\ N_\tau = 4, 6$
- **Now**: preliminary RBC-Bi results with HISQ/tree action and $m_q/m_s = 0.05$, $N_\tau = 6, 8, 12$

$\Longrightarrow$ for a detailed discussion of quadratic fluctuations and correlations see talk by P. Hegde

$\Longrightarrow$ **this talk:** a more qualitative discussion of higher order fluctuations

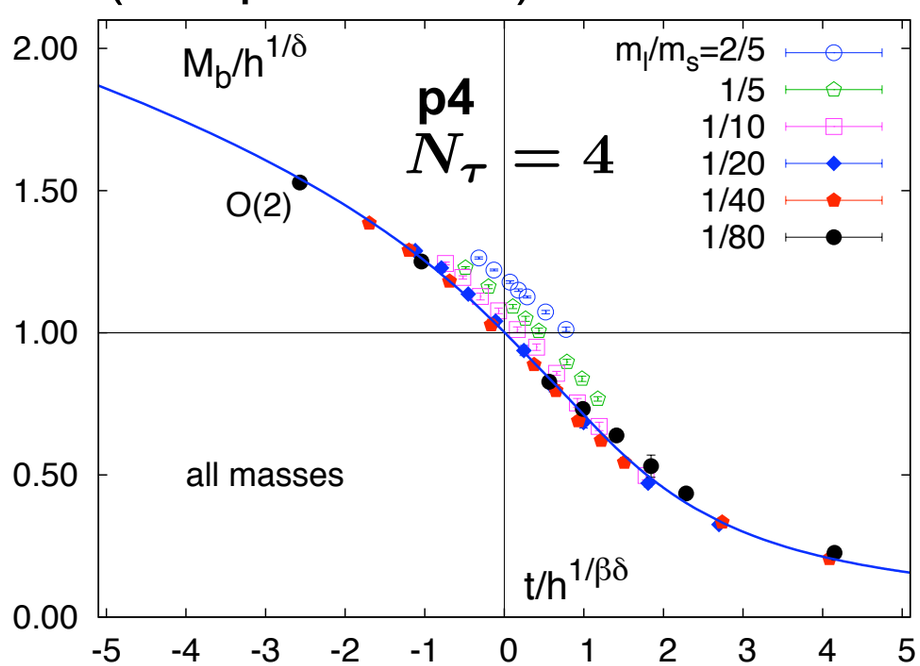
Mapping QCD to the O(N)-universal behavior:

- in the vicinity of $(t,h)=(0,0)$ derivatives of the partition function are given by universal **scaling functions**:

order parameter:

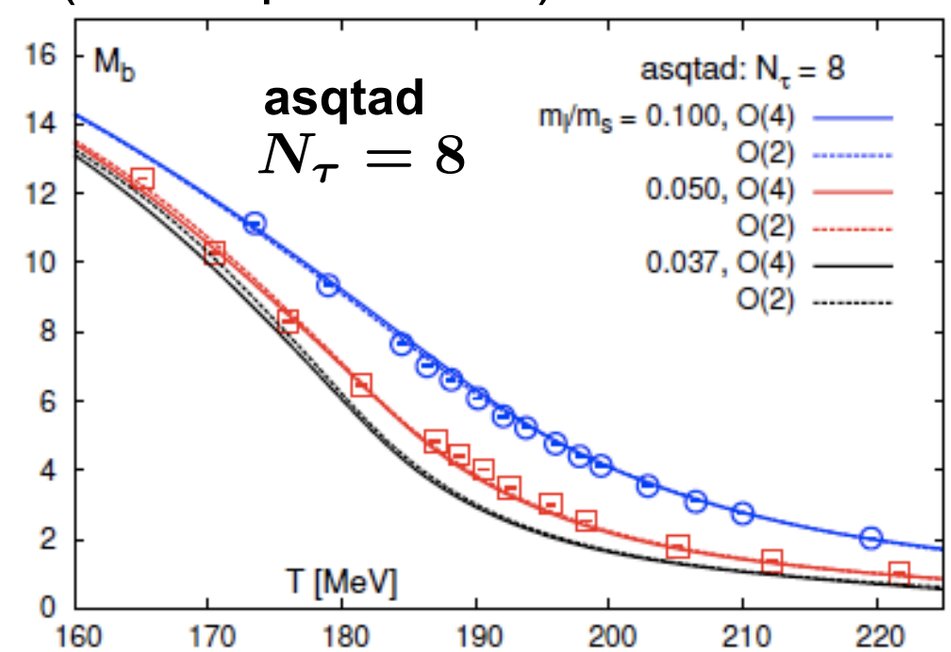
$$M_b = m_s \frac{\langle \bar{\psi}\psi \rangle_l}{T^4} = h^{1/\delta} f_G(z) \qquad \text{with} \qquad z = t/h^{1/\beta\delta}$$

scaling fit w/o regular part:
(3 fit parameter)



Ejiri et al, PRD 80 (2009) 094505

scaling fit w/ leading regular part:
(3+2 fit parameter)



HotQCD preliminary

$\Rightarrow$ **physical point in the scaling region**

2) Analyzing QCD critical behavior

Following the pseudo-critical line:

$$T_c, t_0, h_0 \;\rightarrow\; z_0 = t_0/h_0^{1/\beta\delta}$$

T_c : chiral phase transition at $m = 0$

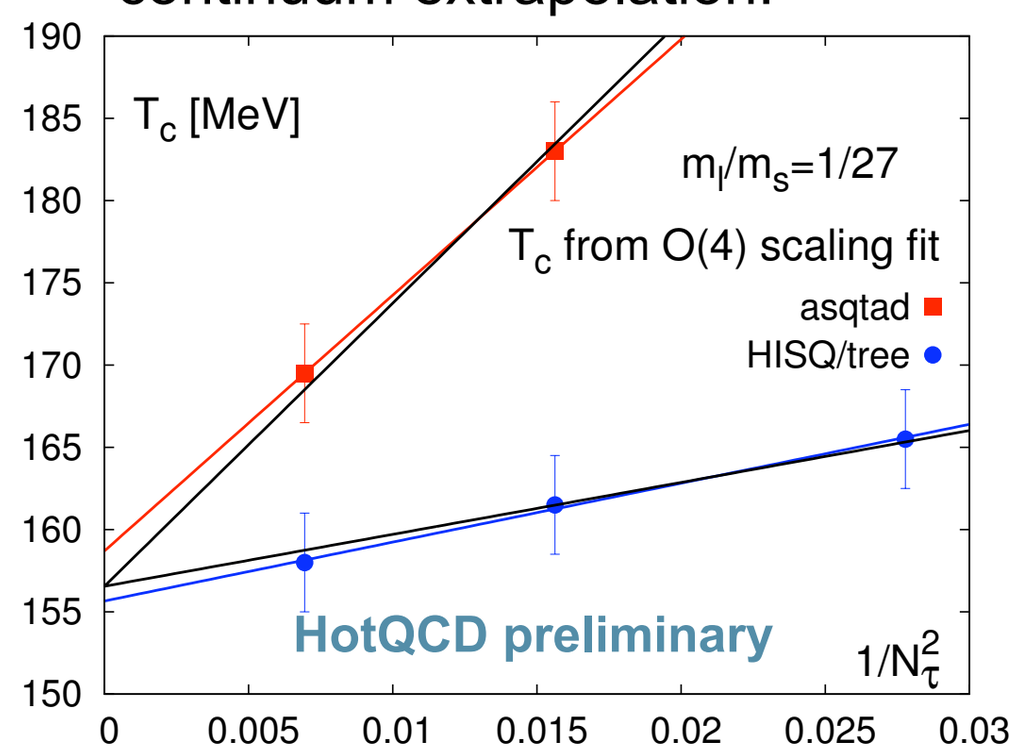
z_0 : **controls mass dependence** $T_{\rm pc}(m_q/m_s)$

pseudo-critical line defined by the peak position z_p in the scaling function of the chiral susceptibility $f_\chi(z)$

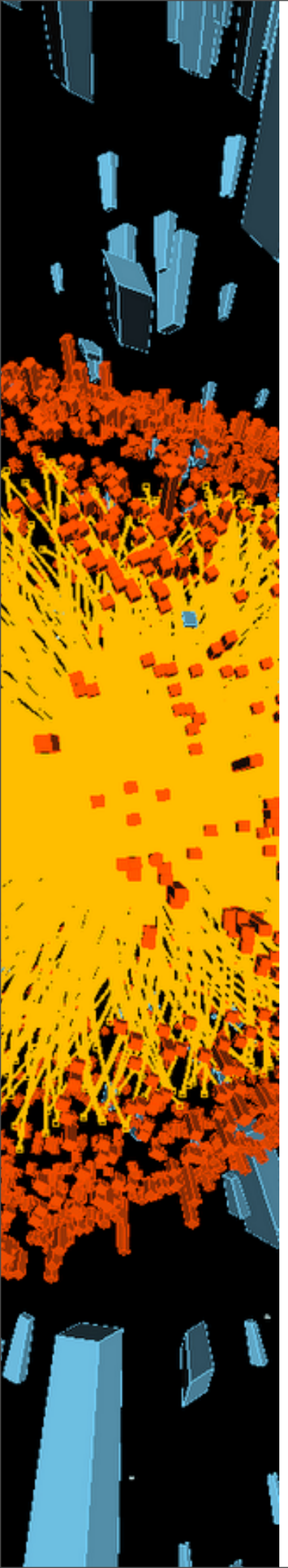
$$\Rightarrow\; t = z_p \left(\frac{m_q}{m_s}\right)^{1/\beta\delta}$$

$$\Rightarrow\; \frac{T_{\rm pc}(m_q/m_s)}{T_c} = 1 - \frac{z_p}{z_0}\left(\frac{m_q}{m_s}\right)^{1/\beta\delta}$$

continuum extrapolation:



$\Rightarrow$ consistent with: **Y. Aoki, et al., PLB 643 (2006) 47**



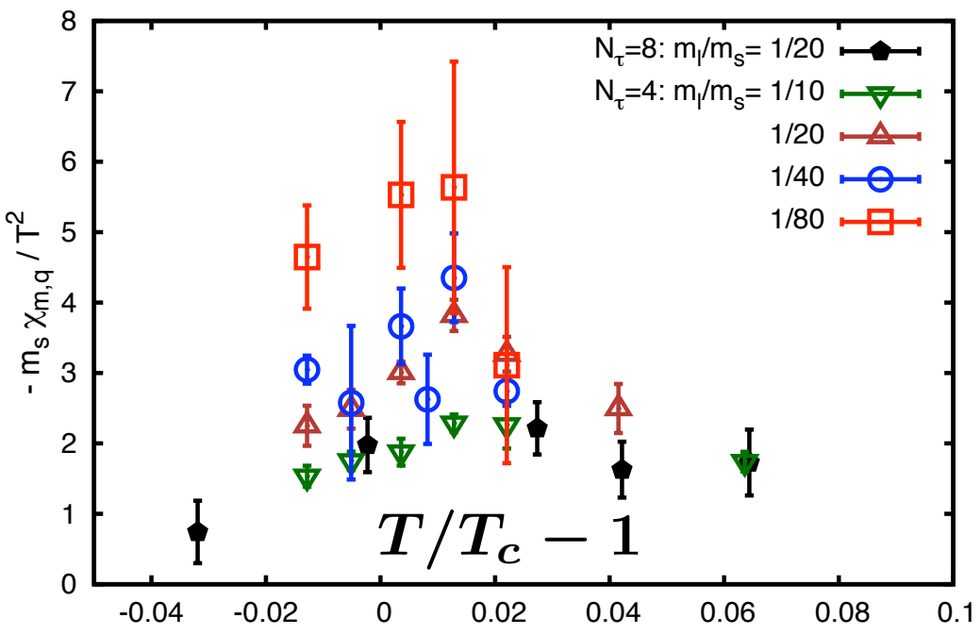
2) Analyzing QCD critical behavior

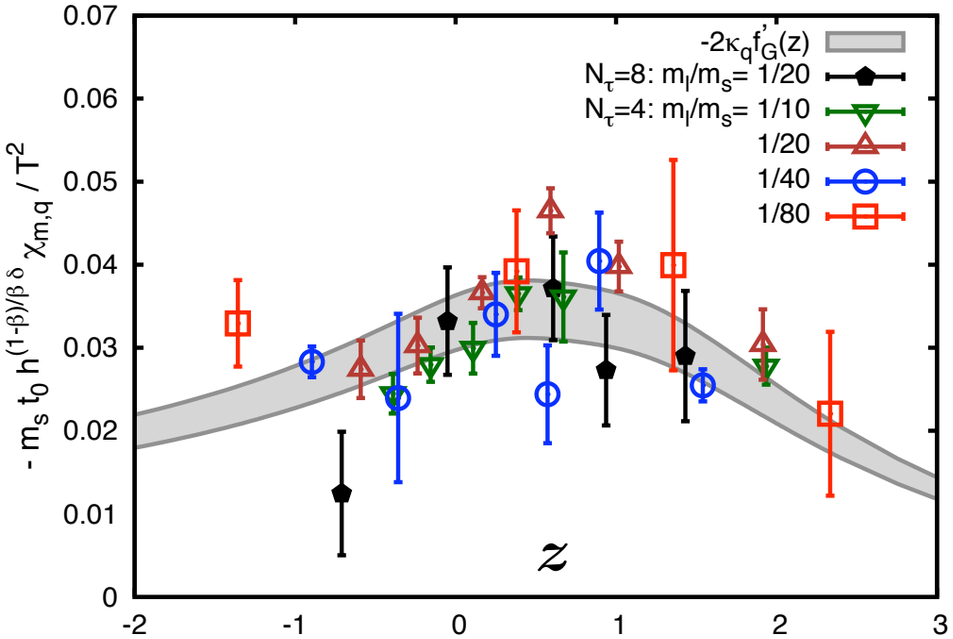
Following the critical line: $\frac{T_c(\mu_q)}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T}\right)^2$

- Three parameters (T_c, t_0, h_0) have been fixed by the magnetic equation of state $M = h^{1/\delta} f_G(z)$
- Determine κ_q by a scaling analysis of the mixed susceptibility

$$\chi_m = \frac{\partial^2 M}{(\partial\mu/T)^2} = \frac{2\kappa_q}{t_0 T_c} h^{(\beta-1)/\beta\delta} f'_G(z) \propto \chi_t$$

$\Longrightarrow$ **one fit parameter:** κ_q



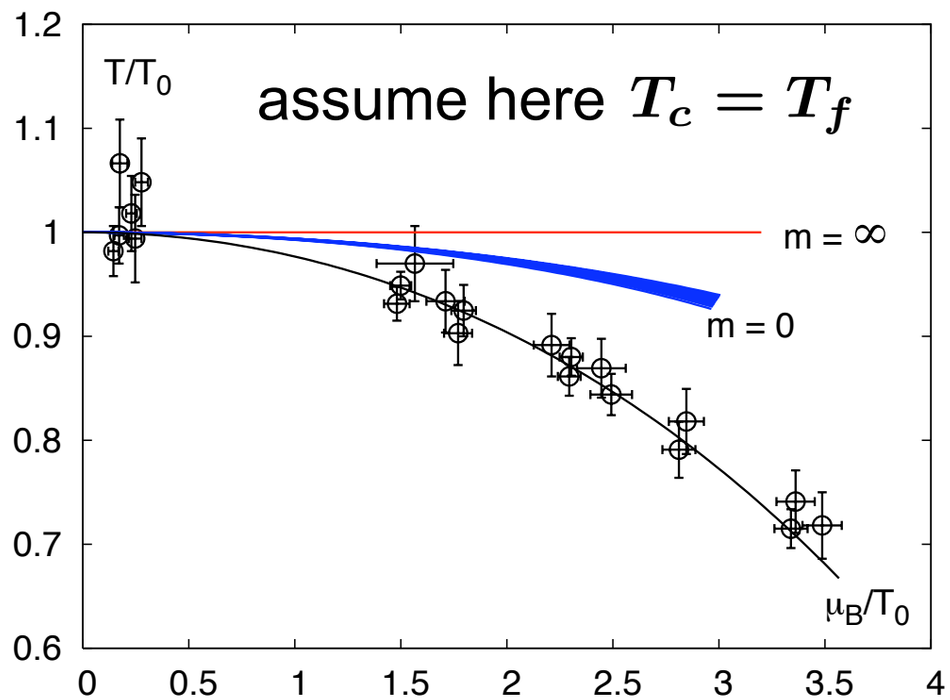


$\Longrightarrow$ **obtain from p4-action,** $N_\tau = 8, 4: \kappa_q = 0.059(6)$

Kaczmarek *et al.*, PRD 83 (2001) 014504

Critical line vs. freeze-out line:

- Statistical models are very successful in describing particle abundances observed in heavy ion collision; use a parametrization of the freeze-out curve



statistical model:

$$\frac{T_c}{T} = 1 - 0.023\left(\frac{\mu_B}{T}\right)^2 - d\left(\frac{\mu_B}{T}\right)^4$$

Cleymans, et al., PRC 73 (2006) 034905

lattice:

$$\frac{T_c}{T} = 1 - 0.0066(7)\left(\frac{\mu_B}{T}\right)^2$$

Kaczmarek *et al.*, PRD 83 (2001) 014504

$\Rightarrow$ **curvature of the freeze-out curve seems to be larger**

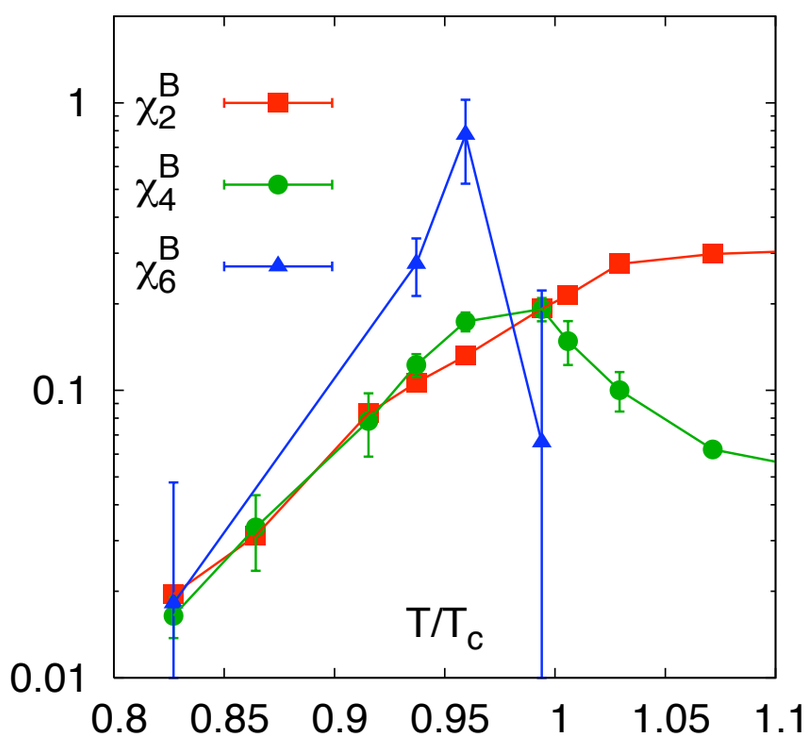
- **open issues**: continuum limit, strangeness conservation, nonzero electric charge

2) Analyzing QCD critical behavior

Analyzing baryon number fluctuation $\chi_n^B = \frac{\partial^n (p/T^4)}{\partial (\mu_B/T)^n}$

- higher moments of baryon number fluctuations are more and more sensitive to the critical behavior.

$16^3 \times 4,\ m_q = m_s/10$



baryonic fluctuations: derivatives of the pressure with respect to μ_B

$$\chi_2^B = \frac{1}{VT^3}\langle N_B^2\rangle$$

$$\chi_4^B = \frac{1}{VT^3}\left(\langle N_B^4\rangle - 3\langle N_B^2\rangle^2\right)$$

$$\chi_6^B = \frac{1}{VT^3}\left(\langle N_B^4\rangle - 15\langle N_B^4\rangle\langle N_B^2\rangle + 30\langle N_B^2\rangle^3\right)$$

$\Rightarrow$ **perform scaling analysis**

$\Rightarrow$ **compare to heavy ion experiments**

2) Analyzing QCD critical behavior

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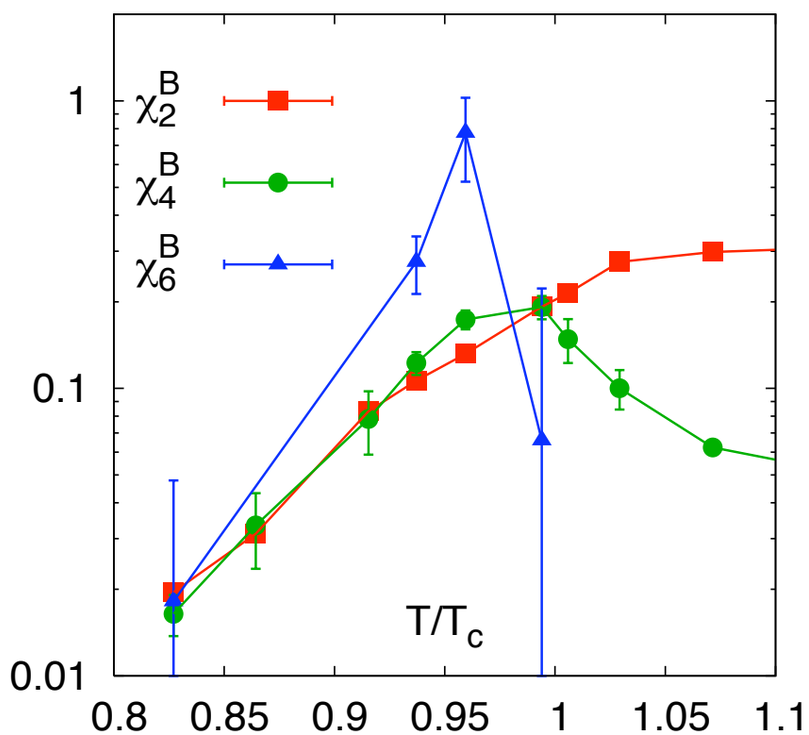

scaling analysis:

scaling field (chiral limit):

$$t = \frac{1}{t_0}\left(\frac{T - T_c}{T_c} + \kappa\left(\frac{\mu_B}{T}\right)^2\right)$$

free energy:

$$f = A_\pm |t|^{2-\alpha} + \text{regular}$$

critical exponent:

$$-0.15 < \alpha < -0.11$$

$$\chi_2^B \sim \mp 2A_\pm(2-\alpha)\kappa\,|t|^{1-\alpha} + \text{regular}$$

$$\chi_4^B \sim -12A_\pm(2-\alpha)(1-\alpha)\kappa^2\,|t|^{-\alpha} + \text{regular} \longrightarrow \text{kink (chiral limit)}$$

$$\chi_6^B \sim \mp 120A_\pm(2-\alpha)(1-\alpha)(-\alpha)\kappa^3\,|t|^{-1-\alpha} + \text{regular} \longrightarrow \text{divergent (chiral limit)}$$

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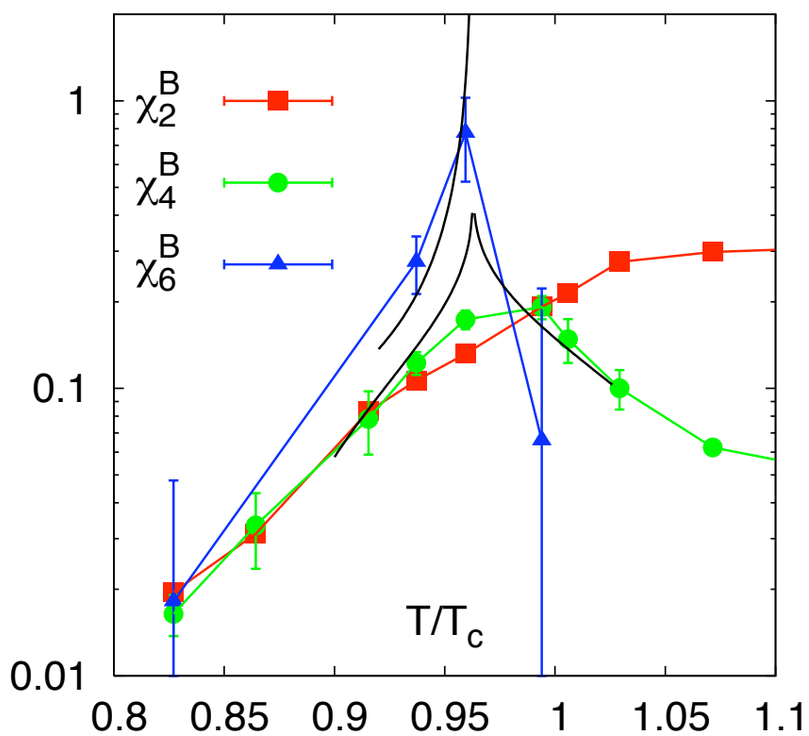

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3) Baryon number fluctuations: lattice vs. HRG

hadron resonance gas:

$$\ln Z(T,V,\mu_B,\mu_S,\mu_Q) = \sum_{i\in hadrons} \ln Z_{m_i}(T,V,\mu_B,\mu_S,\mu_Q)$$

$$\sum_{i\in mesons} \ln Z^B_{m_i}(T,V,\mu_S,\mu_Q) + \sum_{i\in baryons} \ln Z^F_{m_i}(T,V,\mu_B,\mu_S,\mu_Q)$$

mesons:

$$\frac{p_i}{T^4} = \frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 \sum_{l=1}^{\infty}(+1)^{l+1} l^{-2} K_2(lm_i/T)\cosh(lS_i\mu_S/T + lQ_i\mu_Q/T)$$

baryons:

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Boltzmann approximation

ratios are independent of spectrum and volume

$\longrightarrow$ possibly large parts of cut-off effects cancel

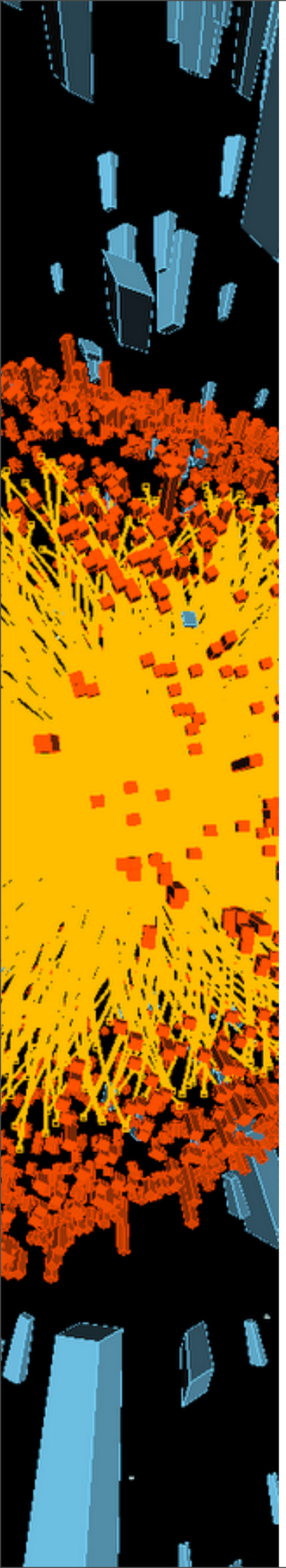
3 ratios:

$$\frac{\chi_4^B}{\chi_2^B} = \kappa\sigma^2 = \frac{B^4}{B^2} = 1$$

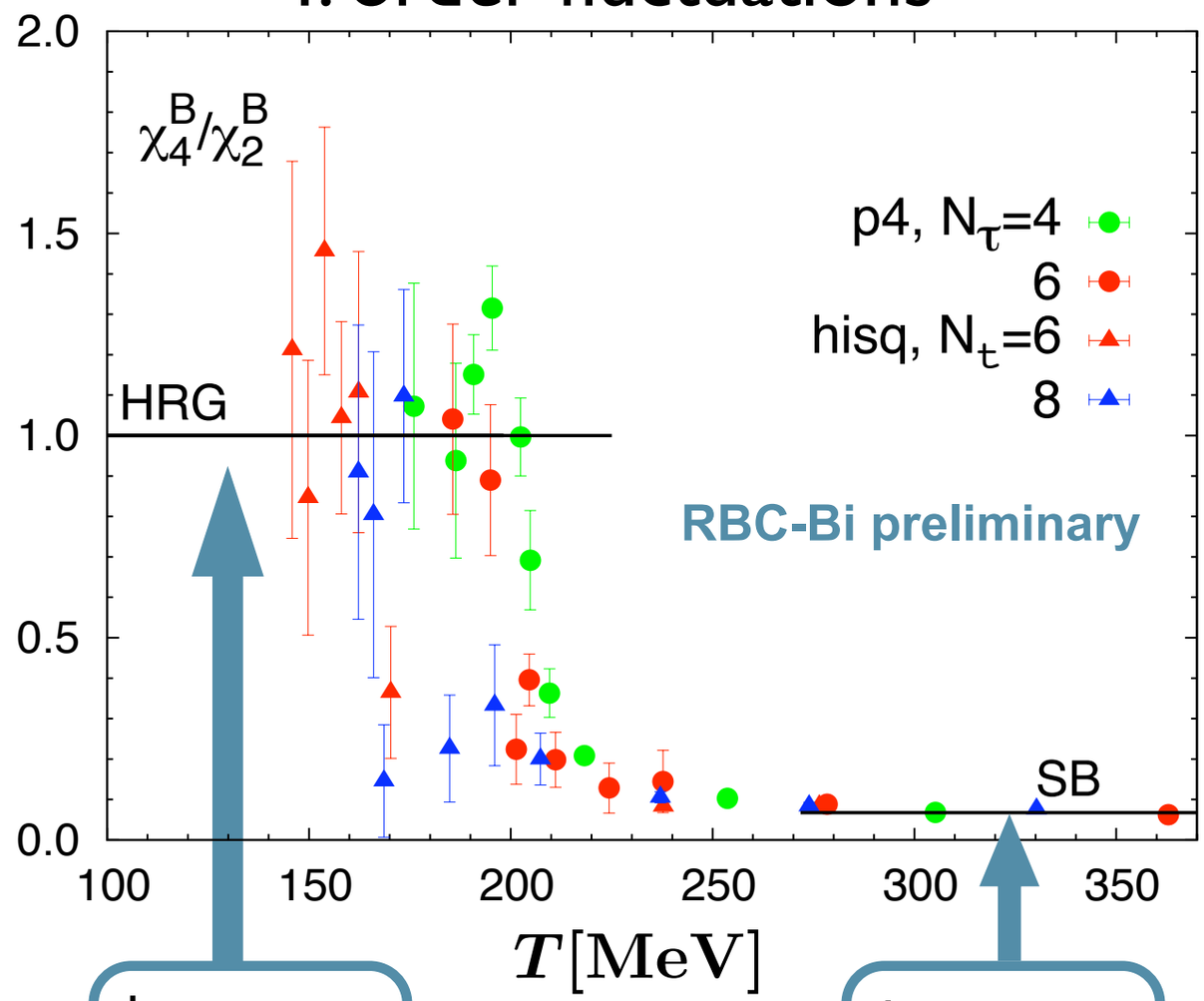
$$\frac{\chi_3^B}{\chi_2^B} = S\sigma = \frac{B^3}{B^2}\tanh(\mu_B/T)$$

$$\frac{\chi_2^B}{\chi_1^B} = \sigma^2/N_B = \frac{B^2}{B^1}\coth(\mu_B/T)$$

3) Baryon number fluctuations: lattice vs. HRG



4. order fluctuations



baryon number carried by **hadrons**

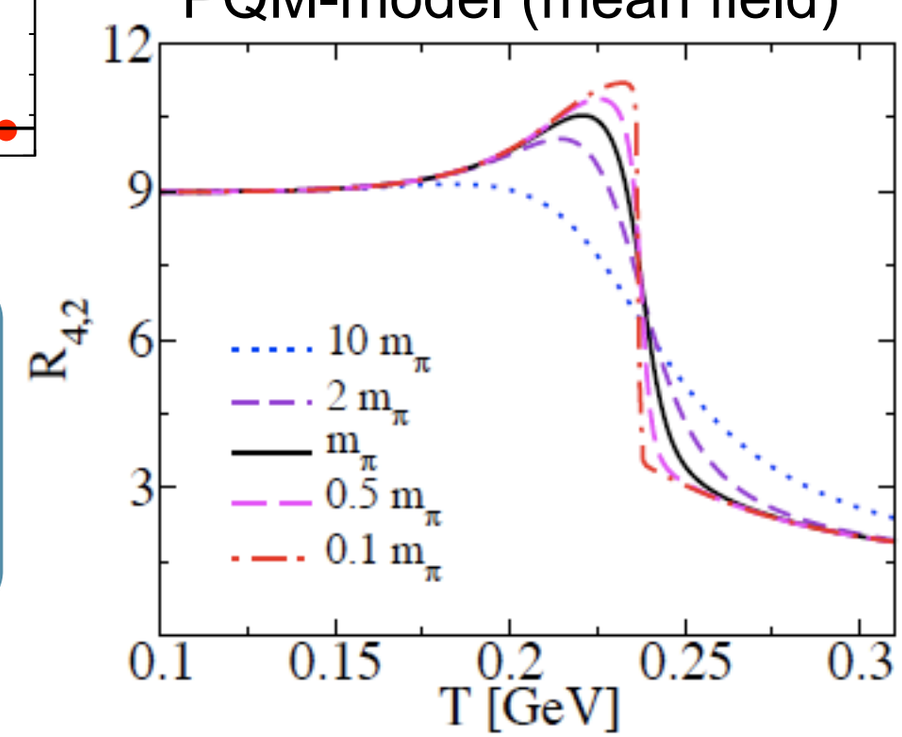
baryon number carried by **quarks**

⇒ cutoff dependence of Tc

⇒ pronounced step function indicating deconfinement

kink at Tc from critical behavior?

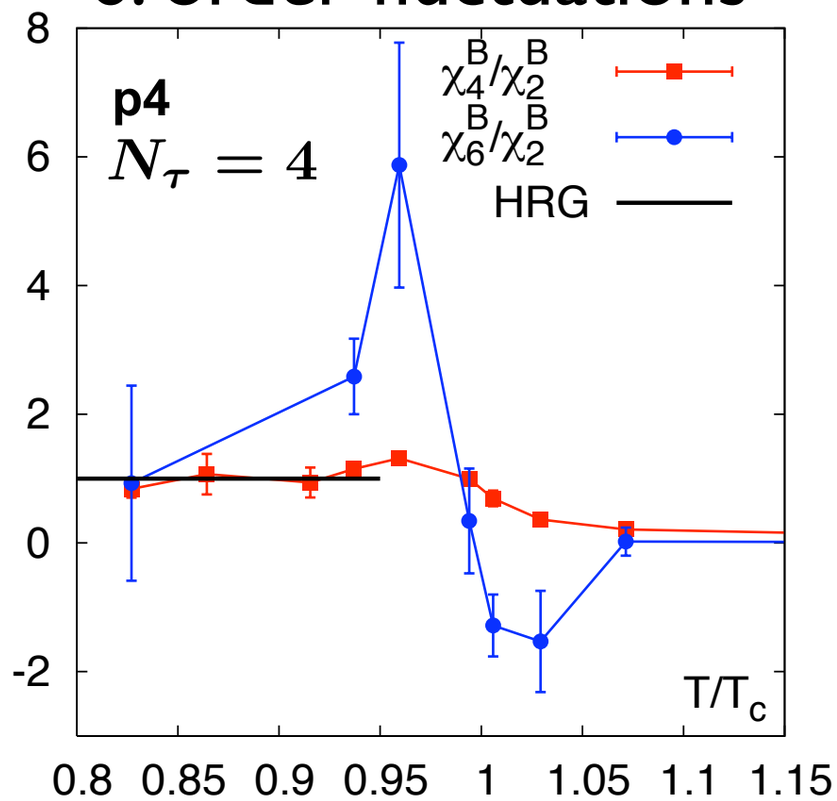
PQM-model (mean field)



Skokov *et al.*, PRD 82 (2010) 034029.

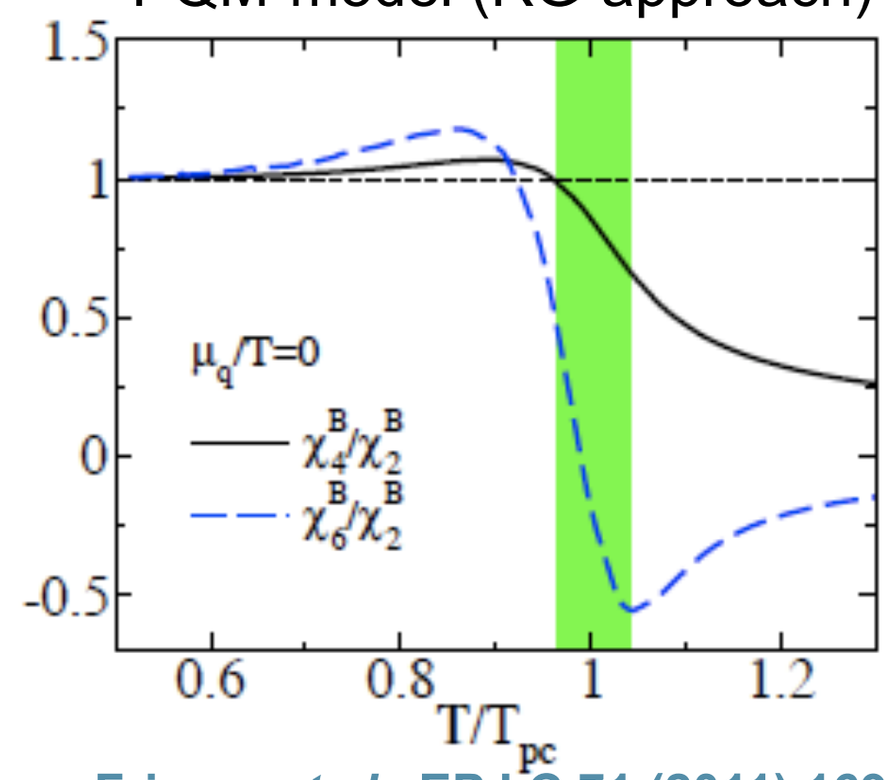
3) Baryon number fluctuations: lattice vs. HRG

6. order fluctuations



CS, Prog. Theor. Phys. Suppl. 186 (2010) 563

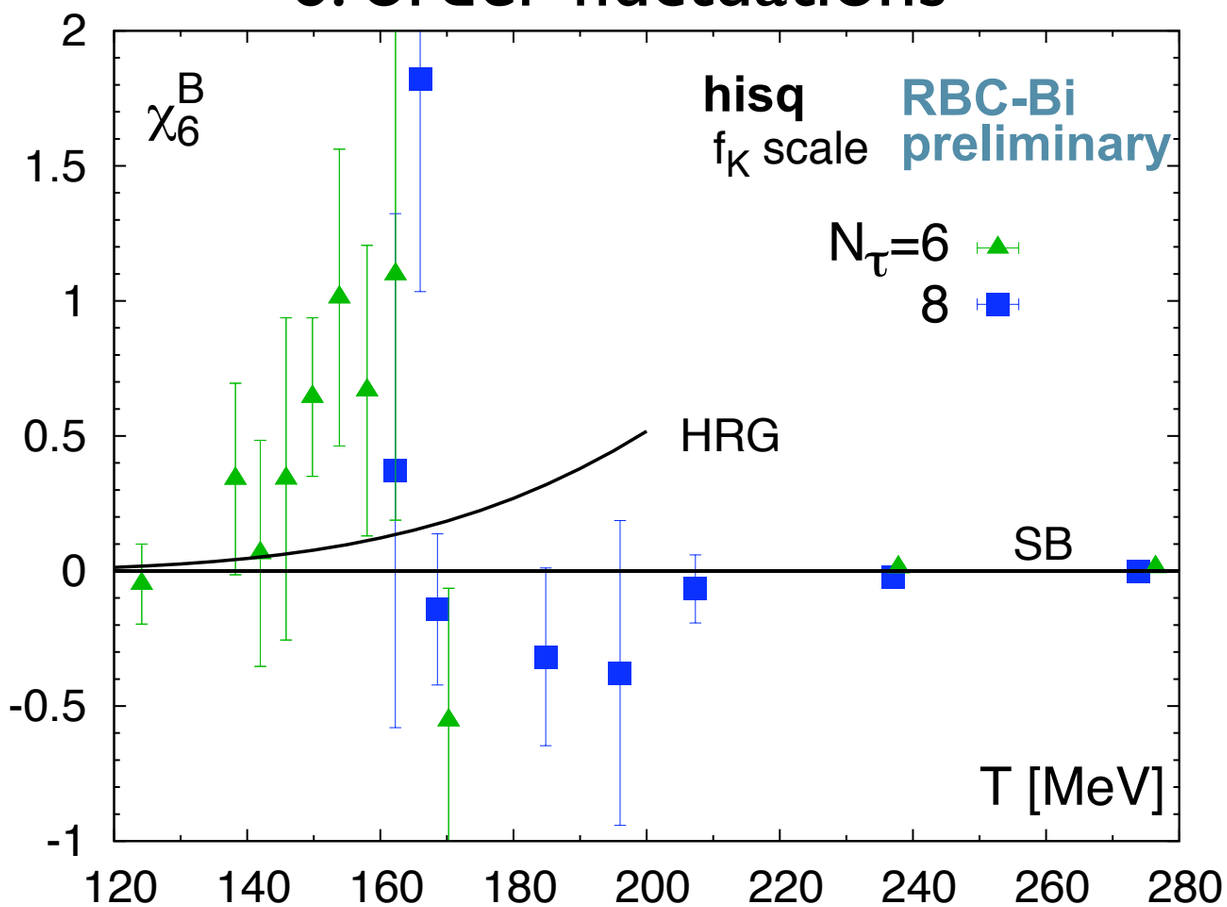
PQM-model (RG-approach)



Friman *et al.*, EPJ C 71 (2011) 1694.

3) Baryon number fluctuations: lattice vs. HRG

6. order fluctuations

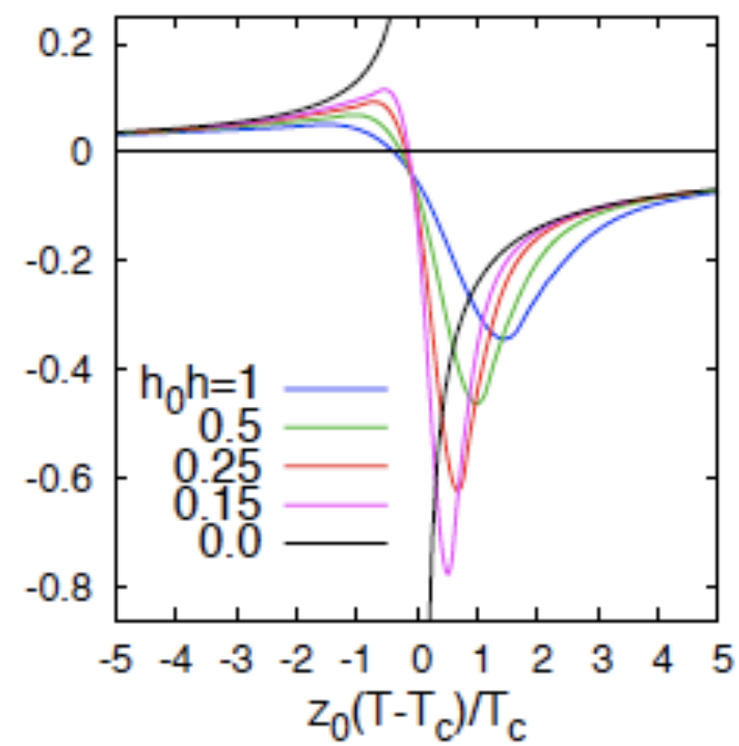


notice for $T < T_c$:

$\Rightarrow$ fluctuations increase over HRG

$\Rightarrow$ fluctuations stay positive

O(4) scaling function



Friman *et al.*, EPJ C 71 (2011) 1694.

3) Q and S fluctuations: lattice vs. HRG

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$$\frac{\chi_4^Q}{\chi_2^Q} = \frac{F^{(Q=\pm1)} + 16F^{(Q=\pm2)}}{F^{(Q=\pm1)} + 4F^{(Q=\pm2)}}$$

contributions from double charged particles, dominated by charged pions for low T

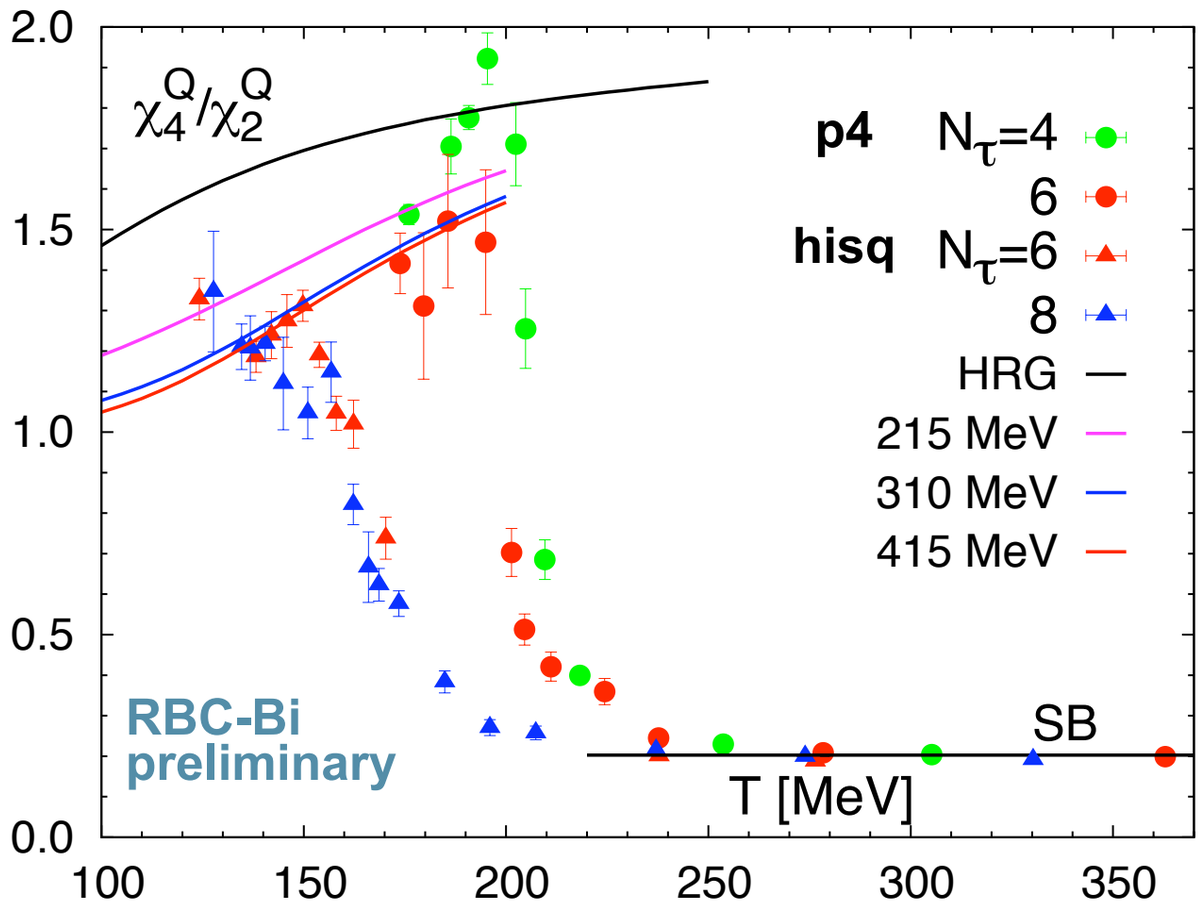
$$\frac{\chi_4^S}{\chi_2^S} = \frac{F^{(S=\pm1)} + 16F^{(S=\pm2)} + 81F^{(S=\pm3)}}{F^{(S=\pm1)} + 4F^{(S=\pm2)} + 9F^{(S=\pm3)}}$$

contributions from double and triple strange particles, dominated by Kaons for low T

$\Longrightarrow$ deviations from unity for $T \gtrsim 100\ \mathrm{MeV}$

3) Electric charge fluctuations: lattice vs. HRG

4. order fluctuations



⇒ cutoff effects can not be absorbed in Tc

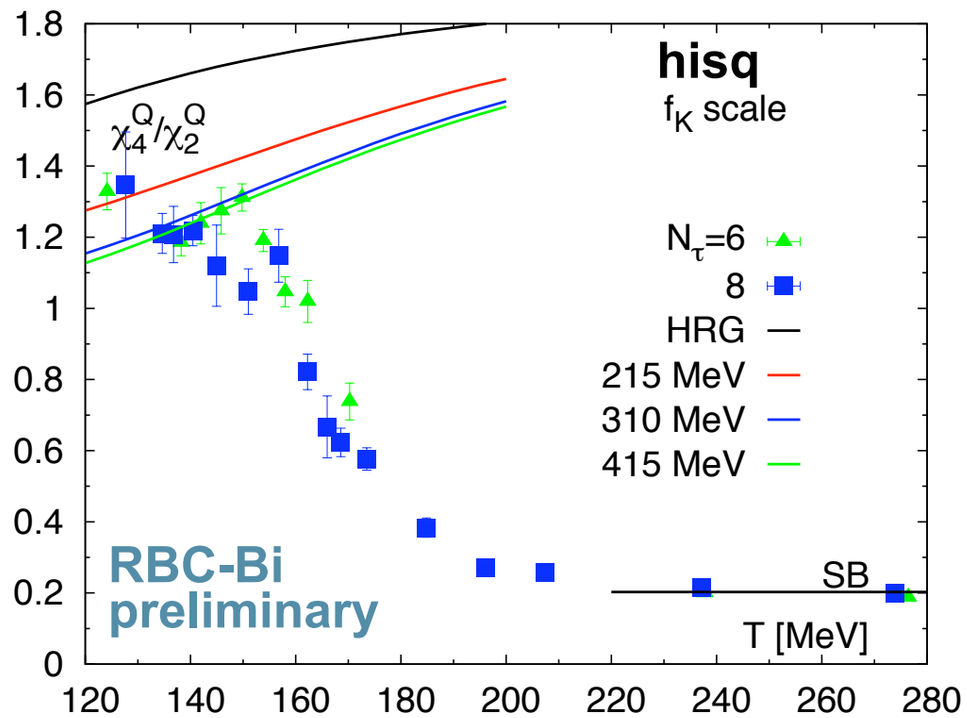
⇒ interpolation between HRG and free quarks

⇒ no or very little sign of critical behavior

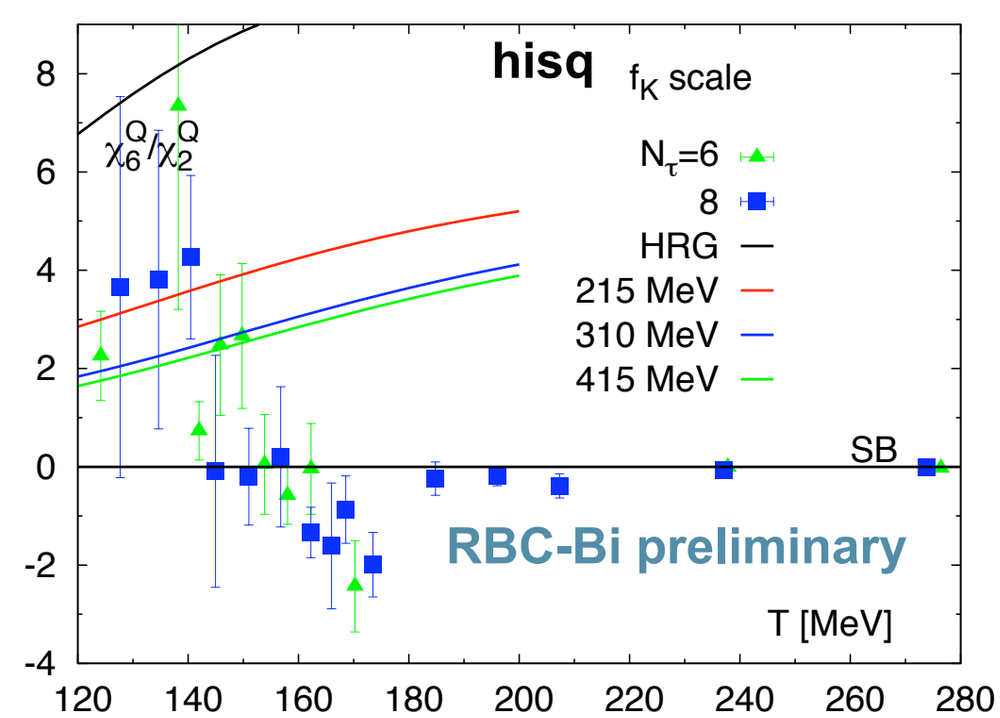
⇒ electric charge fluctuations are sensitive to the light pion sector (which is distorted on the lattice)

3) Electric charge fluctuations: lattice vs. HRG

4. order fluctuations



6. order fluctuations



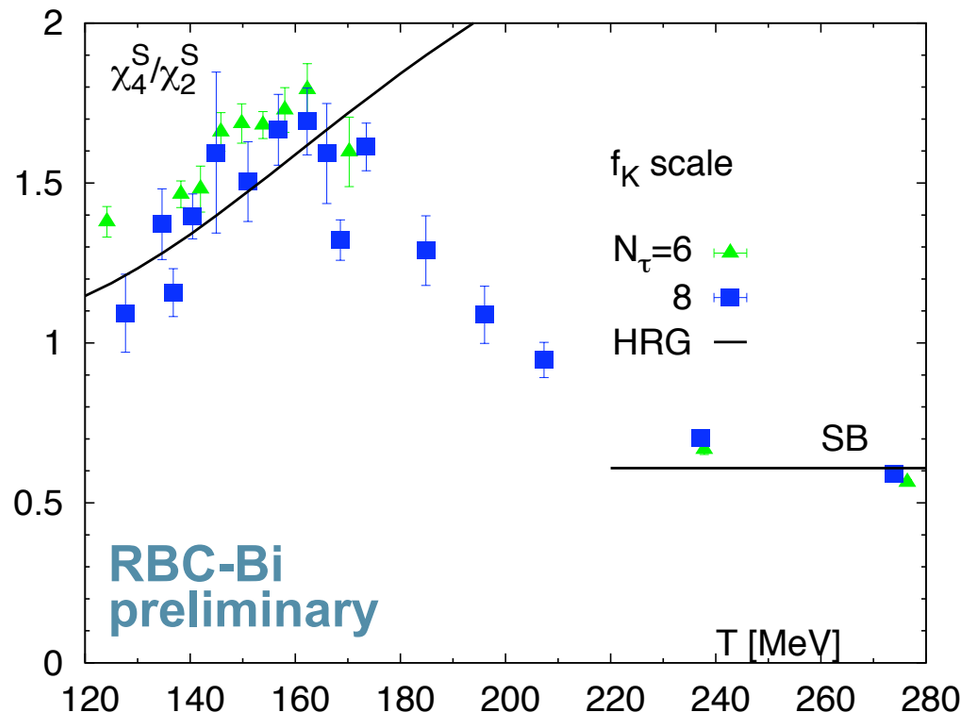
$\Rightarrow$ negative fluctuations above Tc

$\Rightarrow$ electric charge fluctuations are sensitive to the light pion sector (which is distorted on the lattice)

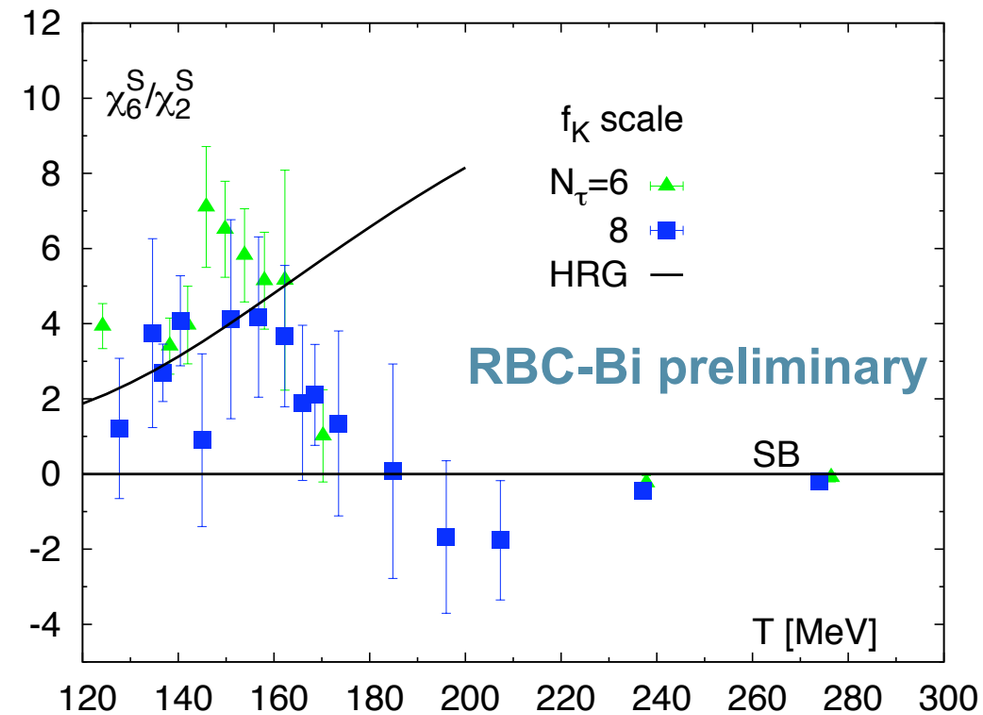
$\Rightarrow$ continuum extrapolations for higher moments of electric charge fluctuations become increasingly difficult

3) Strangeness fluctuations: lattice vs. HRG

4. order fluctuations



6. order fluctuations



$\Rightarrow$ negative fluctuations above Tc

$\Rightarrow$ close to HRG for $T \lesssim 160\ \mathrm{MeV}$ (approach from above)

Approaching a critical end-point at $\mu_B > 0$?

- a Taylor expansion approach:

$$p/T^4 = c_0 + c_2(\mu/T)^2 + c_4(\mu/T)^4 + c_6(\mu/T)^6 + \cdots$$

$$\Rightarrow \quad \frac{\chi_2}{\chi_1} = \frac{2c_2 + 12c_4(\mu/T)^2 + 30c_6(\mu/T)^4 + \cdots}{2c_2(\mu/T) + 4c_4(\mu/T)^3 + 6c_6(\mu/T)^5 + \cdots}$$

$$= \left(\frac{\mu}{T}\right)^{-1} \frac{1 + 6(c_4/c_2)(\mu/T)^2 + 15(c_6/c_2)(\mu/T)^4 + \cdots}{1 + 2(c_4/c_2)(\mu/T)^2 + 3(c_6/c_2)(\mu/T)^4 + \cdots}$$

$\Rightarrow$ similar expansions for χ_4/χ_2, χ_3/χ_2

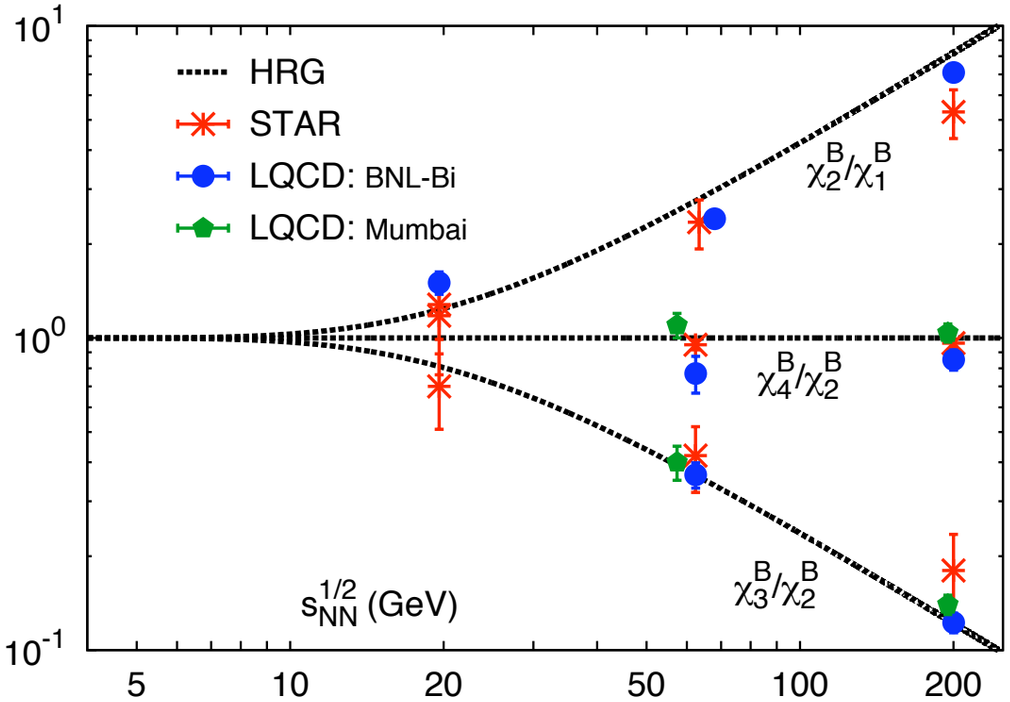
can be used to get a model independent freeze out line, input: continuum extrapolated c4/c2 from lattice QCD

- **for now**: use parametrization of freeze-out line from statistical models in terms of the center of mass energy

3) Hadronic fluctuations: lattice vs. experiment

Approaching a critical end-point at $\mu_B > 0$?

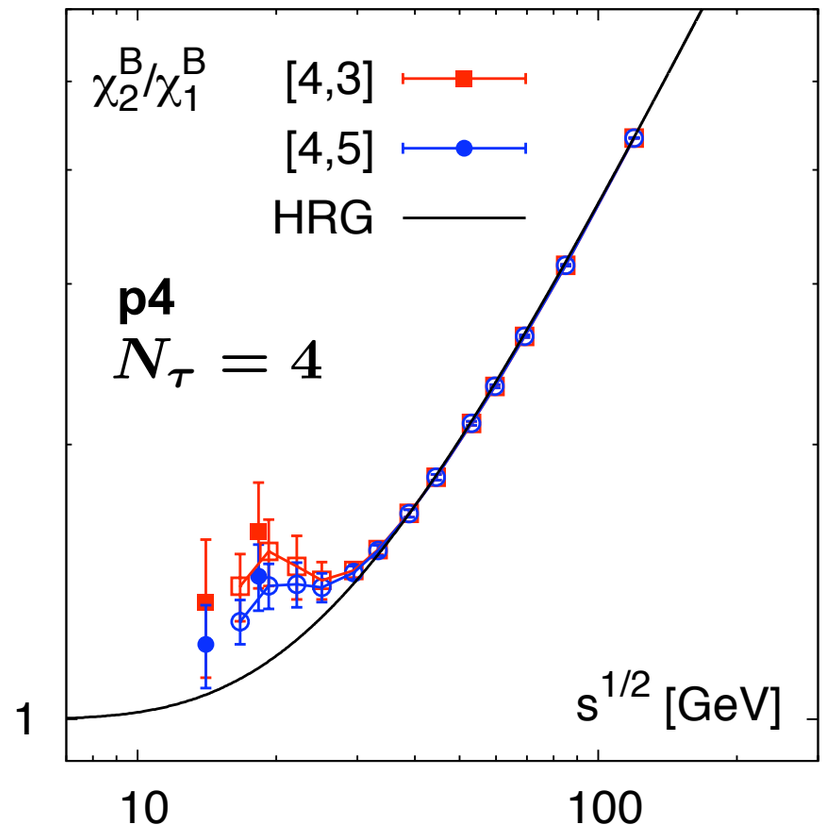
- analyze ratios of moments of baryon number fluctuations
  $\Rightarrow$ **independent of the spectrum, simple dependence on μ_B**
- use parametrization of freeze-out line in the center of mass energy



Mukherjee, QM'11, arXiv:1107.0765

STAR data:
Aggarwal *et al.*, PRL (2010) 022302

$\Rightarrow$ net-proton number fluctuations are close to HRG and lattice QCD (using HRG input)



CS, Prog. Theor. Phys. Suppl. 186 (2010) 563

$\Rightarrow$ systematic and statistical errors increase for $\sqrt{s} \leq 20$ GeV

3) Hadronic fluctuations: lattice vs. experiment

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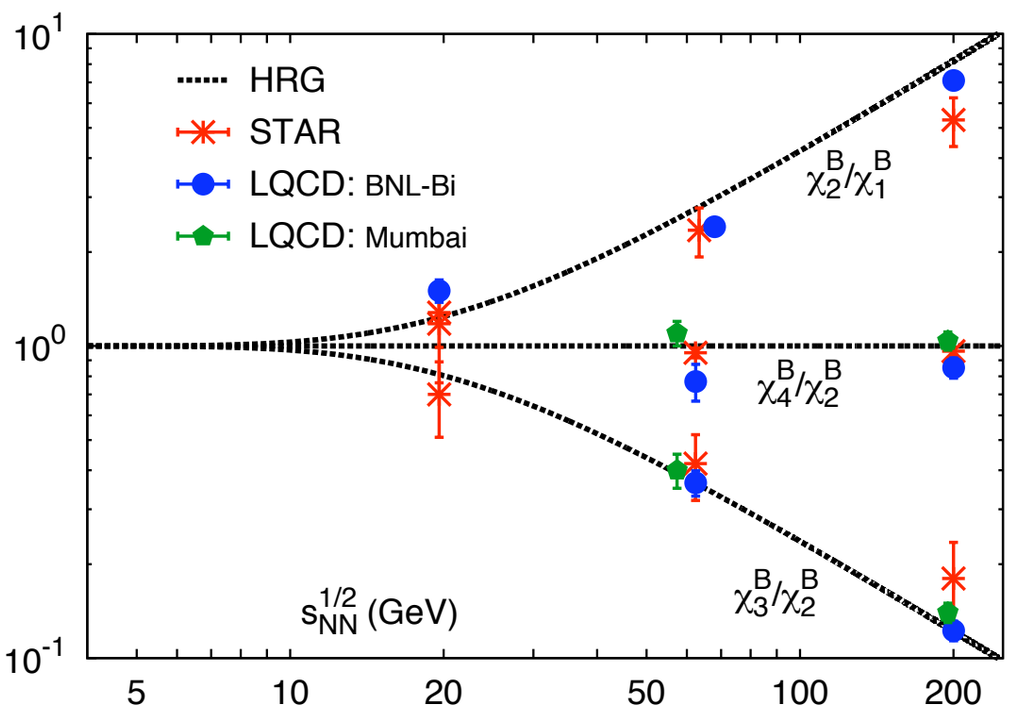

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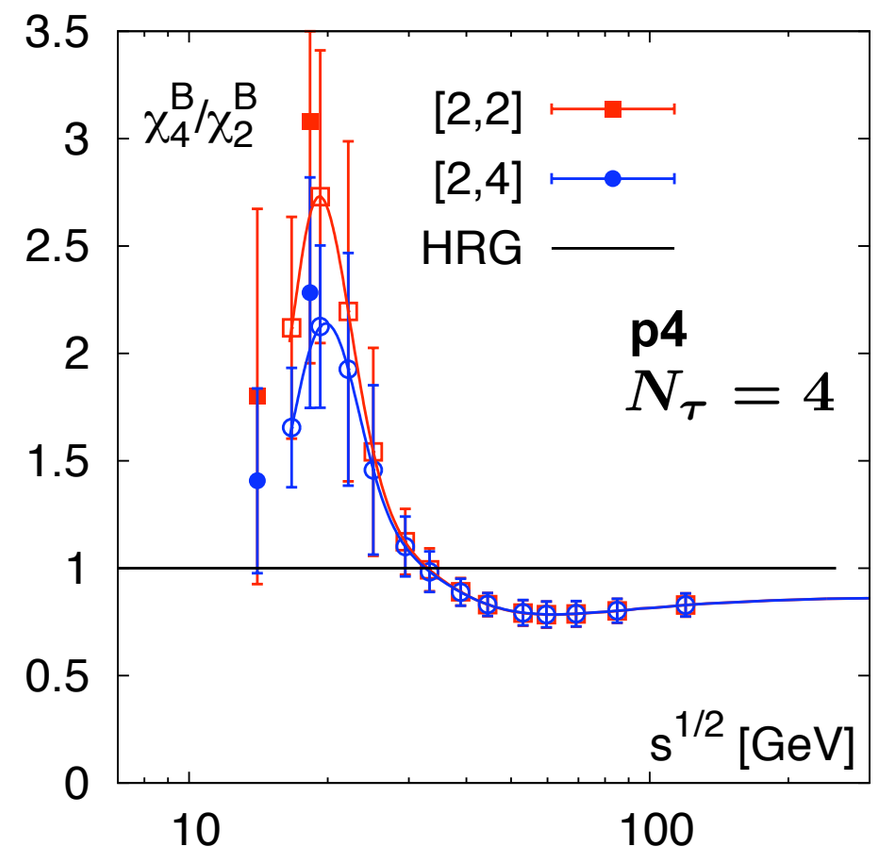
$\Rightarrow$ net-proton number fluctuations can be described by the HRG and lattice QCD



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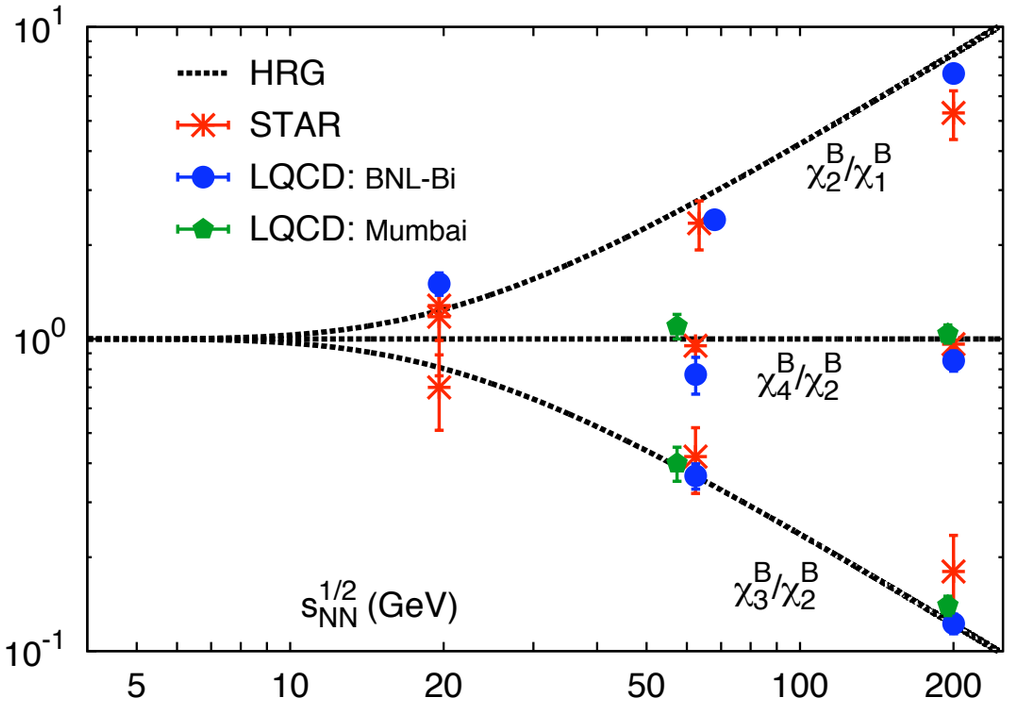

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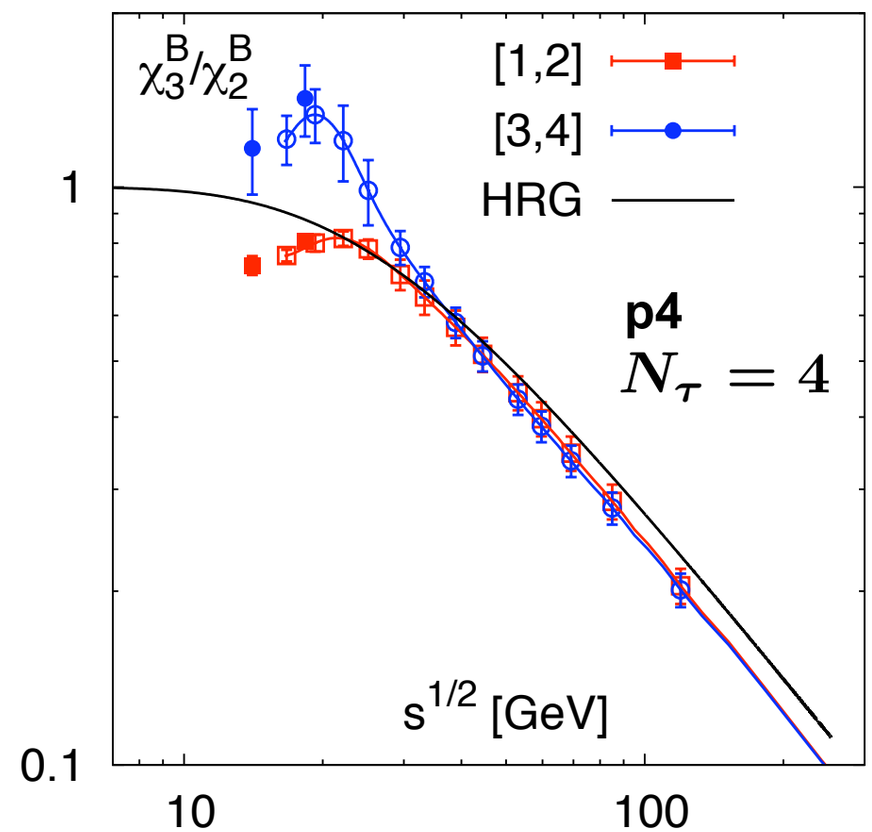

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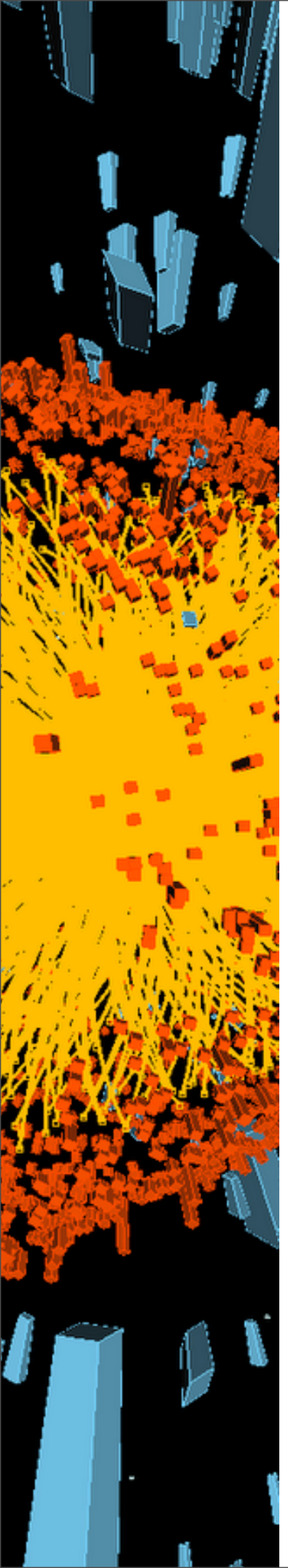

5) Summary

- QCD with physical quark masses is sensitive to universal scaling properties in the chiral limit
- Tc and the curvature of the phase transition line are two important non universal parameters that can be obtained by mapping QCD to O(N) critical behavior

$\Rightarrow$ **Current results suggests that the curvature of the chiral phase transition line is smaller than that of the freeze-out curve.**

$T_c \approx T_f$ **only for small** μ **?**

- We have calculated high order baryonic fluctuations, which are more and more sensitive to the critical behavior of QCD
- Ratios of baryonic susceptibilities are sensitive to the relevant degrees of freedom but are rather independent on the spectrum and volume.
- STAR data on net-proton fluctuations can be reasonably described by lattice data (using HRG input) for $\sqrt{s} \gtrsim 20\ \mathrm{GeV}$

$\Rightarrow$ **results need to be confirmed in the continuum limit w/o using HRG input**