# Longitudinal evolution of small-$x$ gluons

## Opportunities for Exploring Longitudinal Dynamics in Heavy Ion Collisions at RHIC

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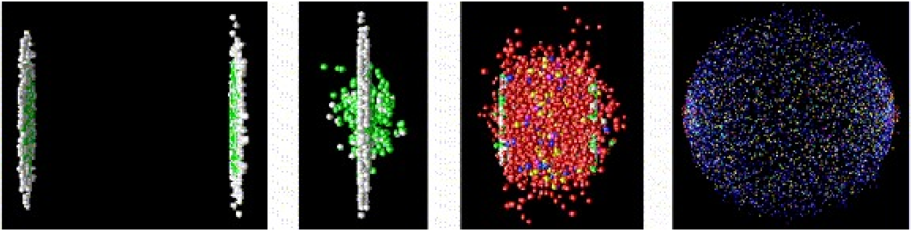
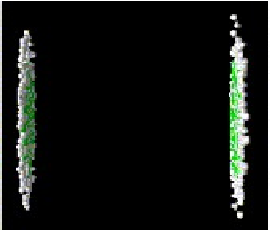
# Standard picture of heavy ion collisions



1. High-density nuclear matter
2. Thermalization
3. Hydrodynamical evolution
4. Freeze out

# High-density nuclear matter



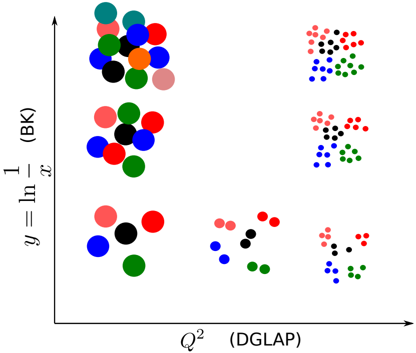
**Kinematics: $x \sim e^y/\sqrt{s}$**

Small-$x$ part of the nuclear wave function controls the initial condition

- Color Glass Condensate

- pA collisions probe initial state more directly

# A short introduction into CGC



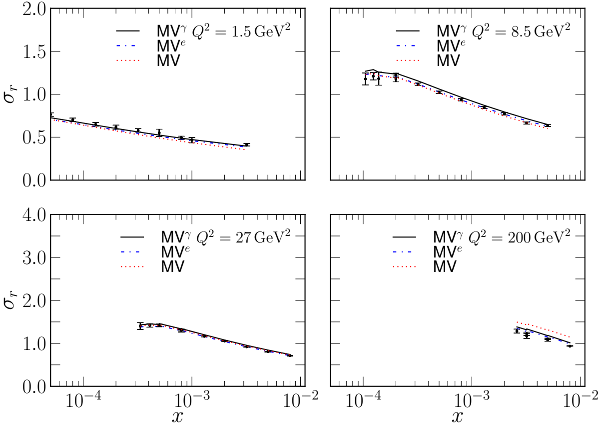

- CGC: QCD at high energies
- QCD favors soft gluon emission $\Rightarrow$ parton density grows at small $x$
- Saturation at high densities

Evolution in $x$ (energy, rapidity) from QCD:

- QCD dynamics included in dipole-target amplitude $N(r_T, x)$
- Evolution in $x$: BFKL/BK/JIMWLK

# CGC at leading order

Fit initial condition for evolution of $N(r_T, x)$ to HERA $F_2$ (or $\sigma_r$) data




T. Lappi, H.M., arXiv:1309.6963

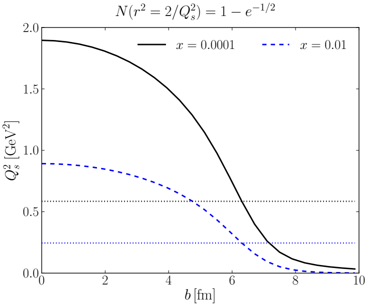
- Small-$x$ evolution in excellent agreement with the data.

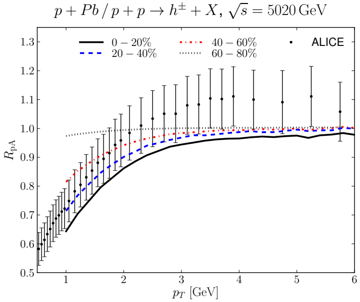
# From protons to nuclei

- Generalize dipole-proton amplitude $N(r_T, x)$ using optical Glauber

$$N^A(r_T, b_T, x_0) = 1 - e^{-AT_A(b_T)\sigma_0 N^p(r_T, x_0)}$$

- Evolve each impact parameter $|b_T|$ independently
- Get $b_T$ dependence of the dipole amplitude, saturation scale $Q_s$, cross sections...

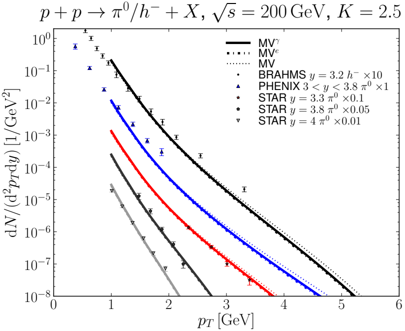



T. Lappi, H.M., arXiv:1309.6963

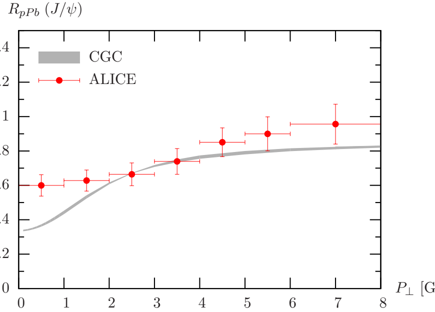
# Good agreement with data, examples

## Single inclusive at RHIC pp



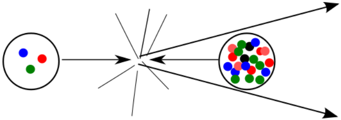

T. Lappi, H.M., arXiv:1309.6963

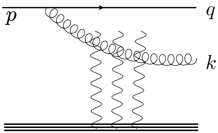
## Inclusive J/Ψ at LHC pA



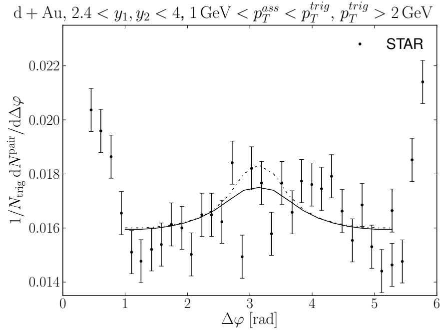

B. Ducloué, T. Lappi, H.M., arXiv:1503.02789

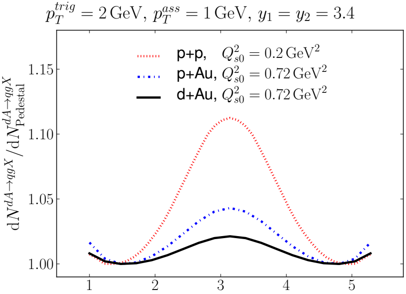
# Two-particle correlations: a clear hint of saturation






- Momentum kick $\sim Q_s \sim A^{1/3}x^{-\lambda}$: back-to-back correlation disappears



# NLO CGC

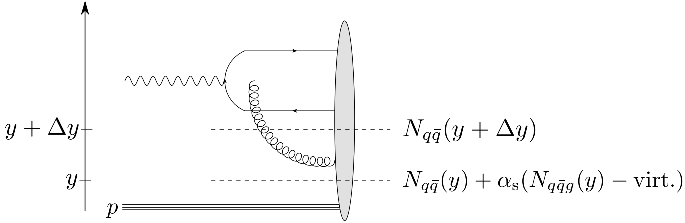
LO CGC calculations are (at least) in qualitative agreement with data.

- Next step: upgrade the saturation picture to NLO accuracy

What is needed:

- **Evolution equation at NLO**
- Cross section at NLO

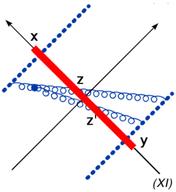
# CGC evolution: BK

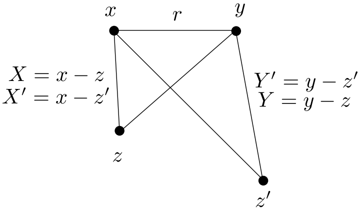



Consider $\gamma^*$ − *hadron* scattering

- $q\bar{q}$ dipole emits a gluon
- The gluon can be counted as a part of
  - $\gamma^*$ wave function
  - Target wave function
- Renormalization group equation for the dipole-target amplitude *N*

# NLO BK equation




$\otimes = \int_{zz'}$

## Balitsky, Chirilli, arXiv:0710.4330 + large-$N_c$:

$$\partial_y S(r) = \frac{\alpha_s}{2\pi^2} K_1 \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 N_c^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$
$$+ \frac{\alpha_s^2 N_f N_c}{8\pi^4} K_f \otimes S(Y)[S(X') - S(X)]$$

$S = 1 - N$

# NLO BK equation, a closer look

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{9} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

$$K_2 = -\frac{2}{(z-z')^4} + \left[\frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4(X^2Y'^2 - X'^2Y^2)} + \frac{r^4}{X^2Y'^2(X^2Y'^2 - X'^2Y^2)} + \frac{r^2}{X^2Y'^2(z-z')^2}\right]\ln\frac{X^2Y'^2}{X'^2Y^2}$$

- Leading order
- Running coupling part ($\sim\beta$): Balitsky prescription in numerics
- Non-conformal double log (diverges at $r \to 0$)

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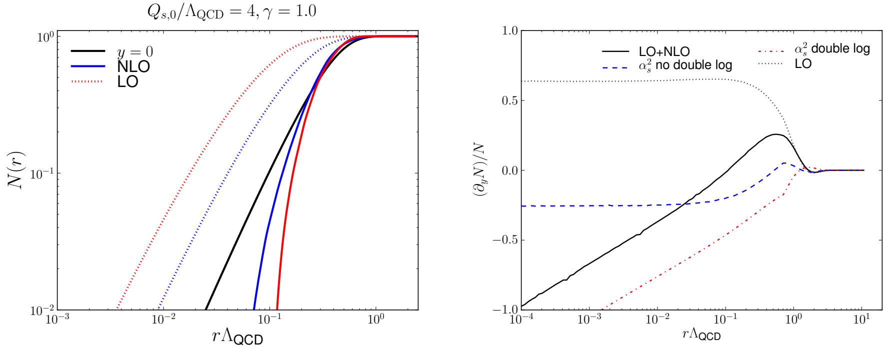
# Solution to NLO BK

Direct numerical solution:

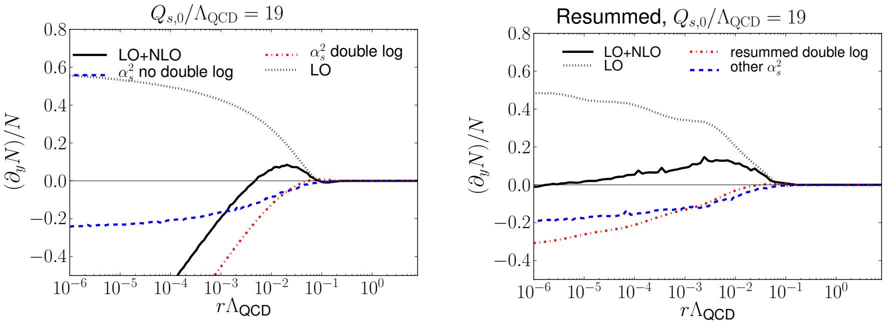



T. Lappi, H.M., arXiv:1502.02400

- Double log $\sim \ln X^2/r^2 \ln Y^2/r^2$ drives the amplitude ($\sim$ UGD) negative
- Negative evolution speed ($\sim$ decreasing UGD when decreasing $x$)

Large double logarithmic corrections:
Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos, arXiv:1502.05642

- Resum gluon emissions that are strongly ordered in momentum
- Removes double log $\ln X^2/r^2 \ln Y^2/r^2$, multiplies LO kernel by an oscillatory factor $K_{\text{resum}}$.
- Partially cures the problem




T. Lappi, H.M., in preparation

# Resummations: single logs

Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos, arXiv:1507.03651

- Large single transverse logarithm $\sim \alpha_s \ln 1/rQ_s$, resum at leading log accuracy
- Source at NLO: $q \to qg$ and $g \to gg$ splittings that are strongly ordered in transverse size

Effect: LO kernel is multiplied by a factor

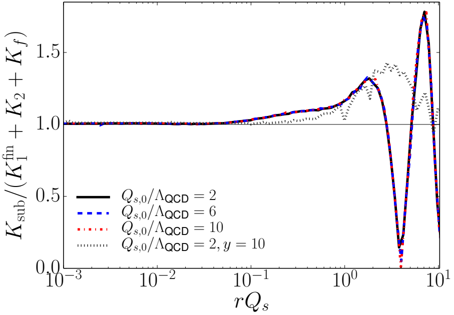
$$K_{\text{STL}} = \left[ \frac{C_{\text{sub}} r^2}{min\{X^2, Y^2\}} \right]^{\pm \bar{\alpha}_s A_1}$$

$\alpha_s^2$ contribution is also included in $K_2$ exactly, subtract it from $K_{\text{STL}}$.

# Subtraction

Resumming logs $\sim \alpha_s \ln 1/rQ_s$ does not fix the constant inside the logarithm $C_{\text{sub}}$.

- Fix $C_{\text{sub}}$ to capture as much as possible of the NLO corrections

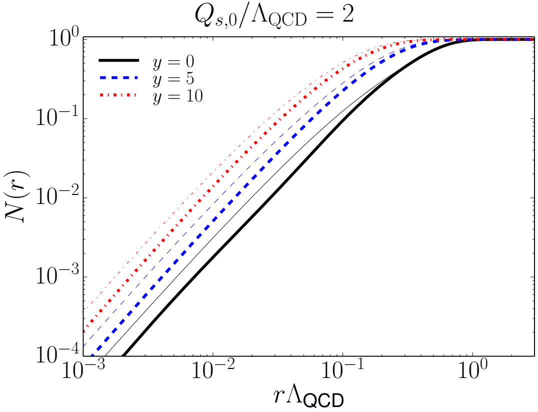



T. Lappi, H.M., in preparation

$$C_{\text{sub}} = 0.65$$

# Results: amplitude

NLO BK with resummations and all terms of the order $\alpha_s^2$
Positive evolution speed and amplitude!

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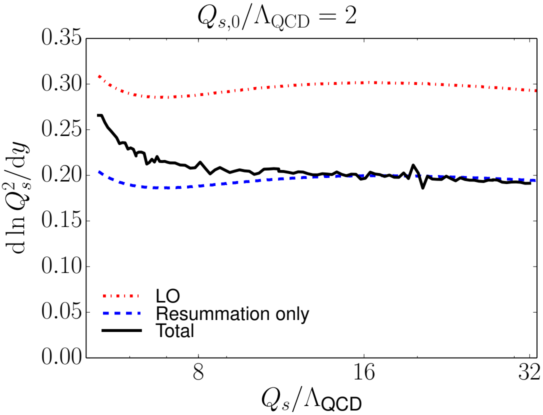
Thin lines: initial condition not resummed

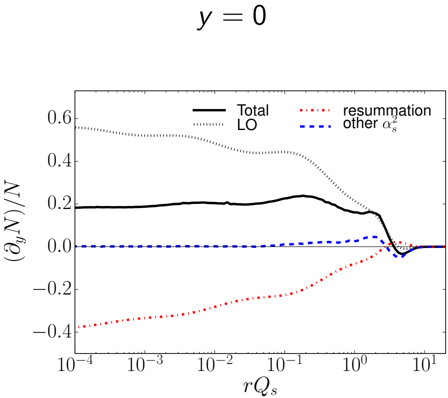
# Results: evolution speed

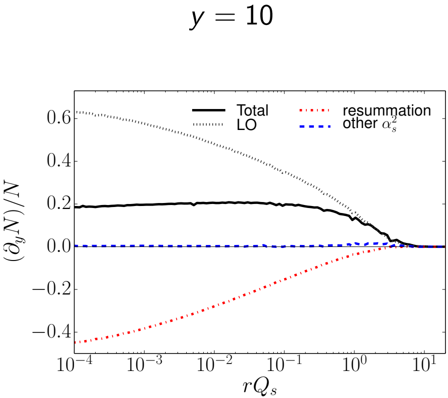



T. Lappi, H.M., in preparation

- Resummation only is good approximation only at very large $Q_s$
- NLO corrections to the small-$x$ evolution are important
- Higher order corrections are negative: good for phenomenology

# Results: contributions to the evolution speed



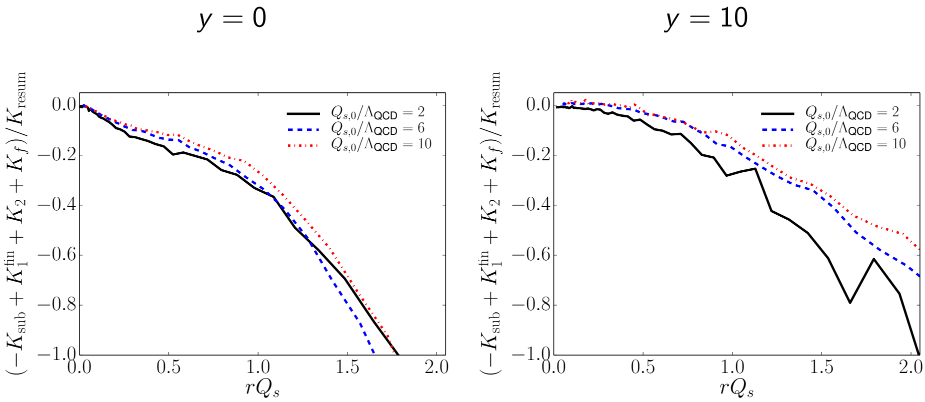




T. Lappi, H.M., in preparation

- Fixed $\alpha_\mathrm{s}^2$ corrections are important around $r \sim 1/Q_s$
- At large rapidities (saturation scales) resummation captures higher order corrections accurately
- Resummation introduces oscillations that are washed out

# Closer look around $r \sim 1/Q_s$

Non-log enhanced NLO contribution compared to leading $\log 1/r$.




- Pure $\alpha_s^2$ terms are important at $r \sim 1/Q_s$

# Conclusions

- Longitudinal evolution of small-$x$ gluons from CGC
- CGC picture is in agreement with current measurements. Want to move beyond LO.
- First solution to the NLO BK equation with
  - Resummation of large double and single logarithms
  - Full NLO contributions
- Unphysical features of fixed $\alpha_{\mathrm{s}}^2$ NLO BK equation are cured by including both resummations
- Fixed order (numerically more demanding to calculate) $\alpha_{\mathrm{s}}^2$ contributions are important at moderate $Q_s$
- At large $Q_s$ the resummation alone is a good approximation

# BACKUPS

# Resummed NLO BK

$$\partial_y S(r) = \frac{\alpha_s}{2\pi^2} K_{\text{Resum}} K_{\text{STL}} K_1 \otimes [S(X)S(Y) - S(r)]$$
$$- \frac{\alpha_s}{2\pi^2} K_1 K_{\text{sub}} \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 N_c^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$
$$+ \frac{\alpha_s^2 N_f N_c}{8\pi^4} K_f \otimes S(Y)[S(X') - S(X)]$$

- $K_{\text{sub}}$: $\alpha_s$ part of $K_{\text{sub}}$, removes double counting

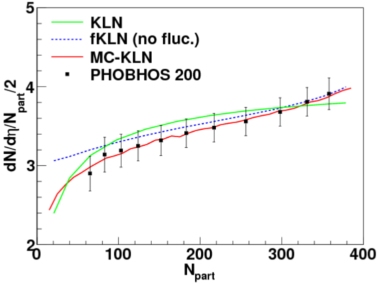
# Connection to hydro

## Example: how to get initial state for hydro (KLN)

$$\frac{\mathrm{d}N_g}{\mathrm{d}y\mathrm{d}^2 p_T} \sim \frac{\alpha_s}{p_t^2} \mathrm{d}^2 k_T \phi_A(x_1, p_T + k_T)\phi_B(x_2, p_T - k_T)$$

UGD $\phi$ can be computed from dipole-target scattering amplitude

- Rapidity $y \sim \ln 1/x$ dependence from CGC evolution of the dipole
- Need initial condition. For protons, fit HERA data.




See also Björn's talk on Friday: IP-Glasma framework.

Albacete, Dumitru, Nara, arXiv:1106.0978