Towards the (3+1)-D Structure of Nuclear Collisions

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RBRC Workshop on Longitudinal Dynamics
BNL, January 20, 2016

3+1 D Structure

- Objects colliding at finite impact parameter $\rightarrow$ interesting spatial structures.
  - Determined by interplay of angular momentum conservation and interactions between particles
- Heavy Ion Physics: state of the art are cylinders with no interesting 3+1 D structure, with some exceptions.
- Is this structure important?
- How to constrain the early stage from QCD?

Overview

- QCD at early times in nuclear collisions: the setup
- Boost invariance and beyond
- Solving Yang Mills
- Phenomenology: angular momentum and directed flow
- Matching to hydro: results and interpretation

Work in collaboration mainly with

- Guangyao Chen (Iowa State University)
- Joe Kapusta, Yang Li (Minnesota)
- Sidharth Somanathan (Texas A&M)
- Sener Ozonder (INT/Seattle)

[RJF, J. Kapusta, Y. Li, nucl-th/0604054

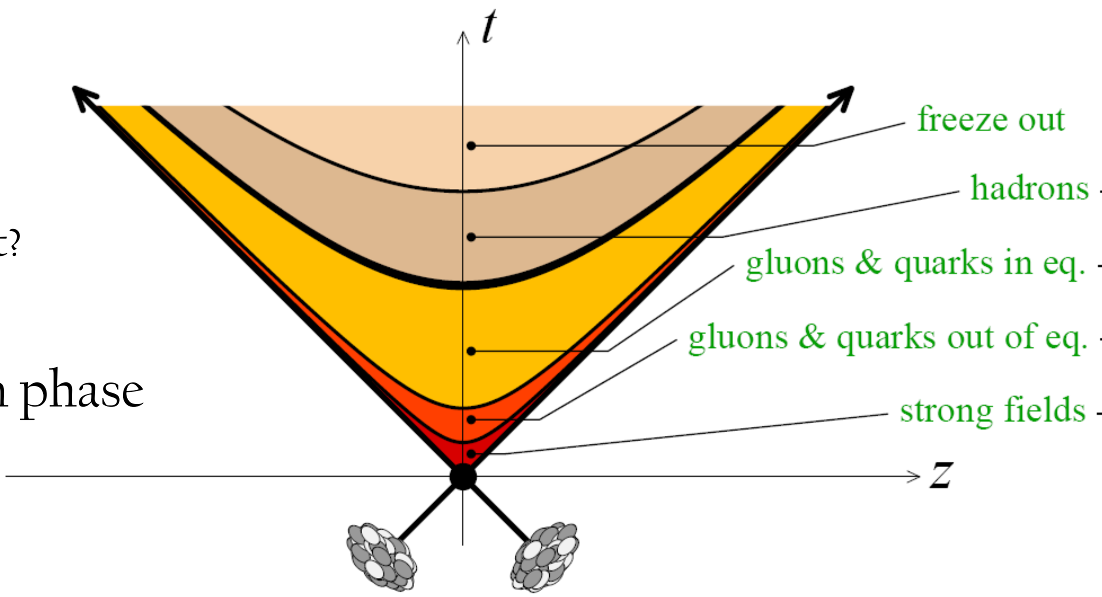
[S. Ozonder, RJF, Phys. Rev. C 89, 034902 (2014)]

[G. Chen, RJF, PLB 723, 417 (2013)]

[G. Chen, RJF, J. Kapusta, Y. Li., Phys. Rev. C 92, 064912 (2015)

Little Bang Evolution

- Thermalized phase: fluid dynamics!
  - Initial conditions and starting time?
- Freeze-out: transport phase
  - Switching conditions?
  - Parameters for hadronic transport?
- Poorly known: pre-equilibrium phase
  - Incoherent N-N collisions?
  - Strings?
  - Strong classical fields?
  - Generic strong coupling arguments, AdS/CFT?

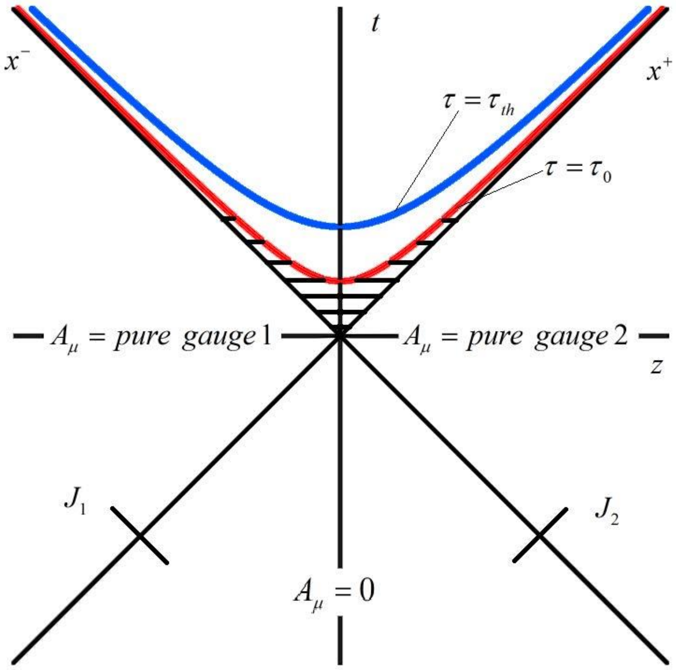


- QCD as guiding principle: color glass condensate (CGC) candidate theory for large energies and large nuclei.

Color Glass Setup

- QCD setup: light cone currents J_1 and J_2 created by color charges ρ_1 and ρ_2.
- Infinite Lorentz contractions → boost symmetry, ρ_1 and ρ_2 transverse densities.
- Transverse gluon density sets a saturation scale Q_s. Classical Yang Mills dynamics.

[L. McLerran, R. Venugopalan]



Classical YM Dynamics

- Solve Yang-Mills equations $\left[D_\mu, F^{\mu\nu}\right] = J^\nu$ for the gluon field $A^\mu(\rho_1, \rho_2)$.
  - Charges ρ_i from Gaussian color fluctuations of a color-neutral nucleus., variance μ_i.

$$\left\langle \rho_i^a(x) \right\rangle = 0$$

$$\left\langle \rho_i^a(x_1)\rho_j^b(x_2) \right\rangle = \frac{g^2}{N_c^2 - 1}\delta_{ij}\delta^{ab}\lambda_i\left(x_1^\mp\right)\delta\left(x_1^\mp - x_2^\mp\right)\delta^2\left(\mathbf{x}_{1T} - \mathbf{x}_{2T}\right) \qquad \mu_i = \int dx^\mp \lambda_i\left(x^\mp\right)$$

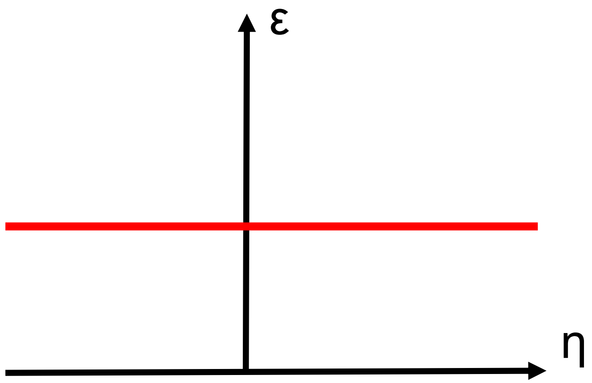
  - Solution for single nuclei A_1^i, A_2^i in light cone gauge well known.
  - Initial conditions for the forward light cone well known. Joining solutions before and after the collision.

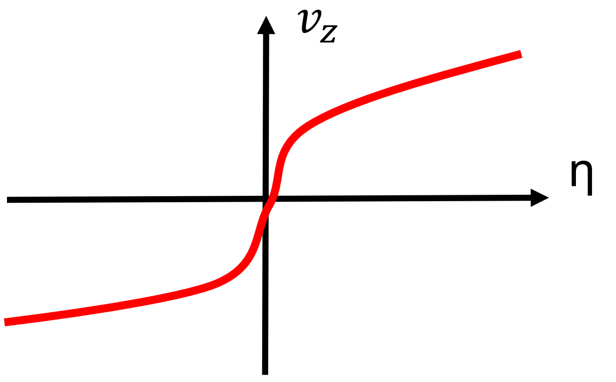
$$A^i_{\perp(0)}(x_\perp) = A^i_1(x_\perp) + A^i_2(x_\perp)$$

$$A_{(0)}(x_\perp) = -\frac{ig}{2}\left[A^i_1(x_\perp), A^i_2(x_\perp)\right]$$

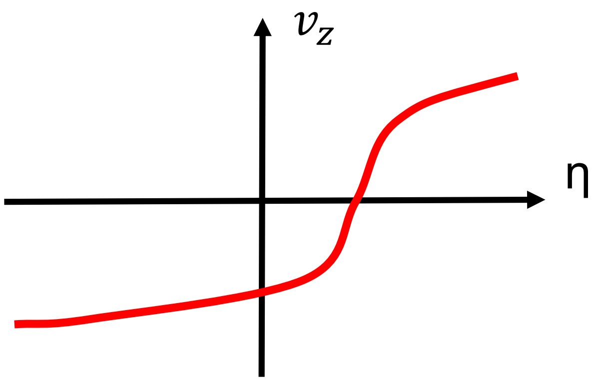
About Boost Invariance

- What do we expect for a simple boost-invariant ansatz?





- What about this?

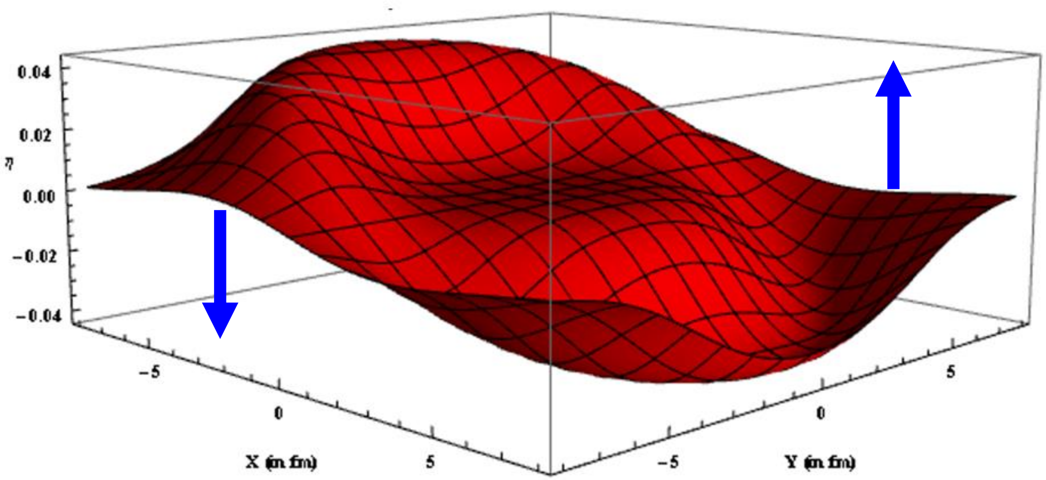


- Yes! Boost invariance symmetry does not have to pick the lab frame for the node $v_z = 0$.
- An asymmetry between ρ_1 and ρ_2 can lead to a shift of the node away from the lab frame.
- Locally determined for each point in the transverse plane.

About Boost Invariance

- This allows for systems with non-zero angular momentum.

SURFACE FOR Vz = 0 (in η units)



- Boost invariance previously interpreted too narrowly.
- One could run a modified 2+1D hydro with all these effects.
  - Run one slice in rapidity with v_z fixed but non-zero. Recover other rapidities through boosts.

Beyond Boost Invariance

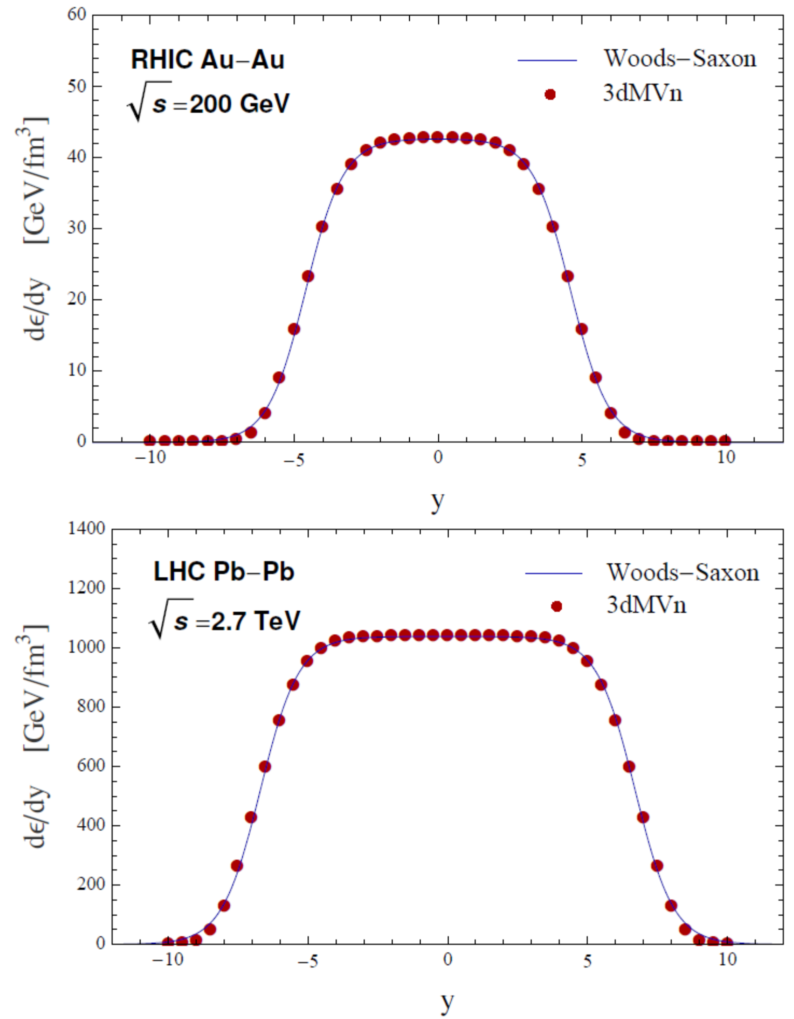
- Real nuclei are slightly off the light cone.
- Classical gluon distributions calculated by Lam and Mahlon.

  [C.S. Lam, G. Mahlon, PRD 62 (2000)]

- Using two approximations valid for $R_A/\gamma \ll 1/Q_s$ we estimated the rapidity dependence of the initial energy density ε_0.

  [S. Ozonder, RJF, Phys. Rev. C 89, 034902 (2014)]

- Parameterizations for RHIC and LHC energies available in the paper.



Back to the Boost-Invariant Case

- Solve the YM equations in the forward light cone

$$\frac{1}{\tau}\frac{\partial}{\partial\tau}\frac{1}{\tau}\frac{\partial}{\partial\tau}\tau^2 A - [D^i,[D^i,A]] = 0\,,$$

$$ig\tau\left[A,\frac{\partial}{\partial\tau}A\right] - \frac{1}{\tau}\left[D^i,\frac{\partial}{\partial\tau}A_\perp^i\right] = 0\,,$$

$$\frac{1}{\tau}\frac{\partial}{\partial\tau}\tau\frac{\partial}{\partial\tau}A_\perp^i - ig\tau^2\left[A,[D^i,A]\right] - \left[D^j,F^{ji}\right] = 0$$

$$F^{+-} = -\frac{1}{\tau}\frac{\partial}{\partial\tau}\tau^2 A,$$

$$F^{i\pm} = -x^\pm\left(\frac{1}{\tau}\frac{\partial}{\partial\tau}A_\perp^i \mp [D^i,A]\right),$$

$$F^{ij} = \partial^i A_\perp^j - \partial^j A_\perp^i - ig[A_\perp^i,A_\perp^j].$$

with given boundary conditions using an expansion in time.

$$A(\tau,\vec{x}_\perp) = \sum_{n=0}^{\infty}\tau^n A_{(n)}(\vec{x}_\perp),$$

$$A_\perp^i(\tau,\vec{x}_\perp) = \sum_{n=0}^{\infty}\tau^n A_{\perp(n)}^i(\vec{x}_\perp)$$

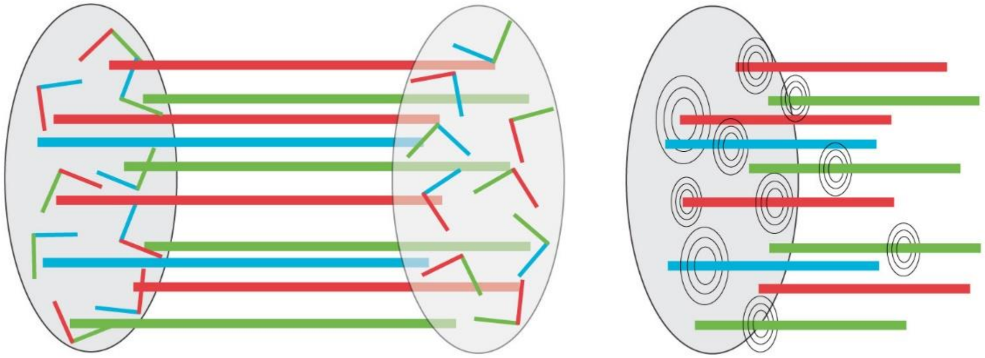
Gluon Fields after Collision

- We find an analytic solutions recursively.

$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D^i_{(k)}, \left[D^i_{(l)}, A_{(m)}\right]\right]$$

$$A^i_{\perp(n)} = \frac{1}{n^2}\left(\sum_{k+l=n-2} \left[D^j_{(k)}, F^{ji}_{(l)}\right] + ig \sum_{k+l+m=n-4} \left[A_{(k)}, \left[D^i_{(l)}, A_{(m)}\right]\right] \right)$$

- Transverse fields in the nuclei before the collision create strong longitudinal chromo-electric and magnetic fields at overlap ($\tau = 0$). Faraday's, Ampere's and Gauss Law start to create transverse fields immediately after $\tau = 0$.



Fields: Into the Forward Light Cone

- With *non-abelian* longitudinal fields E_0, B_0 seeded, the next step in time can be understood in terms of the QCD versions of Ampere's, Faraday's and Gauss' Law.
  - Longitudinal fields E_0, B_0 decrease in both *z* and *t* away from the light cone
- Here *abelian* version for simplicity:
- Gauss' Law at fixed time *t*
  - Long. flux imbalance compensated by transverse flux
  - Gauss: *rapidity-odd radial* field
- Ampere/Faraday as function of *t*:
  - Decreasing long. flux induces transverse field
  - Ampere/Faraday: *rapidity-even curling* field
- Full classical QCD:

$$E^i = -\frac{\tau}{2}\left(\sinh\eta\left[D^i, E_0\right] + \cosh\eta\,\varepsilon^{ij}\left[D^j, B_0\right]\right)$$

$$B^i = \frac{\tau}{2}\left(\cosh\eta\,\varepsilon^{ij}\left[D^j, E_0\right] - \sinh\eta\left[D^i, B_0\right]\right)$$

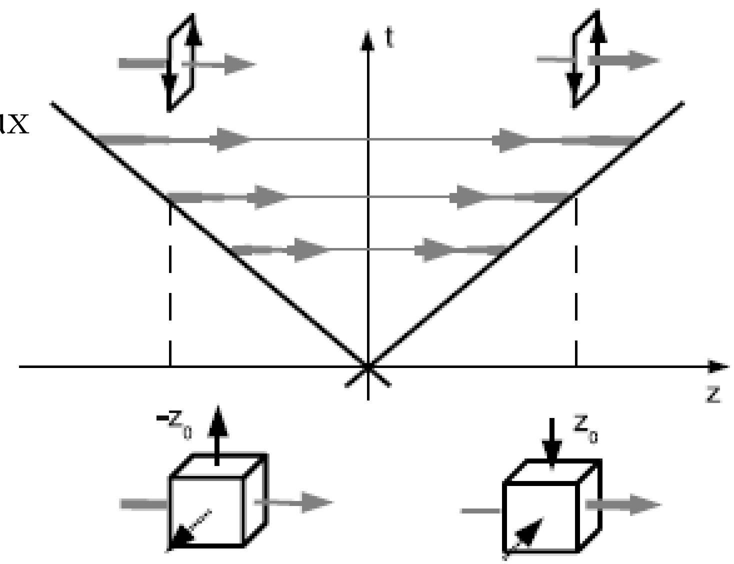


Figure 1: Two observers at $z = z_0$ and $z = -z_0$ test Ampère's and Faraday's Laws with areas a^2 in the transverse plane and Gauss' Law with a cube of volume a^3. The transverse fields from Ampère's and Faraday's Laws (black solid arrows) are the same in both cases, while the transverse fields from Gauss' Law (black dashed arrows) are observed with opposite signs. Initial longitudinal fields are indicated by solid grey arrows, thickness reflects field strength.

 [G. Chen, RJF, PLB 723 (2013)]

Initial Transverse Field: Visualization

- Transverse fields for randomly seeded A_1, A_2 fields (abelian case).
- η = 0: Closed field lines around longitudinal flux maxima/minima
- η ≠ 0: Sources/sinks for transverse fields appear

$\eta = 0$

$\eta = 1$

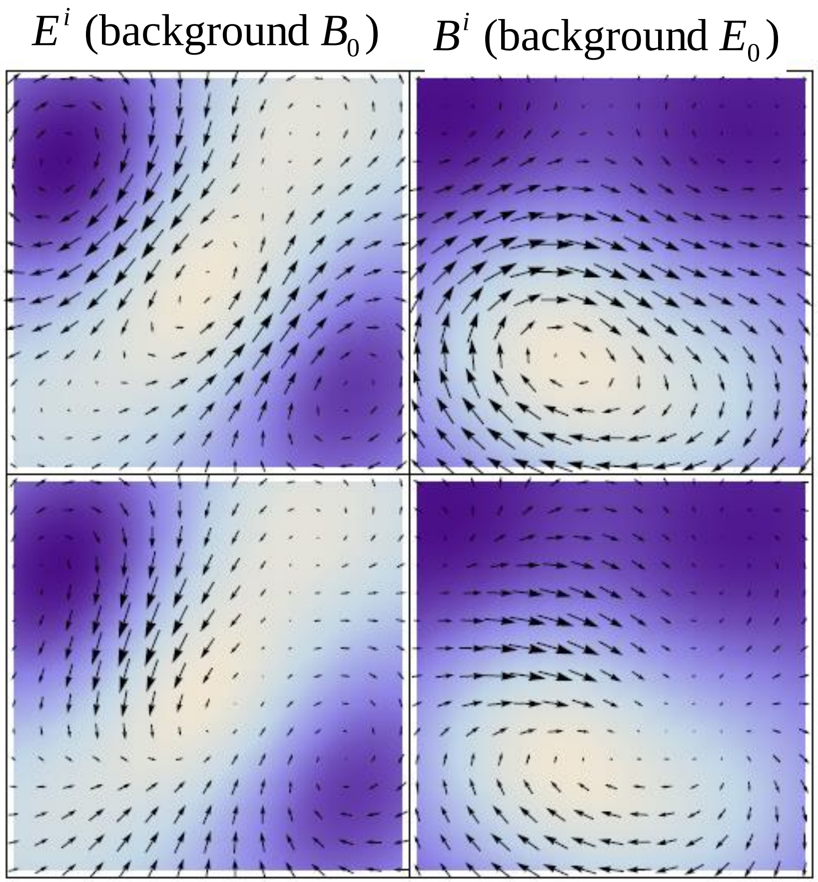


Figure 2: Transverse electric fields (left panels) and magnetic fields (right panels) at $\eta = 0$ (upper panels) and $\eta = 1$ (lower panels) in an abelian example for a random distribution of fields A_1^i, A_2^i. The initial longitudinal fields B_0 (left panels) and E_0 (right panels) are indicated through the density of the background (lighter color = larger values). At $\eta = 0$ the fields are divergence-free and clearly following Ampére's and Faraday's Laws, respectively.

Energy Momentum Tensor

- Flow emerges from pressure at order τ^1:

$$\vec{S} = \vec{E} \times \vec{B}$$

$$T_{\mathrm{f}}^{\mu\nu} = \begin{pmatrix} \varepsilon_0 + O(\tau^2) & \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) \\ \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \varepsilon_0 + O(\tau^2) & O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta \\ \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) & \varepsilon_0 + O(\tau^2) & \alpha^2 \sinh\eta + \beta^2 \cosh\eta \\ O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta & \alpha^2 \sinh\eta + \beta^2 \cosh\eta & -\varepsilon_0 + O(\tau^2) \end{pmatrix}$$

- Transverse Poynting vector gives transverse flow.

[RJF, J.I. Kapusta, Y. Li, (2006)]
[G. Chen, RJF, PLB 723 (2013)]

$$S^i_{\text{even}} = \frac{\tau}{2}\cosh\eta\left(E_0\left[D^i, E_0\right] + B_0\left[D^i, B_0\right]\right) = \alpha^i \cosh\eta$$

$$S^i_{\text{odd}} = \frac{\tau}{2}\sinh\eta\,\varepsilon^{ij}\left(E_0\left[D^j, B_0\right] - B_0\left[D^j, E_0\right]\right) = \beta^i \sinh\eta$$

$$\alpha^i = -\frac{\tau}{2}\nabla^i \varepsilon_0$$

Like hydrodynamic flow, determined by gradient of transverse pressure $P_T = \varepsilon_0$; even in rapidity.

$$\beta^i = \frac{\tau}{2}\varepsilon^{ij}\left(\left[D^j, B_0\right]E_0 - \left[D^j, E_0\right]B_0\right)$$

Non-hydro like; odd in rapidity; from field dynamics

Transverse Flow: Visualization

- Transverse Poynting vector for randomly seeded A_1, A_2 fields (abelian case).

- $\eta = 0$: "Hydro-like" flow from large to small energy density

- $\eta \neq 0$: Quenching/amplification of flow due to the underlying field structure.

$(\text{background} = \varepsilon_0)$

$\eta = 0$
(no odd flow)

$\eta = 1$

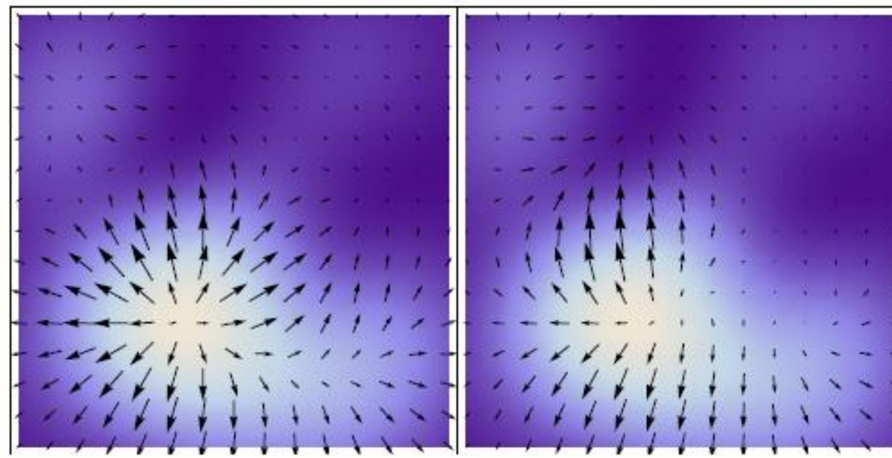


Figure 3: Example for transverse flow of energy for $\eta = 0$ (left panel) and $\eta = 1$ (right panel) in the abelian example for the same random distribution of fields A_1^i, A_2^i as in Fig. 2. The initial energy density T^{00} is shown through the density of the background (lighter color = larger values). At $\eta = 0$ the flow follows the gradient in the energy density in a hydro-like way while away from mid-rapidity energy flow gets quenched in some directions and amplified in others.

Averaged Density and Flow

- Averaging color charge densities. Here no fluctuations!
- Energy density

$$\varepsilon_0 = \frac{g^6 N_c \left(N_c^2 - 1\right)}{8\pi} \mu_1 \mu_2 \ln^2 \frac{Q^2}{m^2}$$

[T. Lappi, 2006]
[RJF, Kapusta, Li, 2006]
[Fujii, Fukushima, Hidaka, 2009]

- "Hydro" flow:

$$\alpha^i = -\tau \frac{g^6 N_c \left(N_c^2 - 1\right)}{64\pi^2} \nabla^i \left(\mu_1 \mu_2\right) \ln^2 \frac{Q^2}{m^2}$$

- "Odd" flow term:

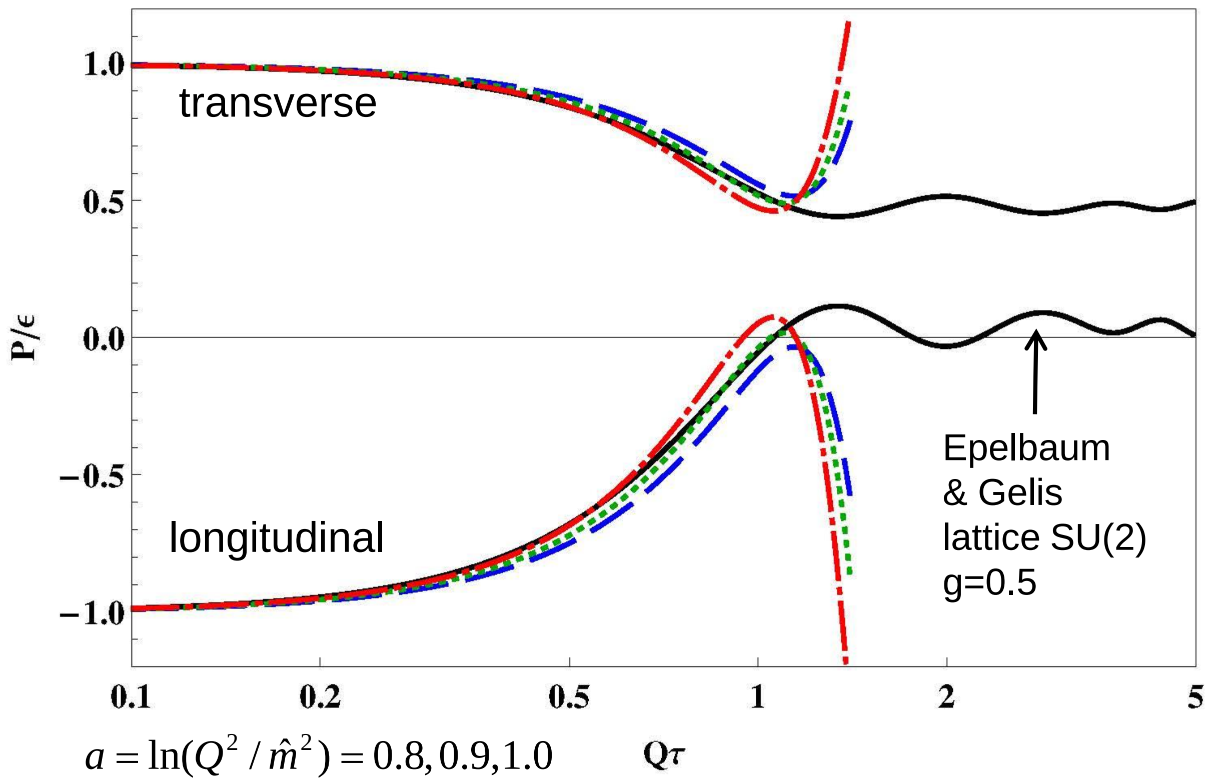
[G. Chen, RJF, PLB 723 (2013)]

$$\beta^i = -\tau \frac{g^6 N_c \left(N_c^2 - 1\right)}{64\pi^2} \left(\mu_2 \nabla^i \mu_1 - \mu_1 \nabla^i \mu_2\right) \ln^2 \frac{Q^2}{m^2}$$

- Order τ^2 terms …

Results: Bulk Variables

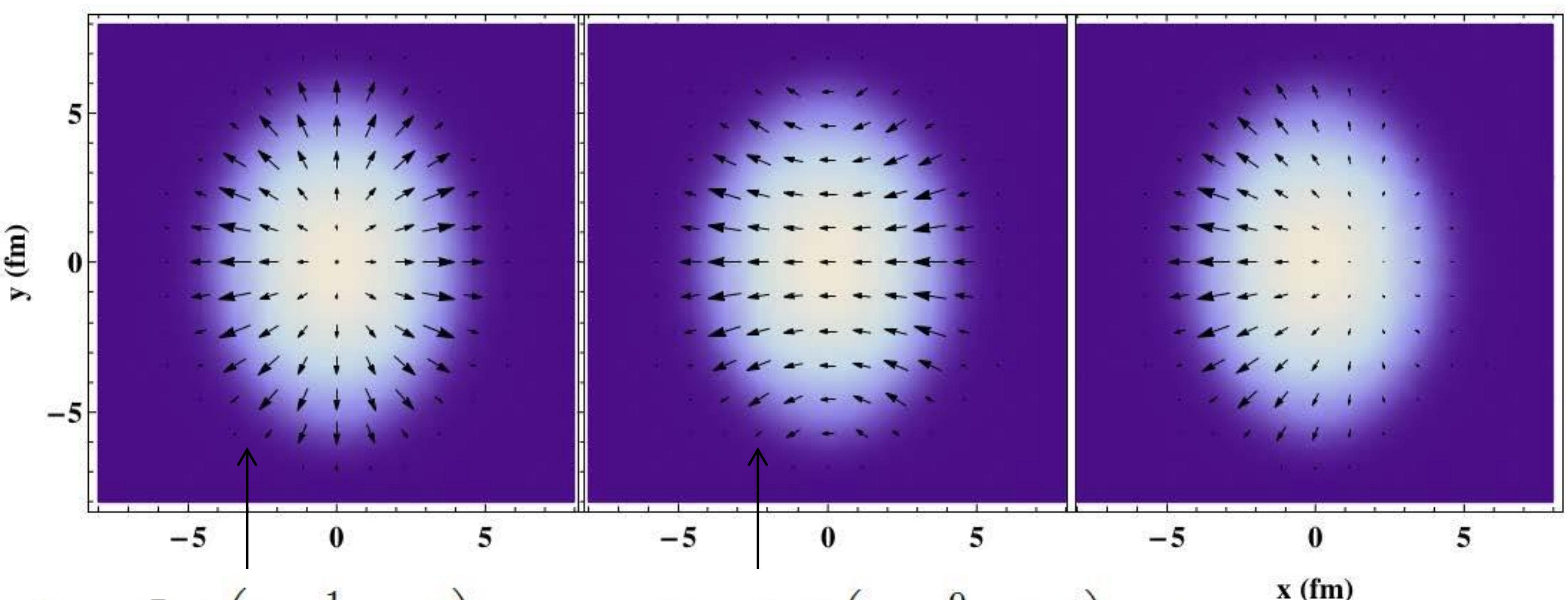
- Our expansion fails around $\tau \sim 1/Q_s$
- Pocket formulas for simple slab nuclei: $\frac{p_L}{p_T} = -\frac{1-\frac{3}{2a}(Q\tau)^2}{1-\frac{1}{2a}(Q\tau)^2} + O(Q\tau)^4$
- Consistent with numerical results



$a = \ln(Q^2/\hat{m}^2) = 0.8, 0.9, 1.0$

Flow Phenomenology: $b \neq 0$

- Odd flow needs asymmetry between sources. Here: finite impact parameter Pb+Pb collision, b = 6 fm.



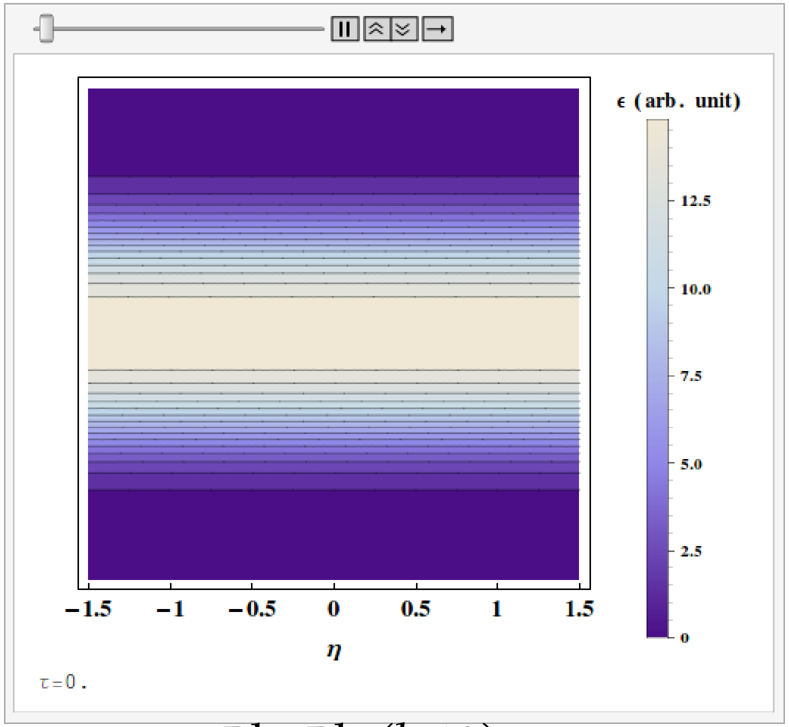
$$T^{0i}_{\text{even}} = \frac{\tau}{2}\alpha^i\left(1 - \frac{1}{2a}(Q\tau)^2\right)\cosh\eta \qquad T^{0i}_{\text{odd}} = \frac{\tau}{2}\beta^i\left(1 - \frac{9}{16a}(Q\tau)^2\right)\sinh\eta$$

sum at $\eta = 1$

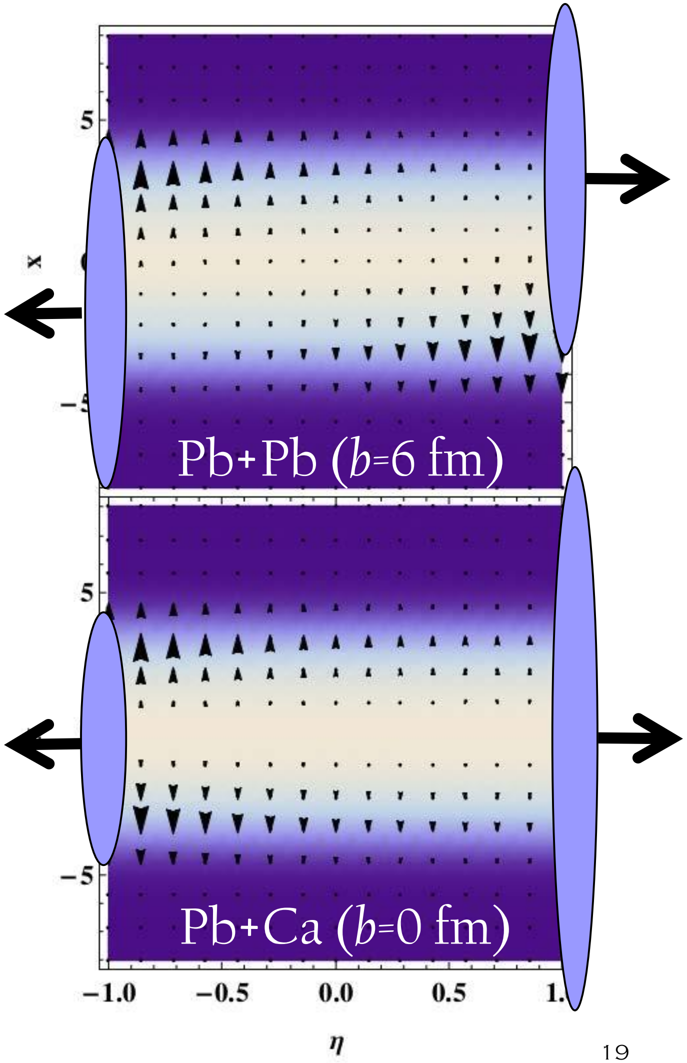
- *Radial* flow following gradients in the fireball at η = 0.
- In addition: *directed* flow away from η = 0.

Flow Phenomenology: $b \neq 0$ and A+B

- For finite impact parameter fireball is rotating, exhibits angular momentum.



Pb+Pb (b=10)



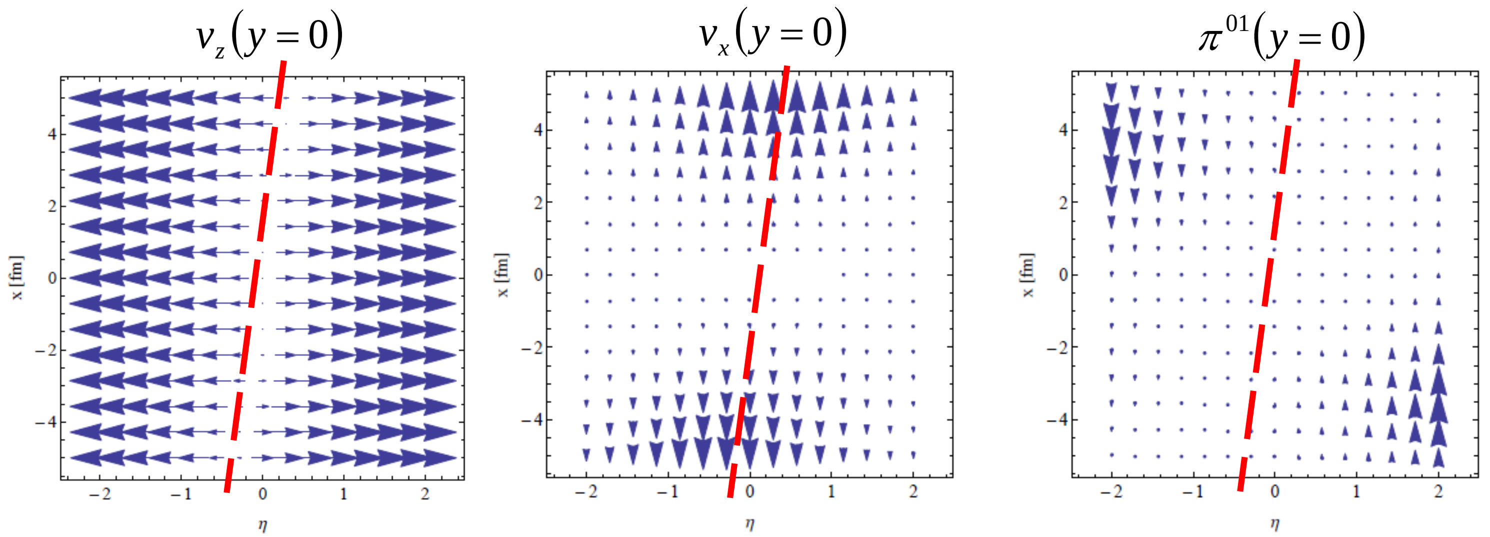
- Conical flow pattern for A+B systems.

Matching to Hydrodynamics

- No dynamic equilibration here; see other talks at this conference.
- Pragmatic solution: instantaneous decomposition of the CGC energy momentum tensor into hydro fields at an early time (0.1 – 0.4 fm).

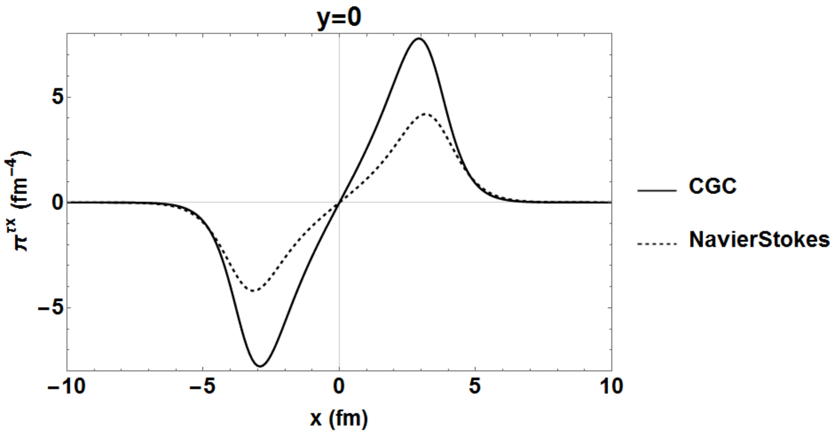
$$T^{\mu\nu} = (\varepsilon + p + \Pi)u^\mu u^\nu - (p + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

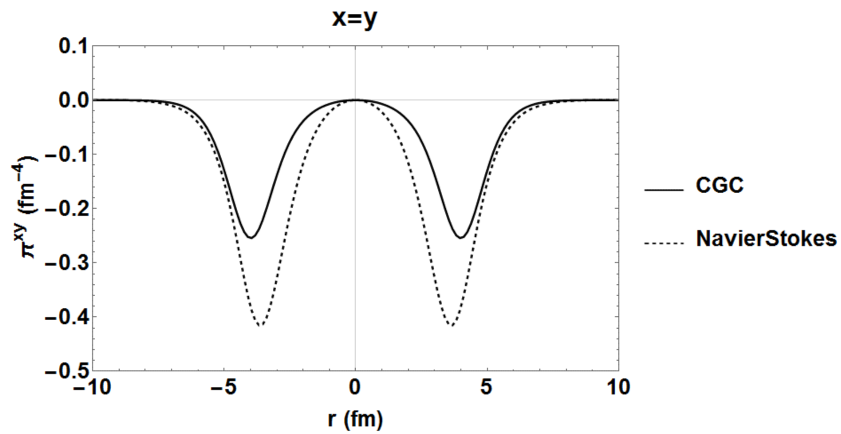
- Keep viscous stress even if it is large.



Matching to Hydrodynamics

- Viscous stress from CGC generally follows Navier-Stokes behavior.
- Samples

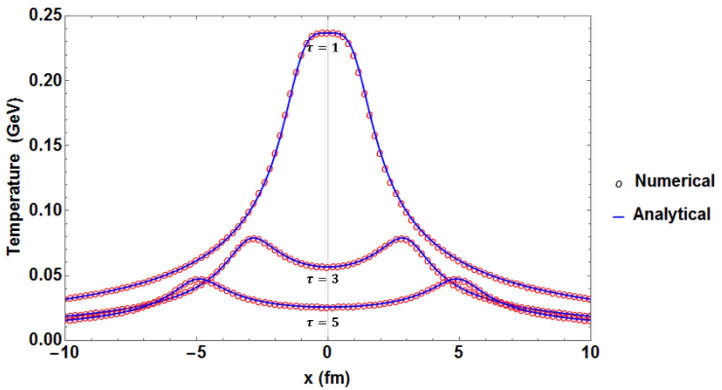


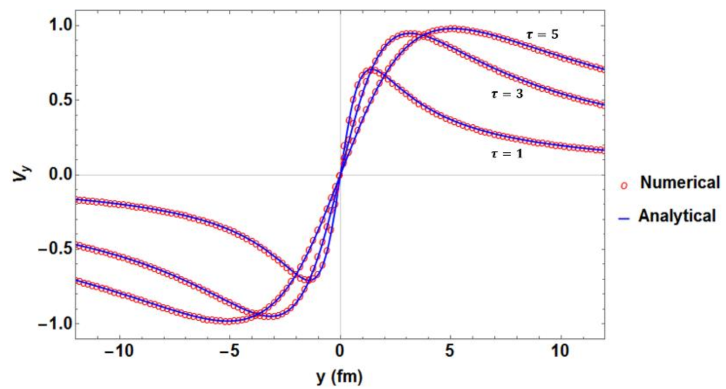


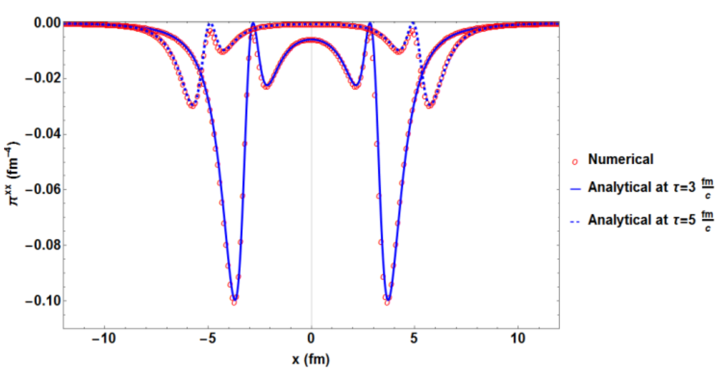
- To be expected?

Texas 3+1D Fluid Dynamics

- KT for fluxes, 5th order WENO for spatial derivatives, 3rd order TVD Runge Kutta for time integration.
- Bulk and shear stress, vorticity
- Gubser test:

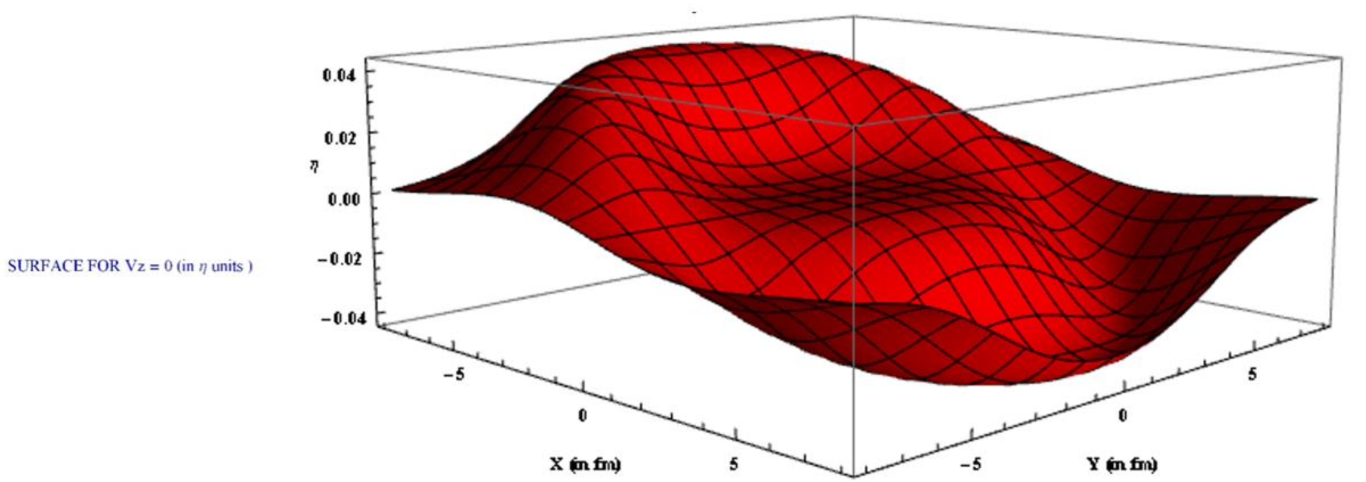






Evolution of Hydro Fields: Nodal Plane

- Viscous hydro evolution:
- Reversal of distortion of the nodal plane → relaxation to "naïve" boost invariant configuration
- Diminishing of angular momentum at midrapidity

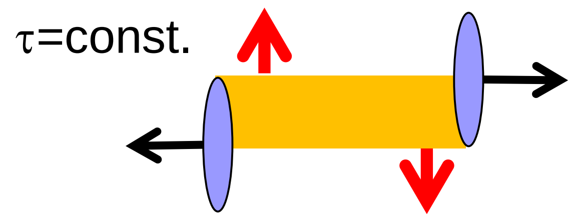


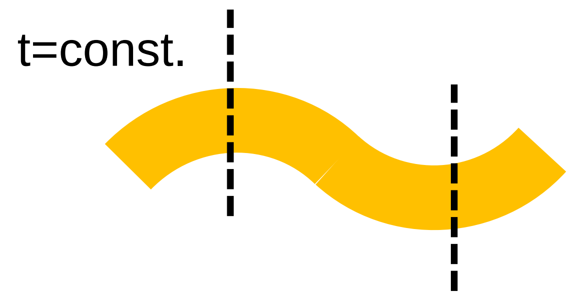
G. Chen and R. J. Fries, Phys. Lett. B 723, 417 (2013)

- Global angular momentum is not conserved: fixed sources on the light cone.
- Need to go to "true 3+1D".

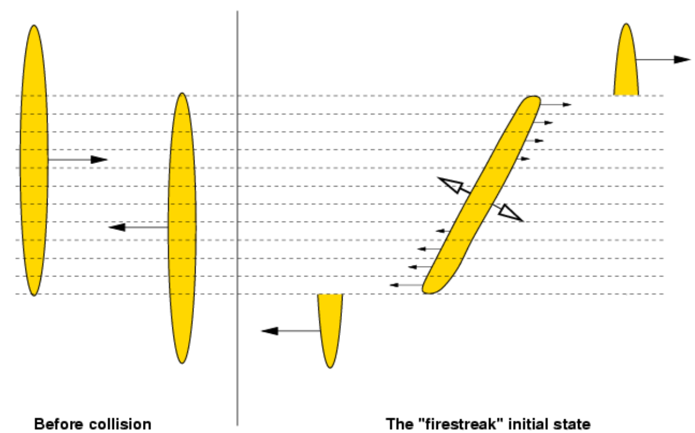
Interpretation

- Gauss' Law and initial longitudinal fields drive rapidity-odd expansion and angular momentum in the CGC phase.
- Switch off Gauss Law: 'boring' boost-invariance wins in the hydro phase maybe with a few exceptions.
- Finite system: global angular momentum must be conserved.
- Easily done in models (fire streak etc.), need to get this into CGC calculation too. Stay tuned.





Cut at ~ beam rapidity!



[Gosset, Kapusta, Westfall (1978)]
[L. Csernai et al. PRC 84 (2011)] …
[Liang, Wang (2005)]

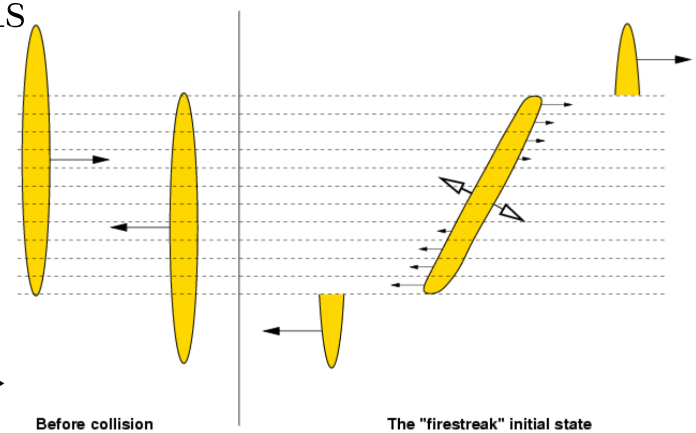
Summary

- Early energy momentum tensor from CGC: analytic results up to times $1/Q_s$ available.
- Interesting features: angular momentum, directed flow, A+B asymmetries, etc.
- Are there signatures unique to CGC?
- Features of gluon energy flow are naturally translated into their counterparts in hydrodynamic fields in a simple matching procedure.
- Fluid dynamics relaxes fireball to the naïve boost-invariant structure.

Backup

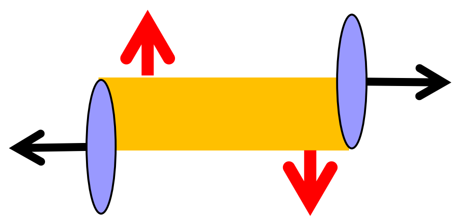
PHENOMENOLOGY: $B \neq 0$

- Angular momentum is natural: some old models have it, most modern hydro calculations don't.
  - Some exceptions
    [L. Csernai et al. PRC 84 (2011)] …
  - Do we determine flow incorrectly when we miss the rotation?

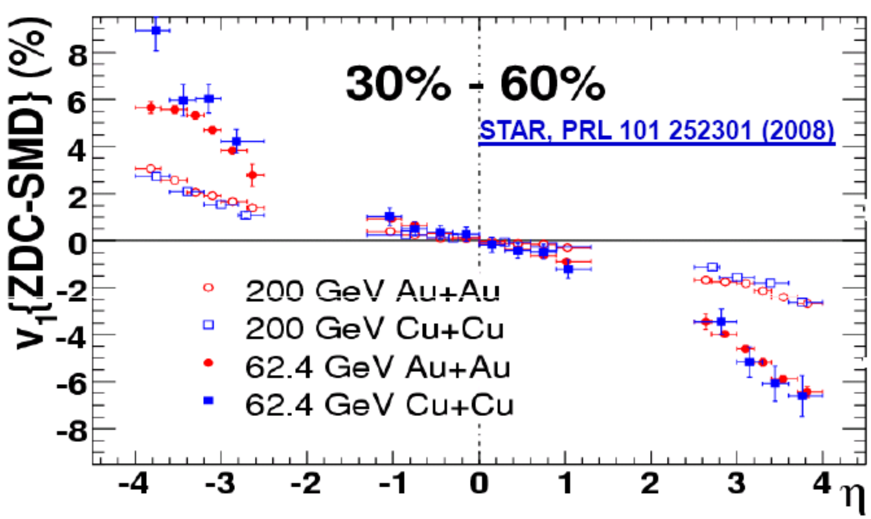


[Gosset, Kapusta, Westfall (1978)]

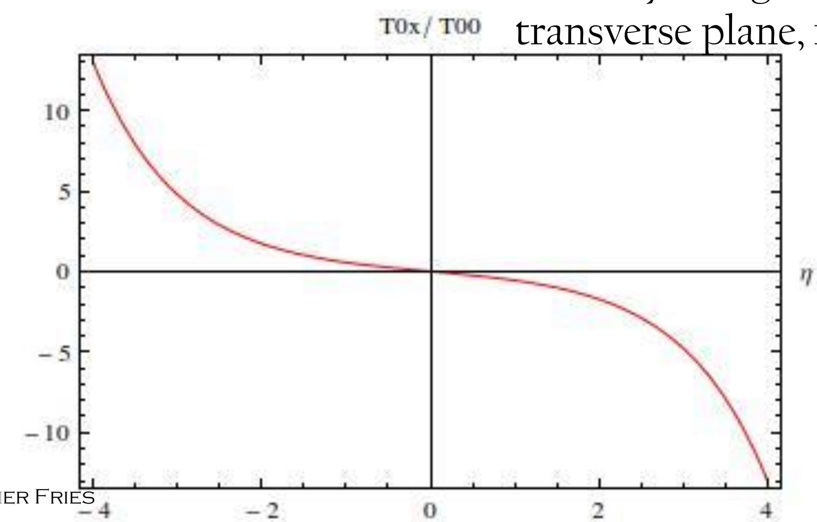
[Liang, Wang (2005)]



- Directed flow v_1:
  - Compatible with hydro with suitable initial conditions.



MV only, integrated over transverse plane, no hydro

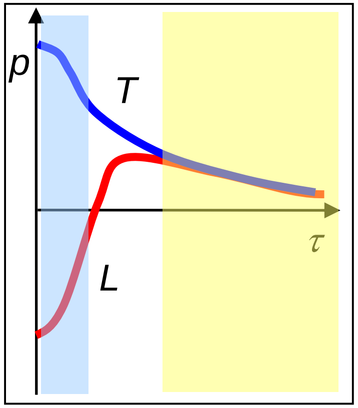


Matching to Hydrodynamics

- No dynamic equilibration here; see other talks at this conference.
- Pragmatic solution: extrapolate from both sides ($r(\tau)$ = interpolating fct.)

$$T^{\mu\nu} = T_{\mathrm{f}}^{\mu\nu} r(\tau) + T_{\mathrm{pl}}^{\mu\nu}\left(1 - r(\tau)\right)$$

- Here: fast equilibration assumption: $r(\tau) = \Theta(\tau_0 - \tau)$



- Matching: enforce $\partial_\mu T^{\mu\nu} = 0$ and

$$\partial_\mu M^{\mu\nu\lambda} = 0 \qquad M^{\mu\nu\lambda} = x^\mu T^{\nu\lambda} - x^\nu T^{\mu\lambda}$$

(and other conservation laws).

- Mathematically equivalent to imposing smoothness condition on all components of $T_{\mu\nu}$.
- Leads to the same procedure used by Schenke et al.