Thermal Abelian Monopoles in the Deconfined Phase of Yang-Mills Theories

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1 – INTRODUCTION

Various mechanisms for color confinement are based on the condensation of topological objects (e.g. monopoles, vortices) in the QCD vacuum.
Such topological objects could be relevant also to explain many strongly interacting properties of the deconfined phase.

Among other topological defects, we are interested here in magnetic monopoles. At $T = 0$, their condensation is thought to induce dual Meissner effect in the dual superconductor model ('t Hooft, 1975, Mandelstam, 1976)
The possible role played around and above T_c by thermal monopoles "evaporating" from the zero T condensate has attracted a lot of attention in the last few years.
(Liao-Shuryak, 2006-2008, Chernodub-Zakharov, 2006-2008, Ratti-Shuryak, 2008).

Monopoles are particle-like defects and therefore it is tempting to describe their properties as those of a particle ensemble.

2 – OUTLINE

- **Abelian projection and Abelian monopoles on the lattice**

- **Thermal monopoles in the Maximal Abelian Gauge: density and interactions**

- **Can we follow their way back to condensation as $T \to T_c$ from above?**

- **Abelian projection dependence and gauge independent properties**

- **More than just magnetic charge?**

3 – Abelian monopoles on the lattice

Magnetic monopoles at work in the dual superconductor model are of Abelian nature.

An abelian subgroup of the gauge group must be fixed (e.g. fix the gauge up to a $U(1)$ residual freedom) to identify the condensing magnetic charge: Abelian Projection

Abelian projection is assigned in terms of an adjoint field (we specialize to SU(2)):

$$\vec{\sigma} \cdot \vec{\phi}(x) \to G(x)(\vec{\sigma} \cdot \vec{\phi}(x))G^\dagger(x)$$

fixing $\hat{\phi} \equiv \vec{\phi}/|\vec{\phi}|$ fixes the gauge up to $U(1)$. The e.m. tensor is the 't Hooft tensor:

$$F_{\mu\nu} = \partial_\mu(\hat{\phi} \cdot \vec{A}_\nu) - \partial_\nu(\hat{\phi} \cdot \vec{A}_\mu) - \frac{1}{g}\hat{\phi} \cdot (\partial_\mu\hat{\phi} \wedge \partial_\nu\hat{\phi})$$

In the gauge where $\hat{\phi} = (0,0,1)$ everywhere then $F_{\mu\nu} = \partial_\mu A^3_\nu - \partial_\nu A^3_\mu$ and the Abelian projection corresponds to taking the diagonal part of gauge links.

There is no natural adjoint field in QCD, but several adjoint fields can be constructed in terms of gauge fields, e.g. in terms of a closed path ordered product of gauge links, like an open Polyakov loop or a spatial or temporal plaquette.

Choice of the adjoint field $\Longrightarrow$ ambiguity in definition of the Abelian projection

A popular choice is the Maximal Abelian Gauge (MAG) projection, where $\hat{\phi} = (0, 0, 1)$ in the gauge where the following functional is maximum:

$$F_{\mathrm{MAG}} = \sum_{\mu,x} \mathrm{Re}\, \mathbf{tr} \left[U_\mu(x) \sigma_3 U_\mu^\dagger(x)\, \sigma_3 \right]$$

$F_{\mathrm{MAG}} \sim$ average squared diagonal part of the gauge links, in that gauge Abelian phases carry large part of the original gauge fields (Abelian dominance)

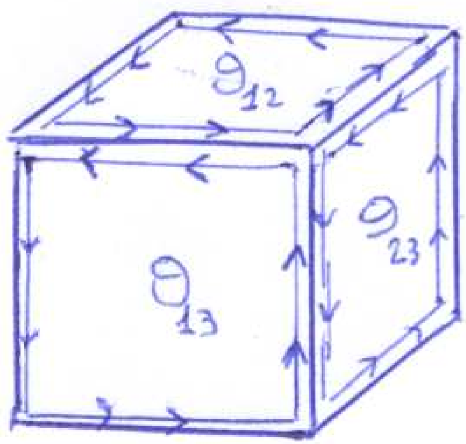
MAG projection has been also recently proposed as a preferred gauge to expose the magnetic content of gauge configurations (Bonati, Di Giacomo, Lepori, Pucci, arXiv:1002.3874)

MAG has been our choice, we discuss later about Abelian projection ambiguities.

In the gauge where $\hat{\phi} = (0,0,1)$, Abelian link phases are extracted as follows:

$$U_\mu = u^0_\mu \mathrm{Id} + i\vec{\sigma}\cdot\vec{u}_\mu \to u^0_\mu \mathrm{Id} + i\sigma_3 u^3_\mu \propto \mathrm{diag}(e^{i\theta_\mu}, e^{-i\theta_\mu})$$

from which Abelian plaquettes can be constructed $\theta_{\mu\nu} \equiv \hat{\partial}_\mu \theta_\nu - \hat{\partial}_\nu \theta_\mu$



Monopole currents are then constructed by the usual De Grand - Toussaint construction.

$$m_\mu = \frac{1}{2\pi}\varepsilon_{\mu\nu\rho\sigma}\hat{\partial}_\nu \overline{\theta}_{\rho\sigma}$$

$$\theta_{\mu\nu} = \overline{\theta}_{\mu\nu} + 2\pi n_{\mu\nu}$$

i.e. we measure the net magnetic flux going out of a cube, modulo Dirac string contributions

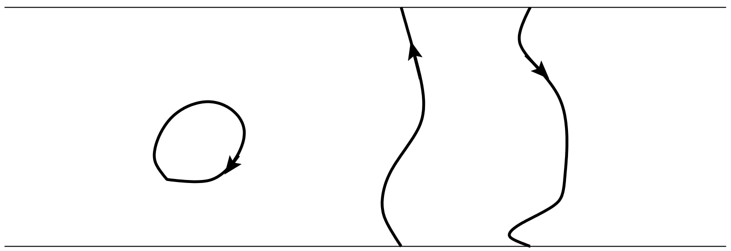
Topological object or UV lattice artifact? Scaling to the continuum is a good criterion, but see later for more discussion

Monopole currents form closed loops, since $\hat{\partial}_\mu m_\mu = 0$. These loops may be either topologically trivial or wrapped around the lattice.

Below T_c typically one big cluster of monopole currents appears, percolating in all directions. Above T_c such cluster disappears, but monopole currents with a non-trivial wrapping in the temporal direction survive

V.G. Bornyakov, V.K. Mitrjushkin and M. Muller-Preussker, Phys. Lett. B284, 99 (1992);

S. Ejiri, Phys. Lett. B376, 163 (1996).



According to a recent proposal, such wrapping currents should be identified with thermal monopoles evaporating from the condensate and becoming a magnetic component of the QGP M. N. Chernodub and V. I. Zakharov, Phys. Rev. Lett. 98, 082002 (2007)

A systematic study of the properties of such thermal monopoles above T_c is the purpose of our investigation.

We take $SU(2)$ pure gauge theory as a reference system. The temperature is $T = 1/N_t a(\beta)$ and we use different values of a and N_t to check the scaling to the continuum limit.

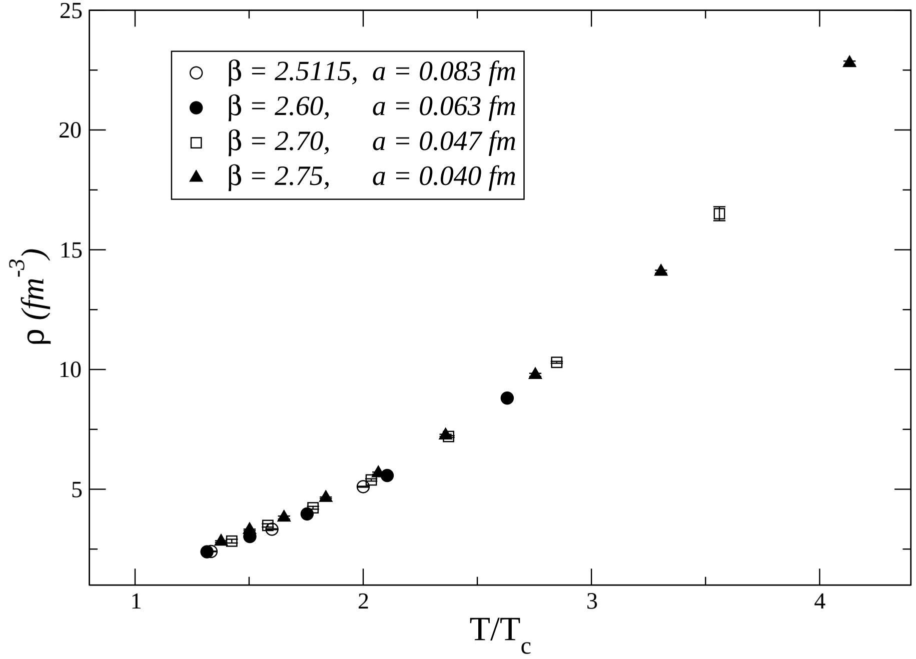
4 – Thermal monopole density and interactions

A. D'Alessandro, M. D., Nucl. Phys. B 799 241 (2008)

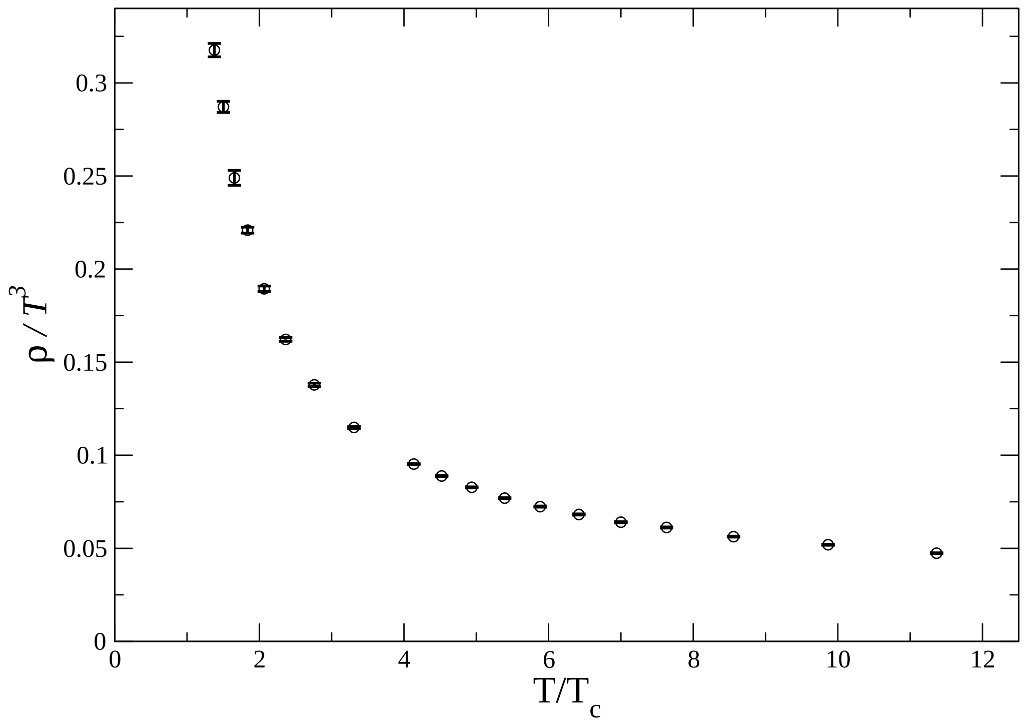
The thermal monopole density is then defined as

$$\rho = \langle \sum_{\vec{x}} |N_{wrap}(m_0(\vec{x},t))| \rangle / V_s$$

$N_{wrap}(m_0(\vec{x},t))$ **is the winding number of the current** $m_0(\vec{x},t)$, $V_s = (L_s a)^3$ **is the spatial volume**



the density of wrapped trajectories scales well to the continuum limit



Density behaviour: not like that of free massless particles ($\rho \sim T^3$).

$$\frac{\rho}{T^3} = \frac{A}{(\log(T/\Lambda_{eff}))^\alpha}$$

Best fit for $T > 2\,T_c$: $A = 0.48(4)$, $T_c/\Lambda_{eff} = 2.48(3)$ and $\alpha = 1.89(6)$

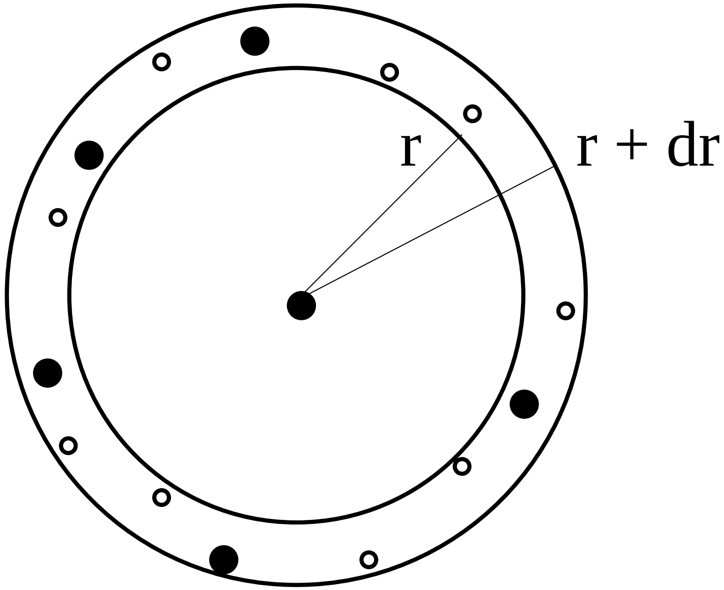
$\alpha = 3$ **(expected exponent in dimensional reduction) works fine for $T > 5\,T_c$.**

Monopoles dominate close to T_c, gluons dominate at very high T.

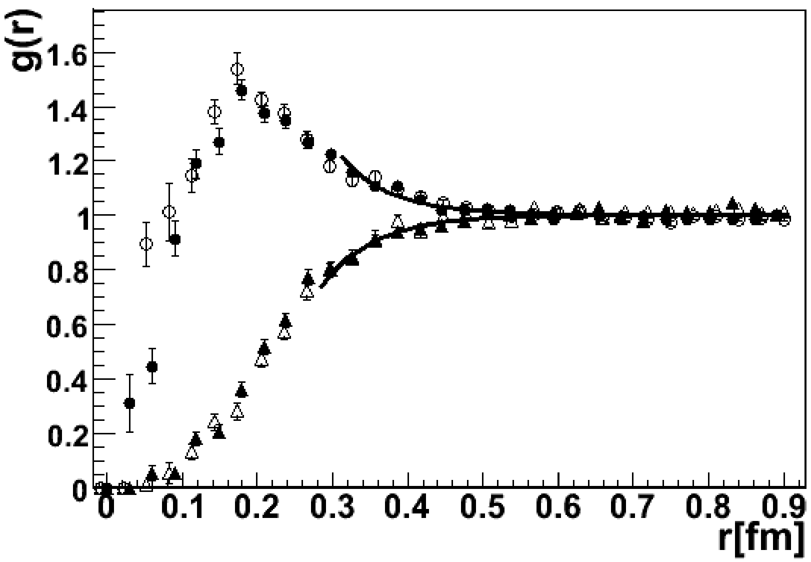
Monopole Interactions

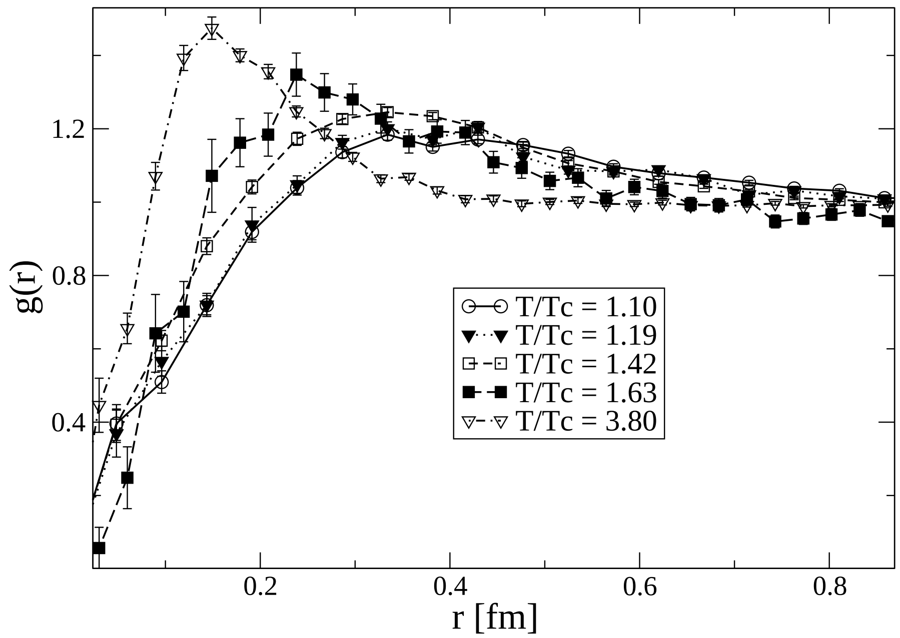
Are wrapped trajectories (thermal monopole) randomly distributed in space? Or do they interact?

$\Longrightarrow$ **fix a reference monopole, count monopoles (antimonopoles) at distance** $\in [r, r+dr]$ **and normalize by the same number expected from random distribution: that gives the correlation function** $g(r)$



- $g(r) = 1 \Longrightarrow$ **no interaction**
- $g(r) \neq 1 \Longrightarrow$ **non-trivial interaction**



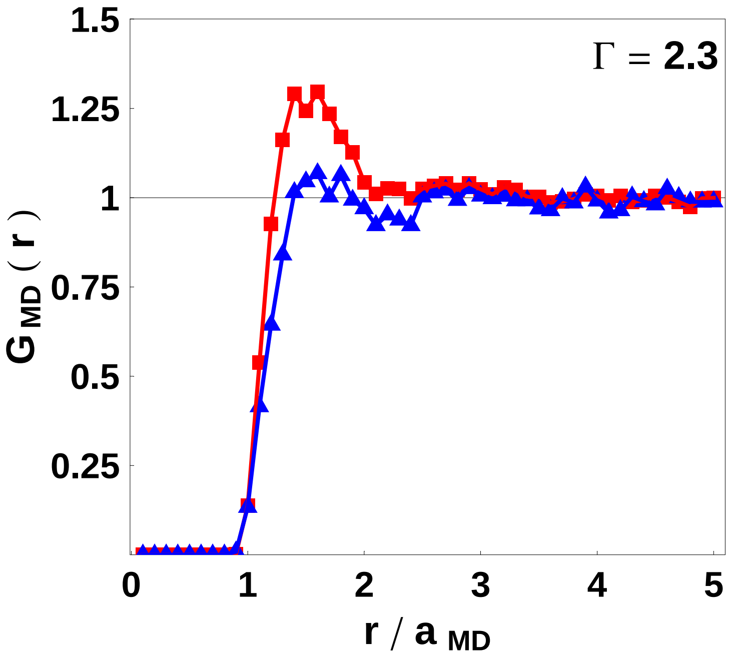


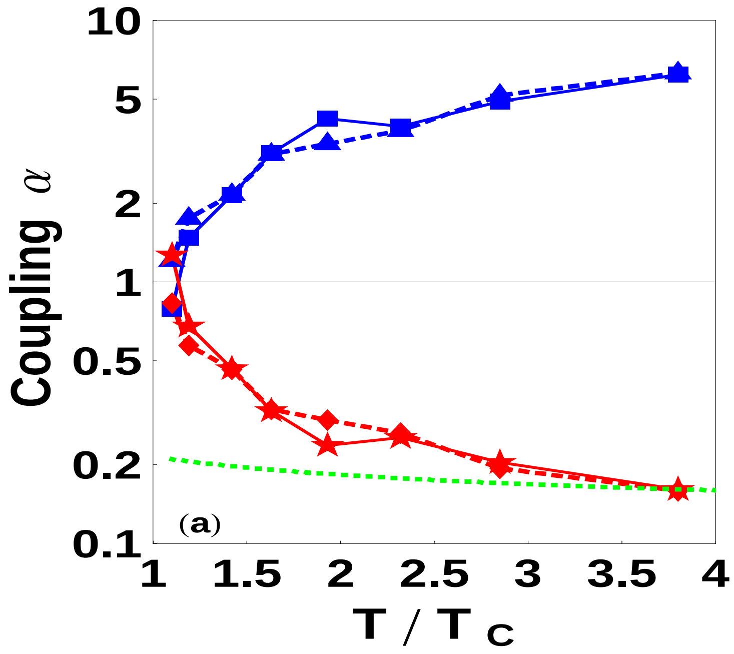
data for $T \simeq 2.85\,T_c$, $\beta = 2.7$ and 2.86

data for mono-antimono $g(r)$, various temperatures

- nice scaling
- monopole-monopole repulsion
- monopole-antimonopole attraction + hard core repulsion
- single peak in $g(r)$: typical of liquid/gas behaviour
- At large distances $g(r) \simeq e^{-V(r)/T}$, where $V(r) = \alpha_M e^{-r/\lambda}/r$ is a screened Coulomb potential, $\lambda \sim$ 0.1 fm.

Further analysis by E. Shuryak and J. Liao from Phys. Rev. Lett. 101, 162302 (2008)





- molecular dynamics results for $g(r)$ similar to lattice results
- magnetic coupling α_M extracted from lattice data grows at high temperatures: duality of electric-magnetic couplings at work.
- $\Gamma = \alpha_M(4\pi\rho/3)^{1/3}/T$ (which gives an estimate of interaction/kinetic energy ratio) stays above 1 down to $T_c \implies$ liquid-like behaviour till close to T_c.

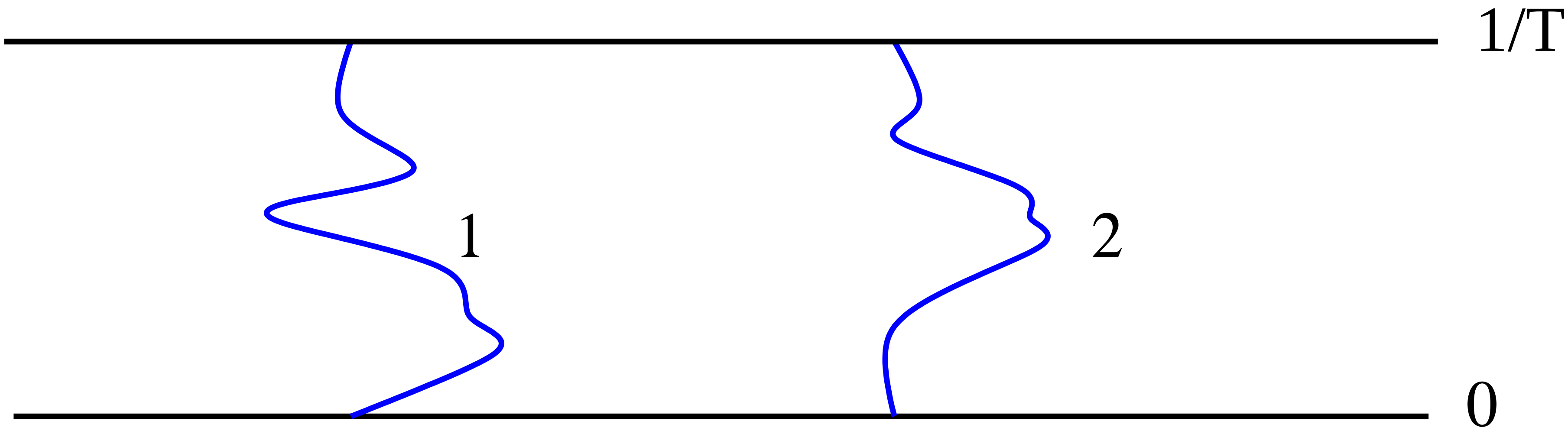
5 – Thermal Monopoles and Confinement

A. D'Alessandro, M.D., E. Shuryak, arXiv:1002.4161

If thermal monopoles above T_c are really the objects condensing below T_c, can we follow their way back to condensation as we approach T_c from above?

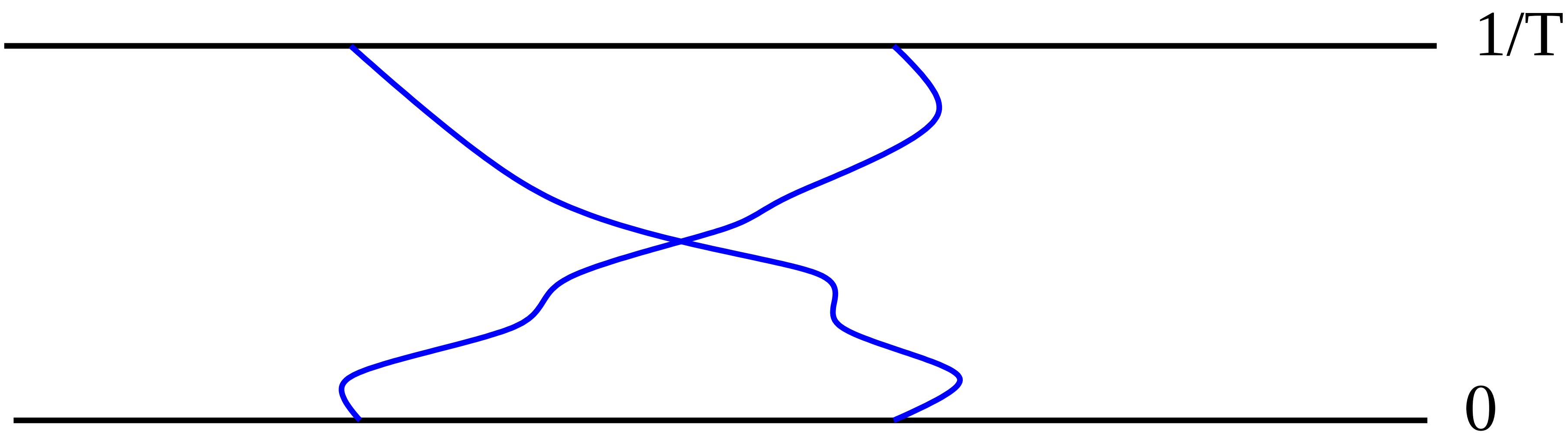
Such an approach complements standard studies about the validity of the dual superconductor model for color confinement which look at the spontaneous breaking of a magnetic symmetry below T_c (Pisa, Bari, Moscow groups).

Condensation is a phenomenon related to the identity of quantum particles. Quantum statistics properties are encoded in monopole trajectories wrapping two or more times in the Euclidean time direction.



What distinguishes the path integral representation of the thermal partition function for two distinguishable particles from that for two identical particles?

For non-identical particles we have to sum over all pairs of periodic (in Euclidean time) disconnected paths ...



... instead for identical particles we have to include doubly wrapping paths corresponding to an exchange of the two particles

$$Z(T) \propto \int d^3x_1 d^3x_2 (\langle x_1 x_2 | e^{-\beta H} | x_1 x_2 \rangle + \langle x_2 x_1 | e^{-\beta H} | x_1 x_2 \rangle)$$

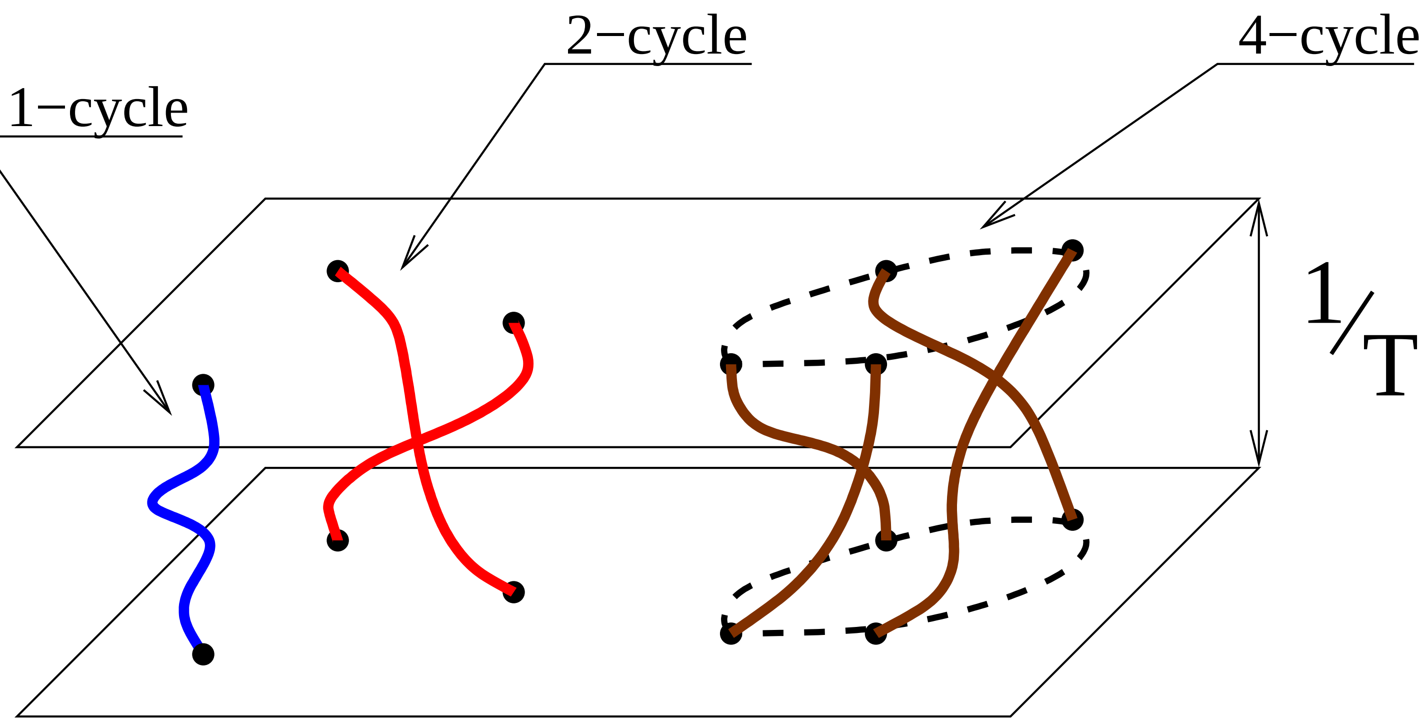
Such paths give an important contribution in the regime in which quantum statistic effects are important

Consider the partition function for N identical particles, e.g. non-interacting bosons

$$Z = \frac{1}{N!} \sum_P \int d^3x_1 \ldots \int d^3x_N \langle x_{P_1} \ldots x_{P_N} | e^{-\beta H} | x_1 \ldots x_N \rangle$$

The sum is now over all possible permutations in the final state. Each permutation can be decomposed into cycles, i.e. grouping together subsets of particles which undergo cyclic permutations and define loops in configuration space.

A k-cycle is a trajectory wrapping k times in time direction.



In the high T limit Boltzmann approximation works well: the largest contribution to the path integral comes from the identical permutation. Non-negligible contributions only from permutations with short cycles.

At low T, large cycles become more and more important, in a critical way as we approach the critical Bose-Einstein condensation (BEC) T (macroscopic cycles appear)

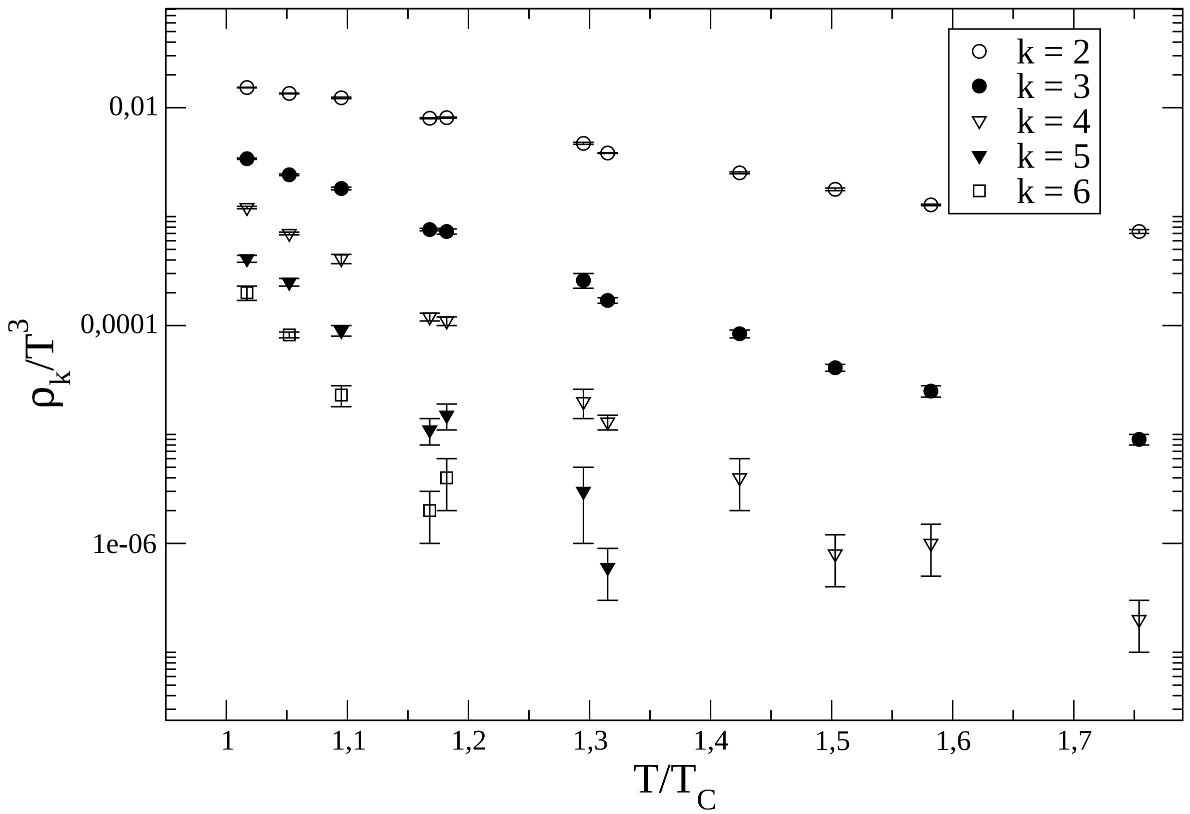
Such an approach goes back to earlier studies about liquid Helium by Feynman in the '50s (see also Elser, 1984).

Can we look at distribution in the number of wrappings of thermal monopoles and understand where they are Boltzmann like or not?

Table of measured densities $^a \longrightarrow a \simeq 0.047$ fm $^b \longrightarrow a \simeq 0.063$ fm

T/T_c	ρ_1/T^3	ρ_2/T^3	ρ_3/T^3	ρ_4/T^3	ρ_5/T^3	ρ_6/T^3
1.017^a	**0.308(2)**	$1.53(1)\ 10^{-2}$	$3.40(5)\ 10^{-3}$	$1.21(3)\ 10^{-3}$	$4.1(3)\ 10^{-4}$	$2.0(3)\ 10^{-4}$
1.052^b	**0.315(5)**	$1.35(1)\ 10^{-2}$	$2.42(4)\ 10^{-3}$	$7.0(2)\ 10^{-4}$	$2.5(2)\ 10^{-4}$	$8.2(5)\ 10^{-5}$
1.095^a	**0.3395(15)**	$1.23(2)\ 10^{-2}$	$1.81(5)\ 10^{-3}$	$4.1(4)\ 10^{-4}$	$0.9(1)\ 10^{-4}$	$2.3(5)\ 10^{-5}$
1.168^b	**0.325(3)**	$8.0(1)\ 10^{-3}$	$7.6(2)\ 10^{-4}$	$1.2(1)\ 10^{-4}$	$1.1(3)\ 10^{-5}$	$0.2(1)\ 10^{-5}$
1.187^a	**0.337(2)**	$8.1(1)\ 10^{-3}$	$7.3(4)\ 10^{-4}$	$1.1(1)\ 10^{-4}$	$1.5(4)\ 10^{-5}$	$0.4(2)\ 10^{-5}$
1.295^a	**0.316(1)**	$4.72(10)\ 10^{-3}$	$2.6(3)\ 10^{-4}$	$2.0(6)\ 10^{-5}$	$0.3(2)\ 10^{-5}$	
1.315^b	**0.297(2)**	$3.83(3)\ 10^{-3}$	$1.7(1)\ 10^{-4}$	$1.3(2)\ 10^{-5}$	$0.6(3)\ 10^{-6}$	
1.424^a	**0.286(1)**	$2.52(5)\ 10^{-3}$	$8.4(7)\ 10^{-5}$	$0.4(2)\ 10^{-5}$		
1.503^b	**0.271(1)**	$1.78(5)\ 10^{-3}$	$4.1(3)\ 10^{-5}$	$0.8(4)\ 10^{-6}$		
1.582^a	**0.252(1)**	$1.28(2)\ 10^{-3}$	$2.5(3)\ 10^{-5}$	$1.0(5)\ 10^{-6}$		
1.754^b	**0.2134(10)**	$7.3(3)\ 10^{-4}$	$9(1)\ 10^{-6}$	$0.2(1)\ 10^{-6}$		
1.780^a	**0.2190(2)**	$6.26(7)\ 10^{-4}$	$8.3(8)\ 10^{-6}$	$0.2(1)\ 10^{-6}$		
2.034^a	**0.1870(4)**	$3.04(10)\ 10^{-4}$	$1.0(4)\ 10^{-6}$			

ρ_k is the density of thermal monopole trajectories wrapping k times



The monopole ensemble is practically Boltzmann like at $T \sim 2\ T_c$, but quantum statistics effect become more and more important as we approach T_c, in agreement with a possible condensation of such objects happening around there

Can we be more quantitative?

For free non-relativistic bosons

$$\rho_k \equiv \frac{\langle n_k \rangle}{V} = \frac{e^{-\hat{\mu}k}}{\lambda^3 \, k^{5/2}} \qquad \rho = \sum_{k=1}^{\infty} k\rho_k = \frac{1}{\lambda^3} \sum_{k=1}^{\infty} \frac{e^{-\hat{\mu}k}}{k^{3/2}} = \frac{2}{\lambda^3 \sqrt{\pi}} \int_0^{\infty} dx \frac{\sqrt{x}}{e^{\hat{\mu}} e^x - 1}$$

$\hat{\mu} \equiv -\mu/T$ **where** μ **is the usual chemical potential for bosons.**

$\hat{\mu} \longrightarrow 0$ **signals the appearance of macroscopic cycles and BEC.**

Can we define also for monopoles a parameter $\hat{\mu}$ **which signals, when it vanishes, the appearance of macroscopically large cycles?**

The thermal monopole ensemble is surely quite far from a free particle ensemble.

Monopole-monopole repulsion is expected to disfavour multiple wrapping trajectories with respect to the free case

Monopole-antimonopole attractive interactions are also expected to play a role.

Taking interactions properly into account is a very difficult task. On general grounds one may expect some finite free energy cost needed to add one particle to a k-cycle plus some interaction dependent contribution:

$$\rho_k = e^{-\hat{\mu} k} f(k)$$

where $f(k)$ is some unknown function decreasing less than exponentially with k.

Our attitude in the following is to give some possible ansatz for $f(k)$ and then try it to fit our data.

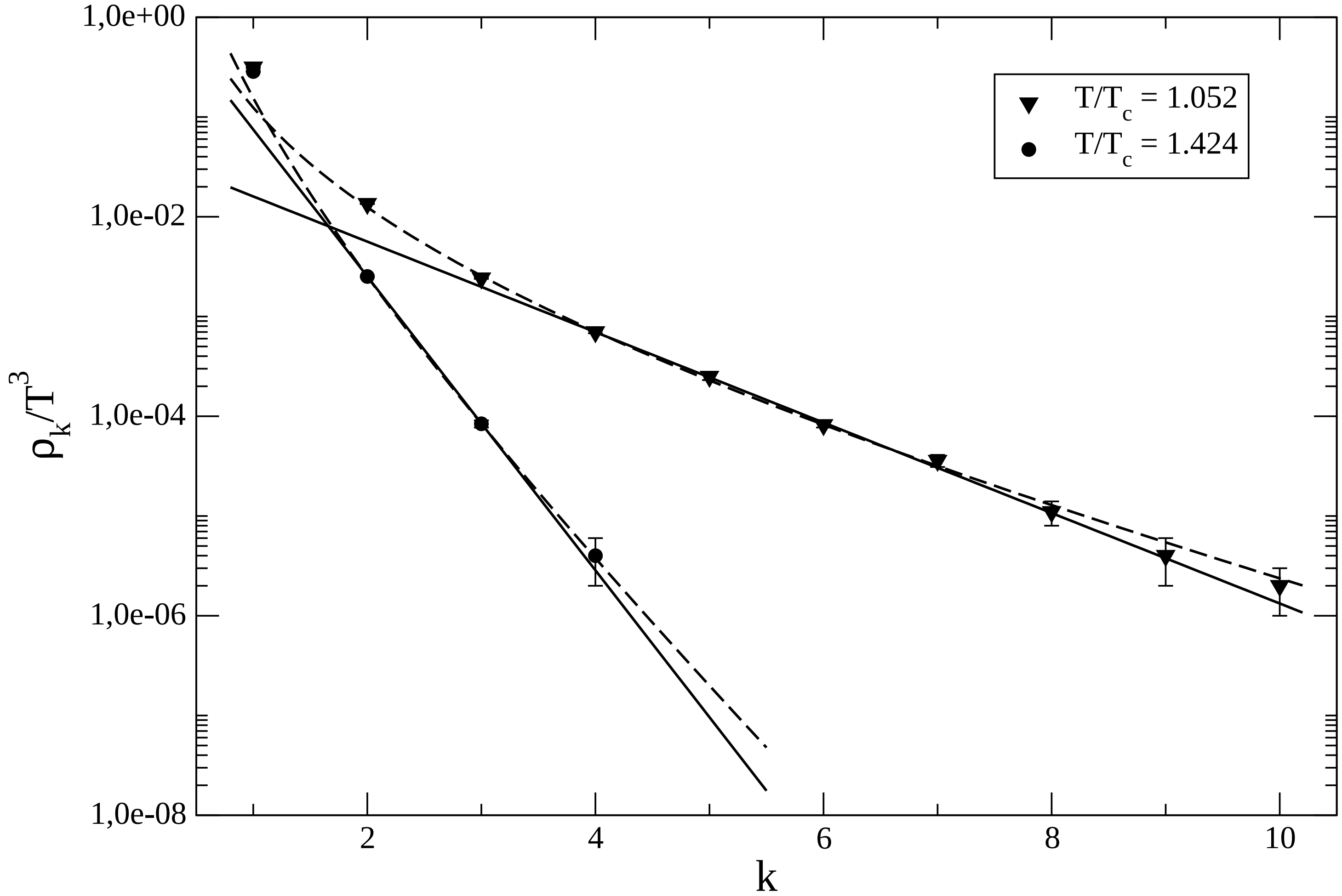
The simplest possibility is that asymptotically (i.e. for large k)

$$f(k) = (\lambda^3 k^{5/2})^{-1}$$

as in the free boson case, with some effective dynamical mass accounting for interactions.

As different possibilities we shall consider more general power law behaviours

$$f(k) \propto 1/k^{\alpha}$$



Fit of the densities ρ_k according to $e^{-\hat{\mu}k}/k^{5/2}$ (dashed line) and according to $e^{-\hat{\mu}k}$ (solid line) for two values of the temperature.

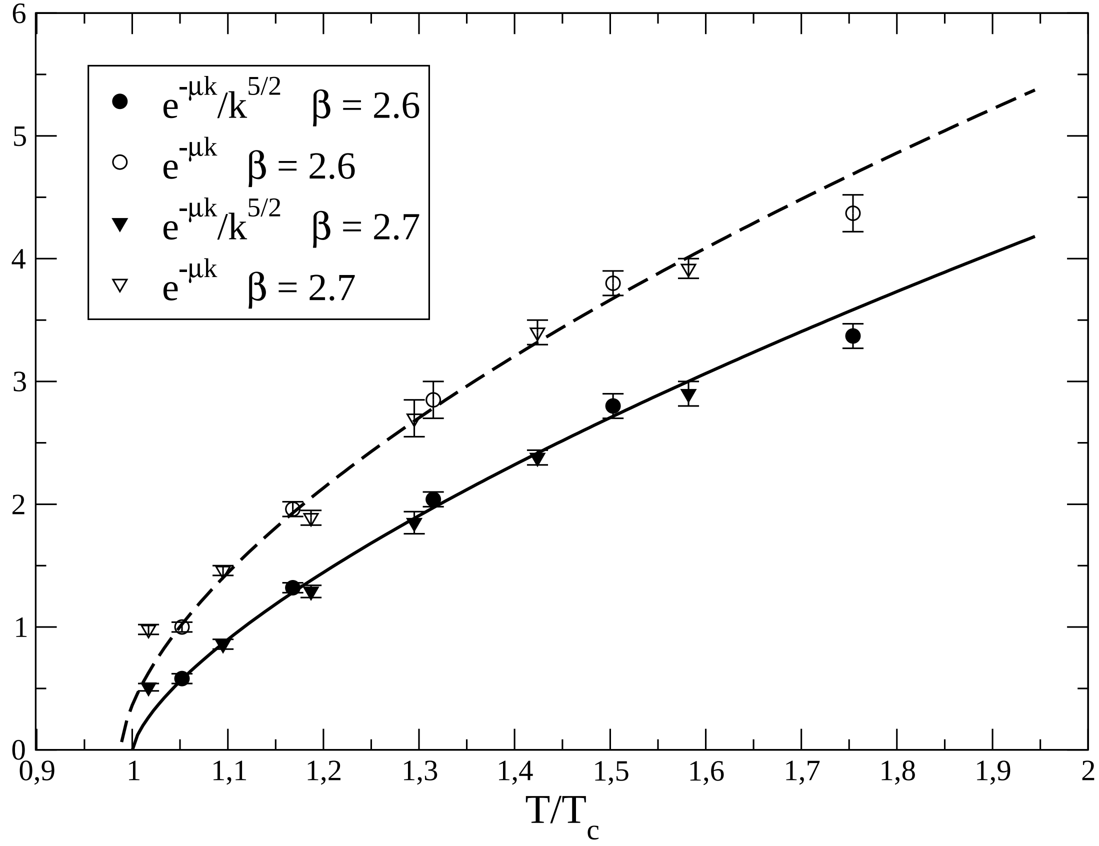
T/T_c	$\mu(\alpha=3)$	$\mu(\alpha=2.5)$	$\mu(\alpha=2)$	$\mu(\alpha=0)$
1.017^a	**0.43(4)**	**0.51(3)**	**0.61(4)**	**0.98(4)**
1.052^b	**0.49(5)**	**0.58(4)**	**0.68(3)**	**1.00(4)**
1.095^a	**0.75(5)**	**0.86(4)**	**1.02(4)**	**1.46(4)**
1.168^b	**1.16(4)**	**1.32(4)**	**1.44(5)**	**1.96(6)**
1.187^a	**1.11(6)**	**1.29(5)**	**1.36(6)**	**1.89(6)**
1.295^a	**1.67(6)**	**1.85(9)**	**2.03(9)**	**2.70(15)**
1.315^b	**1.89(4)**	**2.04(6)**	**2.20(8)**	**2.85(15)**
1.424^a	**2.18(6)**	**2.38(6)**	**2.58(6)**	**3.4(1)**
1.503^b	**2.64(10)**	**2.8(1)**	**3.0(1)**	**3.8(1)**
1.582^a	**2.69(10)**	**2.9(1)**	**3.1(1)**	**3.92(8)**
1.754^b	**3.16(8)**	**3.37(10)**	**3.57(10)**	**4.37(15)**

Depending on the ansatz, different chemical potentials are found

however if one tries to fit data for the chemical potentials according to

$$\hat{\mu} = A\,(T - T_{\mathrm{BEC}})^{\nu'}$$

results are quite independent of the ansatz



α	$T_{\rm BEC}/T_c$	ν'	$\chi^2/{\rm d.o.f.}$
3	**1.005(13)**	**0.71(5)**	**2.24**
2.5	**1.000(12)**	**0.68(5)**	**1.23**
2	**0.989(13)**	**0.68(5)**	**1.72**
0	**0.988(15)**	**0.61(5)**	**2.34**

Our result: thermal monopoles condense at a temperature which coincides with T_c within errors.

Can we also understand the value of the critical index ν' ?

Assume the correlation length ξ grows proportionally to the typical spatial extension of k-cycles.

For a distribution of k-cycles $\rho_k \sim e^{-\hat{\mu}k}/k^{\alpha}$, we have $\langle k \rangle \propto 1/\hat{\mu}$

The typical spatial extension of a k-cycle may grow like k^{ω} where $\omega \sim 1/2$ for a typical random walk behaviour (no interactions) and $\omega \sim 1$ if permutating particles are typically ordered along linear structures by repulsive interactions.

Therefore $\xi \sim \hat{\mu}^{-\omega}$. On the other hand $\xi \sim (T - T_c)^{-\nu}$ ($\nu \sim 0.63$ for $SU(2)$, 3d Ising universality class), hence we expect

$$\hat{\mu} \sim (T - T_c)^{\nu/\omega} \qquad \nu' = \nu/\omega$$

Since $\nu' \sim \nu$, we conclude that $\omega \sim 1$, i.e.:

permutating particles are typically ordered along linear structures by repulsive interactions. A typical cluster of monopole currents wrapping several times in the time direction lies on a time oriented surface.

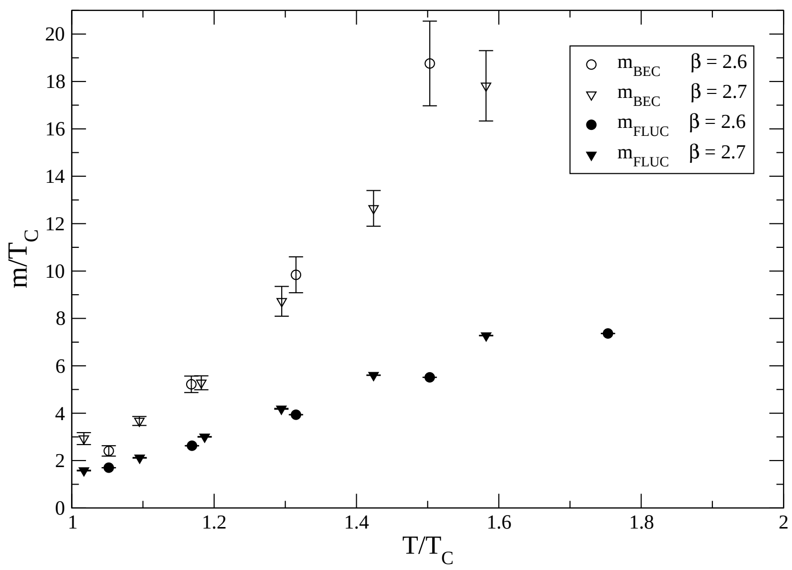
Monopole masses

We have tried two independent definitions of monopole masses:

- **from the thermal de Broglie wavelength λ fitted for our studies of monopole condensation ($m_{\rm BEC}$)**
- **from the fluctuations of thermal monopole trajectories ($m_{\rm FLUC}$)**

$$\Delta x^2 \equiv T \int_0^{1/T} dt \langle (\vec{x}(t) - \vec{x}(0))^2 \rangle \ = 1/(2mT)$$

for free particles



Results are not consistent (would have been for really free particles) apart from very close to T_c, where $m \sim T_c$.

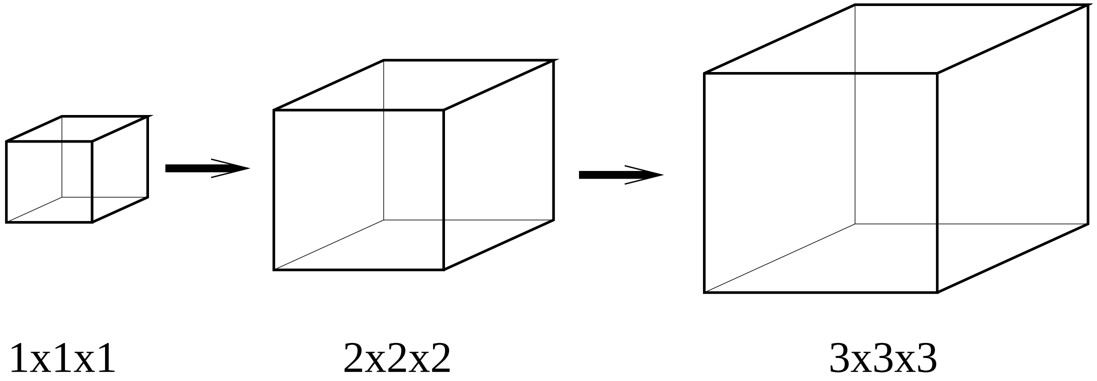
6 – What are MAG thermal Abelian monopoles?

If we change Abelian projection results can change in a drastic way:

- **Landau gauge: no monopoles at all (but can we define an Abelian projection in Landau gauge? U(1) residual degrees of freedom?)**
- **Also fixing Landau and then MAG can change results (Gribov copy problem)**
- **Different Abelian projections, like Polyakov or plaquette, may lead to a factor 10^2 more thermal monopoles: most of them may be just UV artifacts**

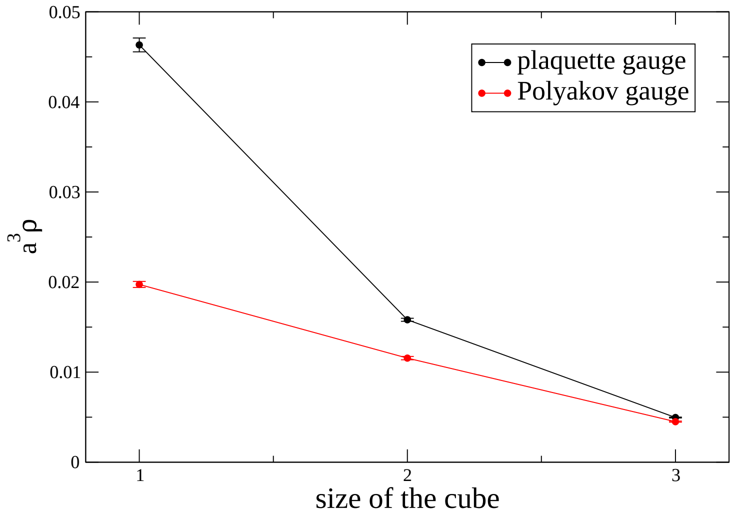
The correct scaling to the continuum limit of results for MAG monopoles makes us confident that they are real topological objects and not UV artifacts.

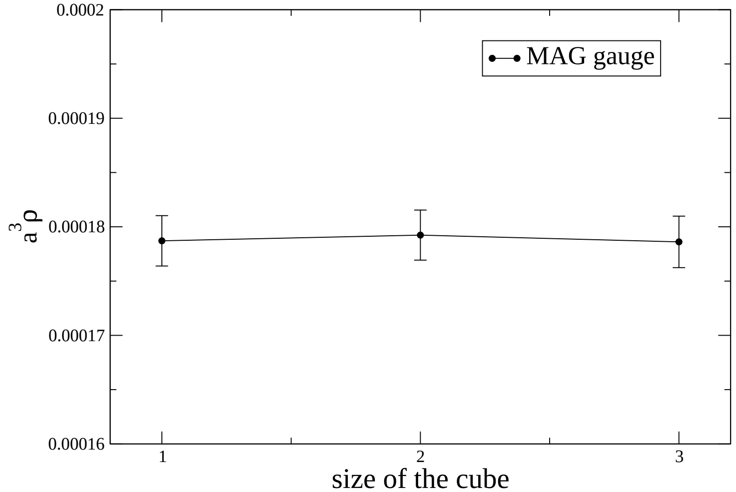
Can we do more tests? For a true monopole the outgoing magnetic field flux should not depend on the surface where we detect it.



do results depend on the size of the cube where the monopole flux is measured?

Not for the MAG monopoles! (results shown for a $48^3 \times 4$ lattice, $T = 1.377\,T_c$)





(D. Dapelo, M.D., in progress; but dependence also for MAG if all monopoles, not just wrapping ones, are considered, L. Del Debbio, A. Di Giacomo, M. Maggiore and S. Olejnik Phys. Lett. B 267, 254 (1991).)

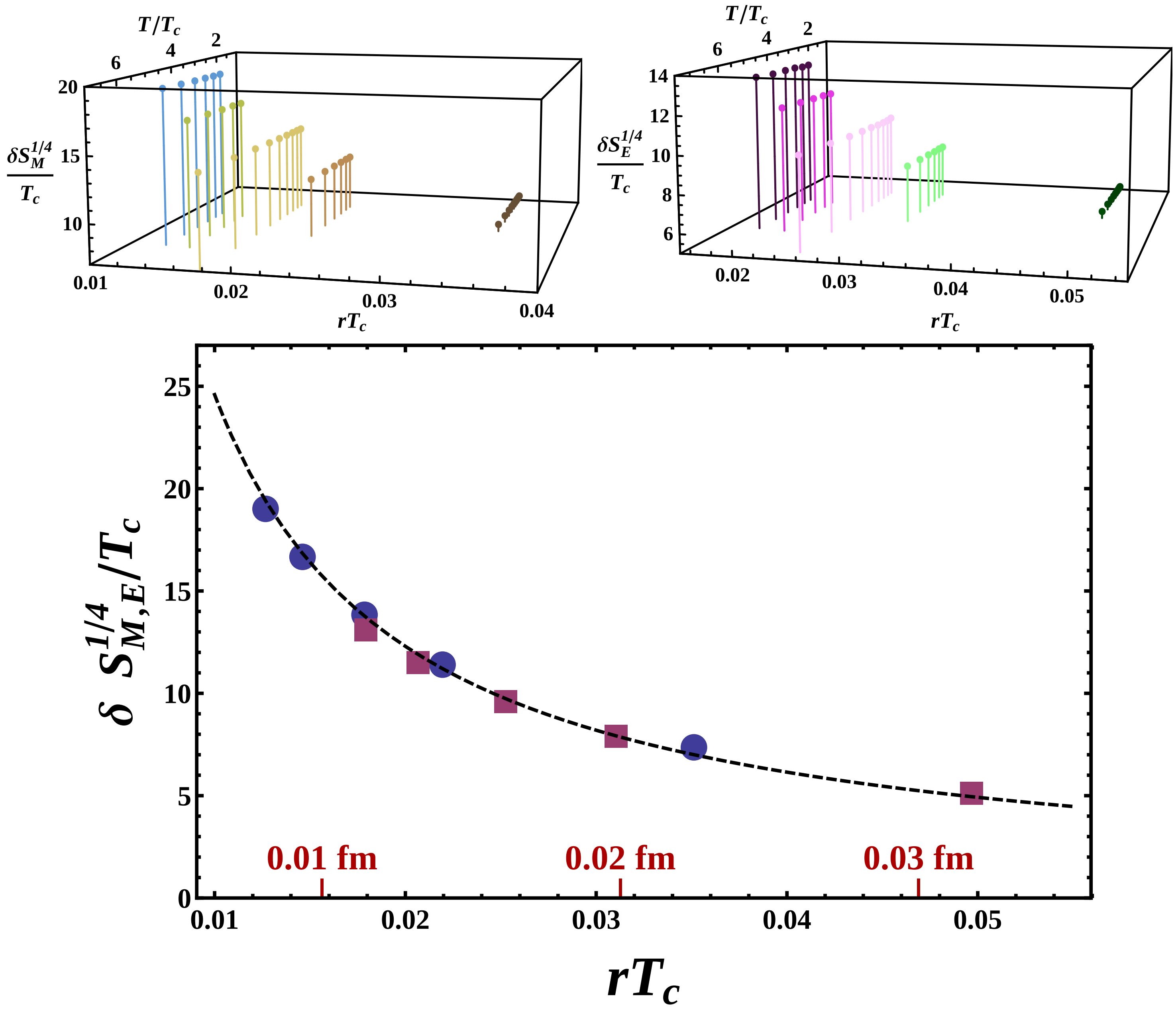
Assuming that MAG projection identifies the correct objects, what non-Abelian gauge configurations are associated with Abelian monopoles and what is their exact physical nature? What other quantum numbers do they carry?

Some steps in this direction have been done

(M. Chernodub, A. D'Alessandro, M.D. and V. Zakharov, arXiv:0909.5441)

It has been shown that a clear correlation exists between the non-Abelian gauge action density and the locations of thermal monopole trajectories; moreover the excess of magnetic action found around monopoles is comparable to that of the electric one:

MAG thermal monopoles, or at least part of them, may be source of both electric and magnetic fields. Can they be source of topological charge fluctuations? May thermal monopoles be similar to dyons?



T/Tc
6
4
2
20
δS_M^{1/4} / T_c
15
10
0.01
0.02
0.03
0.04
rT_c
T/Tc
6
4
2
14
12
δS_E^{1/4} / T_c
10
8
6
0.02
0.03
0.04
0.05
rT_c
δ S_{M,E}^{1/4}/T_c
25
20
15
10
5
0
0.01 fm
0.02 fm
0.03 fm
0.01
0.02
0.03
0.04
0.05
rT_c

7 – CONCLUSIONS

- Topological defects identified as thermal abelian monopoles are an important component of Yang-Mills theories above T_c and it is possible to study their condensation as $T \to T_c$ from above, thus directly relating them to the low temperature magnetic condensate.

- Dependence on the choice of abelian projection requires to better understand their nature from a theoretical point of view and what other (non-abelian) properties may be associated with MAG thermal monopoles. They may be seeds of topological charge fluctuations

- Extension to SU(3) and the interplay of monopoles with fermion degrees of freedom are issues that must be addressed in the future.