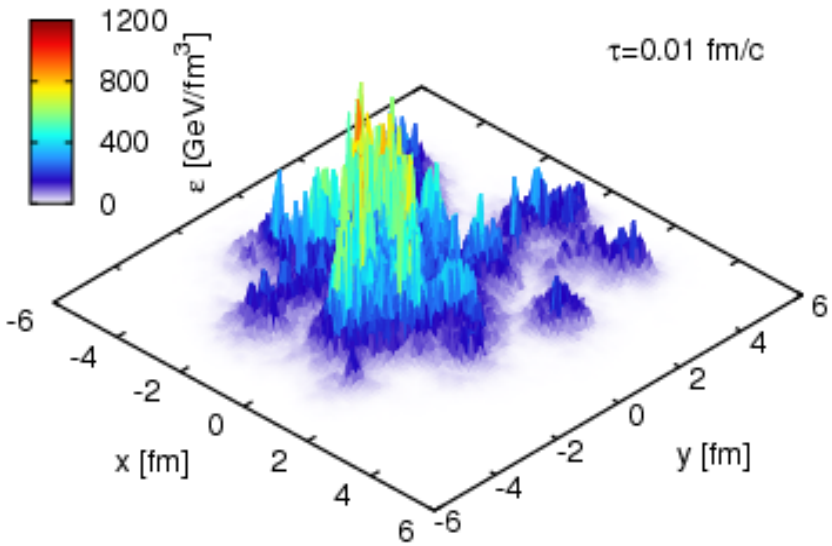
# Recent LHC results on rapidity correlations and fluctuations

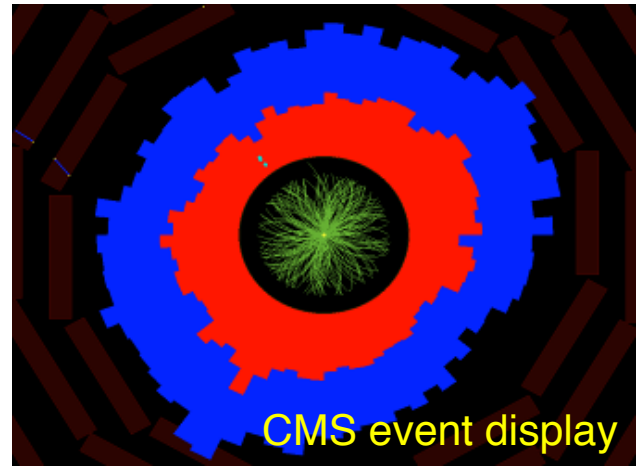
Wei Li (Rice University)

RBRC workshop, BNL, January 20

Initial state → EbE hydro. → Final state




overlap zone in x-y




$$f(p_T,\phi)=\frac{d^2N}{dp_T d\eta}\left\{1+2\sum_n v_n(p_T)\cos\left[n\left(\phi-\Psi_n(p_T)\right)\right]\right\}$$

## QGP dynamics at midrapidity well understood

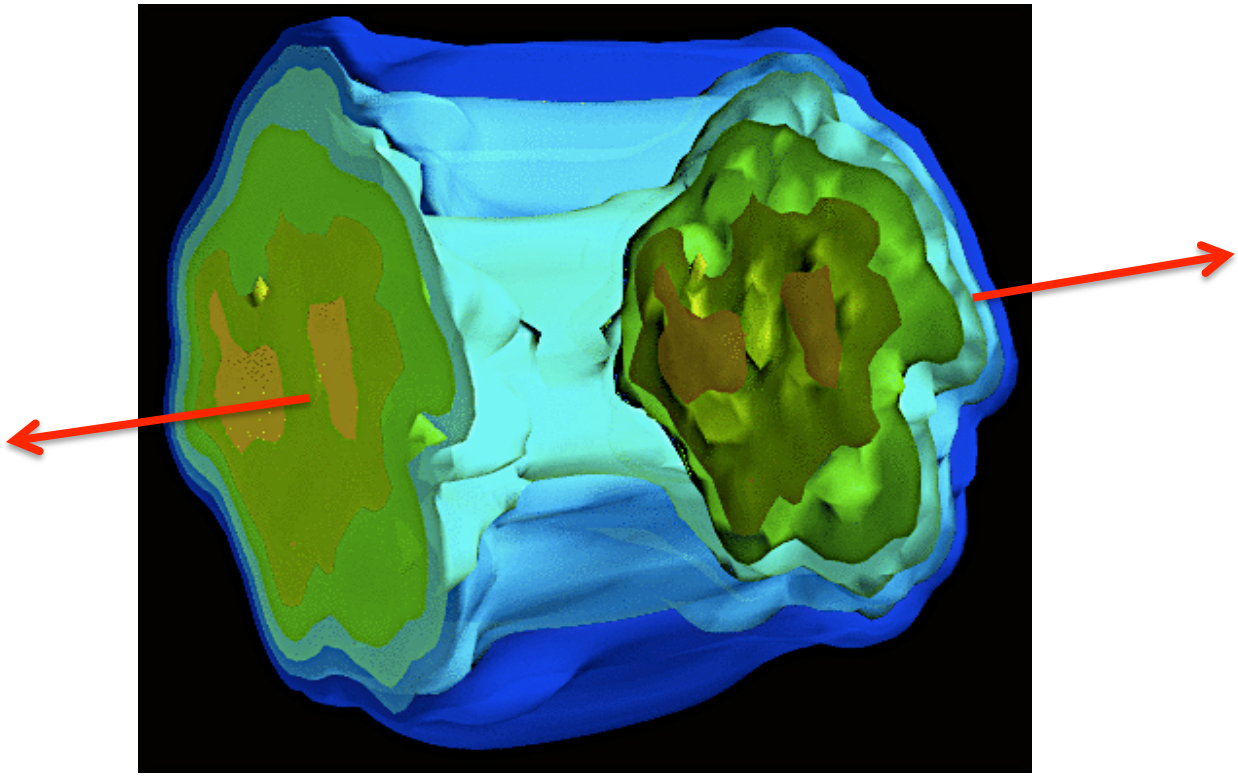
- Multiplicity
- $p_T$ spectra
- PID spectra

- $v_n(p_T)$
- PID $v_n$
- $p(v_n)$
- $v_n$ vs $v_m$
- $\Psi_n$ vs $\Psi_m$

$\Psi_n(p_T)$

- Flow factorization
- PCA

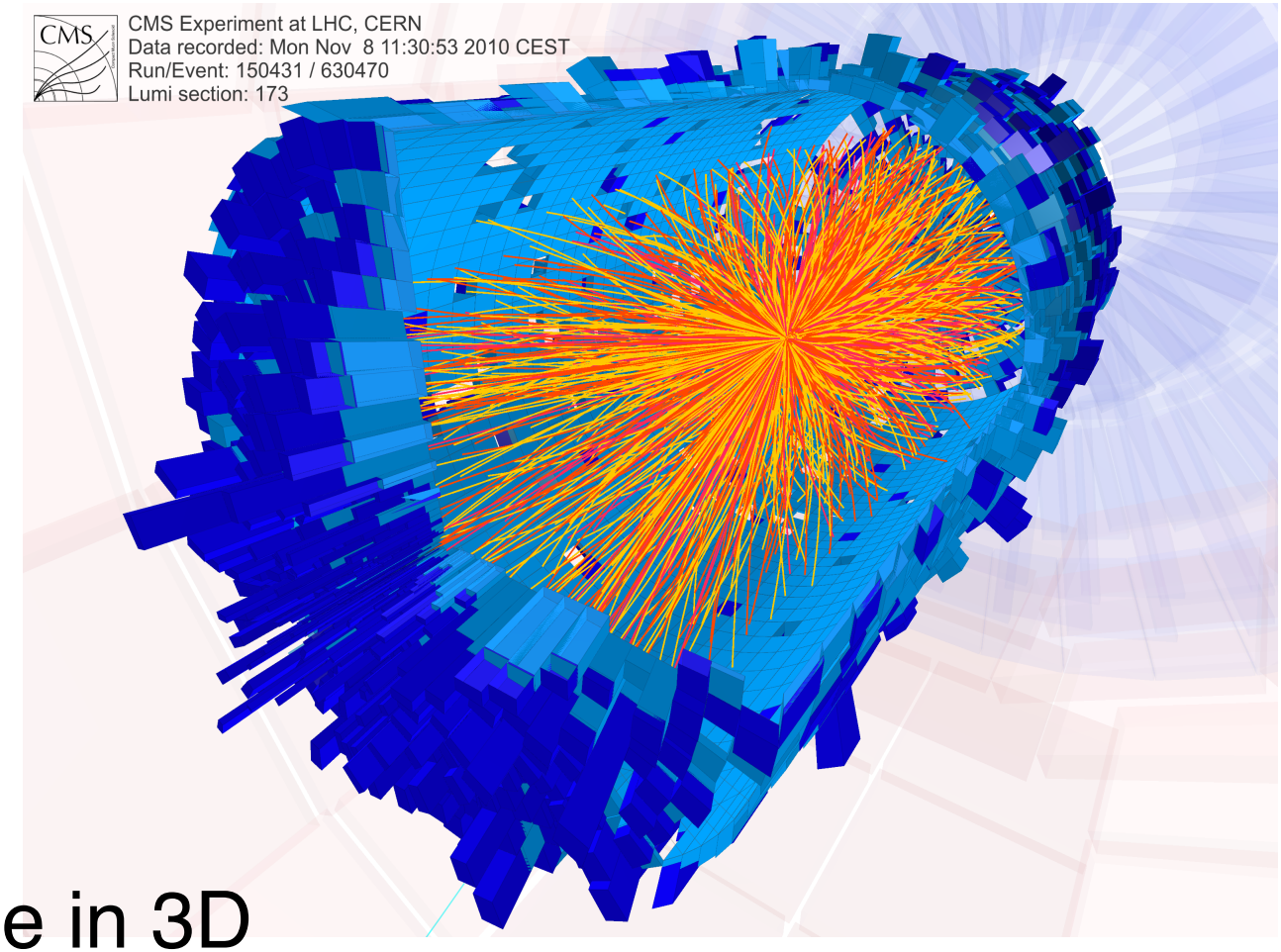
# Longitudinal dynamics: QGP expands in **3D**



Longitudinal dynamics not well understood yet

- How is the initial entropy deposited in 3+1D space-time? How it fluctuates event-by-event?
- What is the role of longitudinal pressure?

# Longitudinal dynamics: QGP expands in 3D




Final-state in 3D

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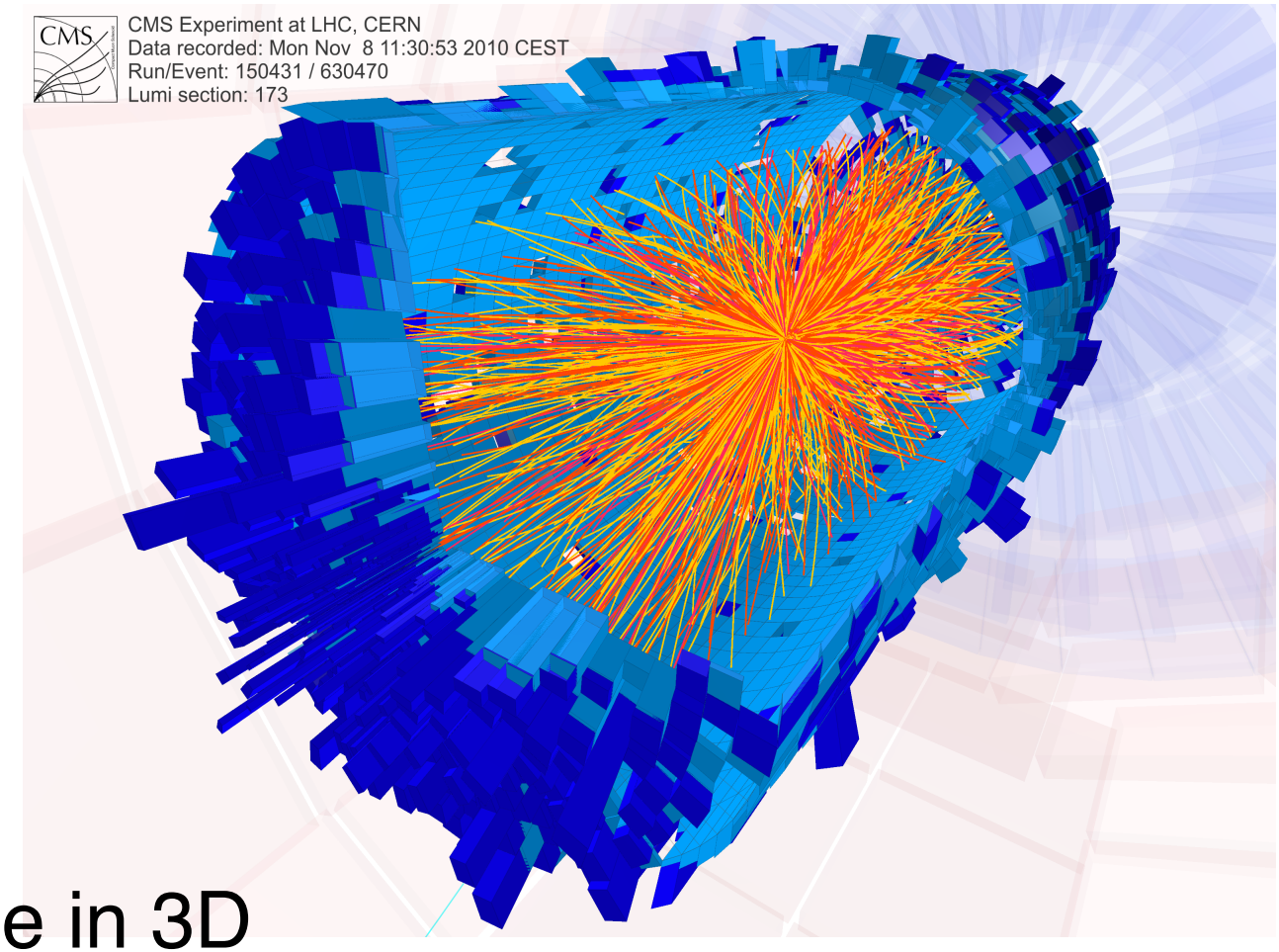
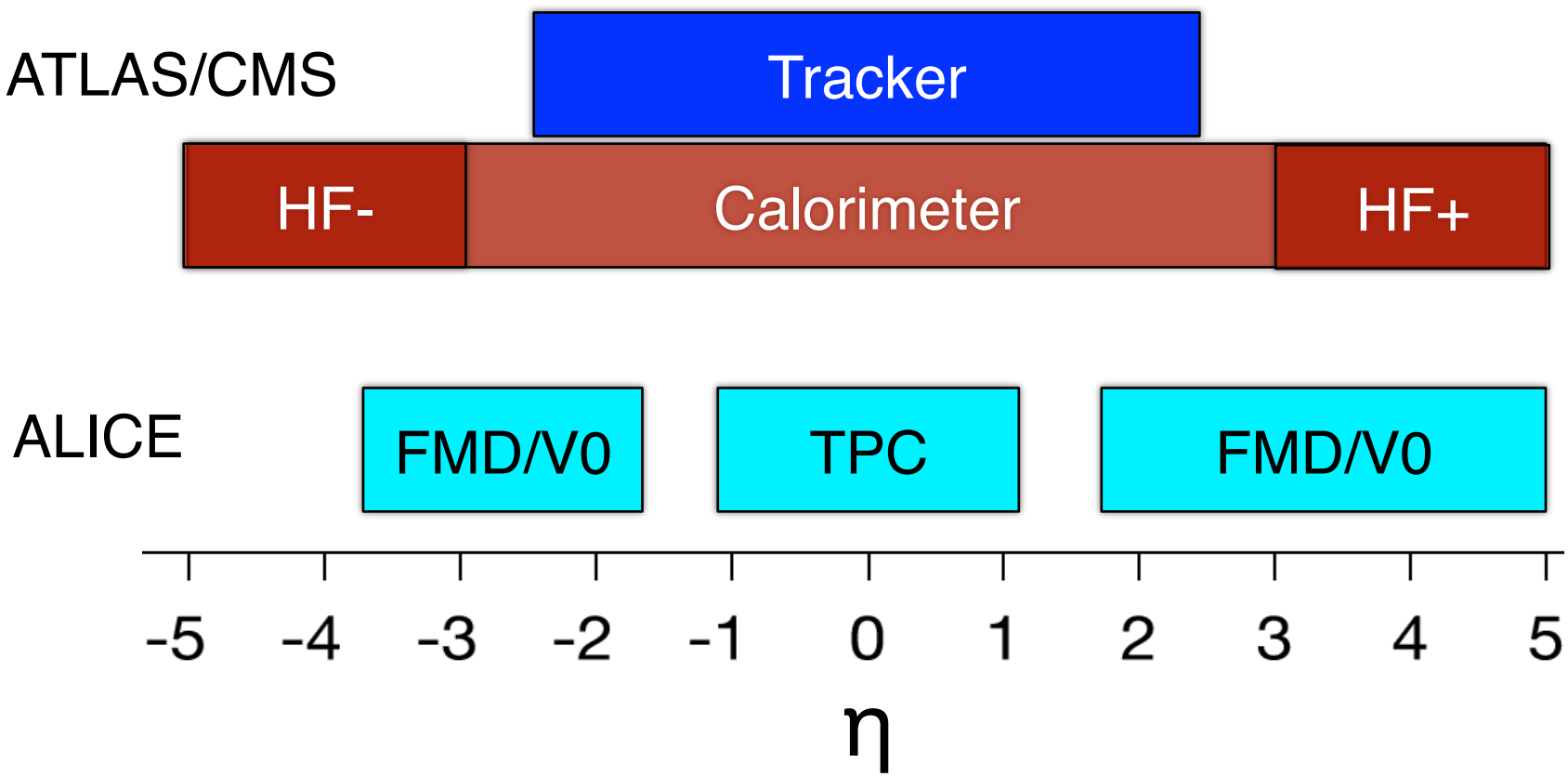



Final-state in 3D

$$f(p_T,\phi,\eta) = \underbrace{\frac{d^2N}{dp_T d\eta}}_{\text{Multiplicity}} \left\{ 1 + 2\sum_n \underbrace{v_n(p_T,\eta)}_{\text{Anisotropy}} \cos\left[ n\left(\phi - \underbrace{\Psi_n(p_T,\eta)}_{\text{Orientation}}\right)\right]\right\}$$

# Large coverage in η is the key …



# What do we measure? **“Two-particle” correlators**

✧ Multiplicity

$$C = \frac{\left\langle N(\eta^a)N(\eta^b)\right\rangle}{\left\langle N(\eta^a)\right\rangle\left\langle N(\eta^b)\right\rangle}$$

✧ Anisotropy flow

$$V_{n\Delta} = \left\langle \vec{Q}_n(\eta^a)\vec{Q}_n^*(\eta^b)\right\rangle \quad \text{n>0}$$

$$\vec{Q}_n(\eta) = \mathrm{v}_n(\eta)e^{in\Psi_n(\eta)}$$

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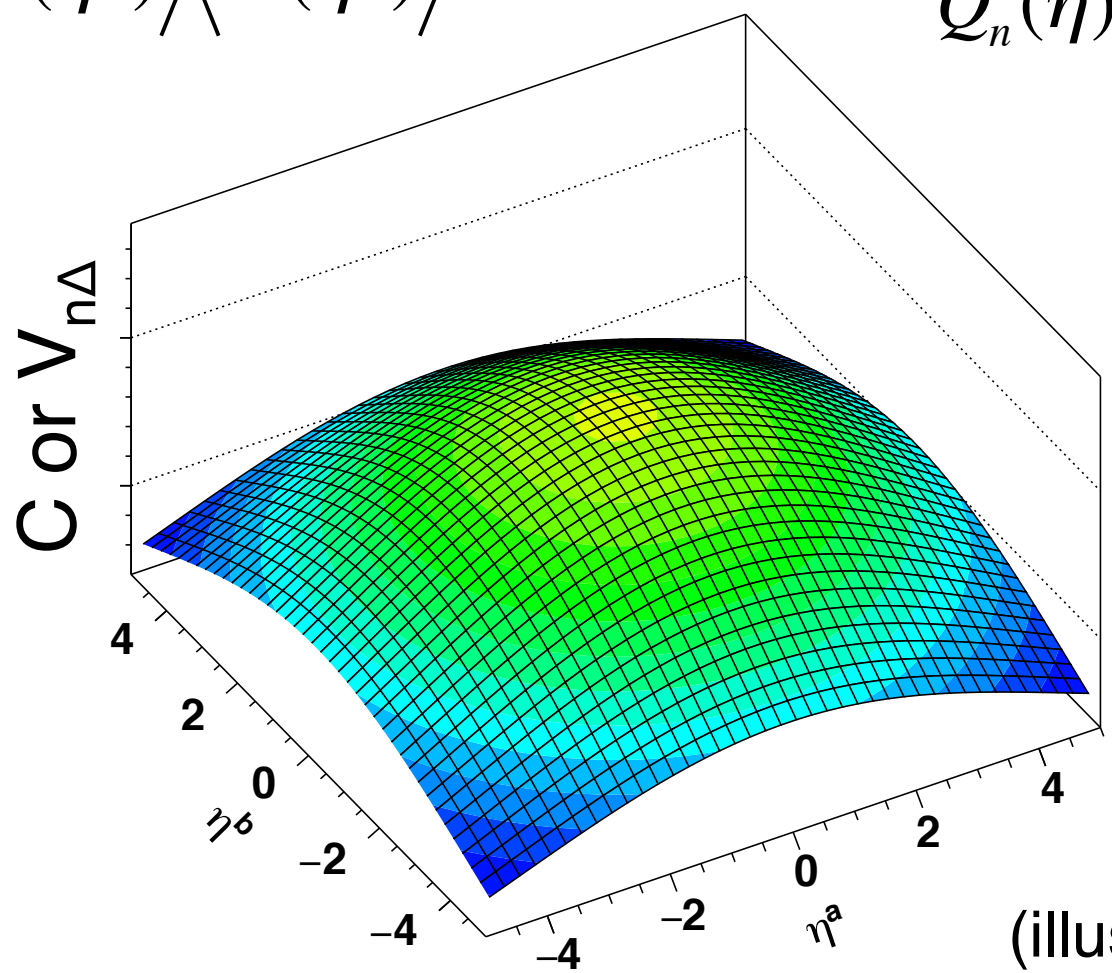


(illustration only)

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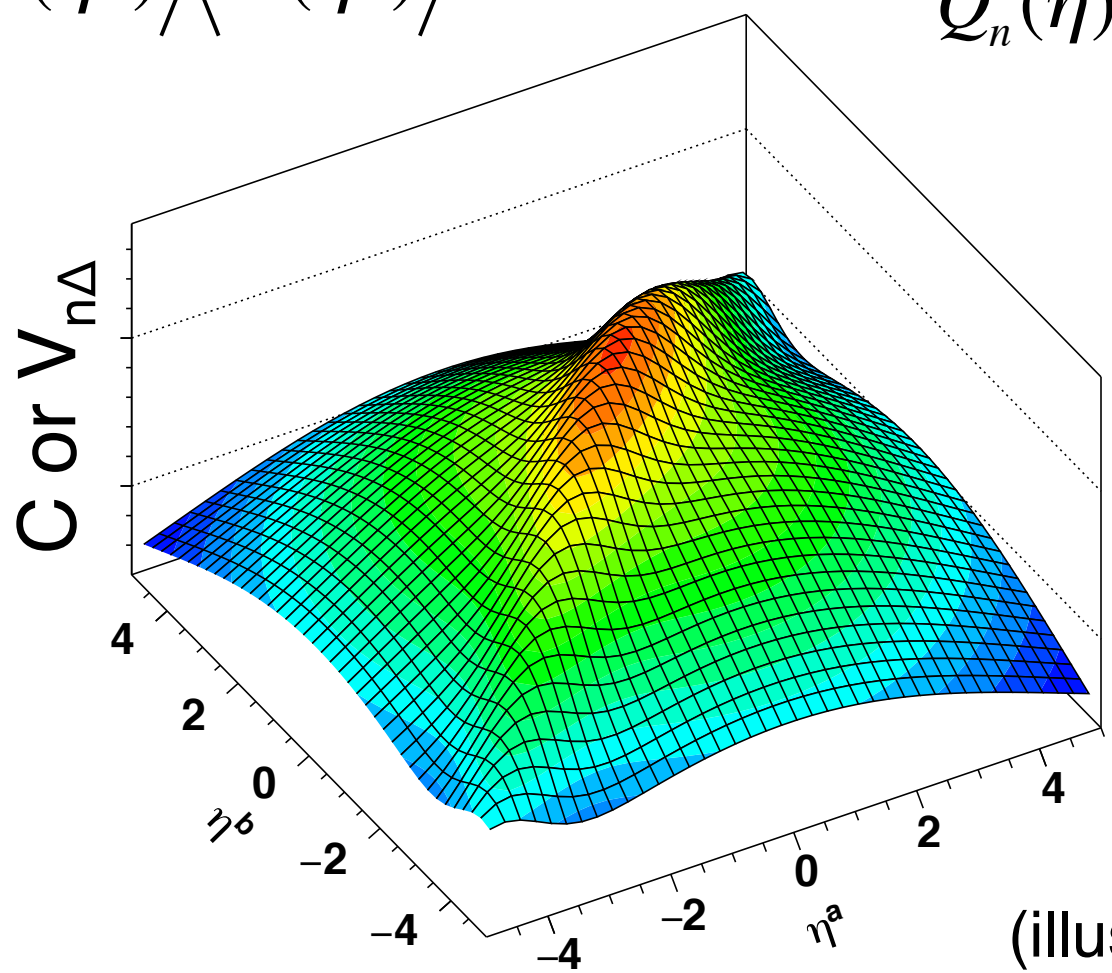


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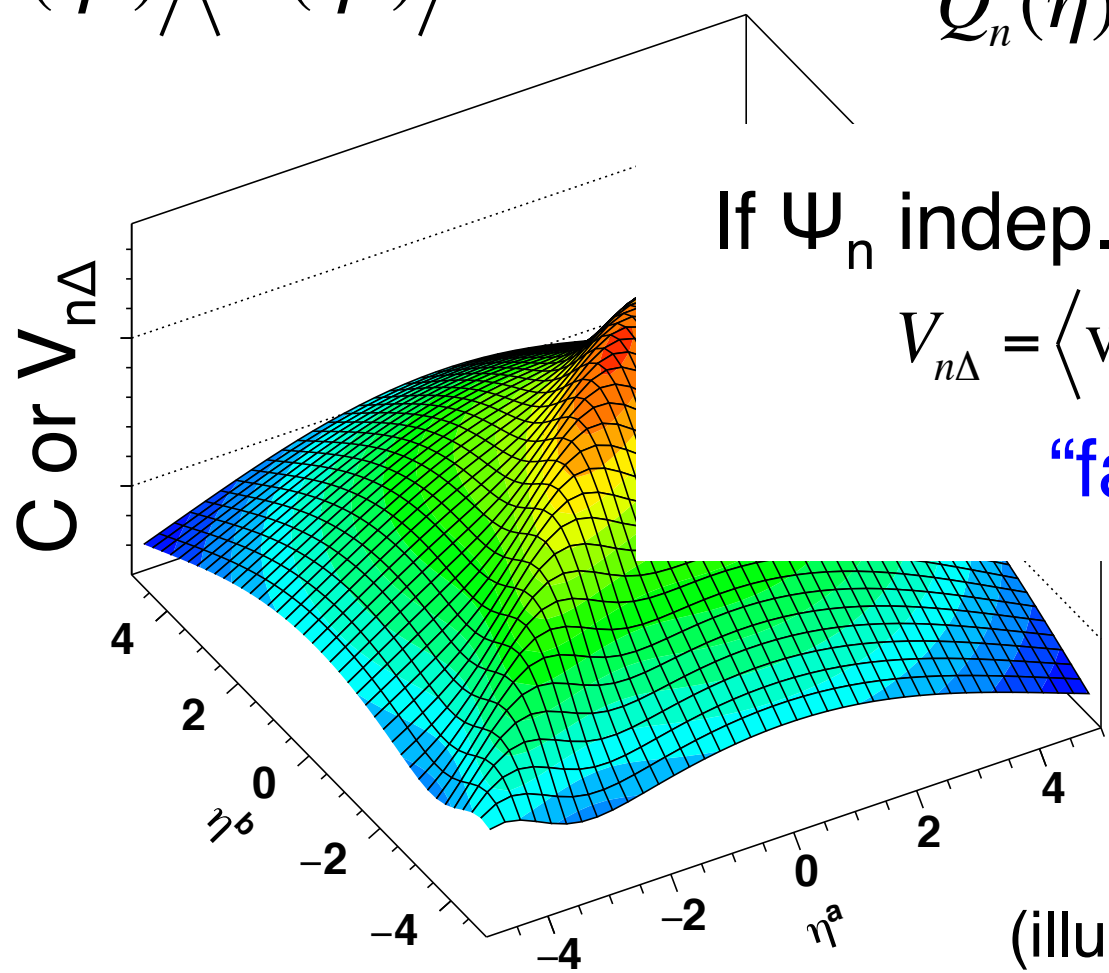



If $\Psi_n$ indep. of η,

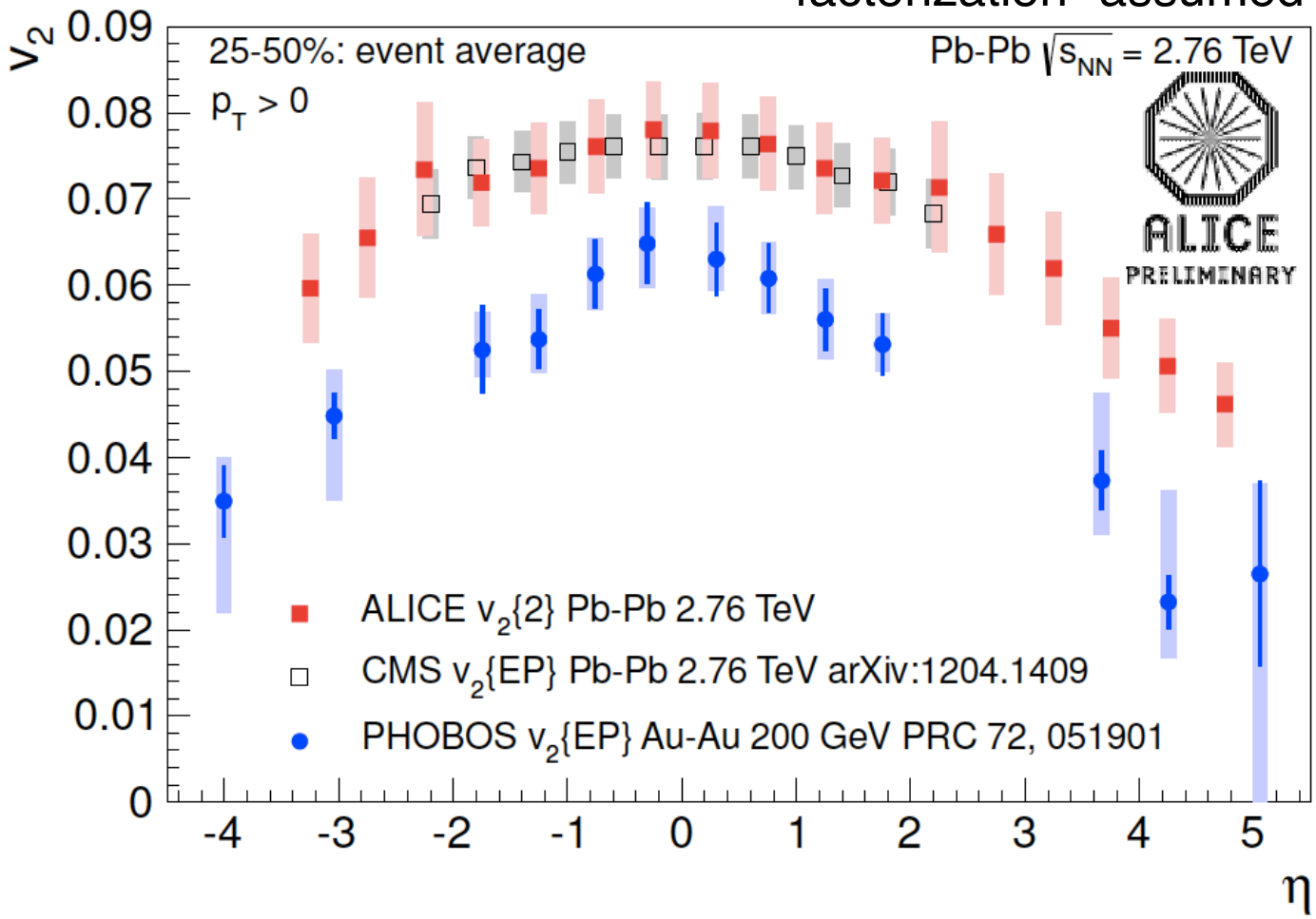
$$V_{n\Delta} = \left\langle \mathrm{v}_n(\eta^a)\mathrm{v}_n(\eta^b)\right\rangle$$

"factorization"

(illustration only)

# Flow is not boost-invariant in rapidity

"factorization" assumed



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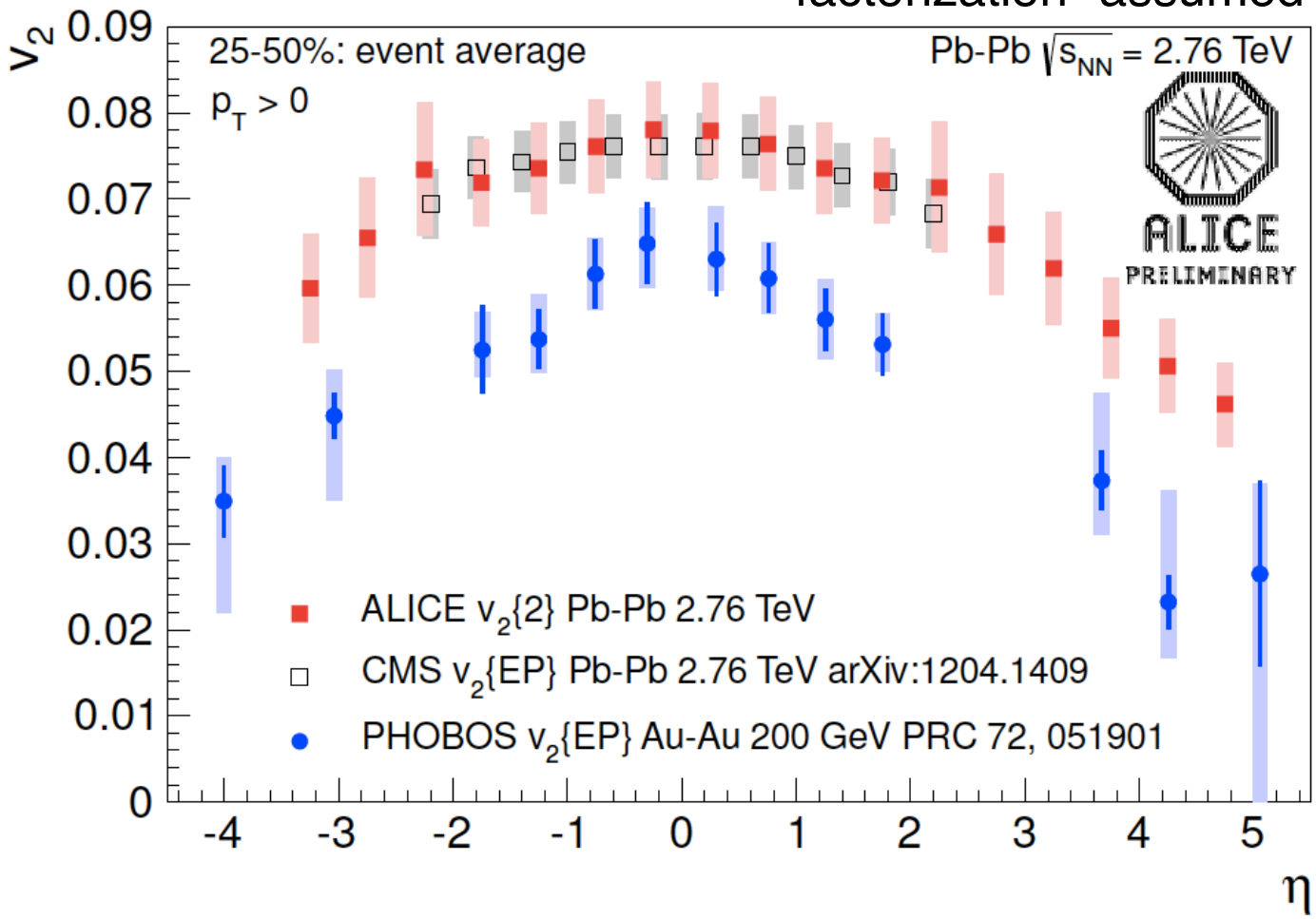


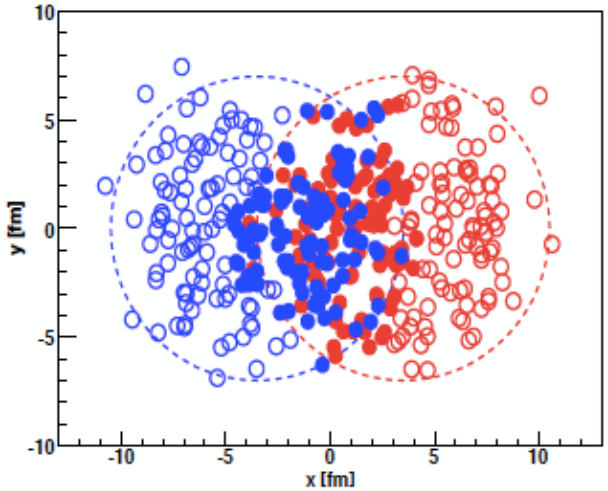

Rapidity dependence of

**$v_n$ magnitude** or **$\Psi_n$ orientation**?

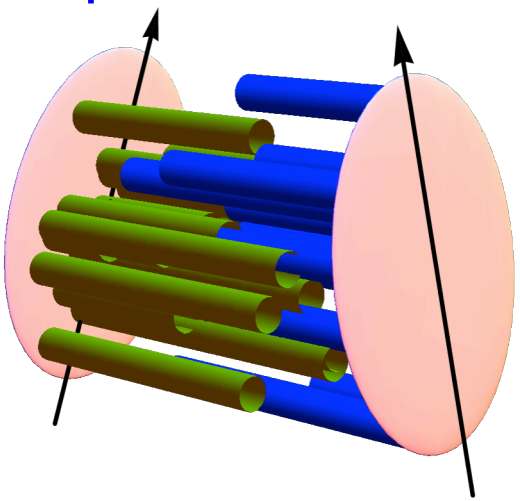
(energy density, η/s) (geometry, initial state)

# Longitudinal structure of QGP

Asymmetry in forward and backward participants




Torqued fireball



Bozek et.al., arXiv:1011.3354

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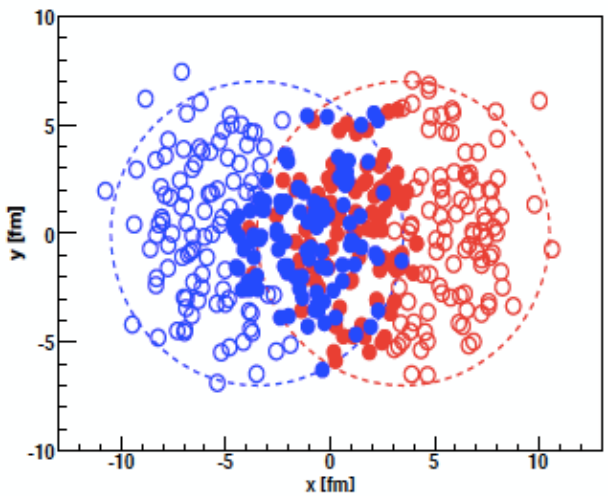


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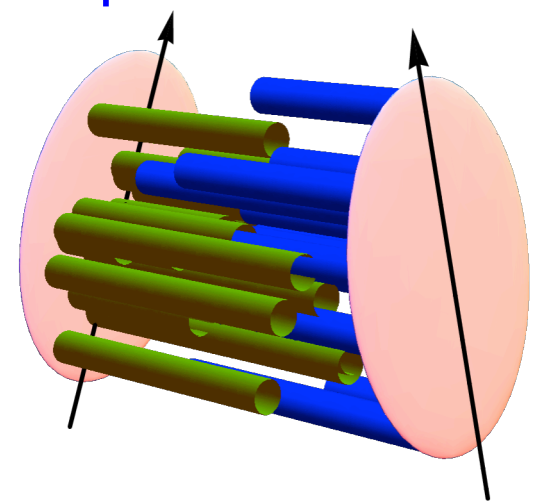

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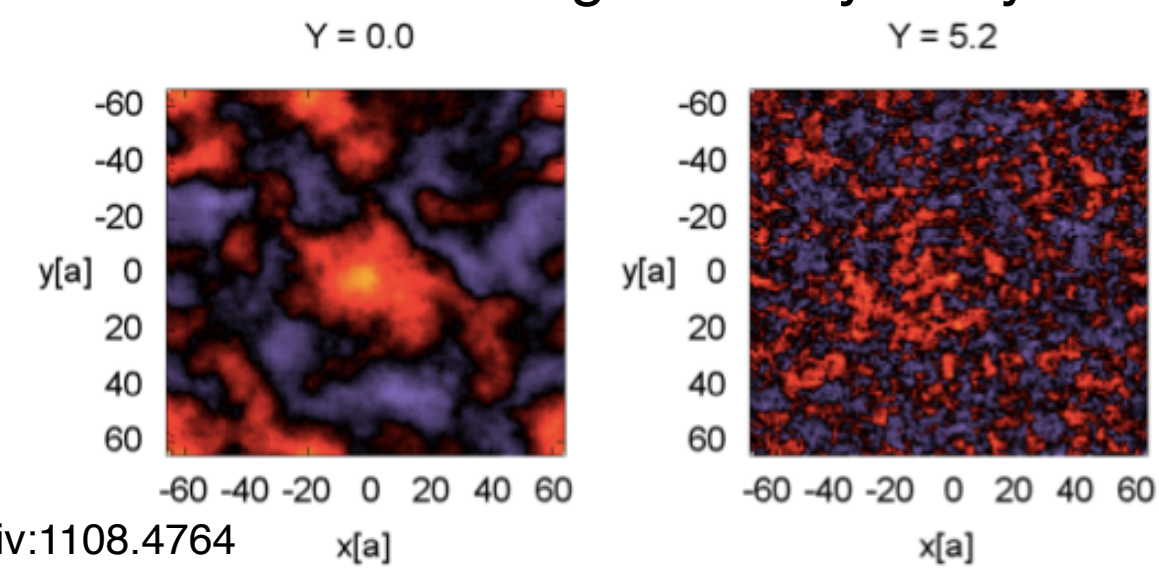
Gluon field granularity v.s. y

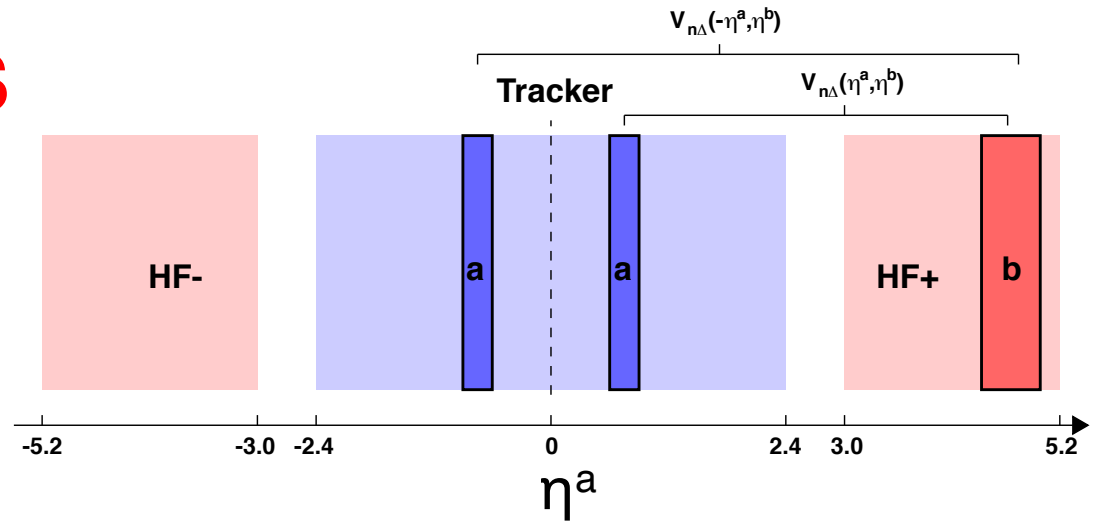
CGC model




Dumitru et. al., arXiv:1108.4764

# Rapidity (de)correlations of anisotropy flow

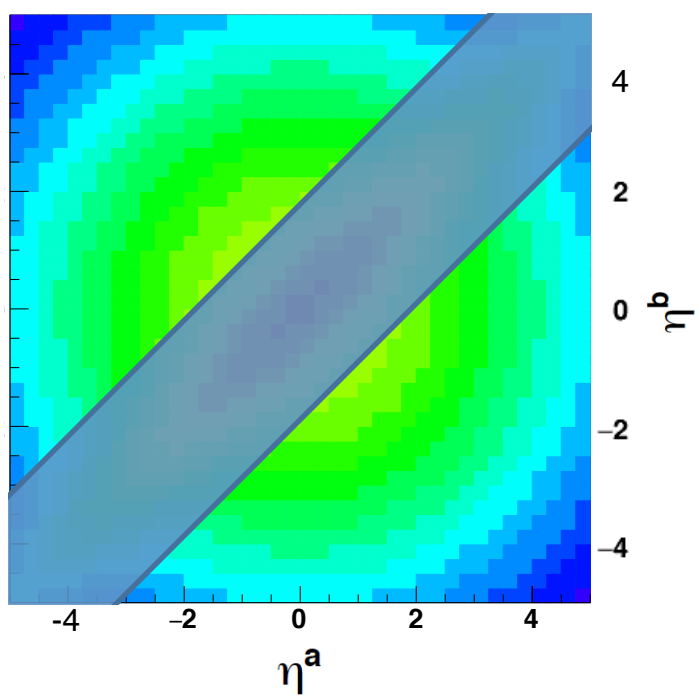
**CMS**




**all pairs with $|\Delta\eta|>2$**

PRC 92 (2015) 034911

$V_{n\Delta}(\eta^a,\eta^b)$



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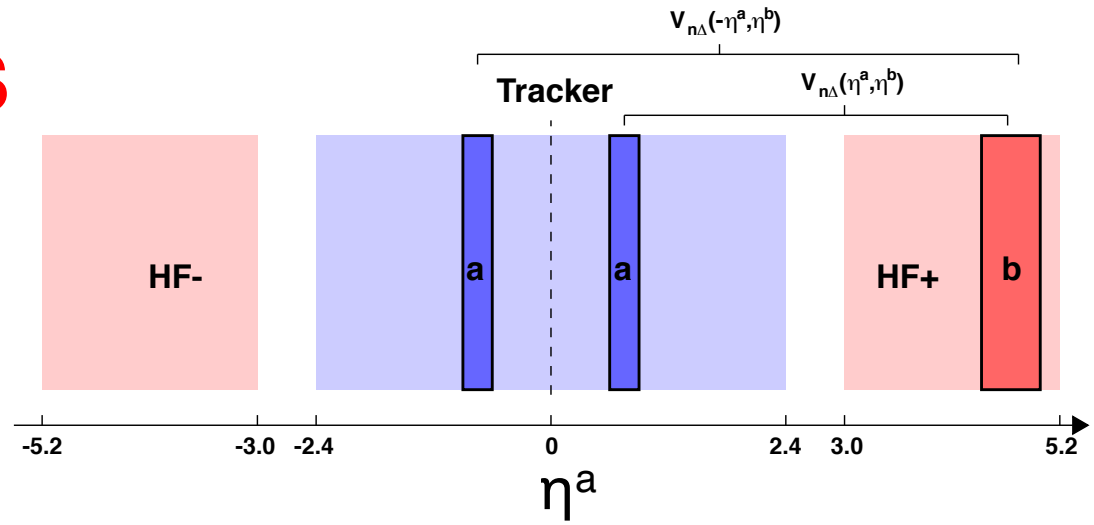


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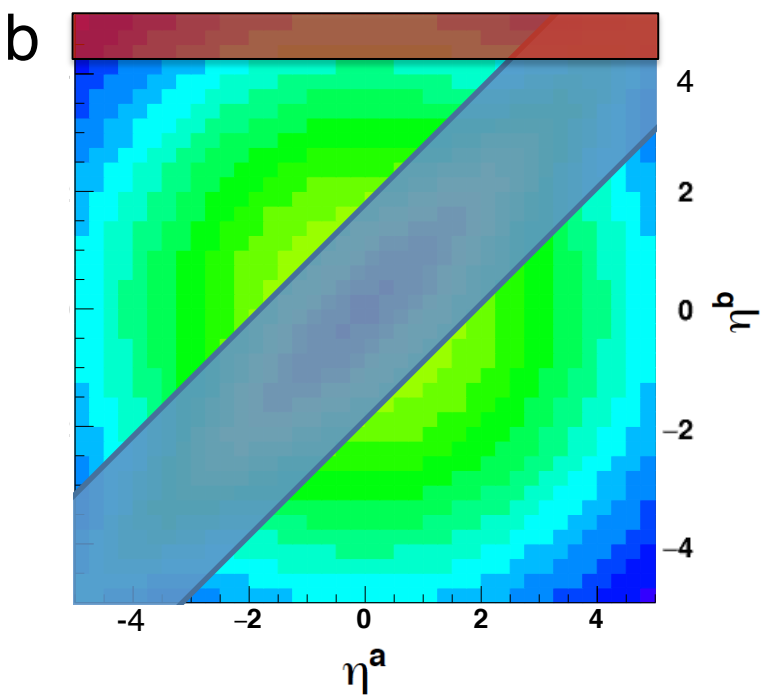

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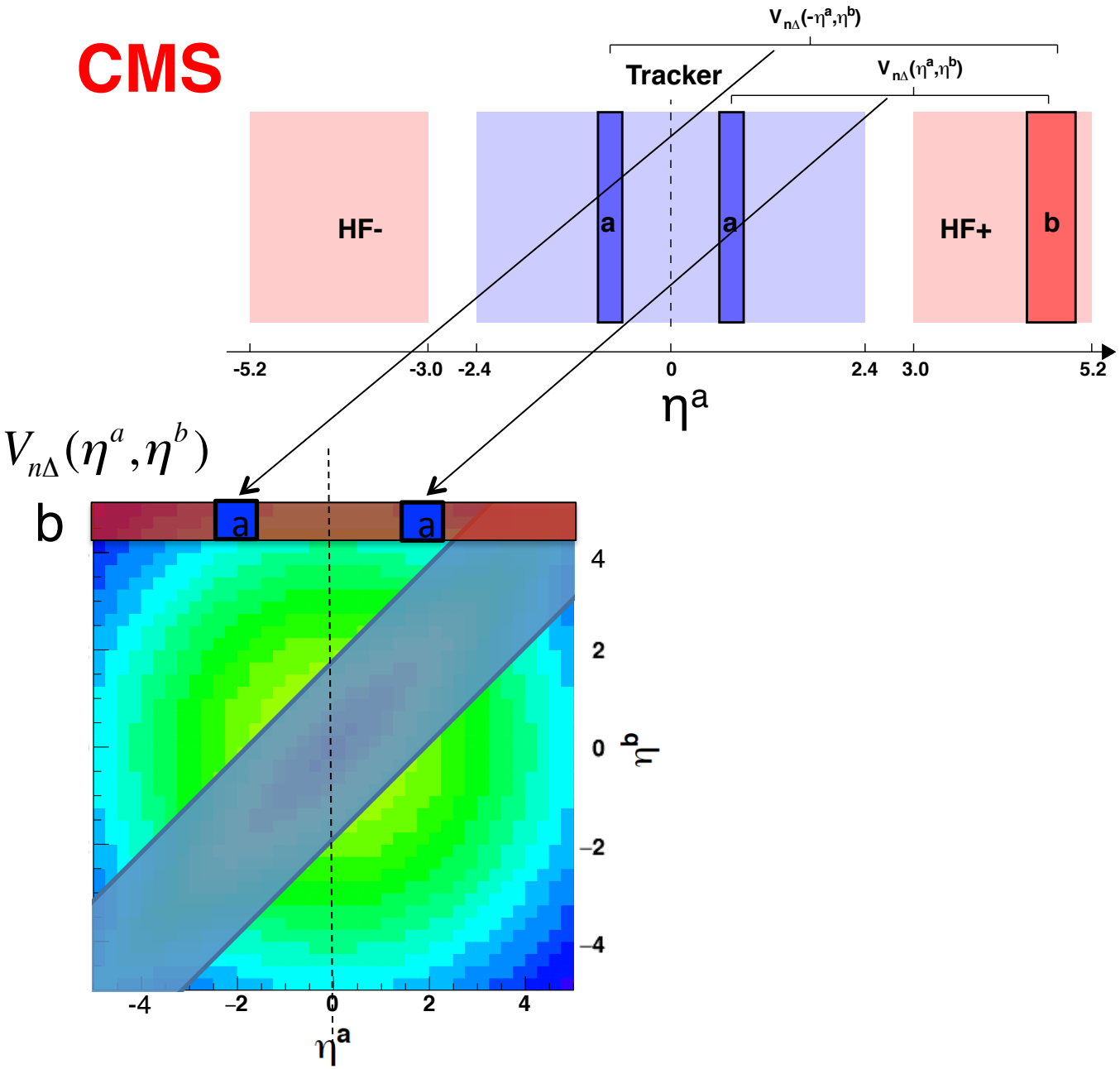


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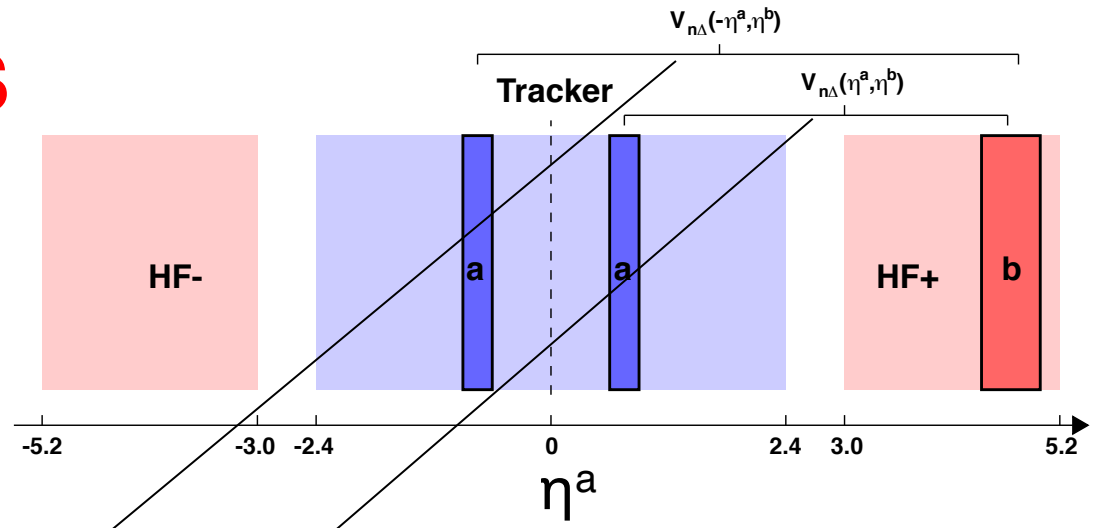


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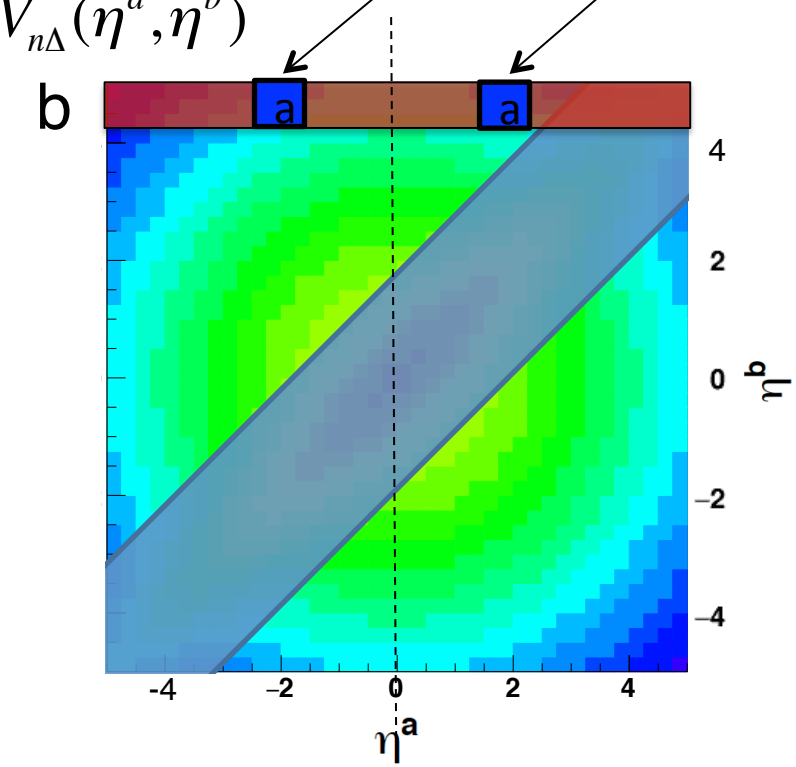


If $\Psi_n(\eta)$ indep. of $\eta$,

$$V_{n\Delta}(\eta^a,\eta^b) = V_{n\Delta}(-\eta^a,\eta^b)$$

(for symmetric system)

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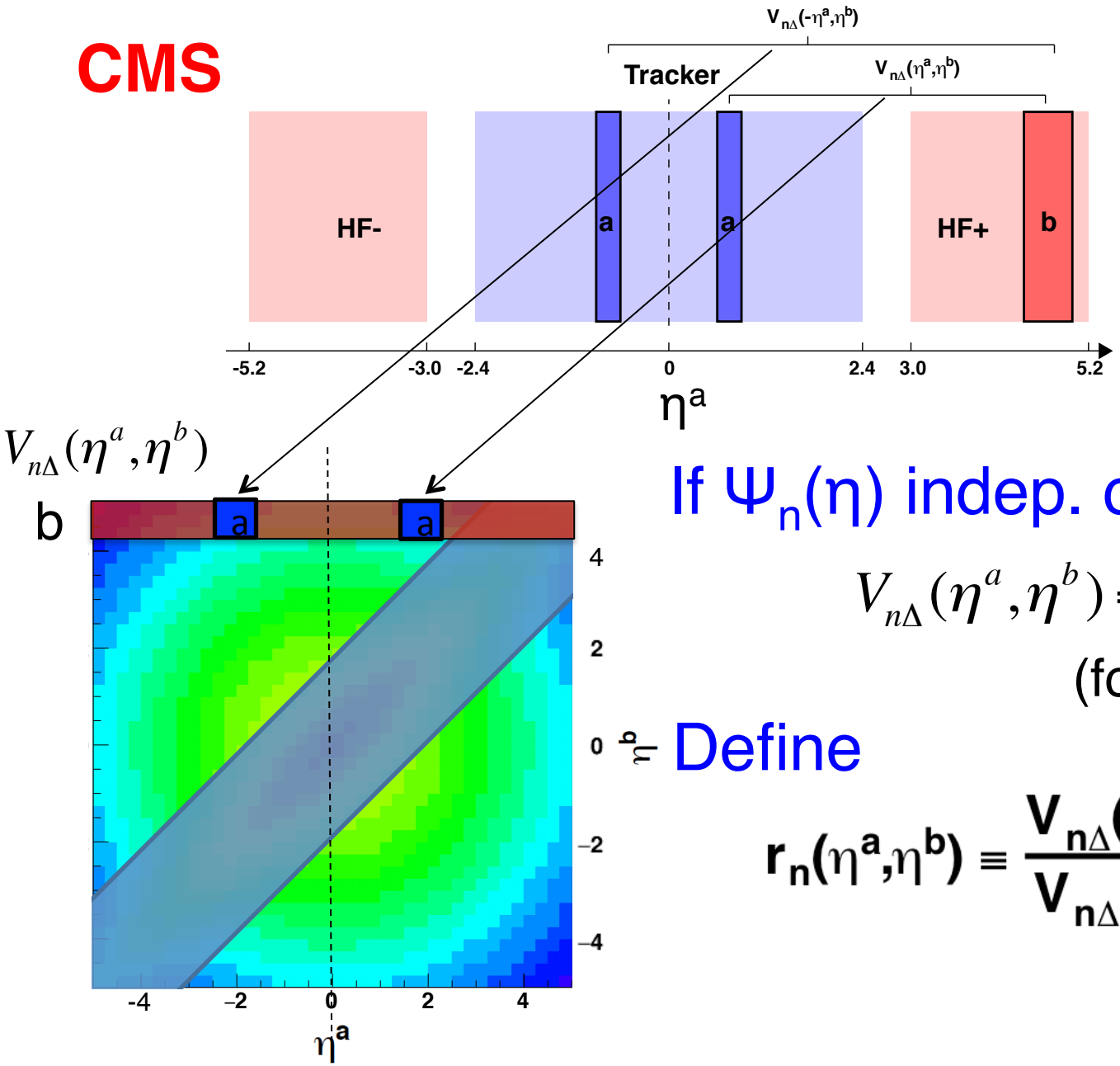


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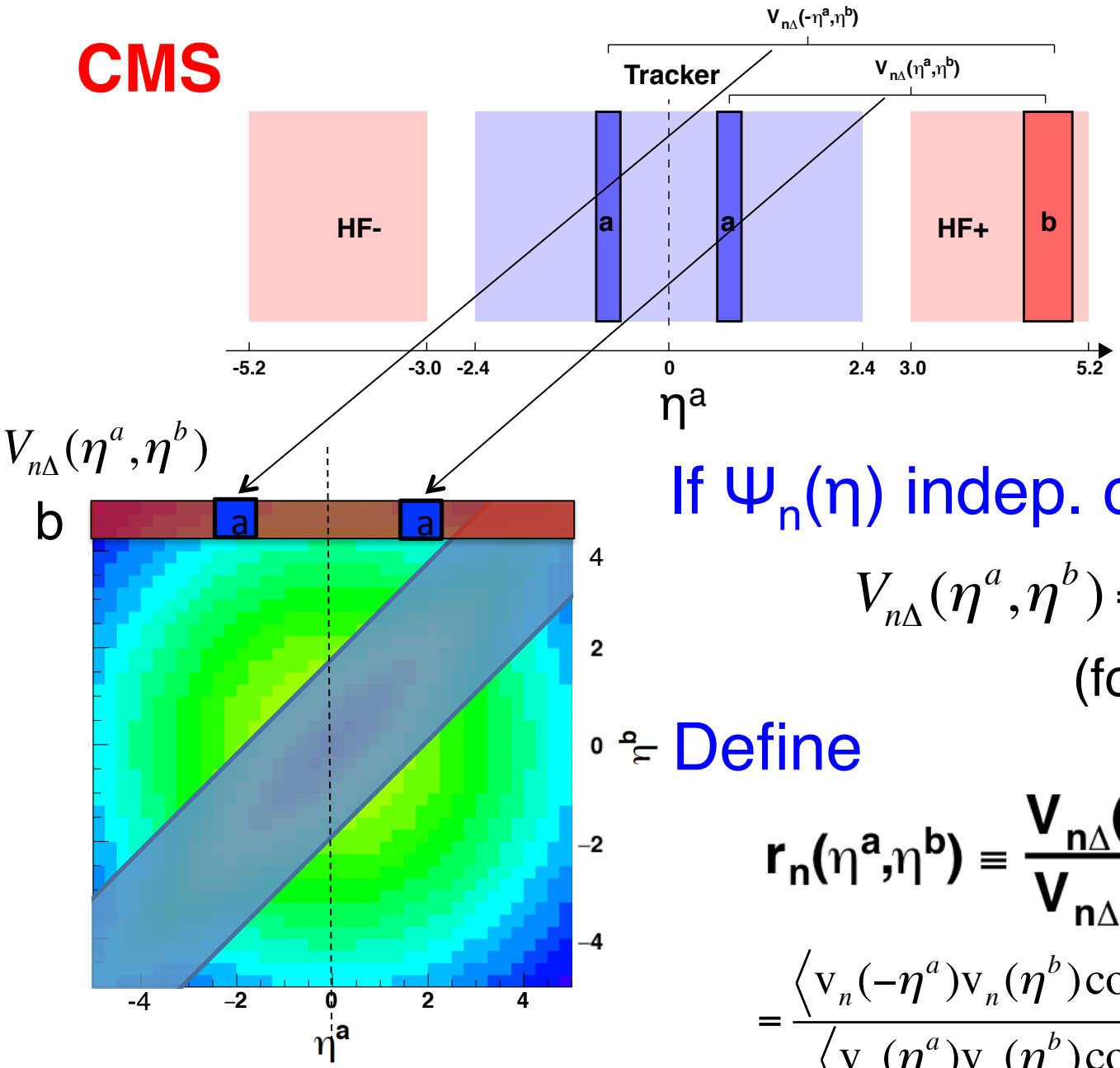


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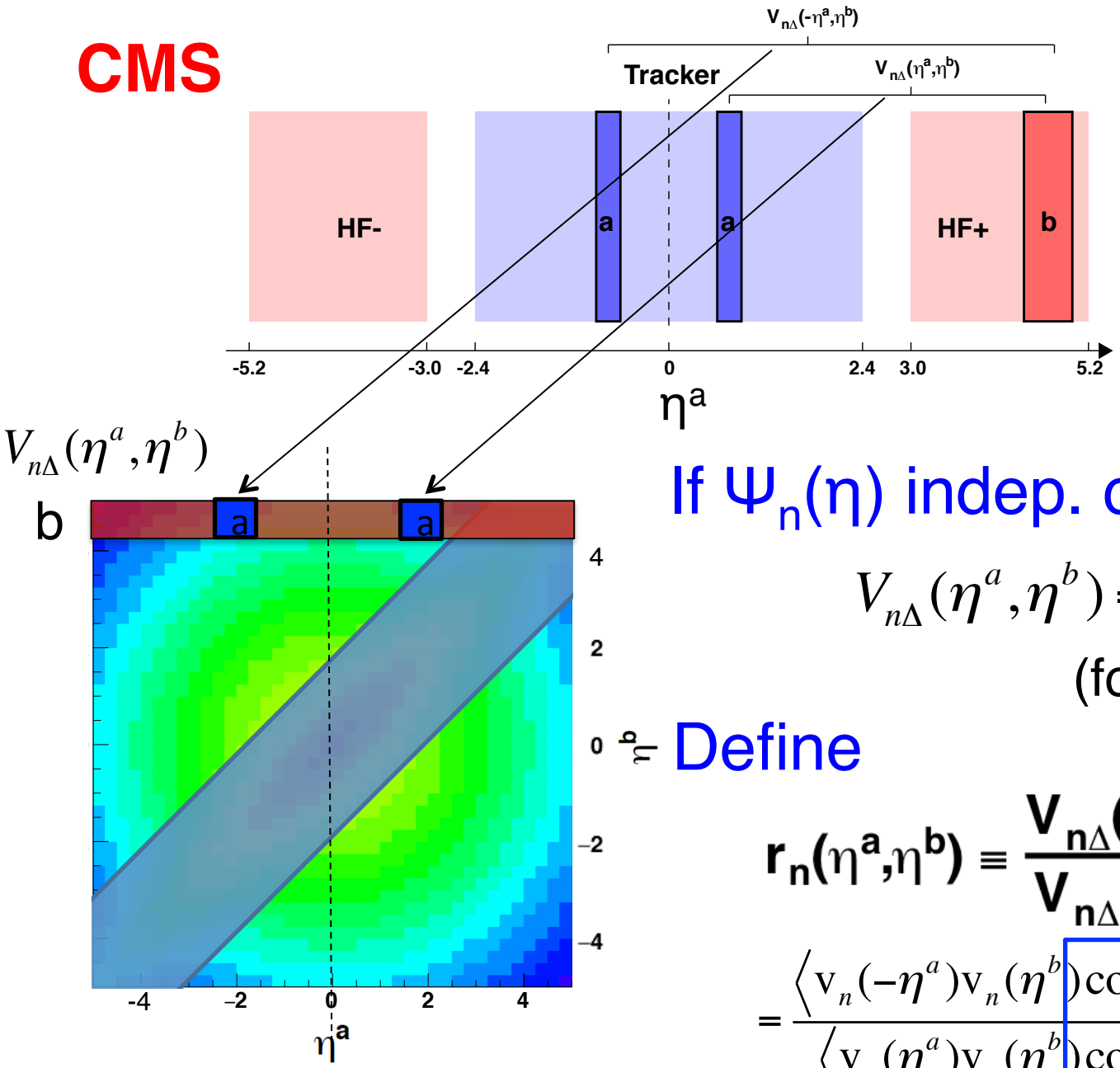


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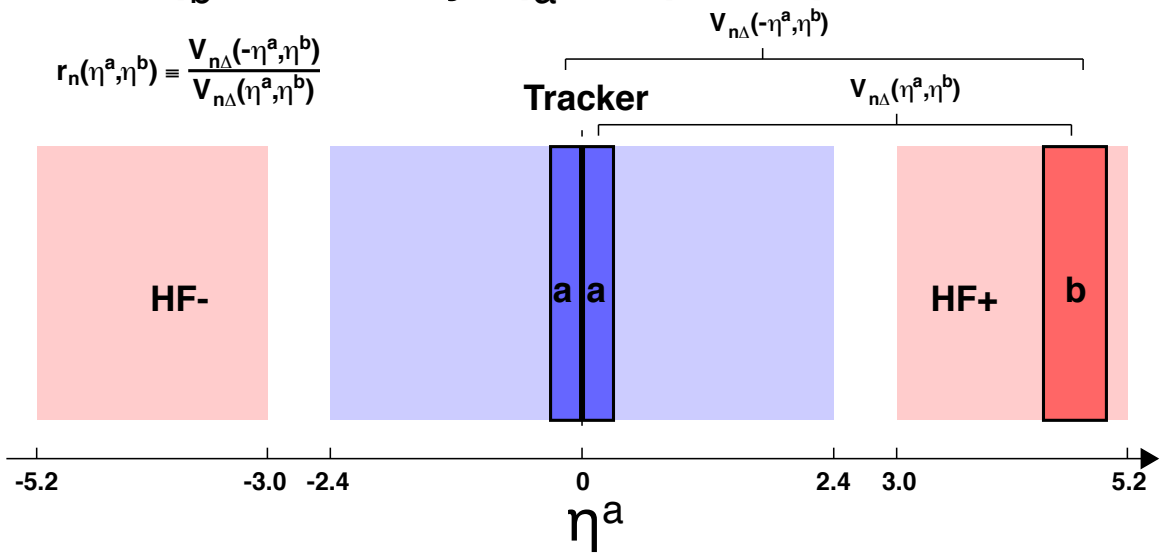


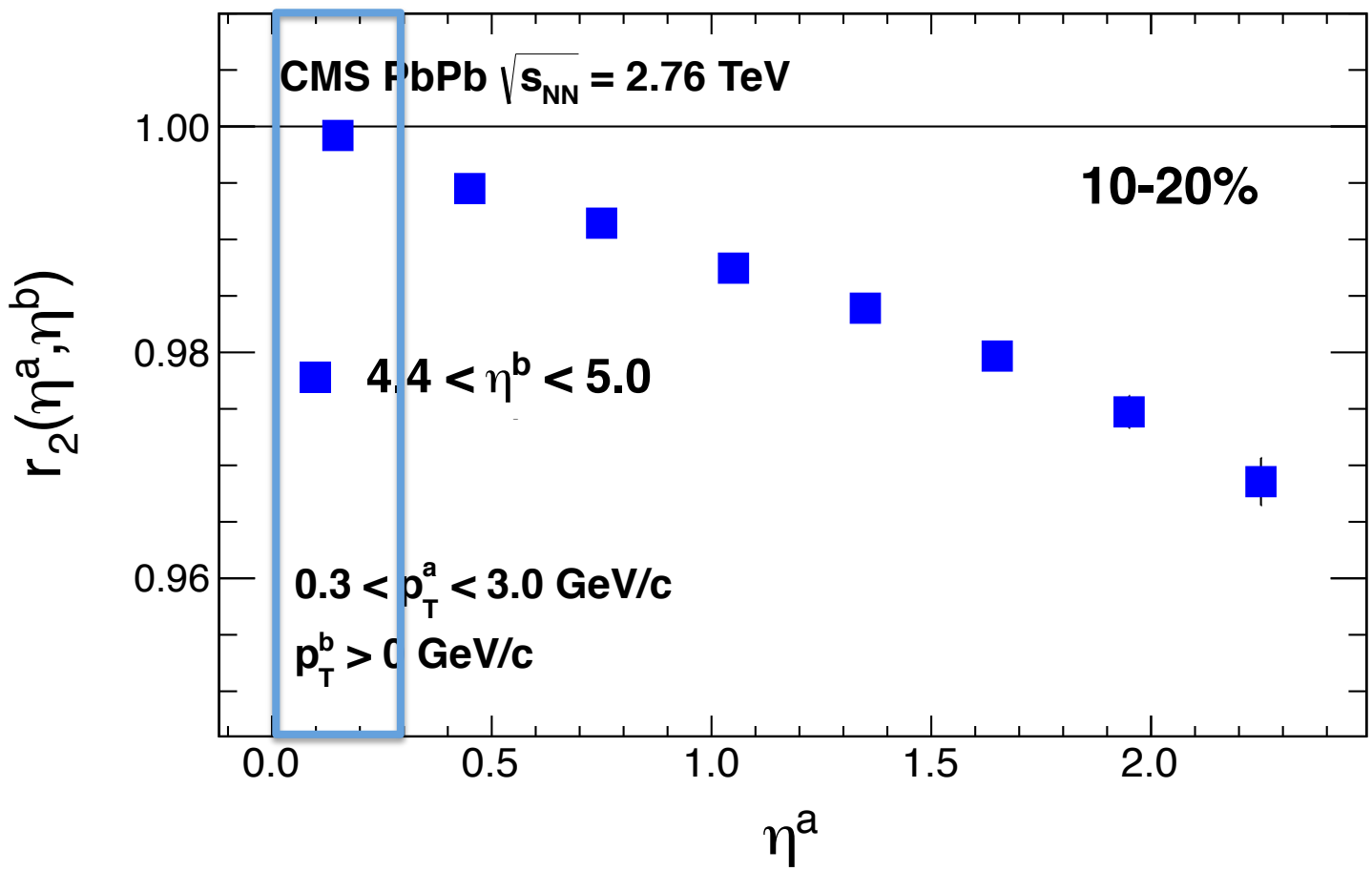

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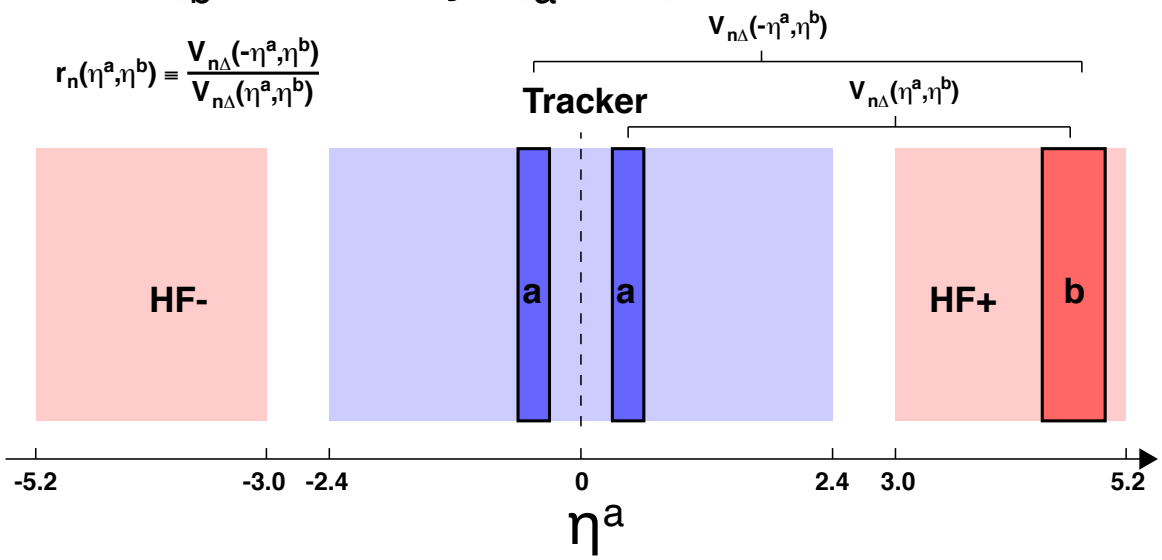
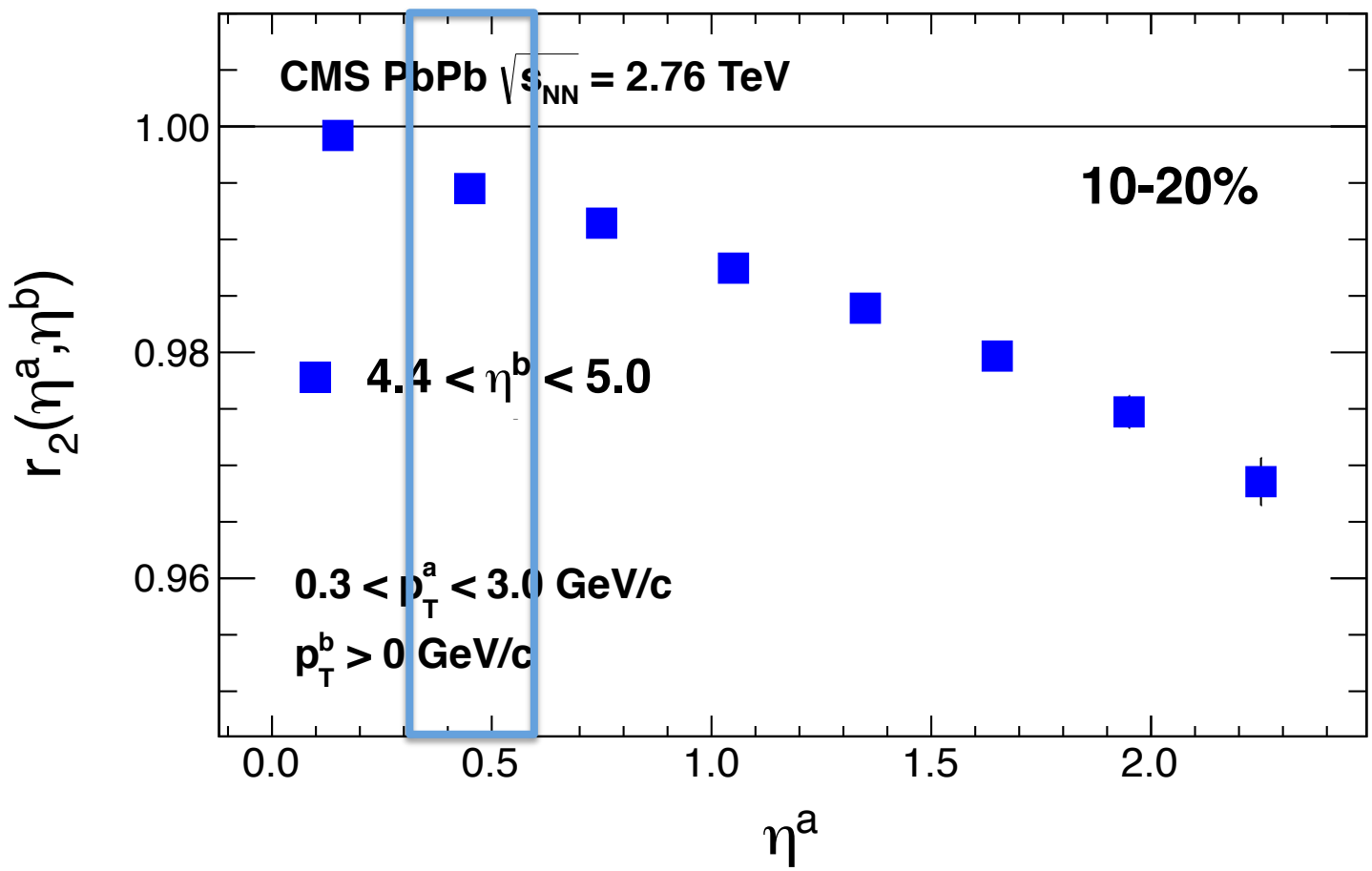



**Decorrelation** of $\Psi_2$ as $\Delta\eta$ increases



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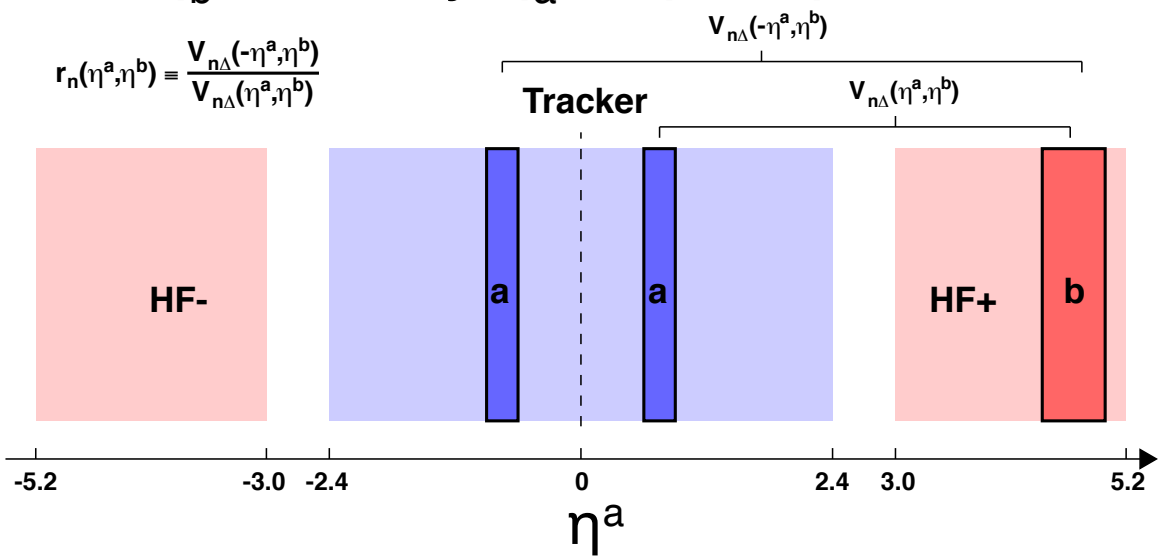


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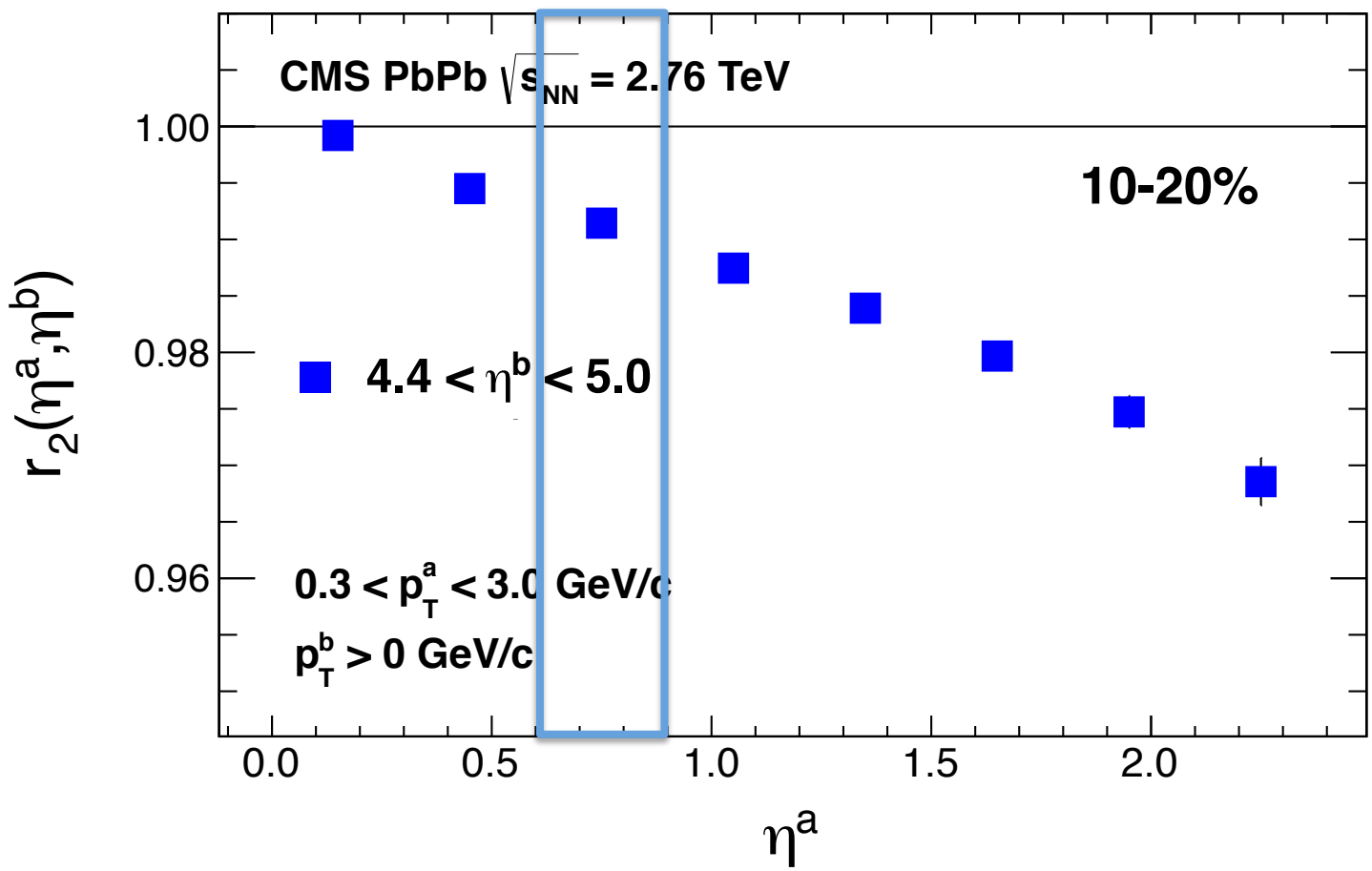

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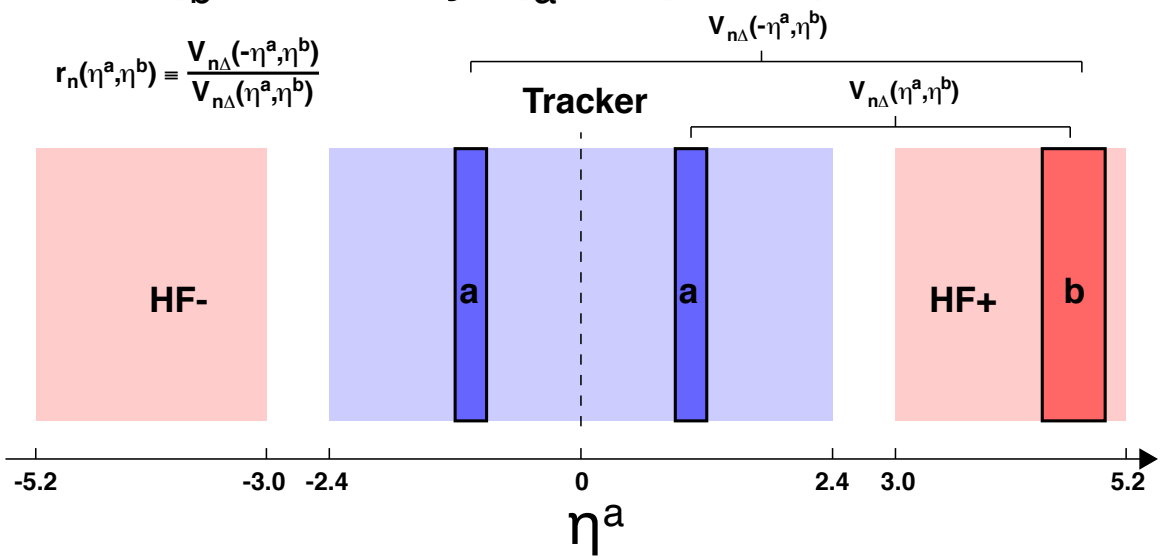


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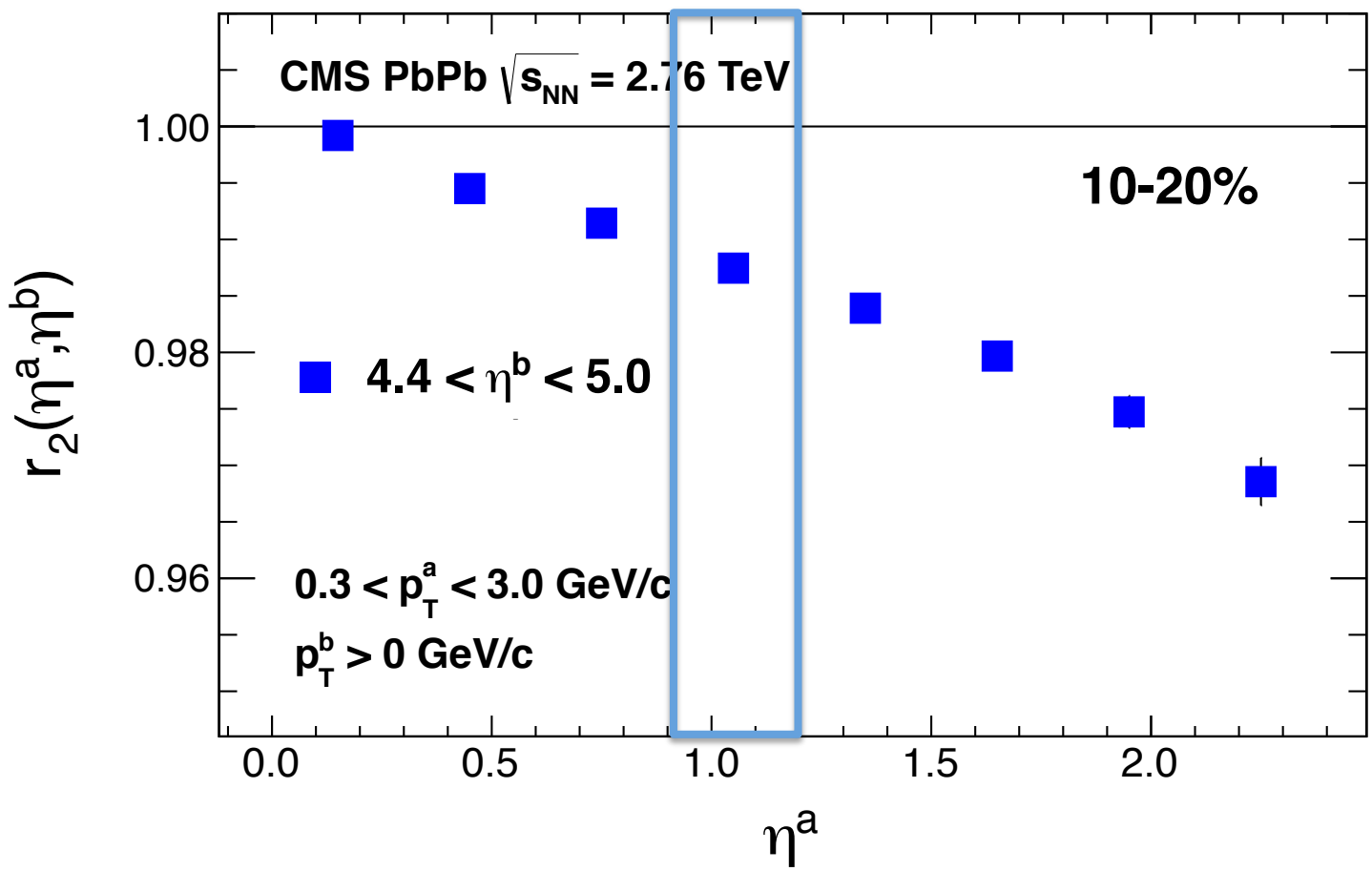

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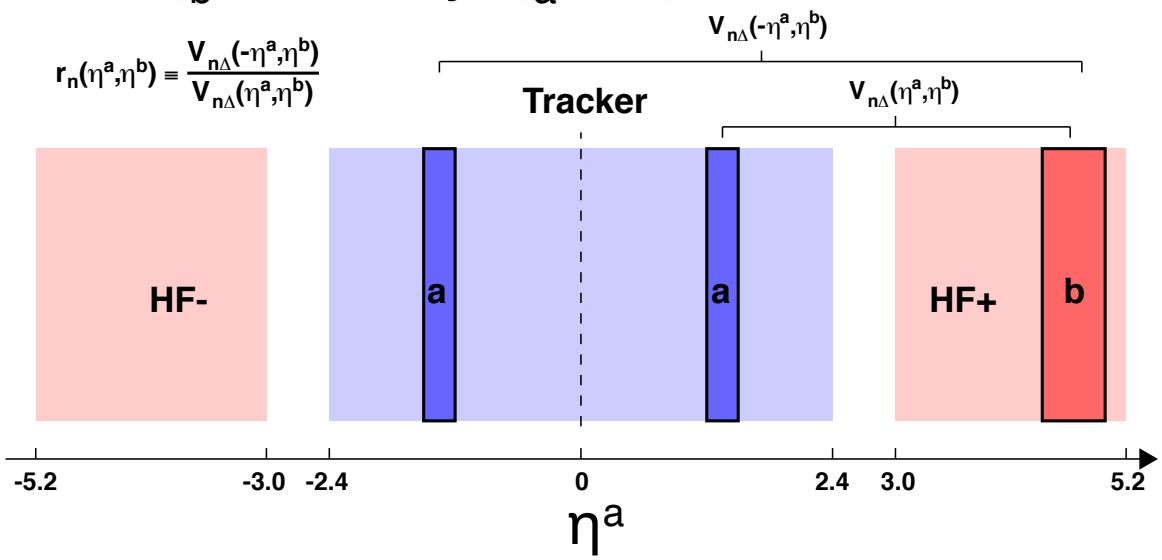


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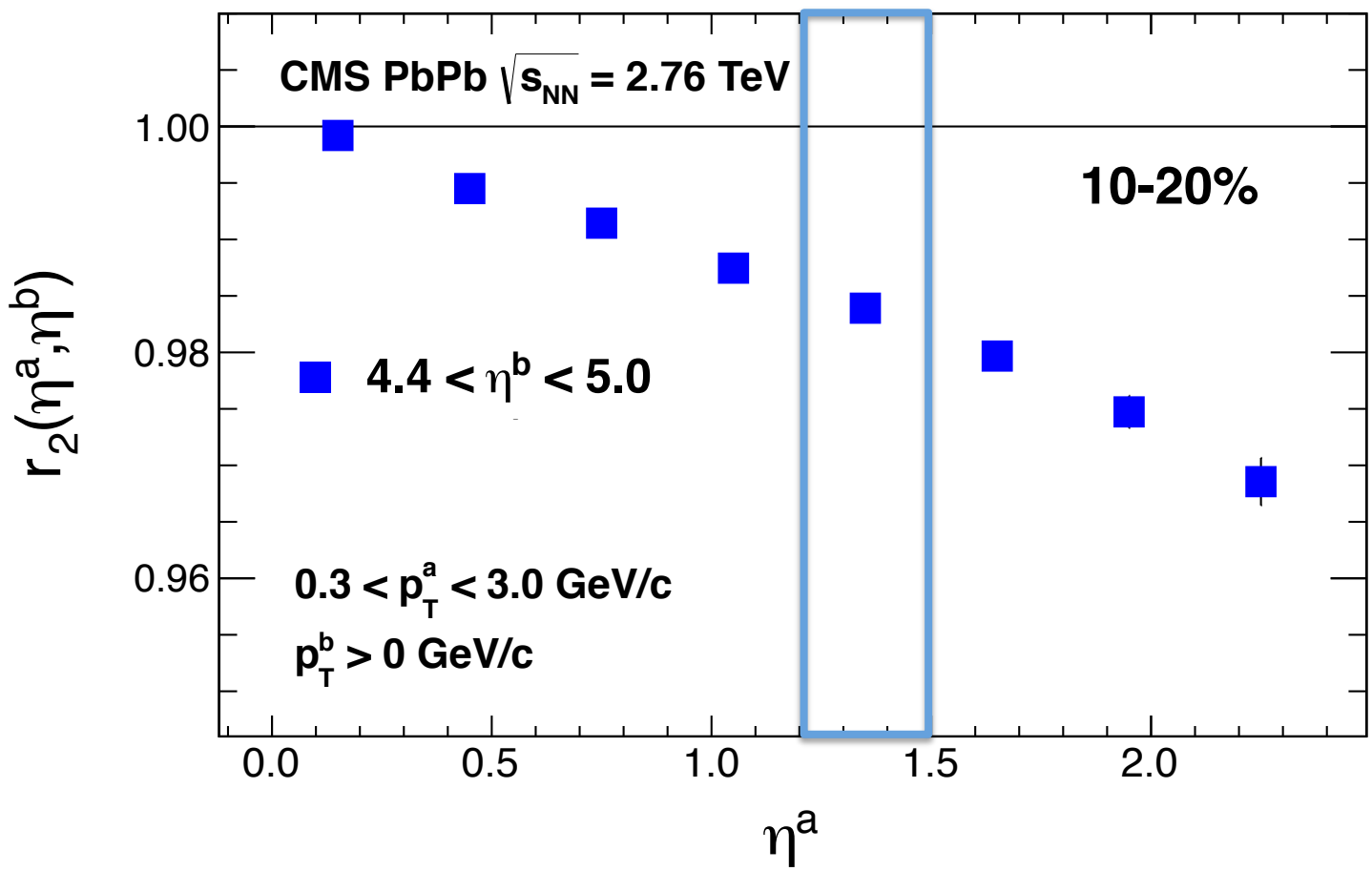

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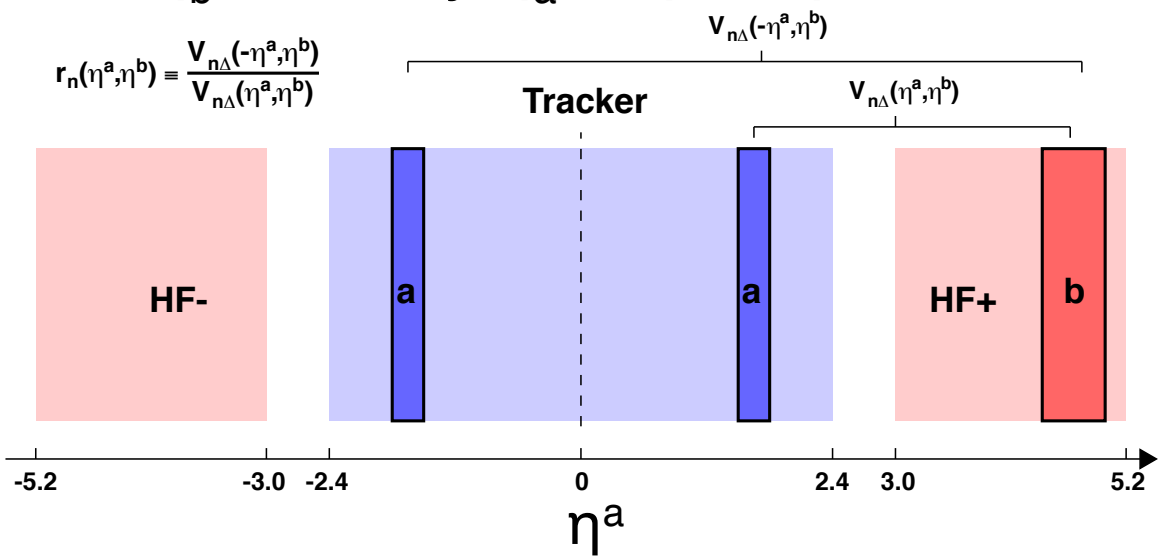


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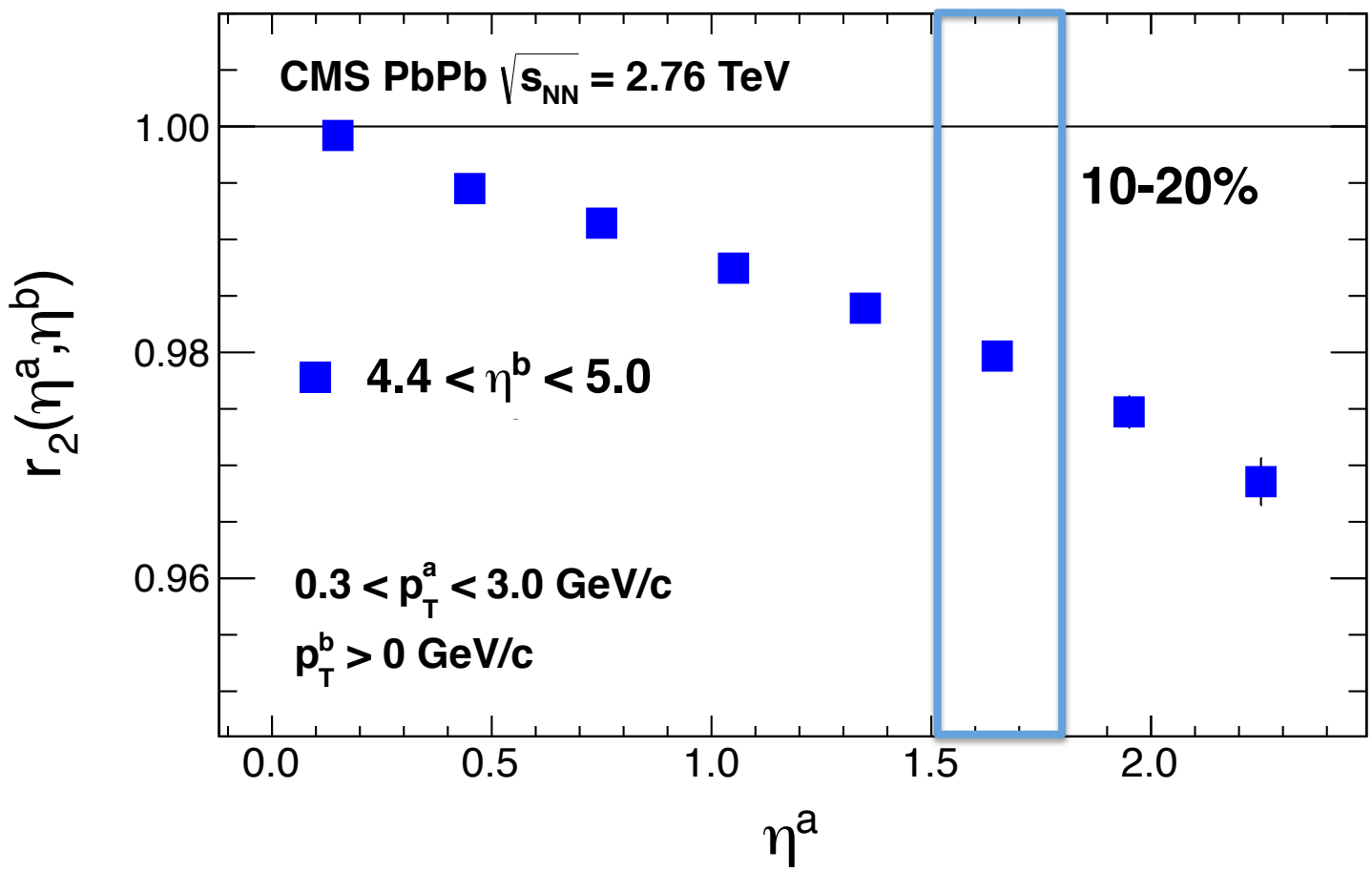

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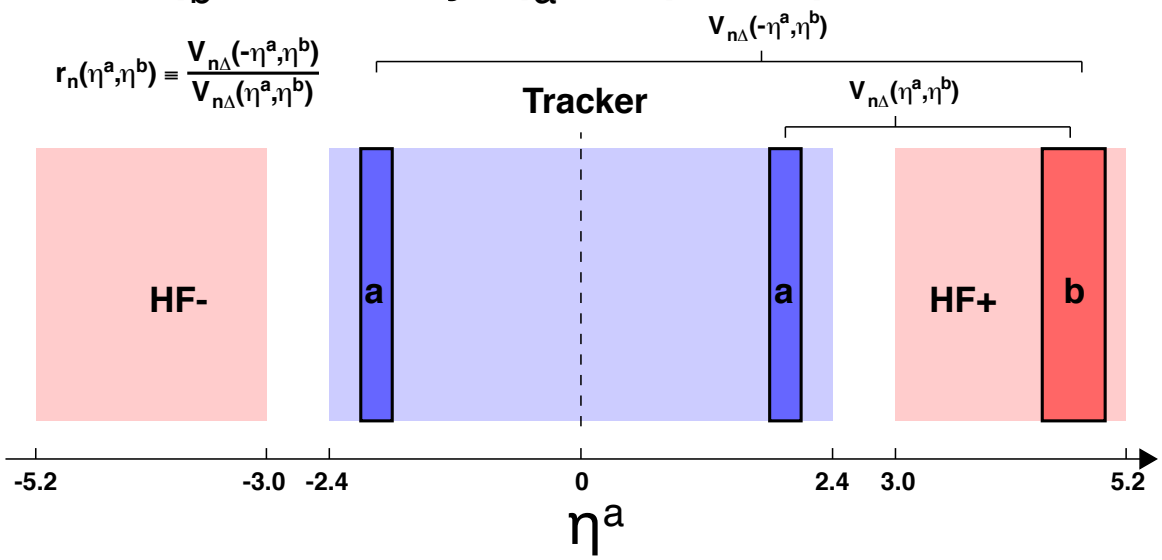


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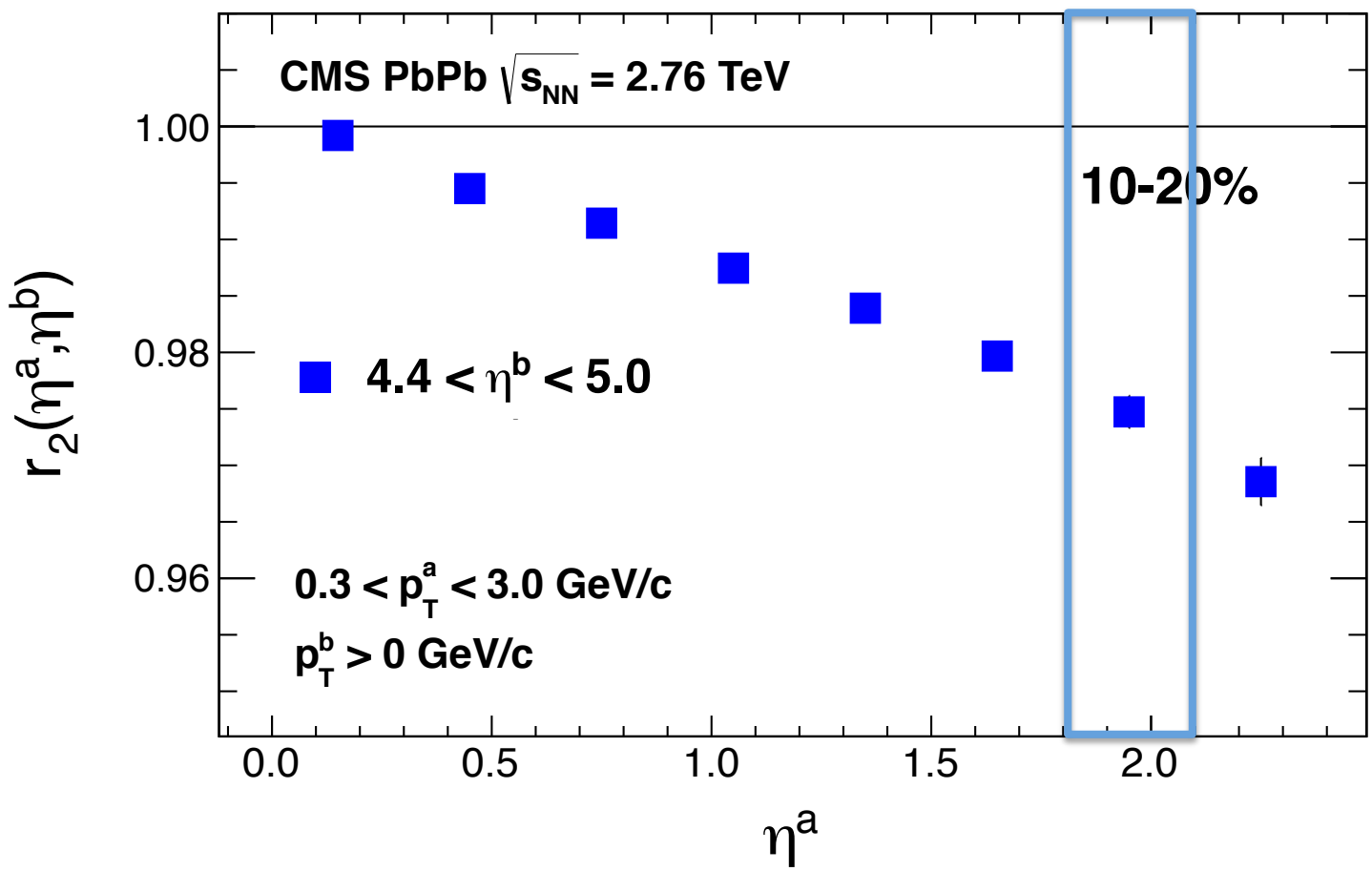

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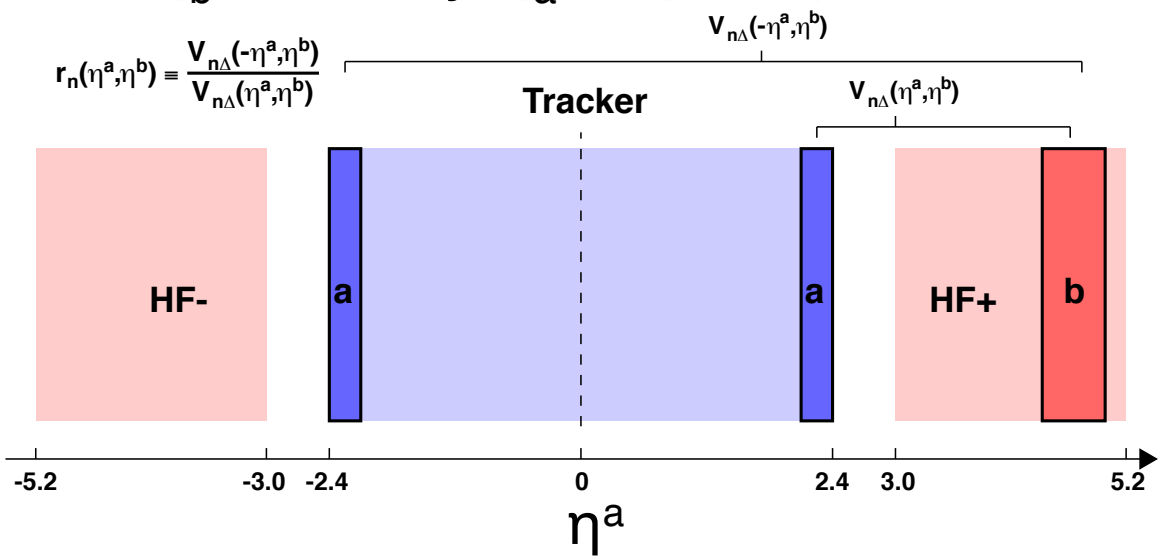
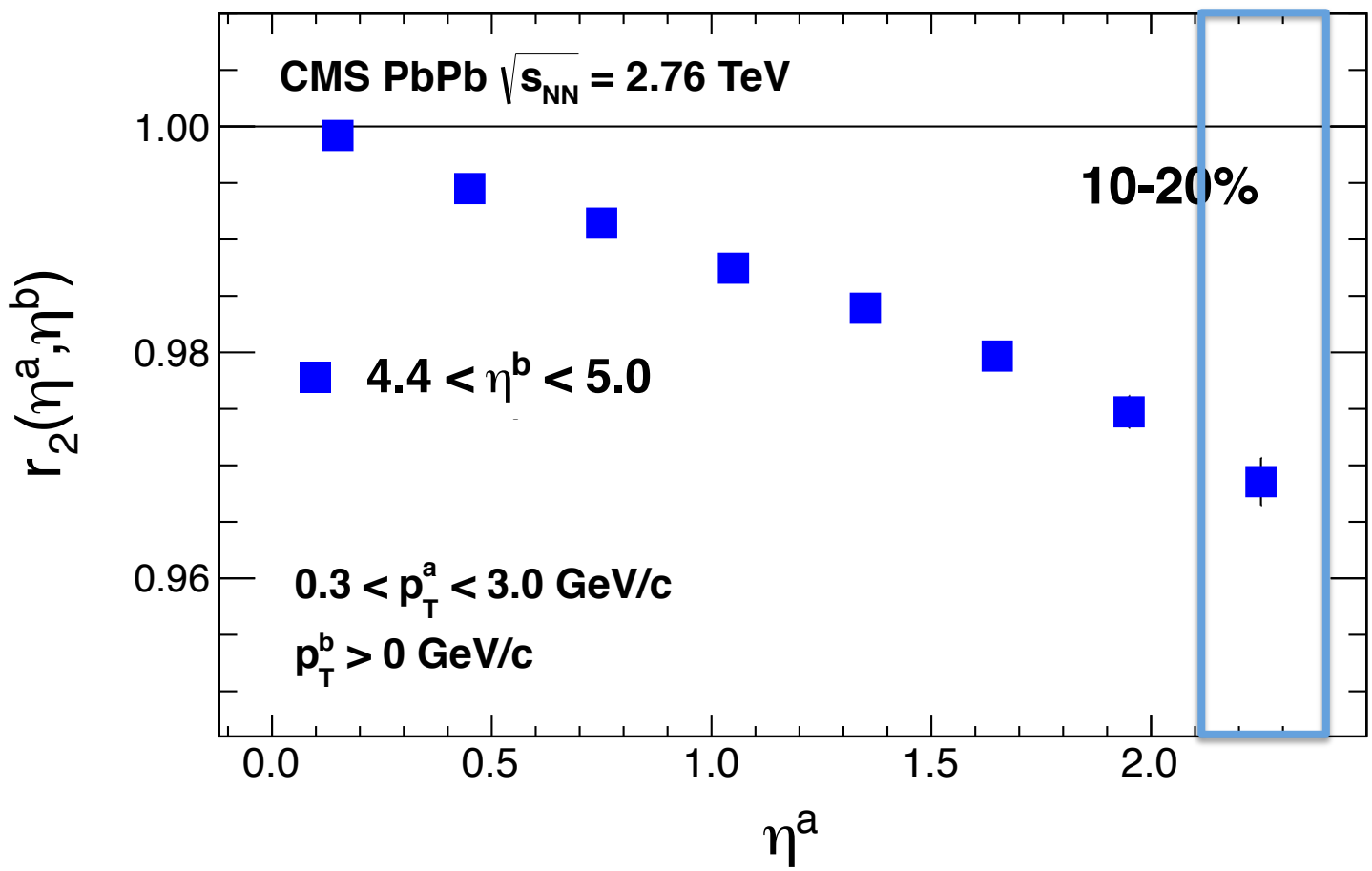



**Decorrelation** of $\Psi_2$ as $\Delta\eta$ increases

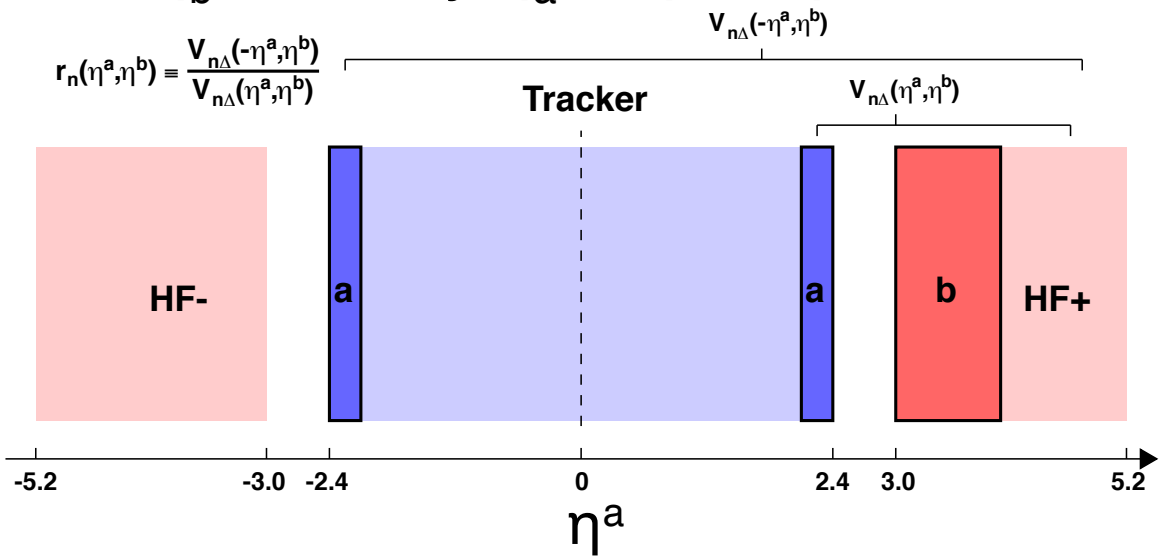
$\eta$ gap $\geq$ 2 units between a and b




~ 3-4 %

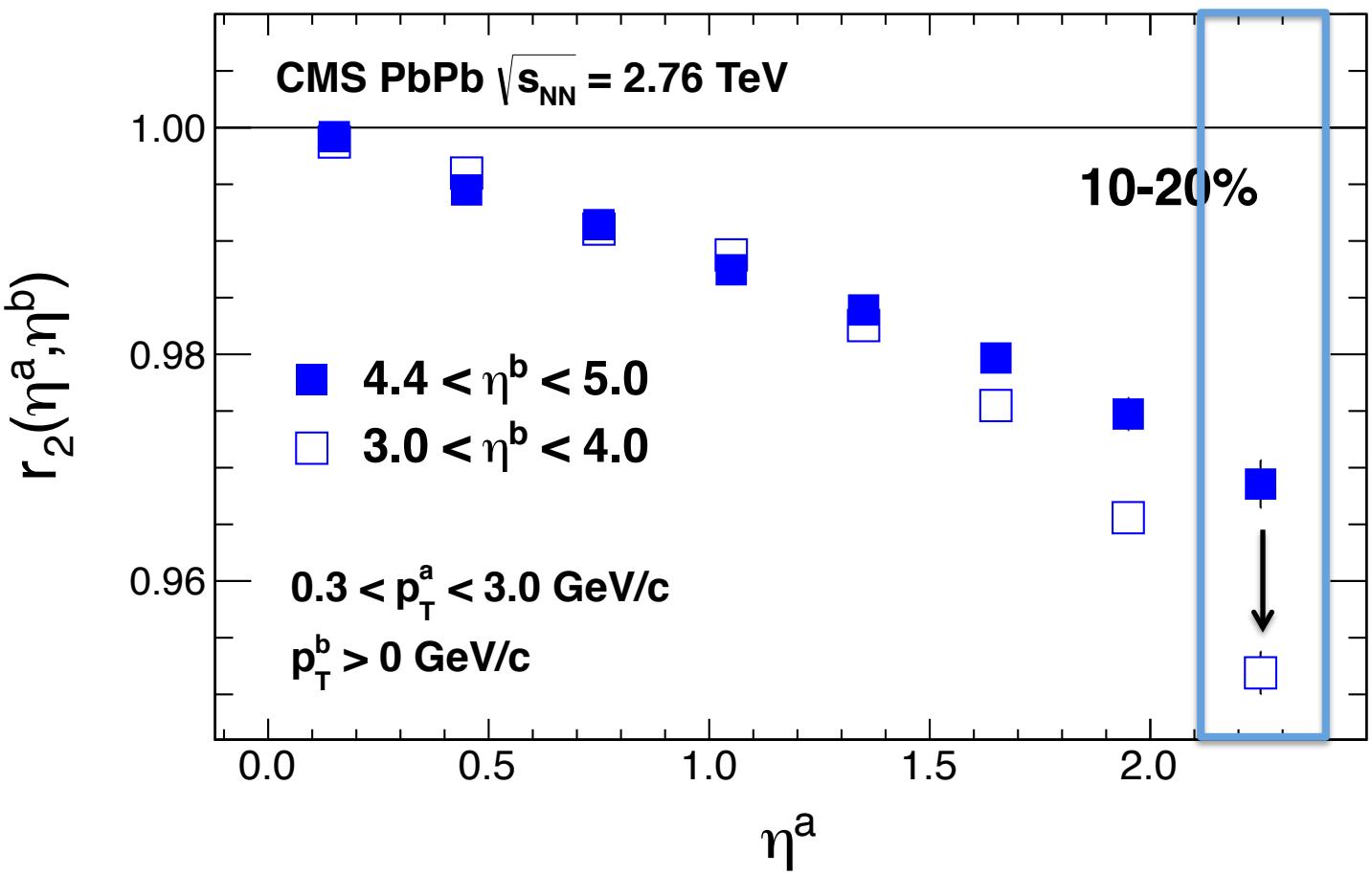
~ linear

# Fix $\eta_b$ and vary $\eta_a$, $\Delta\eta = 2\eta^a$




## Let's vary $\eta_b$ as well

When $\Delta\eta$ is small for denominator, short-range correlations pull $r_2$ down



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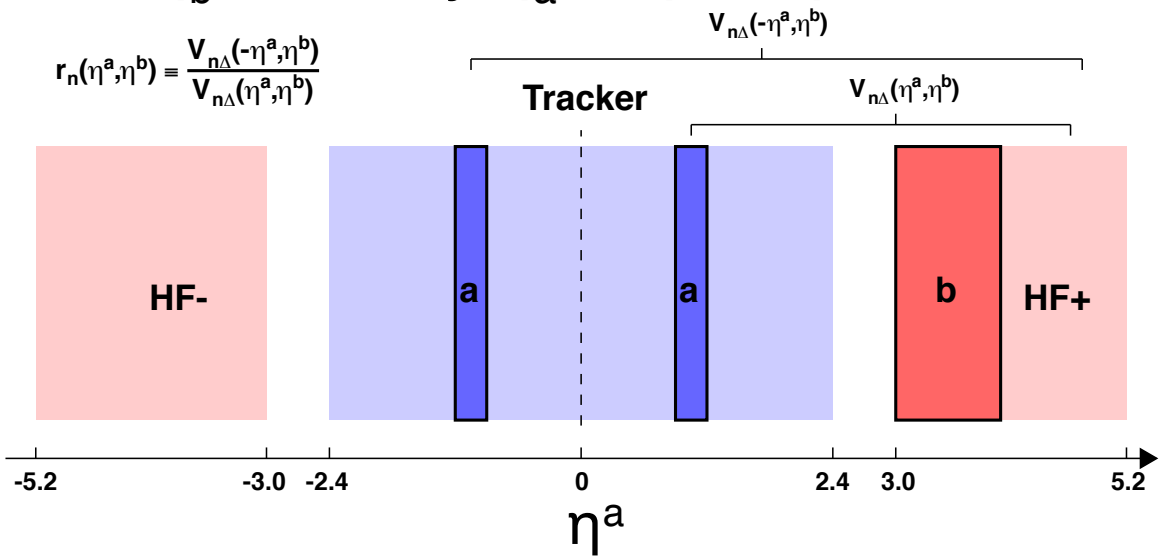
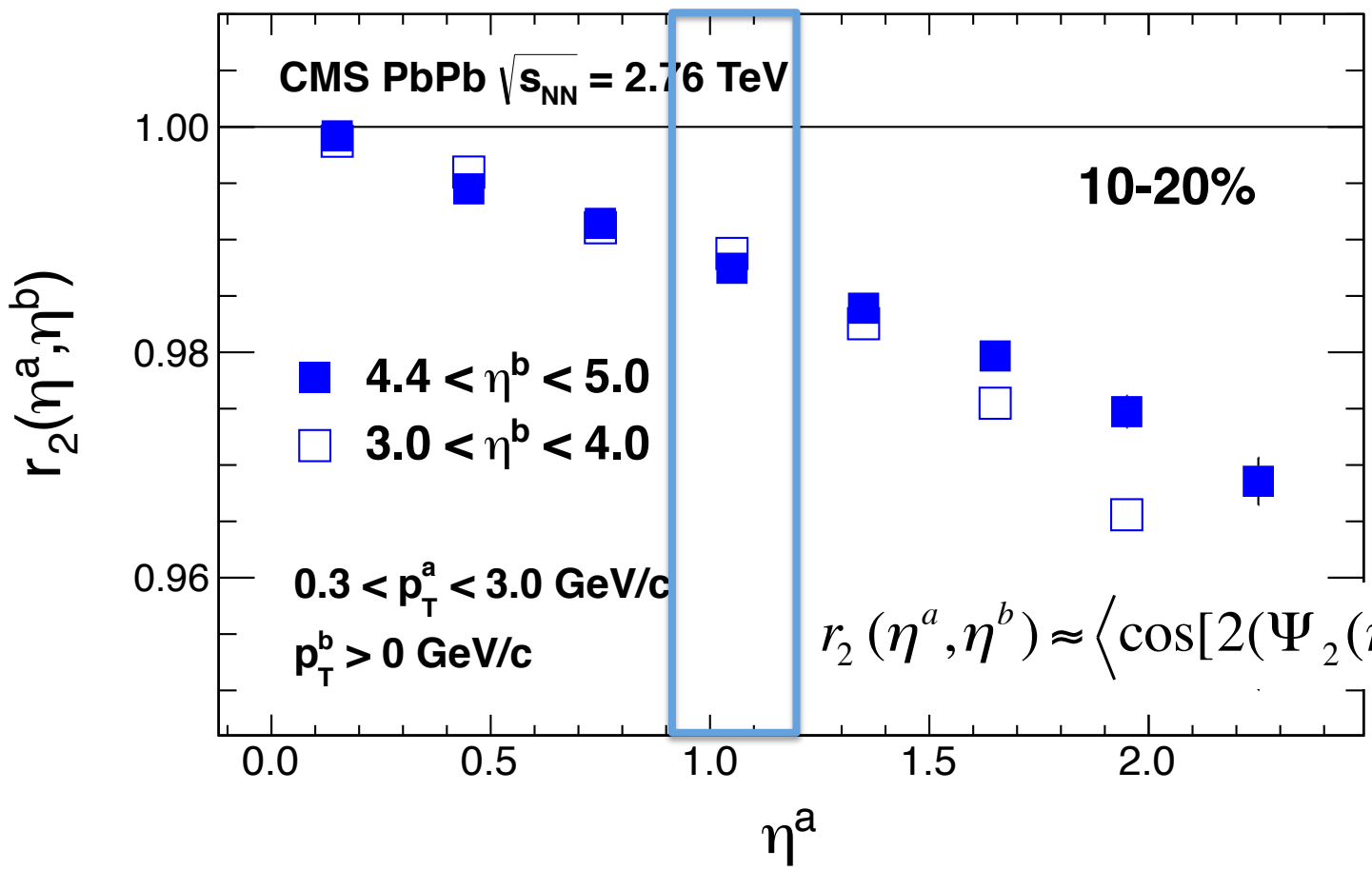



## Let's vary $\eta_b$ as well

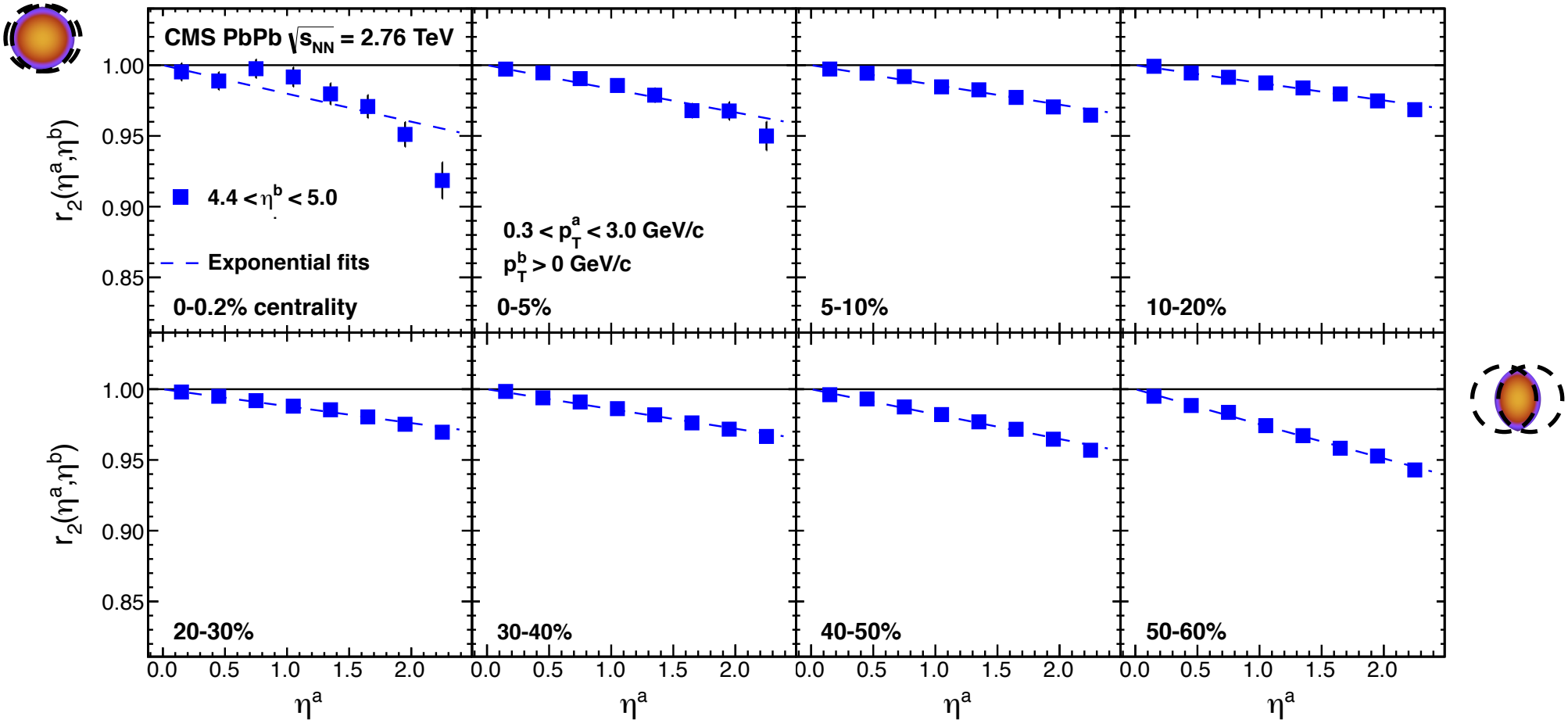
$r_2$ consistent for both $\eta^b$ ranges if $\Delta\eta$ is large

For long-range correlations, $r_2$ is largely $\eta^b$ independent



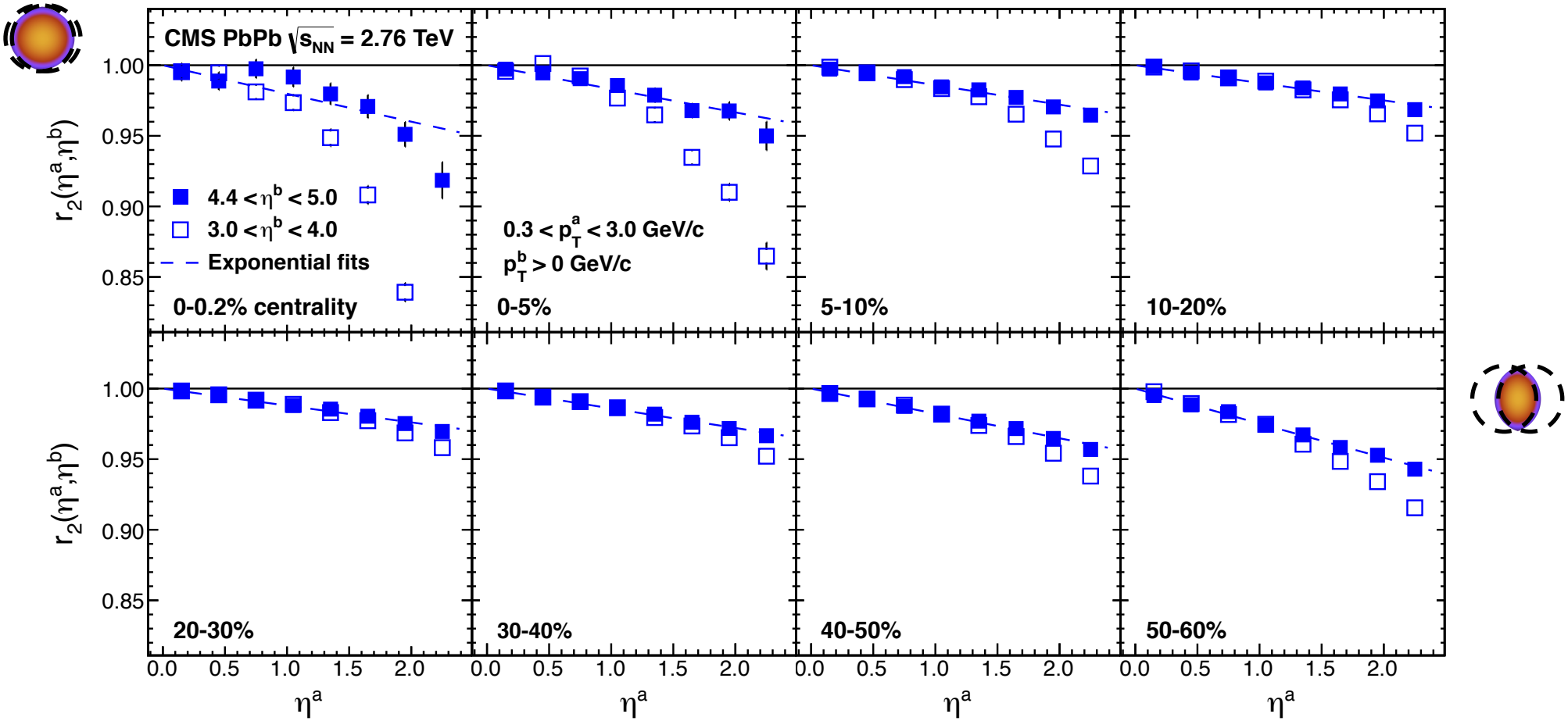

$$r_2(\eta^a,\eta^b) \approx \left\langle \cos[2(\Psi_2(\eta^a) - \Psi_2(-\eta^a))] \right\rangle$$

# Centrality dependence of $r_2(\eta^a,\eta^b)$ in PbPb



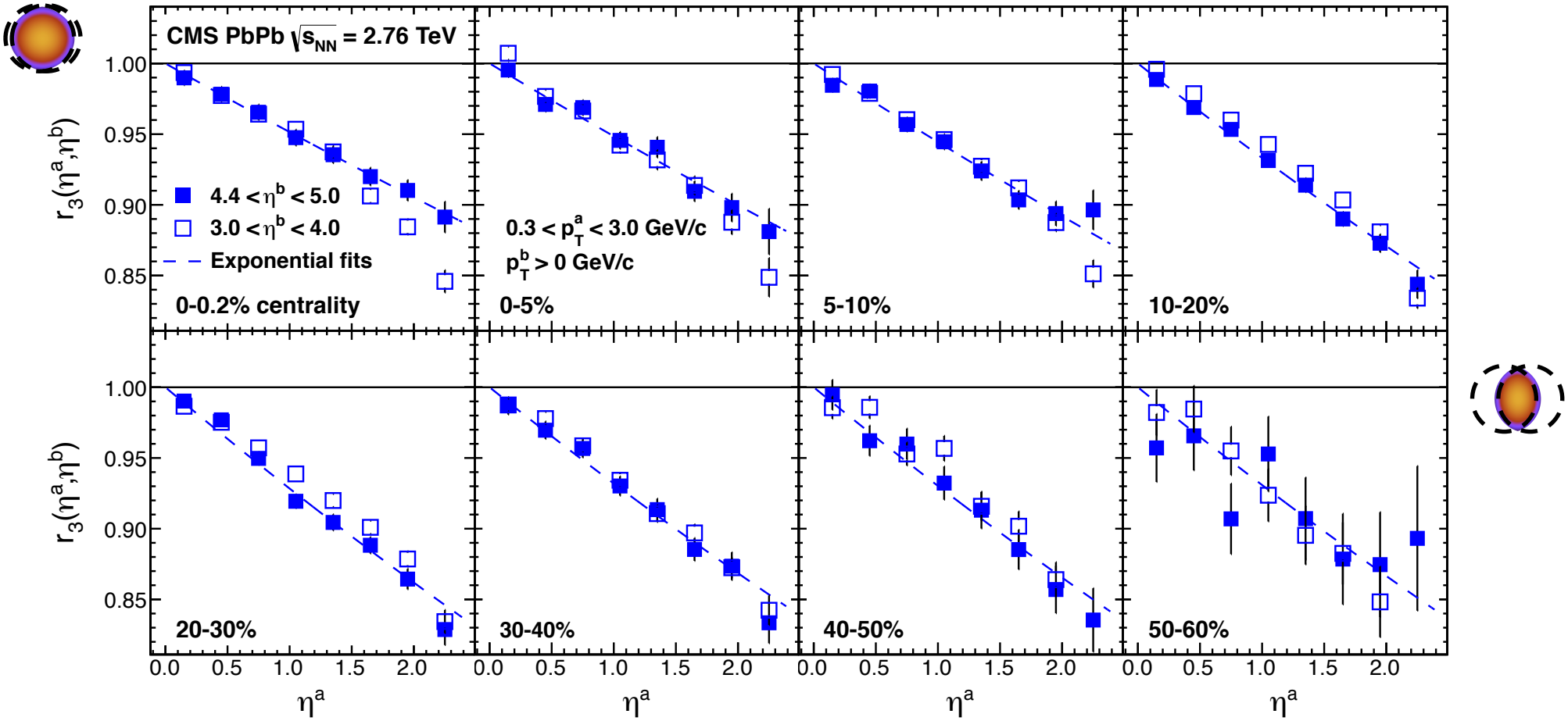

Roughly linear in $\Delta\eta=2\eta^a$, weak dependence on $\eta_b$ except for very central events

# Centrality dependence of $r_2(\eta^a,\eta^b)$ in PbPb



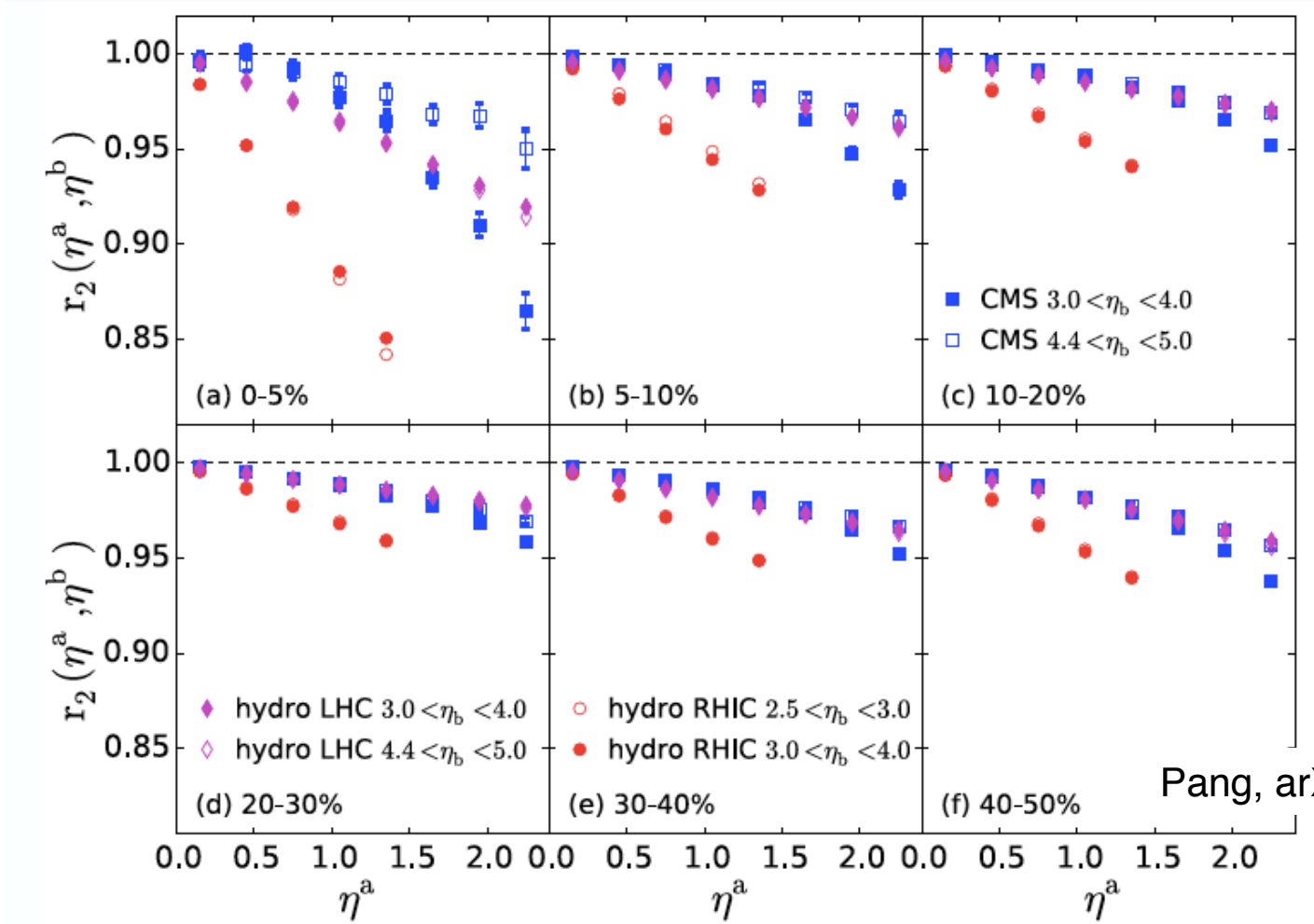

Roughly linear in $\Delta\eta=2\eta^a$, weak dependence on $\eta_b$ except for very central events

# Centrality dependence of $r_3(\eta^a,\eta^b)$ in PbPb




- Stronger effect than $r_2$, dominated by fluctuations
- Weak dependence on $\eta_b$

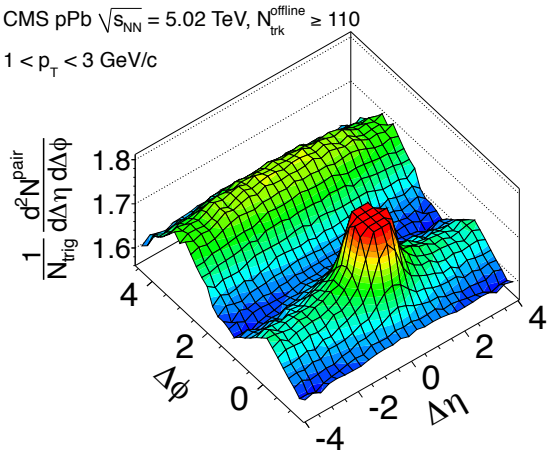
# Comparison with 3+1D hydrodynamics



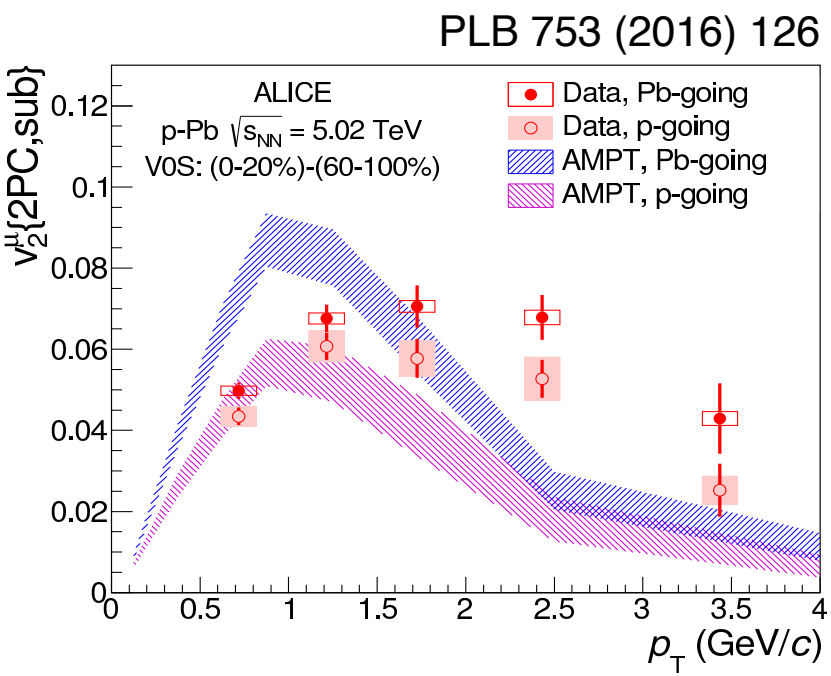

Pang, arXiv:1511.04131

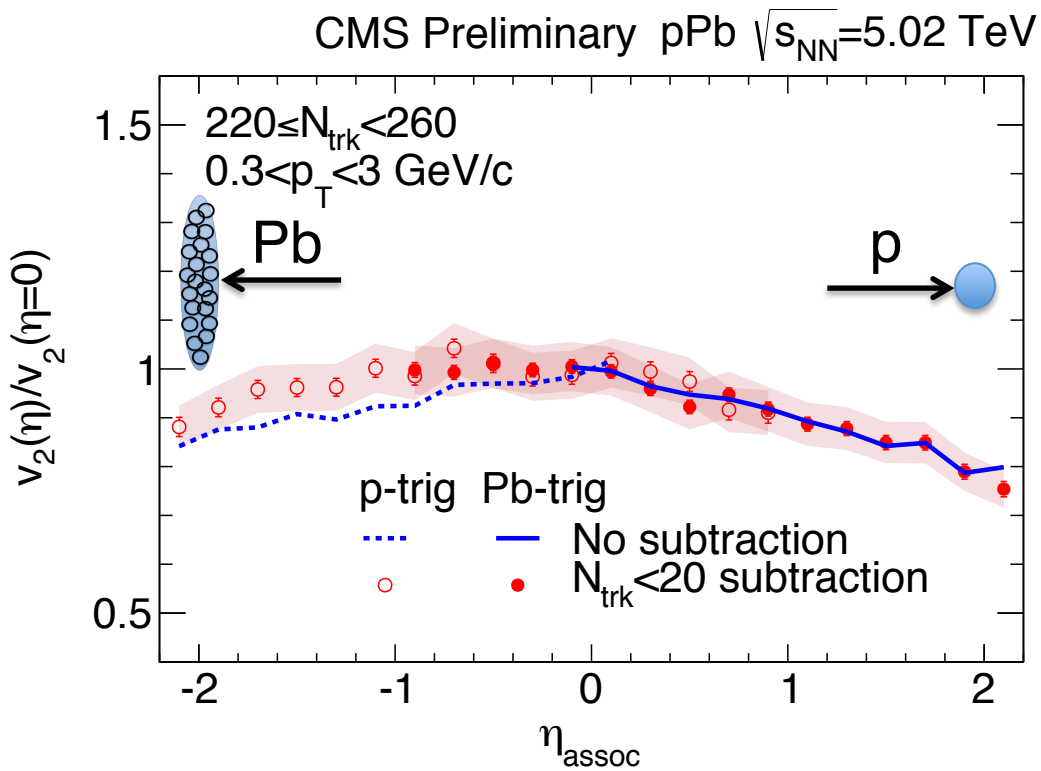
- ✧ Good agreement except for 0-5% (strings overlap?)
- ✧ No strong $\eta_b$ dependent
- ✧ Stronger effect at RHIC energy!

# QGP in small systems (?)



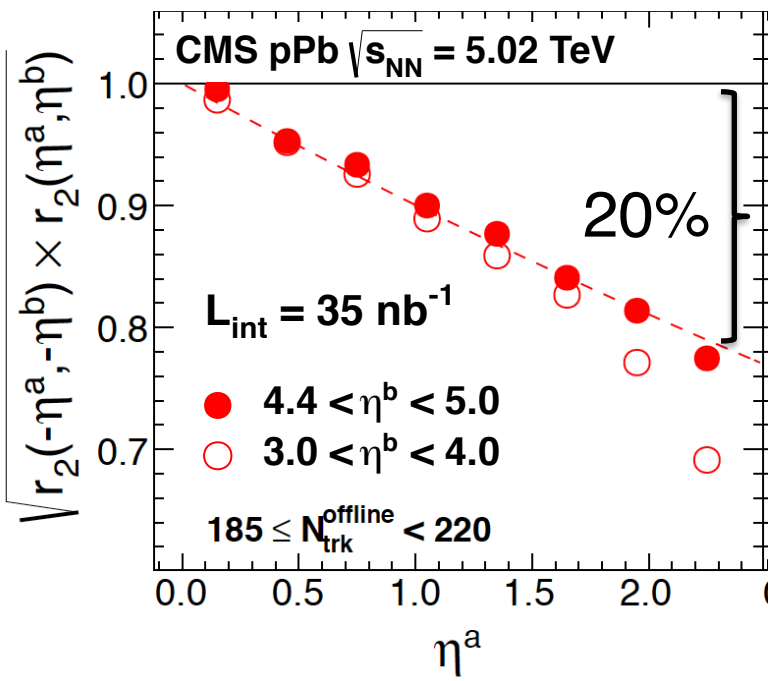

Asymmetric (torqued?) QGP fireball on p- and Pb-going side?



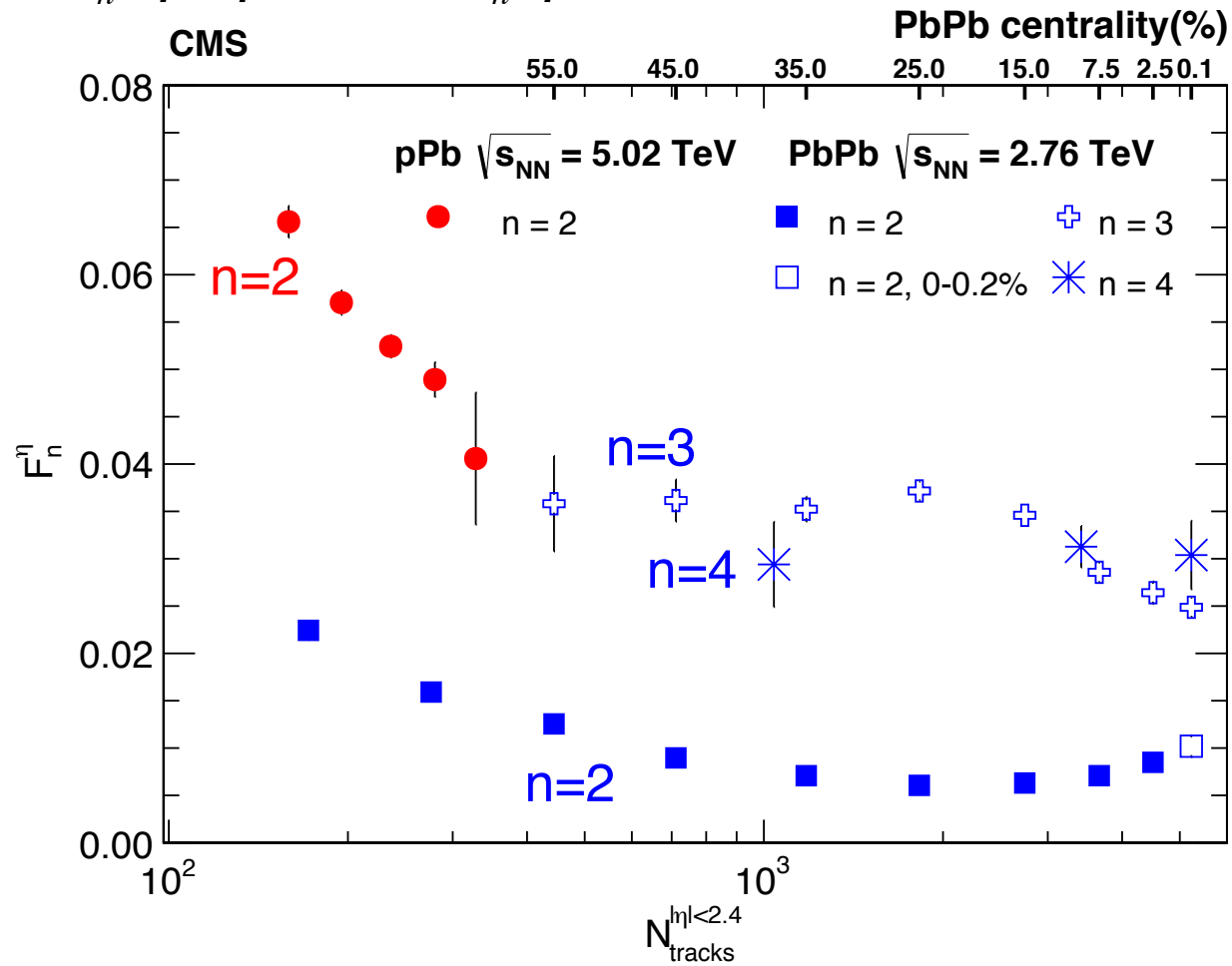

Again, is it η-dependence of $v_n$ or $\Psi_n$?

# $r_2(\eta^a,\eta^b)$ in high-multiplicity pPb




Stronger effect in pPb!

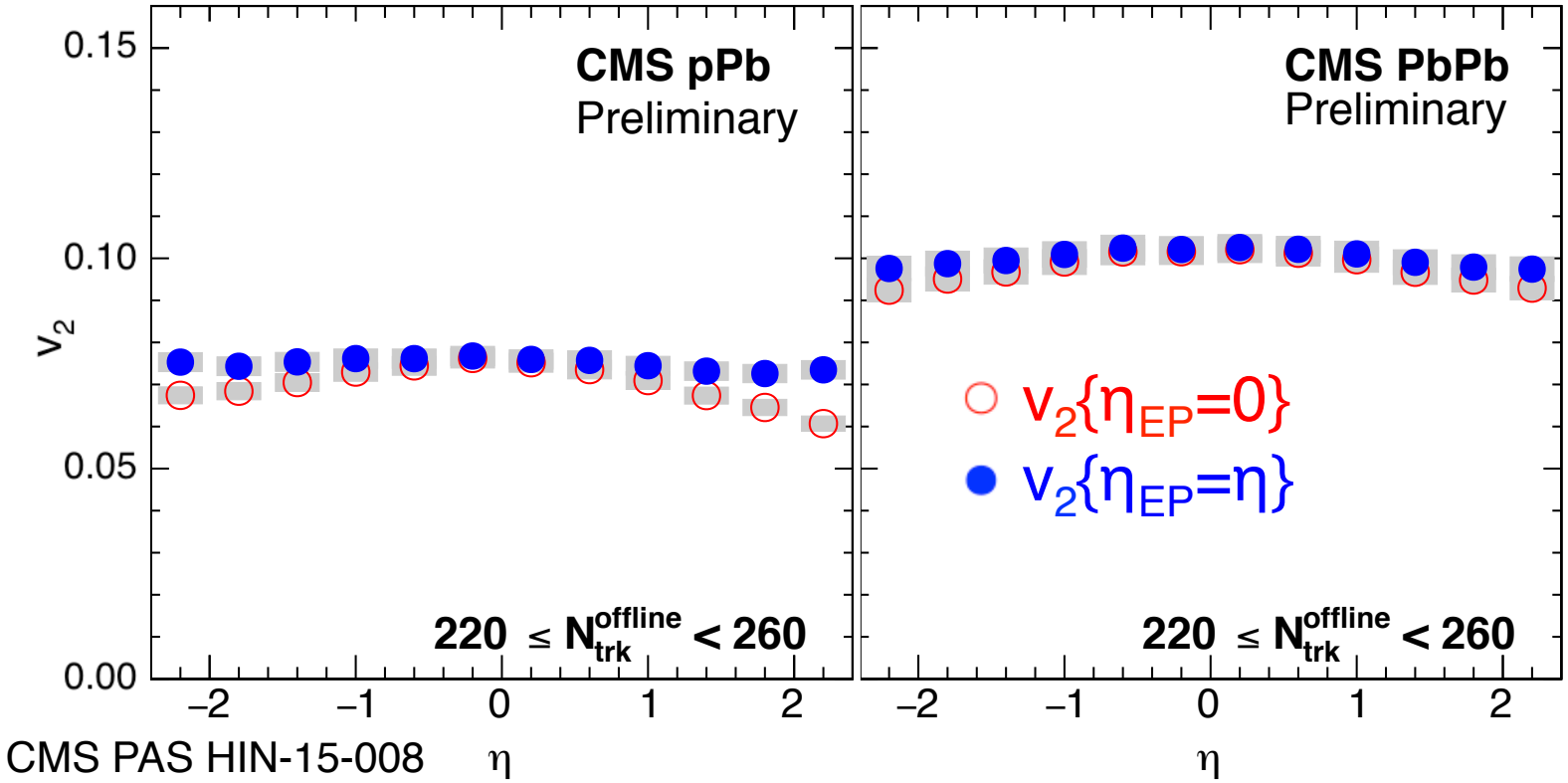
$$r_n(\eta^a,\eta^b) = 1 - 2F_n^{\eta}\eta^a$$




**Lumpiness at subnucleonic scale are crucial to interpret the data! (What about pp?)**

# η dependence of $v_2$ revisited

EP decorrelation "corrected" $v_2\{\eta_{EP}=\eta\}$



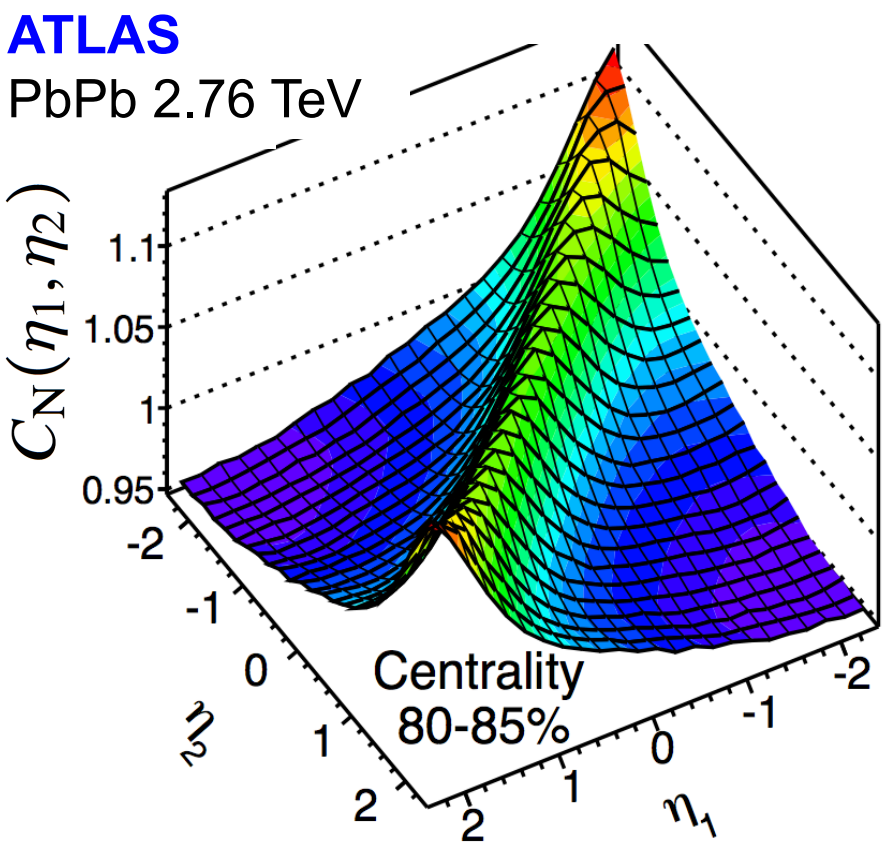

CMS PAS HIN-15-008

Weaker η dependence for $v_2\{\eta_{EP}=\eta\}$

**QGP fluid at forward η may be as "perfect" as mid η**

# Multiplicity fluctuations: $C(\eta_1,\eta_2)$

$$C = \frac{\langle N(\eta_1)N(\eta_2)\rangle}{\langle N(\eta_1)\rangle\langle N(\eta_2)\rangle} = \langle R_S(\eta_1)R_S(\eta_2)\rangle$$




More direct probe of initial entropy fluctuations in y

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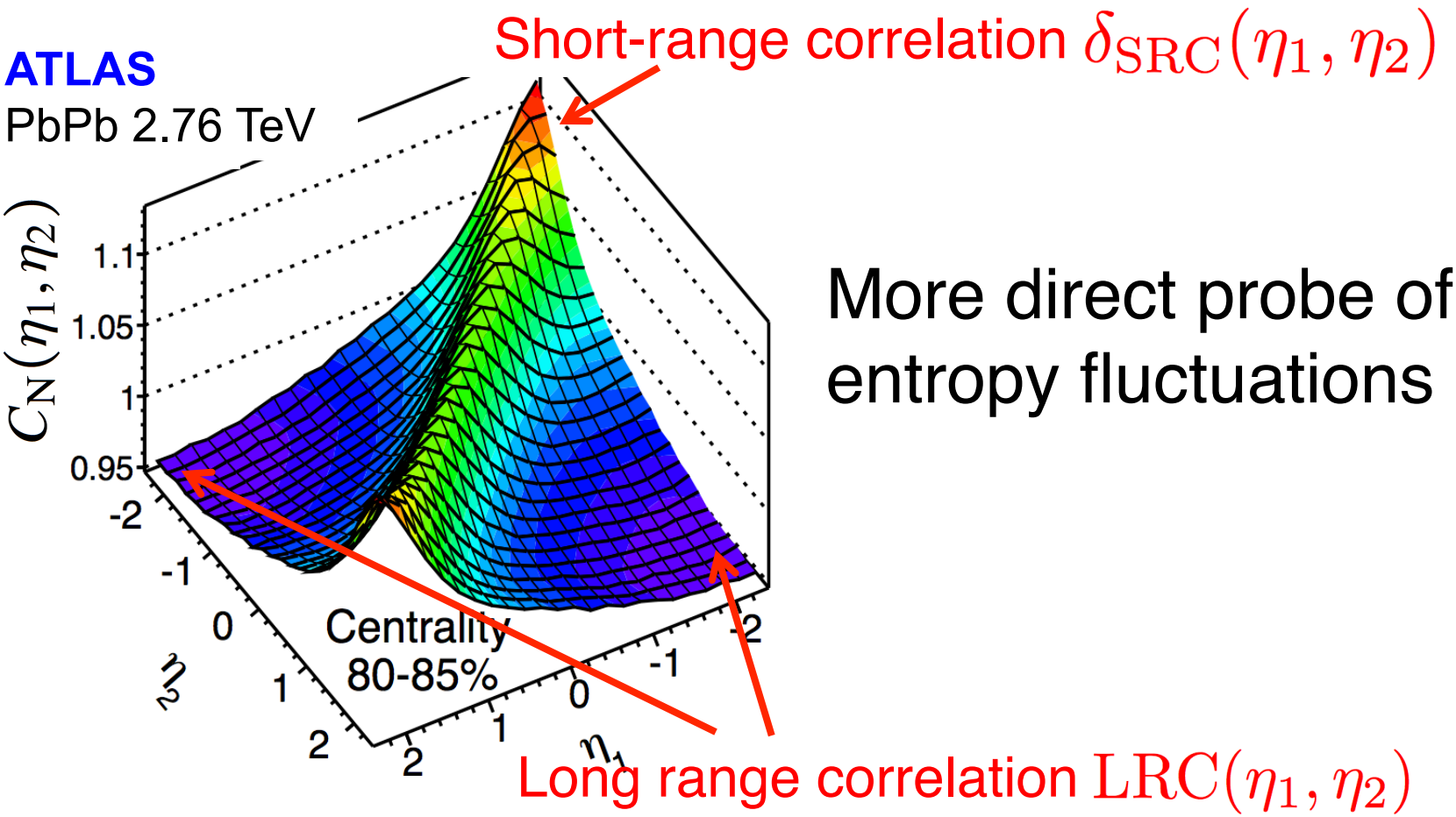



More direct probe of initial entropy fluctuations in y

$$C(\eta_1,\eta_2) = 1 + \delta_{\mathrm{SRC}}(\eta_1,\eta_2) + \mathrm{LRC}(\eta_1,\eta_2)$$

# Multiplicity fluctuations in PbPb, pPb, pp

## ATLAS: decompose SRC and LRC

ATLAS-CONF-2015-051

$$C_N(\eta_1,\eta_2)$$

$$=$$

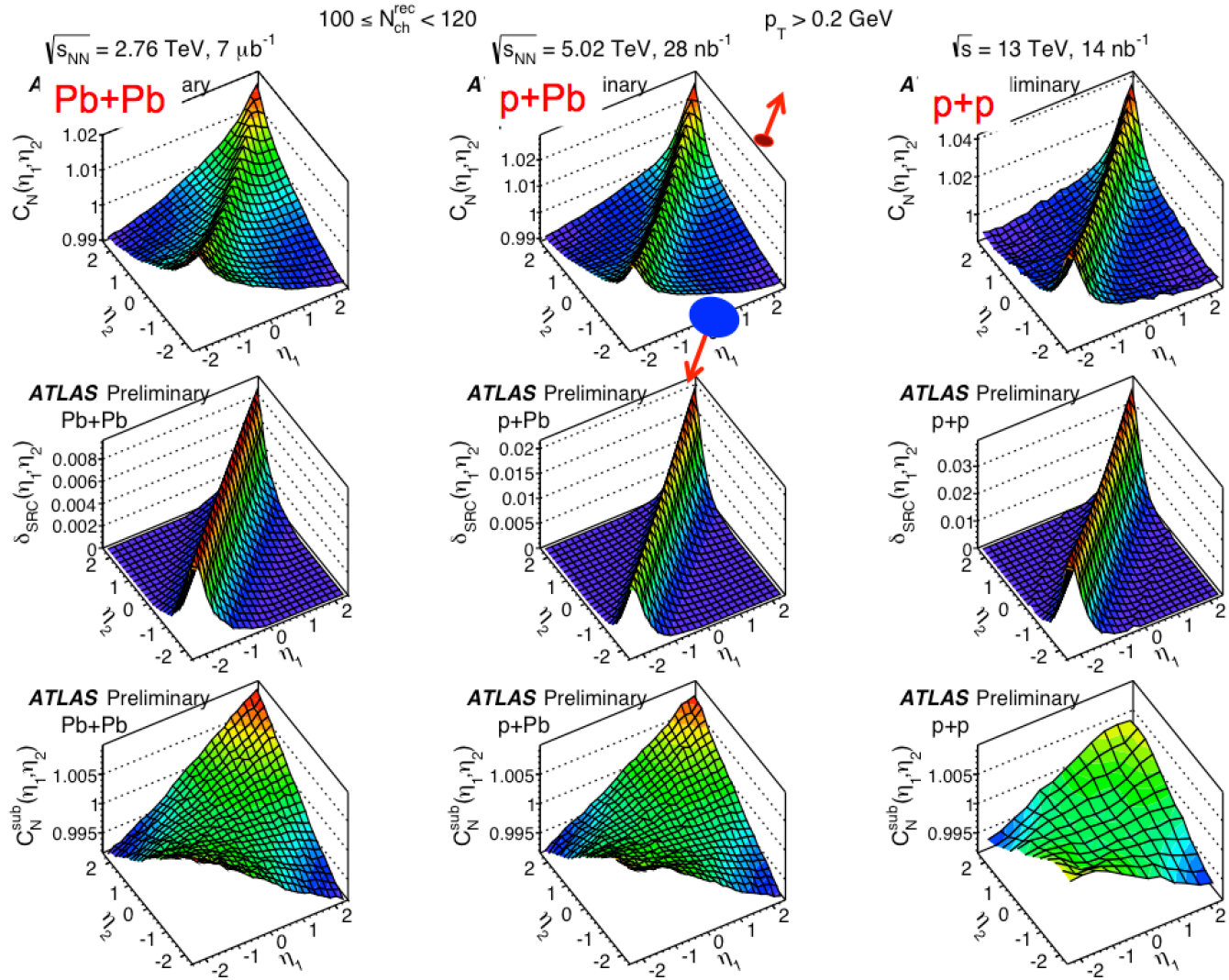
Short-range

$$\delta_{SRC}(\eta_1,\eta_2)$$

$$+$$

Long-range

$$C_N^{sub}(\eta_1,\eta_2)$$



# Multiplicity fluctuations in PbPb, pPb, pp

## ATLAS: decompose SRC and LRC

ATLAS-CONF-2015-051

$C_N(\eta_1, \eta_2)$

=

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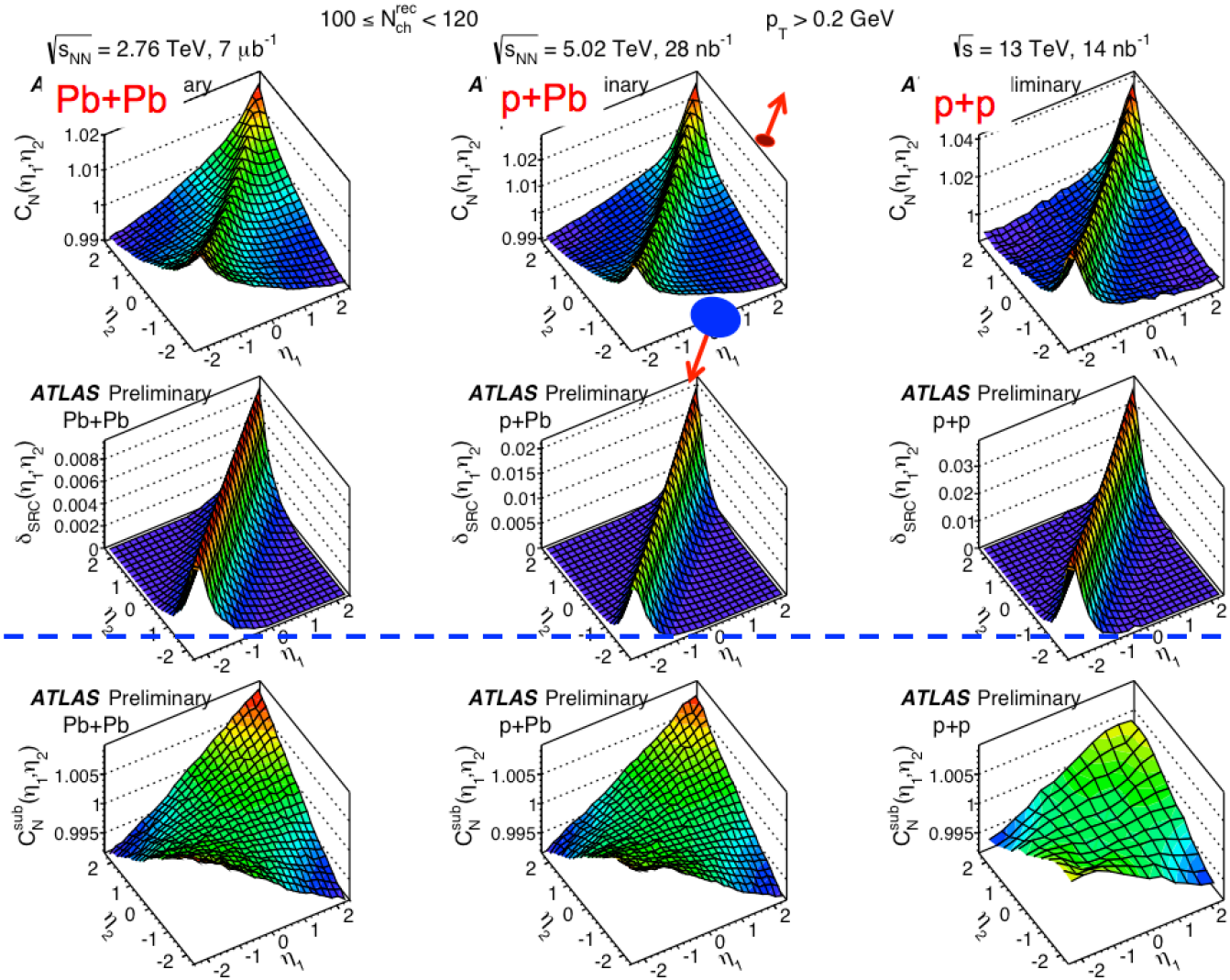
+

Long-range

$C_N^{sub}(\eta_1, \eta_2)$



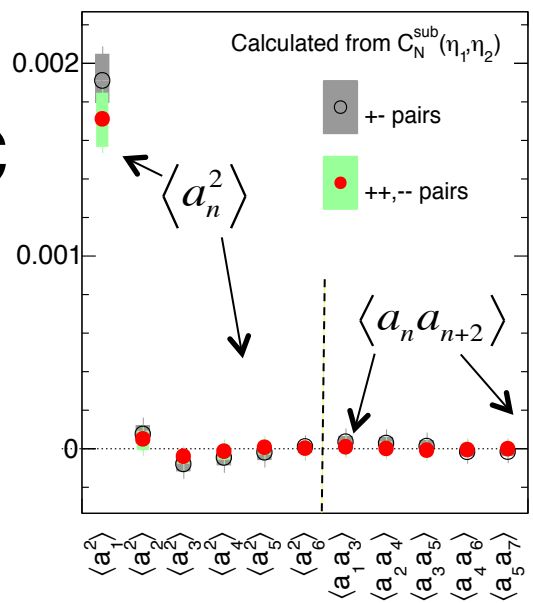

Similar LRC for three systems

# Legendre spectra

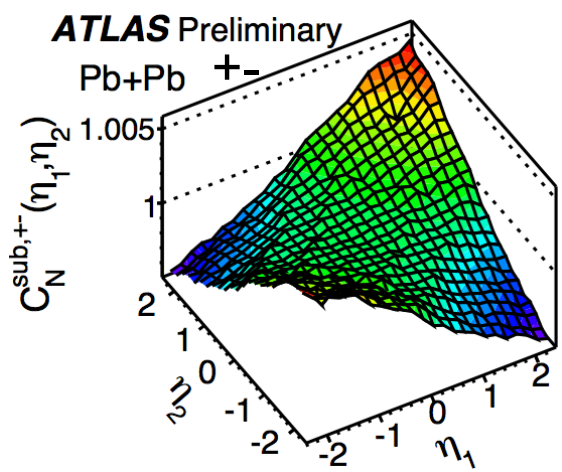
$$C(\eta_1,\eta_2) = 1 + \sum_{n,m=0}^{\infty} \langle a_n a_m \rangle \frac{T_n(\eta_1)T_m(\eta_2) + T_n(\eta_2)T_m(\eta_1)}{2}$$

PbPb

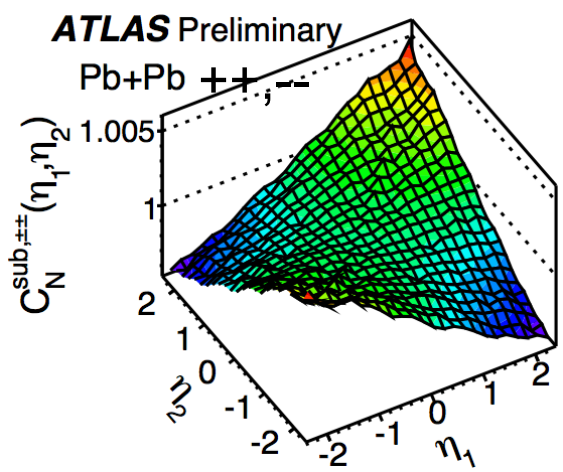
LRC




Unlike-sign




Like-sign



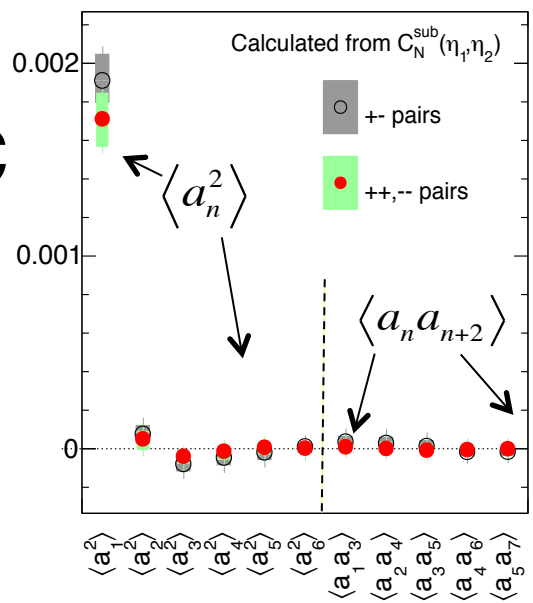

LRC independent of charge combination

# Legendre spectra

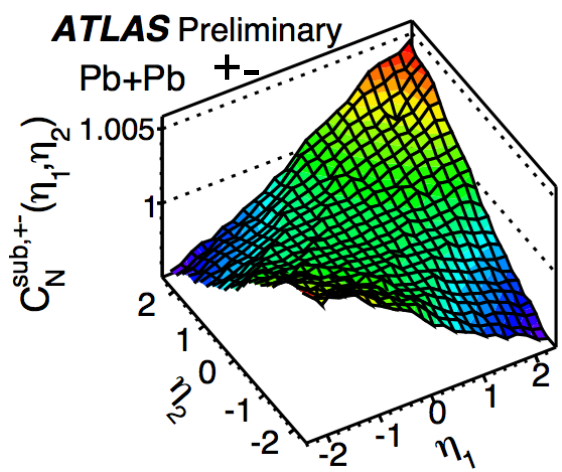
$$C(\eta_1,\eta_2) = 1 + \sum_{n,m=0}^{\infty} \langle a_n a_m \rangle \frac{T_n(\eta_1)T_m(\eta_2) + T_n(\eta_2)T_m(\eta_1)}{2}$$

PbPb

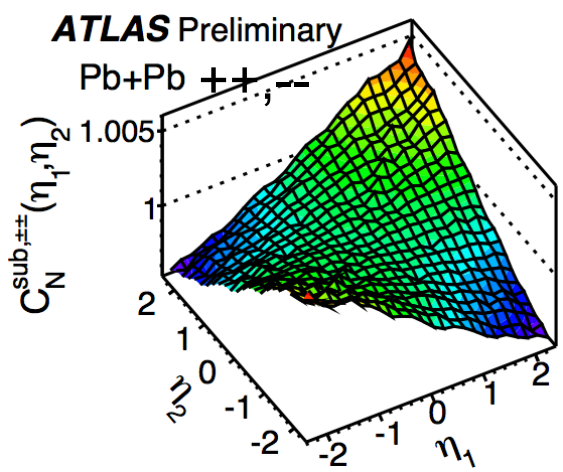
LRC




Unlike-sign




Like-sign




LRC independent of charge combination

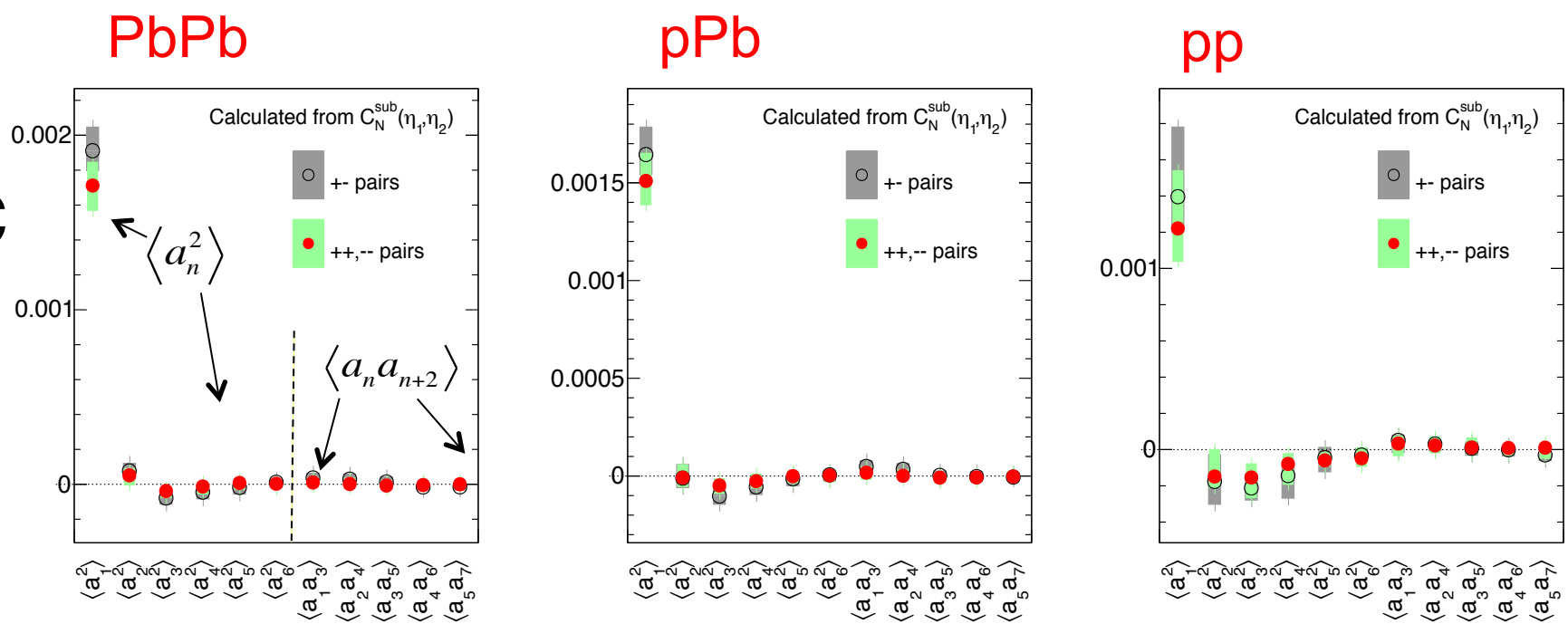
Dominated by $a_1$

$$C_N^{sub}(\eta_1,\eta_2) \approx 1 + \langle a_1^2 \rangle \eta_1 \eta_2 \text{ (LRC)}$$

# Legendre spectra

$$C(\eta_1,\eta_2) = 1 + \sum_{n,m=0}^{\infty} \langle a_n a_m \rangle \frac{T_n(\eta_1)T_m(\eta_2) + T_n(\eta_2)T_m(\eta_1)}{2}$$

LRC



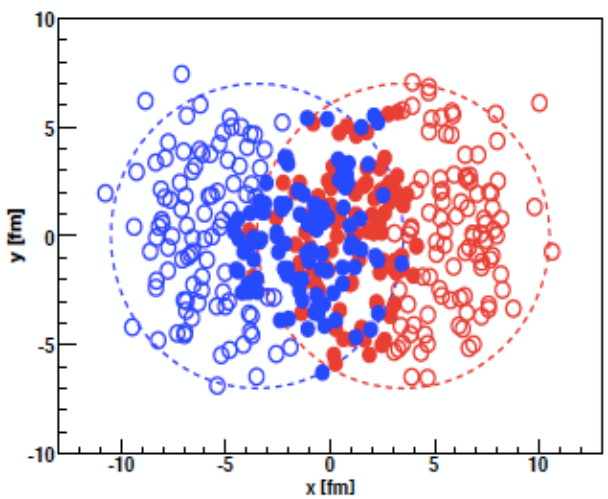

LRC independent of charge combination

Dominated by $a_1$

$$C_N^{sub}(\eta_1,\eta_2) \approx 1 + \langle a_1^2 \rangle \eta_1 \eta_2 \text{ (LRC)}$$

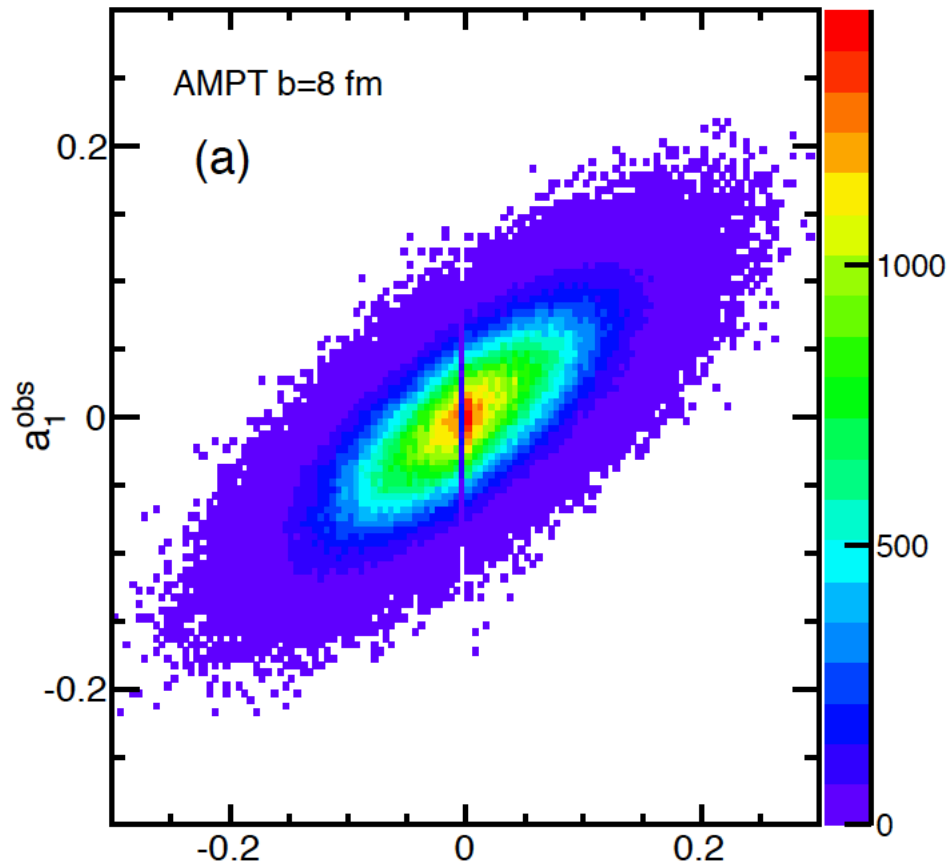
# $a_1$ is related to fluctuations of $N^F_{part}$ v.s. $N^B_{part}$

Asymmetry in participants between F and B regions




$$A_n = \frac{n_f - n_b}{n_f + n_b}, \quad \langle a_1^2 \rangle \propto \langle A_n^2 \rangle$$

AMPT

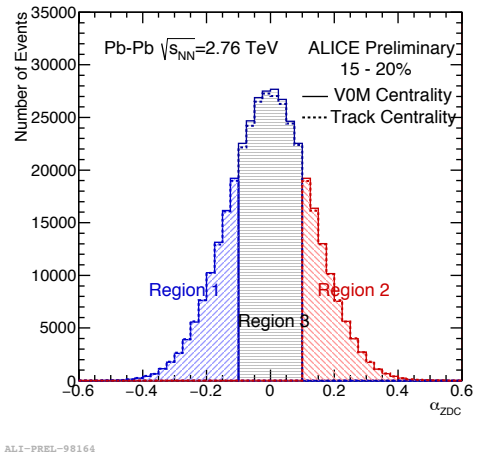



$$A_{part} = \frac{N^F_{part} - N^B_{part}}{N^F_{part} + N^B_{part}}$$

Peng Huo, QM2015

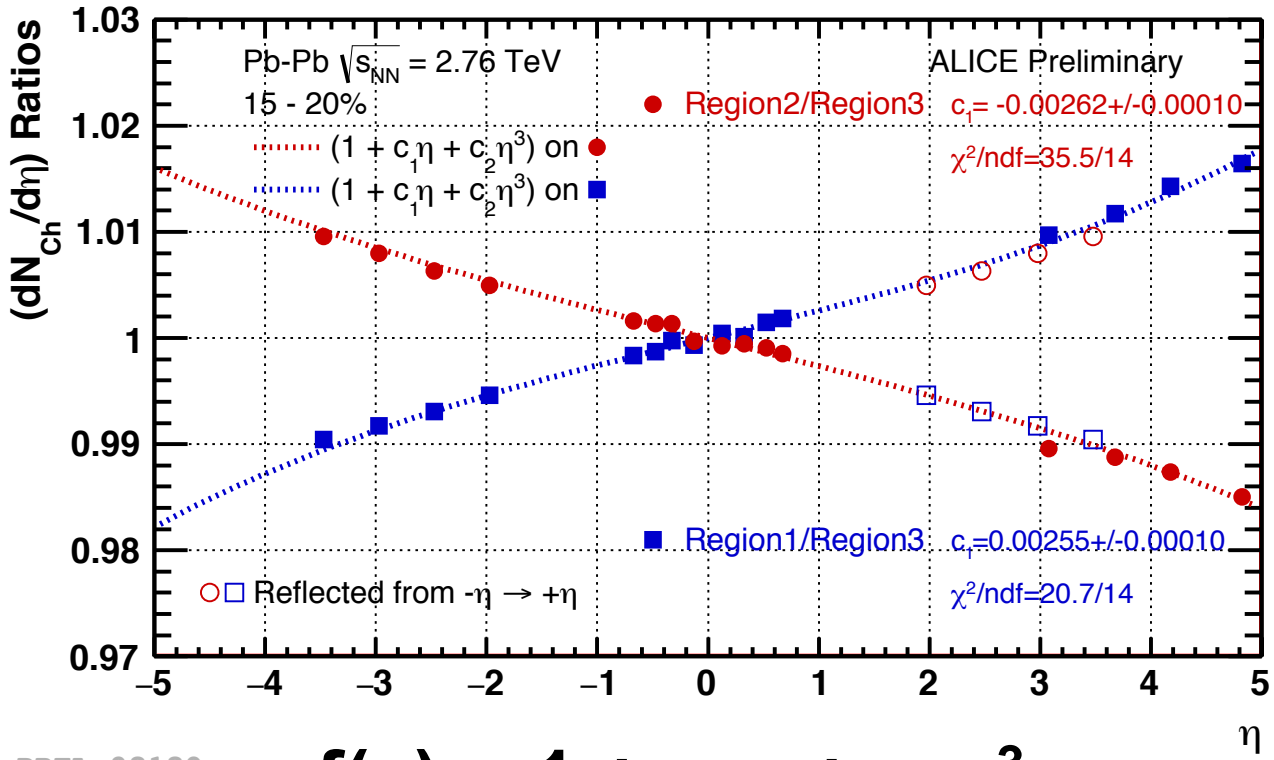
# Engineering F-B participant asymmetry

## ZDC asymmetry




$$\alpha_{ZDC} = \frac{ZDC_+ - ZDC_-}{ZDC_+ + ZDC_-}$$

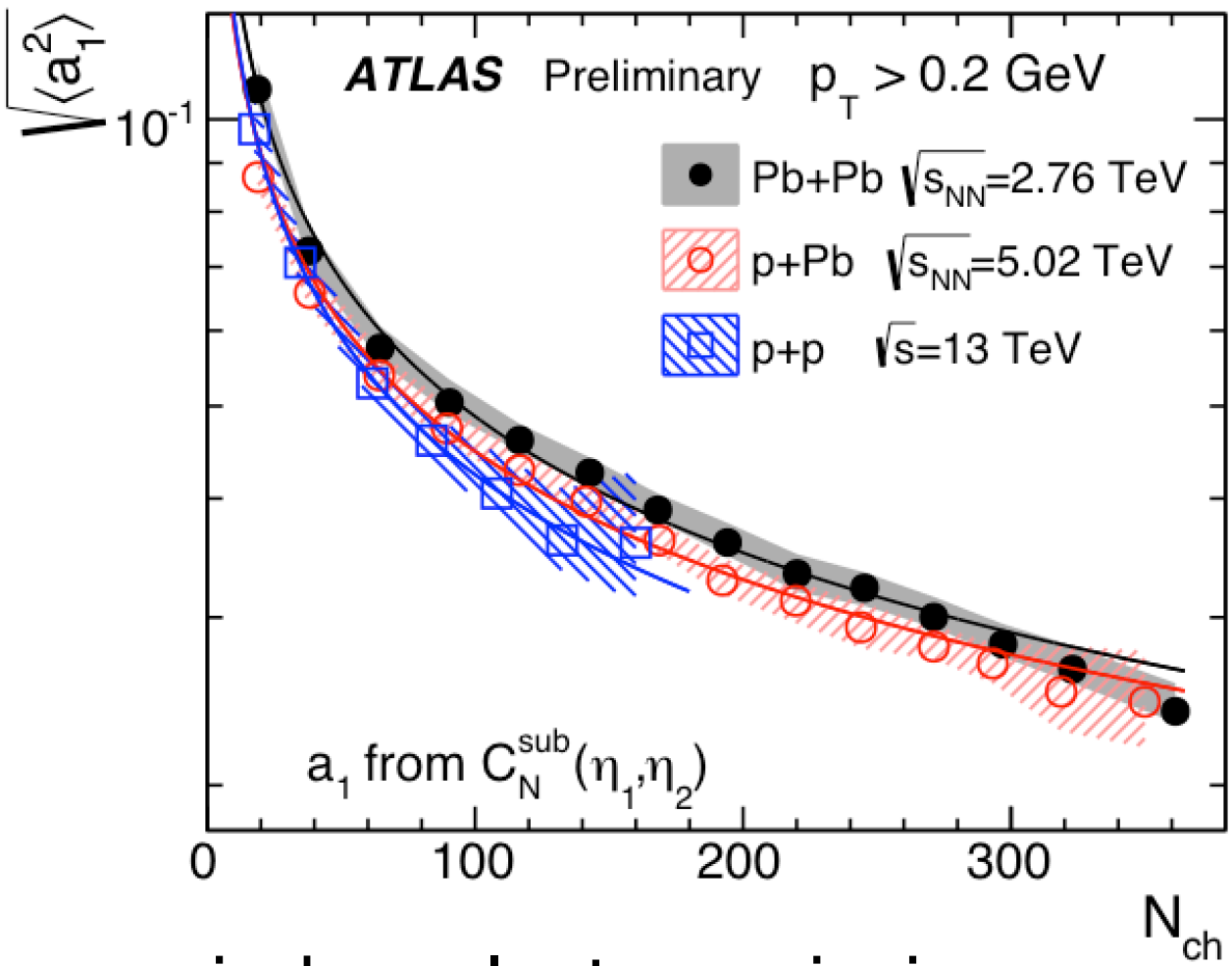
## Ratios of $dN_{ch}/d\eta$ of events with different (a)symmetries.




$$f(\eta) = 1 + c_1\eta + c_2\eta^3$$

$$c_1 \approx \langle y_0/\sigma_\eta^2 \rangle$$

# Multiplicity dependence of $a_1$ for LRC



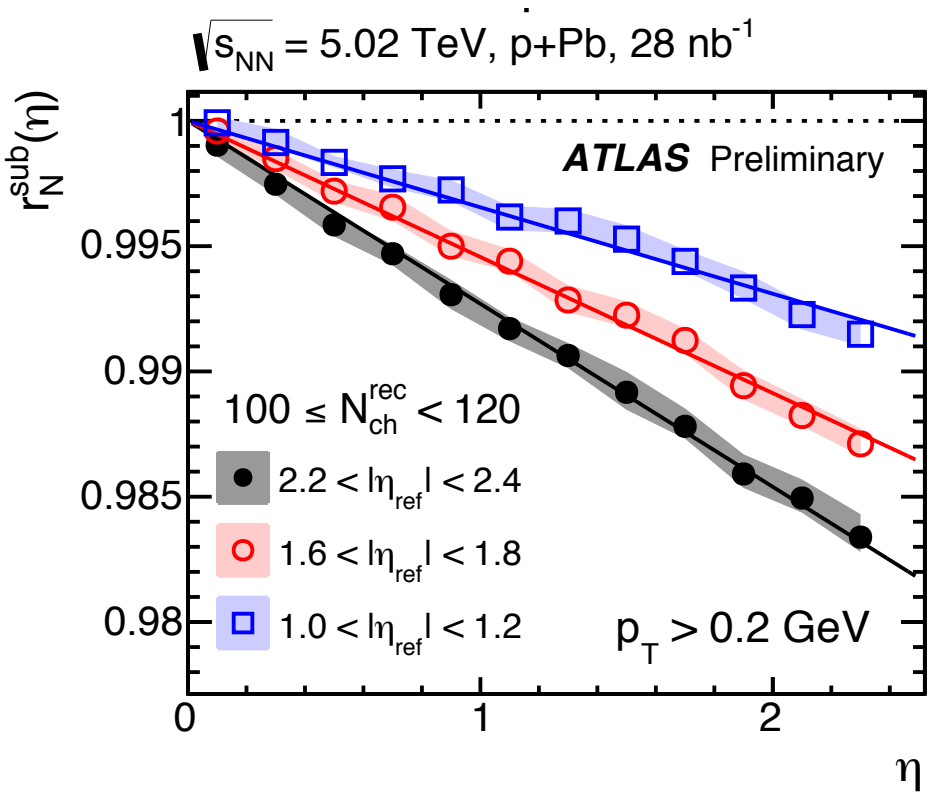

Assume indep. cluster emission

$$\sqrt{\langle a_1^2 \rangle} \sim \frac{1}{N_{ch}^{\alpha}}, \alpha \sim 0.5$$

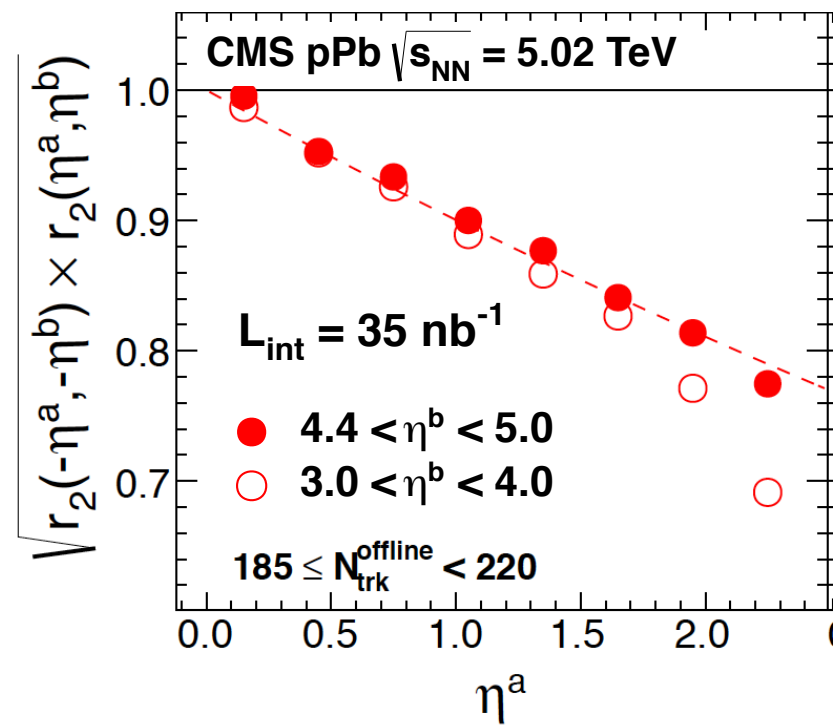
# Multiplicity correlator similar to CMS $r_n$

$$r_{\mathrm{N}}^{\mathrm{sub}}(\eta,\eta_{\mathrm{ref}}) = C_{\mathrm{N}}^{\mathrm{sub}}(-\eta,\eta_{\mathrm{ref}})/C_{\mathrm{N}}^{\mathrm{sub}}(\eta,\eta_{\mathrm{ref}}) = 1 - 2\left\langle a_1^2\right\rangle \eta\eta_{\mathrm{ref}}$$

## Multiplicity fluctuations




## Anisotropy Fluctuations




Similar behavior, but $\eta_b$ dependence is not seen for CMS $r_2$ measurement

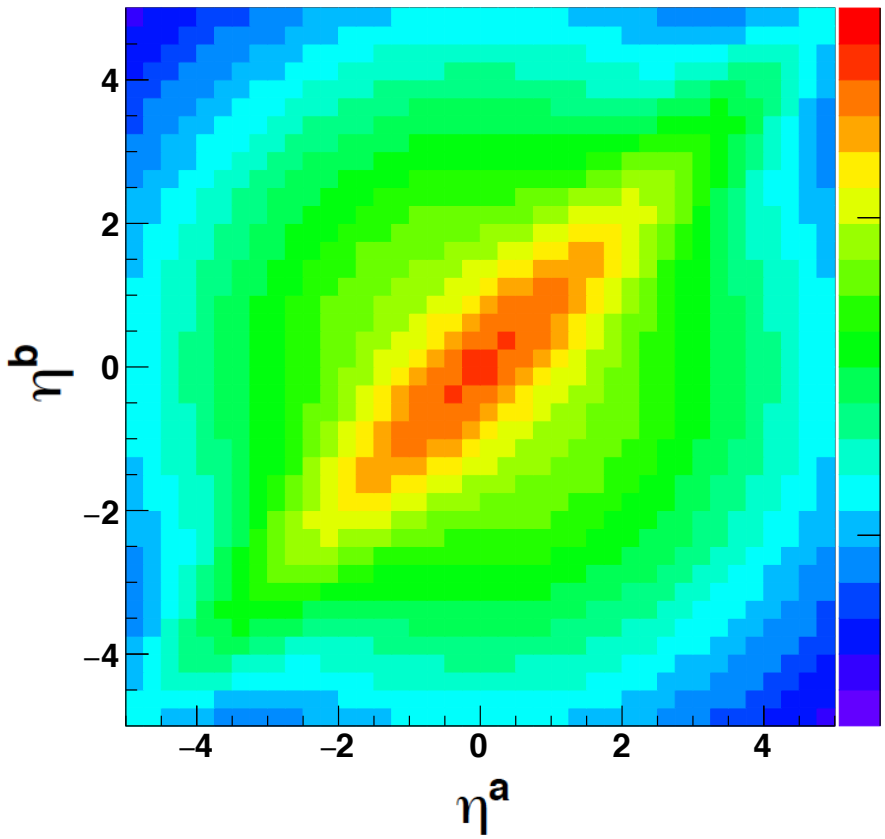
# Summary

- New studies of rapidity correlations and fluctuations at the LHC over a wide range in $\eta$

- Rapidity correlations of anisotropy flow
  - Evidence for EP fluctuations in $\eta$
  - Strong effect in pPb suggests importance of subnucleonic scale fluctuations

- Forward-backward multiplicity fluctuations observed from LRC
  - Consistent with a form $1+\langle a^2_1\rangle\eta_1\eta_2$
  - Scaling among PbPb, pPb and pp

# What's next?

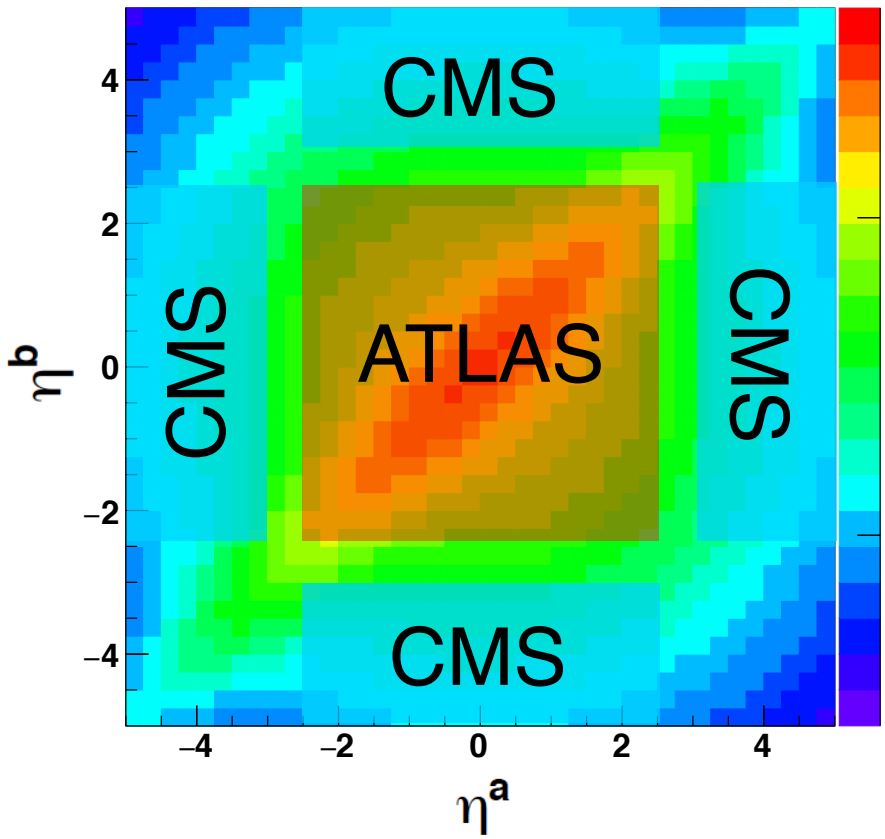



Experiments have 10x10 units coverage in $(\eta^a, \eta^b)$ for

$$C(\eta^a, \eta^b),\ V_{n\Delta}(\eta^a, \eta^b)$$

# What's next?

Presently,

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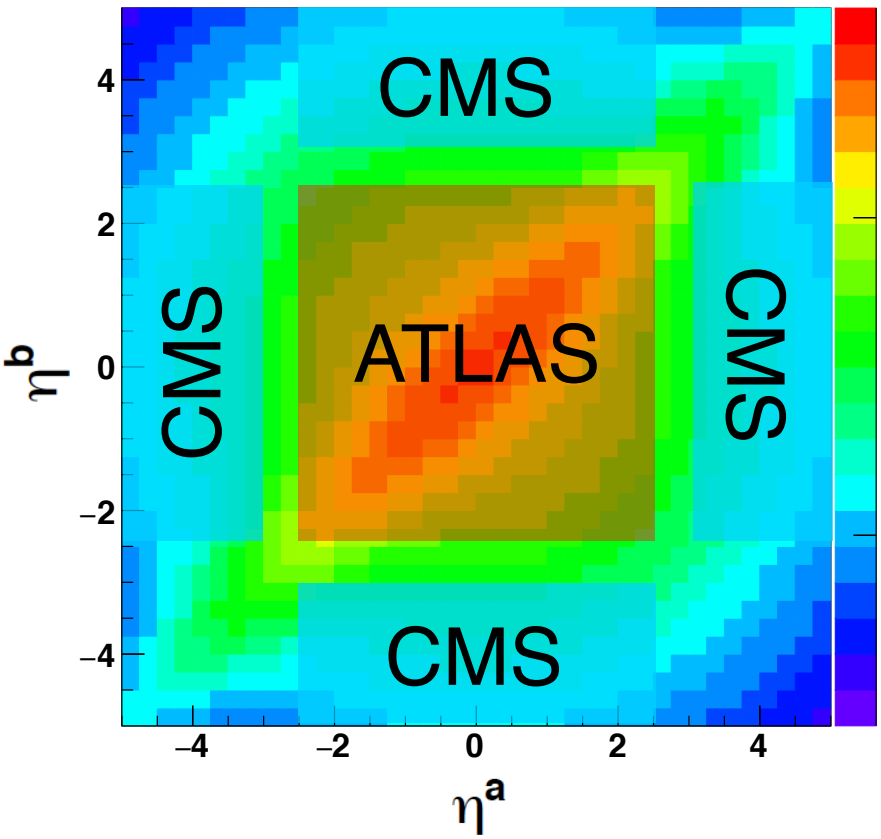
Presently,

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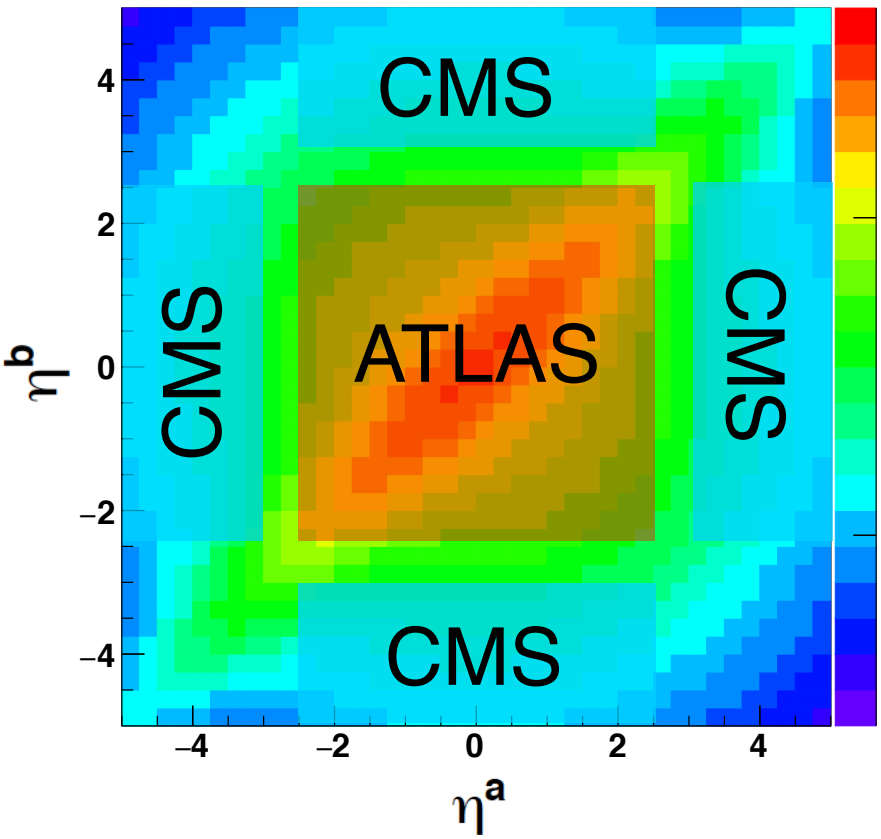
$$C(\eta^a, \eta^b),\ V_{n\Delta}(\eta^a, \eta^b)$$

PCA fit on LRC (e.g., $|\Delta\eta|>2$)

$$V_{n\Delta}(\eta^a,\eta^b) = \sum_{\alpha=1}^{k} V_n^{(\alpha)}(\eta^a) V_n^{(\alpha)*}(\eta^b)$$

# What's next?

Presently,

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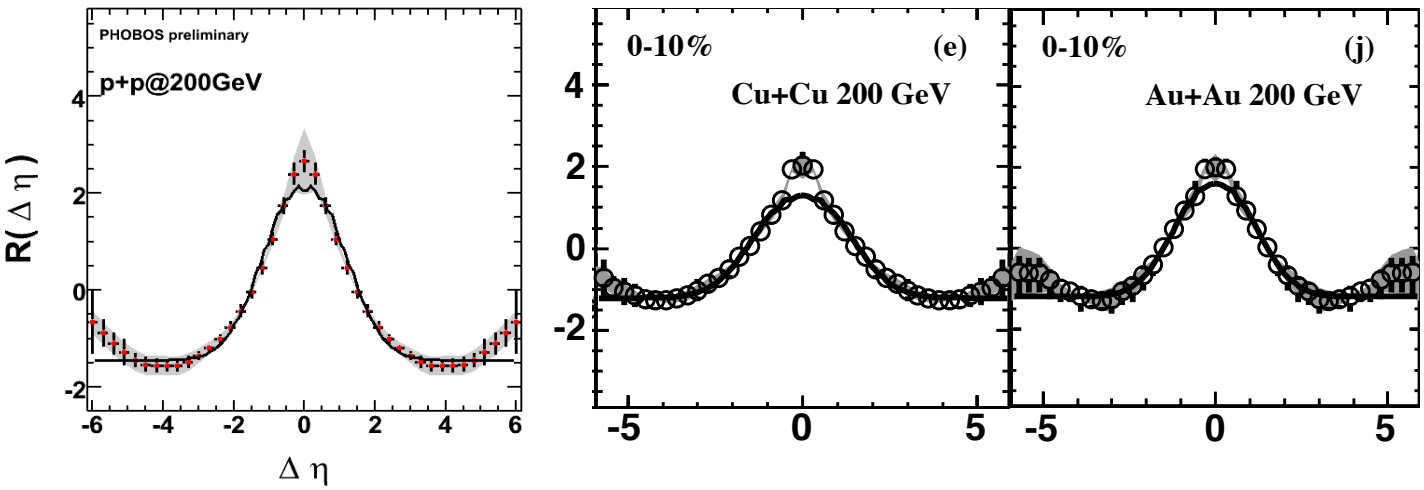
$$V_{n\Delta}(\eta^a,\eta^b) = \sum_{\alpha=1}^{k} V_n^{(\alpha)}(\eta^a) V_n^{(\alpha)*}(\eta^b)$$

- Precise modeling of 3D hydrodynamics
- New insights to collectivity in small systems, internal structure of proton

# For your entertainment …

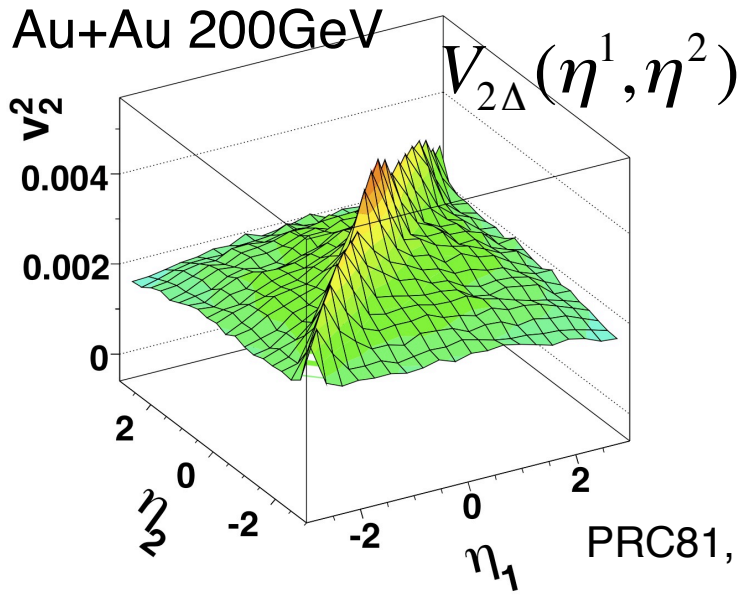
## Some PHOBOS results

Two-particle rapidity correlations




PRC75, 054913 (2007)
PRC81, 024904 (2010)

Au+Au 200GeV

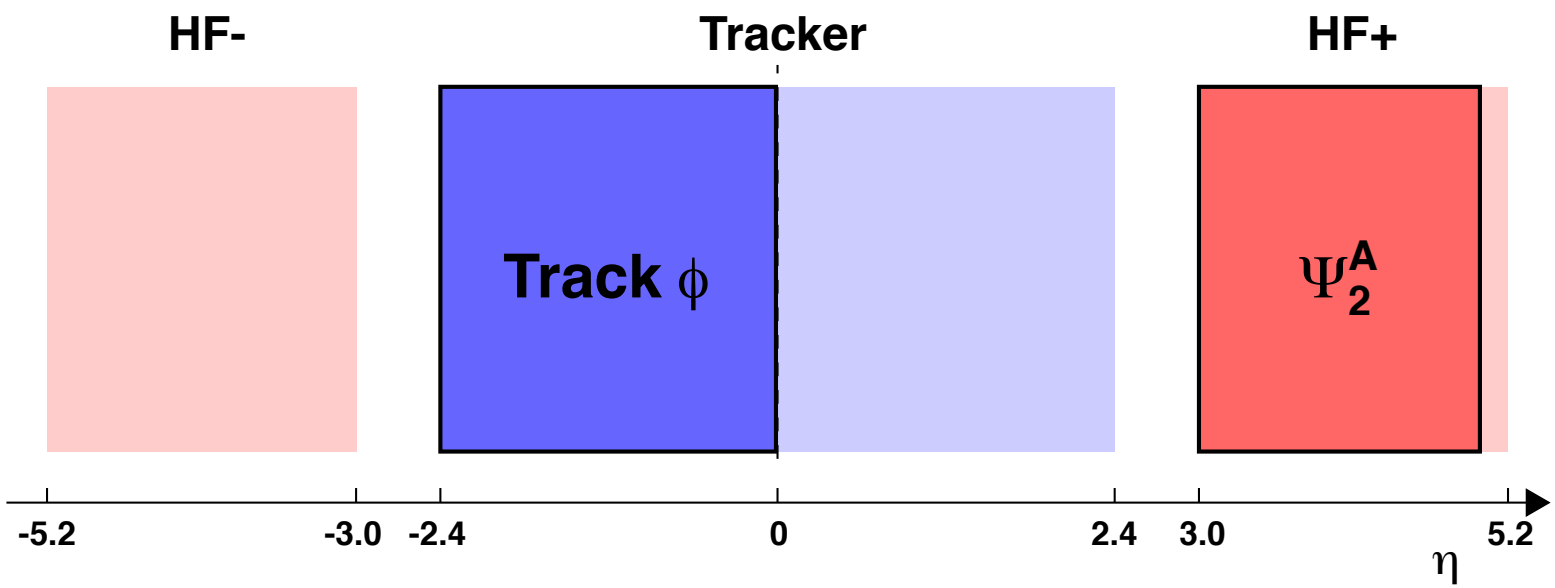



PRC81, 034915 (2010)

Future studies at RHIC will be highly interesting!

# Backups

# How is $v_2\{EP\}$ affected by EP decorrelations?




$$v_2 = \frac{v_2^{obs}}{R_A} = \frac{\left\langle \cos 2(\phi - \Psi_2^A) \right\rangle}{R_A} \quad \Delta\eta \geq 3$$

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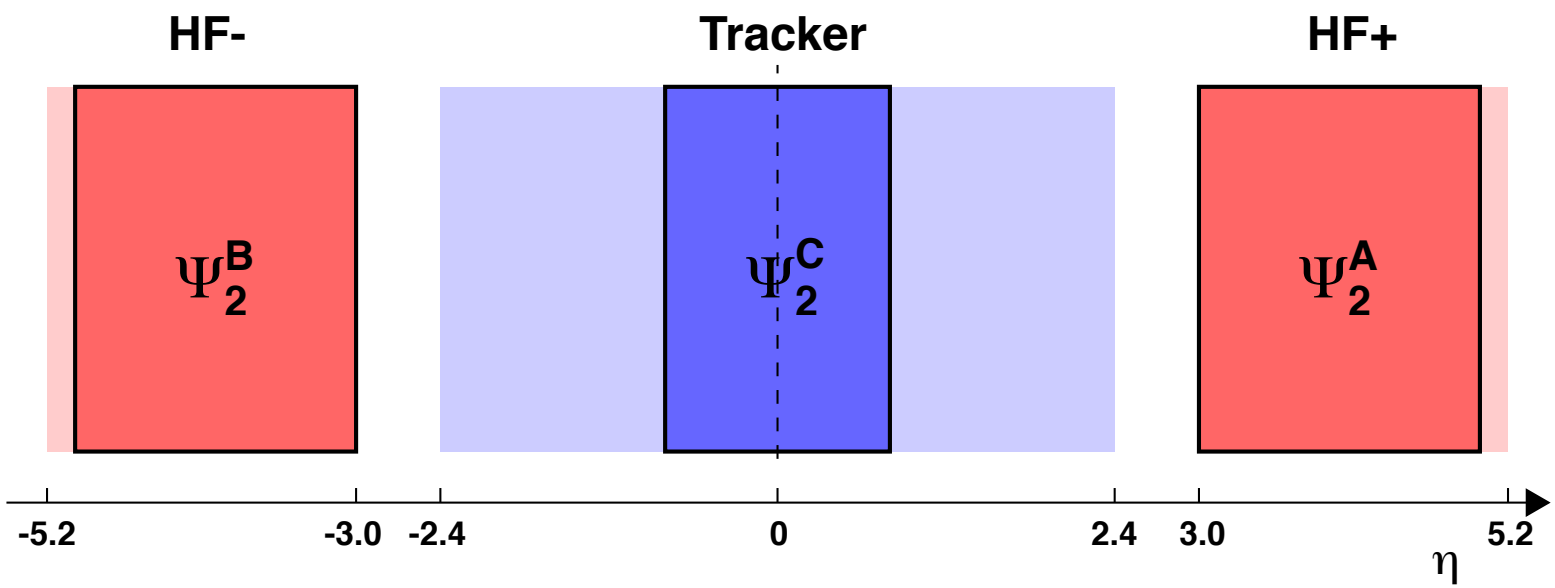
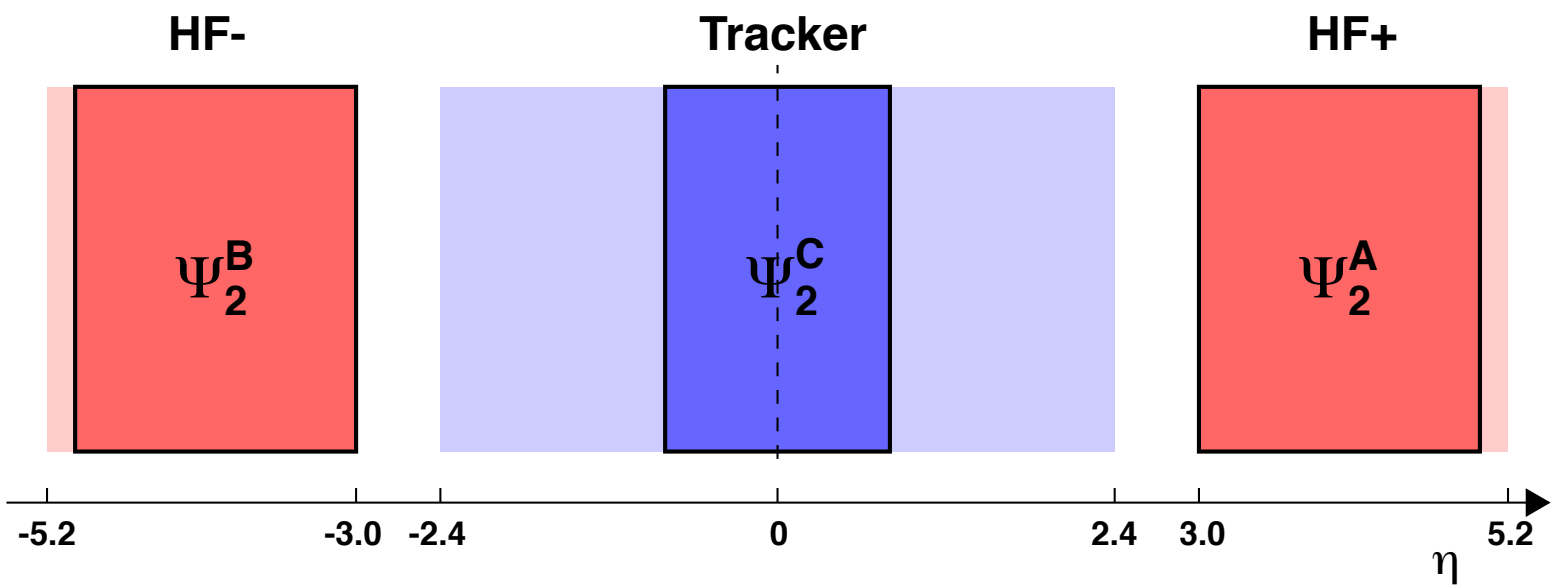



$$v_2 = \frac{v_2^{obs}}{R_A} = \frac{\left\langle \cos 2(\phi - \Psi_2^A) \right\rangle}{R_A} \quad \Delta\eta \geq 3$$

Resolution correction:

$$R_A = \sqrt{\frac{\langle \cos(2(\Psi_2^A - \Psi_2^B)) \rangle \langle \cos(2(\Psi_2^A - \Psi_2^C)) \rangle}{\langle \cos(2(\Psi_2^B - \Psi_2^C)) \rangle}}$$

# How is $v_2\{EP\}$ affected by EP decorrelations?




If $\Psi_2$ depends on η

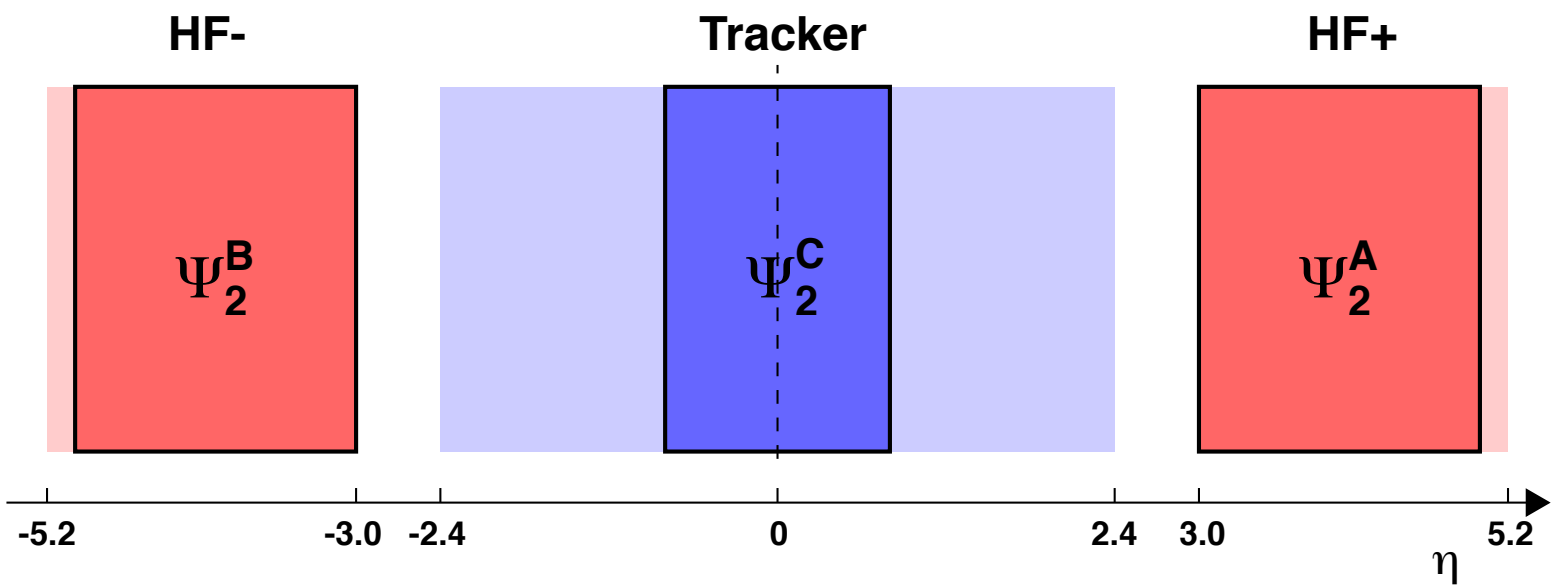
$$R_A = R_A^{res}\sqrt{\frac{\left\langle\cos(2(\Psi_2^+ - \Psi_2^-))\right\rangle\left\langle\cos(2(\Psi_2^+ - \Psi_2^0))\right\rangle}{\left\langle\cos(2(\Psi_2^- - \Psi_2^0))\right\rangle}}$$

$$= R_A^{res}\sqrt{\left\langle\cos(2(\Psi_2^+ - \Psi_2^-))\right\rangle}$$

$$\approx R_A^{res}\left\langle\cos(2(\Psi_2^+ - \Psi_2^0))\right\rangle$$

$\Psi_2^+,\ \Psi_2^-,\ \Psi_2^0$ : real EPs

# How is $v_2\{EP\}$ affected by EP decorrelations?




$$v_2 = \frac{\left\langle \cos 2(\phi - \Psi_2^A) \right\rangle}{R_A}$$

$$\approx \frac{\left\langle \cos 2(\phi - \Psi_2^A) \right\rangle}{R_A^{res}} \frac{1}{\left\langle \cos(2(\Psi_2^+ - \Psi_2^0)) \right\rangle}$$

$$\approx \frac{\left\langle \cos 2(\phi - \Psi_2^+) \right\rangle}{\left\langle \cos(2(\Psi_2^+ - \Psi_2^0)) \right\rangle} \approx \left\langle \cos 2(\phi - \Psi_2^0) \right\rangle$$

EP method measures $v_2$ w.r.t midrapidity EP

# How $r_n(\eta^a,\eta^b)$ is related to factorization and $\Psi_n(\eta)$ fluctuations?

**If $V_{n\Delta}$ factorizes or $\Psi_n(\eta)$ indep. of $\eta$,**

$$r_n(\eta^a,\eta^b) = \frac{\left\langle v_n(-\eta^a)v_n(\eta^b)\right\rangle}{\left\langle v_n(\eta^a)v_n(\eta^b)\right\rangle} = 1 \quad \text{(for symmetric system)}$$

**Otherwise,**

$$r_n(\eta^a,\eta^b) = \frac{\left\langle v_n(-\eta^a)v_n(\eta^b)\cos[n(\Psi_n(-\eta^a)-\Psi_n(\eta^b))]\right\rangle}{\left\langle v_n(\eta^a)v_n(\eta^b)\cos[n(\Psi_n(\eta^a)-\Psi_n(\eta^b))]\right\rangle}$$

$$\sim \frac{\left\langle \cos[n(\Psi_n(-\eta^a)-\Psi_n(\eta^b))]\right\rangle}{\left\langle \cos[n(\Psi_n(\eta^a)-\Psi_n(\eta^b))]\right\rangle} \quad \text{(for symmetric system)}$$

$$\sim \left\langle \cos[n(\Psi_n(\eta^a)-\Psi_n(-\eta^a))]\right\rangle$$

(two EPs separated a gap of $2\eta^a$)

A subtlety in pPb as $\mathrm{v}_n(-\eta^a) \neq \mathrm{v}_n(\eta^a)$

$$r_n(\eta^a,\eta^b) = \frac{\left\langle \mathrm{v}_n(-\eta^a) \times \mathrm{v}_n(\eta^b) \times \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \right\rangle}{\left\langle \mathrm{v}_n(\eta^a) \times \mathrm{v}_n(\eta^b) \times \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \right\rangle}$$

Do not cancel!

Let's take a 'geometric mean'

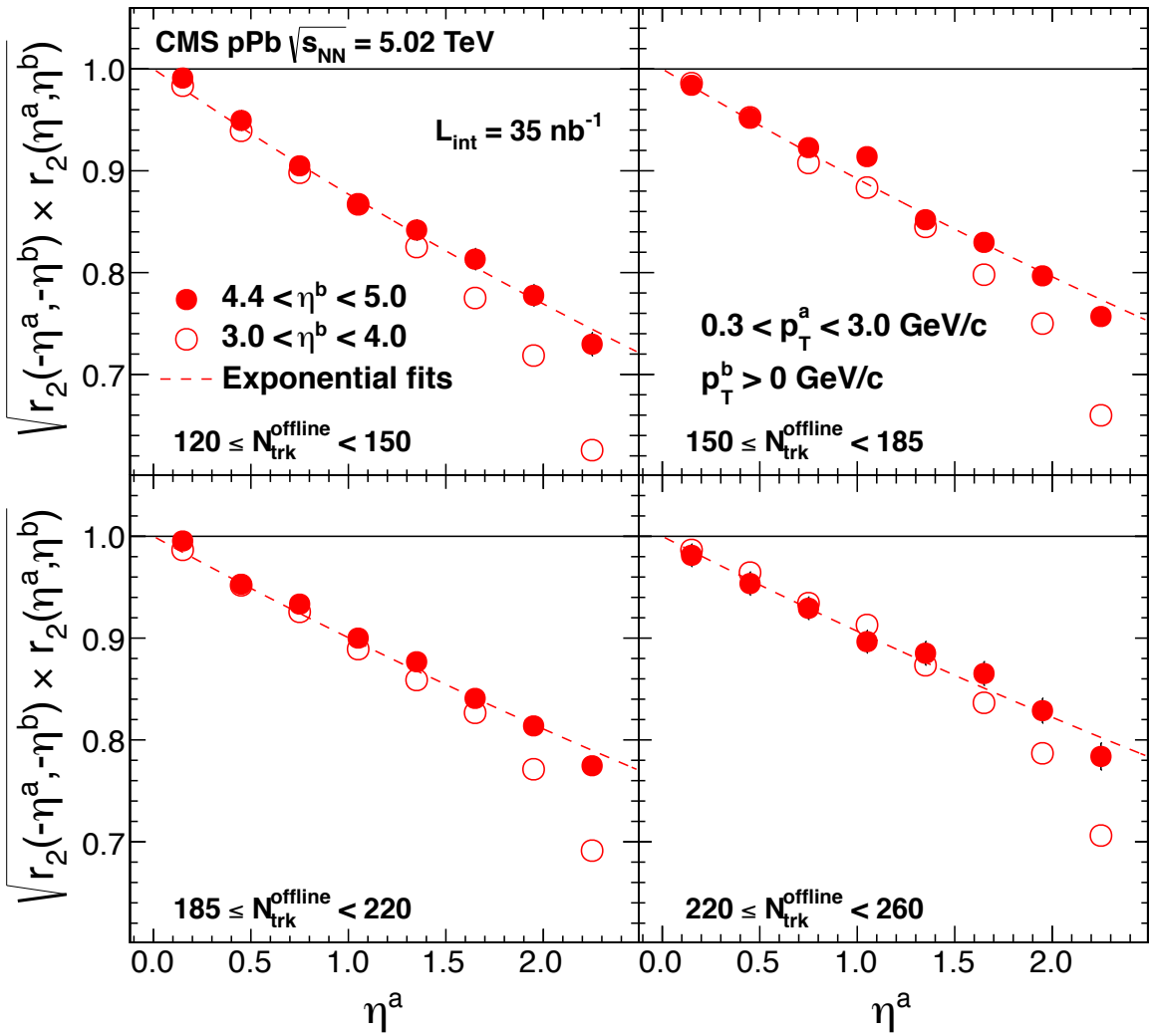
$$\sqrt{r_n(\eta^a,\eta^b) \times r_n(-\eta^a,-\eta^b)} = \sqrt{\frac{V_{n\Delta}(-\eta^a,\eta^b)}{V_{n\Delta}(\eta^a,\eta^b)} \frac{V_{n\Delta}(\eta^a,-\eta^b)}{V_{n\Delta}(-\eta^a,-\eta^b)}}$$

$$= \sqrt{\frac{\left\langle \mathrm{v}_n(-\eta^a)\mathrm{v}_n(\eta^b)\cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \right\rangle}{\left\langle \mathrm{v}_n(\eta^a)\mathrm{v}_n(\eta^b)\cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \right\rangle} \frac{\left\langle \mathrm{v}_n(\eta^a)\mathrm{v}_n(-\eta^b)\cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^b))] \right\rangle}{\left\langle \mathrm{v}_n(-\eta^a)\mathrm{v}_n(-\eta^b)\cos[n(\Psi_n(-\eta^a) - \Psi_n(-\eta^b))] \right\rangle}}$$

$$\sim \sqrt{\frac{\left\langle \cos[n(\Psi_n(-\eta^a) - \Psi_n(\eta^b))] \right\rangle}{\left\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(\eta^b))] \right\rangle} \frac{\left\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^b))] \right\rangle}{\left\langle \cos[n(\Psi_n(-\eta^a) - \Psi_n(-\eta^b))] \right\rangle}} \sim \boxed{\left\langle \cos[n(\Psi_n(\eta^a) - \Psi_n(-\eta^a))] \right\rangle}$$

Drawback: p- and Pb-side averaged ,not differentiable

# Multiplicity dependence of $r_2(\eta^a,\eta^b)$ in PbPb




**Huge effect in pPb!**

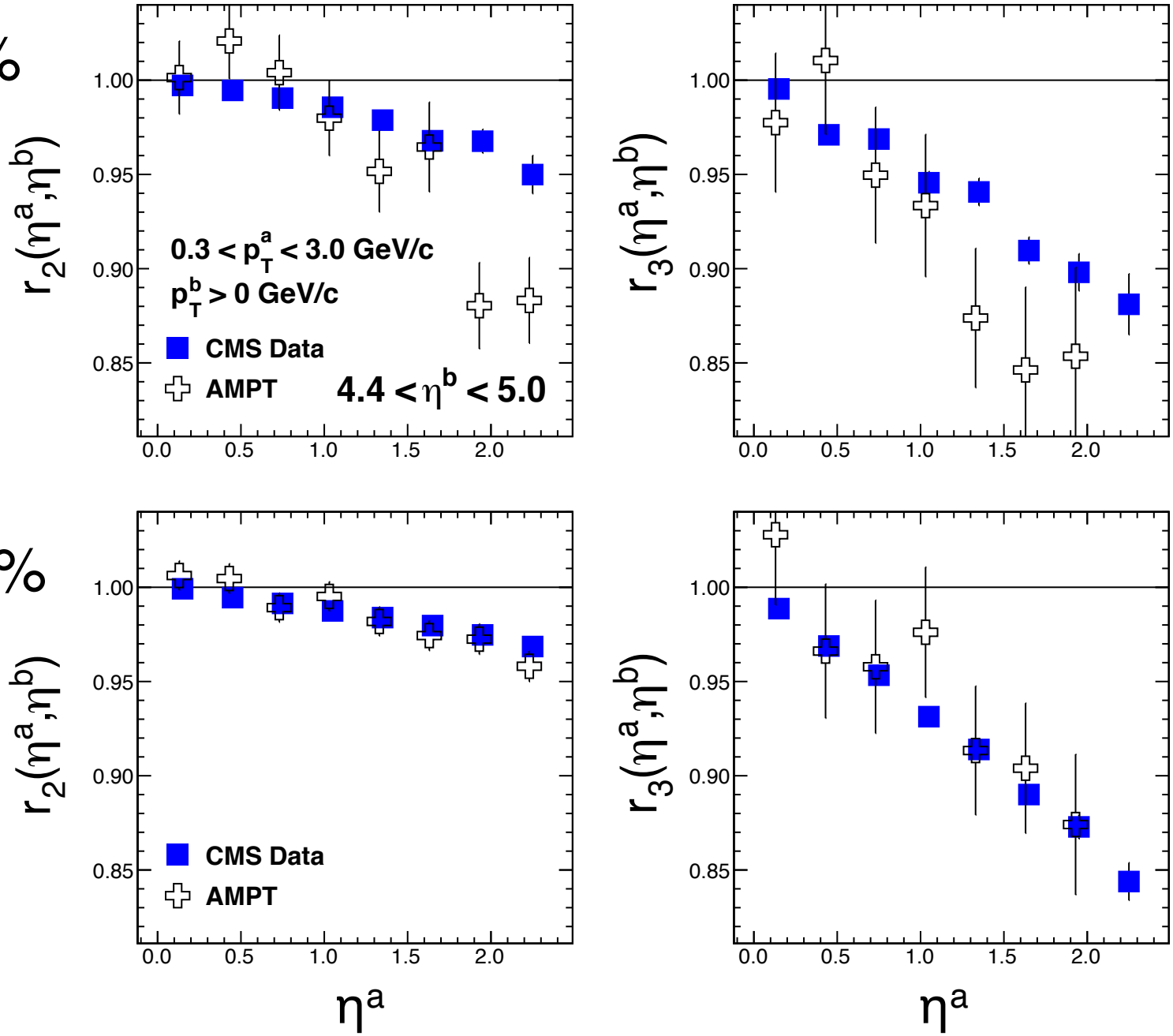
up to 20%

- Intuitively, fluctuations should be larger in pPb
- New constraints on the origin of ridge in pPb

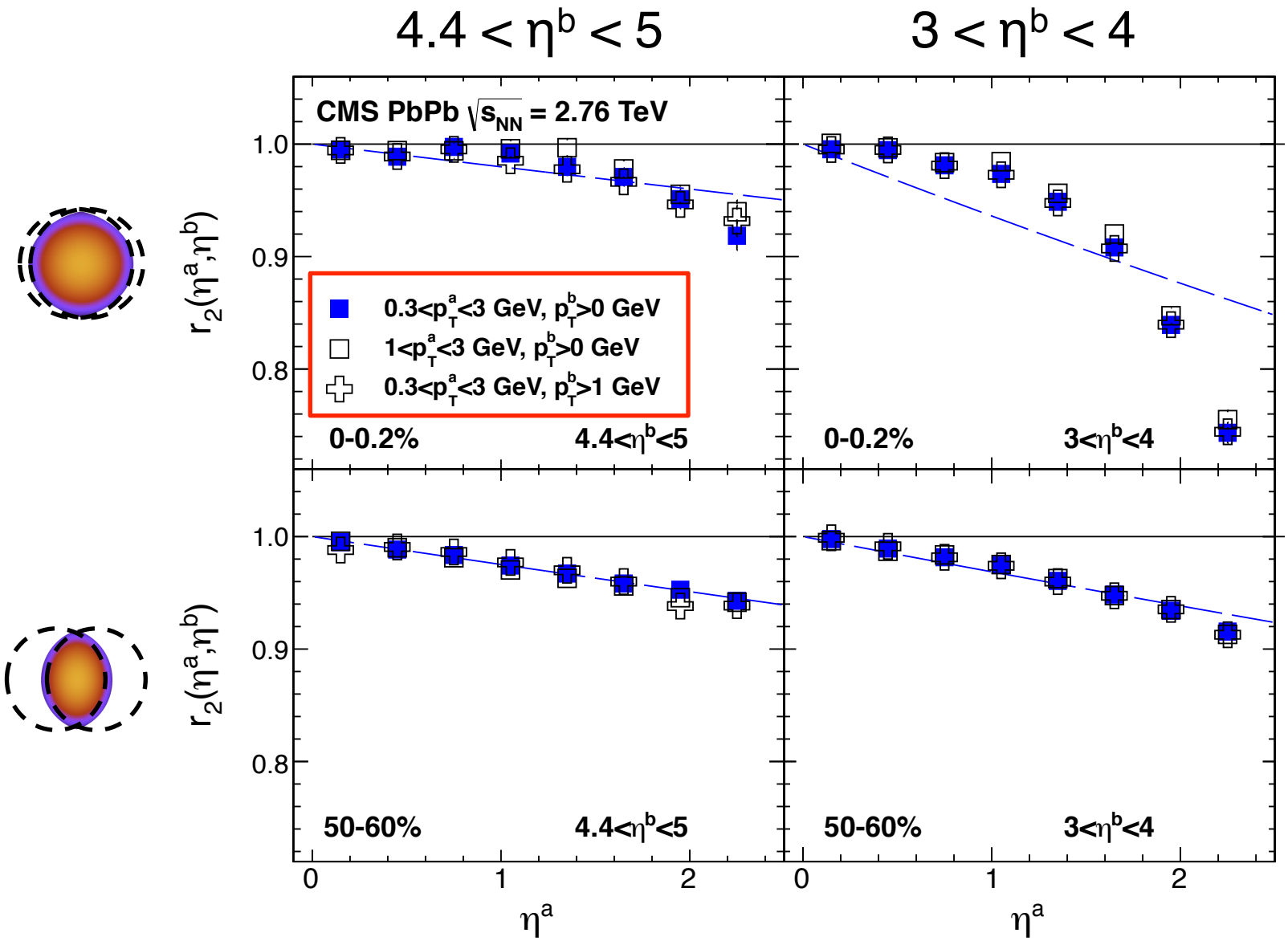
# AMPT comparison

0-5%

10-20%



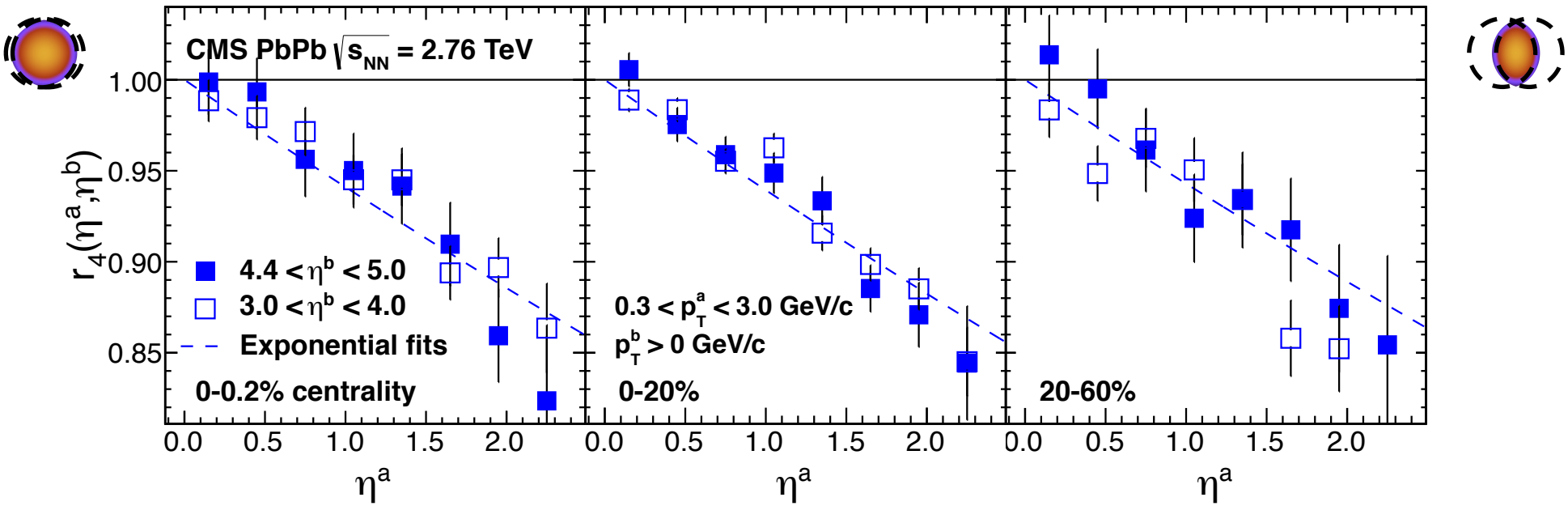
# Nearly no $p_T$ dependence …




Indication of an initial-state effect !?

# Centrality dependence of $r_4(\eta^a,\eta^b)$ in PbPb

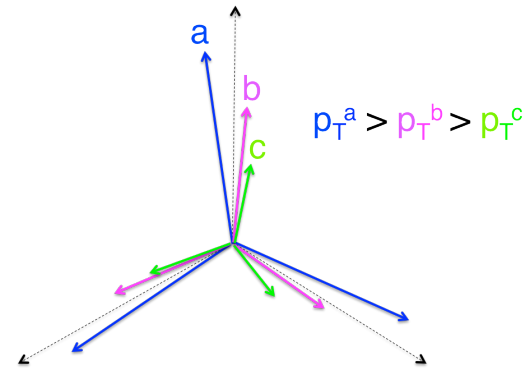
$$r_4(\eta^a,\eta^b) \approx \left\langle \cos[4(\Psi_4(\eta^a) - \Psi_4(-\eta^a))] \right\rangle$$




Also roughly linear increase with η gap

$r_4$ is related to $r_2$, esp. for peripheral events (linear vs non-linear contributions)

# Flow factorization breaking in $p_T$




$$V_{n\Delta}(p_T^a, p_T^b) \neq v_n(p_T^a) \times v_n(p_T^b)$$

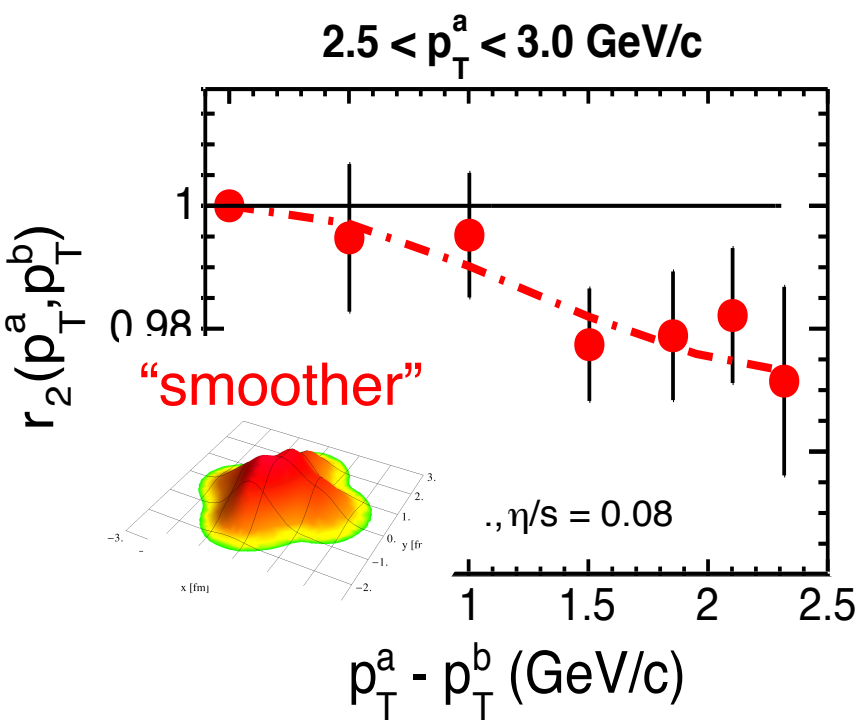
(two-particle) (single-particle)

—— due to EP $\Psi_n(p_T)$, caused by **"lumpy"** initial state

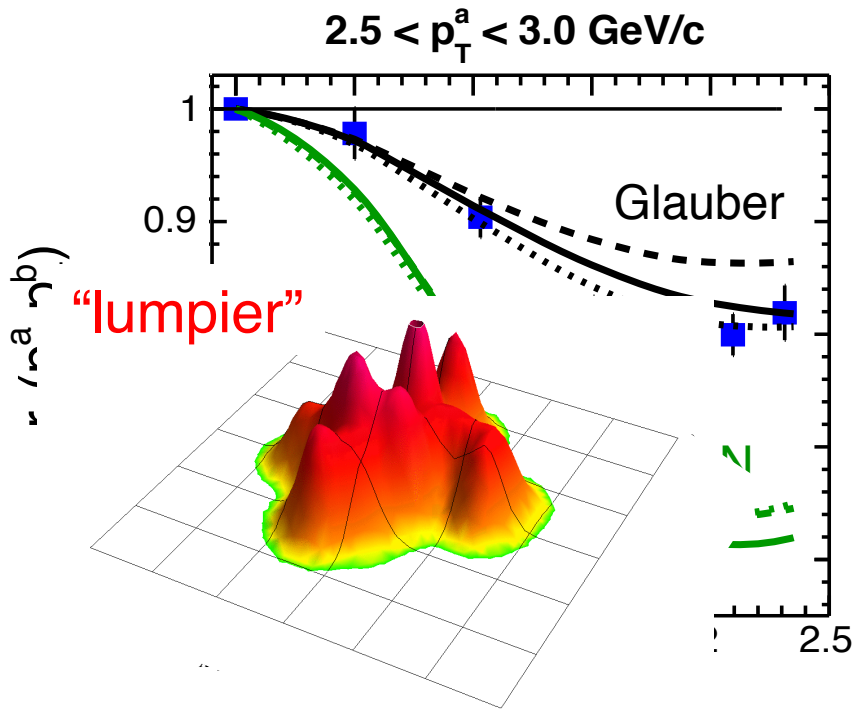
$$r_n(p_T^a, p_T^b) \equiv \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a)}\sqrt{V_{n\Delta}(p_T^b, p_T^b)}} \sim \left\langle \cos[n(\Psi_n(p_T^a) - \Psi_n(p_T^b))] \right\rangle$$

arXiv:1503.01692

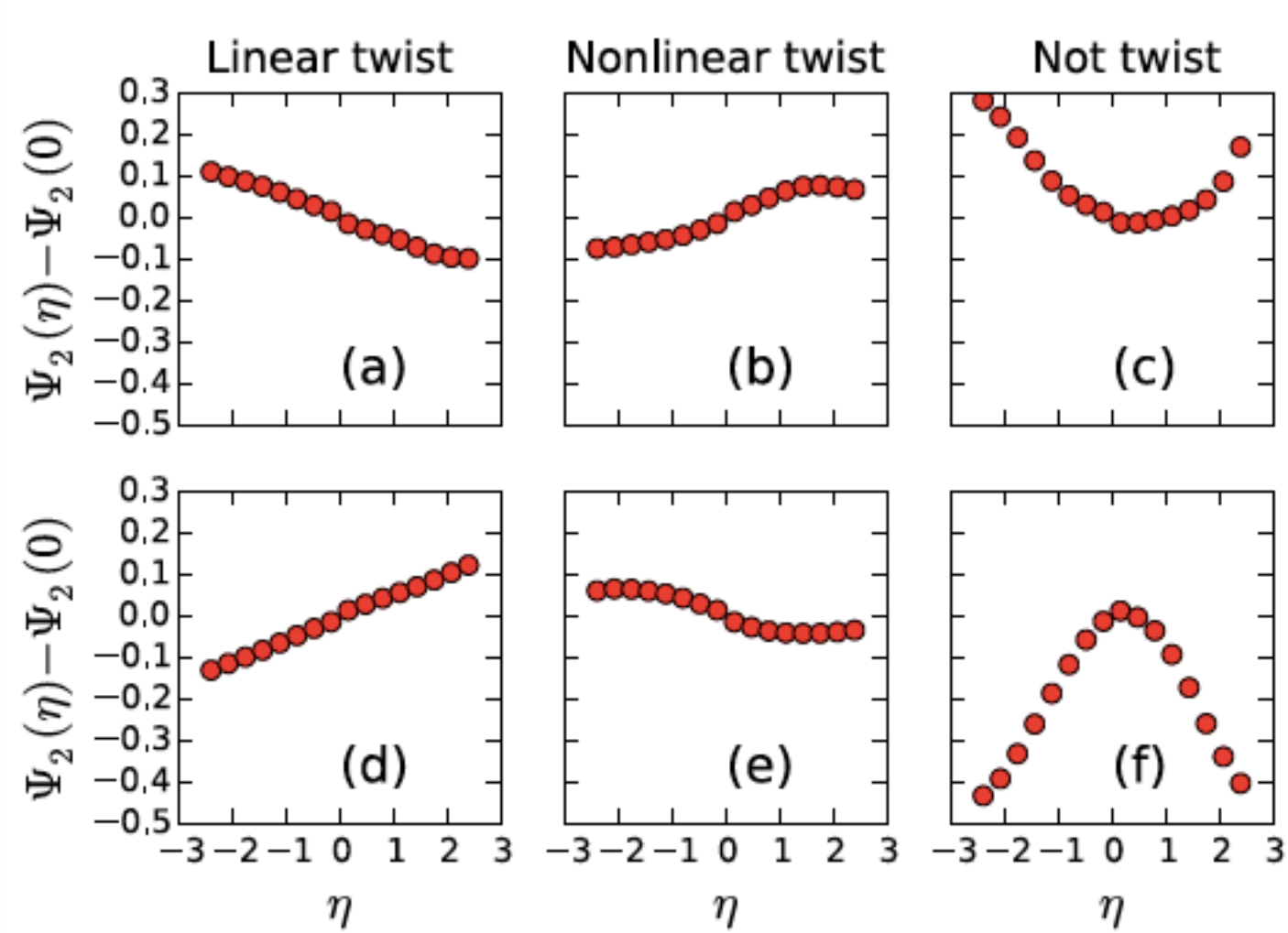
pPb, 220<$N_{trk}$<260




0-0.2% ultra-central PbPb

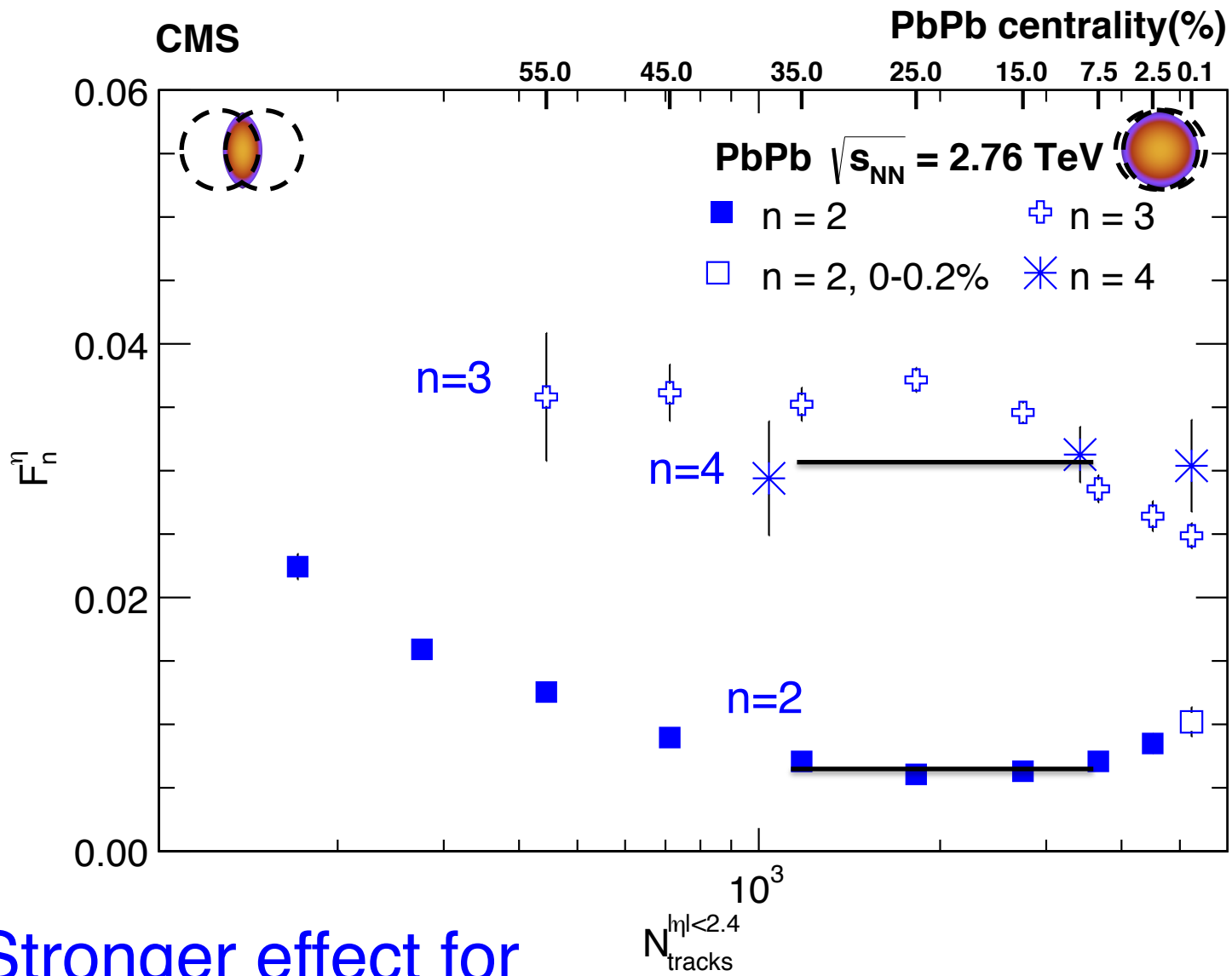


# Twist or random fluctuations (6 typical events)




- $\eta$ dependent event planes from AMPT+Hydro

# Empirical parameterization: $r_n(\eta^a,\eta^b) = 1 - 2F_n^\eta \eta^a$



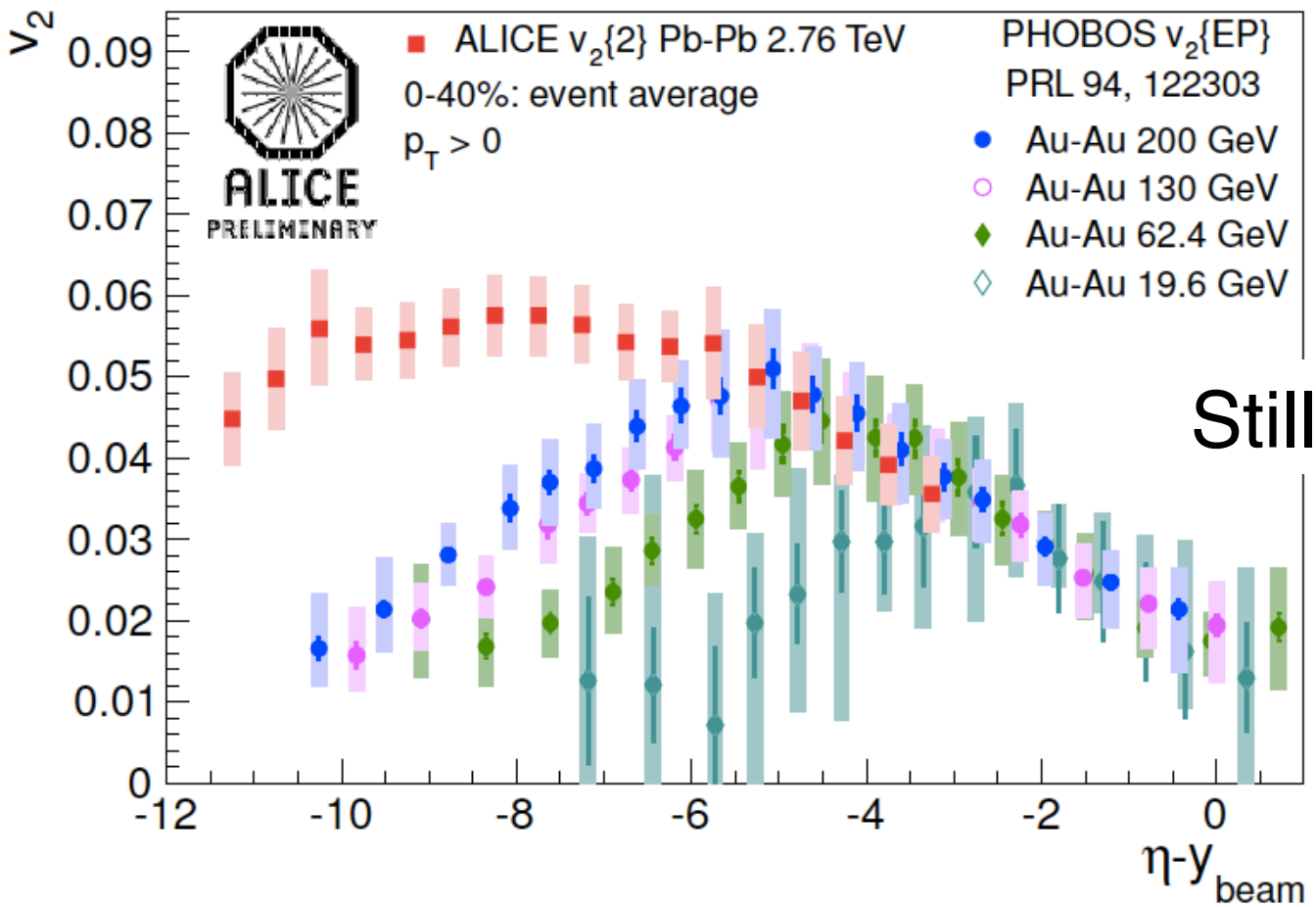

Stronger effect for

- Peripheral events
- higher-order n

$F_4^\eta \approx 4F_2^\eta$ expected from hydro.

# “Extended longitudinal scaling” of $v_2$

“factorization” assumed



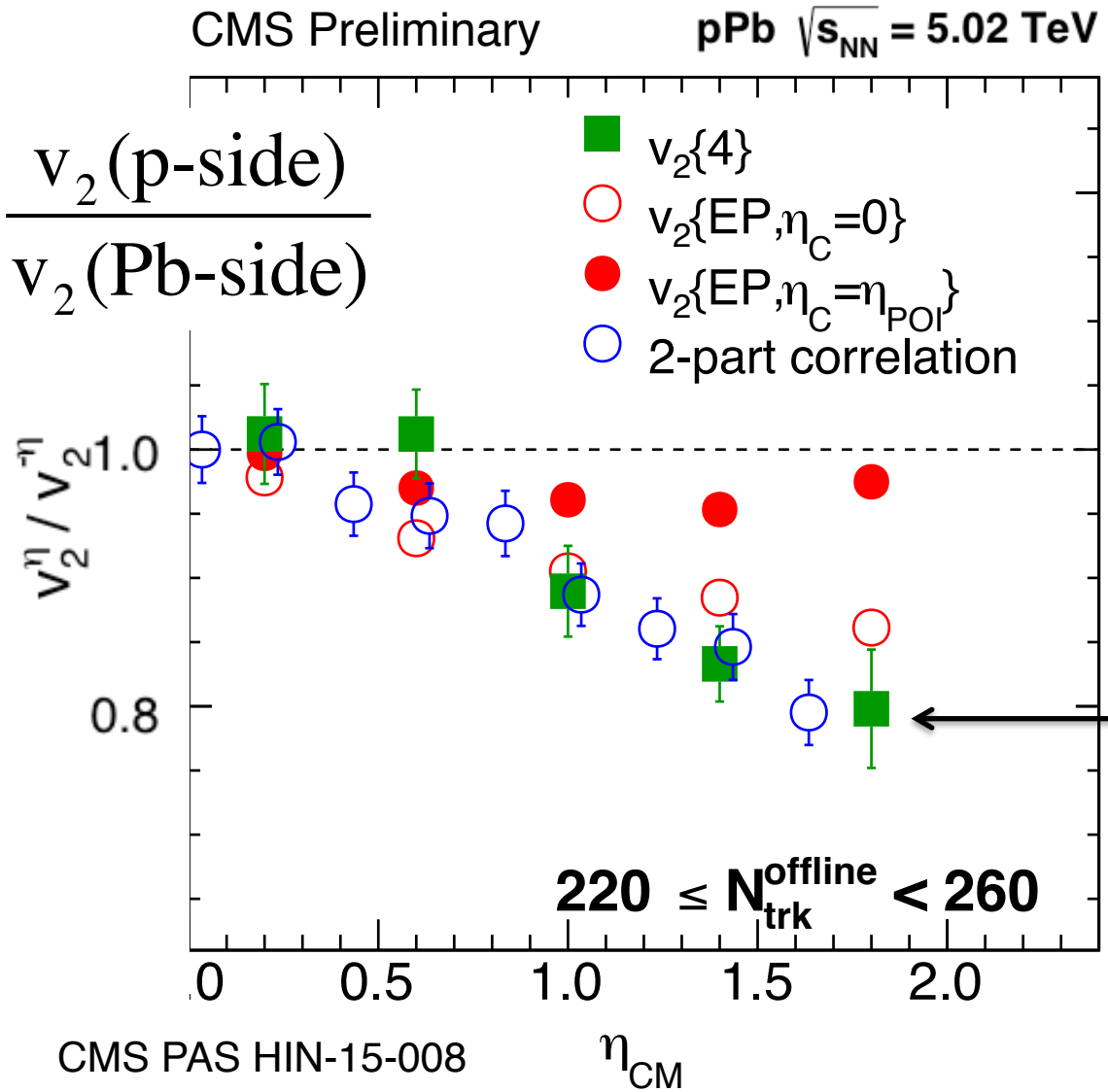

Still a puzzle

Rapidity dependence of

**$v_n$ magnitude** or **$\Psi_n$ orientation**?

(energy density, $\eta/s$) (geometry, initial state)

# η dependence of $v_2$ revisited




CMS PAS HIN-15-008

EP decorrelation also seen in $v_2\{4\}$