CP violation and the nature of the chiral transition

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Is the CP symmetry preserved in QCD at finite temperature?

- Topologically non-trivial configurations of the gauge fields generate a term in the QCD Lagrangian that violates CP symmetry.

- **P violation explicitly manifested:**

The chiral magnetic effect

non-trivial gauge field configurations ⇔ **strong magnetic field**

- How the chiral transition is affected by those factors?

- How is the QCD phase diagram with a "B" axis?

➡ Effective models.

Overview of our work:

- Effects of a strong magnetic background on the chiral transition (E.Fraga and AJM, **PRD78,025016 (2008)**)

- CP violation and the chiral transition

- CP violation and chiral transition on a magnetic background (AJM and E.Fraga, **NPA831,91 (2009)**)

- Chiral and deconfinement transition on a magnetic background (AJM, M.Chernodub and E.Fraga, **HEP-PH/1004.2712**) – E. Fraga presentation

The effective model must contain:

- Chiral transition

  Linear sigma model coupled to quarks

- Non-trivial gauge field configurations

  ‘t Hooft determinant term

- Magnetic background

  Included through the covariant derivative

- Finite temperature

  Matsubara formalism

Effective theory for the chiral transition (LσM)

[Gell-Mann & Levy (1960); Scavenius, Mócsy, Mishustin & Rischke (2001); ...]

- Symmetry: for massless QCD, the action is invariant under $SU(N_f)_L \times SU(N_f)_R$
- "Fast" degrees of freedom: quarks
  "Slow" degrees of freedom: mesons
- Typical energy scale: hundred of MeV
- We choose $SU(N_f=2)$, for simplicity: we have pions and the sigma
- Framework: coarse-grained Landau-Ginzburg effective potential
- $SU(2) \otimes SU(2)$ spontaneously broken in the vacuum
- Also accommodates explicit breaking by massive quarks

The resulting Lagrangian:

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu\partial_\mu - g(\sigma + i\gamma^5\overrightarrow{t}\cdot\overrightarrow{\pi})] + \frac{1}{2}[\partial_\mu\sigma\partial_\mu\sigma + \partial^\mu\overrightarrow{\pi}\partial^\mu\overrightarrow{\pi}] - V(\sigma,\overrightarrow{\pi})$$

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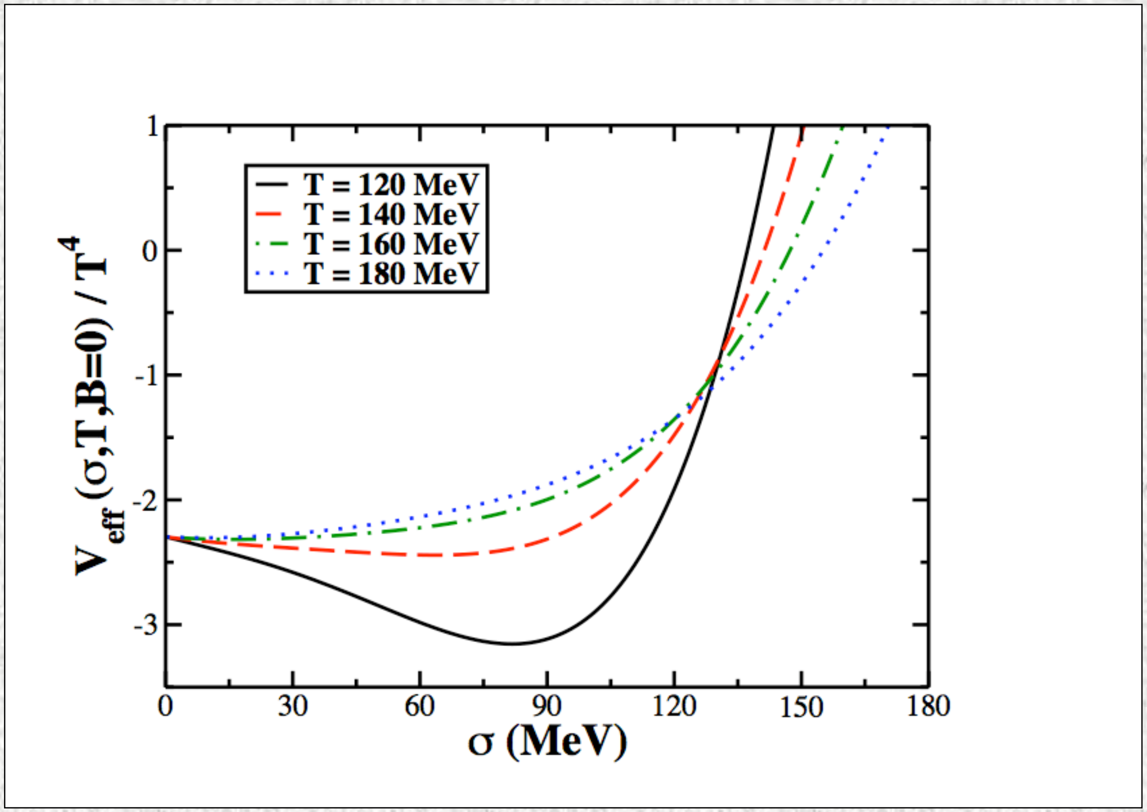
$$V(\sigma,\vec{\pi}) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - h\sigma$$



Effective potential of the linear sigma model. In low temperature the chiral symmetry is broken. Above a specific critical temperature the symmetry is restored.

Incorporating a strong magnetic field background

Let us assume the system is in the presence of a strong magnetic field background that is constant and homogeneous:

$$\boxed{\vec{B} = B\hat{z}} \xrightarrow{\text{choice of gauge}} \boxed{A^\mu = (A^0, \vec{A}) = (0, -By, 0, 0)}$$

- charged mesons (new dispersion relation):

$$\boxed{\begin{array}{c}(\partial^2 + m^2)\phi = 0 \\ \partial_\mu \to \partial_\mu + iqA_\mu\end{array}} \Rightarrow \boxed{p_{0n}^2 = p_z^2 + m^2 + (2n+1)|q|B}$$

Landau levels: $\boxed{\varepsilon_n \equiv \left(\frac{p_{0n}^2 - p_z^2 - m^2}{2m}\right) = \left(n + \frac{1}{2}\right)\omega_B}$ $\boxed{\omega_B \equiv \frac{|q|B}{m}}$

- quarks (new dispersion relation):

$$\boxed{\begin{array}{c}(i\gamma^\mu\partial_\mu - m)\psi = 0 \\ \partial_\mu \to \partial_\mu + iqA_\mu\end{array}} \Rightarrow \boxed{p_{0n}^2 = p_z^2 + m^2 + (2n+1-\sigma)|q|B}$$

$$\boxed{\sigma = \pm 1}$$

The modified effective potential

[E.S.Fraga & AJM, PRD78,025016 (2008)]

Simple mean-field treatment with the following customary simplifying assumptions:

Quarks constitute a thermalized gas that provides a background in which the long wavelength modes of the chiral condensate evolve. Hence:

At $T = 0$ (vacuum: χ symm. broken; reproduce usual LσM & χPT results)

- Quark d.o.f. are absent (excited only for $T > 0$)
- The σ is heavy ($M_\sigma \sim 600$ MeV) and treated classically
- Pions are light: fluctuations in π^+ and π^- couple to B; fluctuations in π^0 give a B-independent contribution (ignored here)

Vacuum effective potential (σ direction):

Classical: $$\boxed{V_{cl} = \frac{\lambda}{4}(\sigma^2 - v^2)^2 - h\sigma}$$

(σ now means $\langle\sigma\rangle$)

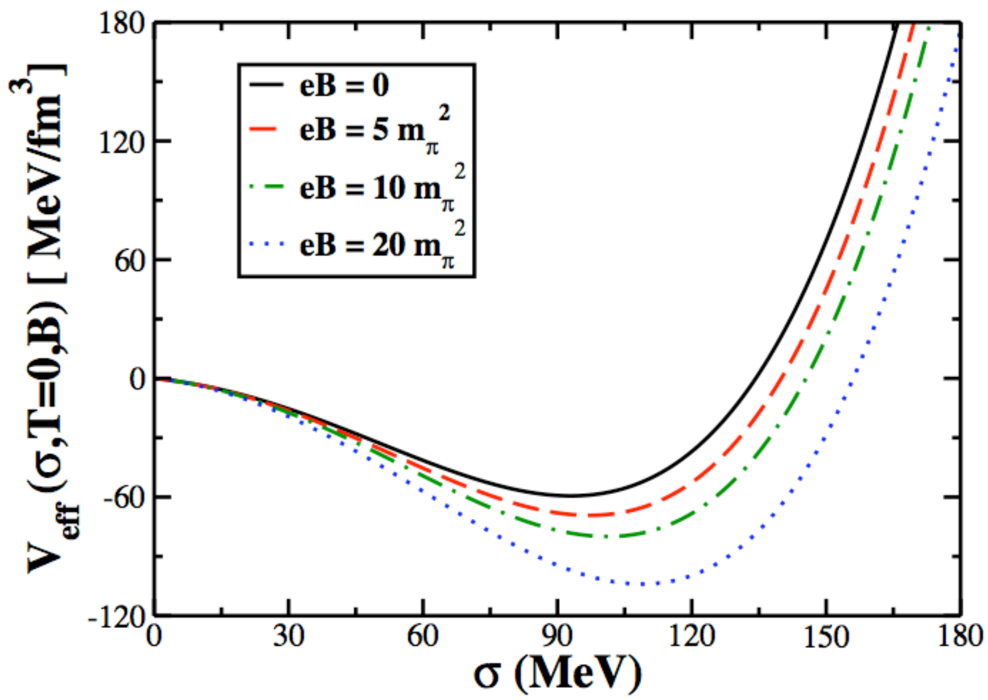
π^+ and π^- fluctuations: $$\boxed{V^V_{\pi^\pm} = \frac{1}{2}\frac{|q|B}{2\pi}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\frac{dk}{2\pi}\left[k^2 + m_\pi^2 + (2n+1)|q|B\right]^{1/2}}$$

Computing the contribution from pions in the MSbar scheme, we obtain (ignoring σ-independent terms):

$$\boxed{V^V_{\pi^+} + V^V_{\pi^+} = -\frac{2m_\pi^2 eB}{32\pi^2}\log 2}$$

using the assumption of large magnetic field, $|q|B \gg m_\pi^{\ 2}$, in the expansion of generalized Zeta functions of the form

$$\boxed{\zeta\left(-1 + \frac{\epsilon}{2}, \frac{m_\pi^2 + |q|B}{2|q|B}\right)}$$



Results in line with calculations in χPT and NJL, as in e.g.

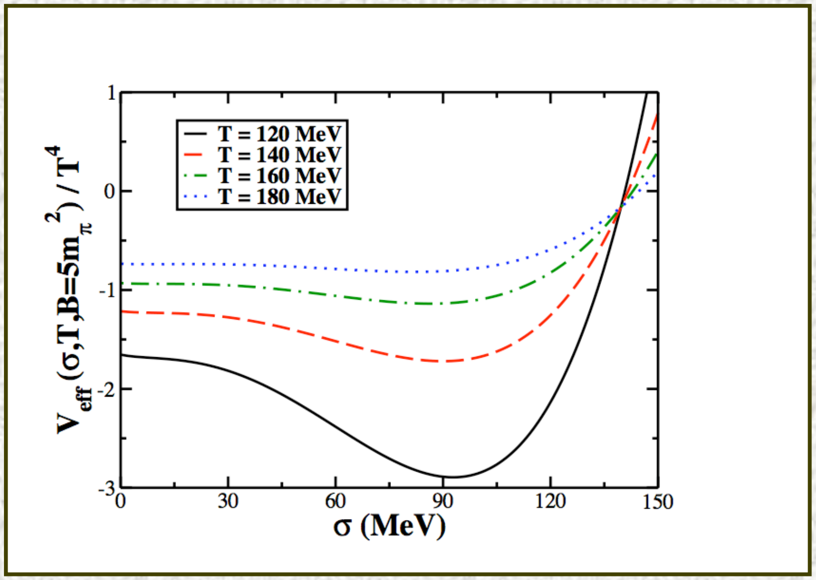
- Shushpanov & Smilga (1997)
- Cohen, McGady & Werbos (2007)
- Hiller, Osipov et al. (2007/2008)
- …

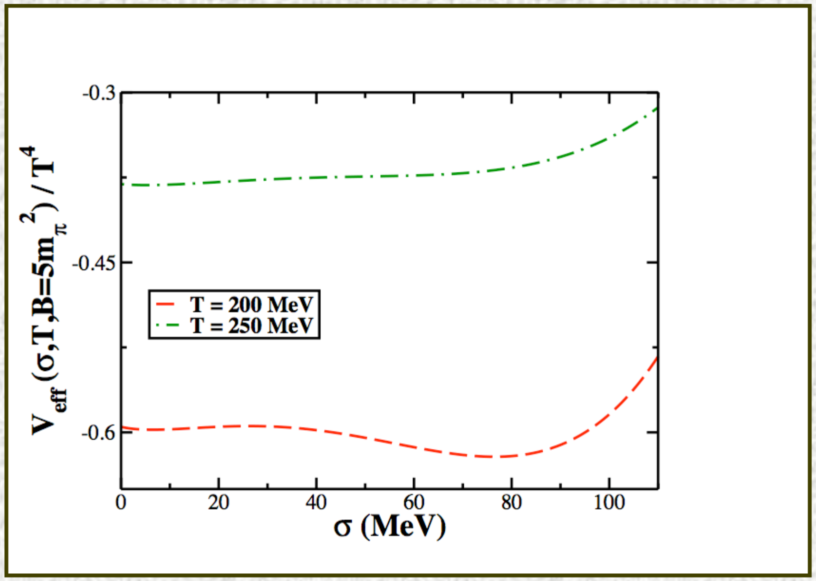
- Condensate grows with increasing magnetic field
- Minimum deepens with increasing magnetic field
- Relevant effects for equilibrium thermodynamics and nonequilibrium process of phase conversion ?

$B \sim 10^{17}$ G

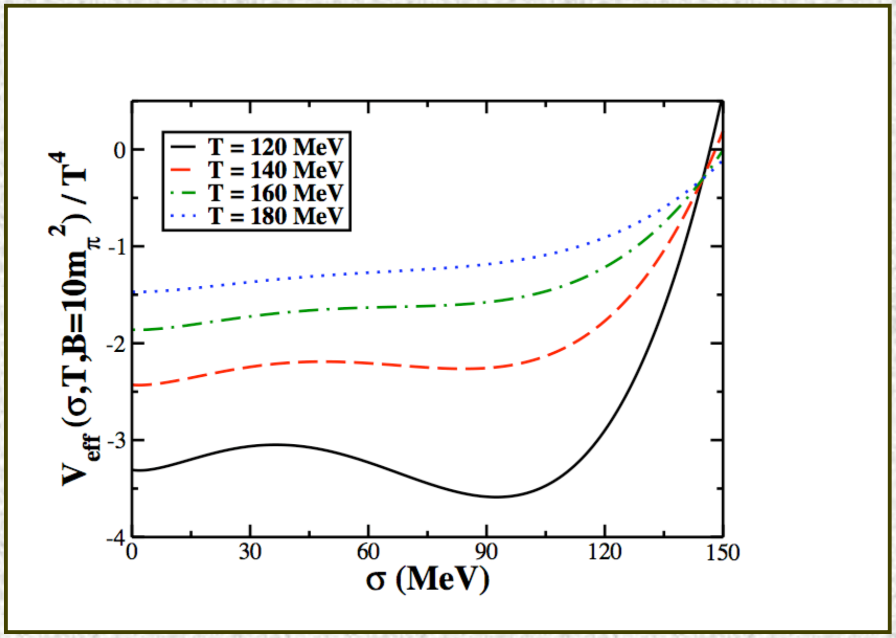
Thermal corrections: the contributions from V_{π^+} e V_{π^-} are exponentially suppressed. From the V_q contribution only one piece remains. The effective potential, including thermal corrections, is:

$$V_{eff}(\sigma,T,B) \approx \frac{\lambda}{4}(\sigma^2-v^2)^2 - h\sigma - 2\frac{m_\pi^2 eB}{32\pi^2}\log 2 - 2N_c\frac{eBT^2}{2\pi^2}\int_0^\infty dx \log\left(1+e^{-\sqrt{x^2+m_f^2/T^2}}\right)$$

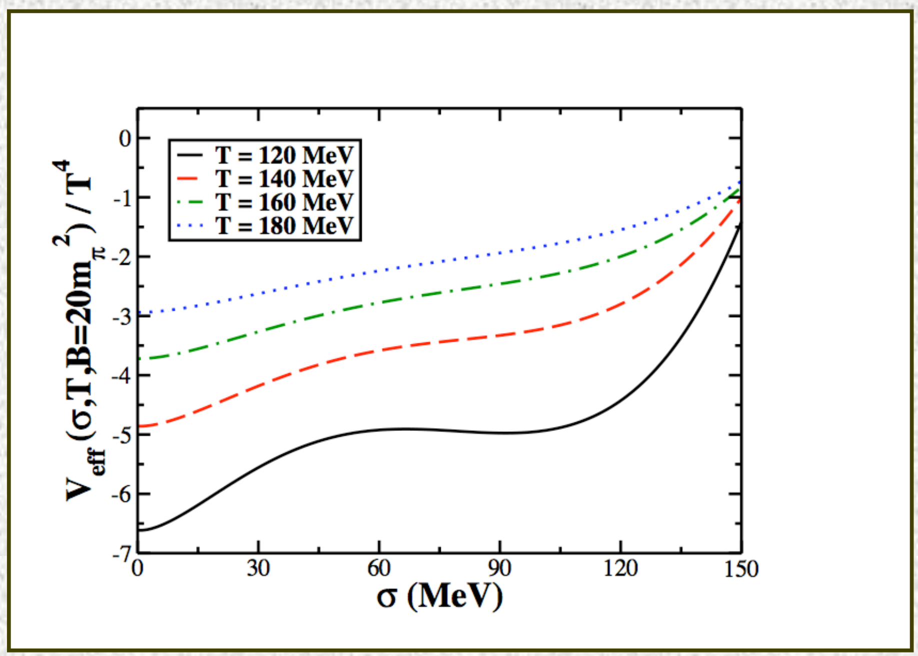




- Weak first order transition
- Including only the first Landau level T_c seems to increase, but considering all the levels it decreases.



- Critical temperature decreases (T_c < 140 MeV)
- The barrier increases: 1st order transition



- The critical temperature keeps decreasing and the barrier increasing

Conclusions from this part of the work

- Estimates for the magnetic field generated in RHIC indicate a magnitude of $5.3m^2_\pi$ and in LHC $6m^2_\pi$. It must be sufficient to change the nature of the phase transition
- In a first order phase transition, part of the system is kept in the false vacuum and bubble formation occurs. The dynamics of the phase transition are different, affecting all the relevant time scales.
- Lattice QCD indicates a crossover. It must change if the magnetic field is considered.
- In this treatment we considered an homogeneous field. A more realistic treatment should consider a variable field.

Including CP violation

(AJM and E.S.Fraga, **NPA831,91 (2009)**)

Effective Lagrangian:

$$\mathcal{L} = \frac{1}{2}Tr(\partial_\mu\phi^*\partial\phi) + \frac{a}{2}Tr(\phi^\dagger\phi) - \frac{\lambda_1}{4}[Tr(\phi^\dagger\phi)]^2$$

$$-\frac{\lambda_2}{4}Tr[(\phi^\dagger\phi)^2] + \frac{c}{2}[e^{i\theta}det(\phi) + e^{-i\theta}det(\phi^\dagger)] + Tr[h(\phi+\phi^\dagger)]$$

Kinetic term

Self interaction

`t Hooft det.

Explicit sym. breaking

Usual treatment: no `t Hooft determinant and

$$\phi = \frac{1}{\sqrt{2}}(\sigma + \vec{\pi}\cdot\vec{\tau})$$

We calculate the LSM potential for a $SU(2_f) X\ SU(2_f)$ field

$$\phi = \frac{1}{\sqrt{2}}(\sigma + i\eta) + \frac{1}{\sqrt{2}}(\vec{\alpha} + i\vec{\pi}) \cdot \vec{\tau}$$

With:

$$\langle\sigma\rangle = \langle\bar{\psi}\lambda_0\psi\rangle \qquad \langle\eta\rangle = \langle\bar{\psi}\lambda_0 i\gamma_5\psi\rangle$$

$$\langle a_0\rangle = \langle\bar{\psi}\lambda\psi\rangle \qquad \langle\pi\rangle = \langle\bar{\psi}\lambda i\gamma_5\psi\rangle$$

Mean field approx. where σ and η are classical fields plus fluctuations

$$V_{cl} = \frac{\lambda}{4}(\langle\sigma\rangle^2 - v_\theta^2)^2 - H\langle\sigma\rangle + \frac{\lambda}{4}(\langle\eta\rangle^2 - u_\theta^2)^2 - c \cdot Sin(\theta)\langle\sigma\rangle\langle\eta\rangle + \frac{\lambda}{2}\langle\sigma\rangle^2\langle\eta\rangle^2 - \frac{\lambda}{4}(v_\theta^4 + u_\theta^4)$$

$$v_\theta^2 \equiv \frac{a + c\ \cos\theta}{\lambda} \quad ; \quad u_\theta^2 \equiv v_\theta^2 - \frac{2c}{\lambda}\cos\theta$$

The interaction of the fermions and mesons is given by a Yukawa term

$$\bar{\psi}[i\gamma^{\mu}\partial_{\mu} - g(\langle\sigma\rangle^2 + \langle\eta\rangle^2)^{\frac{1}{2}}]\psi$$

Integrating over the fermions (heat bath for the long wavelength chiral fields), we obtain an effective thermodynamic potential

$$\Omega(T,\mu,\phi) = V(\phi) - \frac{T}{\mathcal{V}} \ln\det\left[\frac{(G_E^{-1} + M(\phi))}{T}\right]$$

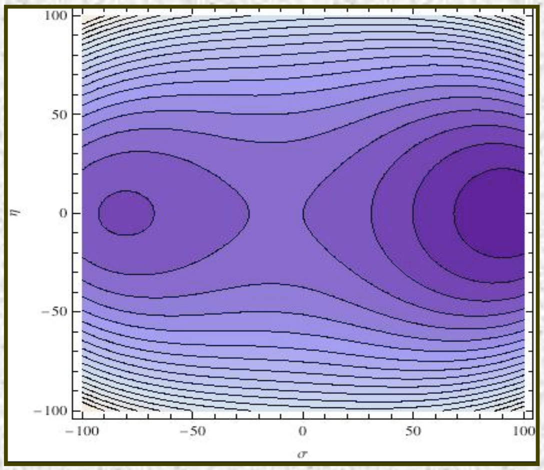
$$V_q \equiv -\nu_q T \int \frac{d^3k}{(2\pi)^3} \ln\left(1 + e^{-E_k(\phi)/T}\right)$$

• Quadratic fluctuations are straightforwardly worked out and provide the means to fix the parameters of the model to meson masses and the pion decay constant in the vacuum in the absence of CP violation.

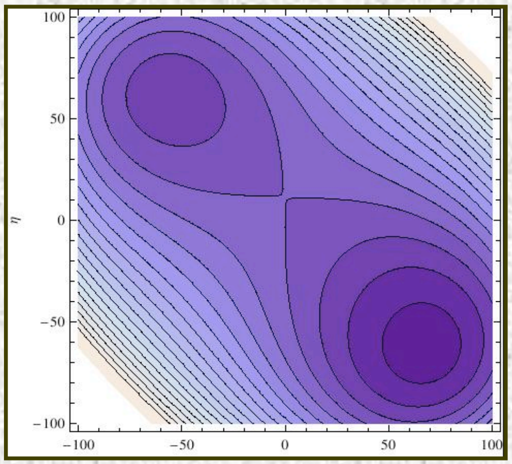
The constants were fixed to reproduce the meson masses in the vacuum (θ=0).

Contour plots for the effective potential [Mizher & ESF (2008,2009)]

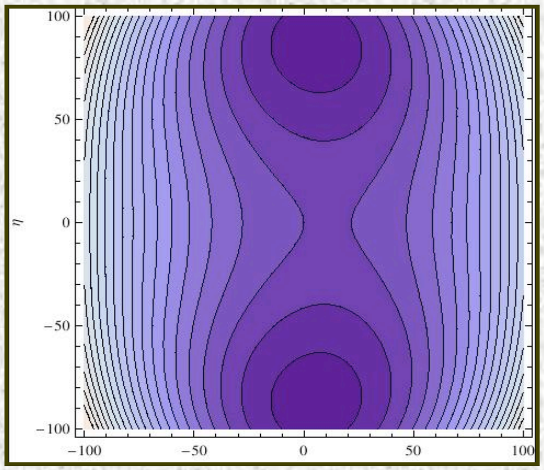
T=0:



$\theta=0$



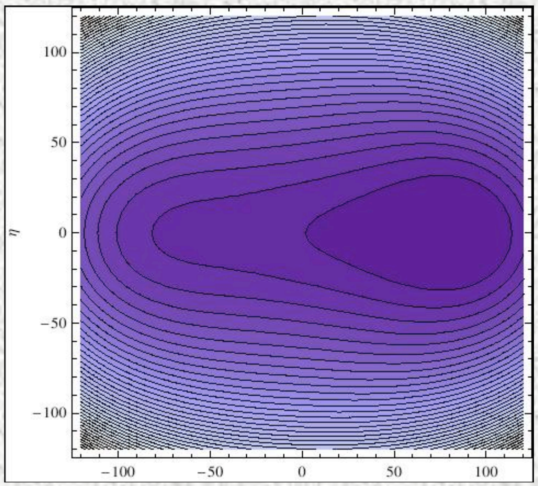
$\theta=\pi/2$



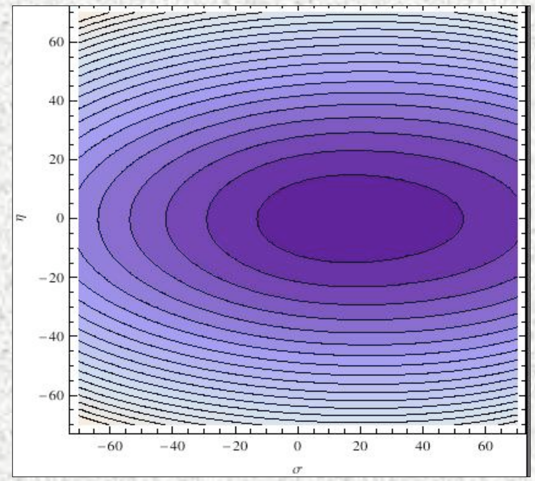
$\theta=\pi$

Increasing θ the positions of the minima, local and global, rotate. For $\theta=\pi$ the global minimum is almost in the η direction, evidencing the relation between a non-vanishing θ and a η condensate.

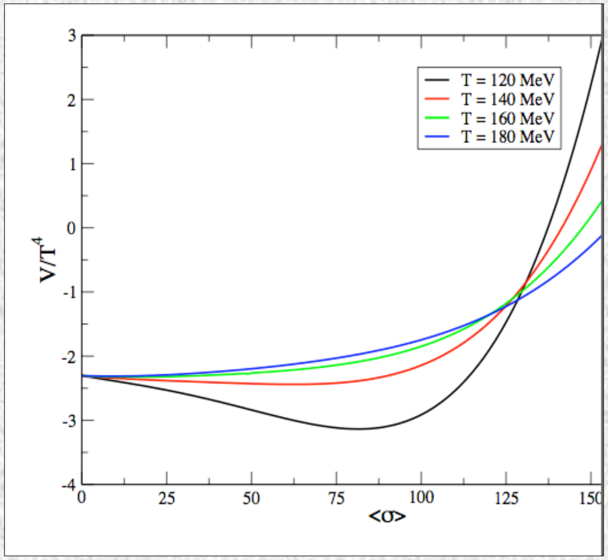
Keeping θ=0 the model reproduces the features of the usual linear sigma model: at low temperature the symmetry is broken and there is a critical temperature where it is restored.



T=120 MeV



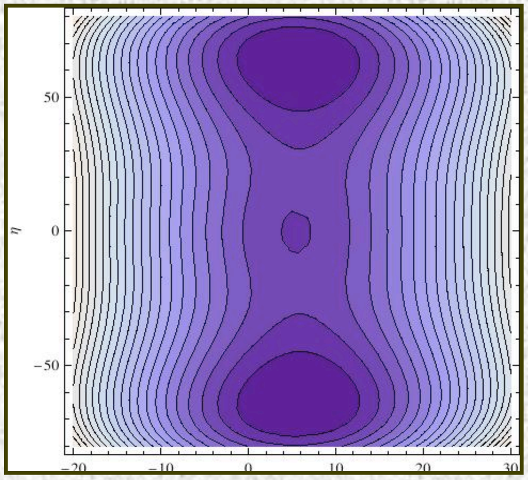
T=160 MeV



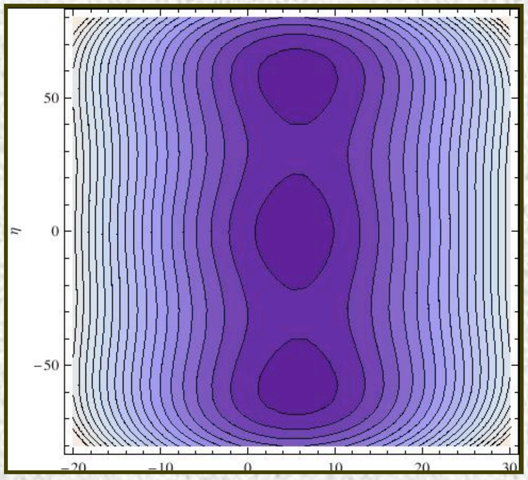
V *vs.* T in the σ direction to several temperatures.

T > 0 , $\theta=\pi$:

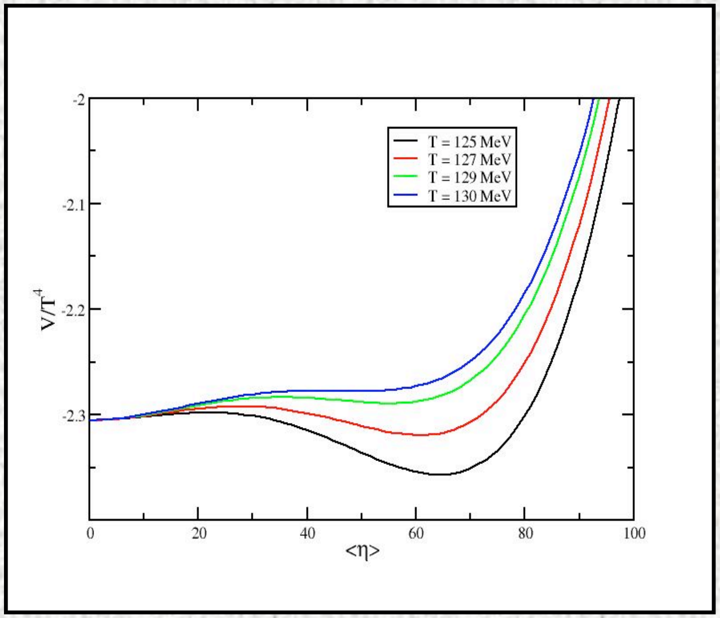
For $\theta=\pi$ the minima are almost in the η direction. As the temperature raises a new minimum appears at $\eta=0$, separated by a barrier, signaling a first-order transition. The critical temperatures for melting the two condensates are different, so that three phases are allowed.



T=125 MeV

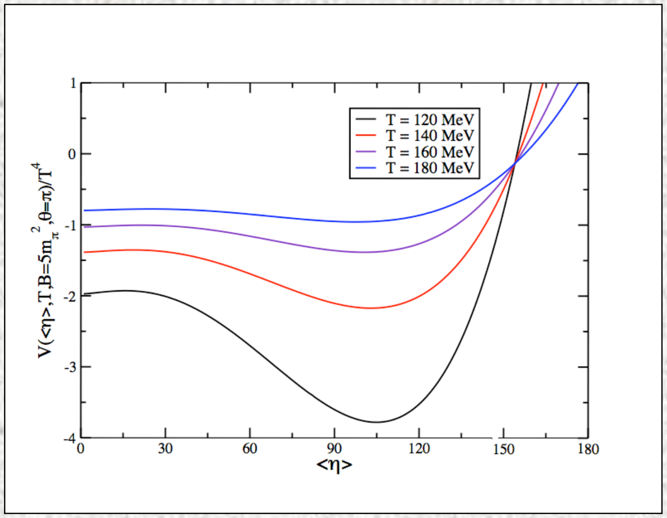


T=128 MeV

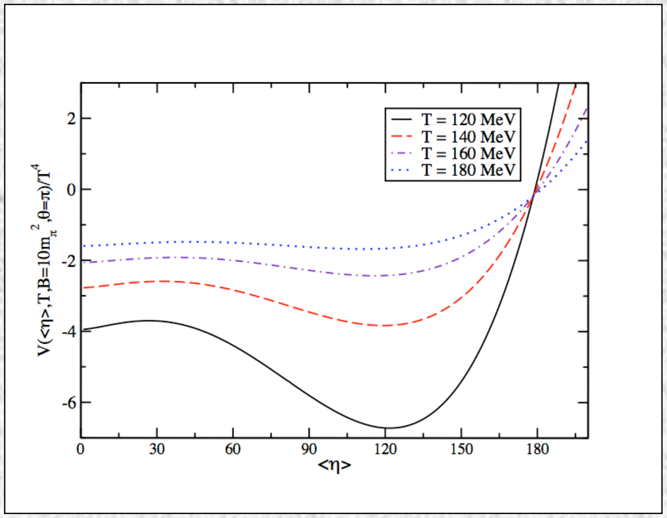


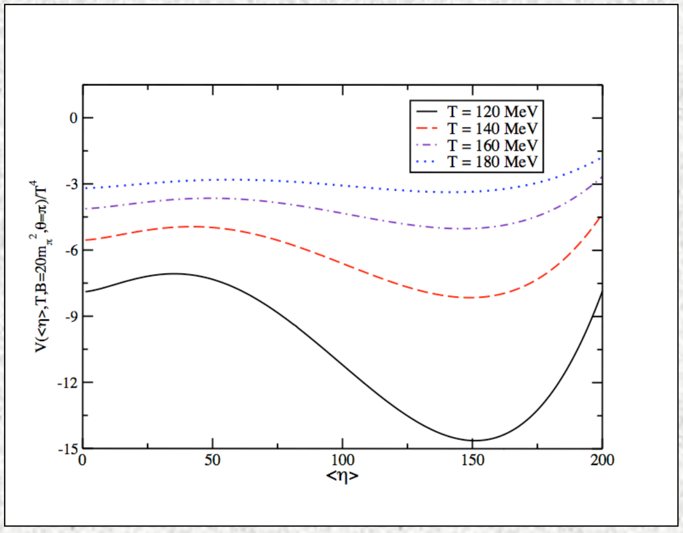
V *vs.* η for several temperatures around the transition.

Adding the magnetic field effect to the CP odd linear sigma model:

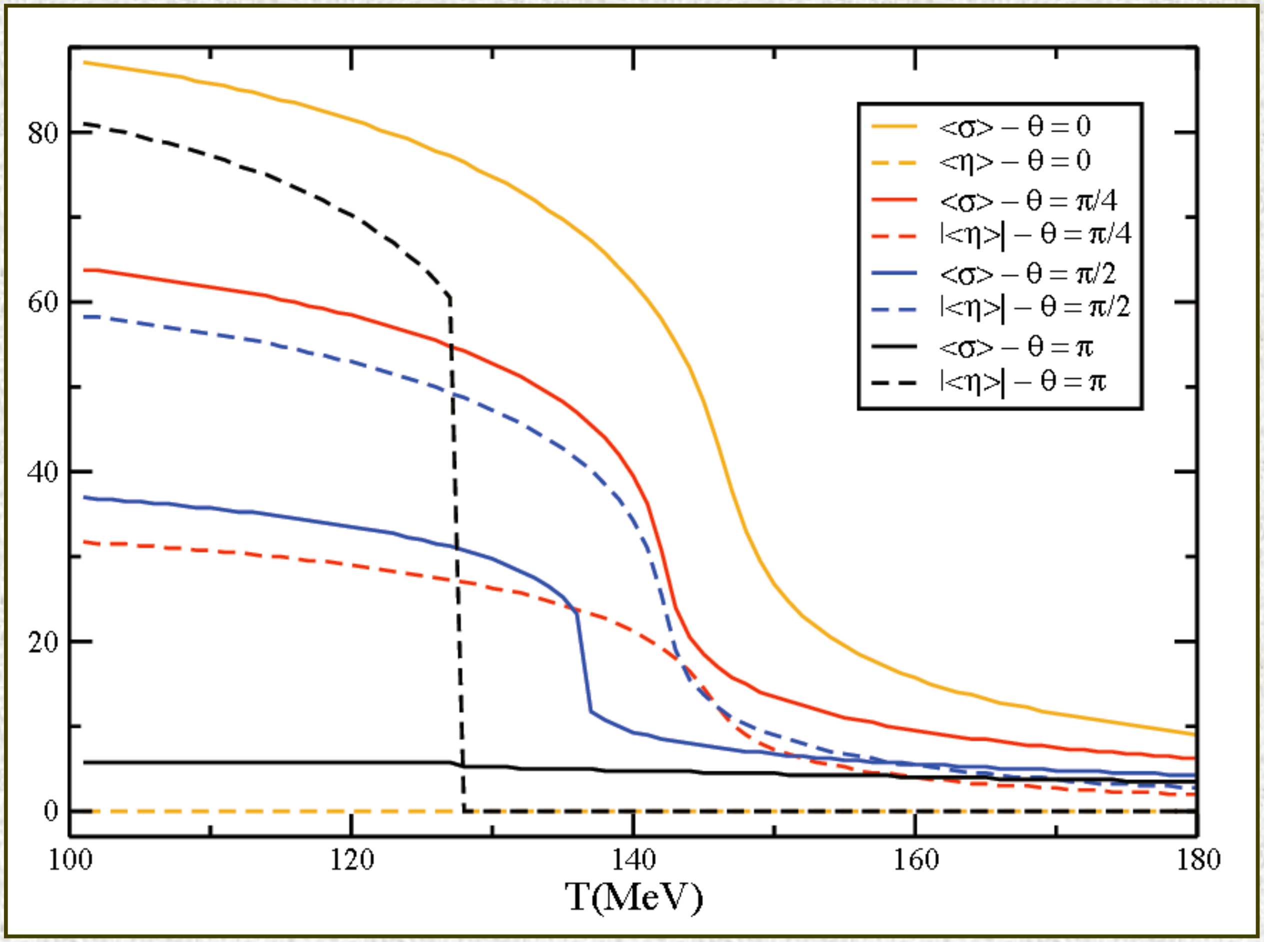


The magnetic field makes the critical temperature higher, as well as the barrier, making the first order transition stronger. As expected, the effect on σ is the same as before.





Condensates:



Linear sigma model X NJL model

- Comparing our results to the work by D.Boer and J.Boomsma [**PRD78,054027 (2008)**], where they used the NJL model, we found a different order for the phase transition.

- The difference is due to the absence of the quark degrees of freedom in the vacuum in our model (J.Boomsma and D.Boer **PRD80,034019 (2009)**). This is crucial also in the case of chiral/deconfinement transition.

- Work in progress: to better understand the differences between the two models.

Conclusions

- The proposal for observation of CP violation in heavy ion collisions requires a strong magnetic field to make the effect observable.

- In this work we model the effects of the magnetic field, and see that its presence in our model can change the order of the phase transition, and consequently, the dynamics of the system.

- We include the possibility of CP violation, through an external controlled parameter.

- We saw that CP violation is related to an η condensate. The critical temperature to the η parameter is different from the one for the σ, yielding three different phases.

- The effect of the magnetic field in our model is to drive both transitions to first order, with the height of the barrier depending on the magnitude of the field.

Perspectives

- Magnetic field inhomogeneous in space.
- Dynamical processes - magnetic field inhomogeneous in time, phase conversion, time scales.
- Application in the early universe.
- High density systems - compact stars.

Thank you