Chiral Magnetic Spiral

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RBRC Workshop: P- and CP-odd Effects in Hot and Dense Matter
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G.Başar, GD & D. Kharzeev, arXiv:1003.3464
G.Başar & GD, arXiv:0803.1501, PRL **100**, 200404 (2008)
arXiv:0806.2659, PRD **78**, 065022 (2008)
G.Başar, GD & M.Thies, arXiv:0903.1868, PRD **79**, 105012 (2009)

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outline

- what is a “chiral spiral”
- application to chiral magnetic effect

physics = magnetic catalysis+ topological charge fluctuations

1+1 dim. Gross-Neveu Models

Gross/Neveu, 1974

GN_2 $$\mathcal{L}_{\mathrm{GN}} = \bar{\psi}\, i\, \partial\!\!\!/ \psi + \frac{g^2}{2} \left(\bar{\psi}\psi\right)^2 \qquad \psi \to \gamma^5\, \psi$$

$\chi\mathrm{GN}_2$
NJL_2 $$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}\, i\, \partial\!\!\!/ \psi + \frac{g^2}{2} \left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi} i \gamma^5 \psi\right)^2\right] \qquad \psi \to e^{i\alpha\,\gamma^5}\, \psi$$

- renormalizable at large N_f
- asymptotically free
- chiral symmetry breaking
- dynamical mass generation $\quad m = \Lambda\, e^{-\frac{\pi}{N_f\, g^2}}$
- self-bound baryonic states $\quad m_B = \frac{2}{\pi}\, m$

DHN, 1975

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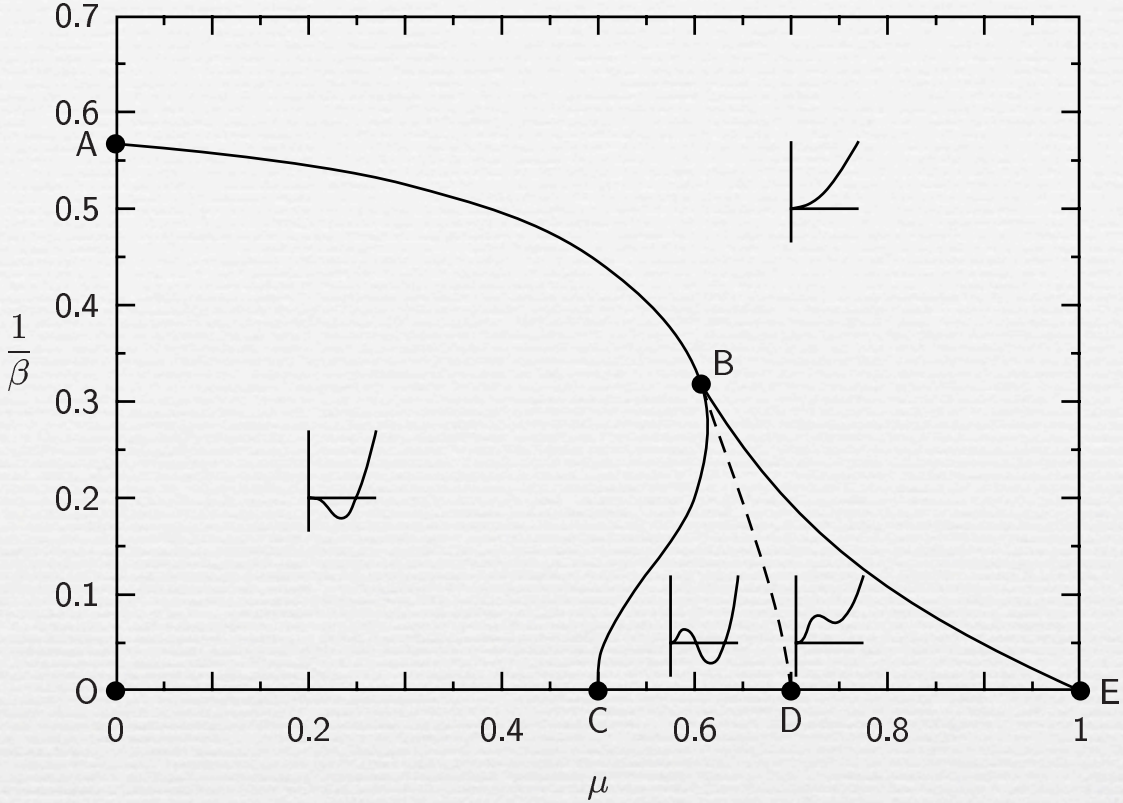
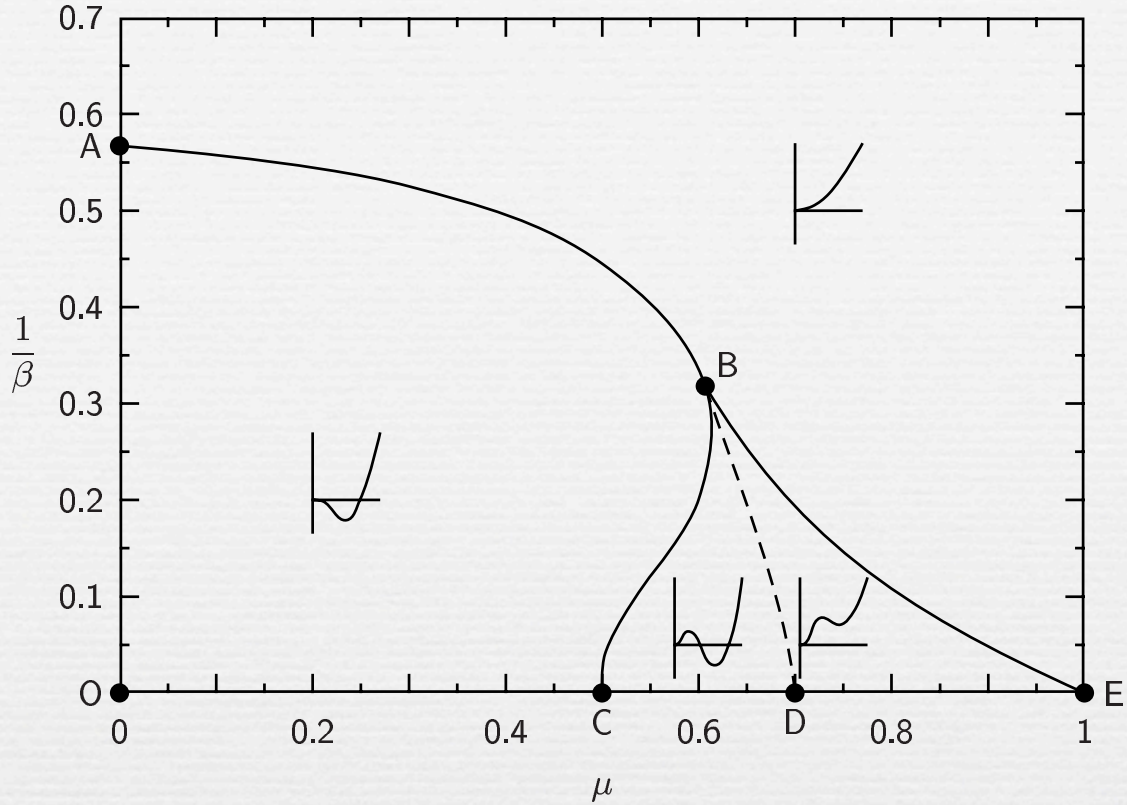
(T, μ) phase diagram?



Phase diagram of Gross-Neveu model

uniform condensate $\sigma = \langle \bar{\psi}\, \psi \rangle$

Wolff, 1985



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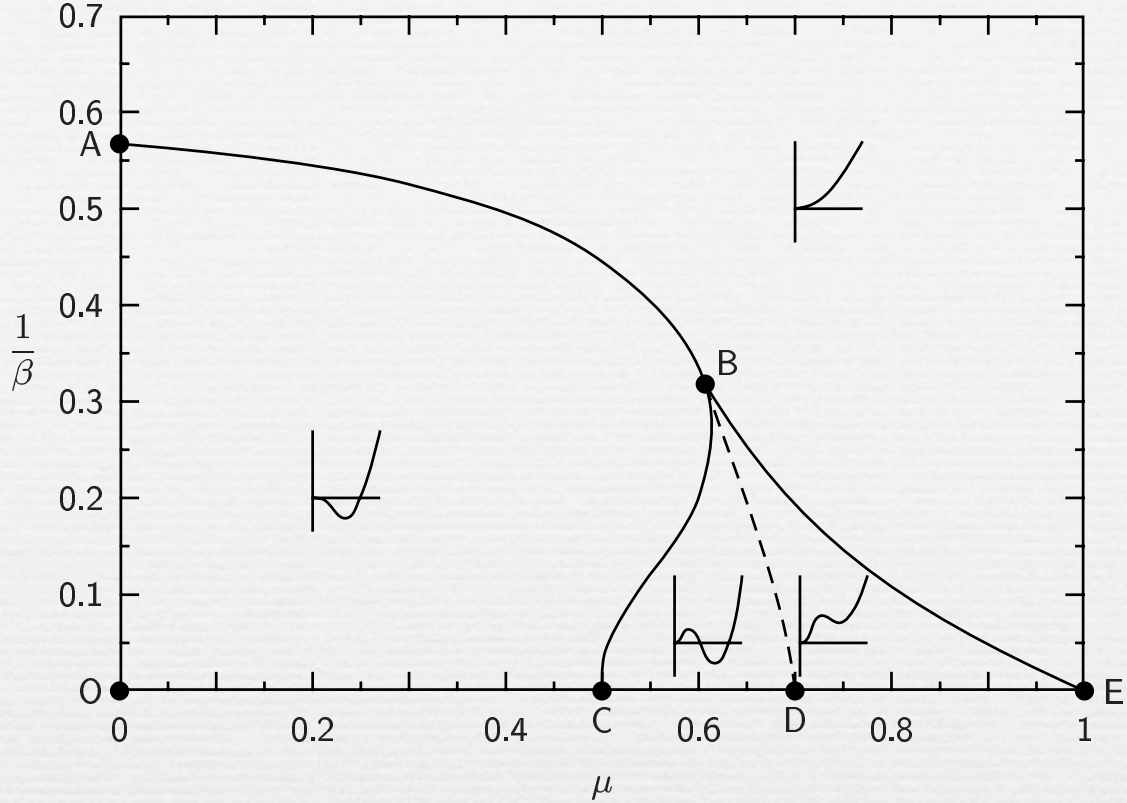

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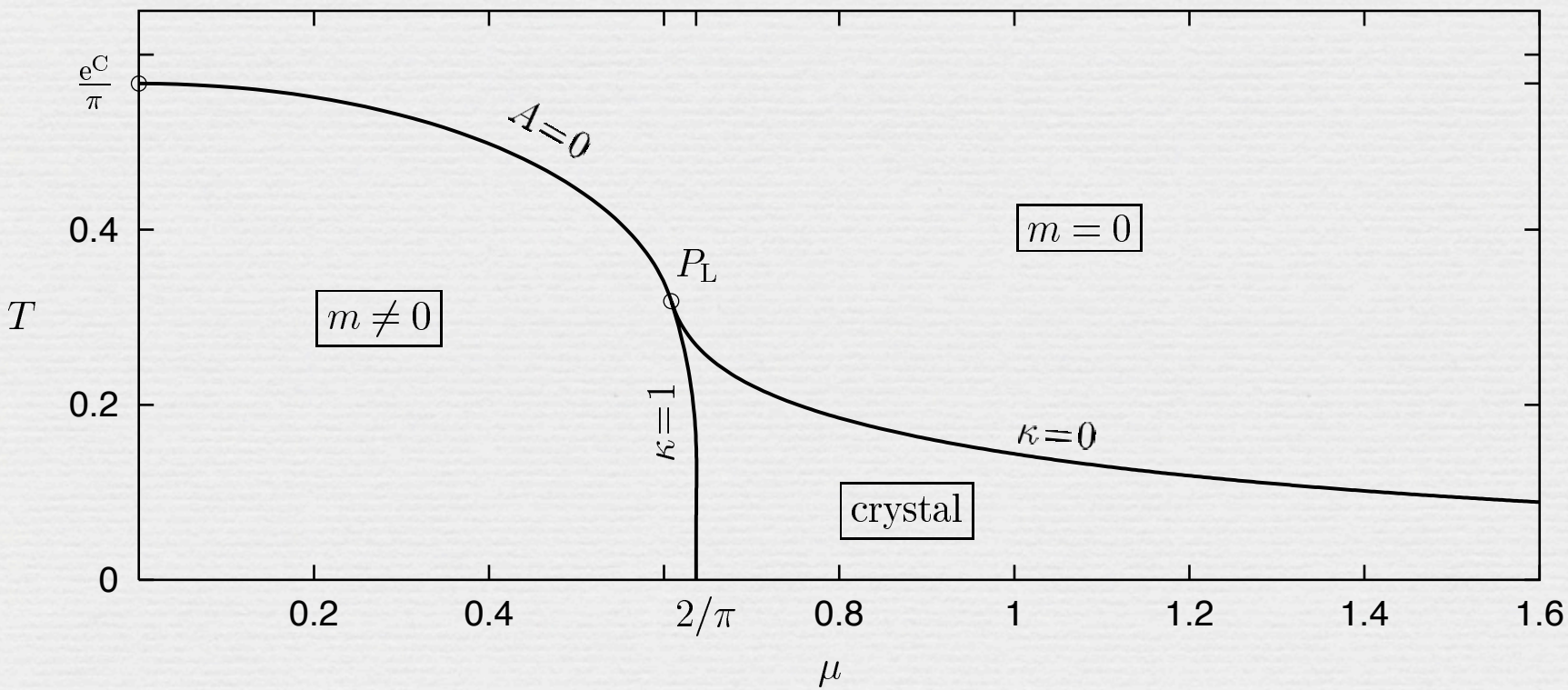

Thies & Urlichs, 2005

Basar, GD, Thies, 2009

periodic, crystalline, phase

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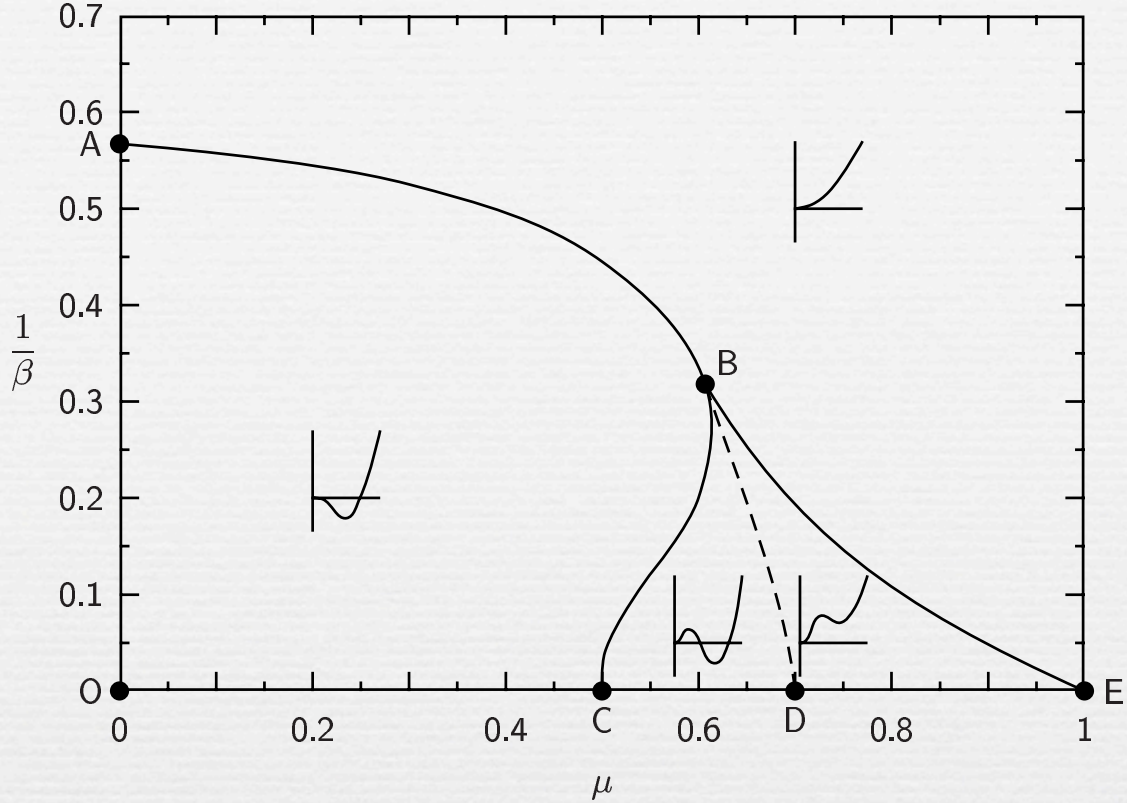

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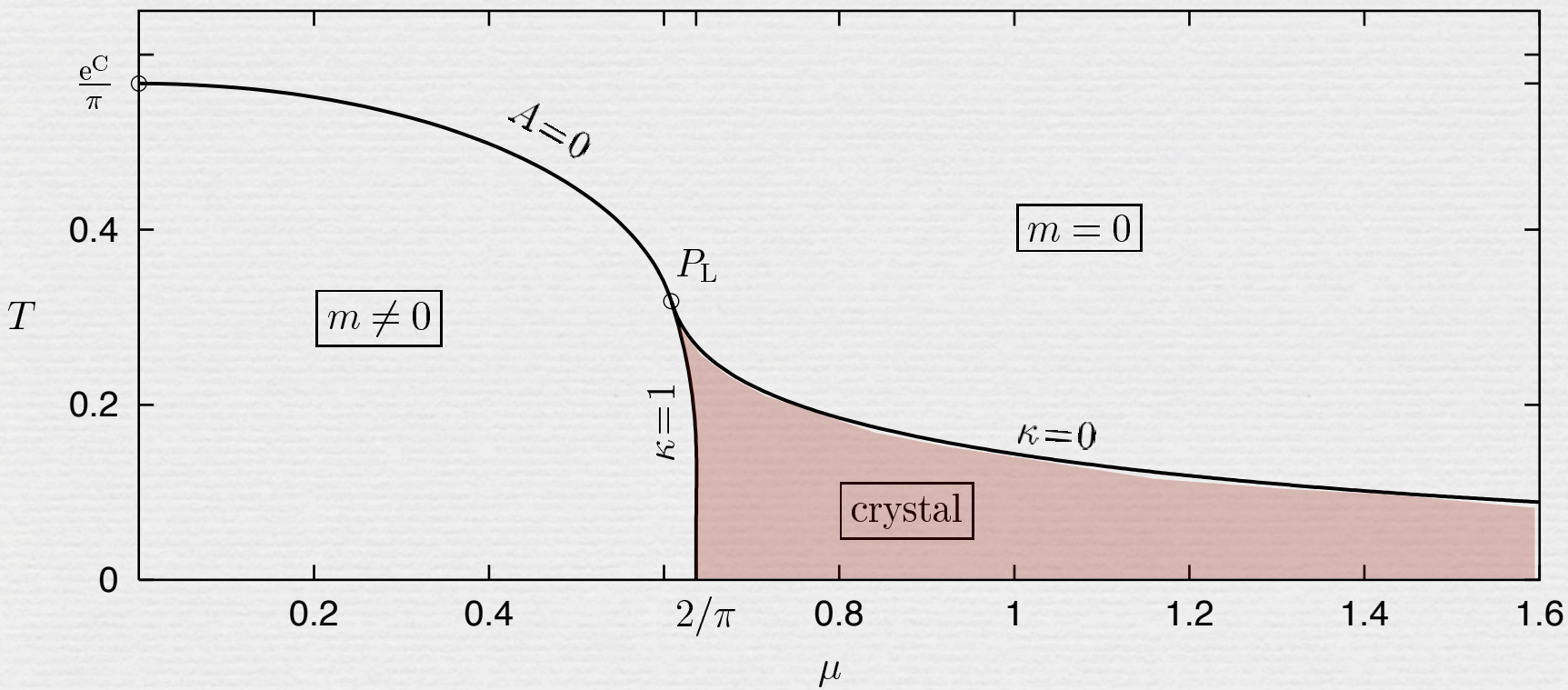

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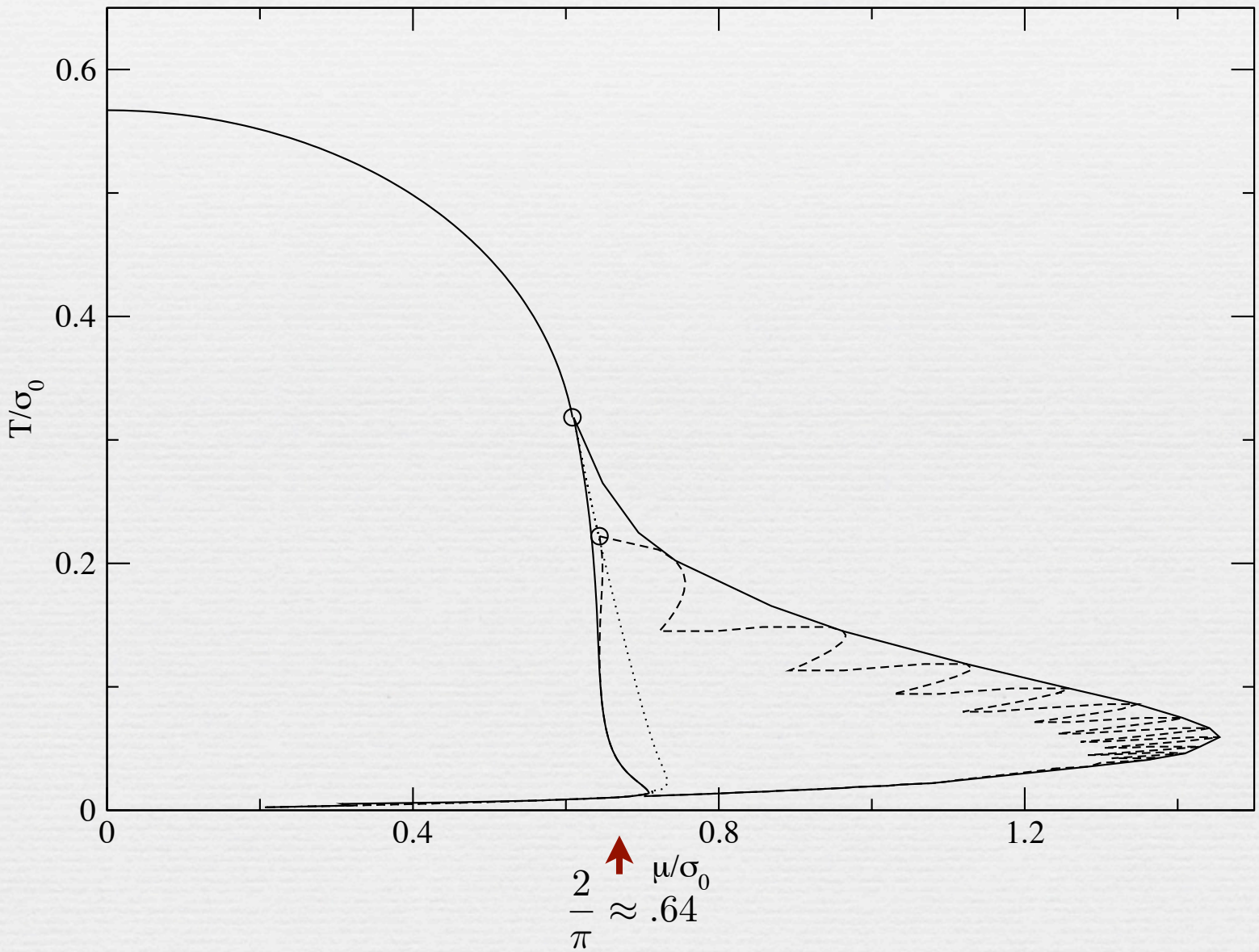
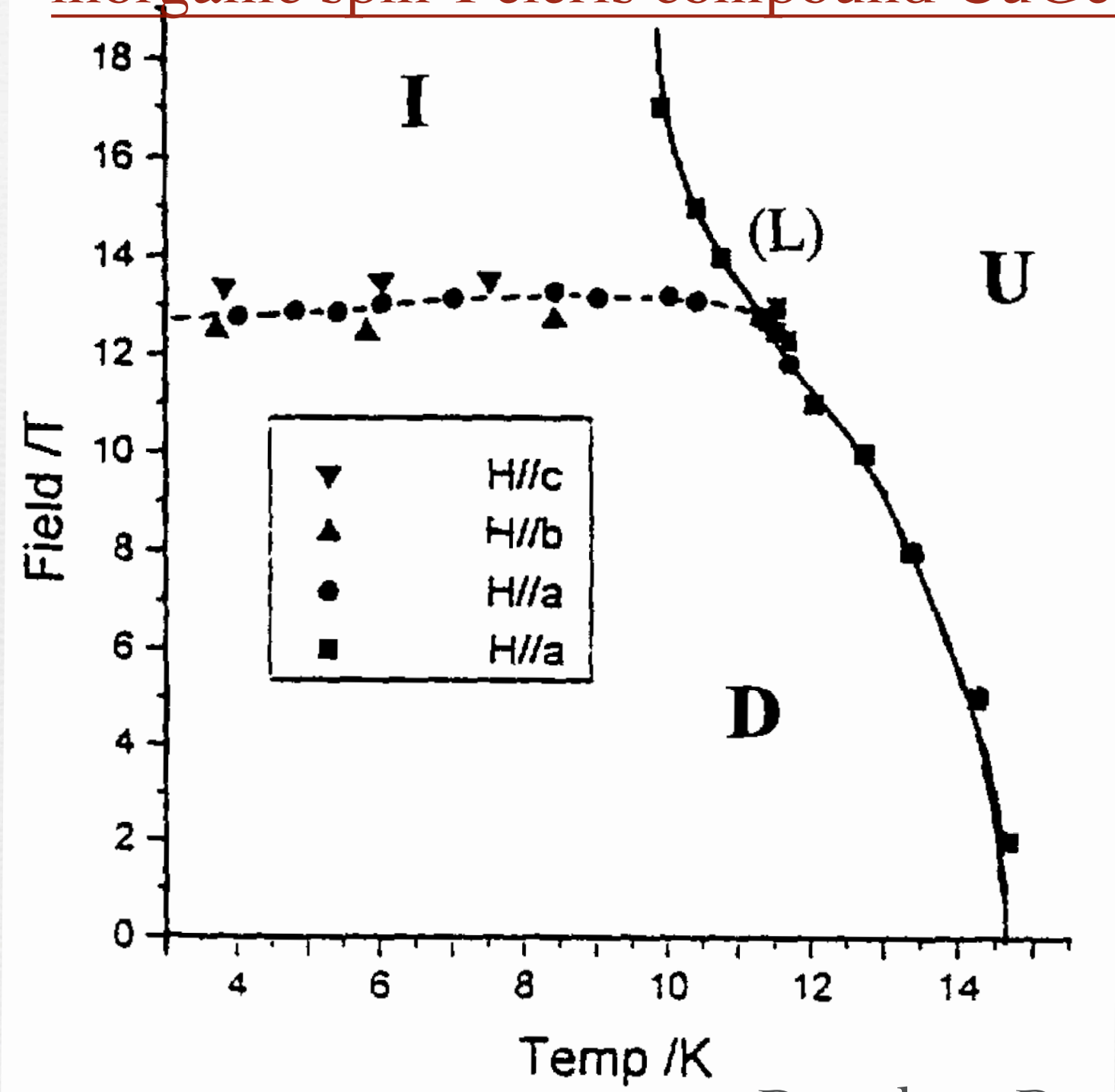
periodic, crystalline, phase

lattice GN_2 model

de Forcrand/Wenger 2006



inorganic spin-Peierls compound $CuGeO_3$



Boucher, Reynault, 1996

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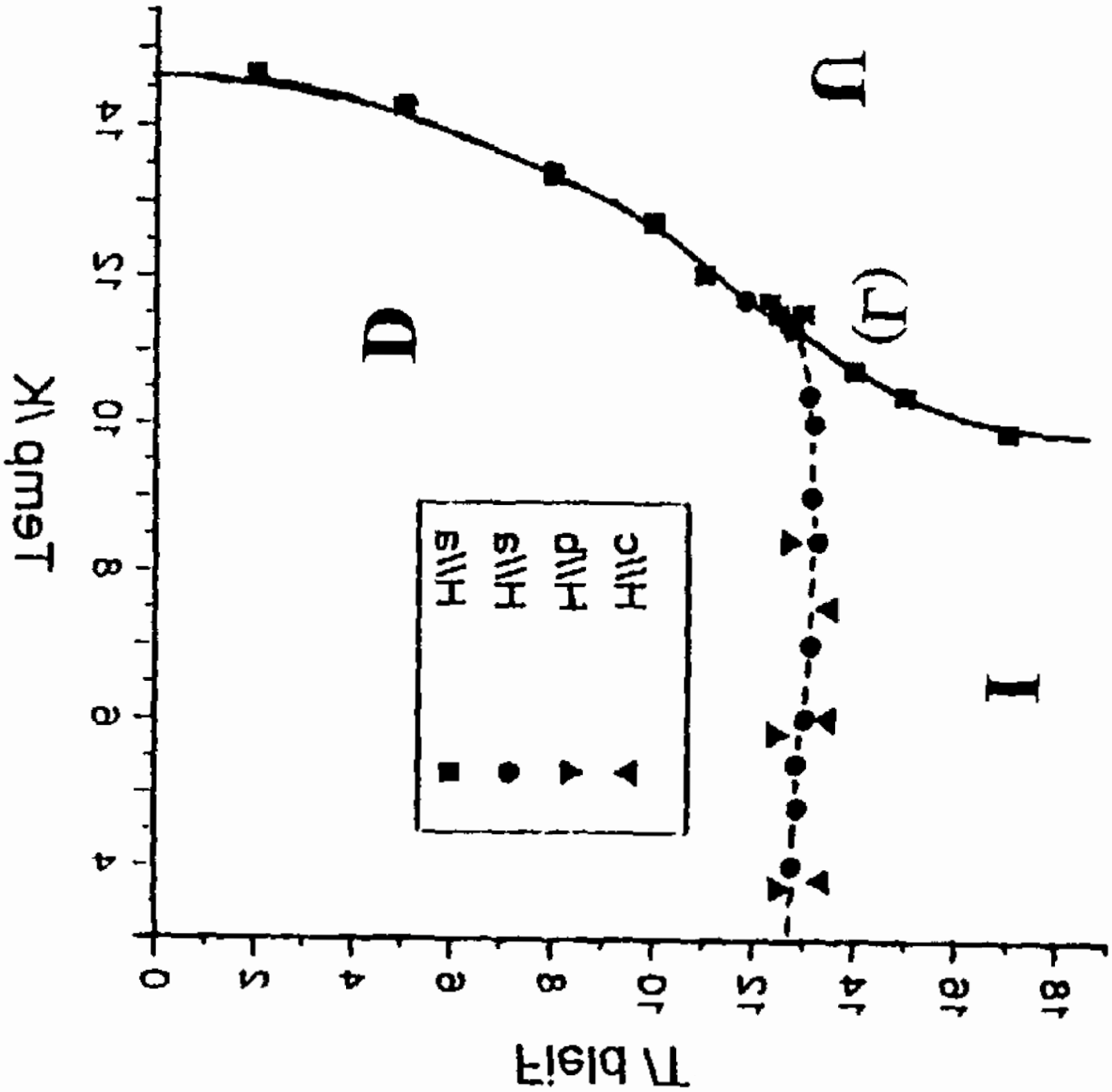

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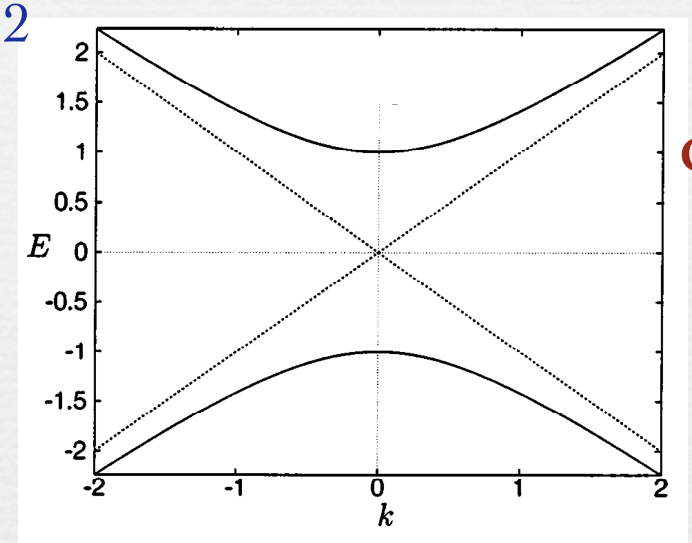
Peierls Instability

one dimension: gap formation at the Fermi surface can lead to breakdown of translational symmetry

GN_2



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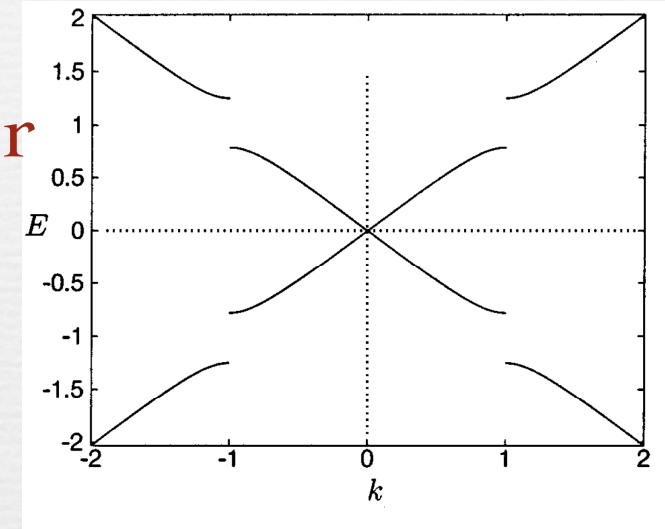
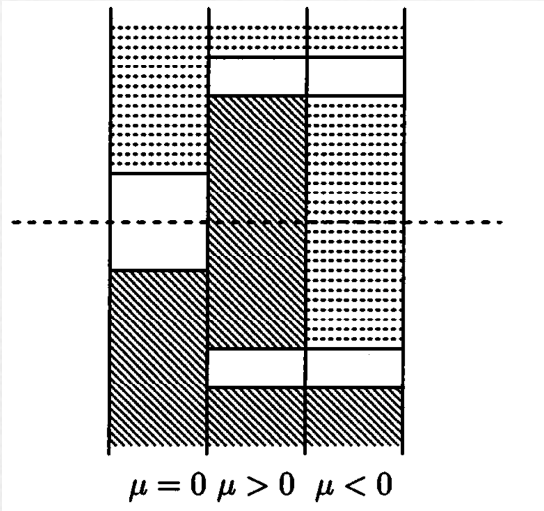
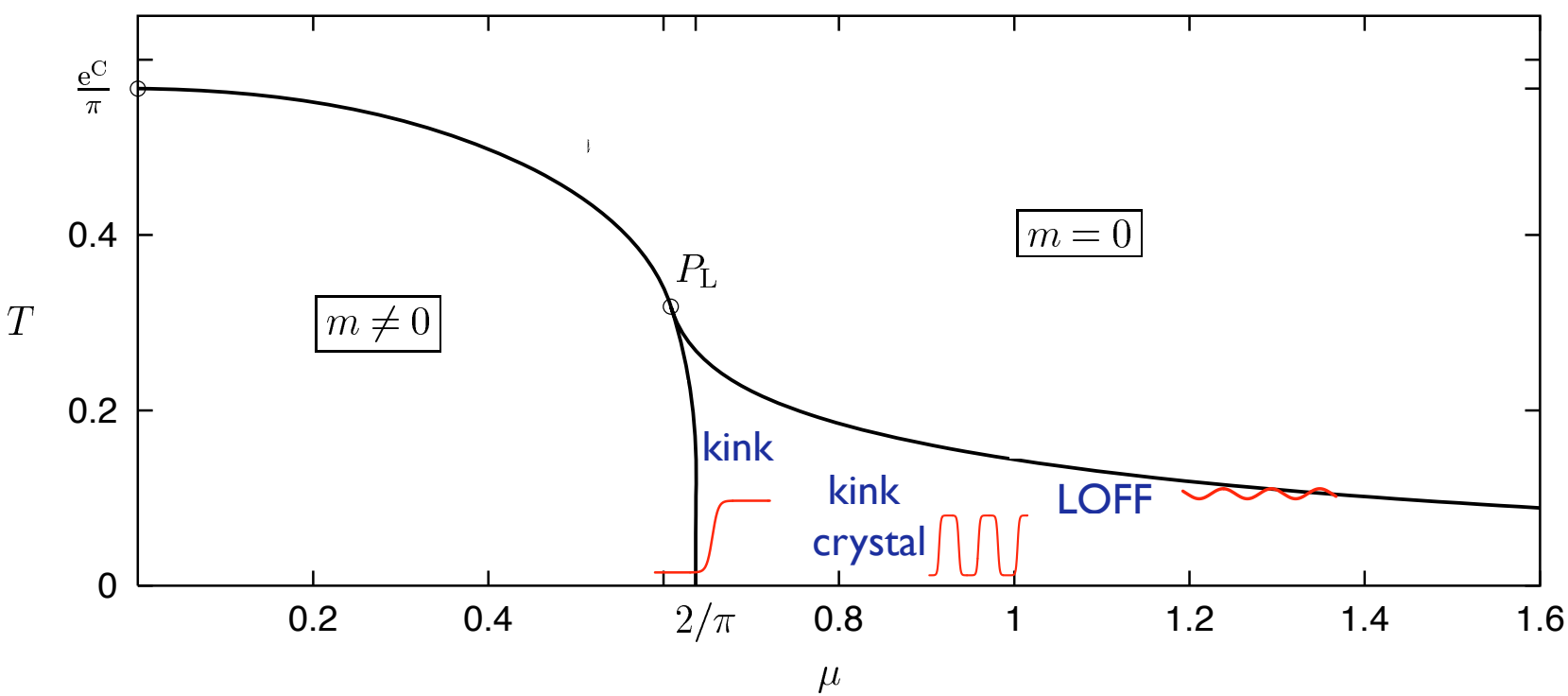




gap equation solution is "finite gap" :

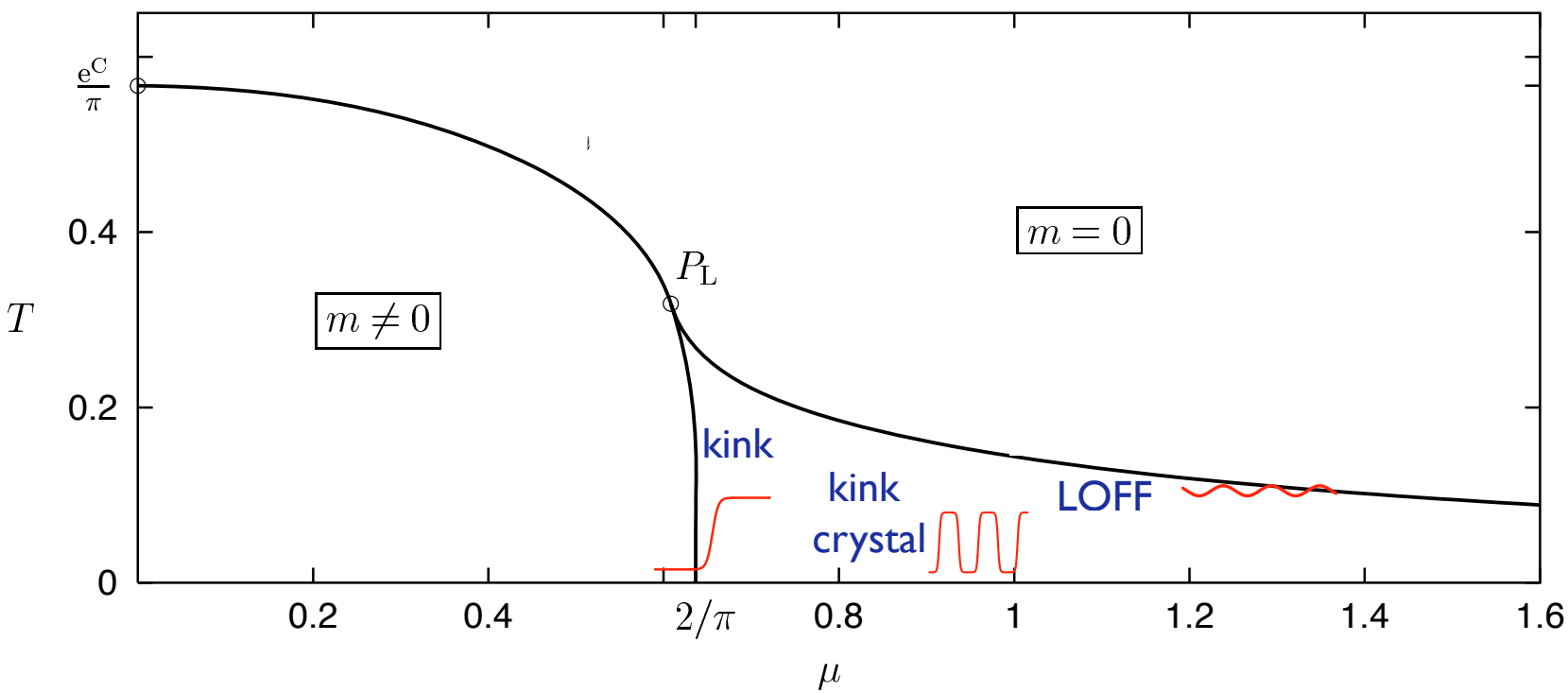
$$\sigma(x) = m\sqrt{\nu}\,\mathrm{sn}(m\,x;\nu)$$

Thies, Urlichs, 2003


$\frac{e^C}{\pi}$
0.4
0.2
0
T
$m \neq 0$
$m = 0$
P_L
kink
kink
crystal
LOFF
0.2
0.4
$2/\pi$
0.8
1
1.2
1.4
1.6
μ

all gap

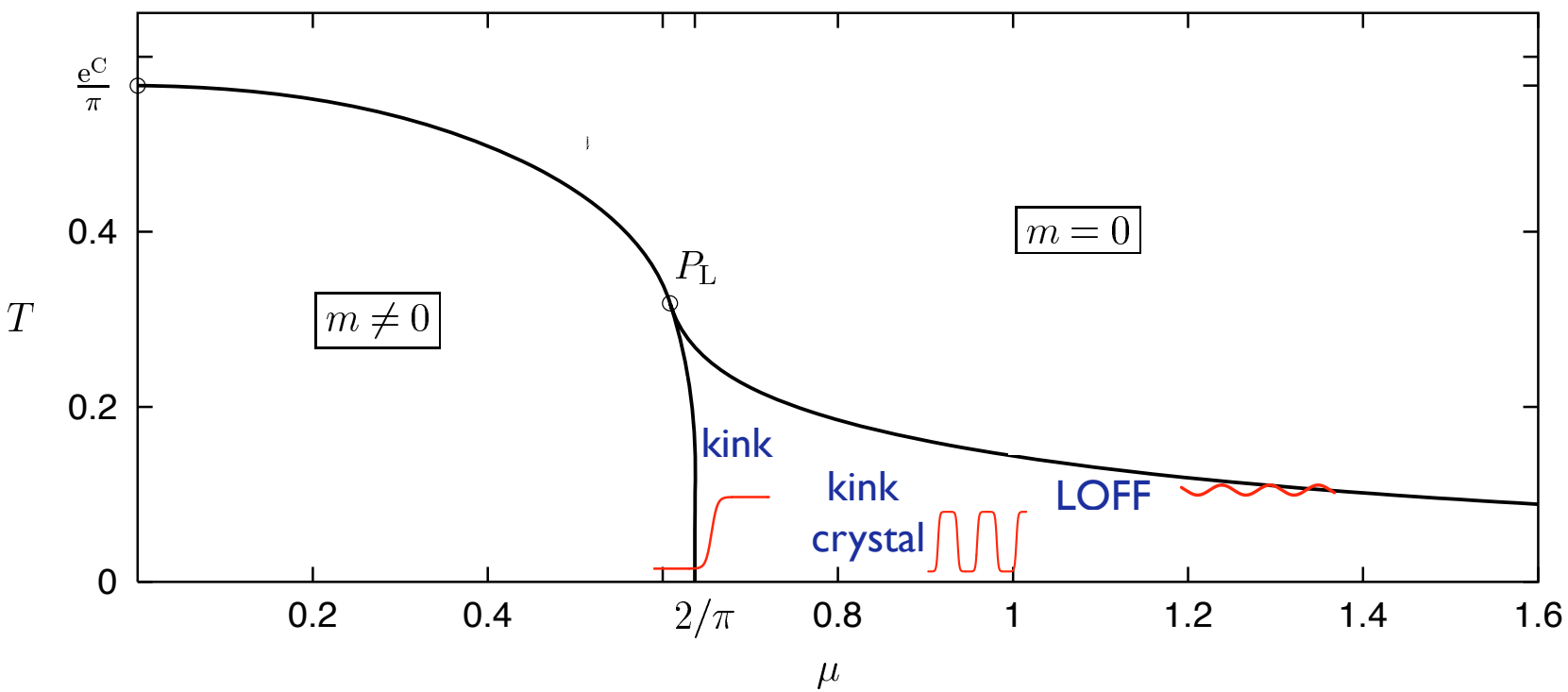
no gap

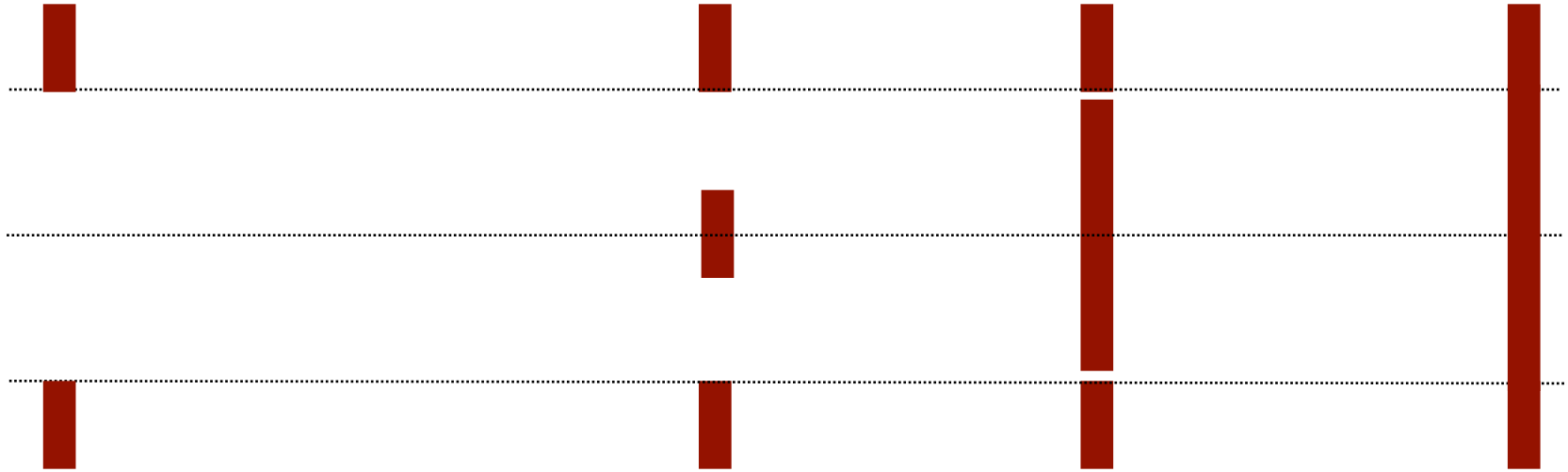


all gap

small gap

no gap



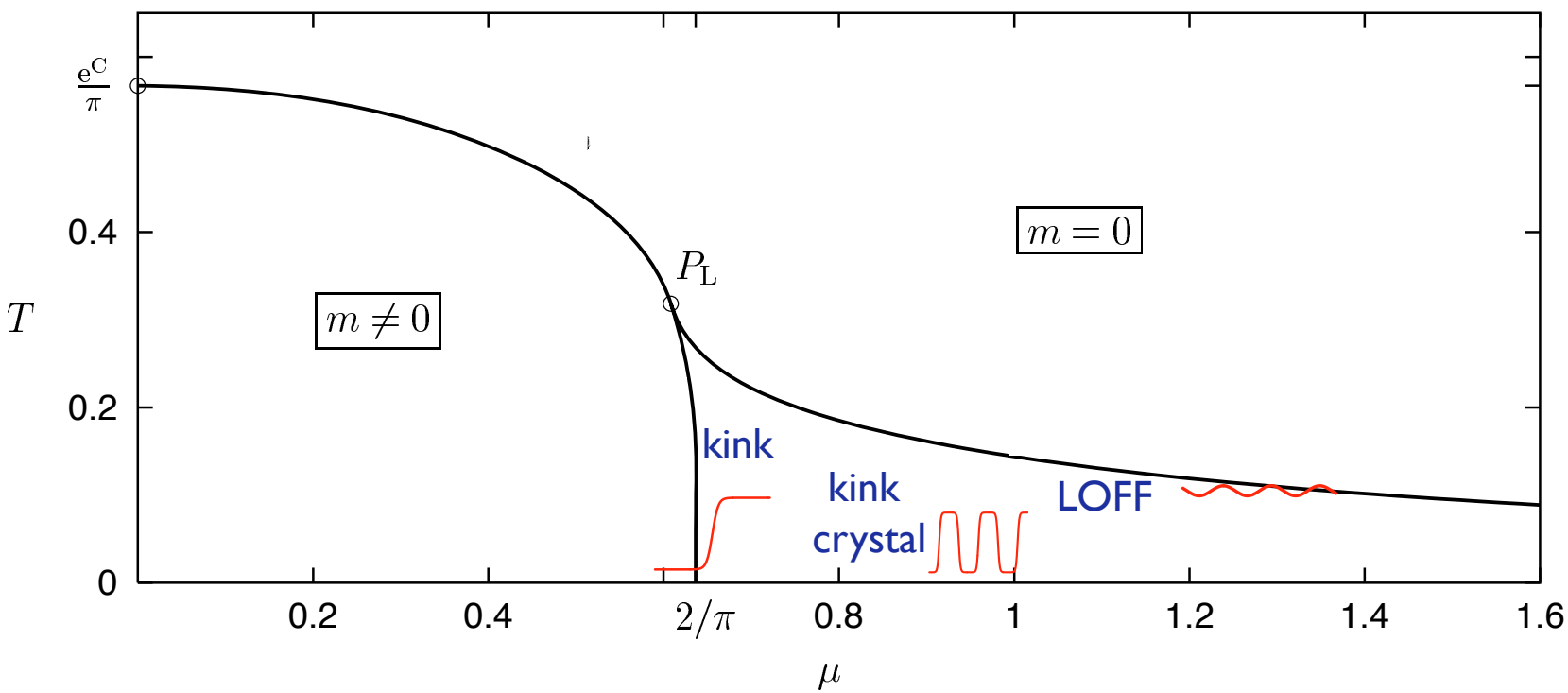


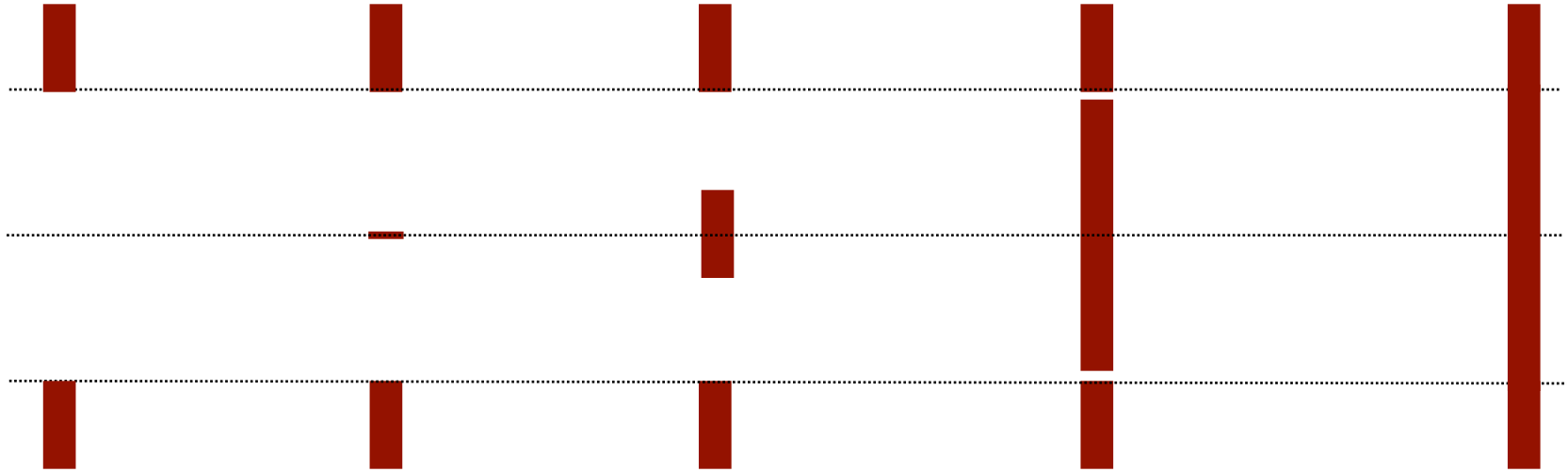
all gap

medium gap

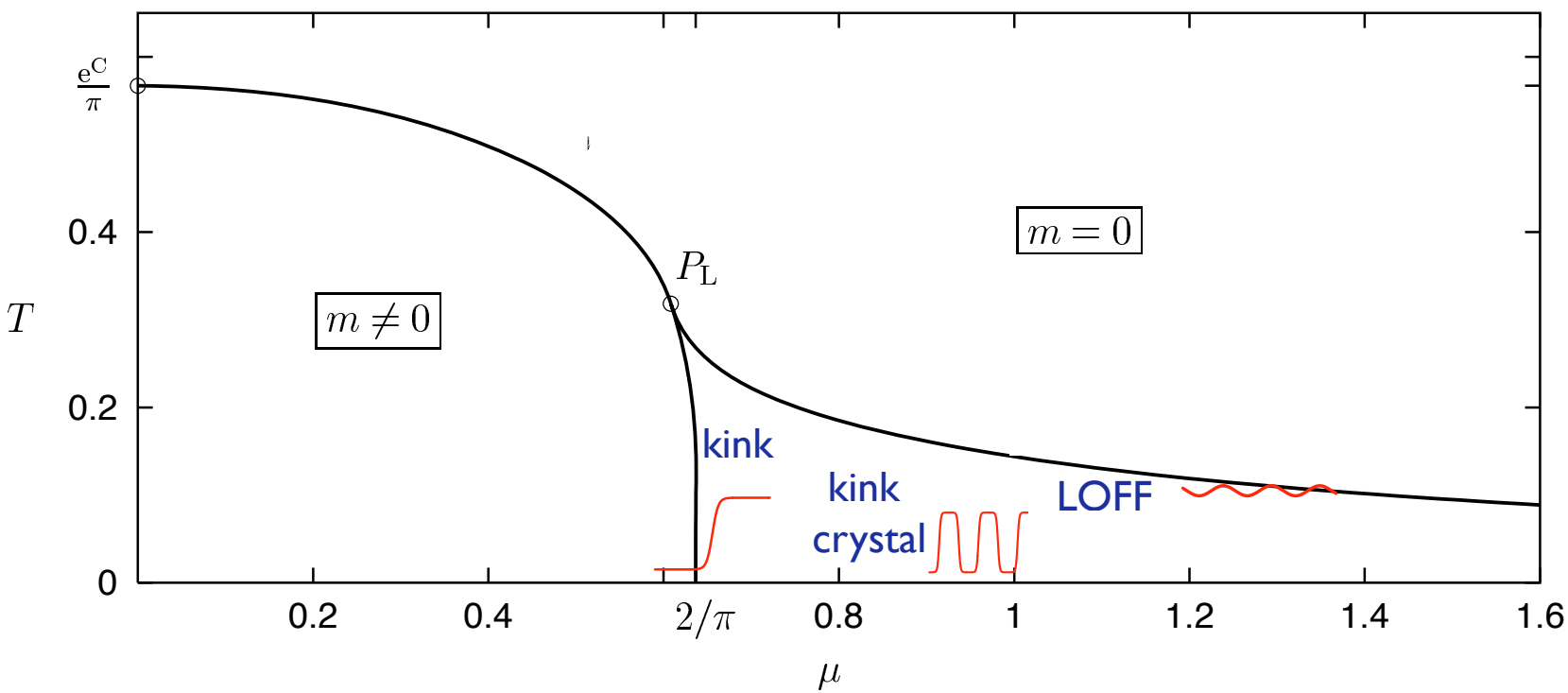
small gap

no gap

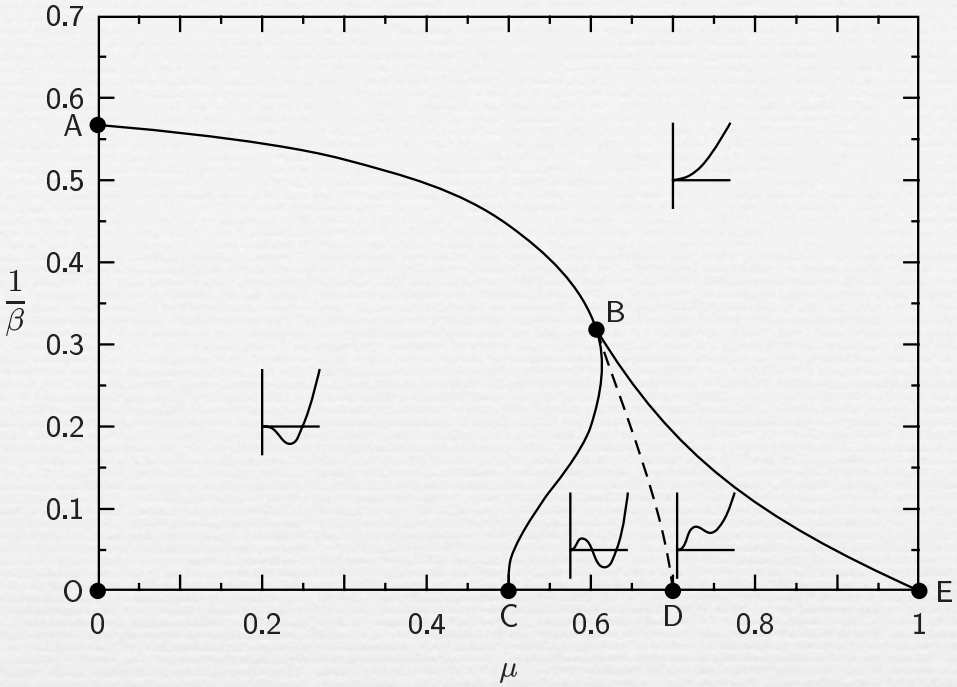




all gap | large gap | medium gap | small gap | no gap



Phase diagram of NJL$_2$ model



uniform condensates
(same as GN$_2$)

$$\sigma = \langle \bar{\psi}\, \psi \rangle$$

$$\pi = \langle \bar{\psi}\, i\gamma_5 \psi \rangle$$

Wolff, 1985
Barducci et al, 1995

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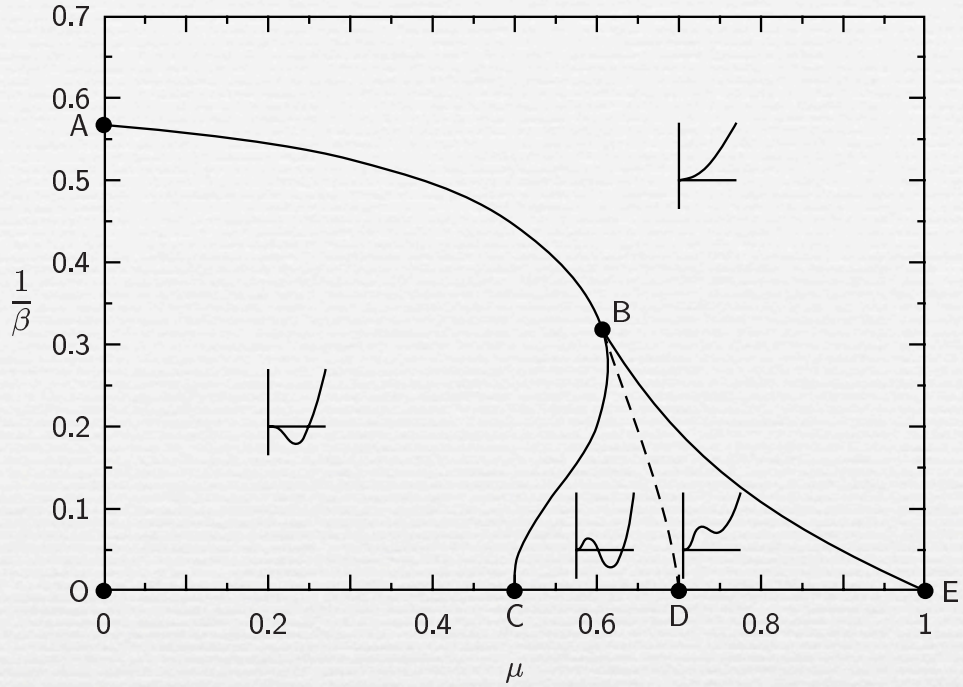

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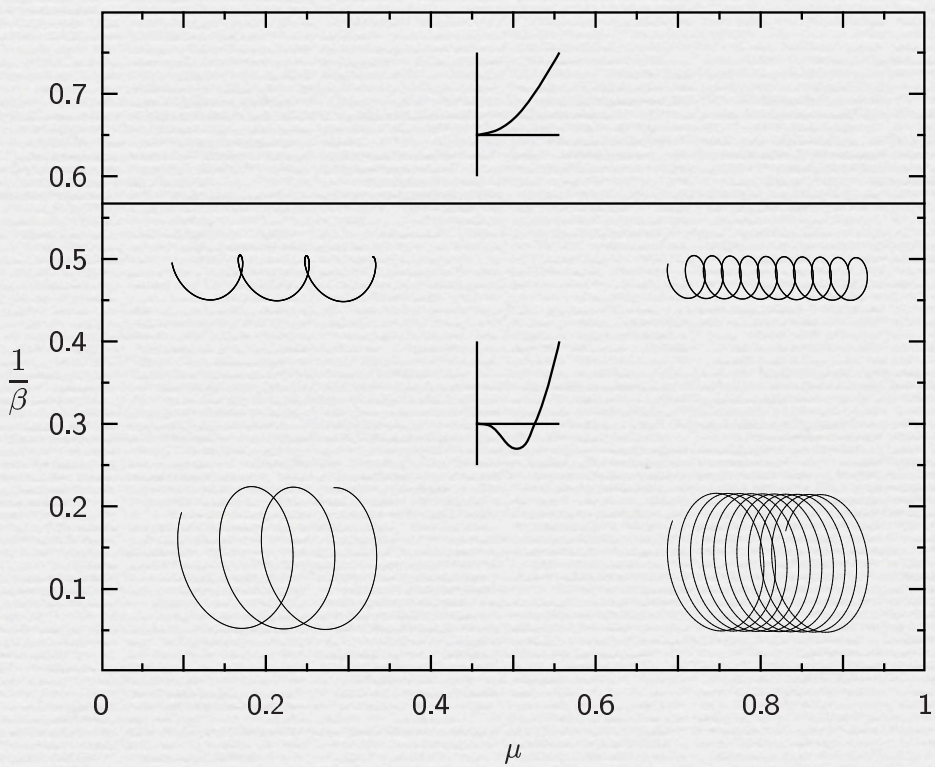


"chiral spiral"

$$\sigma(x) - i\pi(x) = A\, e^{2i\mu x}$$

Schön, Thies, 2000

Basar, GD, Thies, 2009

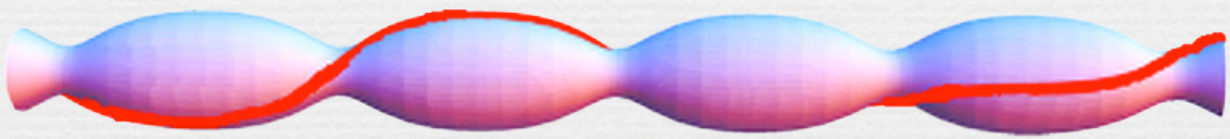
complex gap equation : NJL_2

$$\frac{\Delta(x)}{g^2 N} = \frac{\delta}{\delta \Delta^*(x)} \ln \det \left[\partial\!\!\!/ + (\sigma(x) - i\gamma^5 \pi(x)) \right]$$

$$\Delta = \sigma - i\pi$$

reducible to NLSE: general bounded solution

Basar, GD, 2008



twisted kink crystal

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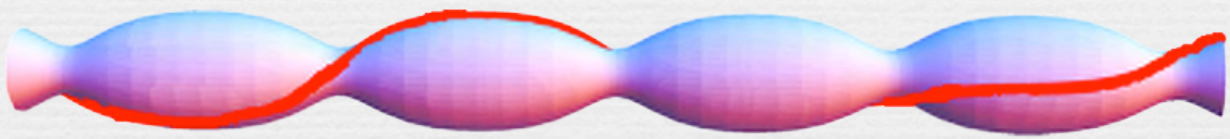

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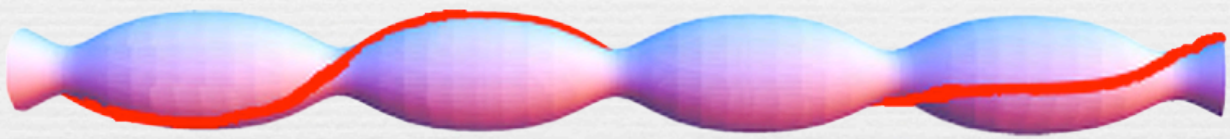

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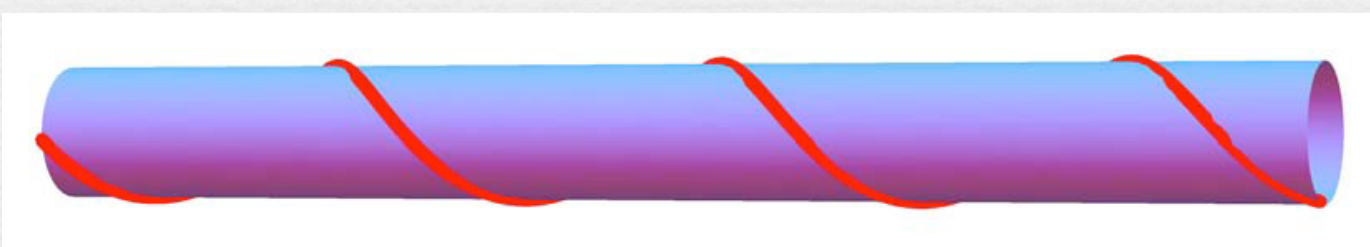

spiral crystal

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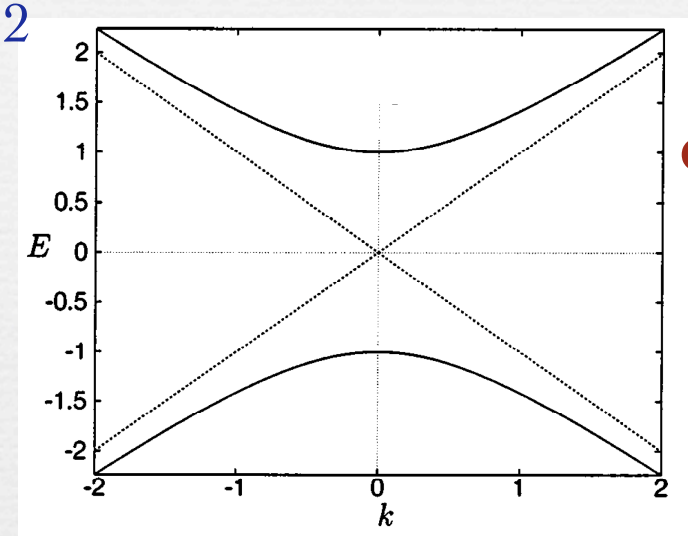

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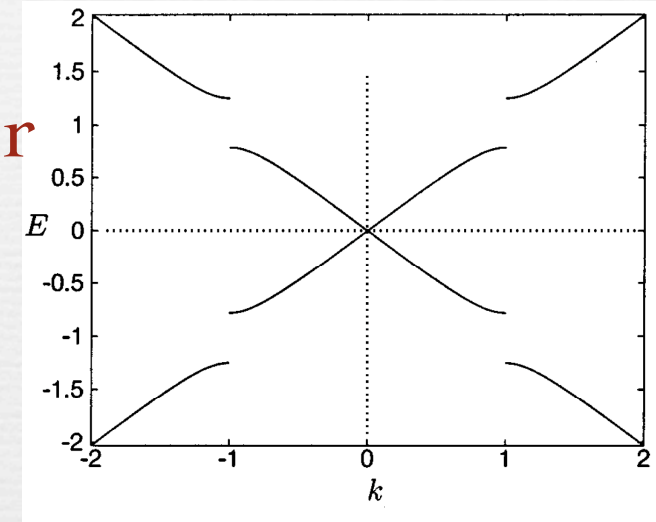
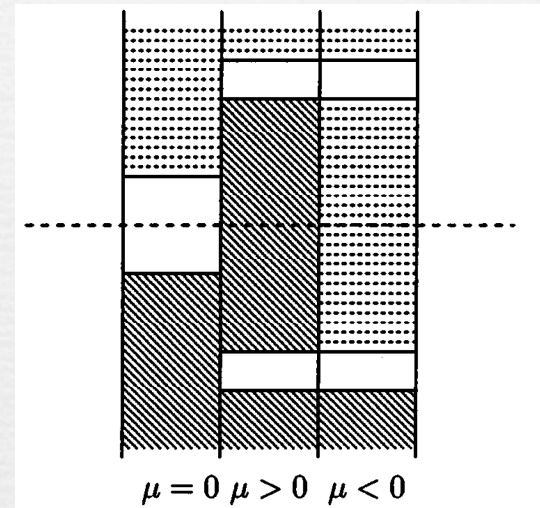
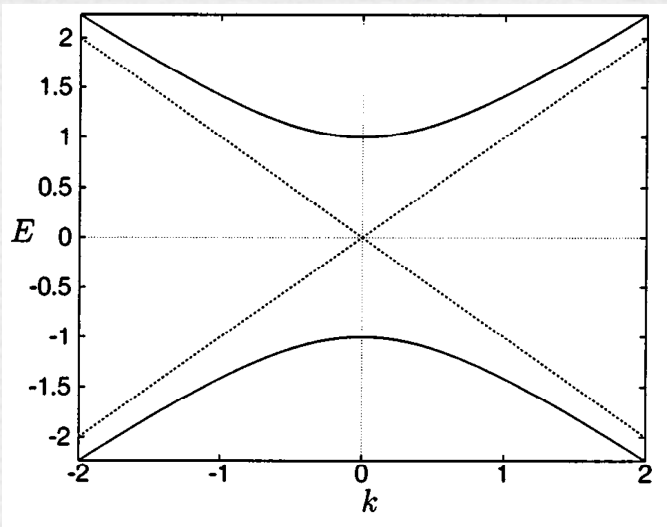
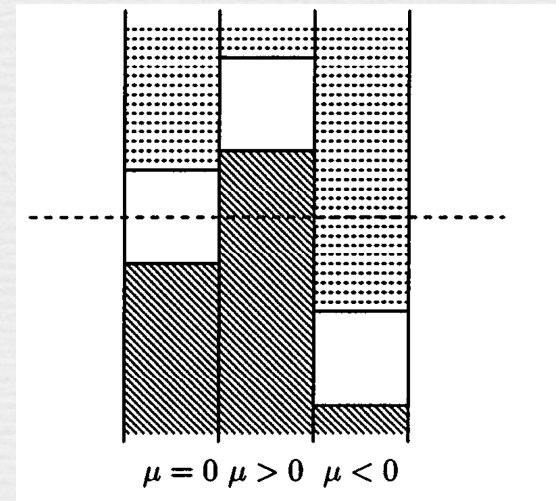


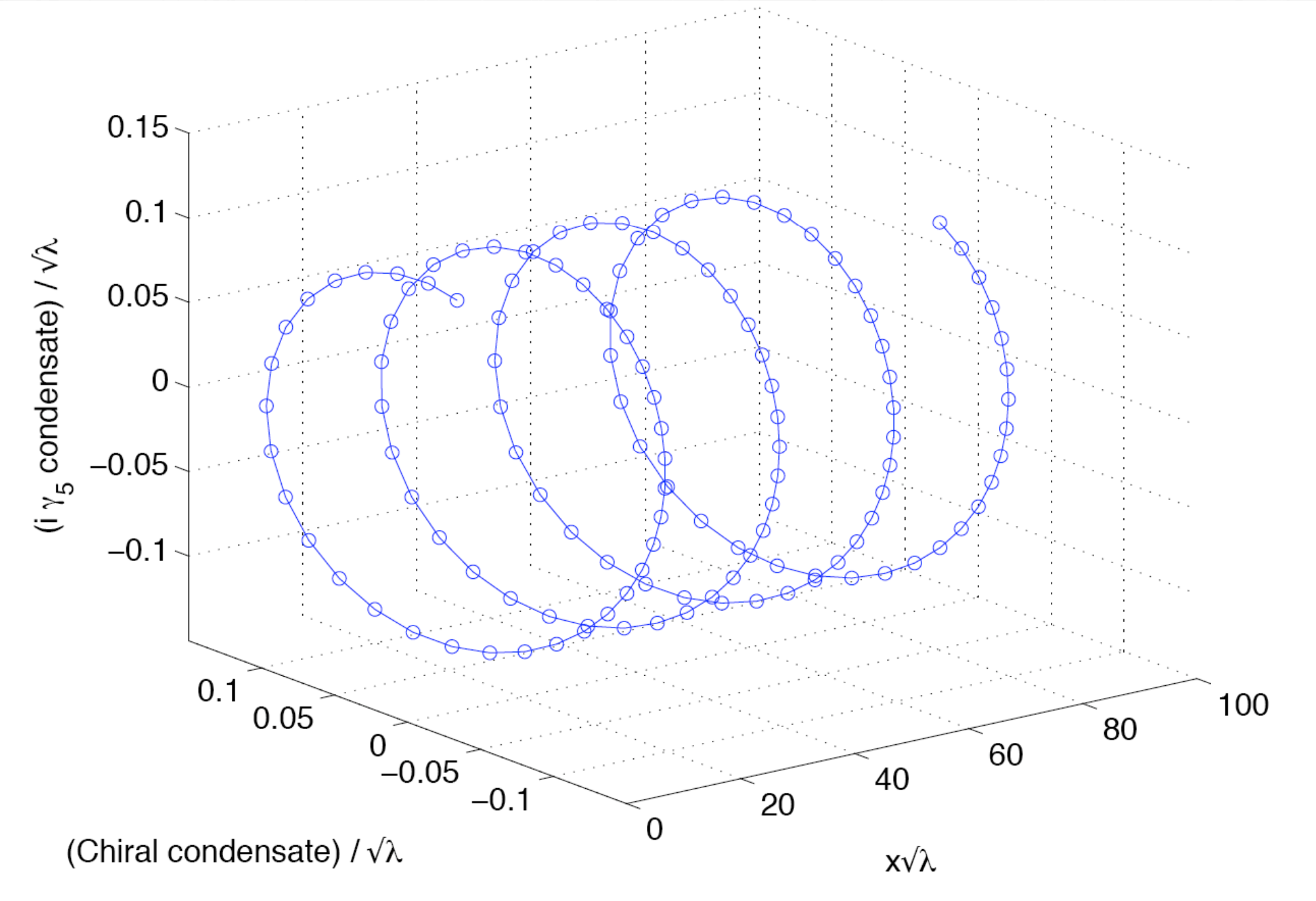


NJL_2



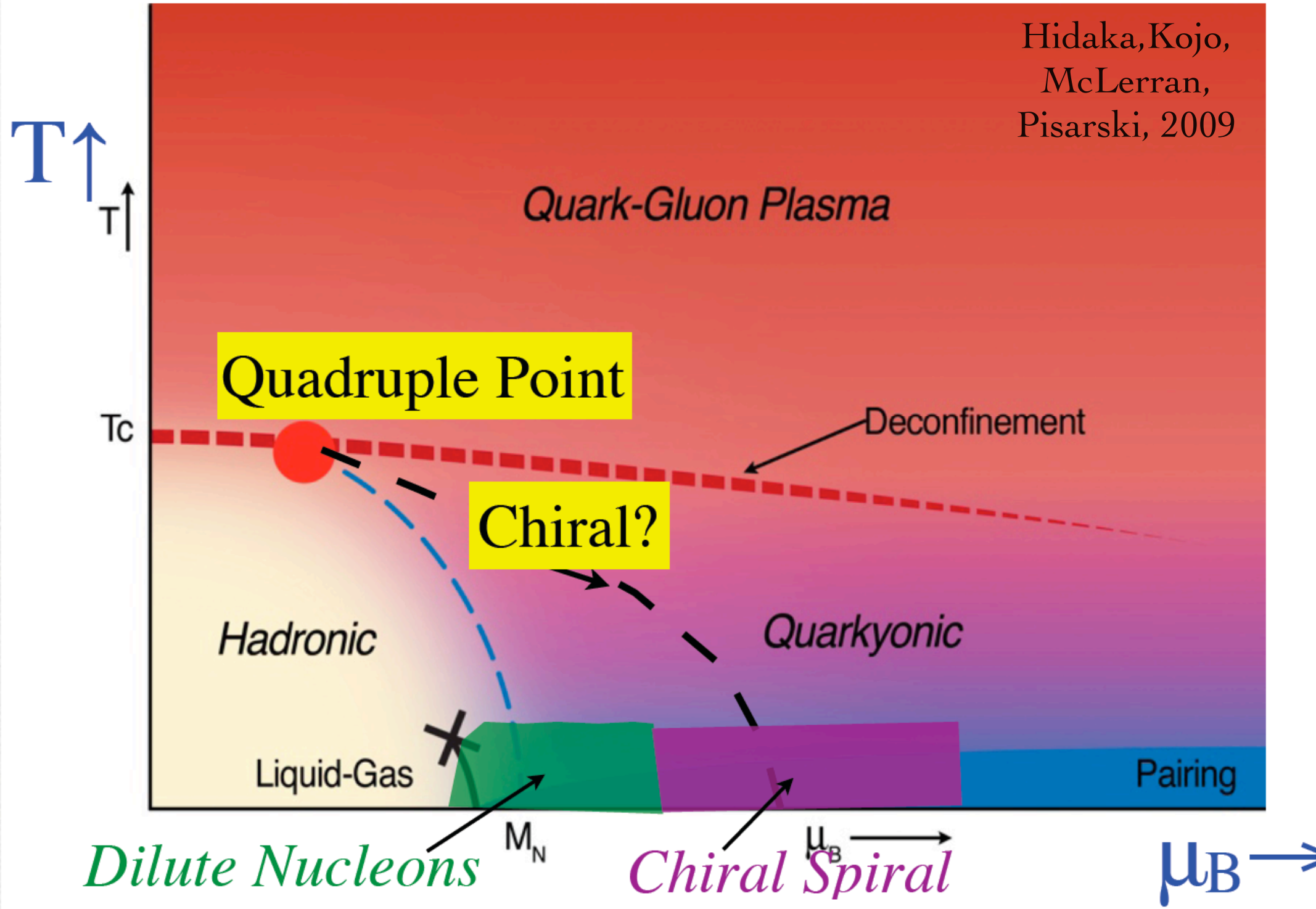


Thies, Urlichs, 2003



1+1 ‘t Hooft model

Bringoltz, PRD 2009


Hidaka,Kojo,
McLerran,
Pisarski, 2009
T
T
Quark-Gluon Plasma
Quadruple Point
Tc
Deconfinement
Chiral?
Hadronic
Quarkyonic
Liquid-Gas
Pairing
Dilute Nucleons
M_N
μ_B
Chiral Spiral
μ_B

chemical potential and chiral spiral

special property of 1+1 dimensions

$$\psi \to e^{i\,\mu\,x\,\gamma_5}\psi$$

$$\bar{\psi} i \partial\!\!\!/ \psi \to \bar{\psi} i \partial\!\!\!/ \psi + \mu\,\bar{\psi}\gamma_0\psi$$

chemical potential $\leftrightarrow$ linear local chiral rotation

chemical potential and chiral spiral

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chemical potential $\leftrightarrow$ linear local chiral rotation

local chiral phase rotation:

$$\psi \to e^{i\alpha(x)\gamma_5}\psi$$

$$A_0(x) = \alpha'(x)$$

$$\mathcal{E}(x) = -\alpha''(x)$$

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chemical potential $\leftrightarrow$ linear local chiral rotation

$$\bar{\psi}\psi \to \bar{\psi} e^{2i\mu\,x\,\gamma_5}\psi$$

$$\bar{\psi}\, i\gamma_5\psi \to \bar{\psi}\, i\gamma_5 e^{2i\mu\,x\,\gamma_5}\psi$$

chemical potential and chiral spiral

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$$\bar{\psi}\, i\gamma_5\psi \to \bar{\psi}\, i\gamma_5 e^{2i\mu\,x\,\gamma_5}\psi$$

$$\Delta(x) \equiv \bar{\psi}\psi - i(\bar{\psi}\, i\gamma_5\psi) \to e^{2i\,\mu\,x}\Delta(x)$$

chemical potential $\leftrightarrow$ spiral condensate

chemical potential and chiral spiral: thermodynamics

$$\Omega = -\frac{1}{\beta}\int_{-\infty}^{\infty} dE\, \rho(E)\, \ln\left(1 + e^{-\beta(E-\mu)}\right)$$

$$\Delta(x) \to e^{2iqx}\Delta(x) \qquad\qquad E \to E + q$$

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renormalization :

$$\Omega[e^{2iqx}\Delta;T,\mu] = \Omega[\Delta;T,\mu-q] + \frac{(\mu-q)^2-\mu^2}{2\pi}$$

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spiral : $$\Omega[e^{2i\mu x}\Delta;T,\mu] = \Omega[\Delta;T,0] - \frac{\mu^2}{2\pi}$$

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spiral : $$\Omega[e^{2i\mu x}\Delta;T,\mu] = \Omega[\Delta;T,0] - \frac{\mu^2}{2\pi}$$

density : $$\rho = -\frac{\partial\Omega}{\partial\mu} \quad\Rightarrow\quad \rho = \frac{\mu}{\pi}$$

currents and condensates

uniform charge density:

$$\langle j^0 \rangle = \langle \psi^\dagger \psi \rangle = \frac{\mu}{\pi}$$

$$\partial_\mu \langle j_5^\mu \rangle = 0$$

chiral spiral condensate:

$$\langle \bar{\psi} \psi \rangle - i \langle \bar{\psi} i \gamma^5 \psi \rangle = A\, e^{2i\mu x}$$

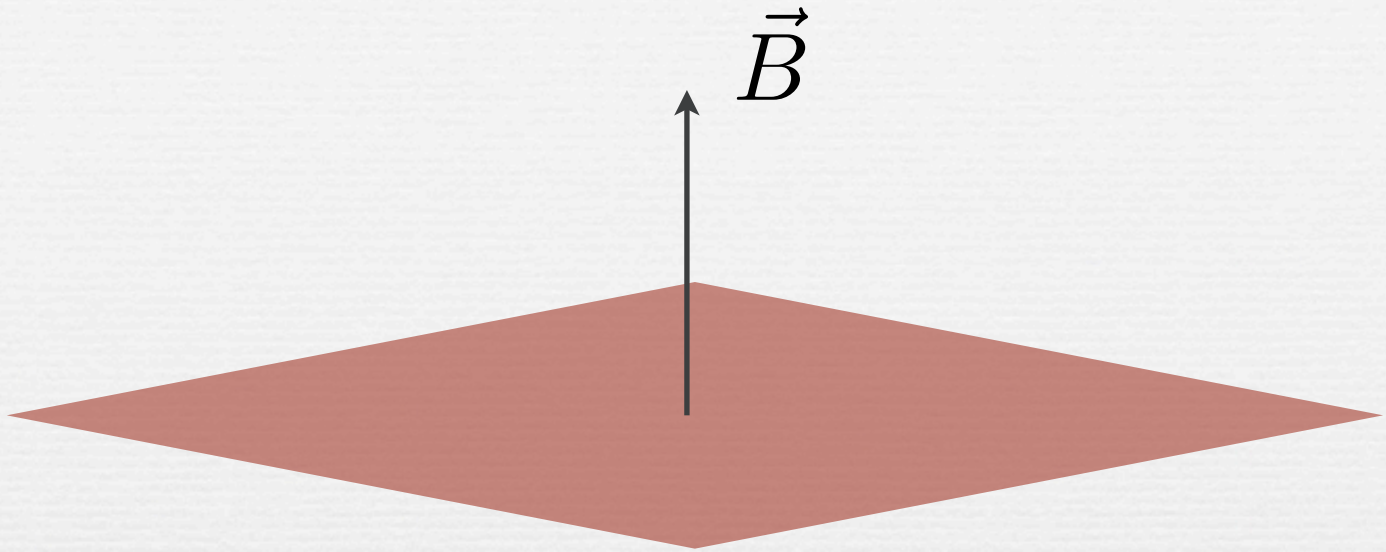
chiral spiral is essentially unavoidable in 1d system
with chiral physics and nonzero μ

relation to the chiral magnetic effect ?

3+1 dimensions ... 1+1 dimensions

chirality

G.Başar, GD & D. Kharzeev, arXiv:1003.3464



$$\left(\vec{p} - \vec{A}\right)^2 \longrightarrow \quad p_{\parallel}^2 + (p_{\perp} - A_{\perp})^2$$

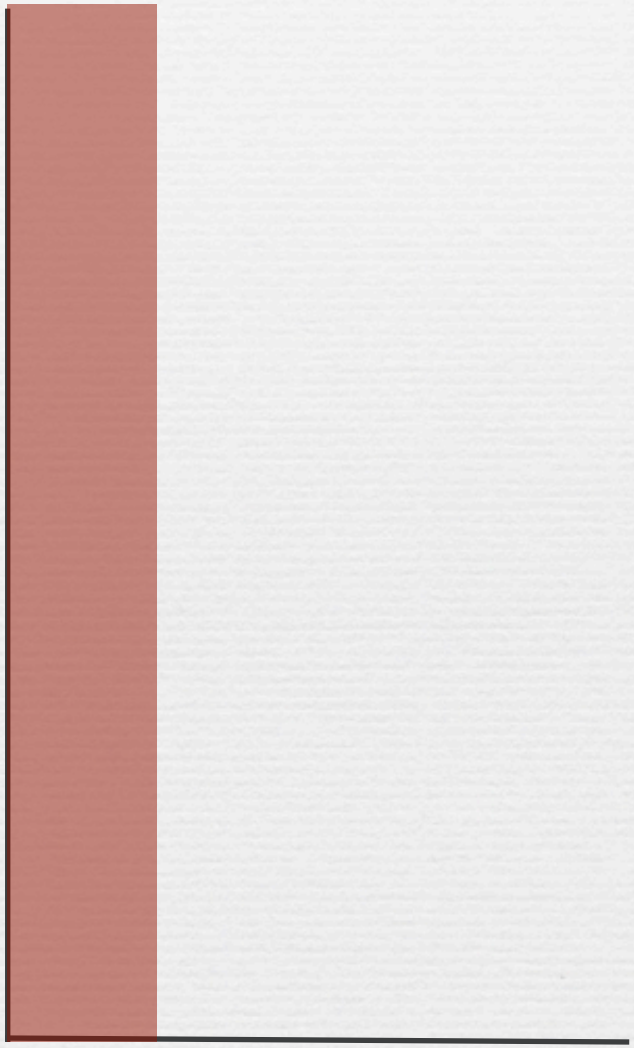
Landau level spectrum

dimensional reduction: $3+1 \quad \longrightarrow \quad 1+1$

Gusynin, Miransky, Shovkovy

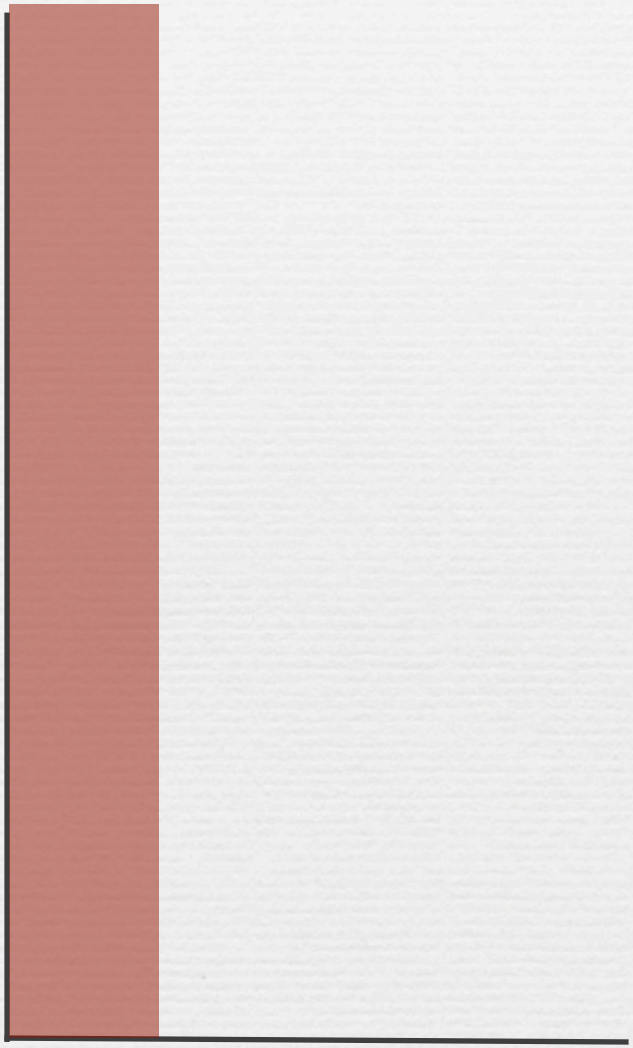
scalar

$$E_n = (2n+1)B$$



spinor

$$E_n = (2n+1)B \mp B$$



scalar

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spinor

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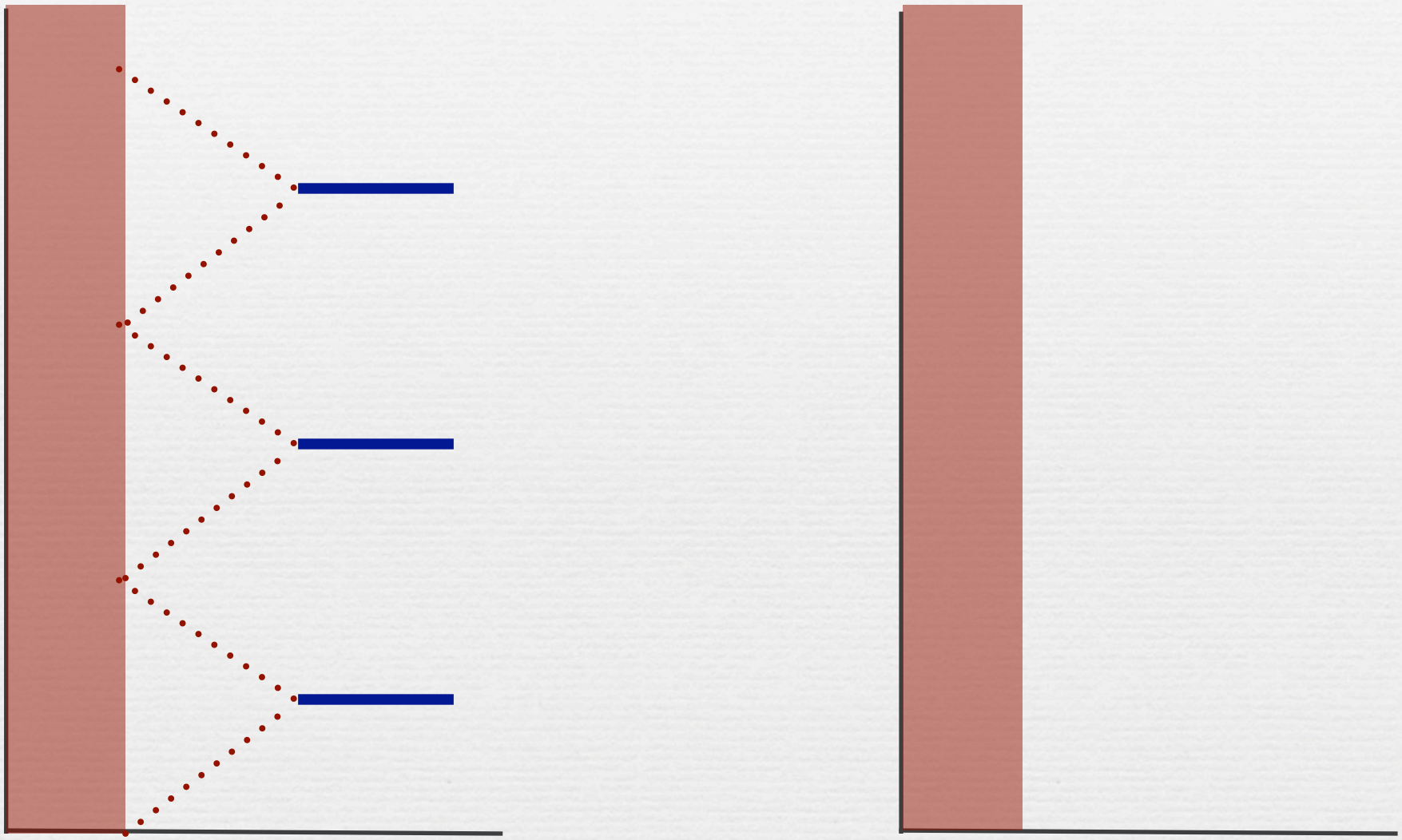

Landau degeneracy $\dfrac{eB}{2\pi}$

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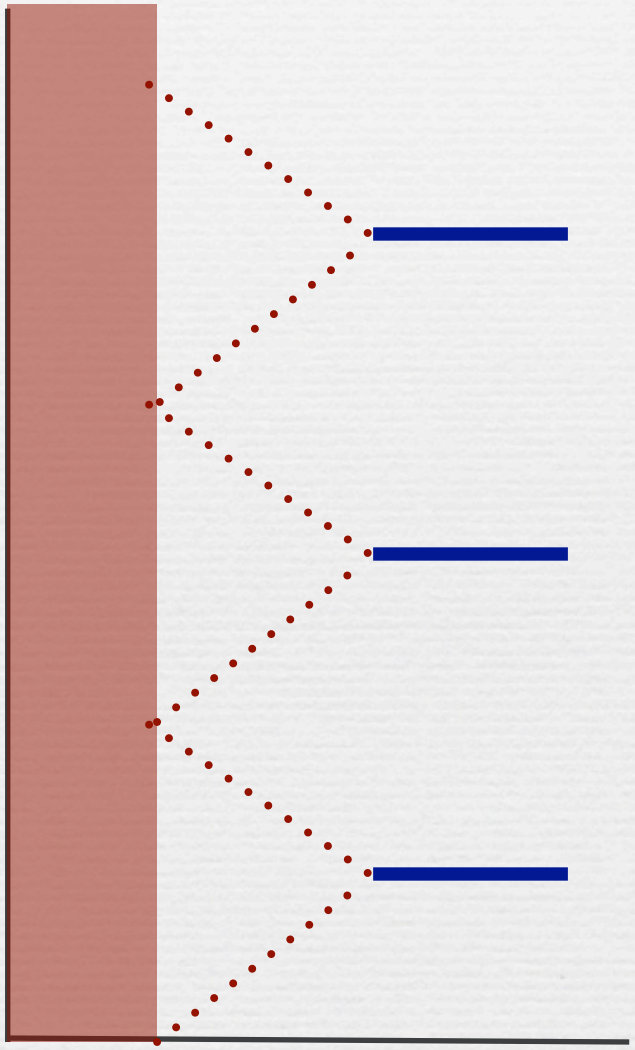

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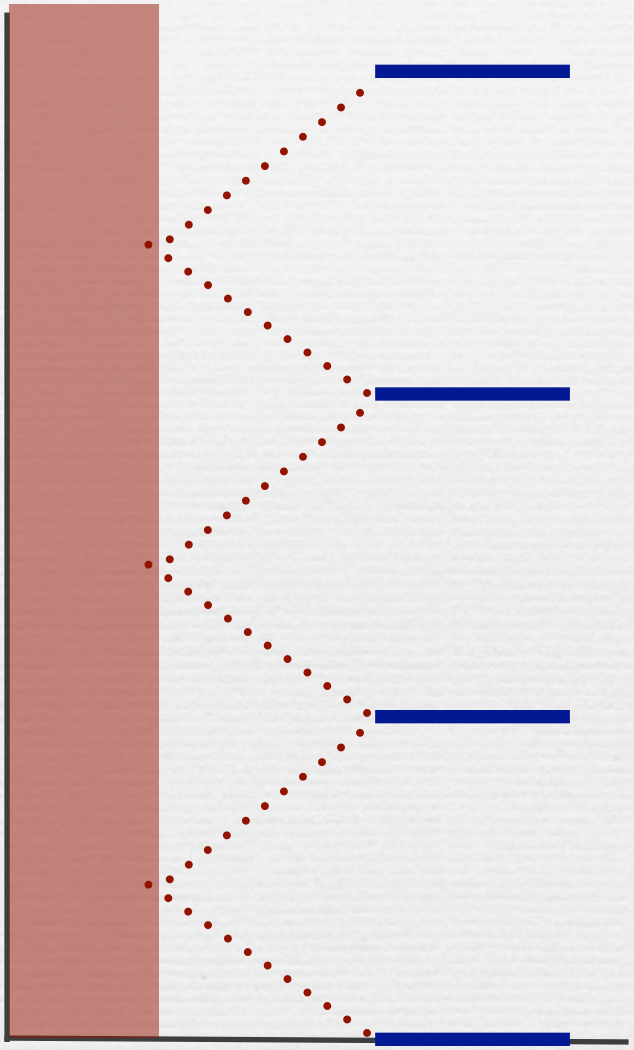

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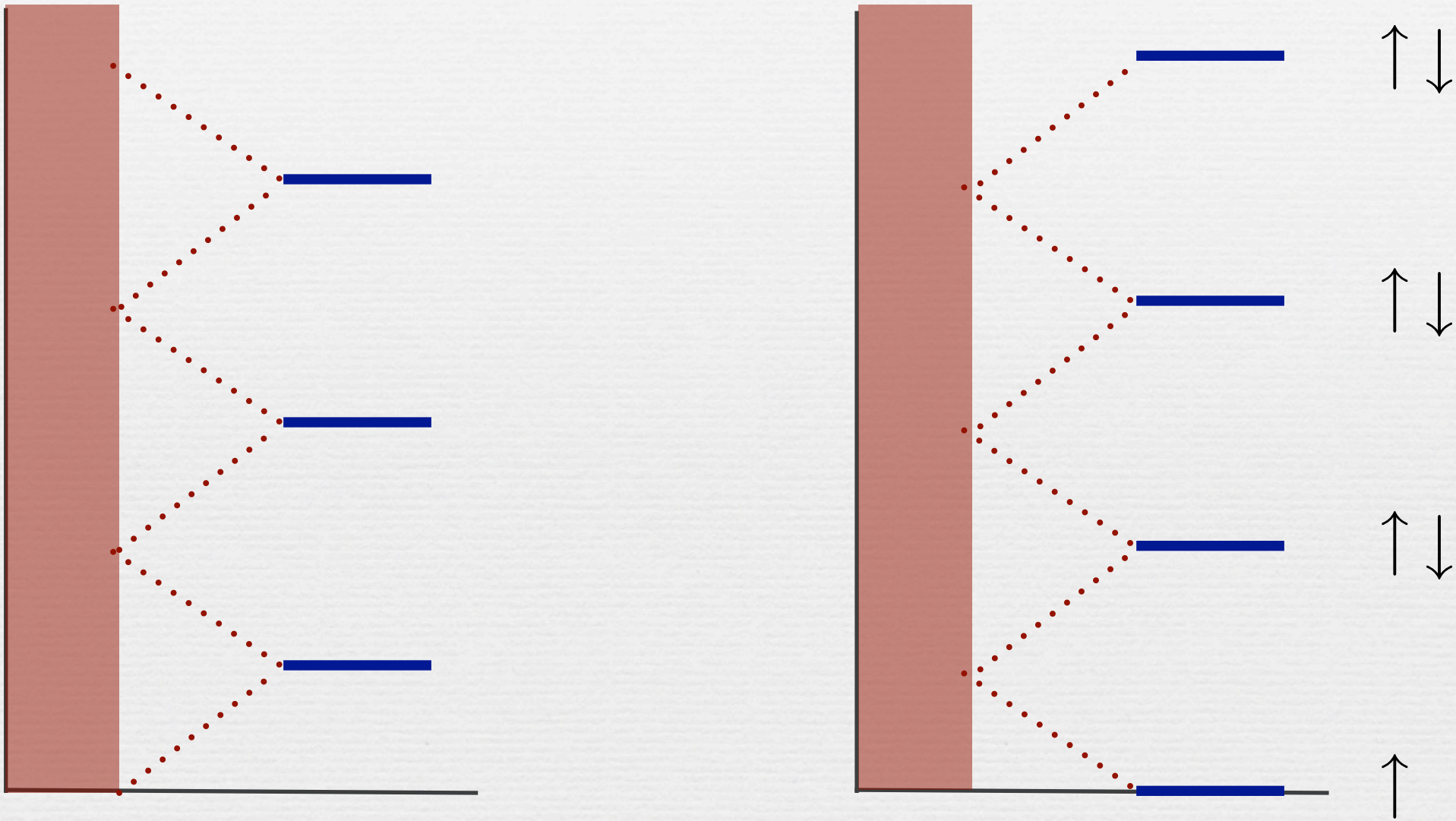

Landau degeneracy $\dfrac{eB}{2\pi}$

some details of dimensional reduction/decomposition

important projectors: chirality
spin along B
momentum along B

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spin along B
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$$P_{R,L} = \frac{1}{2}(\mathbf{1} \pm \gamma^5)$$

$$P_{\uparrow,\downarrow} = \frac{1}{2}(\mathbf{1} \pm \Sigma^3)$$

$$P_{+,-} = \frac{1}{2}(\mathbf{1} \pm \gamma^0\gamma^3)$$

$$\Psi = \begin{pmatrix} R_+ \\ R_- \\ L_- \\ L_+ \end{pmatrix}$$

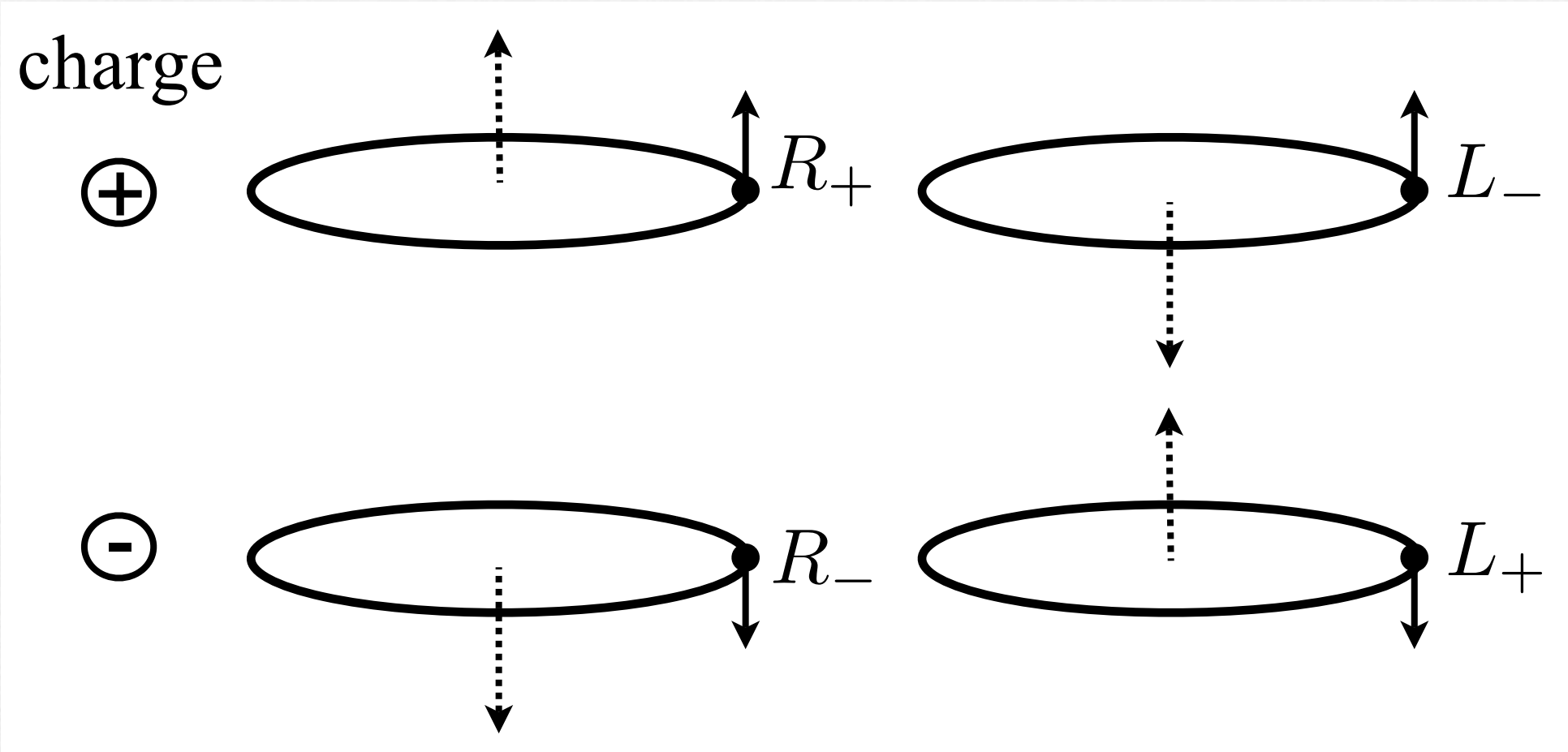
Dirac matrices in 3+1 and 1+1 dimensions

$$\gamma^0 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix} \qquad \gamma^j = \begin{pmatrix} 0 & -\sigma^j \\ \sigma^j & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$$

$$\Gamma^0 = \sigma^1 \qquad \Gamma^z = -i\,\sigma^2 \qquad \Gamma^5 = \sigma^3$$

$$\Psi = \begin{pmatrix} R_+ \\ R_- \\ L_- \\ L_+ \end{pmatrix} \longrightarrow$$

$$\mathcal{R} = \begin{pmatrix} R_+ \\ R_- \end{pmatrix} \quad , \quad \mathcal{L} = \begin{pmatrix} L_+ \\ L_- \end{pmatrix} \quad , \quad \phi_\uparrow = \begin{pmatrix} R_+ \\ L_- \end{pmatrix} \quad , \quad \phi_\downarrow = \begin{pmatrix} L_+ \\ R_- \end{pmatrix}$$


charge
⊕
⊖
R_+
L_-
R_-
L_+

current projections

$$\Psi = \begin{pmatrix} R_+ \\ R_- \\ L_- \\ L_+ \end{pmatrix} \longrightarrow \mathcal{R} = \begin{pmatrix} R_+ \\ R_- \end{pmatrix} \quad , \quad \mathcal{L} = \begin{pmatrix} L_+ \\ L_- \end{pmatrix} \quad , \quad \phi_\uparrow = \begin{pmatrix} R_+ \\ L_- \end{pmatrix} \quad , \quad \phi_\downarrow = \begin{pmatrix} L_+ \\ R_- \end{pmatrix}$$

axial current:

$$\begin{aligned}
J_5^0 &= \mathcal{R}^\dagger \mathcal{R} - \mathcal{L}^\dagger \mathcal{L} = -i\bar{\phi}_\uparrow \Gamma^z \phi_\uparrow + i\bar{\phi}_\downarrow \Gamma^z \phi_\downarrow \\
J_5^1 &= \bar{\mathcal{R}}\,\mathcal{R} + \bar{\mathcal{L}}\,\mathcal{L} = \bar{\phi}_\uparrow \phi_\downarrow + \bar{\phi}_\downarrow \phi_\uparrow \\
J_5^2 &= i\bar{\mathcal{R}}\,\Gamma^5 \mathcal{R} - i\bar{\mathcal{L}}\,\Gamma^5 \mathcal{L} = -i\bar{\phi}_\uparrow \phi_\downarrow + i\bar{\phi}_\downarrow \phi_\uparrow \\
J_5^3 &= \bar{\mathcal{R}}\,\Gamma^z \mathcal{R} - \bar{\mathcal{L}}\,\Gamma^z \mathcal{L} = \phi_\uparrow^\dagger \phi_\uparrow - \phi_\downarrow^\dagger \phi_\downarrow
\end{aligned}$$

charge current:

$$\begin{aligned}
J^0 &= \mathcal{R}^\dagger \mathcal{R} + \mathcal{L}^\dagger \mathcal{L} = \phi_\uparrow^\dagger \phi_\uparrow + \phi_\downarrow^\dagger \phi_\downarrow \\
J^1 &= \bar{\mathcal{R}}\,\mathcal{R} - \bar{\mathcal{L}}\,\mathcal{L} = -\bar{\phi}_\uparrow \Gamma^5 \phi_\downarrow + \bar{\phi}_\downarrow \Gamma^5 \phi_\uparrow \\
J^2 &= i\bar{\mathcal{R}}\,\Gamma^5 \mathcal{R} + i\bar{\mathcal{L}}\,\Gamma^5 \mathcal{L} = i\bar{\phi}_\uparrow \Gamma^5 \phi_\downarrow + i\bar{\phi}_\downarrow \Gamma^5 \phi_\uparrow \\
J^3 &= \bar{\mathcal{R}}\,\Gamma^z \mathcal{R} + \bar{\mathcal{L}}\,\Gamma^z \mathcal{L} = \bar{\phi}_\uparrow \Gamma^z \phi_\uparrow + \bar{\phi}_\downarrow \Gamma^z \phi_\downarrow
\end{aligned}$$

the usual CME

$$
\begin{aligned}
J_5^0 &= \mathcal{R}^\dagger \mathcal{R} - \mathcal{L}^\dagger \mathcal{L} \to \frac{1}{2}\frac{eB}{2\pi}\frac{e\left(\mu_R - \mu_L\right)}{\pi} \\
J^0 &= \mathcal{R}^\dagger \mathcal{R} + \mathcal{L}^\dagger \mathcal{L} \to \frac{1}{2}\frac{eB}{2\pi}\frac{e\left(\mu_R + \mu_L\right)}{\pi}
\end{aligned}
$$

$$J_5^0 = \mathcal{R}^\dagger \mathcal{R} - \mathcal{L}^\dagger \mathcal{L} \to \frac{1}{2}\frac{eB}{2\pi}\frac{e\,(\mu_R - \mu_L)}{\pi}$$

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L/R imbalance $\mu_R = \mu_5 = -\mu_L$

$$\langle J_5^0 \rangle = \frac{eB}{2\pi}\frac{e\,\mu_5}{\pi} \qquad \langle J^0 \rangle = 0$$

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"CME"

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$$\langle J_5^0 \rangle = \frac{eB}{2\pi}\frac{e\,\mu_5}{\pi} \qquad \langle J^0 \rangle = 0$$

$$\langle J_5^3 \rangle = 0 \qquad \langle J^3 \rangle = \frac{eB}{2\pi}\frac{e\,\mu_5}{\pi}$$

"CME"

$$\langle J_5^\mu \rangle = \epsilon^{\mu\nu}\langle J^\nu \rangle$$

$$J_5^0 = \mathcal{R}^\dagger\mathcal{R} - \mathcal{L}^\dagger\mathcal{L} \to \frac{1}{2}\frac{eB}{2\pi}\frac{e\,(\mu_R - \mu_L)}{\pi}$$

$$J^0 = \mathcal{R}^\dagger\mathcal{R} + \mathcal{L}^\dagger\mathcal{L} \to \frac{1}{2}\frac{eB}{2\pi}\frac{e\,(\mu_R + \mu_L)}{\pi}$$

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"CME"

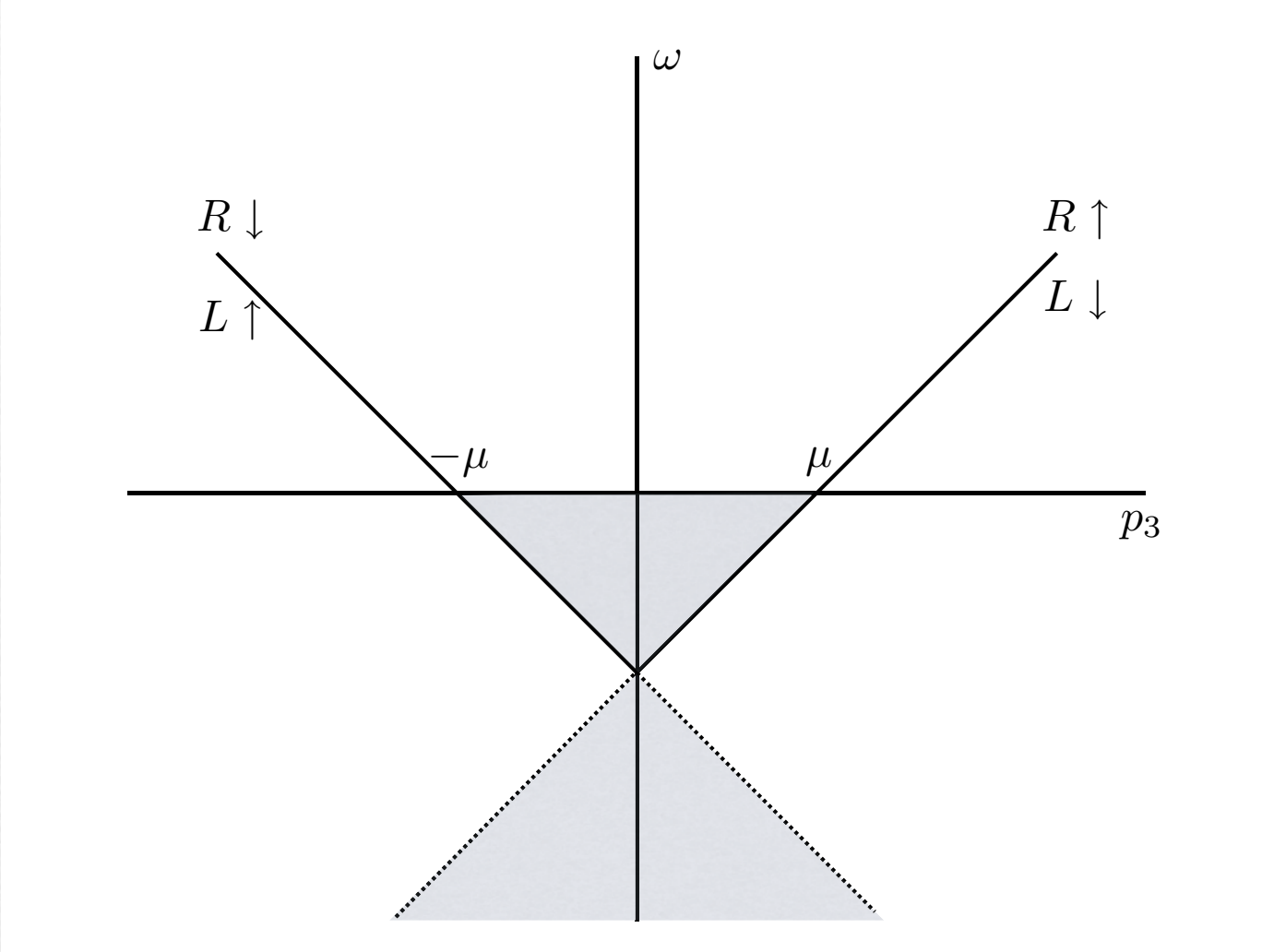
baryon chemical potential $\mu_R = \mu = \mu_L$

$$\langle J_5^0 \rangle = 0 \qquad \langle J^0 \rangle = \frac{eB}{2\pi}\frac{e\mu}{\pi}$$

$$\langle J_5^3 \rangle = \frac{eB}{2\pi}\frac{e\mu}{\pi} \qquad \langle J^3 \rangle = 0$$

Son, Zhitnitsky, 2004
Metlitski, Zhitnitsky, 2005

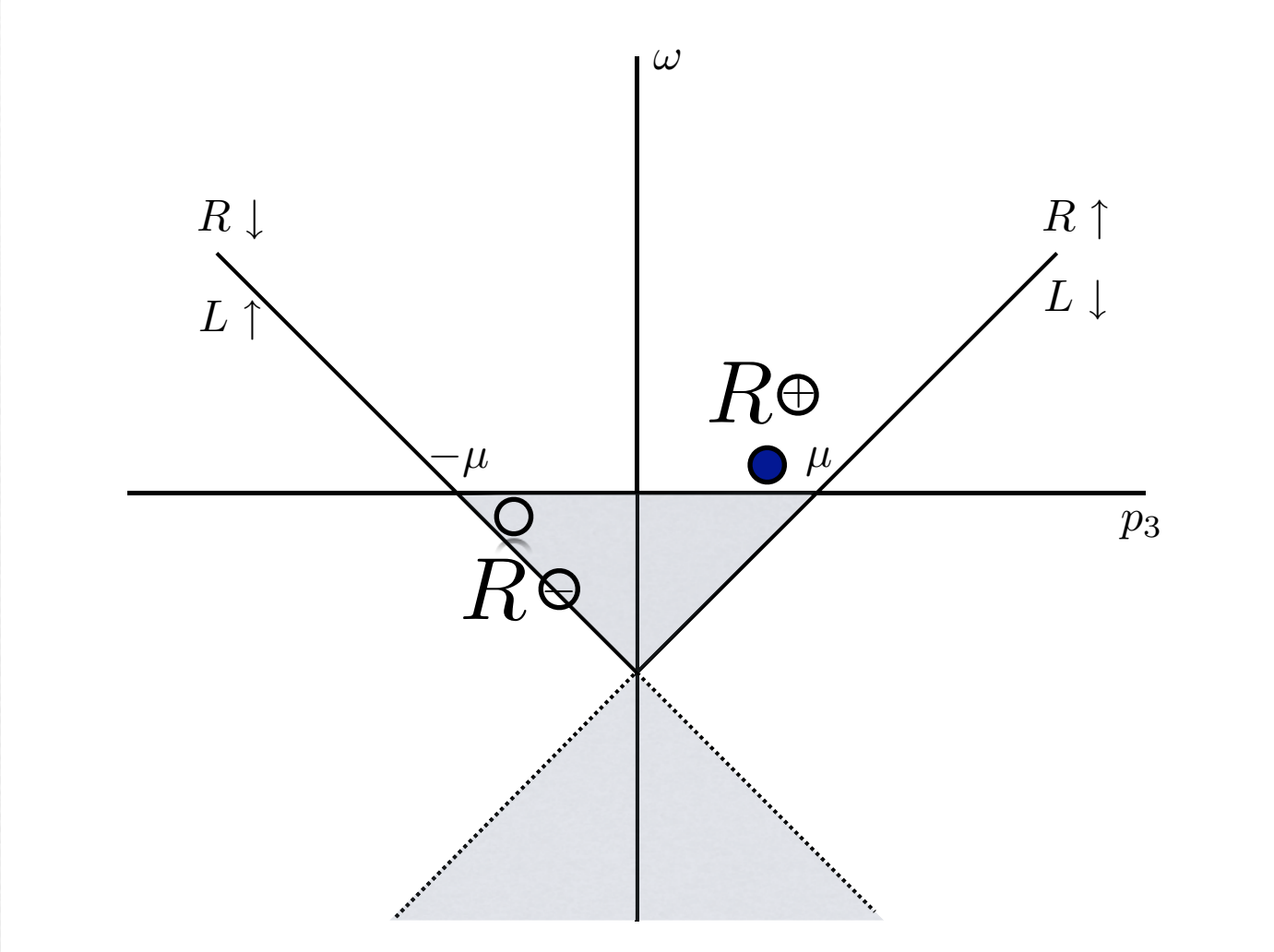
finite baryon density



$$\mu \neq 0 : R\uparrow : \omega = p_3 - \mu \quad , \quad L\uparrow : \omega = -p_3 - \mu$$

$$R\downarrow : \omega = -p_3 - \mu \quad , \quad L\downarrow : \omega = p_3 - \mu$$

finite baryon density



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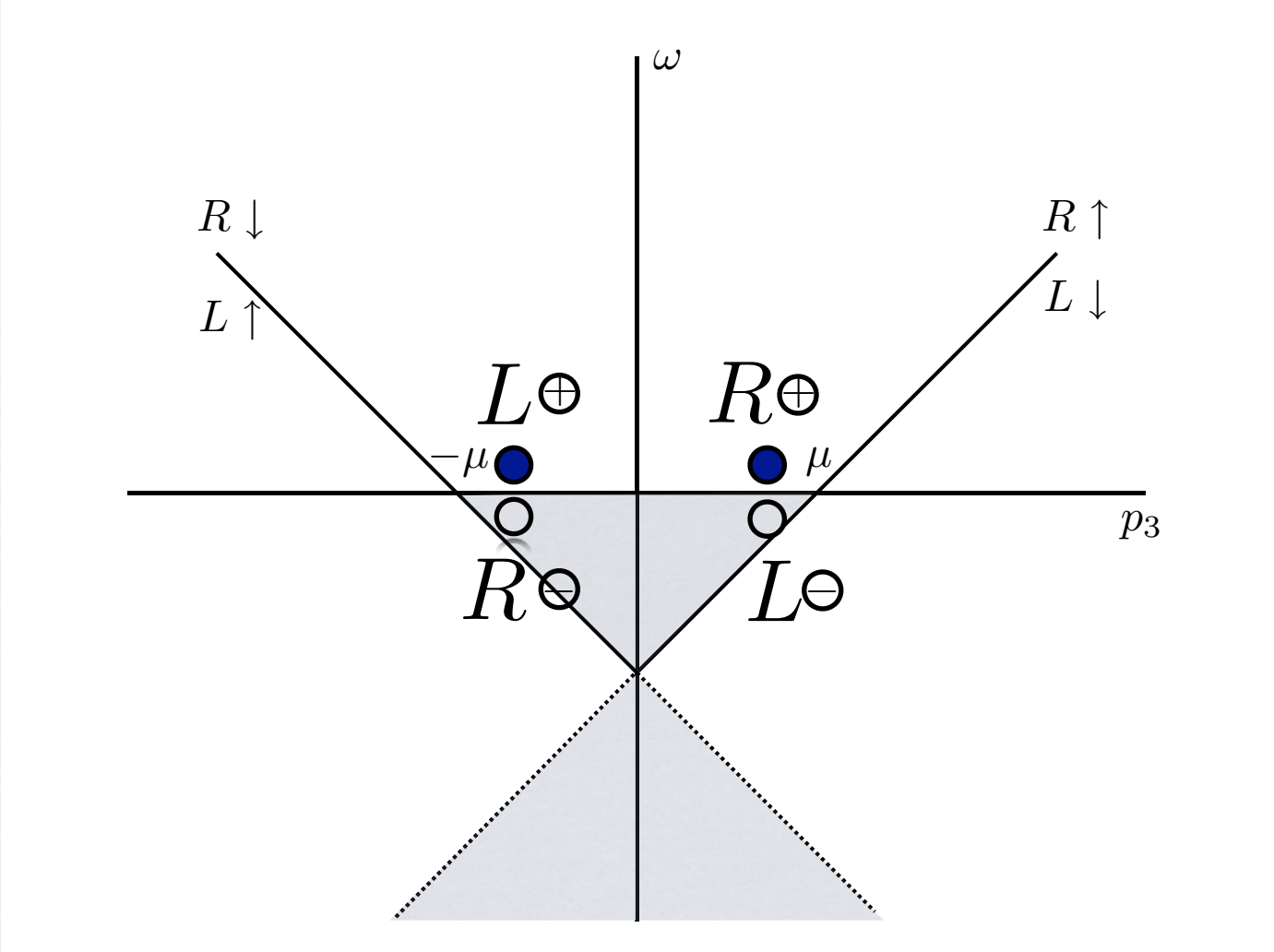

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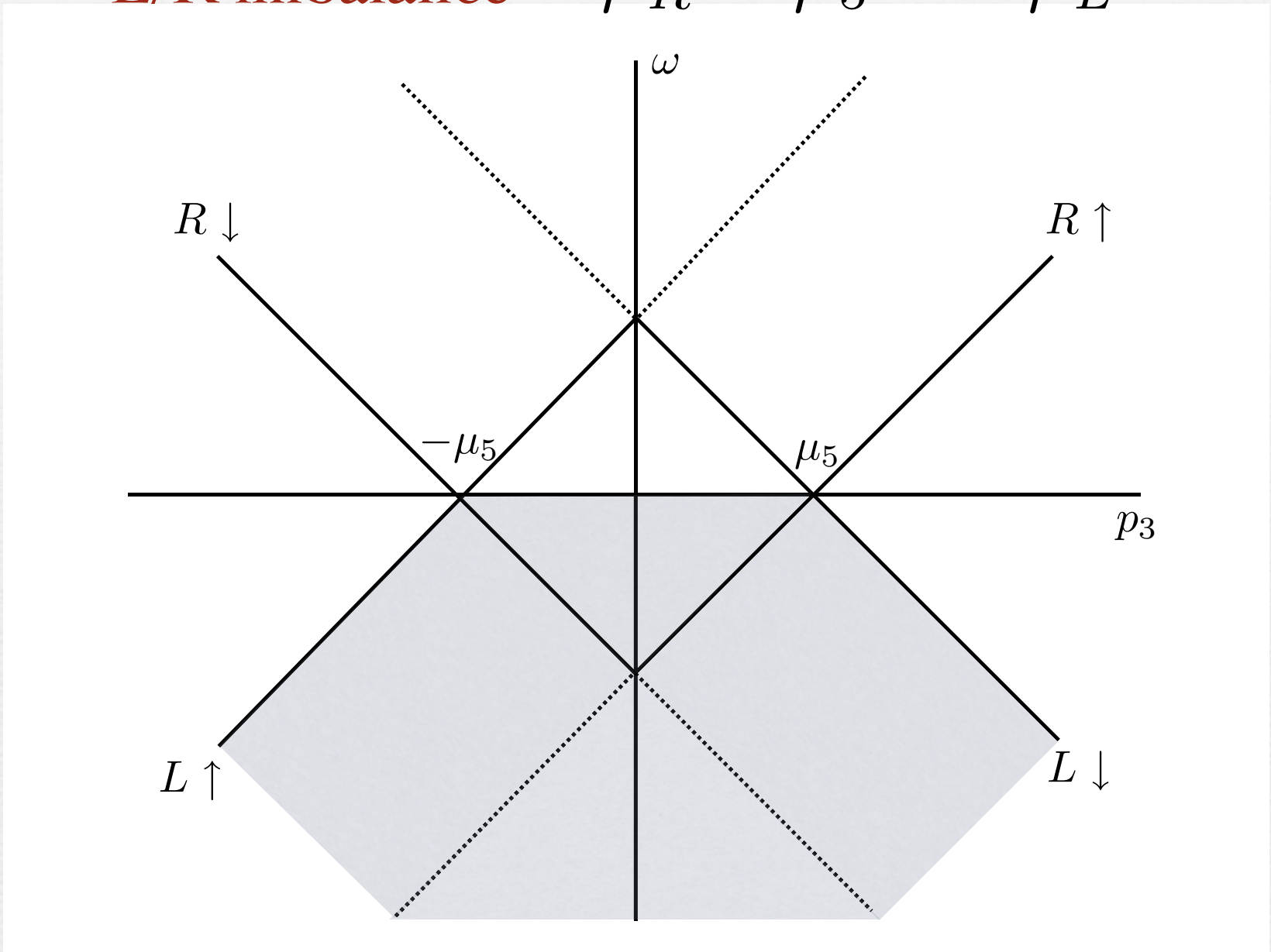

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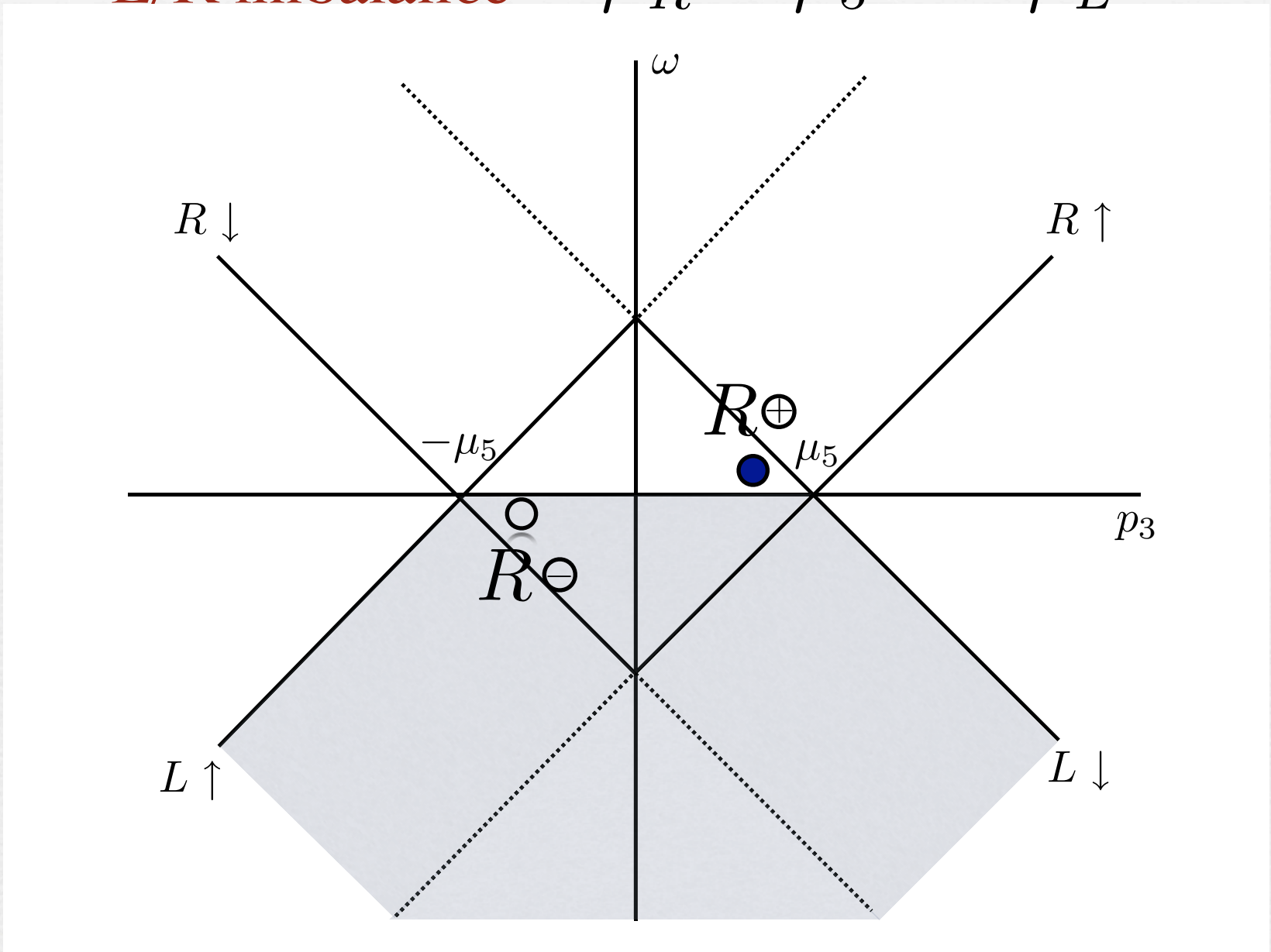

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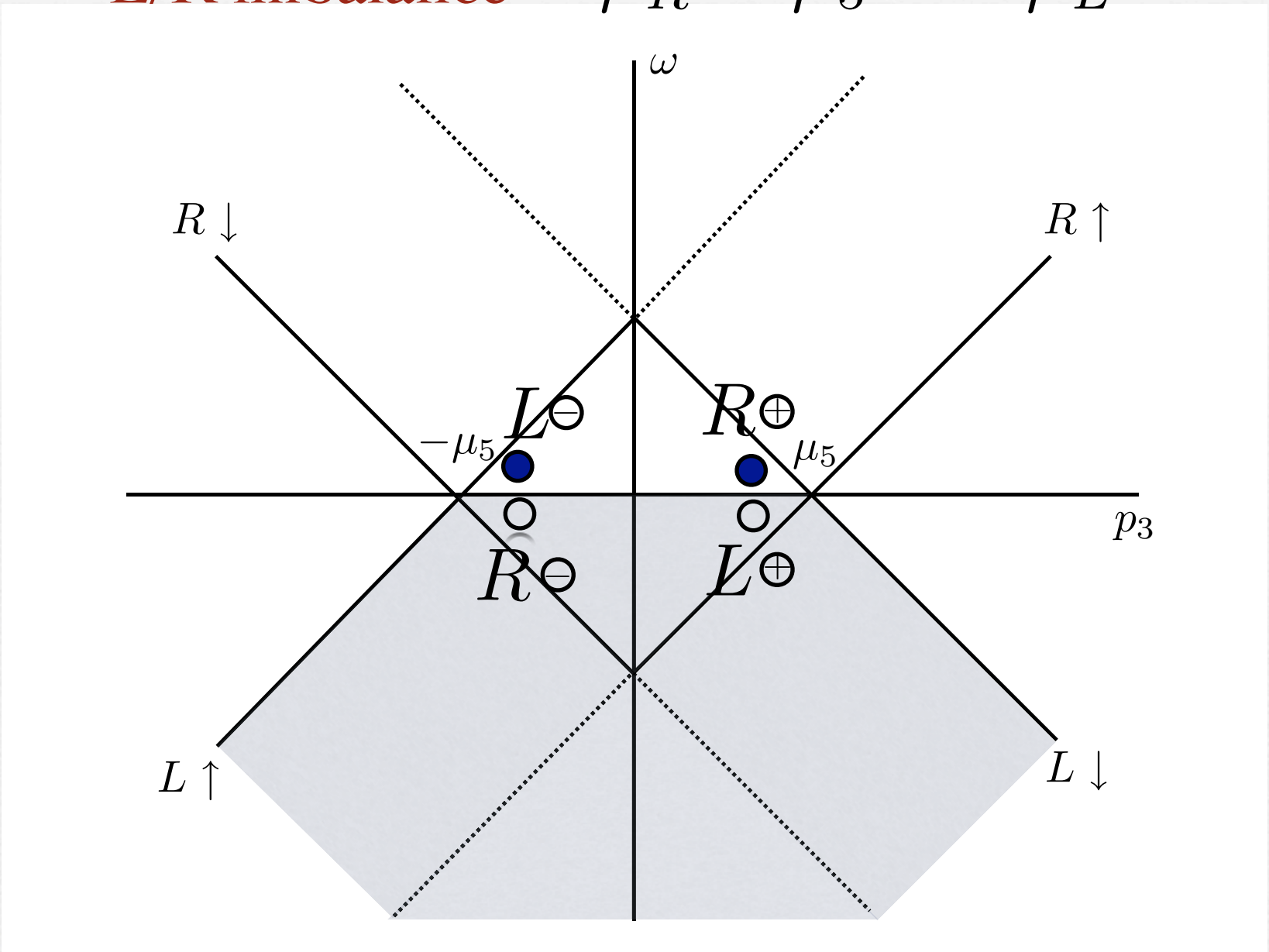

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recall: current projections

$$
\begin{aligned}
J_5^0 &= \mathcal{R}^\dagger \mathcal{R} - \mathcal{L}^\dagger \mathcal{L} \\
J_5^1 &= \bar{\mathcal{R}} \mathcal{R} + \bar{\mathcal{L}} \mathcal{L} \\
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$$

the chiral magnetic spiral

transverse currents : chiral condensate

$$
\begin{aligned}
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L/R imbalance : $\mu_5 \neq 0$ $\mu_R = \mu_5 = -\mu_L$

$$\langle J^{\perp} \rangle \sim \cos(2\mu_5\, z - \phi)$$

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finite density : $\mu \neq 0$ $\mu_R = \mu = \mu_L$

$$\langle J^\perp \rangle \sim \cos(2\mu\, z - \chi)$$

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implication:

chiral magnetic spiral can induce both
in-plane and out-of-plane charge asymmetries

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separation: Bzdak et al, 0912.5050

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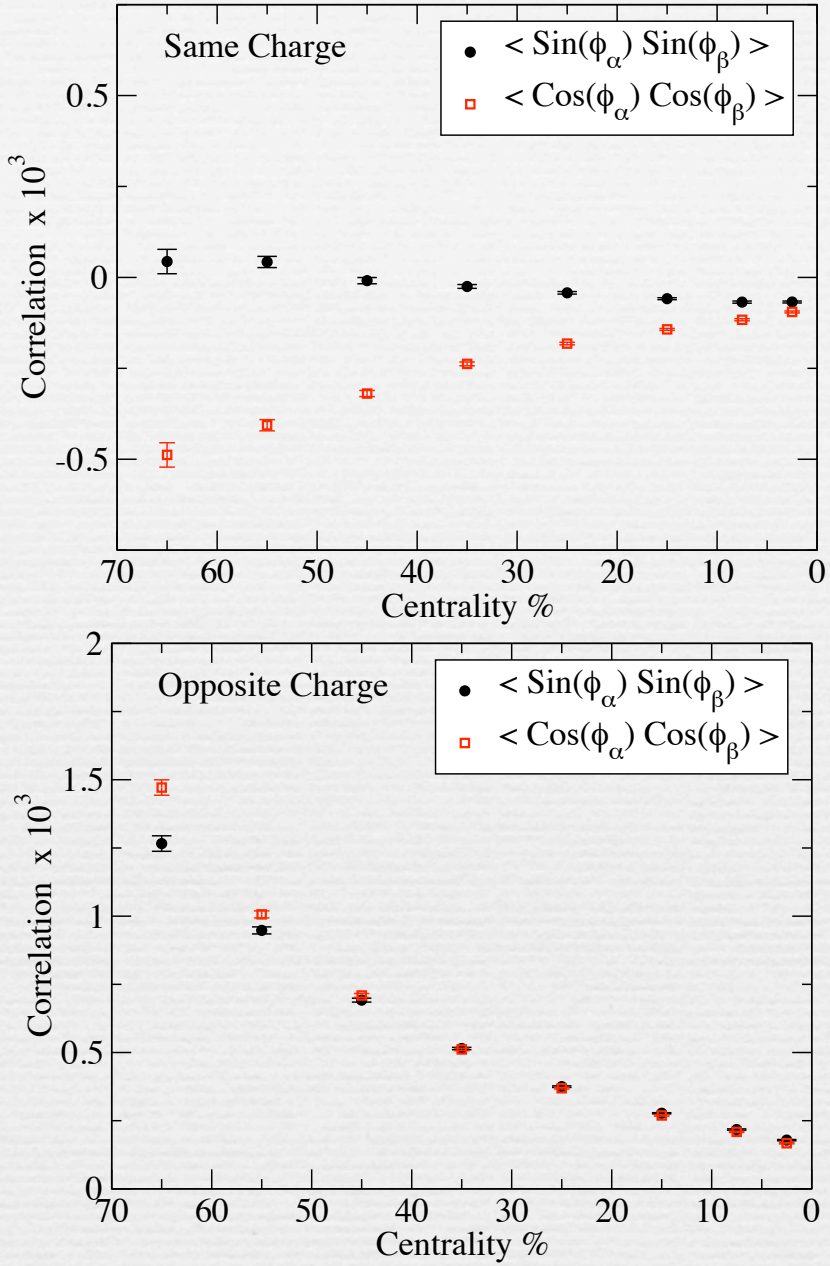

Figure 2: Correlations in-plane $\langle \cos(\phi_\alpha)\cos(\phi_\beta)\rangle$ and out of plane $\langle \sin(\phi_\alpha)\sin(\phi_\beta)\rangle$ for same and opposite charge pairs in $Au + Au$ collisions. As can be seen the correlations for same charge pairs are mainly in-plane.

Conclusions

- spiral condensates arise naturally in 1+1 dim chiral models (NJL, 't Hooft, quarkyonic, ...) at finite μ
- strong B field generates dimensional reduction to 1+1
- generates charge separation along B field ("CME")
- transverse components of currents form chiral spirals ("chiral magnetic spiral")

Conclusions

- spiral condensates arise naturally in 1+1 dim chiral models (NJL, 't Hooft, quarkyonic, ...) at finite μ
- strong B field generates dimensional reduction to 1+1
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- transverse components of currents form chiral spirals ("chiral magnetic spiral")

  - experimental signal at RHIC ?
  - realizable in other (CM, AMO, Astro ...) systems?
  - visible on lattice ?

kink crystal $$\sigma(x) = m\sqrt{\nu}\,\mathrm{sn}(m\,x;\nu)$$

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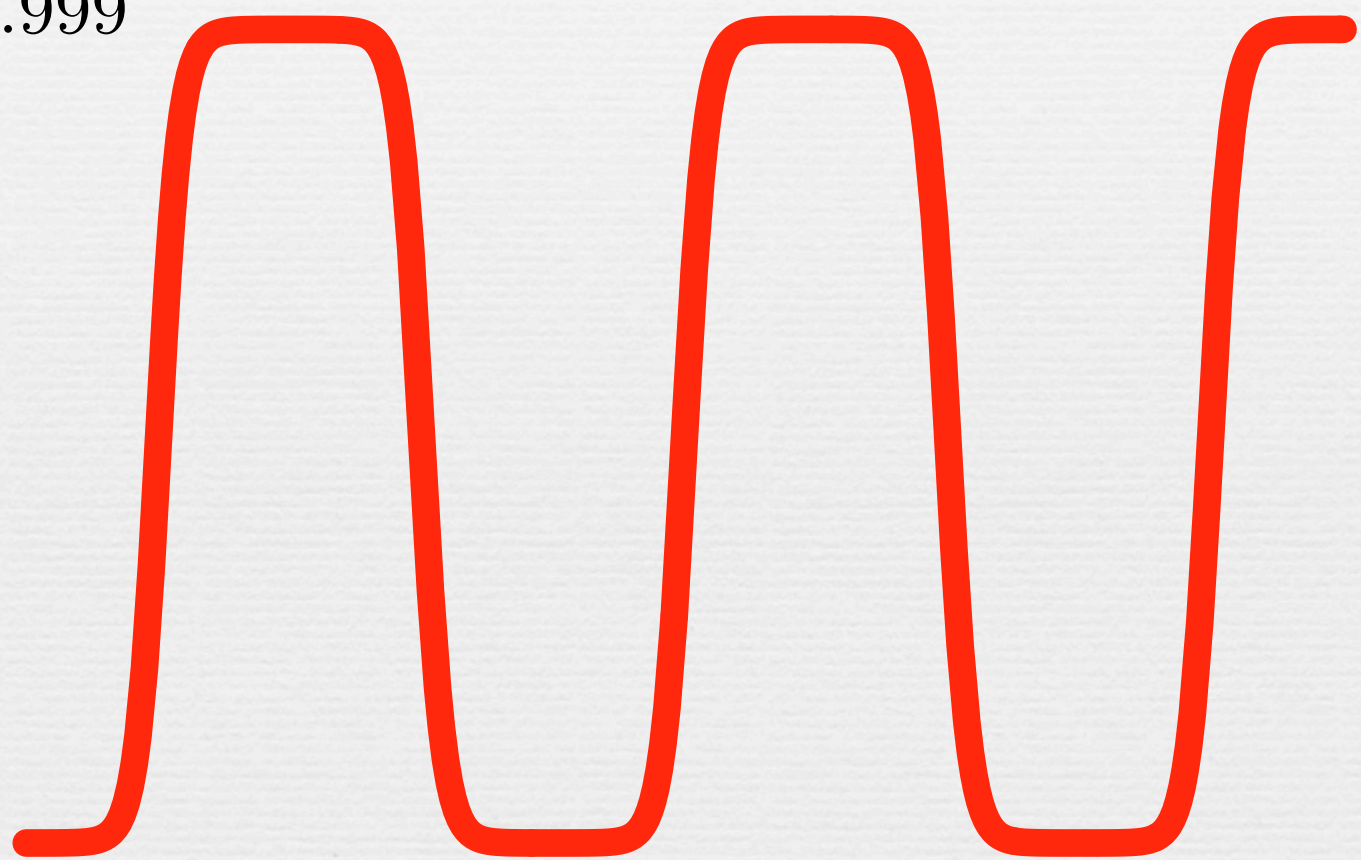
$\nu = 0.999$



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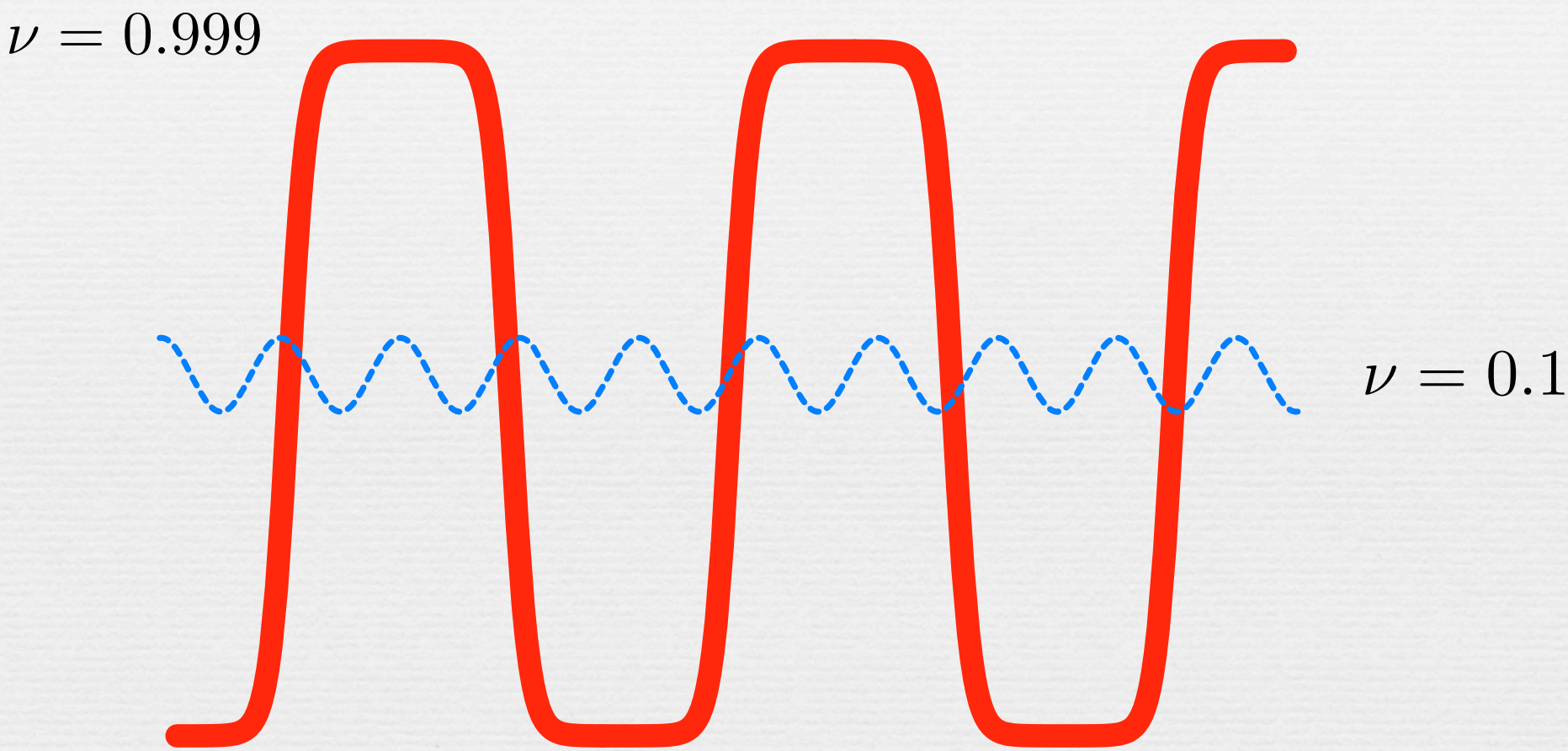

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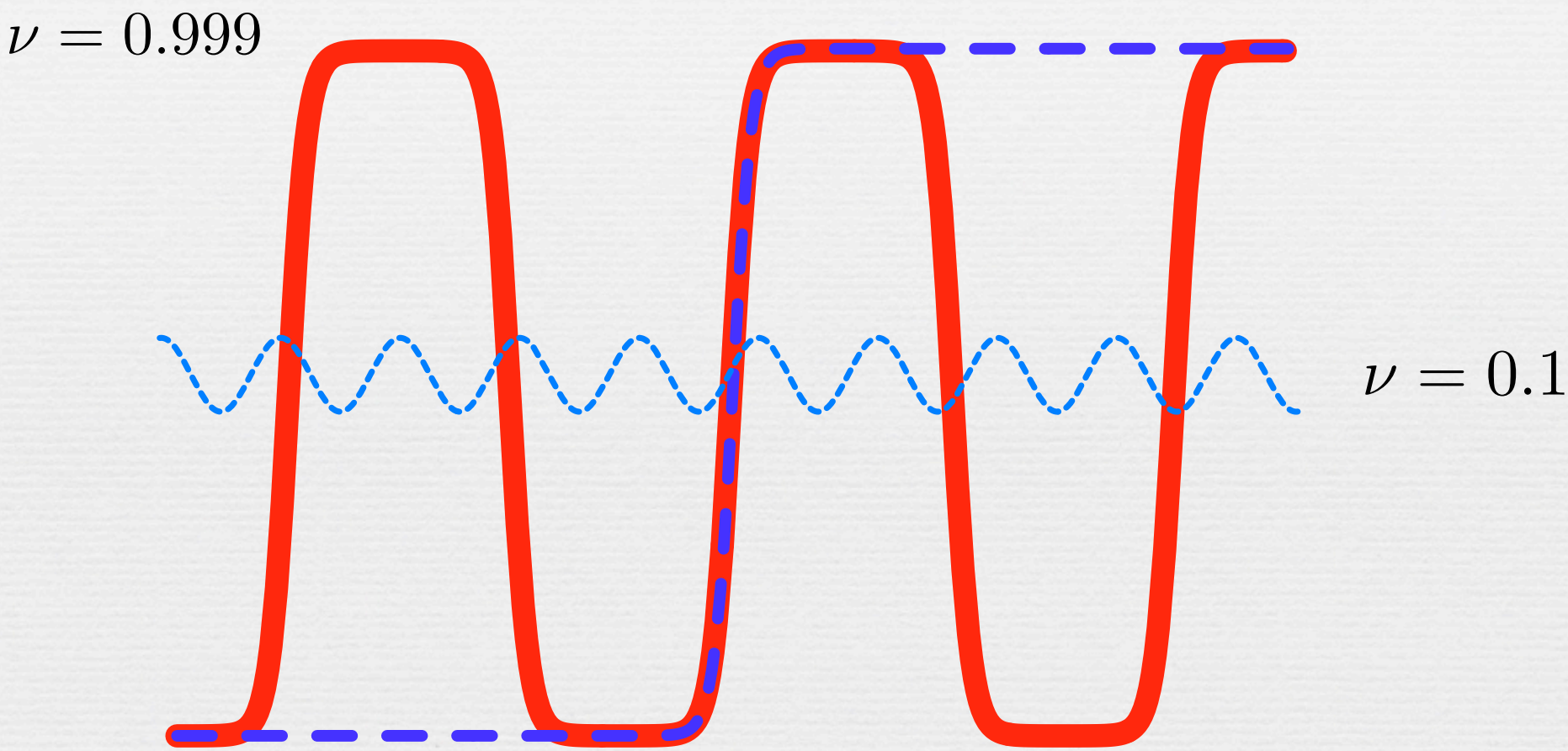

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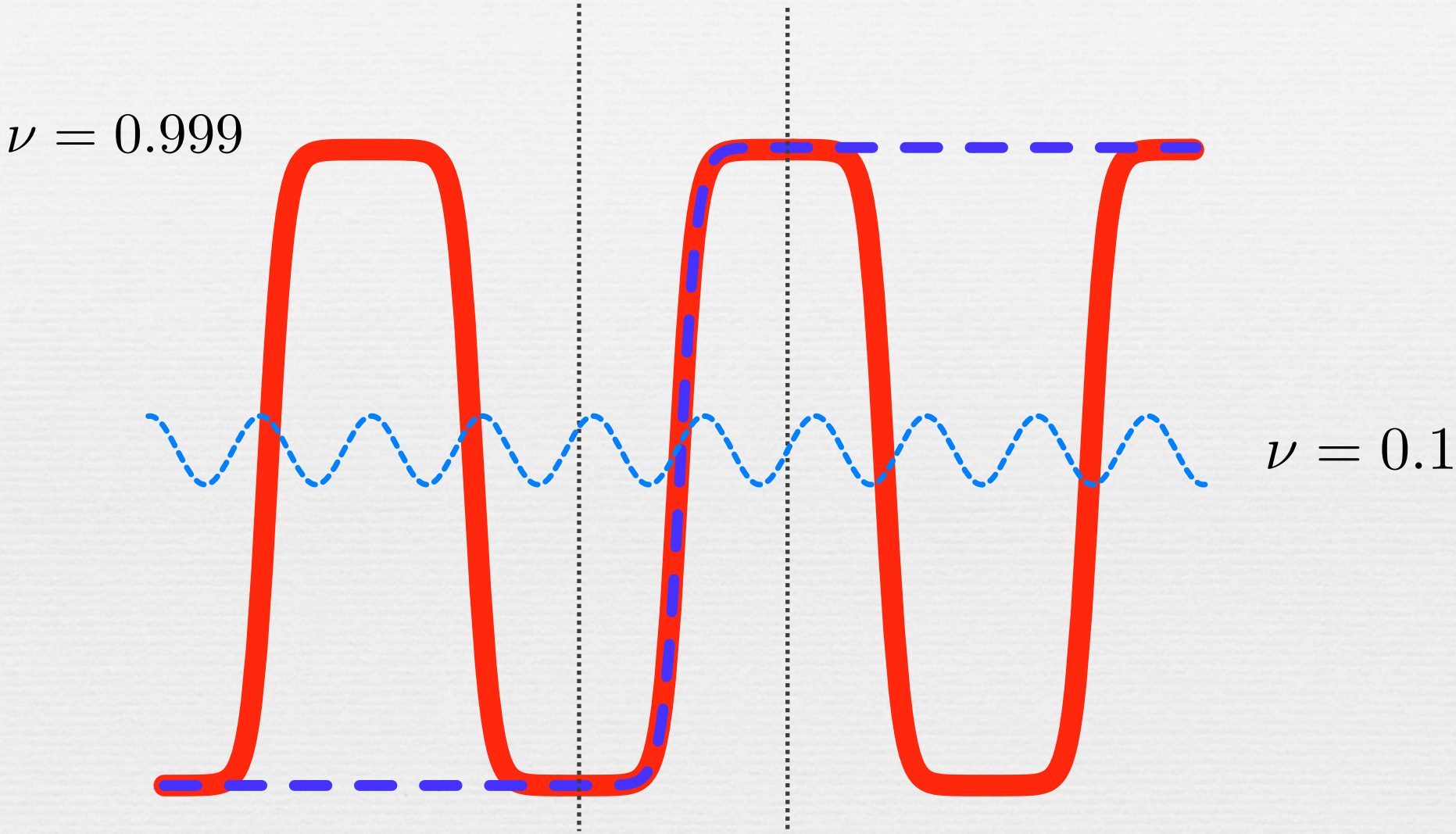

chemical potential and chiral spiral

$$\Omega = -\frac{1}{\beta}\int_{-\infty}^{\infty} dE\,\rho(E)\,\ln\left(1+e^{-\beta(E-\mu)}\right)$$

$$\Delta(x) \to e^{2iqx}\Delta(x) \qquad\qquad E \to E+q$$

$$\Omega[e^{2i\mu x}\Delta;T,\mu] = \Omega[\Delta;T,0] - \frac{\mu^2}{2\pi}$$

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density : $$\rho = -\frac{\partial\Omega}{\partial\mu} \quad\Rightarrow\quad \rho = \frac{\mu}{\pi}$$

gap equation, currents and condensates

$$\langle j^0 \rangle = \langle \psi^\dagger \psi \rangle = \frac{\mu}{\pi}$$

uniform charge density

$$\langle j^1 \rangle = \langle \psi^\dagger \gamma^5 \psi \rangle = 0$$

$$\partial_\mu \langle j^\mu \rangle = 0 = \partial_\mu \langle j_5^\mu \rangle$$

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$$\partial_\mu \langle j_5^\mu \rangle = 2(\sigma(x) \langle \bar{\psi}\, i\gamma^5 \psi \rangle - \pi(x) \langle \bar{\psi} \psi \rangle)$$

for any inhomogeneous condensate

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$$\langle j^1 \rangle = \langle \psi^\dagger \gamma^5 \psi \rangle = 0$$

$$\partial_\mu \langle j^\mu \rangle = 0 = \partial_\mu \langle j_5^\mu \rangle$$

$$\partial_\mu \langle j_5^\mu \rangle = 2(\sigma(x) \langle \bar{\psi}\, i\gamma^5 \psi \rangle - \pi(x) \langle \bar{\psi} \psi \rangle)$$

for any inhomogeneous condensate

vanishes on solution of gap equation

gap equation, currents and condensates

$$\langle j^0 \rangle = \langle \psi^\dagger \psi \rangle = \frac{\mu}{\pi}$$

uniform charge density

$$\langle j^1 \rangle = \langle \psi^\dagger \gamma^5 \psi \rangle = 0$$

$$\partial_\mu \langle j^\mu \rangle = 0 = \partial_\mu \langle j_5^\mu \rangle$$

$$\partial_\mu \langle j_5^\mu \rangle = 2(\sigma(x) \langle \bar{\psi}\, i\gamma^5 \psi \rangle - \pi(x) \langle \bar{\psi}\psi \rangle)$$

for any inhomogeneous condensate

vanishes on solution of gap equation

chiral spiral condensate: $\langle \bar{\psi}\psi \rangle - i \langle \bar{\psi} i \gamma^5 \psi \rangle = A\, e^{2i\mu x}$