Workshop on P- and CP-odd effects in hot and dense matter, BNL, 26-30.April 2010

Photon Instability in Variable Pion/Axion Condensate : A Possible Hint to PHENIX anomaly

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(in collaboration with D. Espriu)

A. Andrianov, D. Espriu, P. Giacconi and R. Soldati, JHEP 09 (2009) 057
A. Andrianov, D. Espriu, F. Mescia and A. Renau, Phys.Lett. B684 (2010) 101

Photon instability in pseudoscalar background

For slowly growing/decreasing neutral pion condensate

$$\eta_\alpha \simeq \partial_\alpha \langle \Pi \rangle \simeq \delta_{\alpha 0} \langle \dot{\Pi}(t) \rangle$$

or isoscalar theta field $\mu_5 = \partial_0 \theta / (2N_f)$ ← **axial chemical potential**

(in central heavy ion collisions)

Induced C-S term

$$\boxed{\Delta \mathcal{L} = \frac{1}{4} \eta \epsilon^{ijk} A_i F_{jk}} \qquad \eta_\alpha = (\eta, 0, 0, 0)$$

Adiabatic approximation: $\dot{\eta}/f_\pi \ll \eta \ll \omega_\gamma$

in units $1/fm \sim f_\pi$ **time derivative of CS vector << photon frequency**

P-and CP-odd condensates

(*Starting from)* **T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974)**

Isoscalar condensate → theta vacuum bubbles

D. Kharzeev, R. D. Pisarski and M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998)

K. Buckley, T. Fugleberg, and A. Zhitnitsky, Phys. Rev. Lett. 84 (2000) 4814

D. Kharzeev and A. Zhitnitsky, Nucl. Phys. A 797, 67 (2007)

D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008)

Isotriplet neutral condensate =“pion” condensate

A.A.Andrianov and D.Espriu, Phys.Lett. B 663 (2008) 450 (*and refs..therein)*

A.A.Andrianov, V.A.Andrianov and D.Espriu, Phys.Lett. B 678 (2009) 416

Photons of different circular polarizations have different dispersion relation between their frequencies and wave vectors

$$k^{\mu}_{\pm} = (\omega_{\mathbf{k}\pm}, \mathbf{k}) \qquad \omega_{\mathbf{k}\pm} = \sqrt{\mathbf{k}^2 + m_\gamma^2 \pm \eta|\mathbf{k}|}$$

Effective photon masses $k^2_{\pm} = m^2_\gamma \pm \eta\,|\mathbf{k}| \simeq \pm\eta\,|\mathbf{k}|$

Photon "-" is a tachyon

Photon "+"decays $\gamma \to l^+ l^-$

for $m_\gamma \ll m_e$

$$|\mathbf{k}| \geq \frac{4m_l^2}{\eta} \equiv k_{\rm th}$$

Threshold hierarchy!

If for electrons/positrons the threshold is of order 100 MeV then for muons it is four orders of magnitude higher , i.e. 1 TeV !

No muon pairs excess in the PHENIX data!? Maybe in NA60?

Five-dimensional interpretation

5-dim abelian gauge field $(A_B) = (A_0, ..., A_3, A_5) \equiv (A_\mu, A_5)$

5-dim Chern-Simons interaction $\mathcal{L}^5_{CS} = c_5 \int d^5x \epsilon^{BCDEF} A_B \partial_C A_D \partial_E A_F$

KK or brane reduction

$$A_B(X_C) \Longrightarrow A_B(x_\mu) \qquad \Longrightarrow \tilde{c}_5 \int d^4x \epsilon^{\mu\nu\sigma\rho 5} A_\mu \partial_\nu A_\sigma \partial_\rho A_5$$

Time dependent fifth component

$$A_5(x_\mu) \Longrightarrow A_5(t) \equiv \theta(t) \qquad \Longrightarrow \int d^4x \, \frac{1}{2} \eta \, \epsilon^{jkl} A_j \partial_k A_l$$

induces electric field $\partial_0 A_5 = E_5 = \dot{\theta}$

Pair creation by a 5-dim Schwinger-like mechanism!

Polarizations $A = T, L, +, -$

$$\varepsilon_T^\mu(k) \equiv \frac{\mathrm{i}\, k^\mu}{\sqrt{k^2}} \qquad (k^2 > 0),$$

$$\varepsilon_L^\mu(k) \equiv (k^2 \mathrm{D})^{-1/2} \left(k^2 \eta^\mu - k^\mu \eta \cdot k\right) \qquad (k^2 > 0)$$

(Almost) circular polarizations of distorted photons

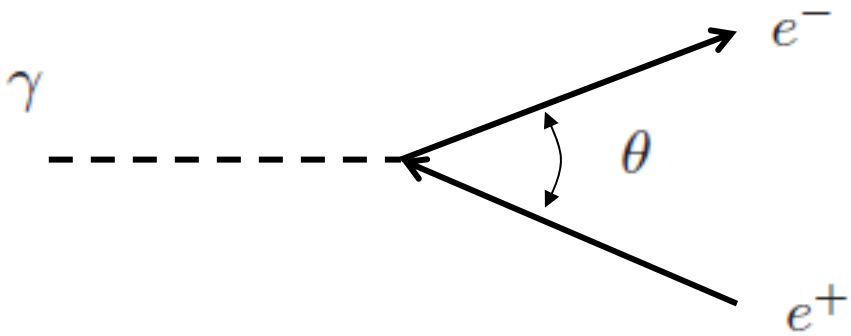
$$\varepsilon_\pm^\mu(k) \equiv \left[\frac{\mathbf{k}^2 - (\epsilon \cdot k)^2}{2\mathbf{k}^2}\right]^{-1/2} P_\pm^{\mu\nu}\, \epsilon_\nu$$

$$P_\pm^{\mu\nu} \equiv \frac{S^{\mu\nu}}{S} \pm \frac{\mathrm{i}}{\sqrt{S/2}} \varepsilon^{\mu\nu\alpha\beta} \eta_\alpha k_\beta \qquad S_\lambda^\nu \equiv \varepsilon^{\mu\nu\alpha\beta} \eta_\alpha k_\beta \varepsilon_{\mu\lambda\rho\sigma} \eta^\rho k^\sigma$$

Polarized decay !! $\gamma \to l^+ l^-$

Kinematics

$$\omega_{\mathbf{k}\pm} = \sqrt{\mathbf{k}^2 + m_\gamma^2 \pm \eta\,|\mathbf{k}|} = \sqrt{\mathbf{p}^2 + m_e^2} + \sqrt{(\mathbf{p}-\mathbf{k})^2 + m_e^2}$$



$$\sin^2\theta \leq \frac{\eta^2}{4m_e^2}\left(1 - \frac{4m_e^2}{\eta k}\right)$$

Narrow cone $\theta_{\max} < \eta/2m_l \qquad \eta \ll 2m_l$

For muons it is plausible but for e^+e^- **??**

Bounds on lepton momenta

for $k \gg k_{\rm th}$

$$\min p_- \simeq \frac{m_e^2}{\eta} \leq p \leq \max p_+ \simeq k - \frac{m_e^2}{\eta}$$

Asymmetric!

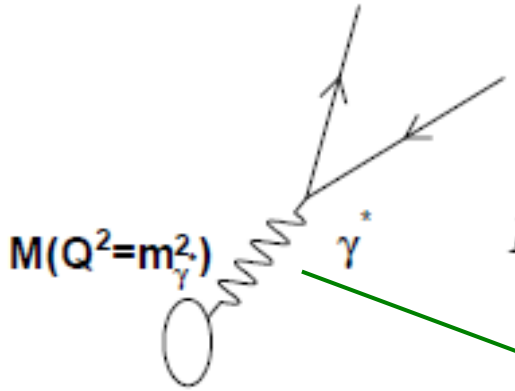
Decay width

$$\Gamma_+ = \tau_+^{-1} \simeq \frac{\alpha\eta}{3}$$

Constant at high energies !

But the distorted photon is not a proper *Breit-Wigner* resonance as its position (effective mass) moves with momentum!

Dilepton pair creation



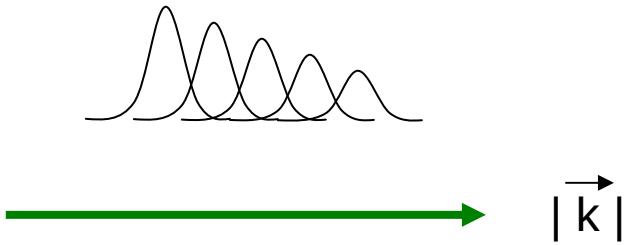
$$M \equiv m_{ee} \equiv m_{\gamma^*}$$

$$\frac{d^2 N_{ee}}{dM^2} = \frac{\alpha}{3\pi}\frac{L(M)}{M^2} dN_{\gamma^*},$$

$$L(M) = \sqrt{1-\frac{4m_e^2}{M^2}}\left(1+\frac{2m_e^2}{M^2}\right)$$

"Giant" resonance
with variable position

$$\frac{1}{\pi}\Theta\left(|\vec{k}| - \frac{4m_e^2}{\eta}\right)\frac{2\Gamma_+\omega_+(|\vec{k}|)}{(M^2-\eta|\vec{k}|)^2 + 4\Gamma_+^2\omega_+^2(|\vec{k}|)}$$



$$\Gamma_+ = \tau_+^{-1} \simeq \frac{\alpha\eta}{3} \qquad \omega_+(|\vec{k}|) = \sqrt{\vec{k}^2 + \eta|\vec{k}|}$$

$$\sim 1/\Gamma_+\omega_+(|\vec{k}|) \sim 1/\alpha M^2 \qquad k \gg \eta$$

"On-shell" enhancement in a range of $|\vec{k}|$!

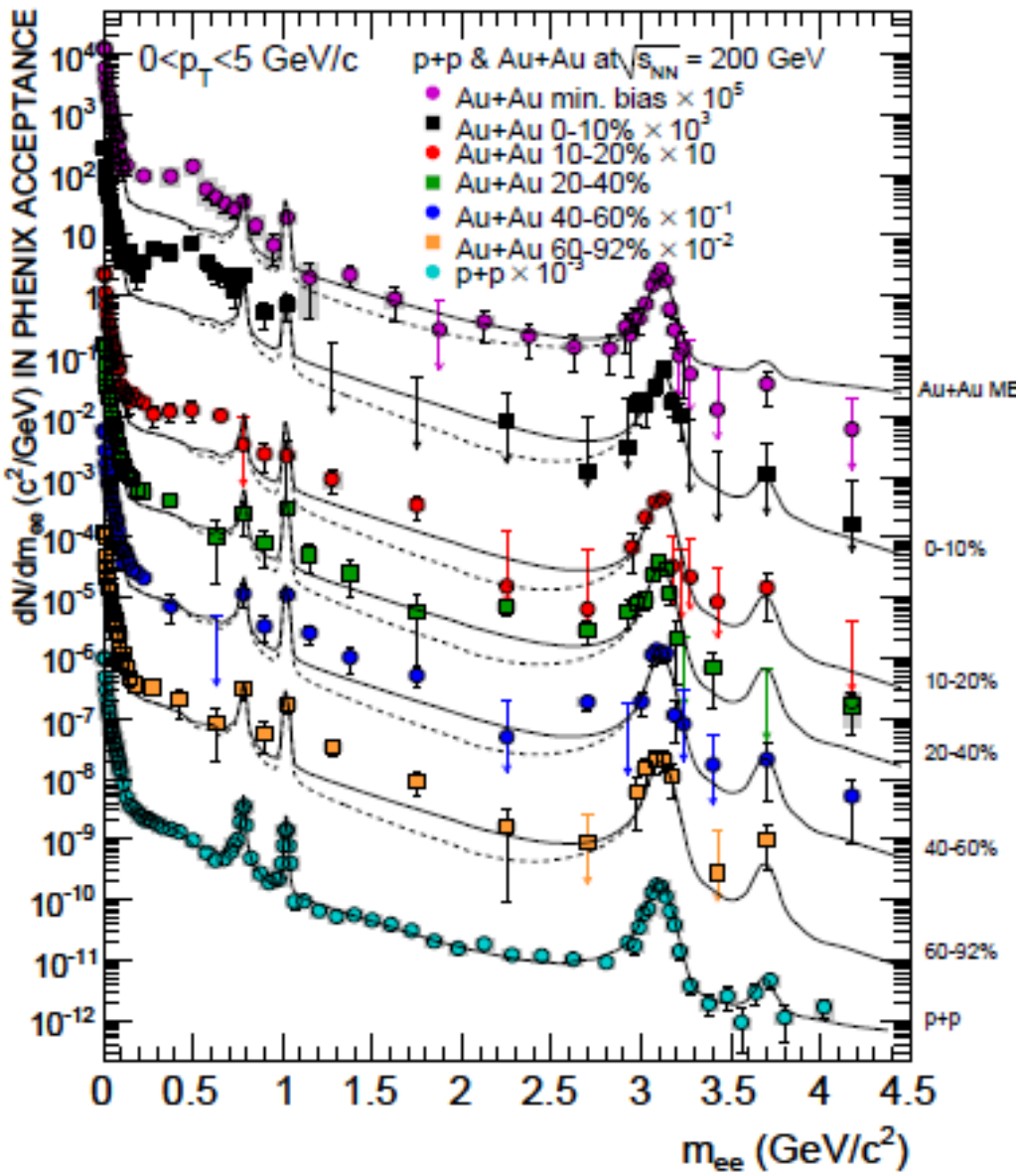
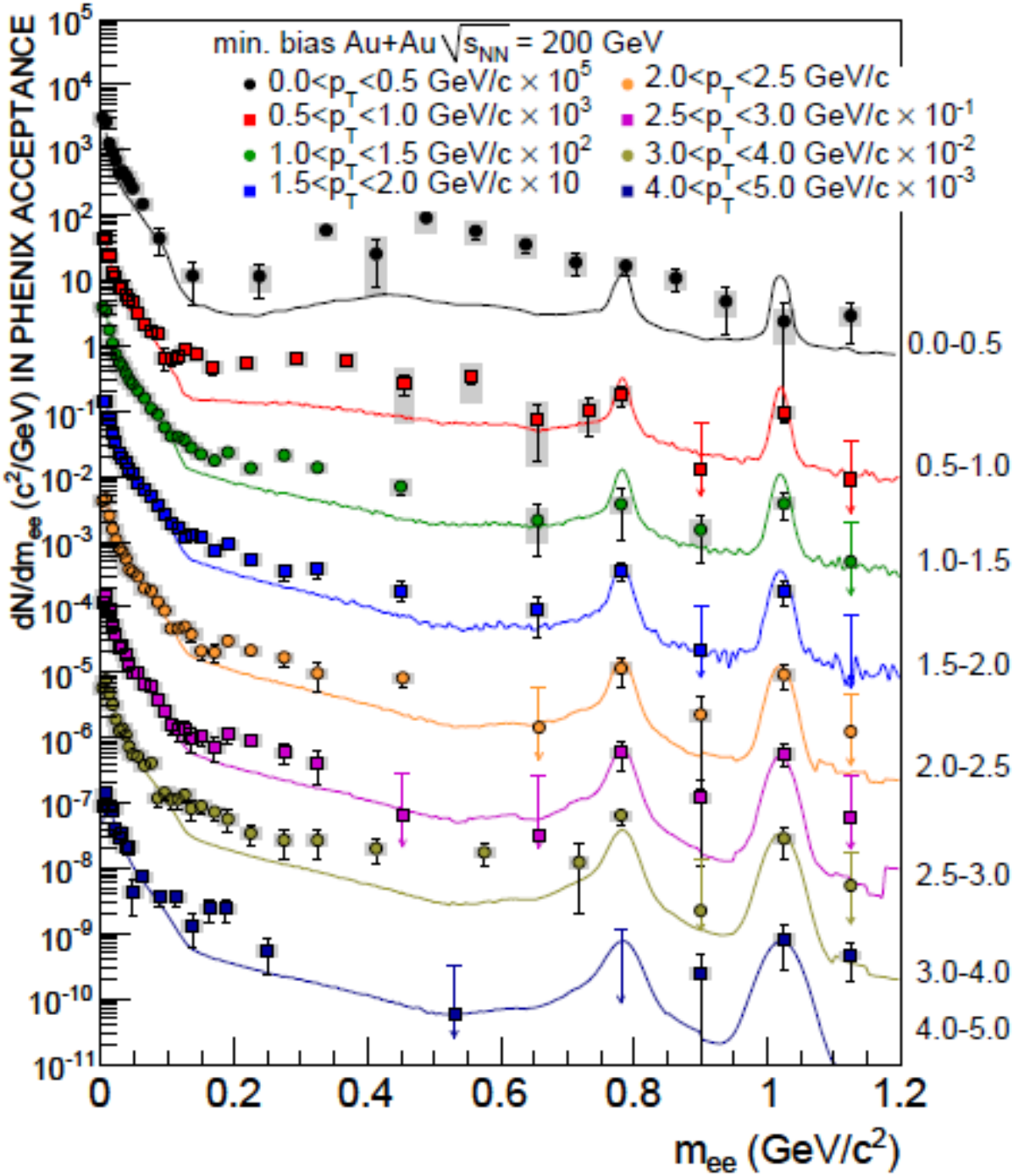


FIG. 27: (Color online) Invariant mass spectrum of e^+e^- pairs inclusive in p_T compared to expectations from the model of hadron decays for $p+p$ and for different Au + Au centrality classes.

From: 0912.0244v1 [nucl-ex]
(PHENIX Collaboration)

TABLE IX: The enhancement factor, defined as the ratio between the measured yield and the expected yield for $0.15 < m_{ee} < 0.75$ GeV/c^2, for different centrality bins. The meaning of the errors is defined in the text.

Centrality	Enhancement (±stat ±syst ±model)
00-10 %	7.6 ± 0.5 ± 1.5 ± 1.5
10-20 %	3.2 ± 0.4 ± 0.1 ± 0.6
20-40 %	1.4 ± 1.3 ± 0.02 ± 0.3
40-60 %	0.8 ± 0.3 ± 0.03 ± 0.2
60-92 %	1.5 ± 0.3 ± 0.001 ± 0.3
Min. Bias	4.7 ± 0.4 ± 1.5 ± 0.9


min. bias Au+Au $\sqrt{s_{NN}}$ = 200 GeV
0.0<p_T<0.5 GeV/c × 10^5
0.5<p_T<1.0 GeV/c × 10^3
1.0<p_T<1.5 GeV/c × 10^2
1.5<p_T<2.0 GeV/c × 10
2.0<p_T<2.5 GeV/c
2.5<p_T<3.0 GeV/c × 10^{-1}
3.0<p_T<4.0 GeV/c × 10^{-2}
4.0<p_T<5.0 GeV/c × 10^{-3}
0.0-0.5
0.5-1.0
1.0-1.5
1.5-2.0
2.0-2.5
2.5-3.0
3.0-4.0
4.0-5.0
dN/dm_{ee} (c^2/GeV) IN PHENIX ACCEPTANCE
m_{ee} (GeV/c^2)

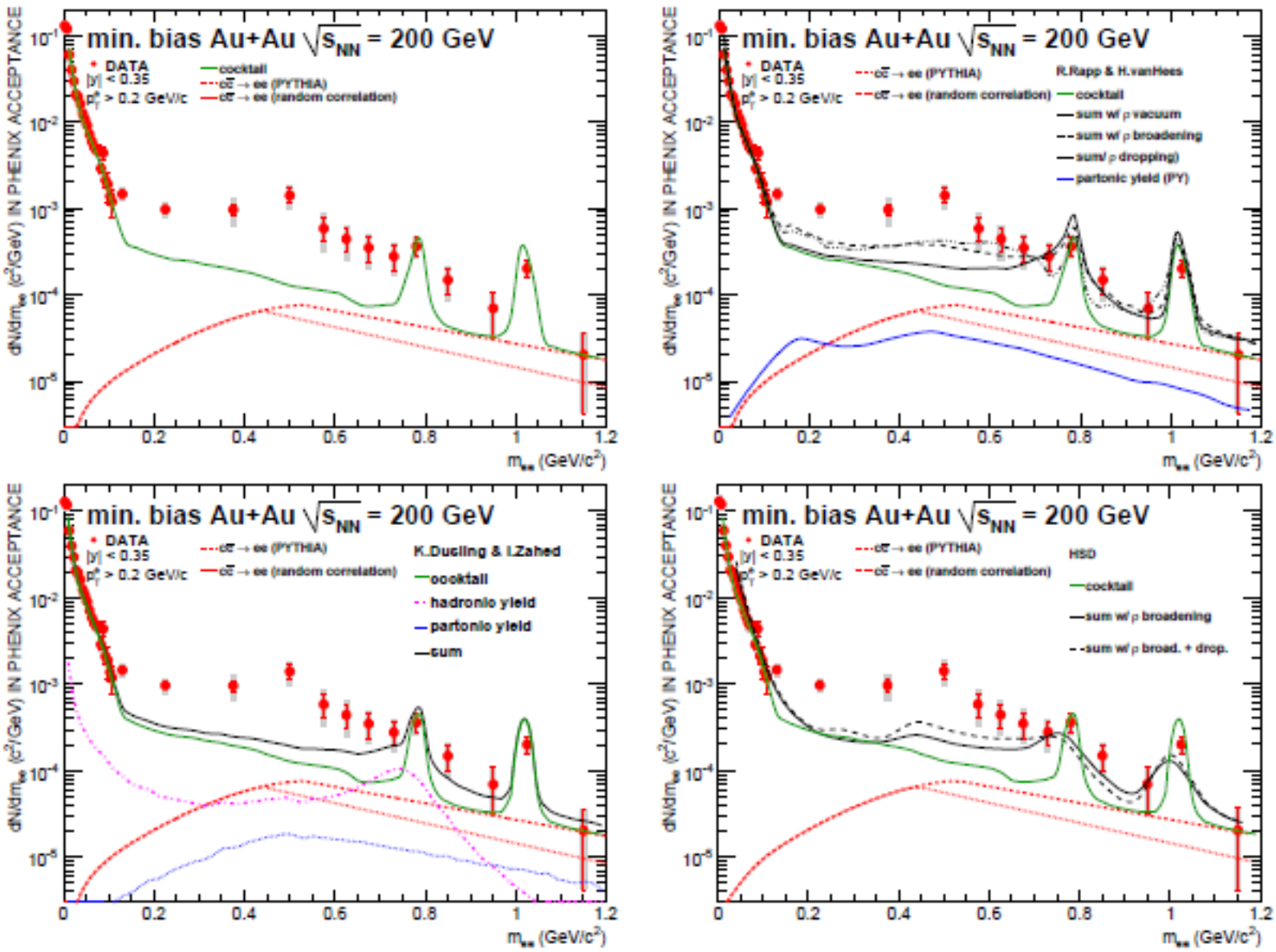


FIG. 42: (Color online) Invariant mass spectra of e^+e^- pairs in Au + Au collisions in the LMR. The data are compared to the sum of cocktail+charm (top left). The data are also compared to the sum of cocktail+charm and hadronic+partonic contributions from different models. The calculations are from The calculations are from (top right) Rapp and van Hees [15, 18, 83], (bottom right) Dusling and Zahed [19, 84, 85], and Cassing and Bratkovskaya [20, 27, 86, 87].

This is an old puzzle!

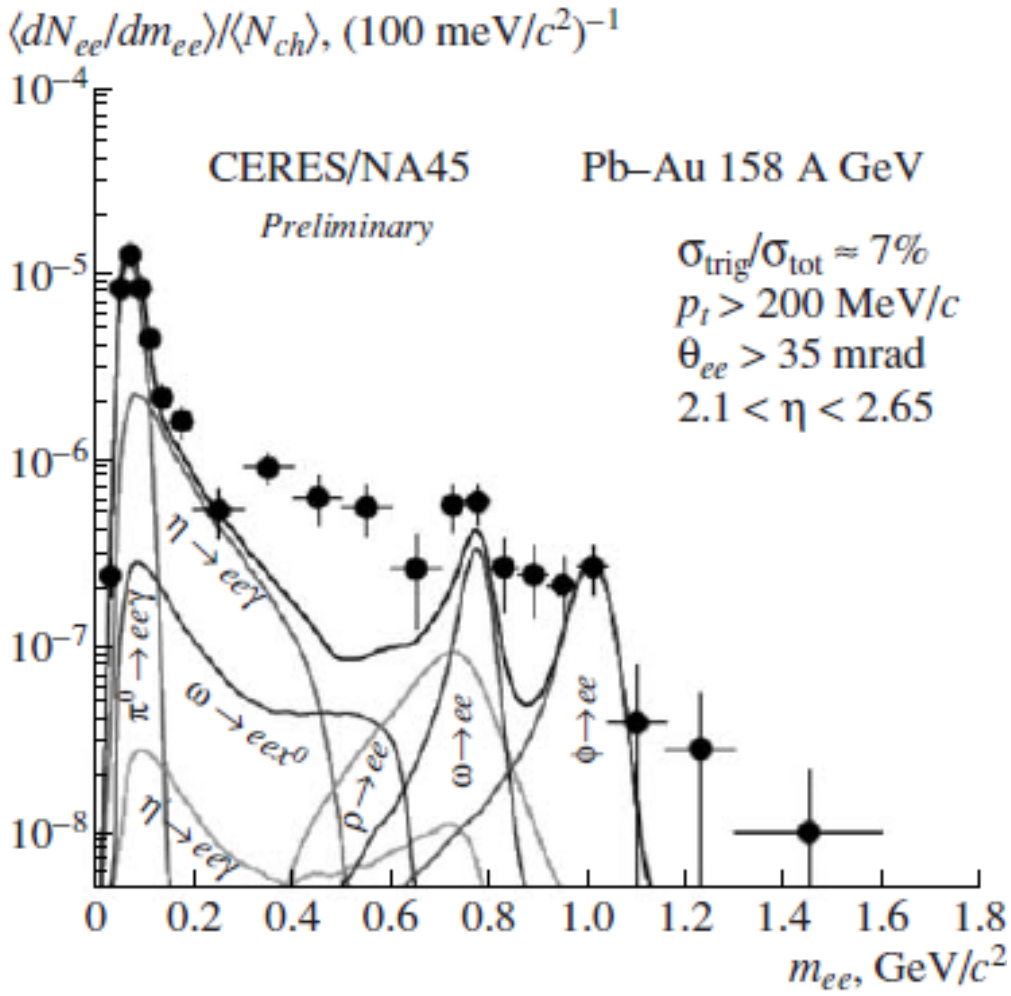
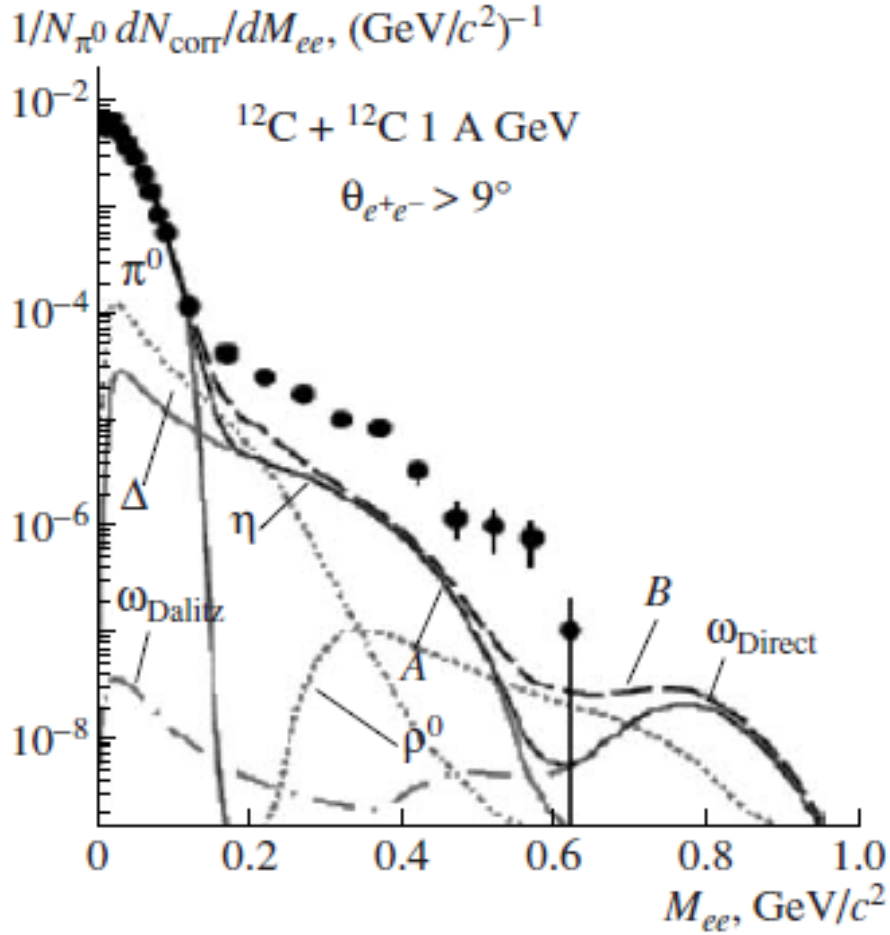


Fig. 2. CERES experiment data collected in 2000 [17].



HADES-experiment data on the dilepton yield

From K. O. Lapidus and V. M. Emel'yanov, Phys. Part. Nucl. 40 (2009) 29

In order to derive the value of η one has to resolve

Mixing with vector mesons $(V_{\mu,a}) \equiv (A_\mu,\, \omega_\mu,\, \rho_\mu)$

In the lagrangian $\mathcal{L}_{mixing}(k) \;=\; \eta\, \epsilon_{jkl}\, V_{j,a}\, N_{ab}\, \partial_k V_{j,b}$

coupling constants N_{ab} are known from the anomaly or, independently, can be derived from radiative decays $\pi_0 \to \gamma\gamma$ $\omega \to \pi_0\gamma$ $\rho_0 \to \pi_0\gamma$

One can, in principle, disentangle the isospin of pseudoscalar condensate:

for isoscalar $\eta \sim \langle\dot{\theta}\rangle$

$$(N_{ab}) \;\simeq\; c_m \begin{pmatrix} \frac{5}{9}e^2 & \frac{1}{6}eg & \frac{1}{2}eg \\ \frac{1}{6} & g^2 & 0 \\ \frac{1}{2}eg & 0 & g^2 \end{pmatrix}$$

for pion condensate $\eta \sim \langle\dot{\pi}_0\rangle$

$$(N_{ab}) \;\simeq\; c_m \begin{pmatrix} \frac{1}{3}e^2 & \frac{1}{2}eg & \frac{1}{6}eg \\ \frac{1}{2} & 0 & g^2 \\ \frac{1}{6}eg & g^2 & 0 \end{pmatrix}$$

where coupling constants

γ	ω	ρ
e	$g_\omega \simeq$	$g_\rho \equiv g$

Mass-shell for transversal polarizations

$$\left[P_\perp^{\mu\nu}(k)\left(k^2\delta_{ab} - (\hat{m}^2)_{ab}\right) + i\, c_m\, \epsilon^{\mu\nu\sigma\rho}\eta_\sigma k_\rho N_{ab}\right] V_{\nu,b}(k) \;=\; 0 \qquad \hat{m}^2 \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_\omega^2 & 0 \\ & 0 & m_\rho^2 \end{pmatrix}$$

Estimation of C.-S. coupling in dense (and hot) nuclear/quark matter

$$\eta \sim \frac{\alpha}{2\pi}\frac{\dot{\varrho}_B}{f_\pi}\frac{\partial\langle\Pi\rangle}{\partial\varrho_B}$$

(+ temperature when it is relevant)

from nuclear evolution

from a model of P-breaking

f_π in-medium pion decay constant

ϱ_B time-dependent baryon density

$\langle\Pi\rangle(\varrho_B)$ density dependent P-breaking condensate (isosinglet $\langle\theta\rangle$ or isotriplet $\langle\pi_0\rangle$)

And vice versa:
when measuring η one can reconstruct the density dependence with a simulated time evolution of density

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Crude estimation (order of magnitude)

$$\eta \sim \alpha\dot{\theta}/f_\pi \sim \alpha f_\pi \sim 1MeV$$

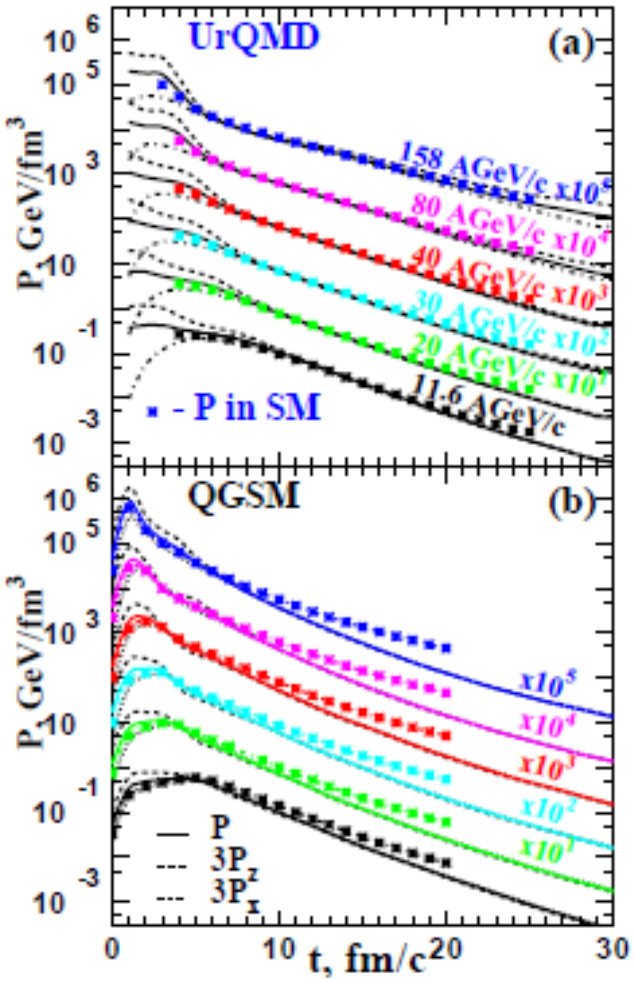


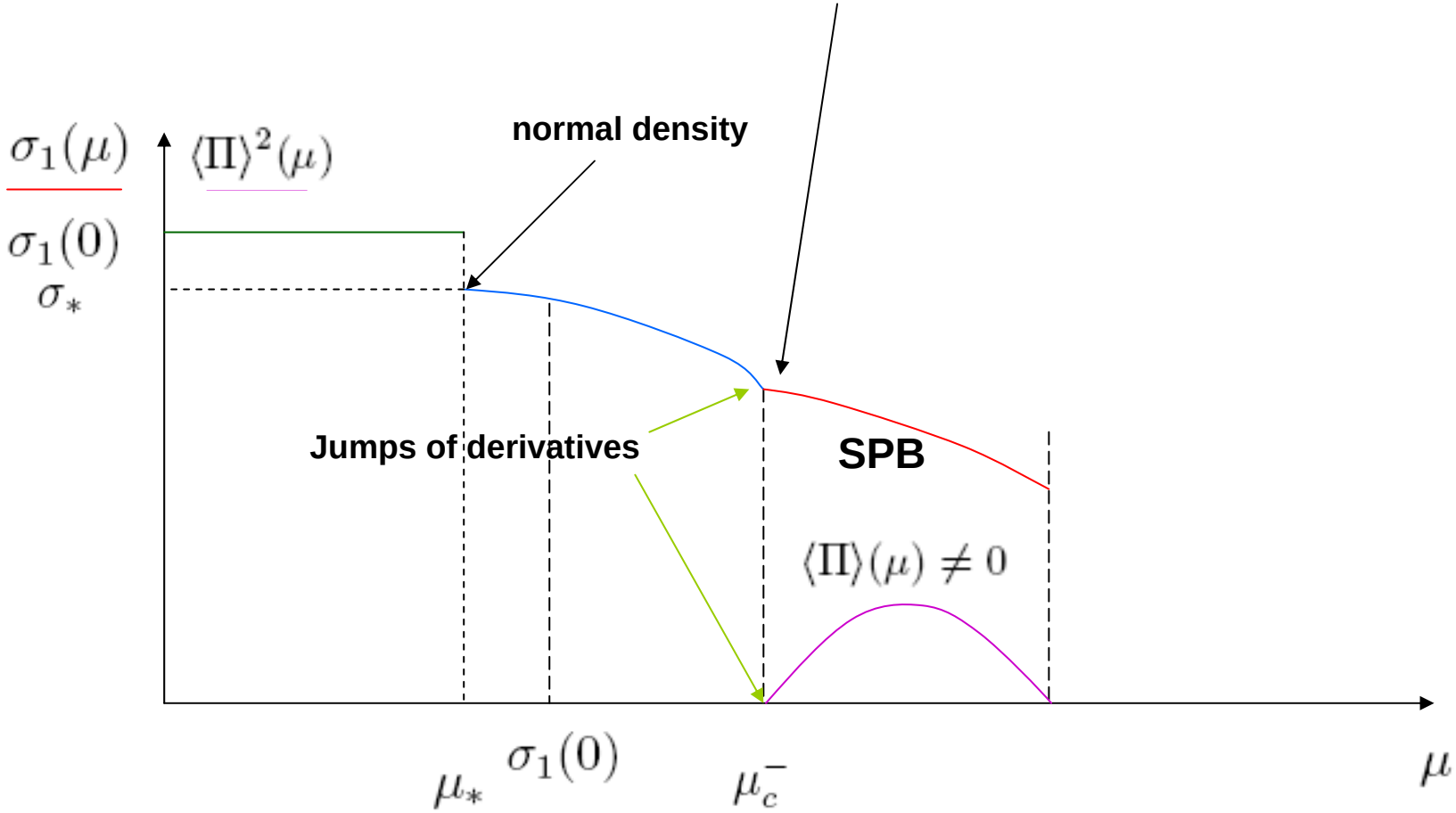
FIG. 2: (Color online) The longitudinal ($3P_z$, dashed curves) and the transverse ($3P_x$, dash-dotted curves) diagonal components of the microscopic pressure tensor in the central 125 fm^3 cell in (a) UrQMD and (b) QGSM calculations of central Au+Au collisions at energies from 11.6A GeV to 158A GeV. Asterisks indicate the pressure given by the statistical model and solid lines show the total microscopic pressure.

From L.Bravina et al arXiv:0804.1484 [hep-ph]

TABLE I: The time evolution of the thermodynamic characteristics of hadronic matter in the central cell of volume $V = 125$ fm^3 in central Au+Au collisions at bombarding energy 20A GeV. The temperature, T, baryochemical potential, μ_B, strange chemical potential, μ_S, pressure, P, entropy density, s, and entropy density per baryon, s/ρ_B, are extracted from the statistical model of ideal hadron gas, using the microscopically evaluated energy density, ε^{cell}, baryonic density, ρ_B^{cell}, and strangeness density, ρ_S^{cell}, as input. Of each pair of numbers, the upper one corresponds to the UrQMD calculations, and the lower one to the QGSM calculations.

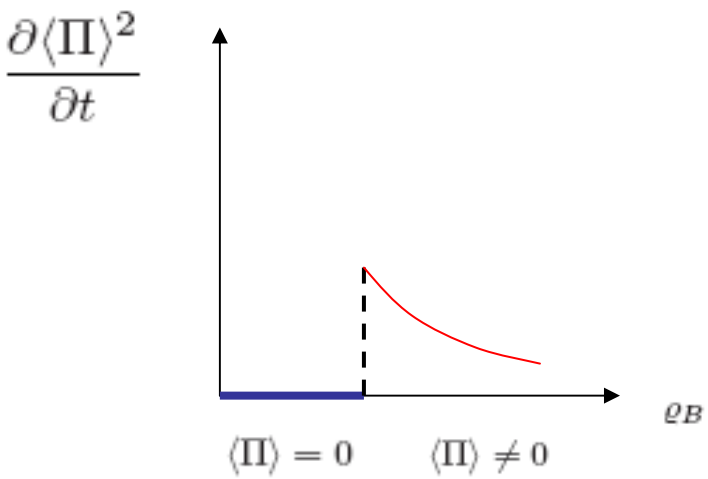
Time fm/c	ε^{cell} MeV/fm^3	ρ_B^{cell} fm^{-3}	ρ_S^{cell} fm^{-3}	T MeV	μ_B MeV	μ_S MeV	P MeV/fm^3	s fm^{-3}	s/ρ_B^{cell}
11	464.2	0.210	-0.0143	144.5	450.5	92.7	59.6	2.97	14.16
	522.6	0.257	-0.0059	150.2	487.8	116.1	73.8	3.13	12.19
12	343.2	0.160	-0.0115	137.9	459.2	86.4	44.0	2.27	14.18
	385.7	0.197	-0.0051	141.9	498.1	109.4	53.1	2.40	12.16
13	255.2	0.124	-0.0093	131.5	469.5	80.4	32.6	1.75	14.15
	286.9	0.153	-0.0046	134.0	609.5	103.1	38.5	1.85	12.09
14	189.9	0.096	-0.0072	124.9	481.7	75.8	24.1	1.34	14.06
	214.2	0.117	-0.0035	127.2	515.9	97.1	28.2	1.43	12.22
15	143.9	0.075	-0.0064	119.2	492.8	68.6	18.1	1.05	13.97
	162.3	0.091	-0.0028	121.0	522.3	91.5	20.1	1.12	12.35
16	108.8	0.059	-0.0052	113.7	502.5	62.7	13.6	0.82	13.97
	125.4	0.072	-0.0025	115.4	529.2	85.4	15.9	0.89	12.43
17	83.6	0.046	-0.0043	108.7	511.0	57.0	10.4	0.65	14.02
	98.3	0.058	-0.0022	110.4	535.9	80.1	12.3	0.72	12.52
18	65.0	0.037	-0.0035	103.5	523.7	52.4	8.0	0.52	13.88
	78.1	0.047	-0.0019	105.9	541.3	75.4	9.6	0.59	12.66
19	50.9	0.030	-0.0029	98.8	534.5	47.6	6.2	0.41	13.82
	62.9	0.039	-0.0016	101.1	552.7	72.2	7.6	0.49	12.52
20	40.6	0.025	-0.0027	94.6	544.2	38.9	4.8	0.34	13.76
	51.0	0.033	-0.0014	97.0	560.1	67.4	6.0	0.40	12.54

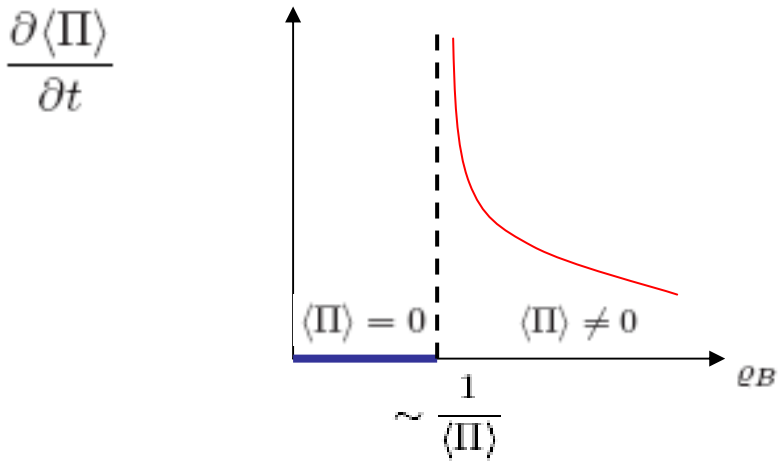
Spontaneous P-parity breaking (IId order phase transition)



A.A.Andrianov and D.Espriu, Phys.Lett. B 663 (2008) 450
A.A.Andrianov, V.A.Andrianov and D.Espriu, Phys.Lett. B 678 (2009) 416

Infinite jump in order parameter





Second order phase transition
in the absence of manifest parity breaking

relevant for anomaly vertex

Signatures and searches of parity breaking

1) Photon of "+" polarization decays in dilepton pair $\gamma \to l^+ l^-$ when

$$|\mathbf{k}| \geq \frac{4m_l^2}{\eta} \equiv k_{\rm th}$$

2) Different thresholds for different dilepton species!
Search for dimuon excess

3) Decay width is energy independent $\Gamma_+ = \tau_+^{-1} \simeq \frac{\alpha\eta}{3}$ for $k \gg k_{\rm th}$

But convolution with photon spectral distribution!

4) Distorted photon resonance enhances considerably the yield of dileptons

5) Position of this resonance moves with photon wave vector k

6) Mixing with vector mesons to disentangle the condensate isospin

Program for RHIC → CBM FAIR + NICA

Accompanying processes

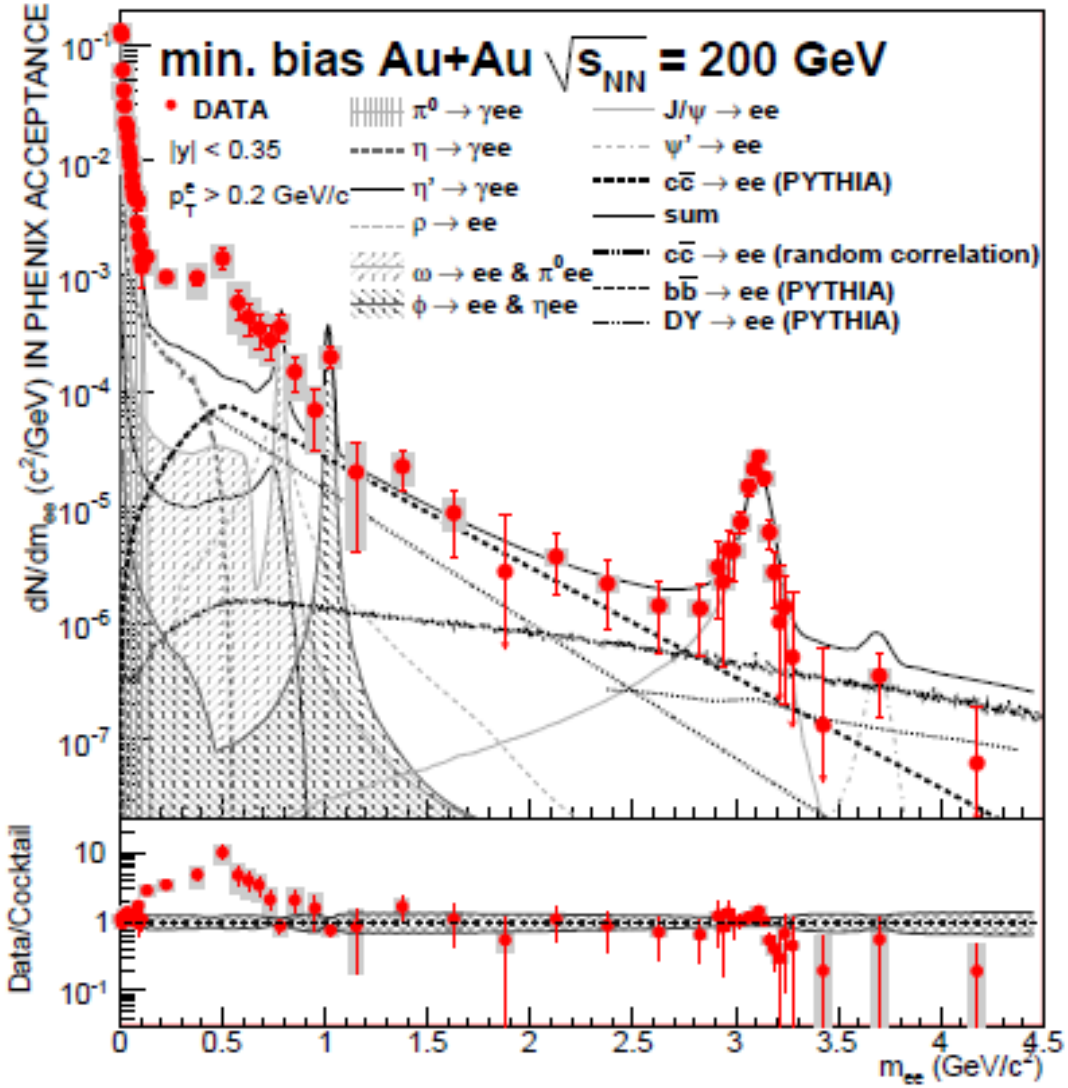
Lepton decays $l^{\pm} \to l^{\pm}\gamma$

A. Andrianov, D. Espriu, F. Mescia and A. Renau, Phys.Lett. B684 (2010) 101

Photon splitting $\gamma \to \gamma_{-}\gamma_{-}\gamma_{-}$ **more rare** $\Gamma_{\gamma\gamma\gamma} \sim \alpha^2$

Red shift of lepton and photon spectra!

Back-up slides



From: 0912.0244v1 [nucl-ex]
(PHENIX Collaboration)

FIG. 26: (Color online) Inclusive mass spectrum of e^+e^- pairs in the PHENIX acceptance in minimum-bias Au + Au compared to expectations from the decays of light hadrons and correlated decays of charm, bottom and Drell-Yan. The

Two-multiplet sigma model

$$V_{\rm eff} = -\sum_{j,k=1}^{2} \sigma_j \Delta_{jk} \sigma_k - \Delta_{22}(\pi_2^a)^2 + \lambda_2\Big((\pi_2^a)^2\Big)^2 + (\pi_2^a)^2\Big((\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big)$$
$$+\lambda_1\sigma_1^4 + \lambda_2\sigma_2^4 + (\lambda_3+\lambda_4)\sigma_1^2\sigma_2^2 + \lambda_5\sigma_1^3\sigma_2 + \lambda_6\sigma_1\sigma_2^3.$$

$$\Delta V_{\rm eff}(m_q) = 2m_q\left[-(d_1\sigma_1 + d_2\sigma_2)\cos\frac{|\pi_1^a|}{F_0} + d_2\frac{\pi_1^a\pi_2^a}{|\pi_1^a|}\sin\frac{|\pi_1^a|}{F_0}\right]$$

Mass-gap equations for spontaneous P-breaking

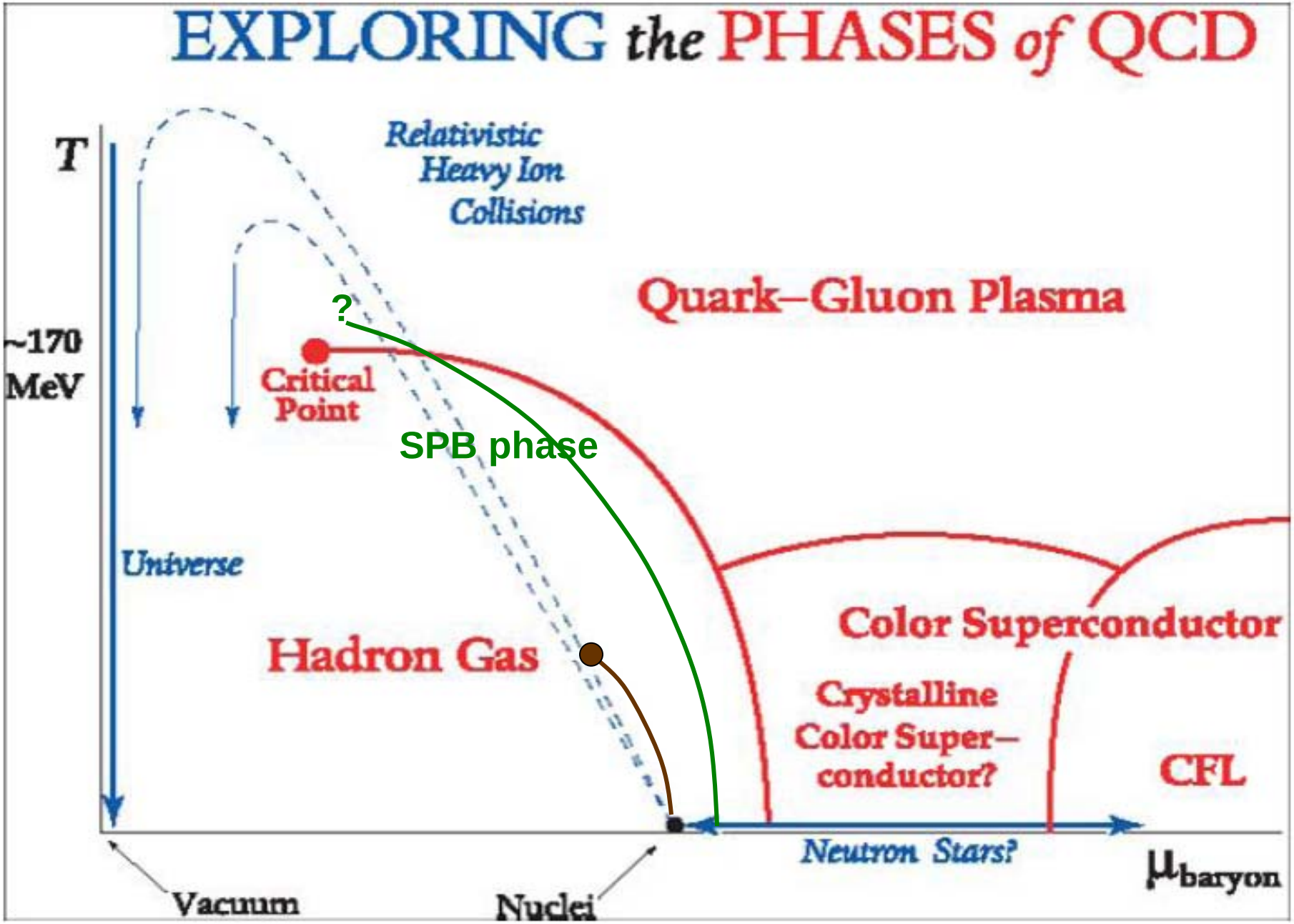
$$\langle\pi_1^a\rangle = \langle\pi^0\rangle\delta^{0a}, \qquad \langle\pi_2^a\rangle = \rho\delta^{0a}.$$

$$(d_1\sigma_1 + d_2\sigma_2)\sin\frac{\langle\pi^0\rangle}{F_0} = -d_2\rho\cos\frac{\langle\pi^0\rangle}{F_0},$$
$$-m_q d_2\sin\frac{\langle\pi^0\rangle}{F_0} = \rho\frac{m_q d_2^2}{(d_1\sigma_1 + d_2\sigma_2)}\cos\frac{\langle\pi^0\rangle}{F_0}$$
$$= \rho\Big(-\Delta_{22} + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 + 2\lambda_2\rho^2\Big)$$

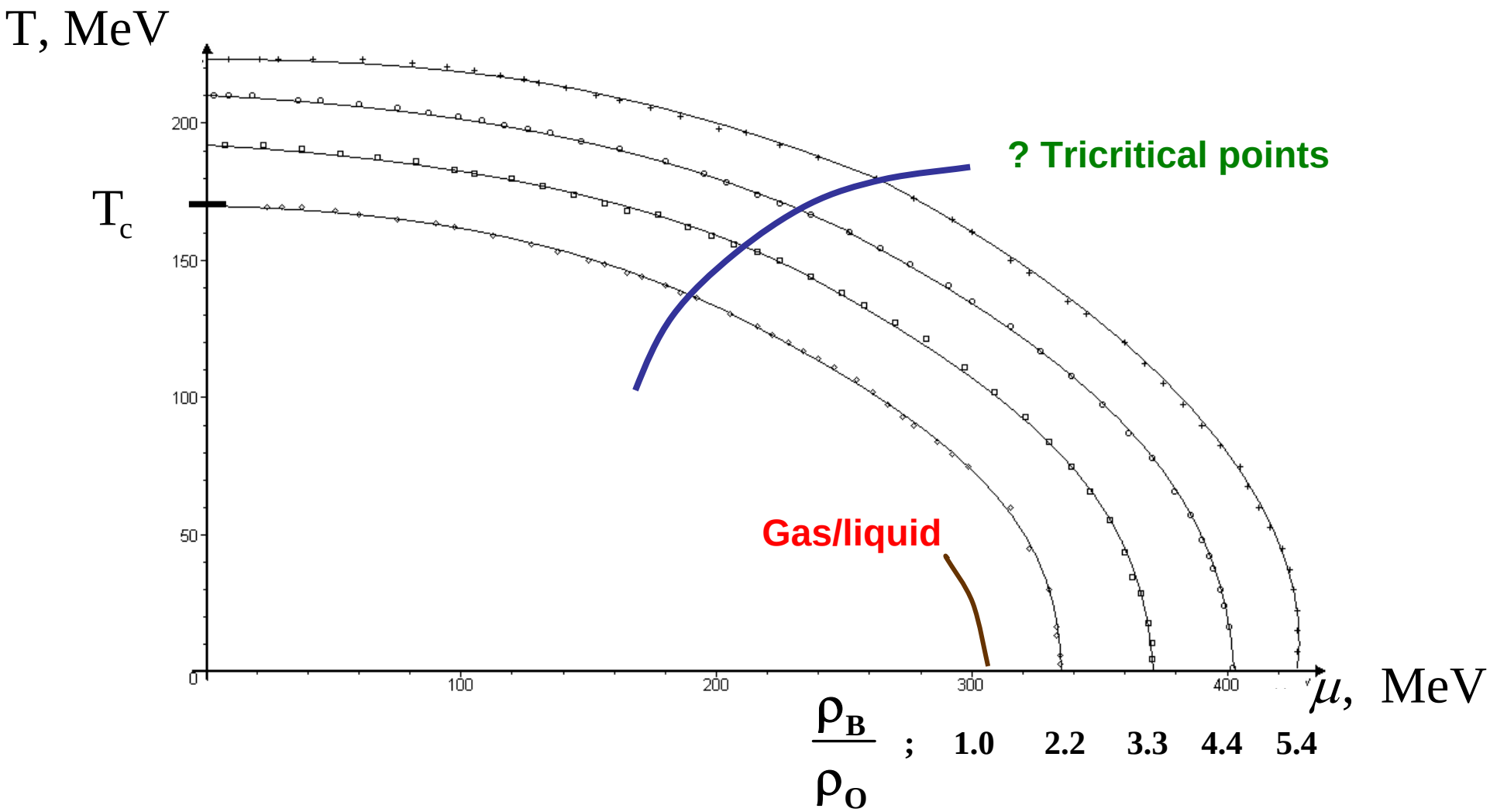
At a critical point typically both v.e.v. emerge simultaneously

for $\mu > \mu_{crit}$

$$\cos\frac{\langle\pi^0\rangle}{F_0} = \frac{d_1\sigma_1 + d_2\sigma_2}{\sqrt{d_2^2\rho^2 + (d_1\sigma_1 + d_2\sigma_2)^2}}$$


EXPLORING the PHASES of QCD
T
~170 MeV
Relativistic Heavy Ion Collisions
Quark–Gluon Plasma
?
Critical Point
SPB phase
Universe
Hadron Gas
Color Superconductor
Crystalline Color Super–conductor?
CFL
Neutron Stars?
μbaryon
Vacuum
Nuclei

Parity breaking phase divide line



Vacuum states for symmetric baryon matter

A pseudoscalar condensate breaking P-parity?

$$\pi_2^a = \delta^{a0}\rho$$

No! (Vafa-Witten theorem) $\rho = 0$ **in QCD at zero quark density**

Eqs. for vacuum states

$$\begin{aligned}
2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) &= 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 \\
&\quad +\rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big), \\
2(\Delta_{12}\sigma_1 + \Delta_{22}\sigma_2) &= \lambda_5\sigma_1^3 + 2(\lambda_3+\lambda_4)\sigma_1^2\sigma_2 + 3\lambda_6\sigma_1\sigma_2^2 + 4\lambda_2\sigma_2^3 \\
&\quad +\rho^2\Big(\lambda_6\sigma_1 + 4\lambda_2\sigma_2\Big), \\
0 &= 2\pi_2^a\Big(-\Delta_{22} + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 + 2\lambda_2\rho^2\Big)
\end{aligned}$$

Necessary and sufficient condition to avoid P-parity breaking in normal QCD vacuum (μ = 0)

$$(\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22} \qquad \lambda_2 > 0$$

positive mass of $\pi(1300)$

Embedding a chemical potential into "quark matter free energy"

$$\langle N^\dagger N \rangle \quad \Longleftrightarrow \quad \int d^3x \mu_B (\bar{N}\gamma_0 N(x) - \varrho_B)$$

Local coupling to quarks

$$\mathcal{L}_{int} = -(\bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R)$$

Superposition of physical meson states

From a quark model

$$\Delta V_{\text{eff}}(\mu) = \frac{\mathcal{N}}{2}\Theta(\mu - |H_1|)\left[\mu|H_1|^2\sqrt{\mu^2 - |H_1|^2} - \frac{2\mu}{3}(\mu^2 - |H_1|^2)^{3/2} - |H_1|^4 \ln\frac{\mu + \sqrt{\mu^2 - |H_1|^2}}{|H_1|}\right]\left(1 + O\left(\frac{\mu^2}{\Lambda^2}\right)\right)$$

$$\mathcal{N} \equiv \frac{N_c N_f}{4\pi^2}$$

Monotonous function

Only the first Eq. for stationary points is modified

$$2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3 + \lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3$$
$$+\rho^2\Big(2(\lambda_3 - \lambda_4)\sigma_1 + \lambda_6\sigma_2\Big) + 2\mathcal{N}\Theta(\mu - \sigma_1)\underbrace{\left[\mu\sigma_1\sqrt{\mu^2 - \sigma_1^2} - \sigma_1^3 \ln\frac{\mu + \sqrt{\mu^2 - \sigma_1^2}}{\sigma_1}\right]}_{\equiv \sigma_1 \mathcal{A}(\sigma_1, \mu)}$$

Extension to Hot and Dense matter:

replace $\mathcal{A}(\sigma_1, \mu)$

$$\mathcal{A}(\sigma_1, \mu, \beta) = \int_{\sigma_1}^{\infty} dE \, \sqrt{E^2 - \sigma_1^2} \, \frac{\cosh(\beta\mu) + \exp(-\beta E)}{\cosh(\beta\mu) + \cosh(\beta E)} \; + \; \mathcal{O}\left(\mu^2/\Lambda^2, \sigma_1^2/\Lambda^2\right).$$

after summing up Matsubara frequences in one quark loop

On classical solutions

$$V_{\rm eff}(\mu) = -\frac{1}{2}\sum_{j,k=1}^{2} \sigma_j(\mu)\Delta_{jk}\sigma_k(\mu) - \frac{1}{2}\Delta_{22}\rho^2(\mu) - \frac{\mathcal{N}}{3}\mu\Big(\mu^2 - \sigma_1(\mu)^2\Big)^{3/2}\theta\Big(\mu - \sigma_1(\mu)\Big)$$

$$+ \Delta V_\omega(\bar{\mu}, \mu)$$

Thermodynamic properties at zero temperature

Pressure $$p(\sigma_j(\mu),\mu) \equiv V_{\rm eff}\left(\sigma_j^0\right) - V_{\rm eff}\left(\sigma_j(\mu),\rho(\mu),\mu\right),$$

$$\boxed{\partial_\mu p = N_c \varrho_B.}$$

Energy density $$\varepsilon = -p + N_c \mu \varrho_B.$$

relation between baryon density, Fermi momenta and the chemical potential is for quark matter

$$\varrho_B = -\frac{1}{N_c}\partial_\mu V_{\rm eff} = \frac{N_f}{3\pi^2}p_F^3 = \frac{N_f}{3\pi^2}(\mu^2 - \sigma_1(\mu)^2)^{3/2}.$$

Pressure vanishes in vacuum and at a minimum of energy per baryon

$$p = \varrho_B^2 \partial_{\varrho_B}\left(\frac{\varepsilon}{\varrho_B}\right),$$

This minimum realizes the stable baryon matter. Since pressure is increasing with density the phase diagram in the p, ϱ_B plane must necessarily exhibit discontinuity – 1st order phase transition "vapor-liquid".

It occurs at $$\bar{\mu}^* < \sigma_1^0, \sigma_j^* \equiv \sigma_j(\bar{\mu}^*)$$

To realize it one needs two competitive minima in the effective potential in order to interchange them at a phase transition point.

P-parity violation strip

Boundary of the P-breaking phase,

$$\mathcal{N}\mathcal{A}(\sigma_1^{\pm}, \mu_c^{\pm}, \beta) = \Delta_{11} - 2\lambda_1(\sigma_1^{\pm})^2 - \lambda_5\sigma_1^{\pm}\sigma_2^{\pm} - (\lambda_3 - \lambda_4)(\sigma_2^{\pm})^2$$

It defines the P-breaking strip in the $T - \mu$ plane.

$\mathcal{A} > 0$ and $\mathcal{A} \to \infty$ when $T, \mu \to \infty$.

It means that for *any* nontrivial solution $r_{\pm}, \sigma_1^{\pm}, \sigma_2^{\pm}$ with $\mathcal{C}^{\pm}(\Delta_{jk}, \lambda_j) > 0$ the P-breaking phase boundary exists.

Thus if the phenomenon of P-breaking is realized for zero temperature it will take place in a strip including lower chemical potentials but higher temperatures.

However within the validity of chiral expansion the temperature cannot be too high!?

Beyond the chiral limit: $m_q \neq 0$

Two new lowest-dimensional operators

$$\frac{1}{2} m_q d_1 \mathrm{tr}(H_1 + H_1^\dagger) \qquad \frac{1}{2} m_q d_2 \mathrm{tr}(H_2 + H_2^\dagger)$$

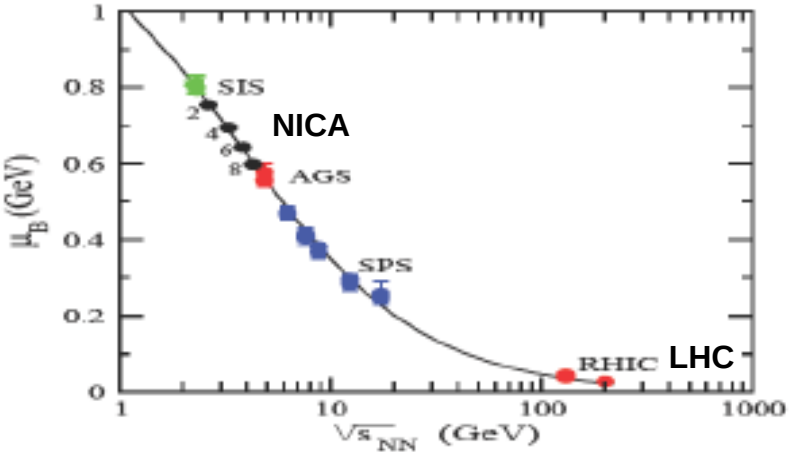
The spectrum in dense matter

$\mu = 0$	$m_\pi^2 \;:\; m_q$	$m_\Pi^2 \;?\; m_\pi^2$	heavy
$\mu = \mu_c$	$m_\pi^2 = 0$	$m_\Pi^2 \;:\; m_q$	very light
$\mu > \mu_c$	$m_{\pi_\pm}^2 = 0$	$m_{\Pi_\pm}^2 \;:\; m_{\pi_0}^2 \;:\; m_q$	light

Two Goldstone bosons of isospin breaking

Three light massive "pions"

Converter of collider energies per NN into phase diagram



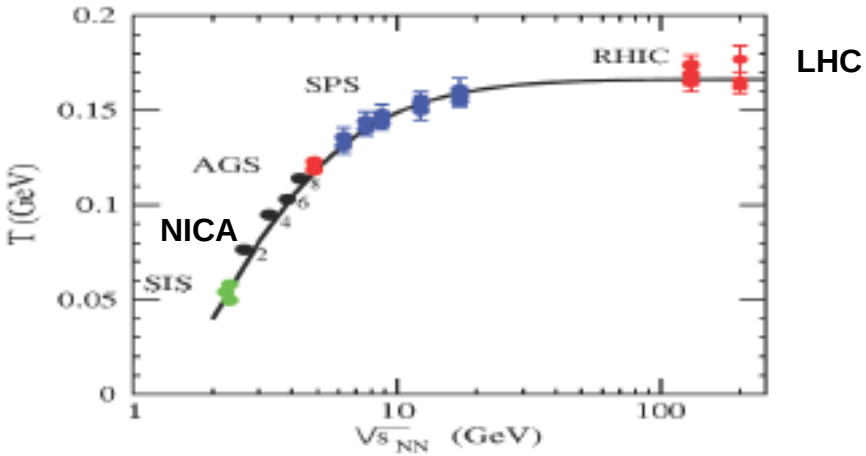


FIG. 1: (Color online) $\sqrt{s_{NN}}$ dependence of μ_B (top panel) and T obtained from chemical fits [20] to particle yield ratios obtained at different accelerator facilities as indicated. The solid lines are fits to these data.

NICA, Dubna,2014?