The chiral magnetic effect from the lattice

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Outline

1. Introduction

2. results for a classical (lattice) instanton

3. results for a 2+1 flavor QCD configuration

4. Summary

Introduction

- Lattice calculation very interesting and useful

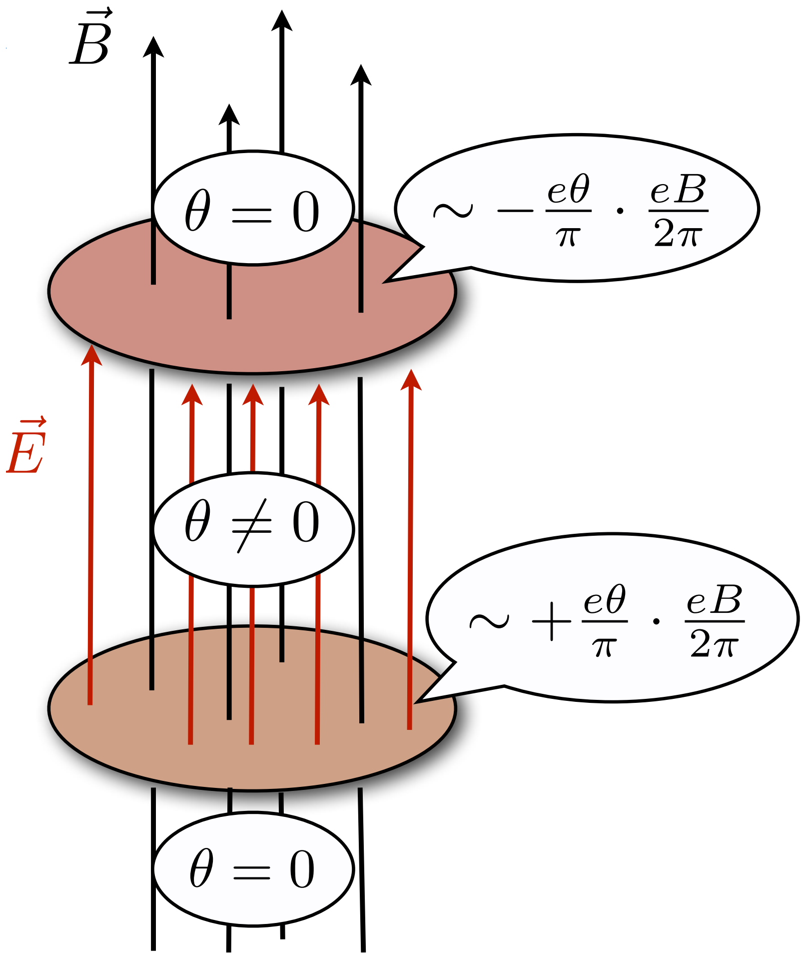
    - Probe equilibrium QCD gauge field configurations with a uniform $\vec{B}$

    - Calculate electric charge separation, and dependence on external $\vec{B}$, T, m_q, χ SB...

- Moscow group [Phys.Rev.D80:054503,2009] (discussion session this afternoon)

- UConn group [PoS (2009) arXiv:0911.1348]

Charge separation (chiral magnetic effect)



In general $\rho = \frac{q^2}{8\pi^2} \vec{\nabla}\theta \cdot \vec{B}$

Take θ static, non-zero only between domain-walls ("parallel-plate capacitor")

"Plates" are charged, with charge density $\pm q^2\theta B/2\pi^2$

$$E = \theta \frac{q^2}{4\pi^2} B$$

(Kharzeev, arXiv:0906:2808;

Kharzeev and Zhitnitsky, 2006)

Zero Modes of $\not{D}$

Useful to work with low modes of the Dirac operator.

Physical picture: $\vec{B}$ polarizes the zero mode(s) associated with the instanton (quark and anti-quark)

Spectral decomposition of Dirac operator

$$\begin{aligned} (\not{D} + m)\psi_\lambda &= (i\lambda + m)\psi_\lambda \\ (\not{D} + m)^{-1} &= \sum_\lambda \frac{\psi_\lambda^\dagger \psi_\lambda}{i\lambda + m} \end{aligned}$$

Calculate eigenvectors of hermitian Domain Wall Fermion operator instead, $\gamma_5(\not{D} + m)$. Zero modes are the same.

Contribution to charge density

$$\begin{aligned}\rho &= \bar{\psi}\gamma_0\psi \\ &= \mathrm{tr}(\not{D} + m)^{-1}\gamma_0 \;=\; \mathrm{tr}\gamma_5\not{D}_H\gamma_0 \\ &= \sum_\lambda \frac{\psi_\lambda^\dagger\gamma_0\gamma_5\psi_\lambda}{\lambda + m}\end{aligned}$$

ψ_λ is eigen-vector of hermitian Dirac operator

contribution to $\rho = 0$ for an *exact* chiral zero-mode, so in presence of $\vec{B}$, zero-mode $\rightarrow$ near-zero mode

(use exactly conserved current for DWF)

Domain wall fermions (aside)

Kaplan (1992), reformulated for QCD by Shamir (1993)

Chiral fermions on the lattice at non-zero lattice spacing

By adding extra-fifth direction for fermions

Chiral zero modes stuck to boundary

Finite size of extra dimension L_s – explicit χ SB

Small additive quark mass, m_{res} (draw picture)

Classical instanton (-like solution)

Put classical, topological charge = 1, instanton on lattice

Chen, *et al*, PRD59 (1999)

$$
\begin{aligned}
A_\mu &= -i \sum_{j=1}^{3} \eta^{j\mu\nu} \lambda_j \frac{x_\nu}{x^2 + \rho^2} \\
\rho(r) &= \rho_0 \left(1 - \frac{r}{r_{\text{max}}}\right) \Theta(r_{\text{max}} - r)
\end{aligned}
$$

Smoothly cutoff instanton as $r \to r_{\text{max}} < L/2$.

Boundary Conditions in presence of $\vec{B}$ (Al-Hashimi, Weise (2008))

In *infinite* volume for $\vec{B} = B\hat{z}$ (z dir), $A_y = Bx$

On torus, BC's in x-y directions are

$$
\begin{aligned}
A_x(x + L_x, y) &= A_x(x, y), \quad A_y(x + L_x, y) = A_y(x, y) + BL_x \\
A_x(x, y + L_y) &= A_x(x, y), \quad A_y(x, y + L_y) = A_y(x, y)
\end{aligned}
$$

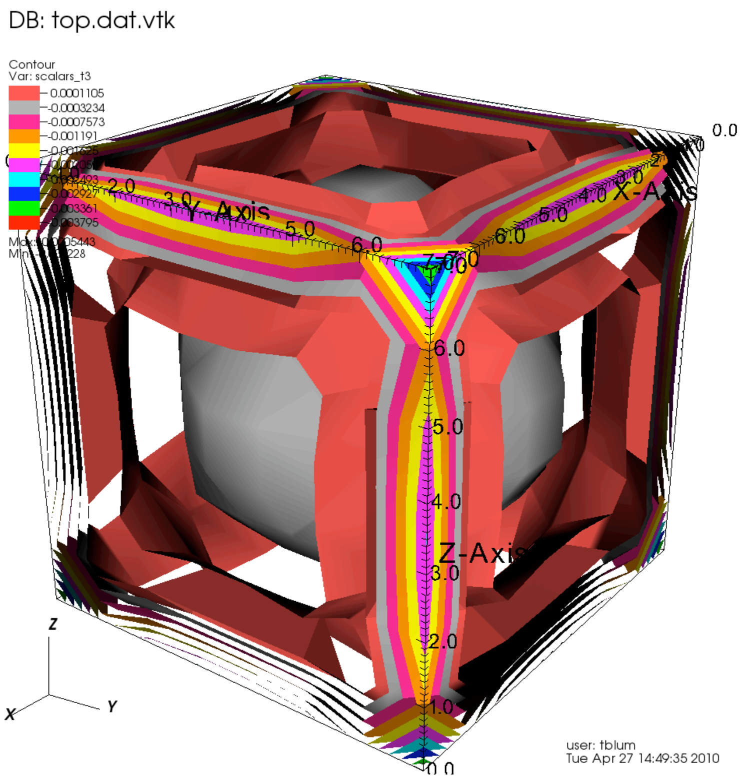
To respect gauge invariance, fermion fields must be gauge-transformed:

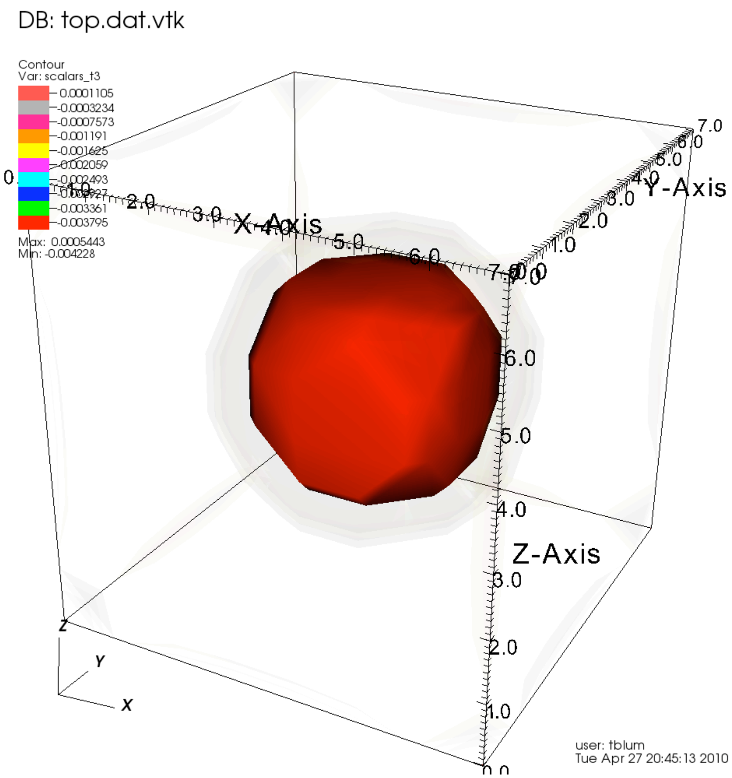
$$
\psi(x + L_x, y) = \exp(-ieBL_x y)\psi(x, y), \quad \psi(x, y + L_y) = \psi(x, y)
$$

which implies $eBL_xL_y = e\Phi_B = 2\pi n$, magnetic flux is quantized on torus!

Classical instanton (-like solution)

8^4 lattice, $\rho_0 = 10$, $r_{max} = 3$





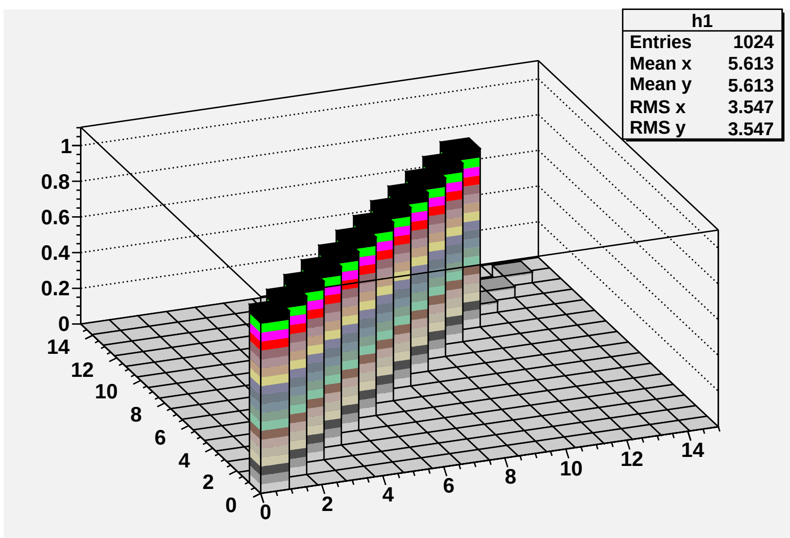
"peeled" view

Classical instanton (-like solution)

$$\gamma_5|\psi_0\rangle = \pm|\psi_0\rangle \ \langle\psi_0|\gamma_5|\psi_0\rangle = \pm 1 \quad \text{(for zero-modes)}$$
$$\gamma_5|\psi_\lambda\rangle = |\psi_{-\lambda}\rangle \ \langle\psi_{-\lambda}|\gamma_5|\psi_\lambda\rangle = 1 \quad \text{(for non-zero-modes)}$$

Same is true for DWF if m_{res} small

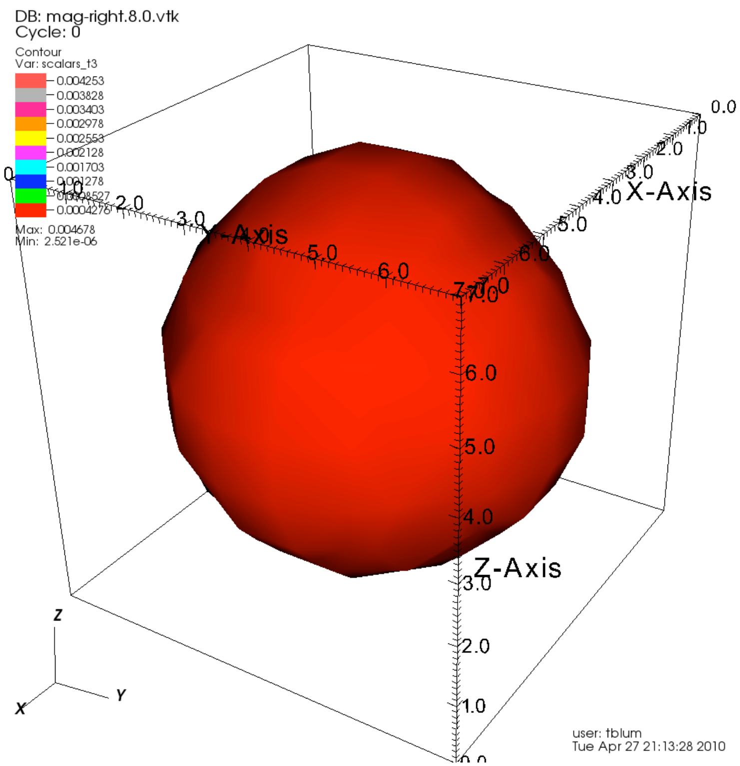
Chirality: $\langle\Psi_i|\Gamma_5|\Psi_j\rangle$ Plot, $\qquad B_z = 0$

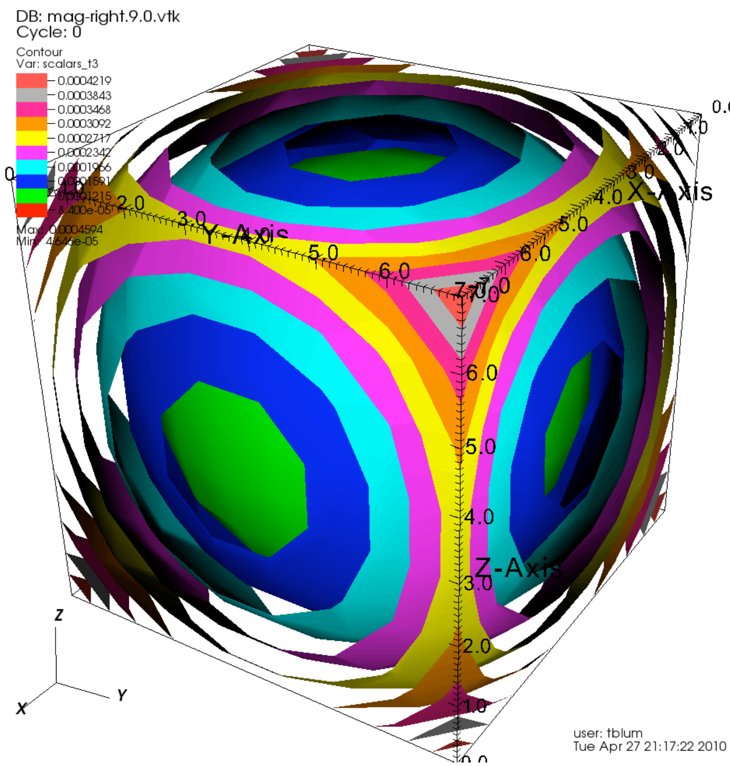


12 Zero modes (4 are plane waves: SU(2) instanton in SU(3))

Classical instanton (-like solution)

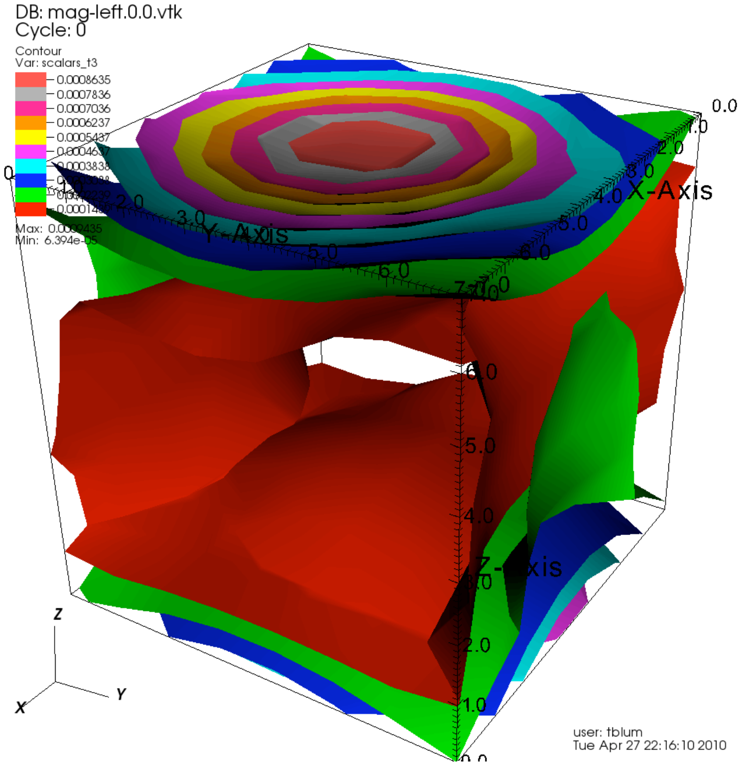
Magnitude of the zero mode(s), $B_z = 0$

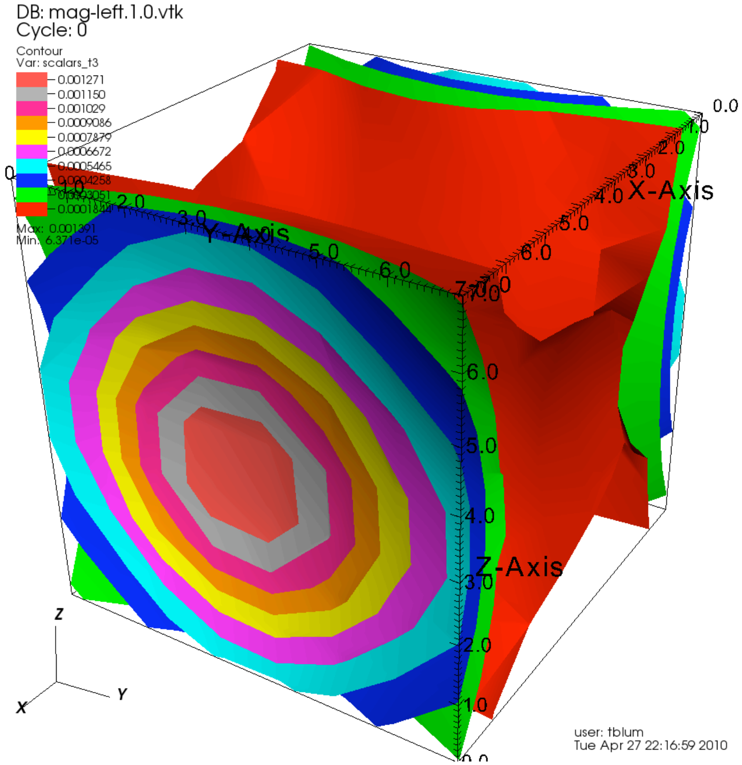




Loc. around "instanton" (1) "Lattice-artifact Instanton" (3)

Classical instanton (-like solution)





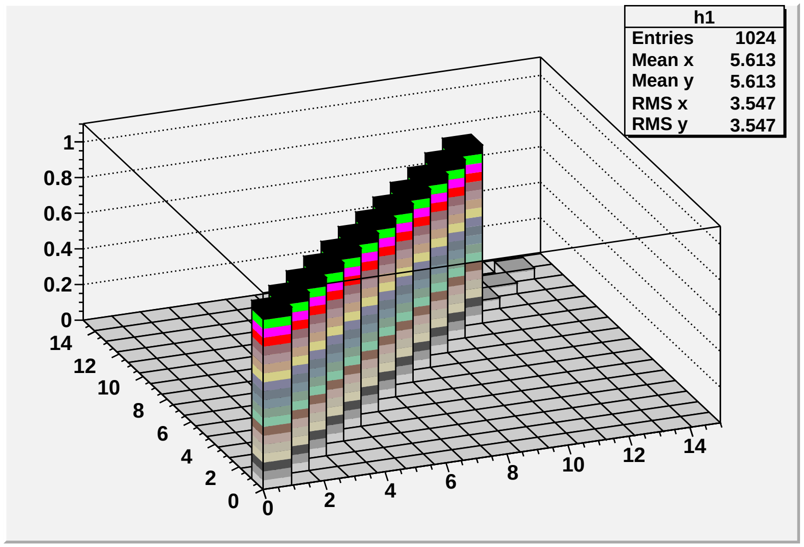
more "lattice-artifact zero-modes" (4 of them)

1+3+4 (+4 plane waves) = 12 zero modes

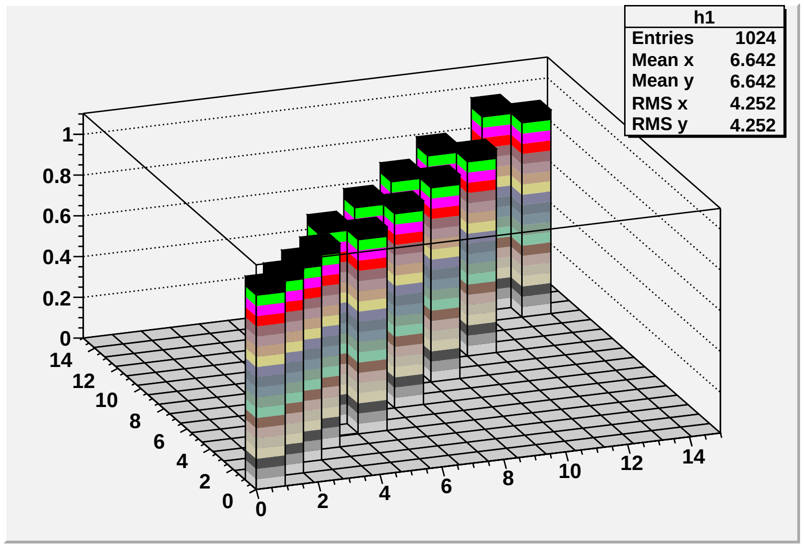
Classical instanton (-like solution)

Apply magnetic field B_z in z-direction

$B_z = 0$



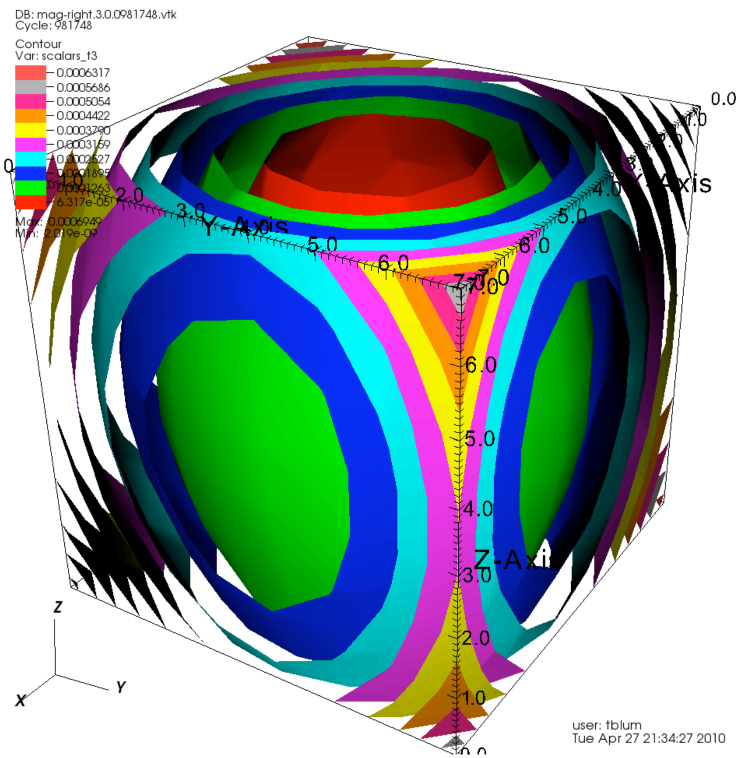
$B_z = 0.0981748$ $(n_\Phi = 1)$

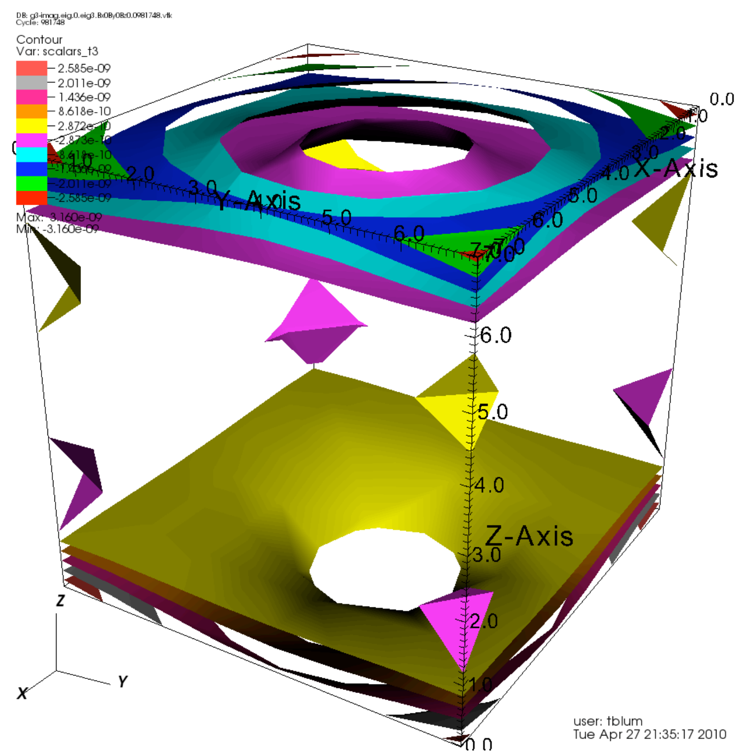


Only 4 Zero modes! (2 are plane waves)

Classical instanton (-like solution)

Magnitude of the zero mode, $B_z = 0.0981748$ $(n_\Phi = 1)$



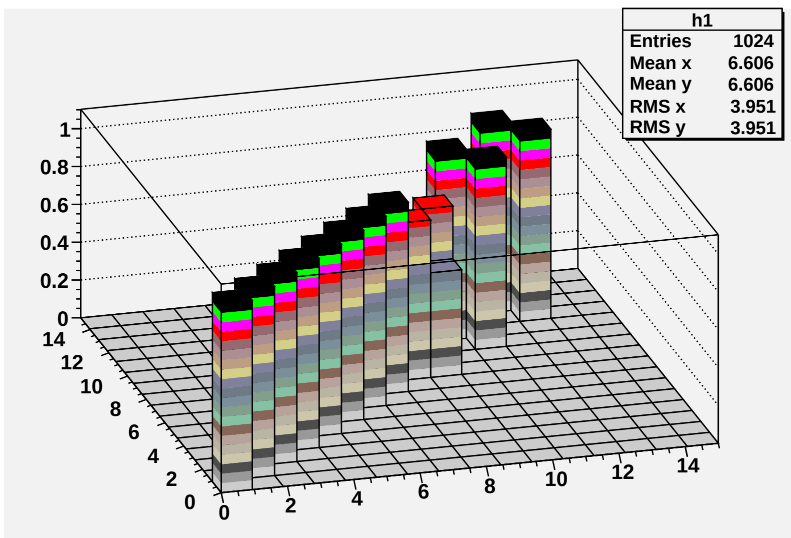


Charge separation!

Classical instanton (-like solution)

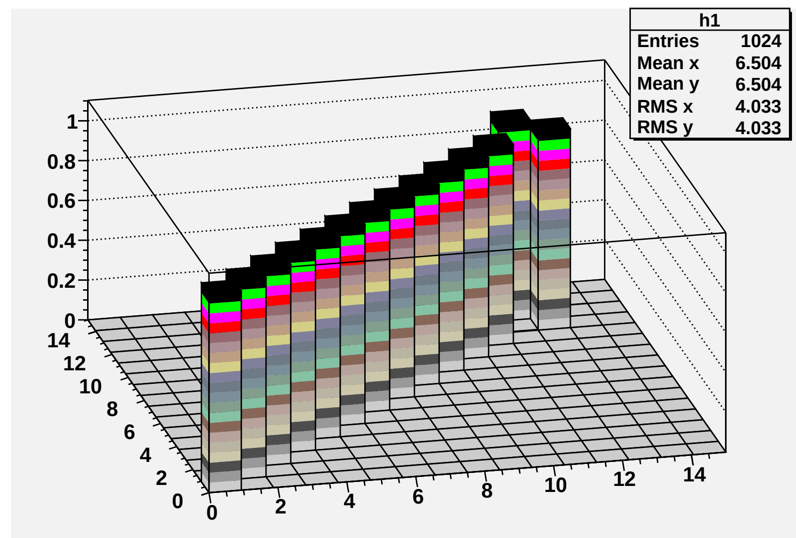
Degeneracy of Landau levels goes like n_Φ:

$B_z = 0.19635$ ($n_\Phi = 2$)



8 Zero modes (4 plane waves)

$B_z = 0.294524$ ($n_\Phi = 3$)

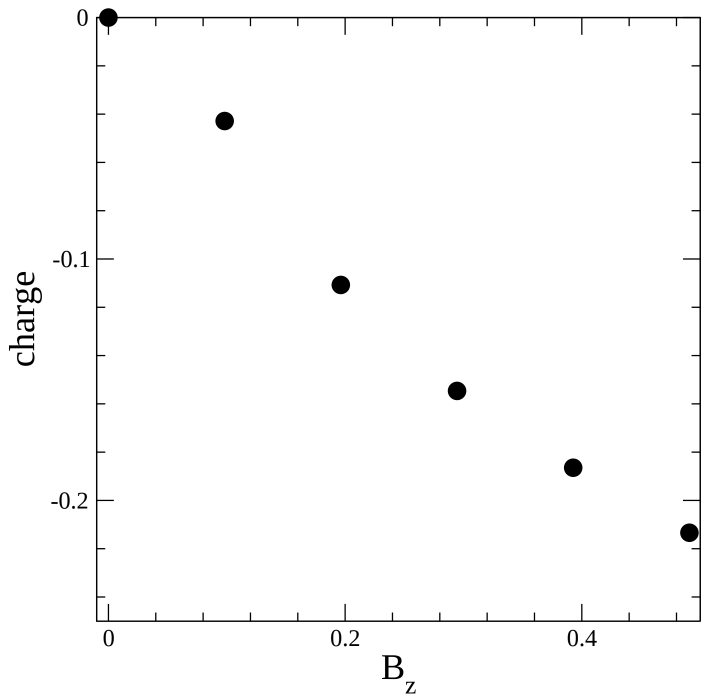


12 Zero modes (6 plane waves)

and so on...

Classical instanton (-like solution) Put it all together.

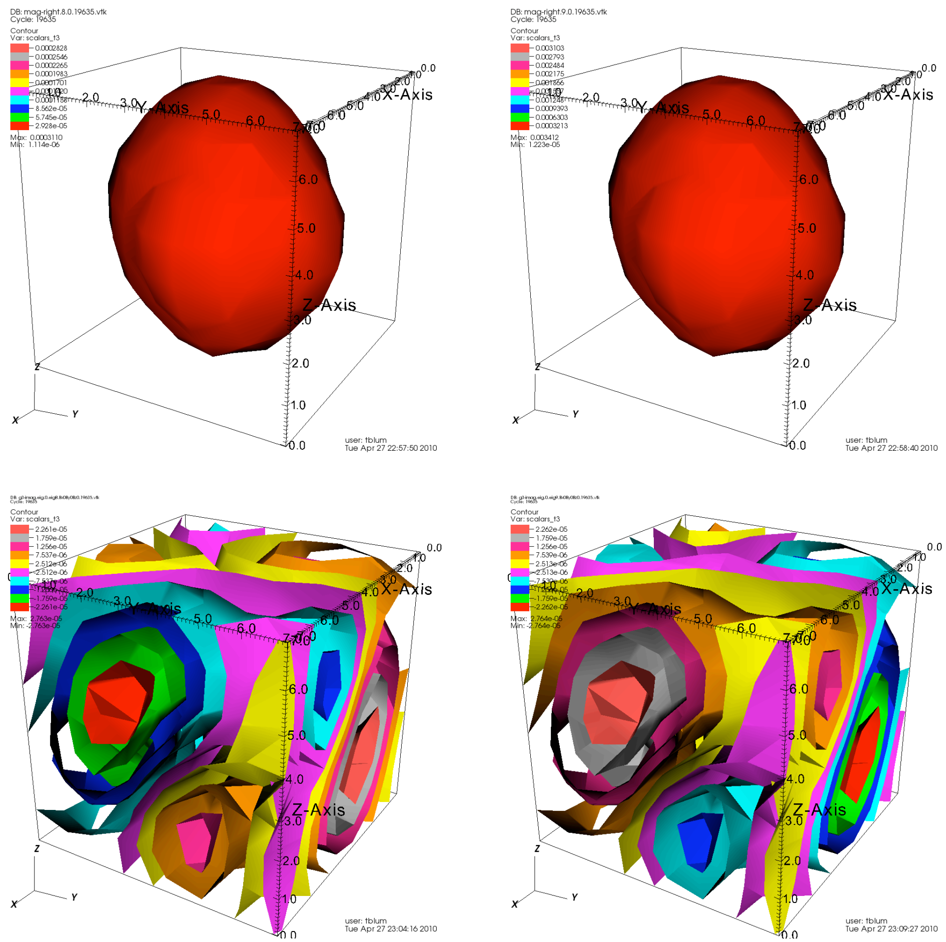
It works...



Charge in top (z-)half of lattice from near-zero-modes.
Dividing in x, y, or t gives zero, effect flips sign under $B_z \to -B_z$

Classical instanton (-like solution)

$B_z = 0.19635$ $(n_\Phi = 2)$



"Instanton-like" zero mode(s). There are two.

$\langle \Psi_i | \Gamma_5 | \Psi_j \rangle \sim \pm 0.8$

Very large charge separation.

Only occurs for this B_z.

QCD+QED Lattice Simulations

- $N_\tau = 8$, N_f=2+1, DWF (RBC+LLNL). Eventually 1+1+1

- Couple sea quarks to QCD and QED

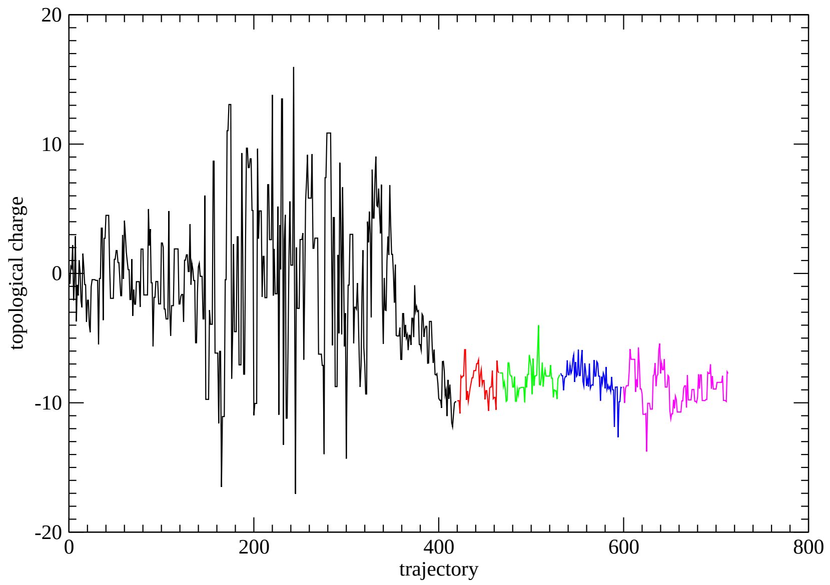
- Include external magnetic field $\vec{B}$ in dynamical evolution

- Work in $fixed$ topological sector(s)

  - use the DSDR method (Vranas, JLQCD, RBC)

Topological Charge History

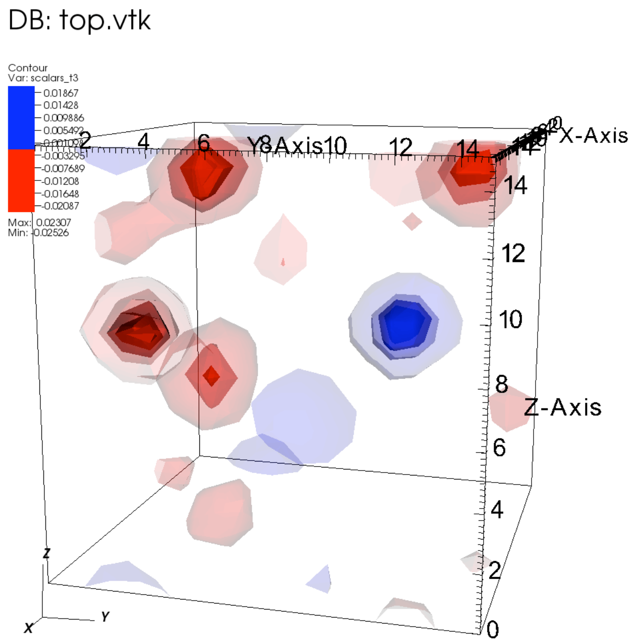
Q from 5li method of de Forcrand, *et al.*, APE smearing

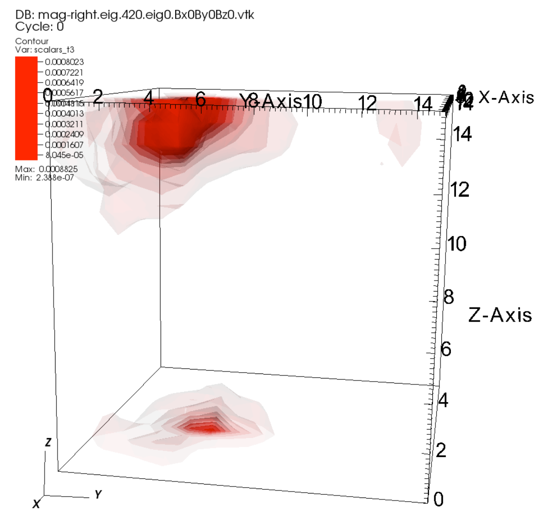


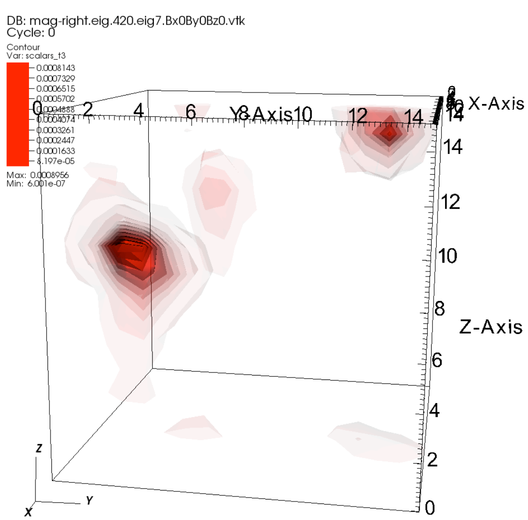
Start with AuxDet $=1$ ($\epsilon_f = \epsilon_b = 0.5$), gradually reduce ϵ_f to 10^{-4}

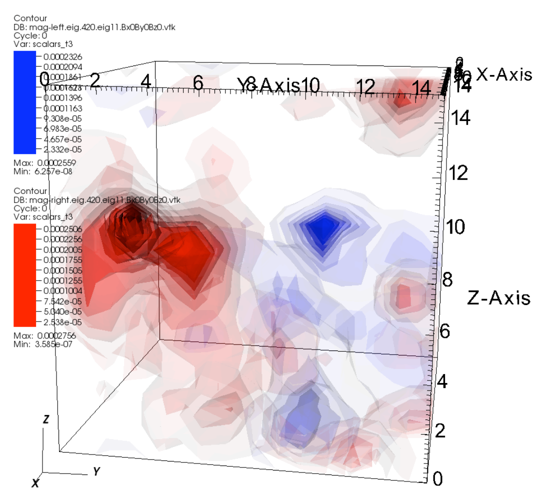
2+1 flavors, $N_t = 8$, $T \gtrsim T_c$

Top. charge and low eigen-modes









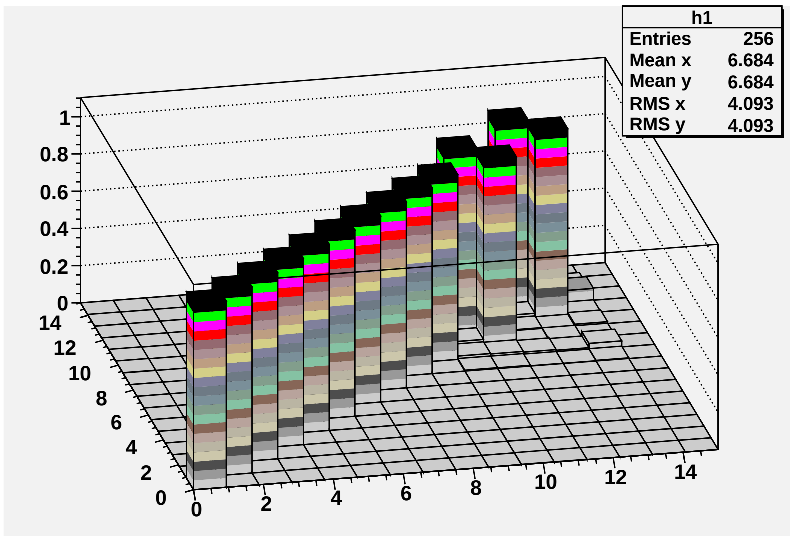
Low eigen-modes correlated with instantons

APE smeared, "5LI" definition of Q. $Q = 9 - 10$ (5li) for config. 420, or 10 from zero-modes (index)

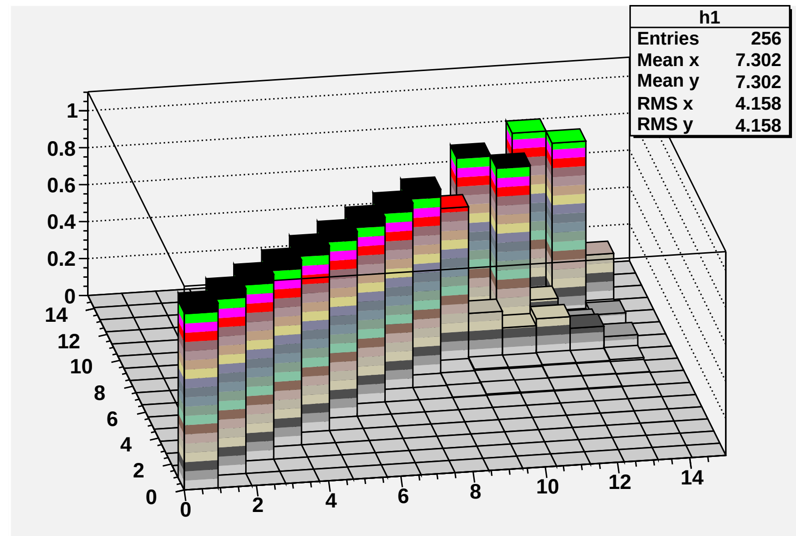
2 "zero-modes", 1 "near-zero mode" shown

2+1 flavor QCD

$B_z = 0$ (10 zero modes)



$B_z = 1.22718$ (9 zero modes)

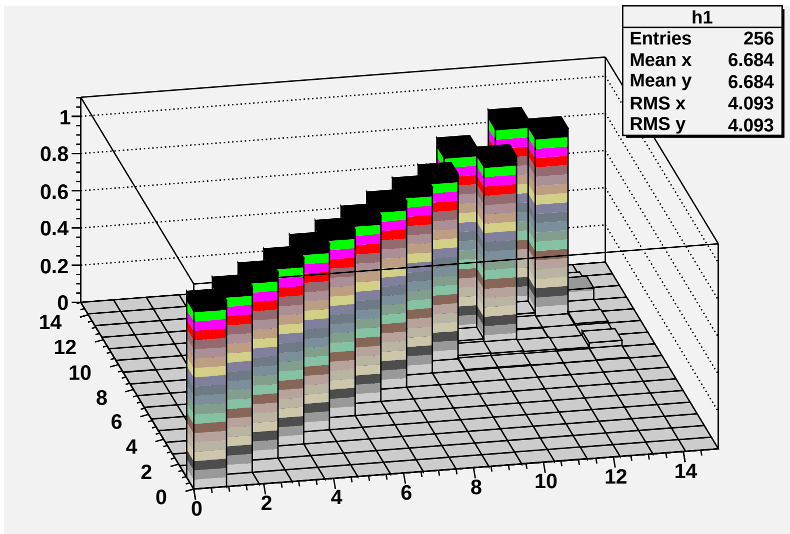


10th mode: $\langle \Psi_i | \Gamma_5 | \Psi_j \rangle \sim 0.8$

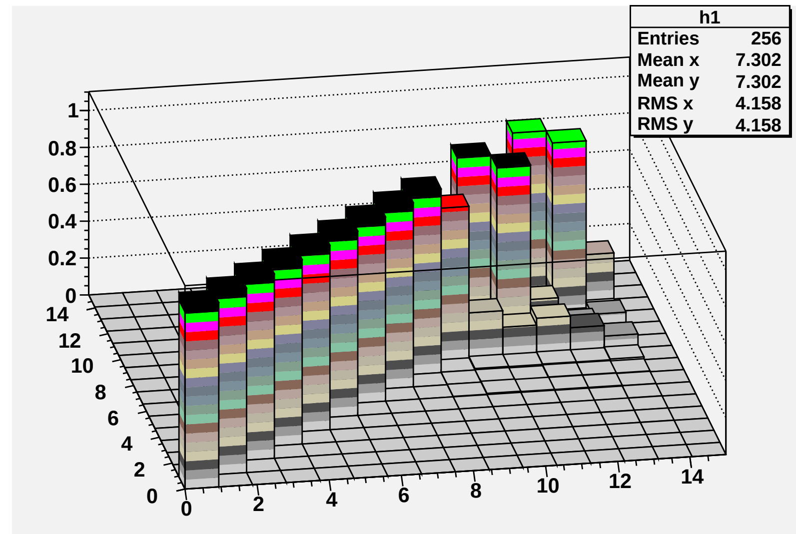
$\langle \Psi_i | \Gamma_5 | \Psi_j \rangle \sim 0.999998, 0.9998, 0.993,$ 0.823
for $B_z = 0.490874$-1.22718

2+1 flavor QCD

$B_z = 0$ (10 zero modes)



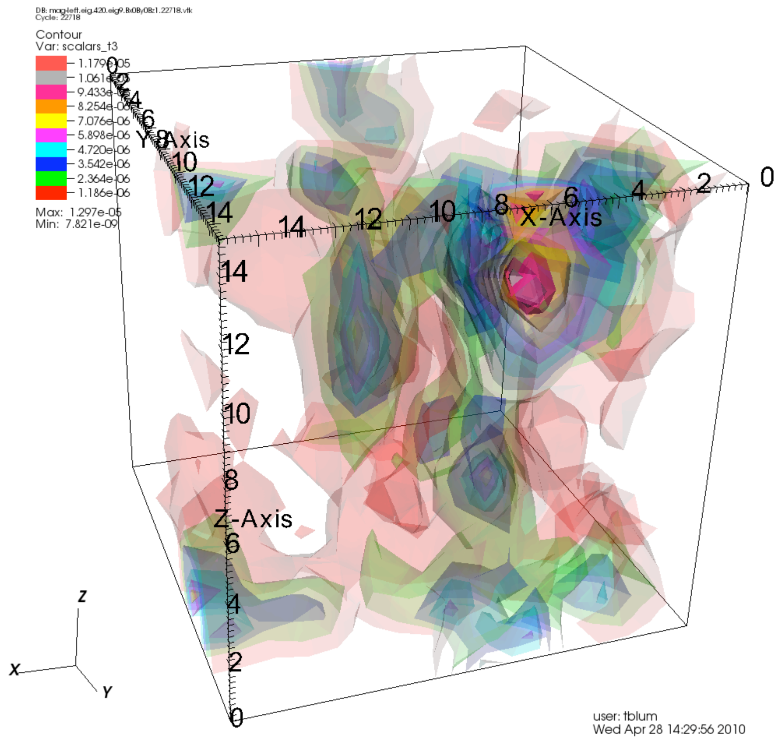
$B_z = 1.22718$ (9 zero modes)

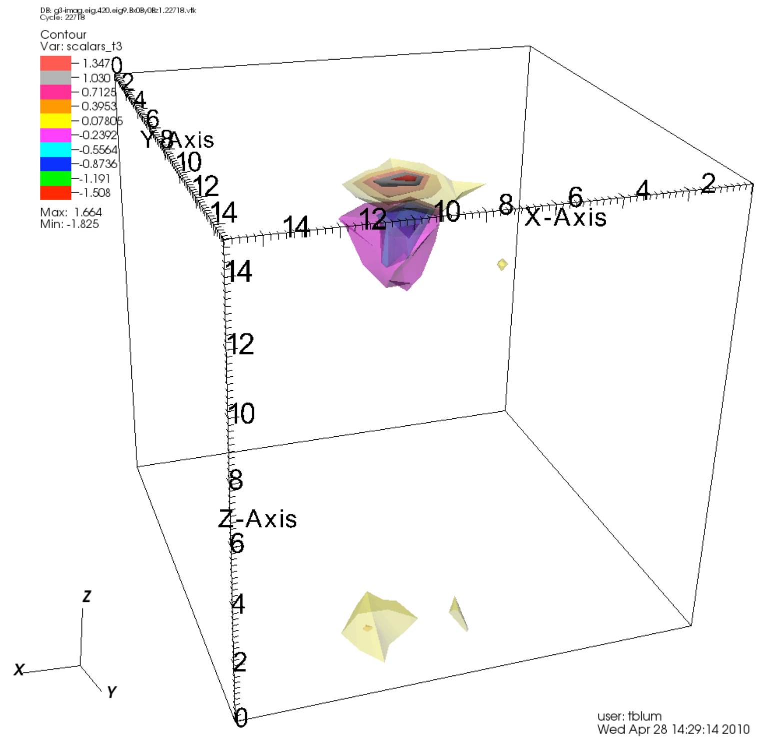


10th mode: $\langle \Psi_i | \Gamma_5 | \Psi_j \rangle \sim 0.8$

For 10th mode, $\langle \Psi_i | \Gamma_5 | \Psi_j \rangle \sim 0.999998, 0.9998, 0.993,$ 0.823 for $B_z = 0.490874$-1.22718

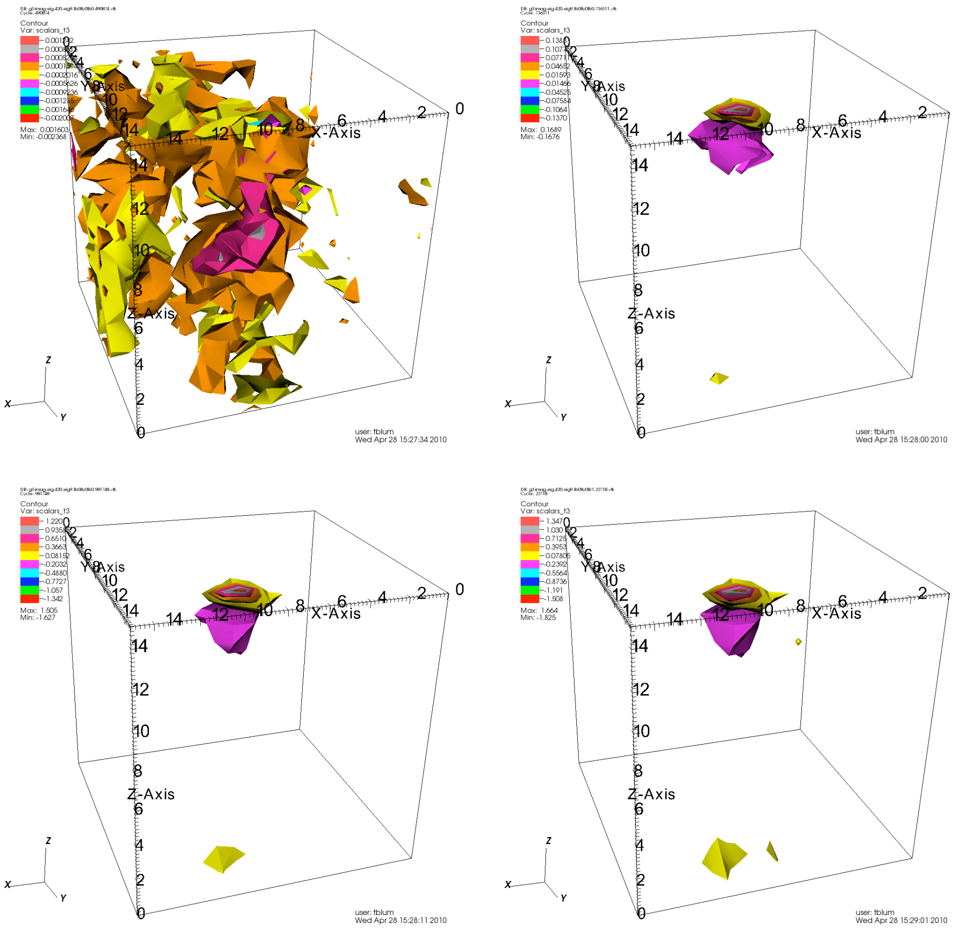
Charge density (from zero modes)





Charge separation, but localized around instanton?

Charge density (from zero modes)

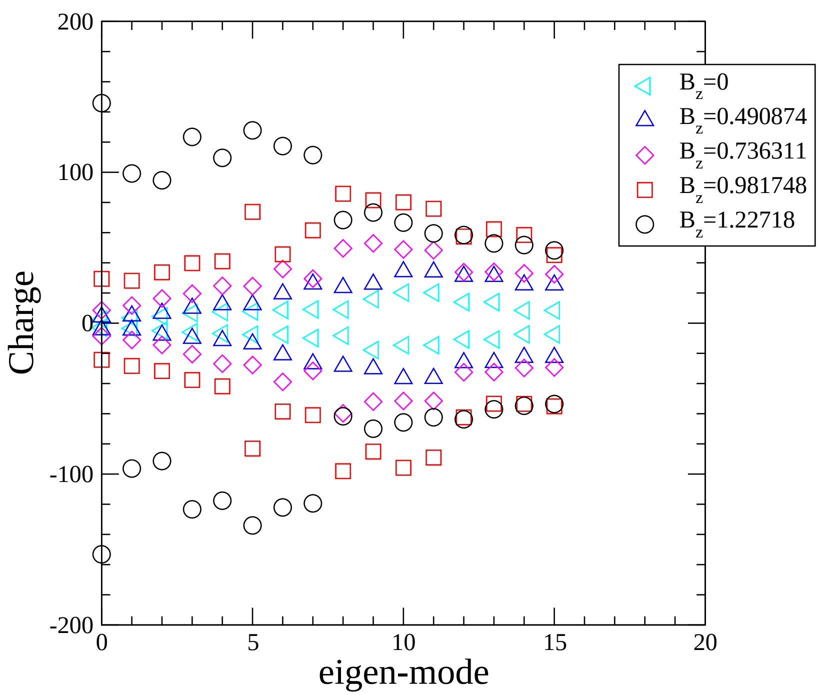


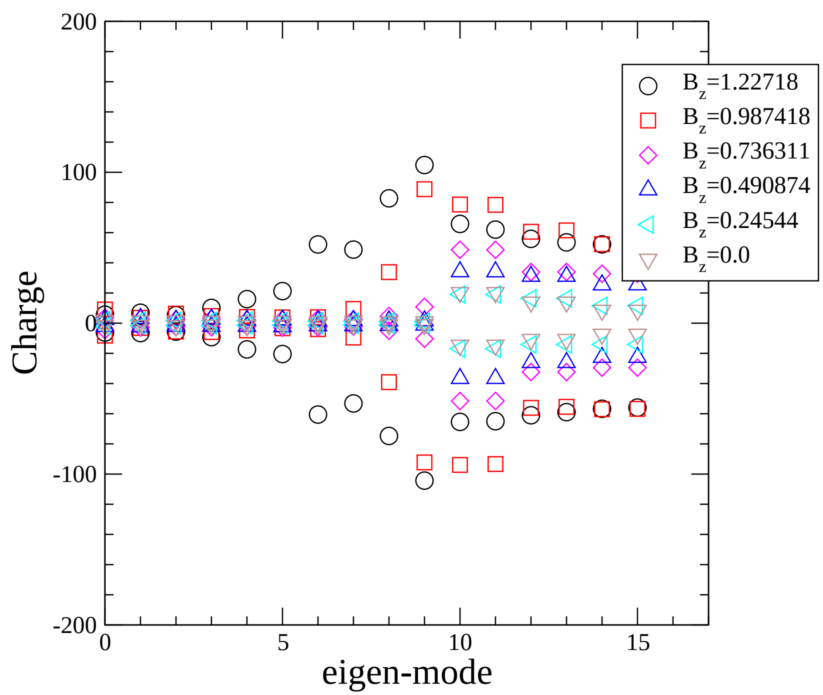
$B_z = 0.490874, 0.736311$

$B_z = 0.981748, 1.22178$

$|\rho_{\max}| = 0.002, 0.167, 1.627, 1.825$

Charge separation (from zero modes)





Charge separation for large B_z, $n_\Phi = 30$ to 50
Depends on L_s (lattice artifact χ SB – expensive)

Charge separation (from zero modes)

How large is large?

$a^2 eB_z = 2\pi/(L_x L_y) n_\Phi$

$T \approx T_c$, so $a^{-1} \approx 1.4 - 1.5$ GeV ($\sim$0.14 fm)

$B_z \approx 1.5 - 2$ GeV^2

if $r_{\text{inst}} \approx (1-2)a$, $(L/r_{\text{inst}})^2 = 16 - 32$

quenched studies: $\langle r_{\text{inst}} \rangle \approx 0.3$ fm

Charge separation (from non-zero modes)

Is the vacuum lumpy? (Phys. Rev. D 65 (2001) and others)

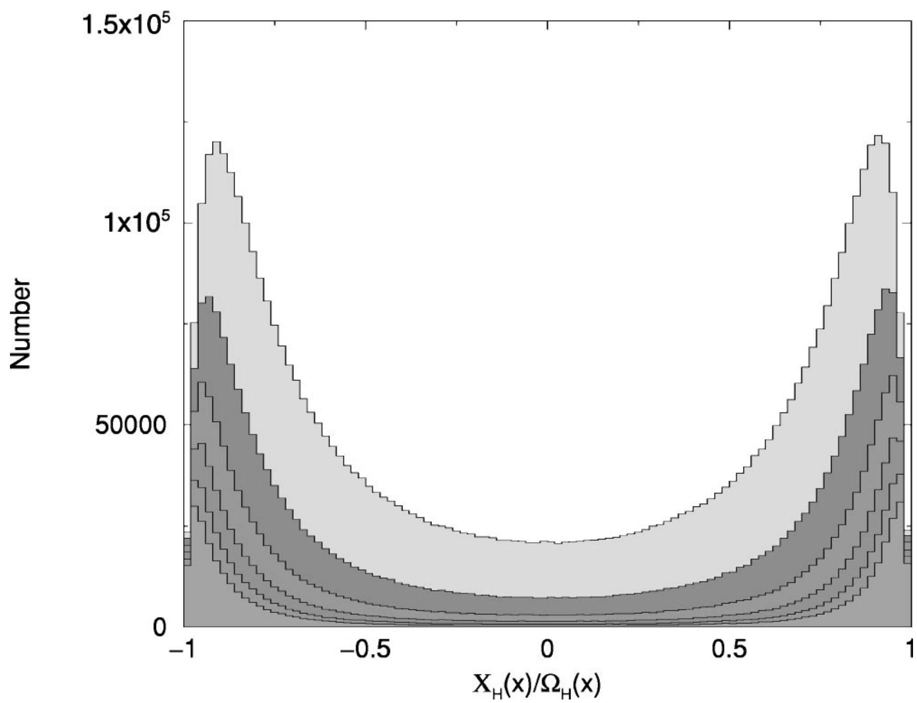


FIG. 5. The same quantities as in Fig. 4, but for nonzero mode eigenvectors and again for the Iwasaki action. The double-peak structure is a feature expected in instanton-dominated models of the QCD vacuum.

local chirality, $\psi^\dagger\gamma_5\psi(x)$ will be peaked around ± 1 if it is

So near-zero modes probably contribute to charge-separation as well

But they will "screen" each other

Correlate with local chirality?

Summary

- 2+1 (1+1+1) QCD+QED simulations to investigate chiral magnetic effect
- Initial results for classical instanton (-like) and QCD configurations show that it really works!
- Need T, $\vec{B}$, m_q scans
- "Unfreeze" topology (Q) of gauge field
- Exploit dynamical QED+QCD configurations
- Important for understanding the recent results from RHIC

Calculations done on NY blue and QCDOC supercomputers at Brookhaven National Lab.

Thanks to Dima Kharzeev for useful discussions and Massimo Di Pierro for help with 3d plots