Quarkyonic Chiral Spirals

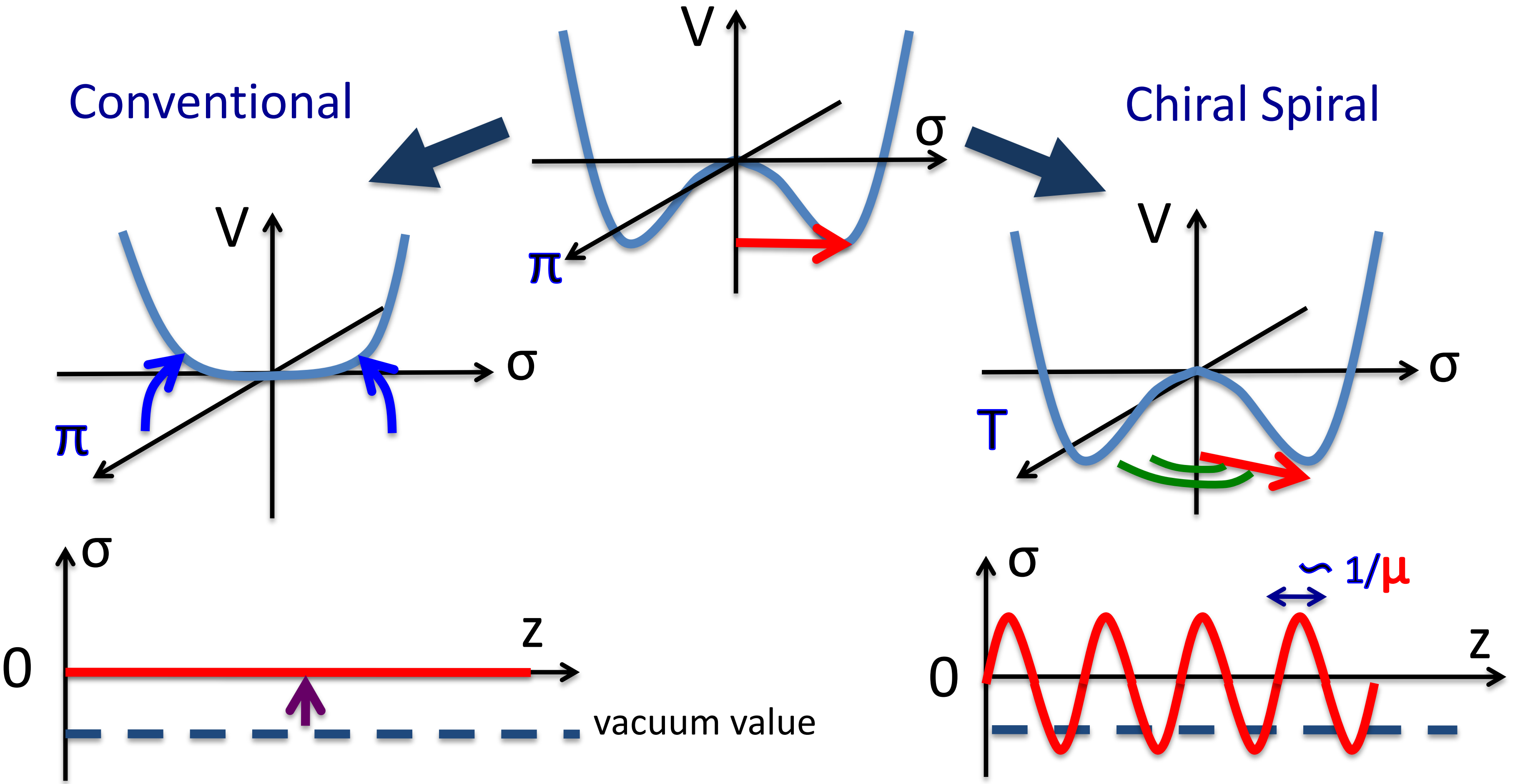
Toru Kojo (RBRC)

based on the works

T.K., Y. Hidaka, L. McLerran, R. Pisarski ; arXiv:0912.3800 [hep-ph]
(to be published in NPA)

T.K., R. Pisarski, A.M. Tsvelik ; in preparation

Local violation of chiral sym. in dense quark matter



zero everywhere.

nonzero locally, but globally zero

Local P, CP violation

(cf: G. Dunne’s talk)

Contents

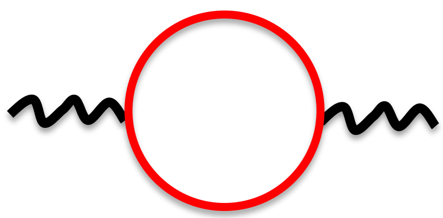
1, Introduction

Quarkyonic matter, chiral pairing phenomena

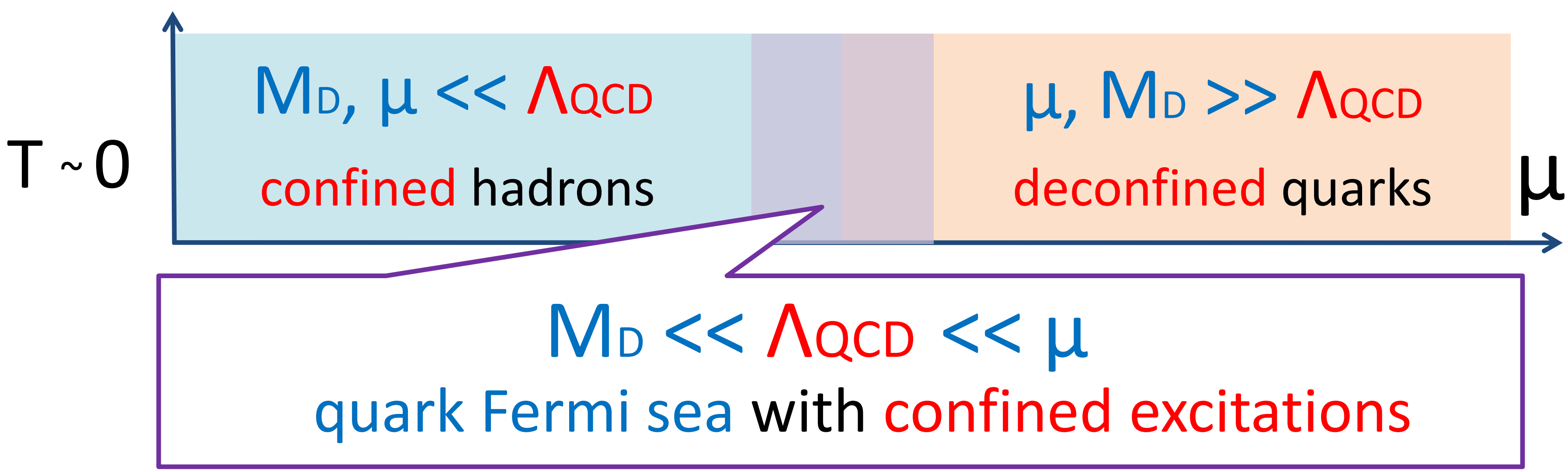
Dense QCD at T=0 : Confining aspects

Allowed phase space differs → Different screening effects

e.g.)



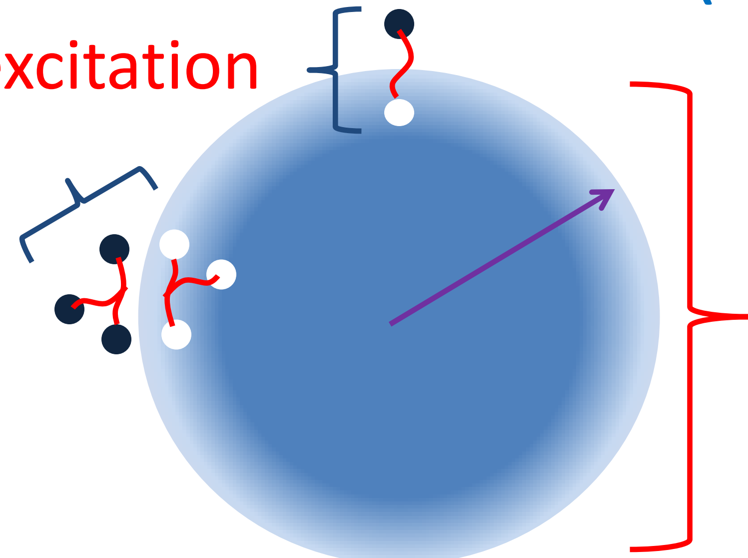
$$M_D \sim N_c^{-1/2} \times f(\mu)$$



Quarkyonic Matter

McLerran & Pisarski (2007) $(M_D \ll \Lambda_{QCD} \ll \mu)$

hadronic excitation



As a total,
color singlet
→ deconfined quarks

Quark Fermi sea + baryonic Fermi surface → Quarkyonic

- Large Nc: $M_D \backsim N_c^{-1/2} \rightarrow 0$

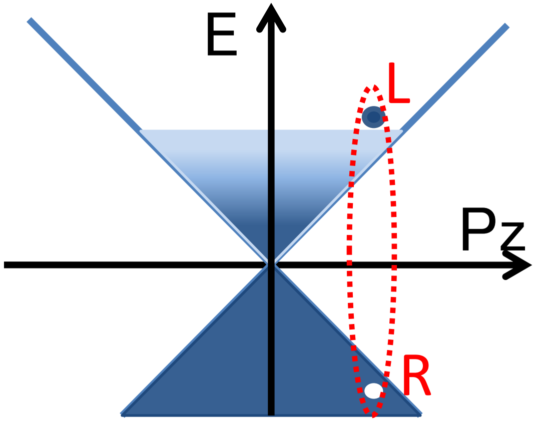
Quarkyonic regime always holds.

(so we can use vacuum gluon propagator)

How is chiral symmetry realized ?

- Candidates which spontaneously break Chiral Symmetry

Dirac Type

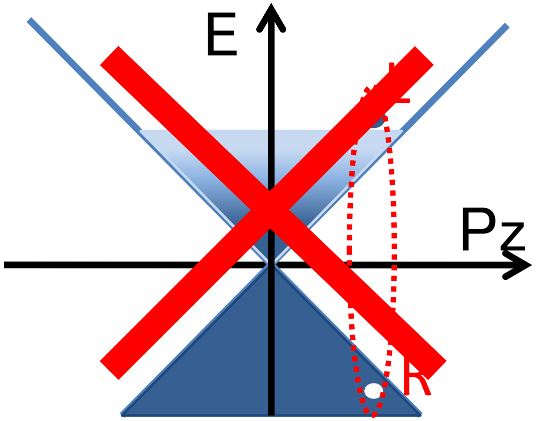


$P_{Tot}=0$ (uniform)

Chiral Pairing Phenomena

▪Candidates which spontaneously break Chiral Symmetry

Dirac Type



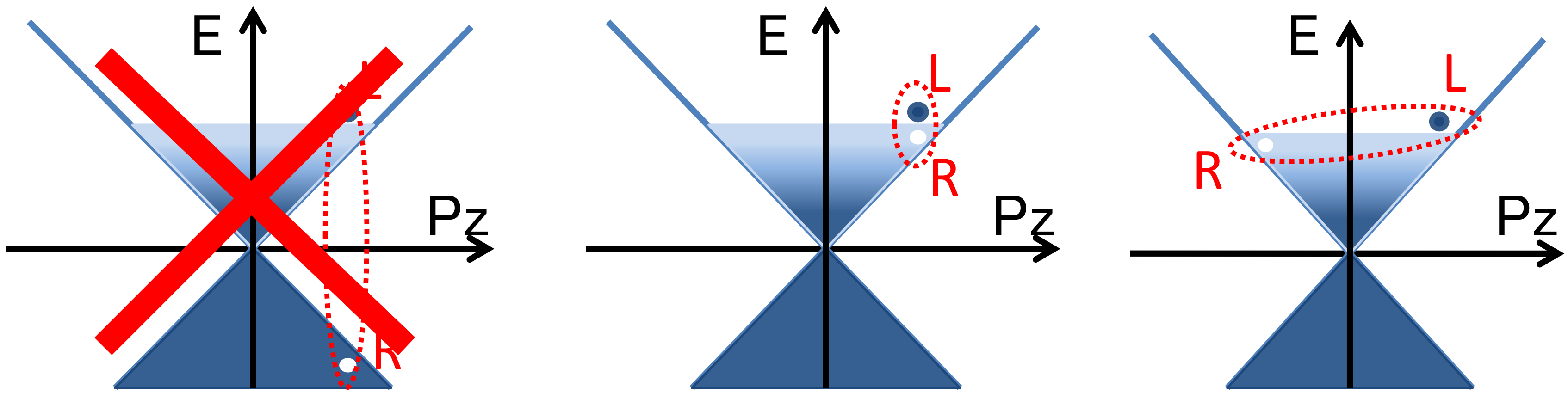
$P_{Tot}=0$ (uniform)

It costs large energy,
so does not occur spontaneously.

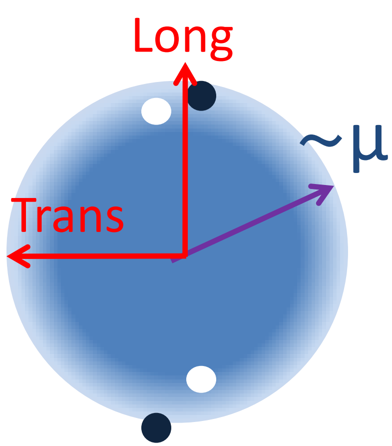
Chiral Pairing Phenomena

- Candidates which spontaneously break Chiral Symmetry

Dirac Type | Exciton Type | Density wave



$P_{Tot}=0$ (uniform) | $P_{Tot}=0$ (uniform) | $P_{Tot}=2\mu$ (nonuniform)



We will identify the most relevant pairing:
Exciton & Density wave solutions
will be treated and compared simultaneously.

2, How to solve

Dimensional reduction from (3+1)D to (1+1)D

Preceding works on the chiral density waves

(Many works, so incomplete list)

- **Nuclear matter or Skyrme matter:**

  Migdal '71, Sawyer & Scalapino '72.. : effective lagrangian for nucleons and pions

- **Quark matter:**

  Perturbative regime with Coulomb type gluon propagator:

  Deryagin, Grigoriev, & Rubakov '92: Schwinger-Dyson eq. in large Nc

  Shuster & Son, hep-ph/9905448: Dimensional reduction of Bethe-Salpeter eq.

  Rapp, Shuryak, and Zahed, hep-ph/0008207: Schwinger-Dyson eq.

  Effective model:

  Nakano & Tatsumi, hep-ph/0411350, D. Nickel, 0906.5295

  Nonperturbative regime with linear rising gluon propagator:

  → Present work

Set up of the problem

- Confining propagator for quark-antiquark:

$$D_{\mu\nu} = C_F \times g_{\mu 0} g_{\nu 0} \times \frac{\sigma}{(\vec{p}^2)^2}$$ (linear rising type)

cf) leading part of Coulomb gauge propagator (ref: Gribov, Zwanziger)

But how to treat conf. model & non-uniform system??

- 2 Expansion parameters in Quarkyonic limit

$1/N_c \to 0$: Vacuum propagator is not modified

(ref: Glozman, Wagenbrunn, PRD77:054027, 2008;
Guo, Szczepaniak,arXiv:0902.1316 [hep-ph]).

$\Lambda_{QCD}/\mu \to 0$: Factorization approximation

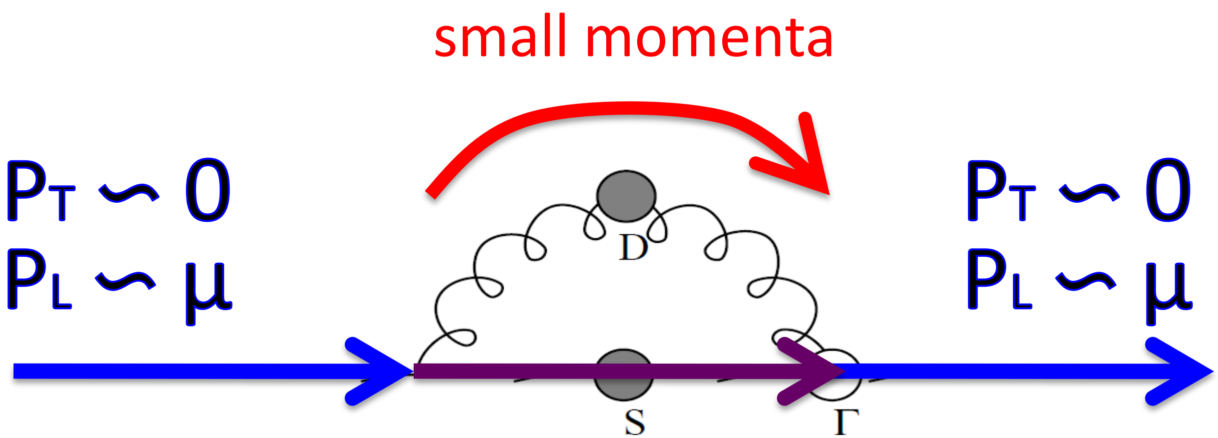
We will perform the dimensional reduction of nonperturbative self-consistent equations, Schwinger-Dyson & Bethe-Salpeter eqs.

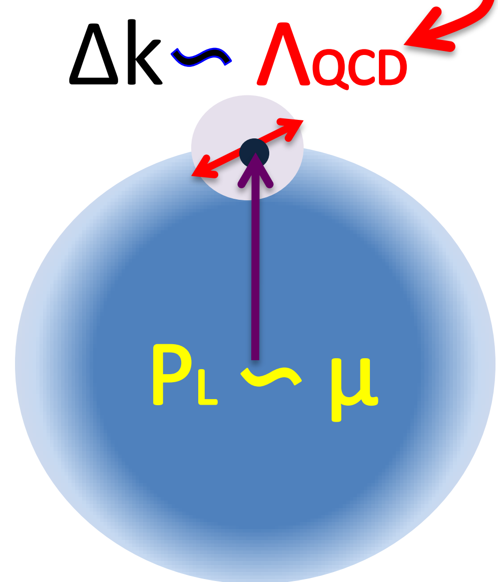
e.g.) Dim. reduction of Schwinger-Dyson eq.

quark self-energy　　　　　　　　　　including Σ

$$\not{\Sigma}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4} \gamma_4 S(k) \gamma_4 \frac{\sigma}{|\vec{p} - \vec{k}|^4}$$

- Note1: Mom. restriction from **confining** interaction.





e.g.) Dim. reduction of Schwinger-Dyson eq.

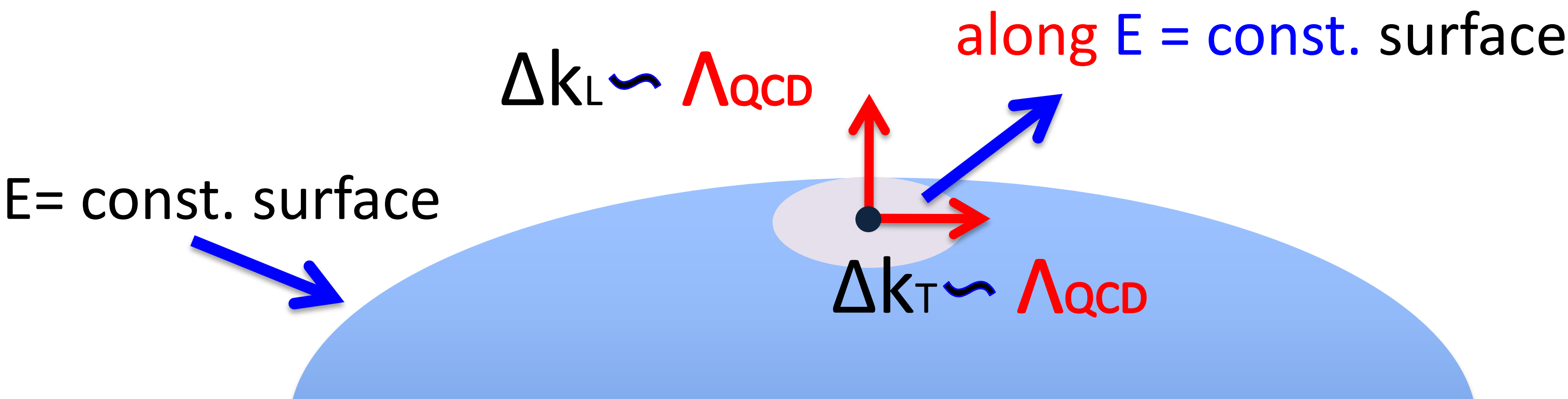
quark self-energy

$$\Sigma\!\!\!/(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4} \gamma_4 \underline{S(k)} \gamma_4 \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

- Note2: Suppression of transverse part:

$$S(k) = \gamma_0 S_0 - \underbrace{\gamma_z S_z}_{\sim \mu} - \underbrace{\vec{\gamma}_T \cancel{\vec{S}_T}}_{\sim \Lambda_{QCD}} + S_m$$

- Note3: quark energy is insensitive to small change of k_T :



e.g.) Dim. reduction of Schwinger-Dyson eq.

insensitive to kT

$$\not\Sigma(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4} \gamma_4 S(k) \gamma_4 \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

factorization

$$\gamma_4 \Sigma_4 + \gamma_z \Sigma_z = \int \frac{dk_4 dk_z}{(2\pi)^2} \gamma_4 S(k_4, k_z, \underline{\vec{0}_T}) \gamma_4 \otimes \int \frac{d\vec{k}_T}{(2\pi)^2} \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

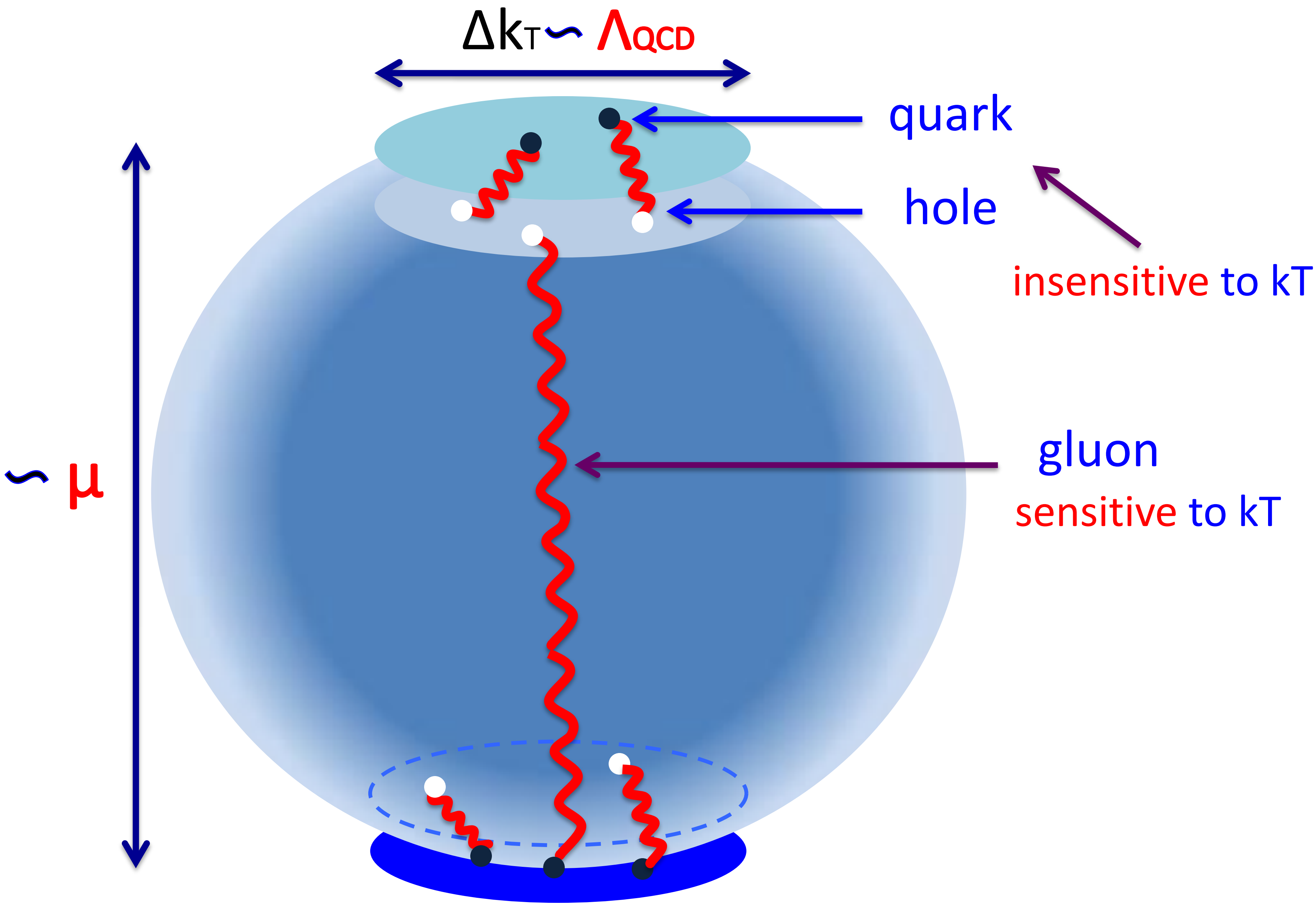
smearing

confining propagator in (1+1)D: $\boxed{\dfrac{\sigma}{2\pi} \dfrac{1}{|p_z - q_z|^2}}$

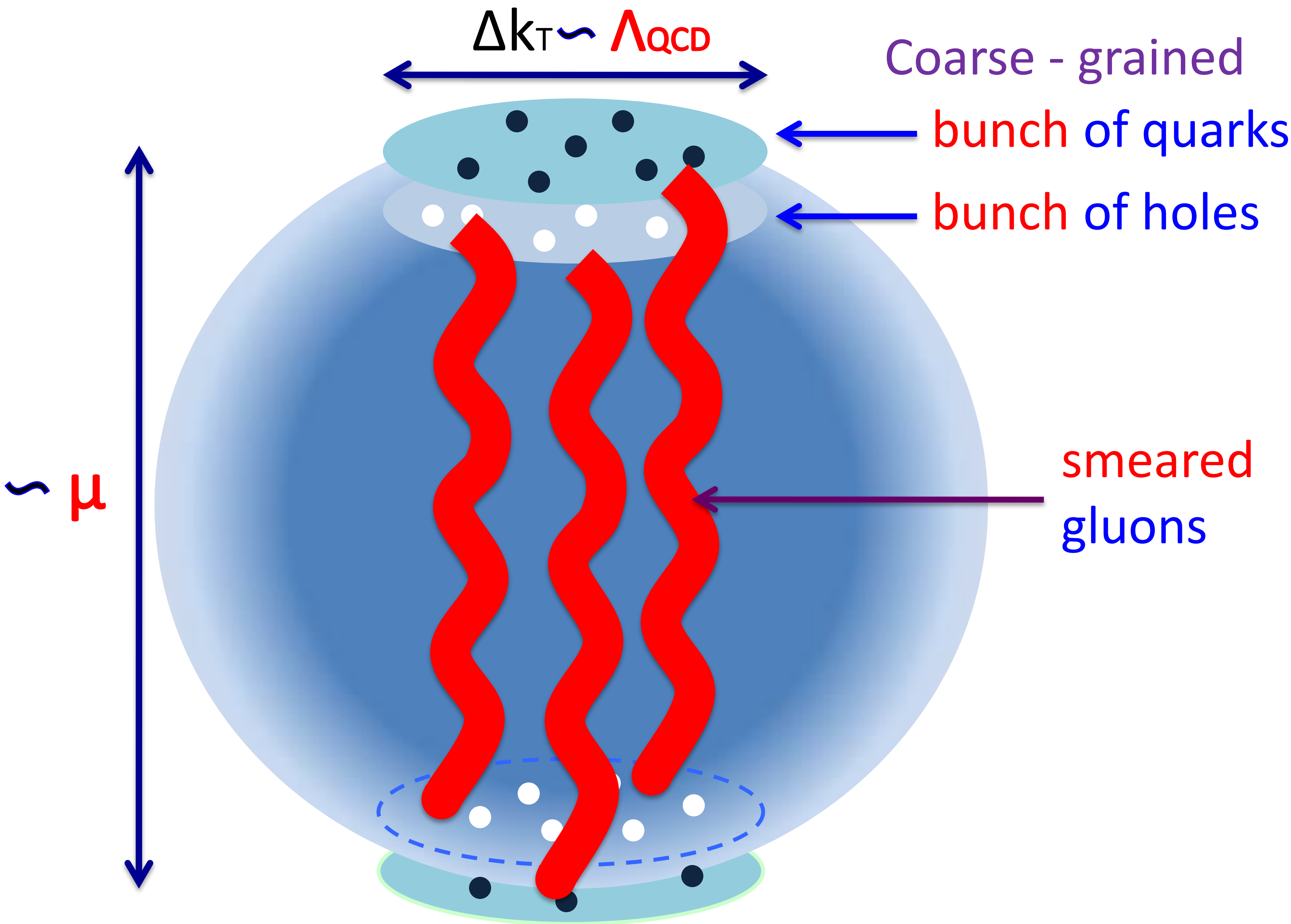
Schwinger-Dyson eq. in (1+1) D QCD in $A_1=0$ gauge

Bethe-Salpeter eq. can be also converted to (1+1)D

Catoon for Pairing dynamics before reduction



1+1 D dynamics of patches after reduction



3, Two Dictionaries

- between (3+1)D & (1+1)D quantum numbers
- between $\mu = 0$ & $\mu \neq 0$ condensates

Flavor Doubling

- At leading order of $1/N_c$ & Λ_{QCD}/μ

Dimensional reduction of Non-pert. self-consistent eqs :

4D "QCD" in Coulomb gauge $\longleftrightarrow$ 2D QCD in $A_1=0$ gauge

(confining model)

- One immediate nontrivial consequence: $P_T/P_L \to 0$

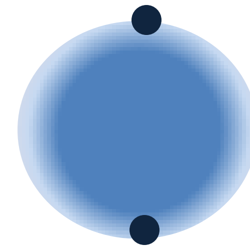
Absence of γ_1, γ_2 → Absence of spin mixing

suppression of spin mixing		no angular d.o.f in (1+1) D
spin SU(2) x SU(Nf)	$\longleftrightarrow$	SU(2Nf)
(3+1)-D side		(1+1)-D side

cf) Shuster & Son, NPB573, 434 (2000)

Flavor Multiplet

particle near north & south pole

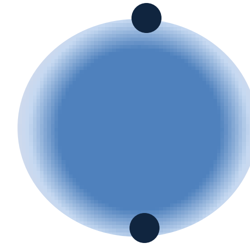


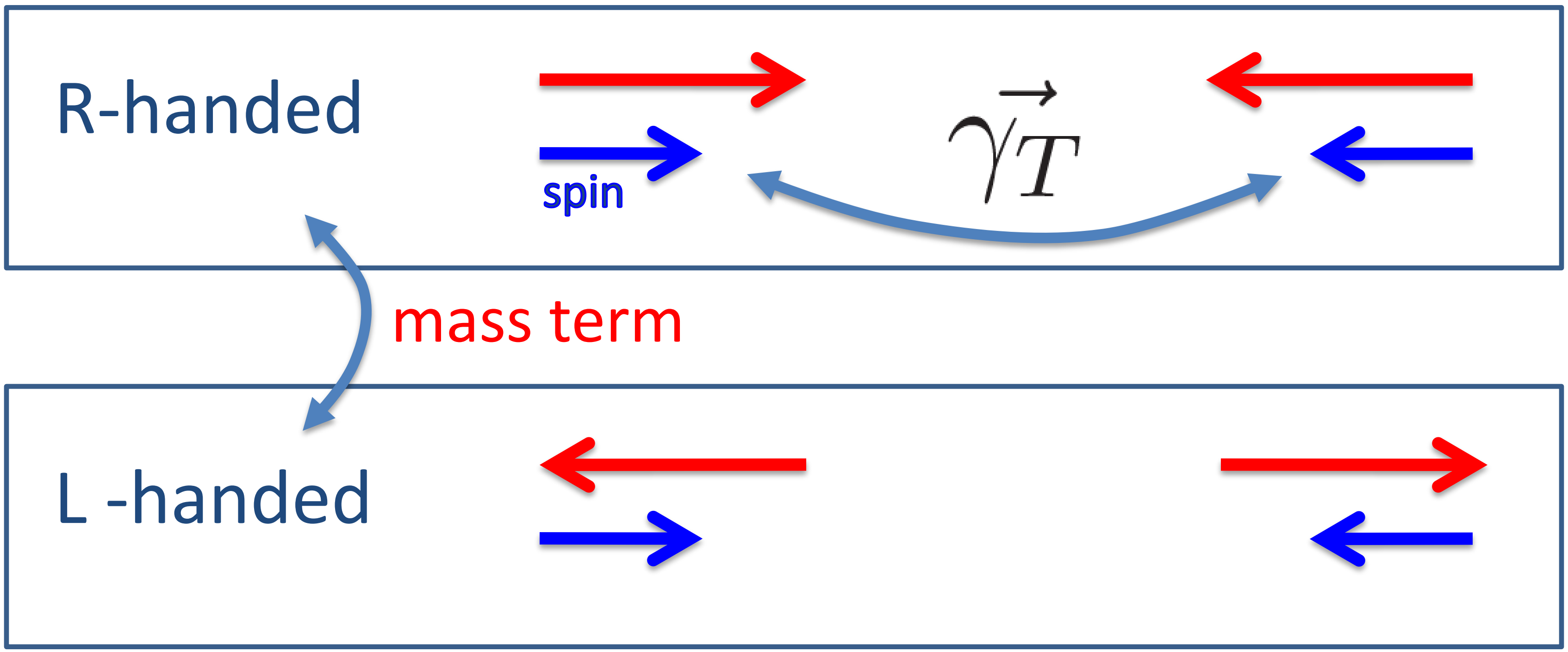
R-handed

L -handed

Flavor Multiplet

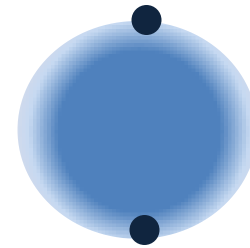
particle near north & south pole

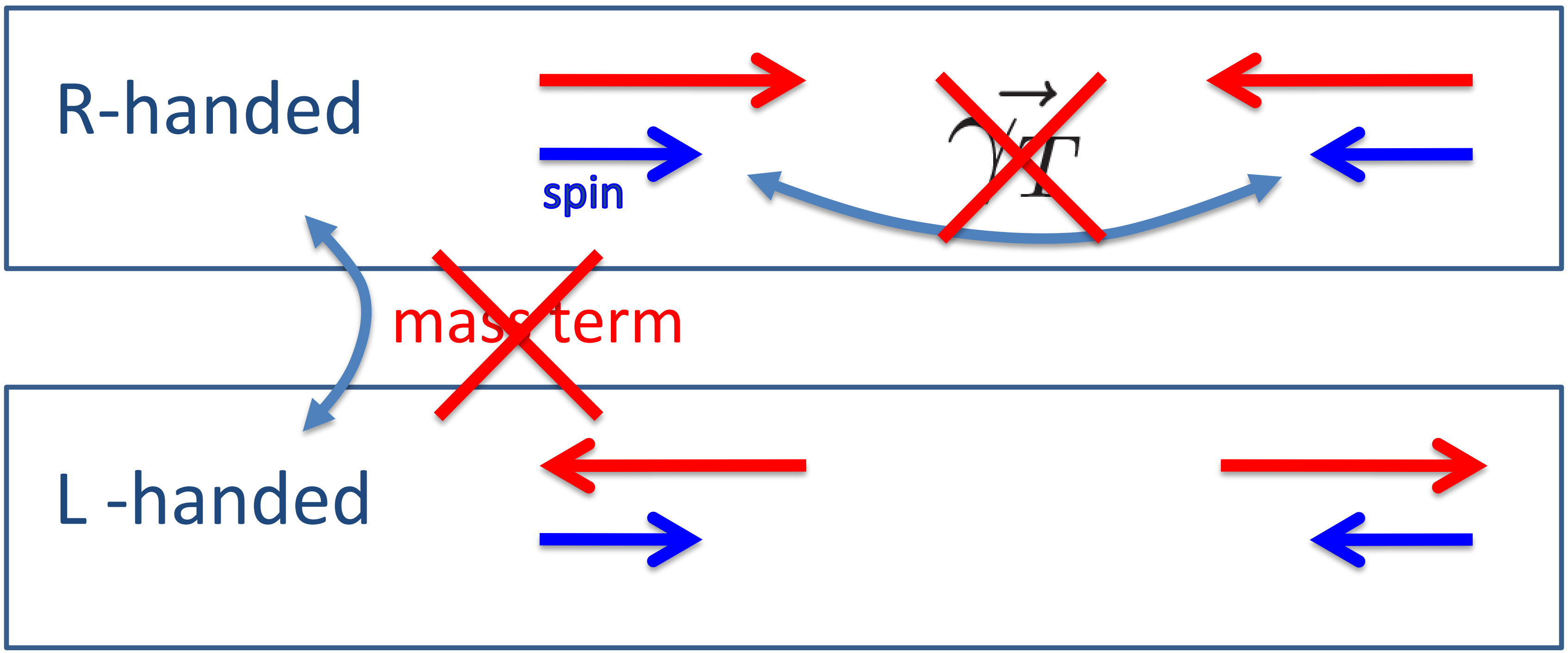




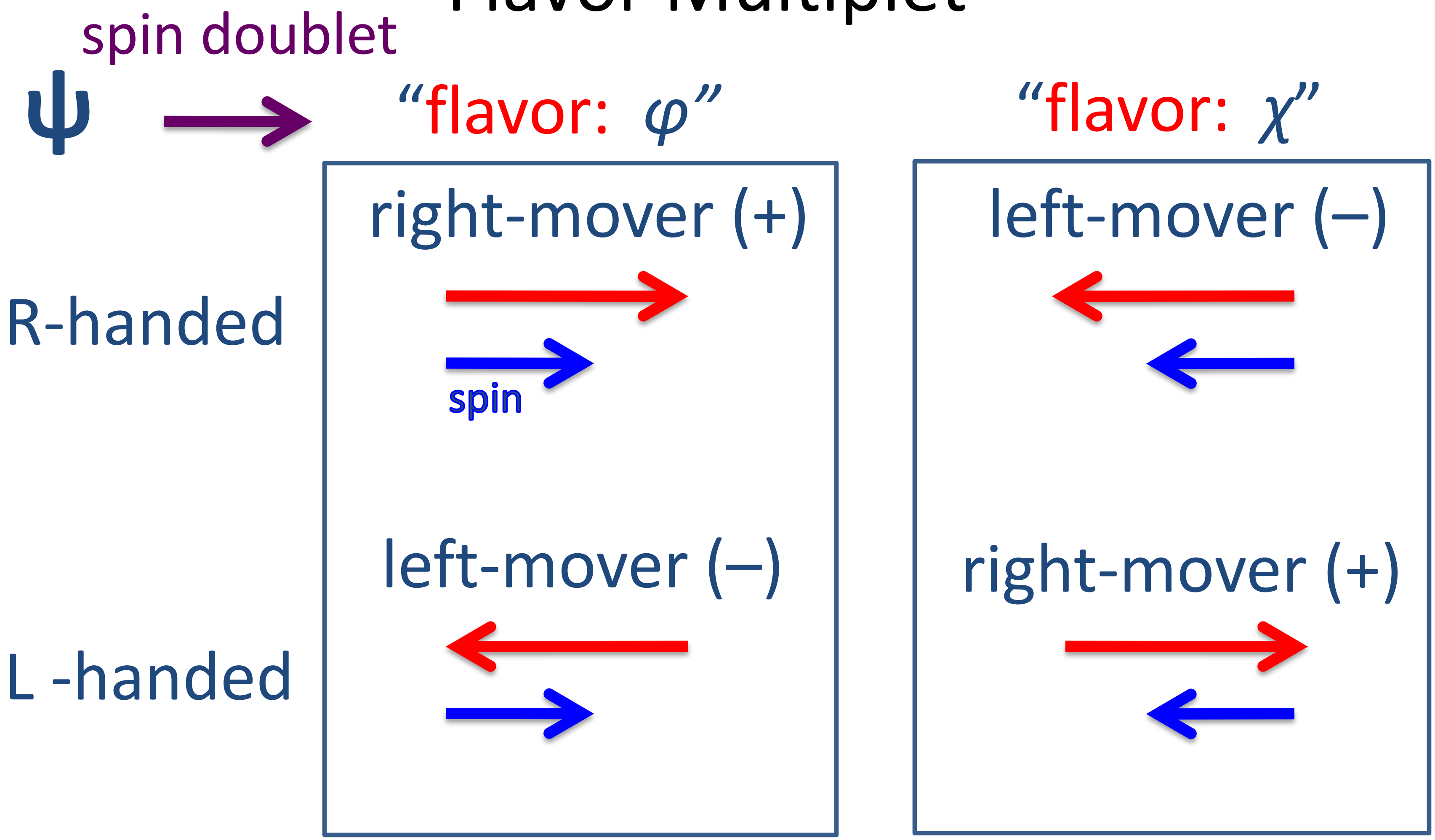
Flavor Multiplet

particle near north & south pole





Flavor Multiplet



Moving direction: (1+1)D “chirality”

(3+1)D – CPT sym. directly convert to (1+1)D ones

Relations between composite operators

- 1-flavor (3+1)D operators without spin mixing:

$$\bar{\psi}\psi \quad \bar{\psi}\gamma^0\psi \quad \bar{\psi}\gamma^z\psi \quad \bar{\psi}\gamma^0\gamma^z\psi$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\bar{\Psi}\Psi \quad \bar{\Psi}\Gamma^0\Psi \quad \bar{\Psi}\Gamma^z\Psi \quad \bar{\Psi}\Gamma^5\Psi$$

Flavor singlet in (1+1)D

- All others have spin mixing:

ex) $\bar{\psi}\gamma^5\psi \longrightarrow \bar{\Psi}\tau_3\Psi$

$\bar{\psi}\gamma^1\psi \longrightarrow \bar{\Psi}\tau_2\Psi$

(They will show no flavored condensation)

Flavor non-singlet in (1+1)D

2nd Dictionary: μ = 0 & μ ≠ 0 in (1+1)D

- μ ≠ 0 2D QCD can be mapped onto μ = 0 2D QCD

$$\Psi' = e^{i\mu z \Gamma_5}\Psi$$: chiral rotation (or boost)

$$\boxed{\underbrace{\bar{\Psi}\left[i\Gamma^\mu \partial_\mu + \mu\Gamma^0\right]\Psi}_{(\mu \neq 0)} \rightarrow \underbrace{\bar{\Psi}' i\Gamma^\mu \partial_\mu \Psi'}_{(\mu = 0)}}$$

(due to special geometric property of 2D Fermi sea)

- Dictionary between μ = 0 & μ ≠ 0 condensates:

μ = 0 | μ ≠ 0

$$\langle \bar{\Psi}'\Psi' \rangle \rightarrow \langle \bar{\Psi}\Psi \rangle \cos(2\mu z) + \langle \bar{\Psi} i\Gamma_5 \Psi \rangle \sin(2\mu z)$$

$$\underbrace{\langle \bar{\Psi}'\Gamma^0\Psi' \rangle}_{(\,=0\,)} \rightarrow \underbrace{\langle \bar{\Psi}'\Gamma^0\Psi' \rangle}_{(\,=0\,)} + \underline{\frac{\mu}{2\pi}}$$

induced by anomaly

"correct baryon number"

Why Chiral Rotation in (1+1)D ?

- Key observation: Moving direction = (1+1)D Chirality

$$\langle \bar{\varphi}_- \varphi_+ \rangle = \Delta e^{\underline{2i\mu z}} \qquad \langle \bar{\varphi}_+ \varphi_- \rangle = \Delta e^{\underline{-2i\mu z}}$$

Opposite phase

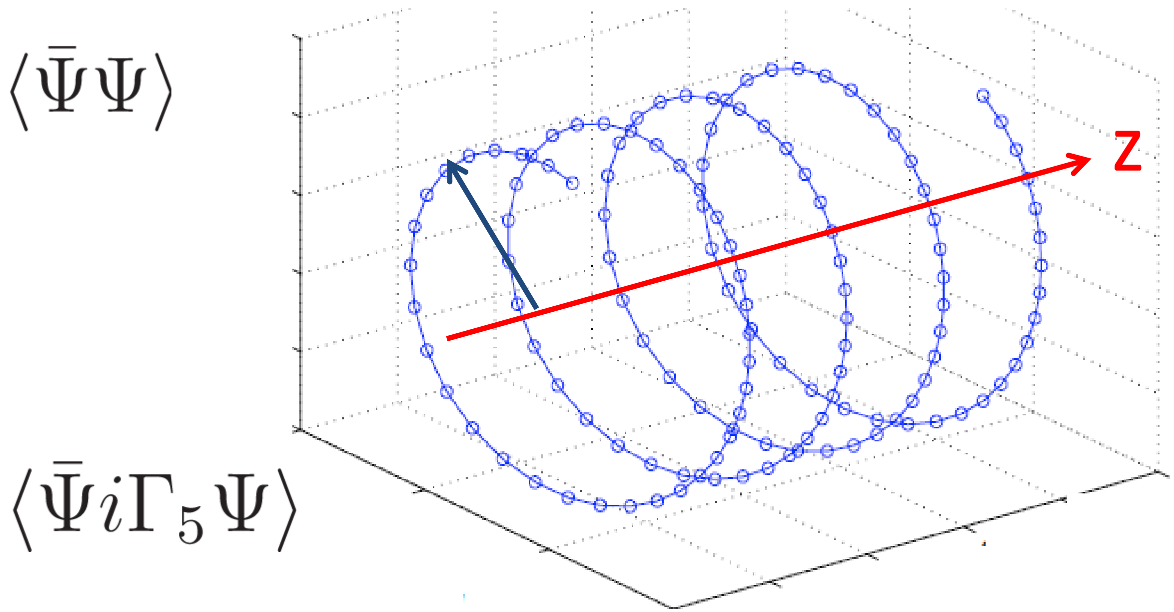
$$\rightarrow \boxed{\langle \bar{\varphi}\Gamma_5 \varphi \rangle = \langle \bar{\varphi}_- \varphi_+ \rangle - \langle \bar{\varphi}_+ \varphi_- \rangle = \Delta i \sin 2\mu z \neq 0}$$

Density wave of $\bar{\Psi}\Psi$ inevitably accompanies $\bar{\Psi}\Gamma^5\Psi$

(1+1)D: Chiral Density wave → Chiral Spiral

Chiral Spirals in (1+1)D

- At μ ≠ 0: periodic structure (crystal) which oscillates in space.



cf)
- Chiral Gross Neveu model (with continuous chiral symmetry)

  Schon & Thies, hep-ph/0003195; 0008175; Thies, 06010243

  Basar & Dunne, 0806.2659; Basar, Dunne & Thies, 0903.1868

- `tHooft model, massive quark (1-flavor)

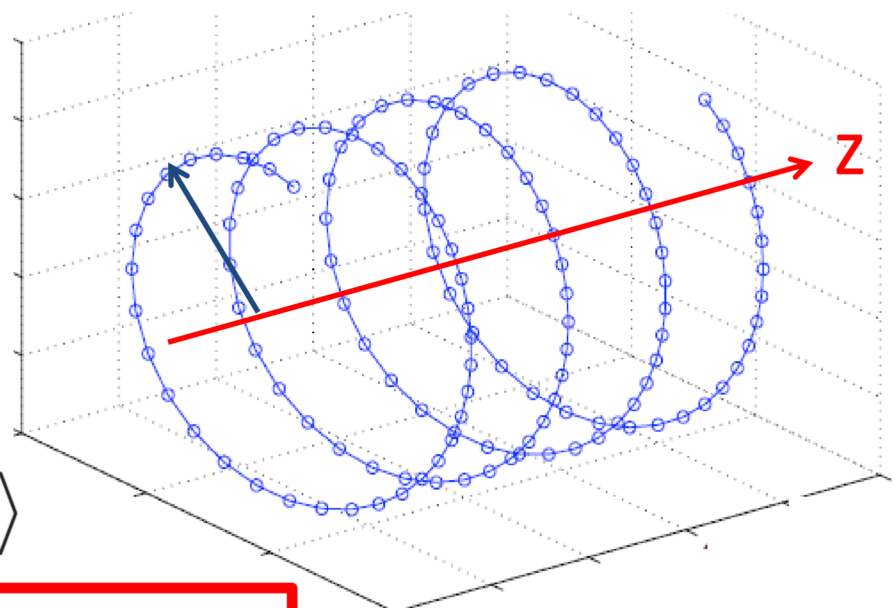
  B. Bringoltz, 0901.4035

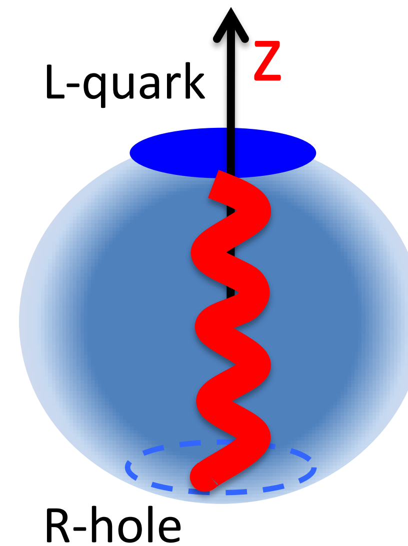
Quarkyonic Chiral Spirals in (3+1)D

- Chiral rotation evolves in the longitudinal direction:

$\langle \bar{\Psi}\Psi \rangle$

$$\boxed{\langle \bar{\psi}\psi \rangle}$$





$\langle \bar{\Psi} i\Gamma_5 \Psi \rangle$

$$\boxed{\langle \bar{\psi}\gamma^0\gamma^z\psi \rangle}$$

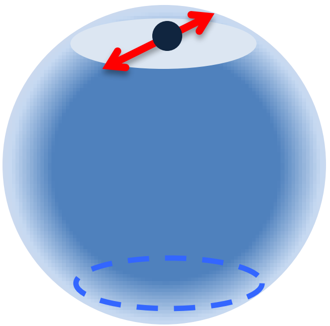
(not conventional pion condensate in nuclear matter)

- Quarkyonic limit:
  - Baryon number is spatially constant.
  - No other condensates.

Summary

- 1; Quarkyonic regime: $\Lambda_{QCD}/\mu \ll 1$, but confining int.
  - Self-consistent eqs. for 1-patch are saturated within small momentum region.

  → (3+1) D → (1+1) D confining model for 1-patch



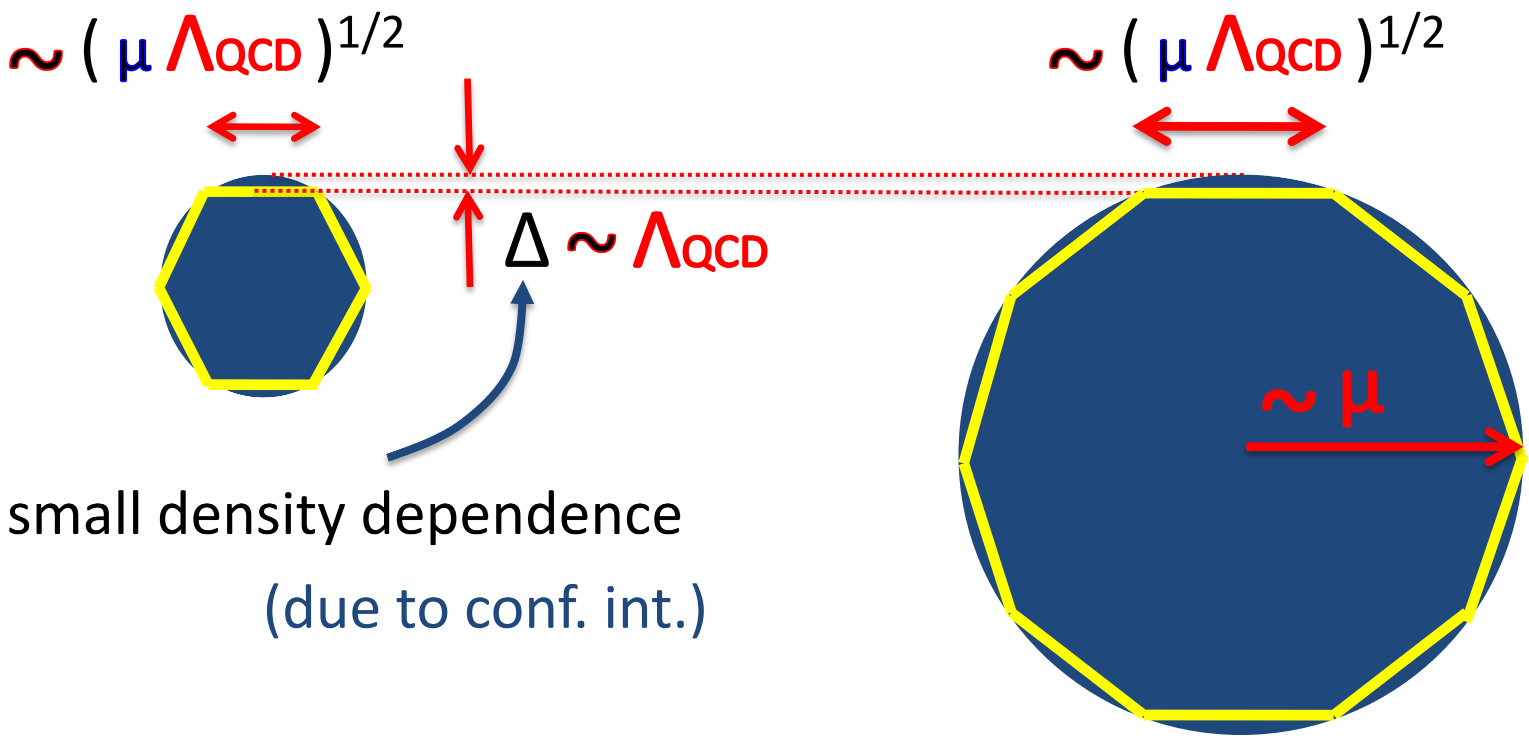
- 2; Nonzero V.E.V. of amp. field → Local violation of sym.

$$\langle \bar{\varphi}_- \varphi_+ \rangle = \underline{\Delta} e^{2i\mu z}$$

nonzero due to conf. int.

- 3; CDW accompanies Spiral structures: $\langle \bar{\Psi} i \Gamma_5 \Psi \rangle \neq 0$ (1+1)D
  - Due to mismatch of phases between Right and Left moving pairs.
- 4; Spirals accompany P & CP odd condensate: $\langle \bar{\psi} \gamma^0 \gamma^z \psi \rangle$ (3+1)D

Changes in topology of Fermi surface



Sequential phase transitions occur as μ increases

$\Delta\varepsilon$ ~ incoherent sum of many patch contributions

Back up

Contributions from other QCSs

- e.g.) Quark propagator in the presence of many QCSs

Sum over all Chiral spirals

Space-dependent mass self-energy

$$\sum_{i=1}^{N_p} \int \frac{d^4p}{(2\pi)^4} \bar{\psi}(p - \boldsymbol{Q}_i) M(\underline{p;} \boldsymbol{Q}_i) \psi(p)$$

- Hypothesis :

  quarks with high virtuality does not feel Chiral Sym. breaking effects

For both of p^2 and $(p - \mathbf{Q_i})^2$ to be close to Minkovski region:

Angle between **p** and $\mathbf{Q_i}$ $\longrightarrow$ $|\theta|, |\theta - \pi| < \Lambda_{\mathrm{QCD}}/p_F$

Only perturbations from nearest neighbor patches are relevant

$\longrightarrow$ Each QCS can be treated incoherently.

Excitation modes in quarkyonic limit

- Fermionic action for (1+1)D QCD:

$$S = \int d^2x [\Psi_+ i\partial_- \Psi_+ + \Psi_- i\partial_+ \Psi_-] \quad + \text{gauge int.}$$

- Bosonized version:

U(1) free bosons & Wess-Zumino-Novikov-Witten action :

(Non-linear σ model + Wess-Zumino term)

"Charge – Flavor – Color Separation"

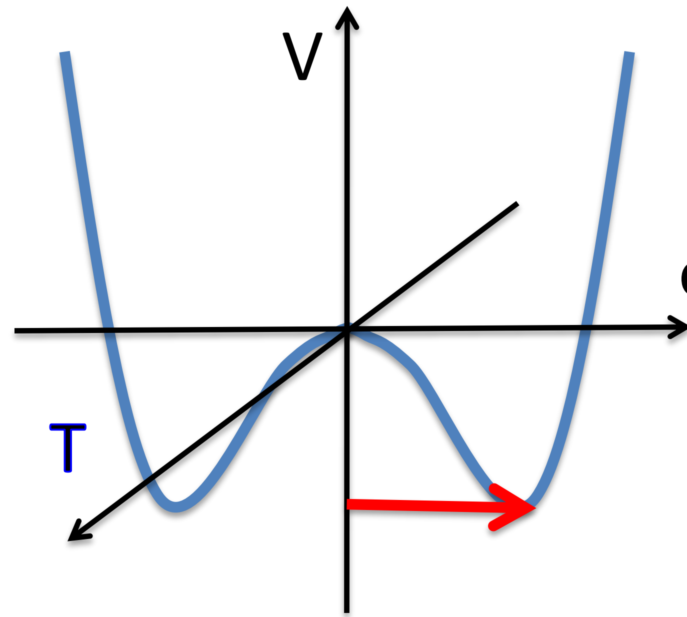
$$S = \underbrace{S_{U(1)}[\phi] + S^{flavor}_{k=N_c}[g]}_{\text{conformal inv.}} + \underbrace{S^{color}_{k=N_f}[h] \quad + \text{gauge int.}}_{\text{dimensionful}}$$

$\downarrow$ $4N_f^2$ gapless phase modes

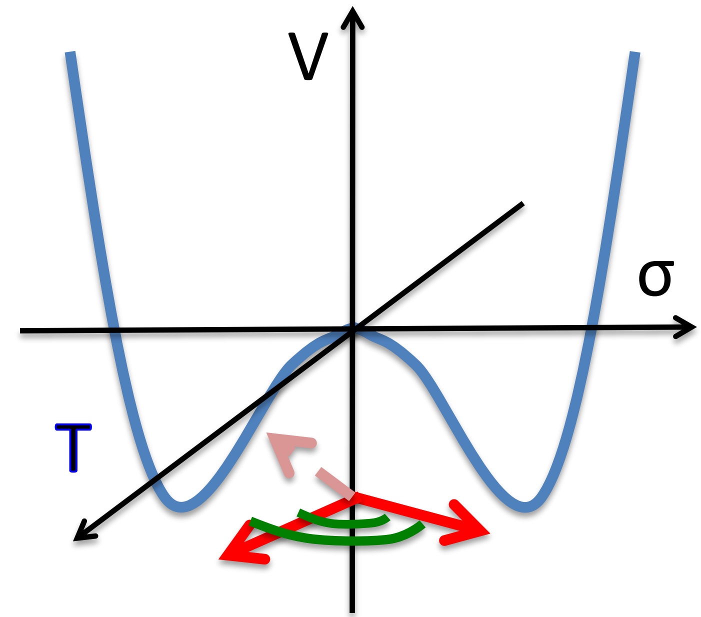
$\downarrow$ gapped phase modes

Coleman's theorem ?

- Coleman's theorem: No Spontaneous sym. breaking in 2D



$\langle \rho \rangle \neq 0$



$\langle e^{i\theta} \rangle \neq 0$ (SSB)

$\langle e^{i\theta} \rangle = 0$ (No SSB)

IR divergence in (1+1)D phase dynamics

- Phase fluctuations belong to:

Excitations (physical pion spectra)	ground state properties (No pion spectra)

Quasi-long range order & large Nc

- Local order parameters:

gapless modes　　gapped modes

$$\langle \bar{\Psi}_+ \Psi_- \rangle \sim \underbrace{\langle e^{i\sqrt{4\pi/N_c N_f}\,\phi} \rangle}_{0} \otimes \underbrace{\langle \mathrm{tr}\, g \rangle}_{0} \otimes \underbrace{\langle \mathrm{tr}\, h \rangle}_{\text{finite}}$$

due to IR divergent phase dynamics

But this does not mean the system is in the usual symmetric phase!

- Non-Local order parameters:

$$\langle \bar{\Psi}_+ \Psi_-(x) \bar{\Psi}_- \Psi_+(0) \rangle \sim \begin{cases} \cancel{e^{-m|x|}} & : \text{symmetric phase} \\ \cancel{\langle \bar{\Psi}_+ \Psi_- \rangle^2} & : \text{long range order} \\ |x|^{\underline{-C/N_c}} & : \text{quasi-long range order} \end{cases}$$

(including disconnected pieces)

(power law)

Quasi-long range order & large Nc

- Local order parameters:

gapless modes　　gapped modes

$$\langle \bar{\Psi}_+ \Psi_- \rangle \sim \langle e^{i\sqrt{4\pi/N_c N_f}\,\phi} \rangle \otimes \langle \mathrm{tr}\, g \rangle \otimes \langle \mathrm{tr}\, h \rangle$$

due to IR divergent phase dynamics → 0　　0　　finite

But this does not mean the system is in the usual symmetric phase!

- Non-Local order parameters:

$$\langle \bar{\Psi}_+ \Psi_-(x) \bar{\Psi}_- \Psi_+(0) \rangle \sim \begin{cases} \cancel{e^{-m|x|}} & \text{: symmetric phase} \\ \langle \bar{\Psi}_+ \Psi_- \rangle^2 & \text{: long range order} \\ |x|^{-C/N_c} & \text{: quasi-long range order} \end{cases}$$

(including disconnected pieces)

large Nc limit (Witten `78)

(power law)

How neglected contributions affect the results?

- Neglected contributions in the dimensional reduction:

$$S(k) = \gamma_0 S_0 - \gamma_z S_z - \vec{\gamma}_T \vec{S}_T + S_m$$

$\sim \mu$ $\sim \Lambda_{QCD}$

(3+1)D (1+1)D

spin mixing → breaks the flavor symmetry explicitly

mass term → acts as mass term

Expectations:

1, Explicit breaking regulate the IR divergent phase fluctuations, so that quasi-long range order becomes long range order.

2, Perturbation effects get smaller as μ increases, but still introduce arbitrary small explicit breaking, which stabilizes quasi-long range order to long range order.

(As for mass term, this is confirmed by Bringoltz analyses for massive `tHooft model.)

Final results should be closer to our large Nc results!

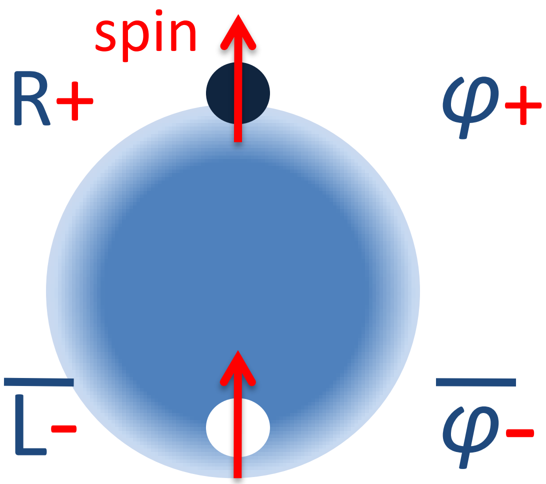
Chiral Density Wave VS Chiral Exciton

IF dimensionally reduced models respect "flavor" symmetry

→ there is no chiral exciton condensates:

Chiral Density Wave

→ $R+\ \overline{L-}$ pairing



same spin in 3+1 D

↕

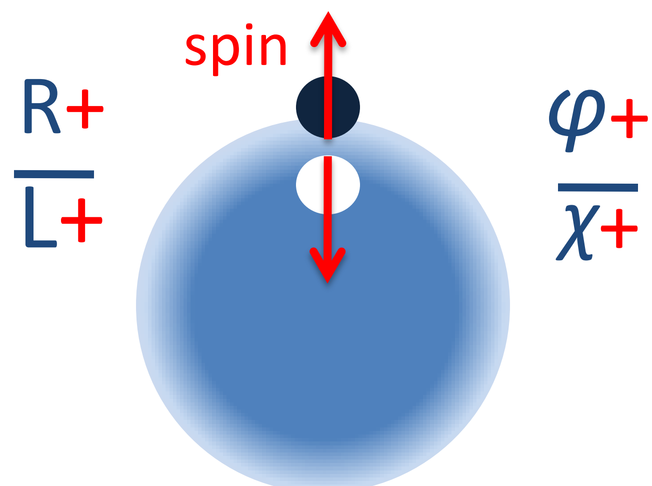
same "flavor" in 1+1 D

Possible (1+1)D flavor singlet

$$\bar{\Psi}\Psi \qquad \bar{\Psi}\Gamma^5\Psi$$

Chiral Exciton

→ $R+\ \overline{L+}$ pairing



No flavor singlet pairing

→ No chiral exciton!