Hadronic and Partonic Mechanisms of Charge Asymmetry Fluctuations

Berndt Müller

In collaboration with:
M. Asakawa (Osaka)
A. Majumder (OSU)
arXiv:1002.2436

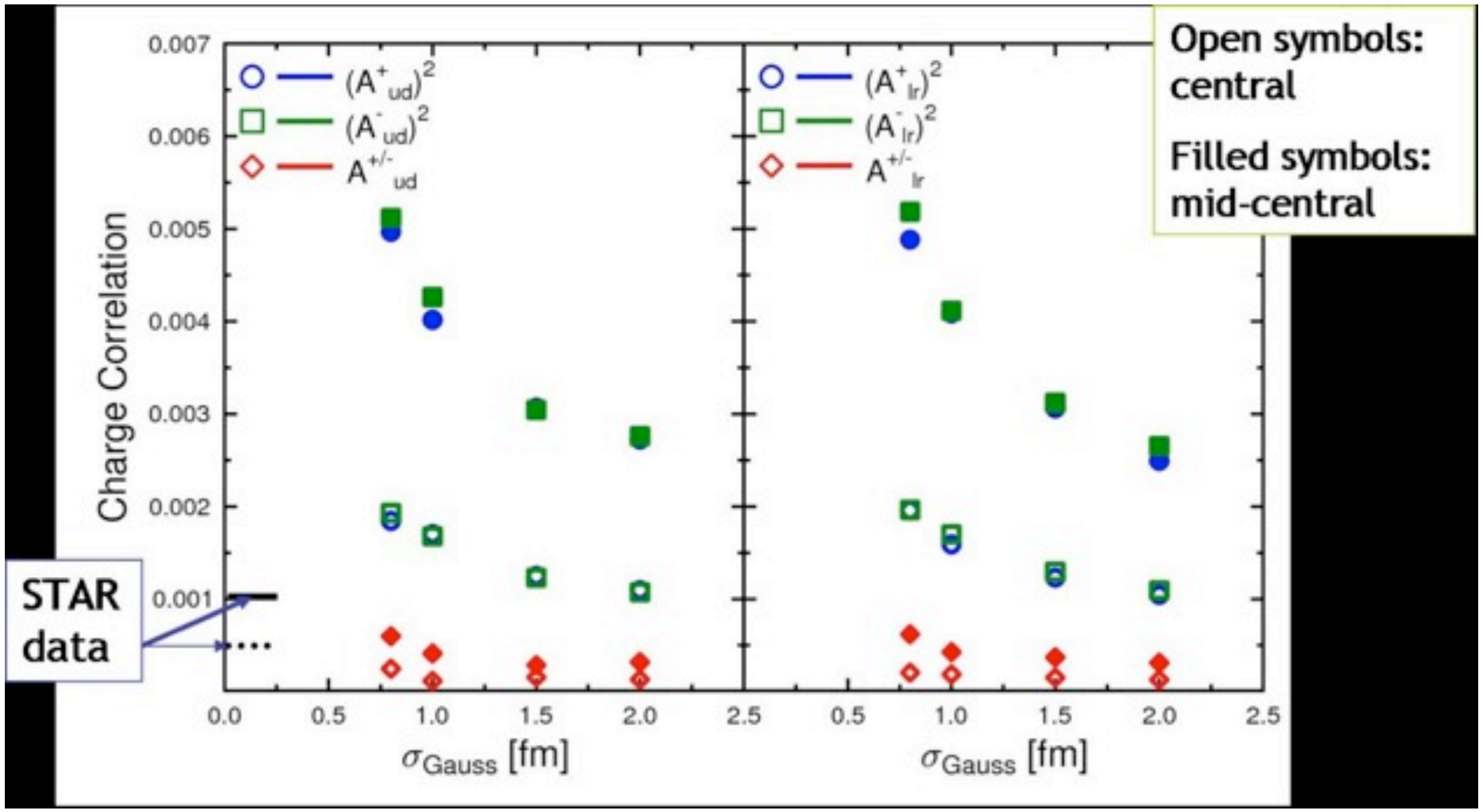
P- and CP-odd Effects in Hot and Dense Matter
BNL, 26-30 April 2010

Overview

- This talk is about:
  - *P* or -*CP* conserving processes that contribute to the dynamical fluctuations of the electric charge asymmetry with respect to the reaction plane
  - The influence of final state interactions on the magnitude of dynamical charge fluctuations
- Also some comments on:
  - The effective strength of the magnetic field produced in relativistic heavy ion collisions and its energy dependence
- If there is time left, some comments on:
  - What is “local” symmetry violation?

Disclaimer

- This talk is not about the chiral magnetic effect.
- This talk is not about the experimental data, which may have an entirely different origin.
  - See, e.g., talk by H. Petersen on Wednesday.



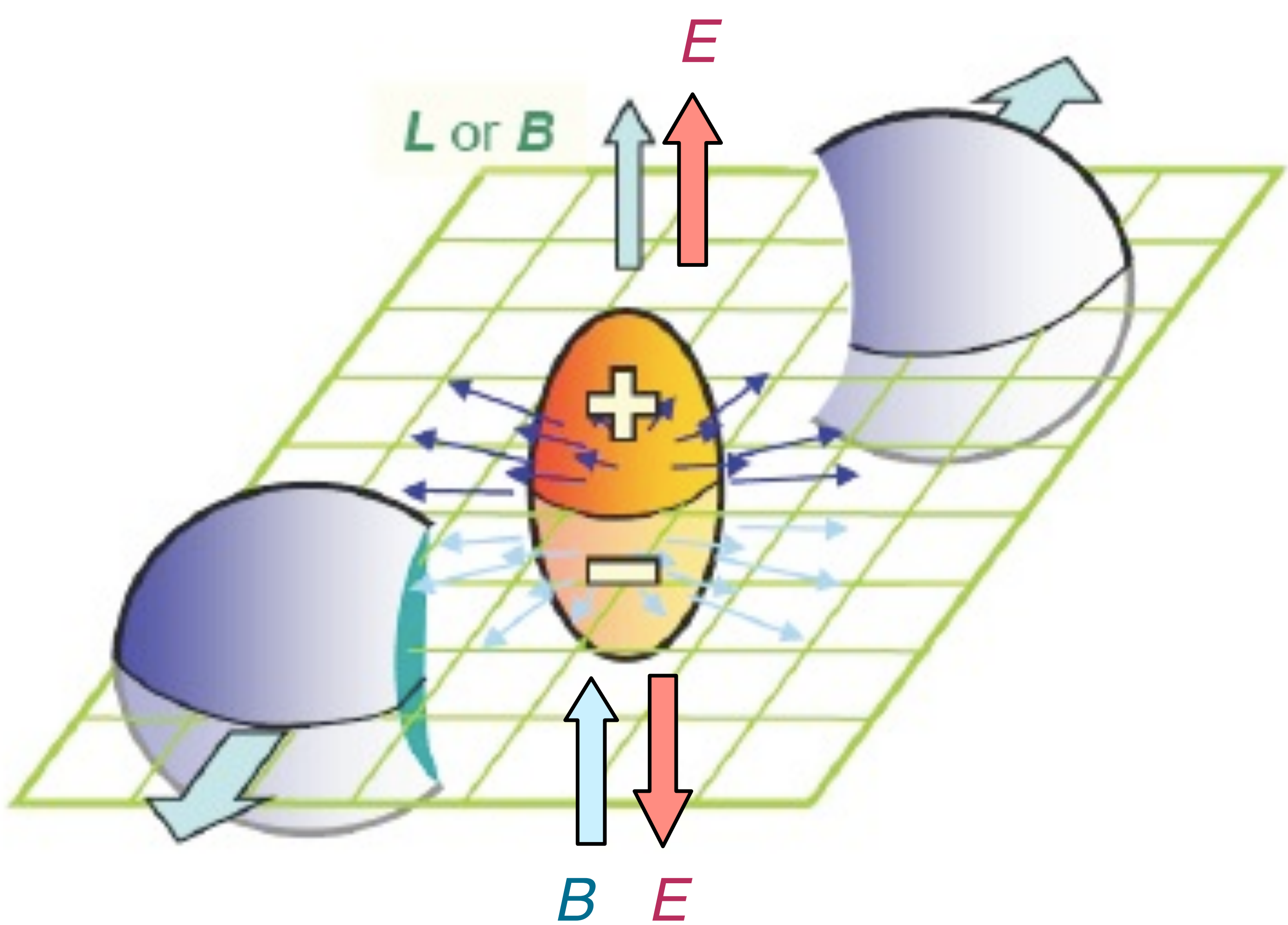
Strong *E*,*B* fields

In heavy ion collisions, ***E·B*** is positive (negative) above (below) the reaction plane, as defined by the direction of the angular momentum ***L***.

The peak electric and magnetic fields of fast nuclei ($\gamma = 100$) in the CM frame are very large:

$$|e\boldsymbol{E}| \approx |e\boldsymbol{B}| \approx Z\alpha\gamma/R^2 \approx (200\ \mathrm{MeV})^2$$

Can they influence the nuclear reaction?



Interactions of interest involve the Lorentz invariant $\mathcal{G} = \boldsymbol{E}\cdot\boldsymbol{B} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}$, which has symmetry $J^{PC} = 0^{-+}$, i.e. is parity and time-reversal negative, and thus can probe the *P*-odd, *CP*-odd sector of QCD. Effective interactions with QCD matter are of the form $\mathcal{L}_{\rm eff} = \mathcal{P}\,(\mathbf{E}\cdot\mathbf{B})$, where $\mathcal{P}$ is an electrically neutral pseudoscalar.

The anomalous current

$$\mathcal{L}_{\rm eff} = \mathcal{P}\,(\mathbf{E}\cdot\mathbf{B}) = \tfrac{1}{4}\mathcal{P}\,F_{\mu\nu}\tilde{F}^{\mu\nu}$$ implies a conserved, “anomalous” current

$$j^\mu = \frac{\partial\mathcal{L}_{\rm eff}}{\partial A_\mu} = \frac{\partial\mathcal{P}}{\partial x^\nu}\,\tilde{F}^{\mu\nu}$$

Vector components: $\boxed{\mathbf{j} = (\partial\mathcal{P}/\partial t)\,\mathbf{B}}$ (Compare with $\mathbf{j} = \sigma\,\mathbf{E}$)

$\partial\mathcal{P}/\partial t$ can be thought of as parity-odd electric conductivity (“chiral conductivity”)

A nonzero expectation value $\langle\mathcal{P}\rangle \neq 0$ would imply parity violation in the QCD sector, but nonzero transition matrix elements $\langle f|\mathcal{P}|i\rangle \neq 0$ are easily allowed. They give rise to anomalous current fluctuations:

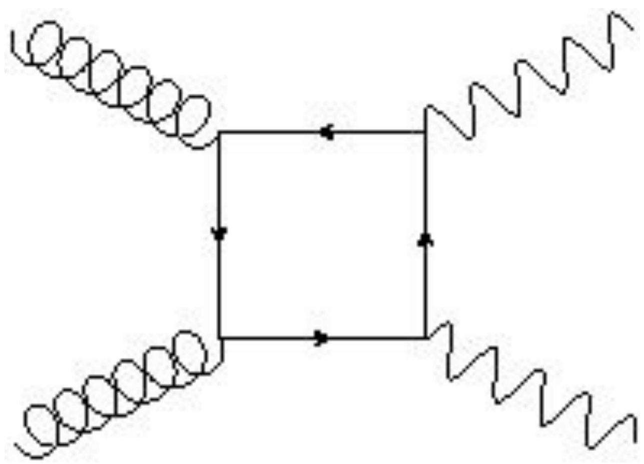
$$\boxed{\langle\, j_i(x) j_k(x')\,\rangle = \langle\,\partial_t\mathcal{P}(x)\,\partial'_t\mathcal{P}(x')\,\rangle\, B_i(x) B_k(x')}$$

QECD - EFT (I)

There exist two kinds of pseudoscalar QCD interactions with ***E·B***

Gluonic interactions

$\mathbf{E}^a \cdot \mathbf{B}^a$ interacts with ***E·B*** via a rectangular quark loop:



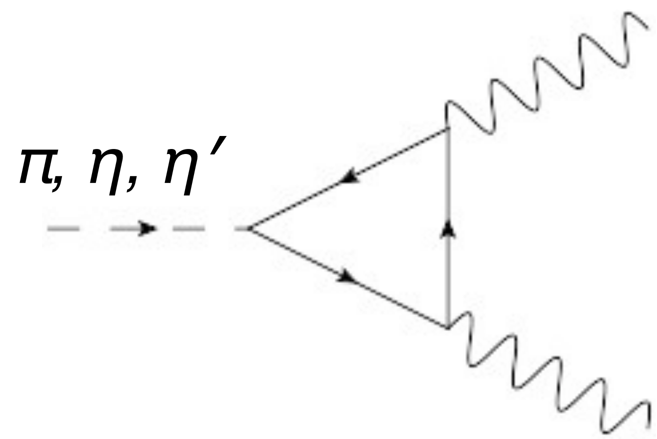
Lowest order contribution:

$$\mathcal{L}_{\mathrm{QECD}} = \kappa\alpha\alpha_s(\mathbf{E}^a \cdot \mathbf{B}^a)\,(\mathbf{E} \cdot \mathbf{B})$$

"Chiral magnetic effect" may be encoded in nonperturbative (in α_s) contributions

Hadronic interactions

Pseudoscalar mesons interact with ***E·B*** via a triangular quark loop:

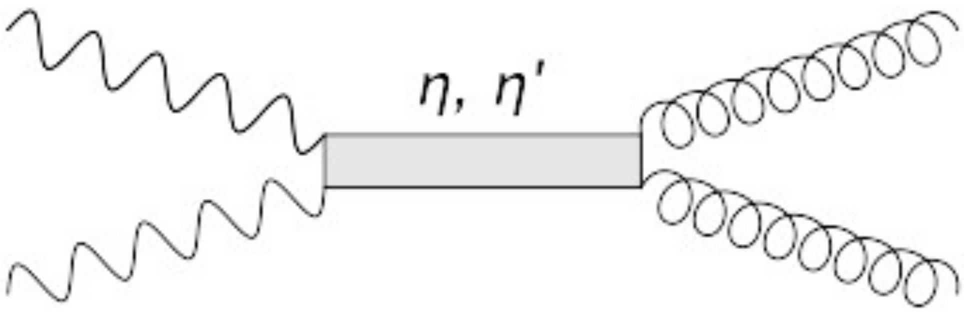


$$\mathcal{L}'_{\mathrm{QECD}} = \sum_{i=\pi^0,\eta,\eta'} \frac{\alpha}{\pi f_i}\phi_i\,\mathbf{E} \cdot \mathbf{B}$$

f_i is the meson decay constant

QECD - EFT (II)

Both mechanisms can be combined into an effective pseudoscalar QECD interaction via η, η' meson exchange:



The $\eta(\eta')gg$ and $\eta(\eta')\gamma\gamma$ vertices are known from data and model independent hadronic relations:

$$\begin{aligned}
\mathcal{L}^{(\text{eff})}_{\text{QECD}} &= \left[1 + \frac{\tan\theta}{2\sqrt{2}} - \frac{m^2_{\eta'}}{m^2_{\eta}}\left(\frac{\tan\theta}{2\sqrt{2}} - \tan^2\theta\right)\right] \\
&\quad \times \frac{\alpha\alpha_s\cos^2\theta}{\pi^2 f^2_\eta m^2_{\eta'}}(\mathbf{E}^a\cdot\mathbf{B}^a)(\mathbf{E}\cdot\mathbf{B}) \\
&\approx 1.46\,\frac{\alpha\alpha_s}{\pi^2 f^2_\eta m^2_{\eta'}}(\mathbf{E}^a\cdot\mathbf{B}^a)(\mathbf{E}\cdot\mathbf{B})
\end{aligned}$$

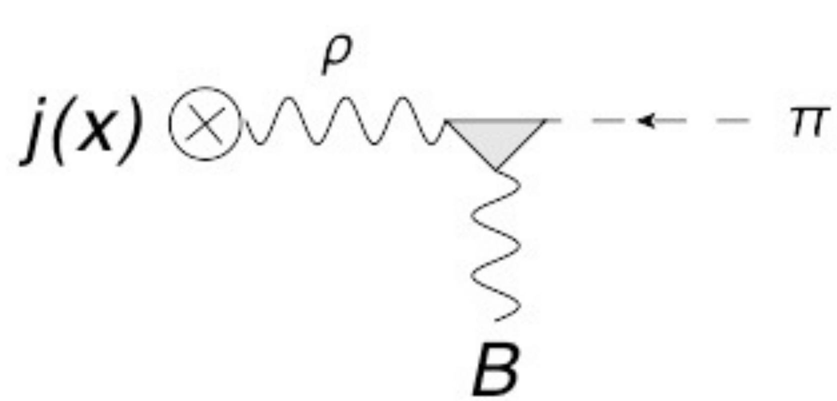
$\theta \approx -20°$ is the η–η' mixing angle, and $f_\eta \approx 157$ MeV.

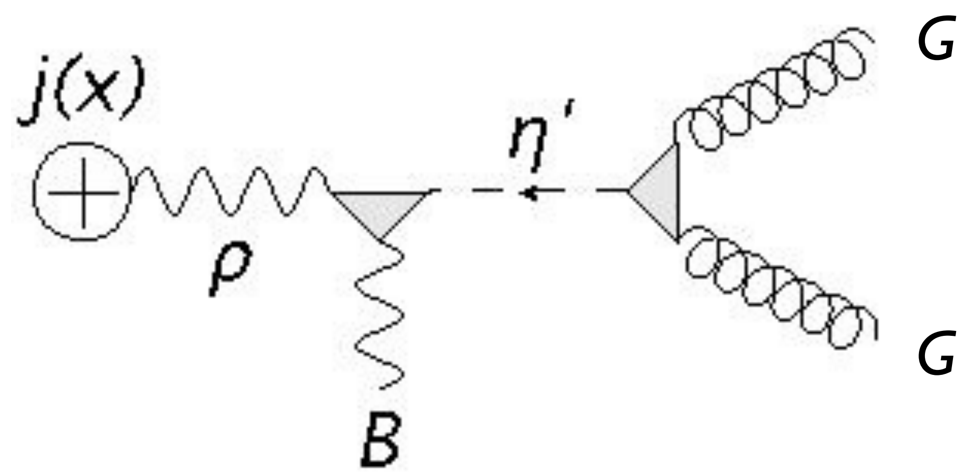
"Anomalous" current

Vector meson dominance (VMD) relates the electromagnetic hadronic current to the neutral rho-meson field:

$$j^\mu = -\frac{e\, m_\rho^2}{g_{\rho\pi\pi}}\, \rho^\mu$$

Thus, the B-field generates an "anomalous" current by converting a pseudoscalar meson (π, η, η') into a rho-meson:





The relevant interactions are experimentally known from radiative decays:

$$\Gamma_{\rho^0\to\pi^0\gamma} = 3\alpha\, g_{\rho\pi\gamma}^2\, \frac{p_{\rm cm}^3}{m_\rho^2} \approx 90 \pm 12 \text{ keV}$$

$$\Gamma_{\eta'\to\rho\gamma} = \alpha\, g_{\rho\eta'\gamma}^2\, \frac{p_{\rm cm}^3}{m_{\eta'}^2} \approx 60 \pm 5 \text{ keV}$$

$$g_{\rho\pi\gamma} = 0.58$$

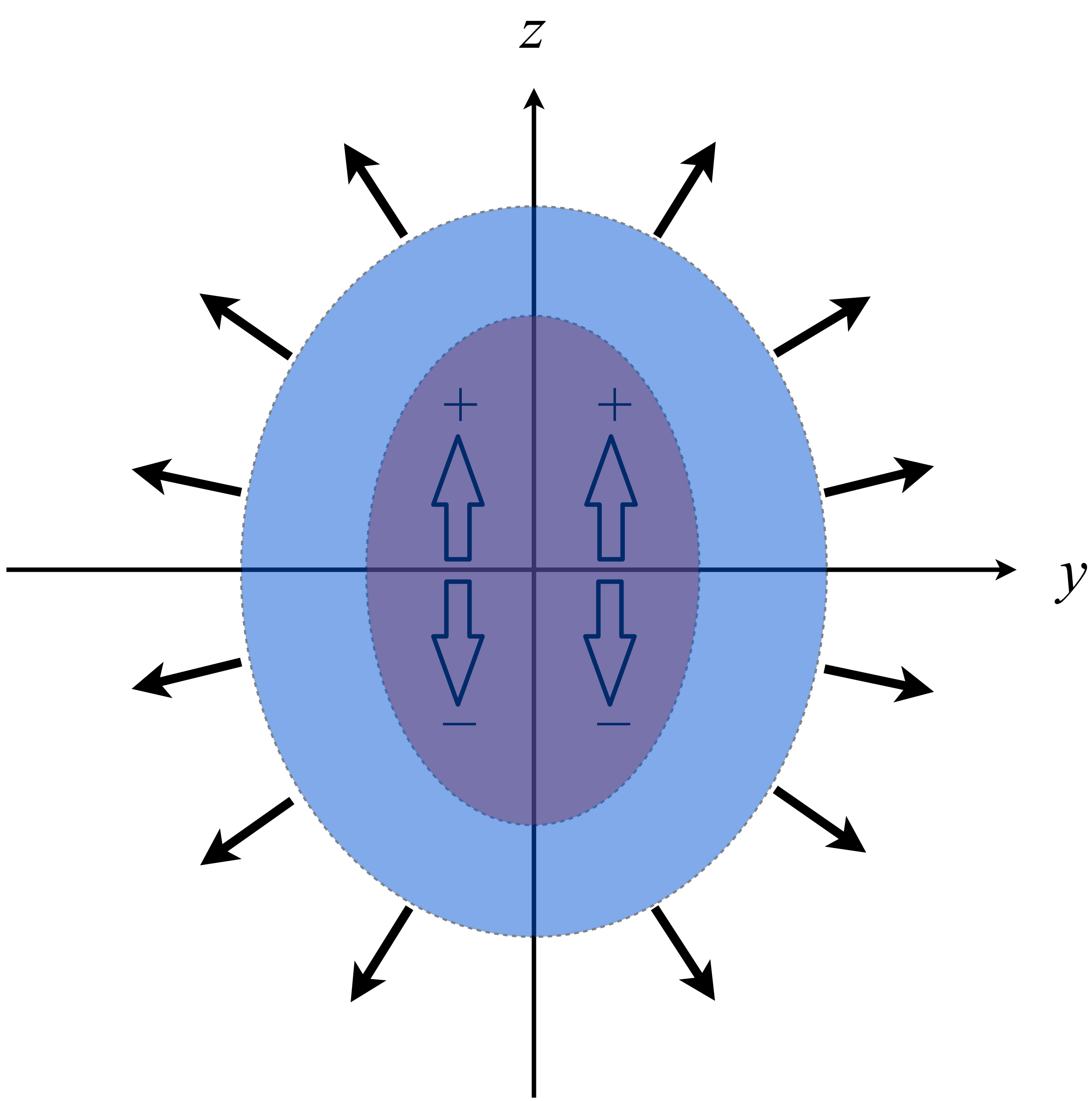
$$g_{\rho\eta'\gamma} = 1.31$$

Charge separation mech's (I)

- *CGC mechanism*: Two gluons from the initial nuclei fuse in the pseudo-scalar channel and generate an anomalous current during the peak phase of the magnetic field;
- *Glasma mechanism*: Gluons in the "glasma" generate an anomalous current in the strong magnetic field via a winding number fluctuation;
- *QGP mechanism*: Gluons in the equilibrated quark-gluon plasma generate an anomalous current in the strong magnetic field via a winding number fluctuation ("sphaleron");
- *Corona mechanism*: A neutral pion in the hadronic corona generates an anomalous current by converting into a rho-meson in the strong magnetic field;
- *Hadronic gas mechanism*: A neutral pion in the final hadronic gas phase generates an anomalous current by converting into a rho-meson in the strong magnetic field.

Final state interactions

Charge transport



The separated charge must be transported from its original location to the freeze-out hypersurface. Not all charges located above the reaction plane will be emitted in the upward direction.

A correlation between the original position and the final momentum can be created by:

- Transverse flow
- Geometric opacity

From J to ΔQ (I)

Anomalous current is not directly observable. What is observed is the final charged particle distribution given by the Cooper-Frye formula:

$$E\frac{dN_i}{d^3p} = \int_{\Sigma_f} d\sigma_\mu\, p^\mu f_i(\mathbf{x},\mathbf{p})\theta(\sigma_\mu p^\mu)$$

The net charge in the upper hemisphere ($z > 0$) is:

$$\Delta Q = \int d^3p \int_{\Sigma_f} \frac{d\sigma_\mu p^\mu}{E} \sum_i e_i f_i(\mathbf{x},\mathbf{p})\,\mathrm{sgn}(p_z)\theta(\sigma_\mu p^\mu)$$

For <u>isochronous freeze-out</u>, a position-momentum correlation requires collective flow.

$$\Delta Q = \int d^3x \int d^3p \sum_i e_i f_i(\mathbf{x},\mathbf{p};\tau_f)\mathrm{sgn}(p_z)$$

where $\quad f_i(\mathbf{x},\mathbf{p};\tau_f) = \exp\left[-u_\mu(\mathbf{x})p^\mu/T_f + e_i\mu_Q(\mathbf{x})/T_f\right]$

This finally leads to:

$$\boxed{\Delta Q = \int d^3x\, \rho(\mathbf{x},\tau_f)\, \frac{\int d^3p \sum_i e_i^2 f_i^{(0)}(\mathbf{x},\mathbf{p};\tau_f)\mathrm{sgn}(p_z)}{\int d^3p \sum_i e_i^2 f_i^{(0)}(\mathbf{x},\mathbf{p};\tau_f)}}$$

From J to ΔQ (II)

Two ways to proceed:

1. For weak flow, expand $f^{(0)}$ in first order in **v**. For charged pions only one finds the simple result:

$$\Delta Q \approx \tfrac{3}{2} \int d^3x \, \rho(\mathbf{x}, \tau_f) \, v_z(\mathbf{x})$$

then use the continuity equation $\partial_t \rho = -\nabla\cdot \mathbf{j}_{an}$ to obtain:

$$\begin{aligned} \left\langle (\Delta Q)^2 \right\rangle \quad &\approx \quad \frac{9}{4} \int_0^{\tau_f} dt\, dt' \int d^3x \int d^3x' \\ &\times \quad \nabla v_z(\mathbf{x}, \tau_f) \cdot \langle \mathbf{j}_{\rm an}(\mathbf{x}, t) \mathbf{j}_{\rm an}(\mathbf{x}', t') \rangle \cdot \nabla' v_z(\mathbf{x}', \tau_f). \end{aligned}$$

2. In the geometric approximation, one simply assumes that all charges in the upper hemisphere are emitted upwards, and vice versa:

$$\Delta Q = \int_{z>0} d^3x \, \rho(\mathbf{x}) = \int d^4x \, \delta(z) j_z(x)$$

From J to ΔQ (III)

The weak flow approximation differs from the geometric approximation by a factor

$$\Theta = \frac{\langle (\Delta Q)^2 \rangle_{\rm flow}}{\langle (\Delta Q)^2 \rangle_{\rm geo}} \approx v_f^2\, \xi_j / R$$

where ξ_j is the anomalous current correlation length.

For $\xi_j \approx 1$ fm, $v_f \approx 0.5$ and $R \approx 7$ fm, one has $\Theta \approx 0.035$ $\Rightarrow$ large uncertainty.

A more complete treatment of final state effects needs to include advective effects and charge diffusion in the continuity equation:

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho - D_{\rm ch} \nabla^2 \rho = -\nabla \cdot \mathbf{j}_{\rm an}.$$

This requires the numerical calculation of the charge asymmetry transport in the framework of relativistic hydrodynamics (or other transport models).

Magnetic field

Magnetic field (I)

Consider two colliding, homogeneously charged nuclei of charge Z, moving along the x-direction. The magnetic field component perpendicular to the reaction plane is:

$$eB_z^{(\pm)} = \begin{cases} Z\alpha v\gamma \left(\frac{b}{2} \mp y\right) \frac{\tilde{r}_\pm}{R^4} & (\tilde{r}_\pm \le R), \\ Z\alpha v\gamma \left(\frac{b}{2} \mp y\right) \frac{1}{\tilde{r}_\pm^3} & (\tilde{r}_\pm > R), \end{cases} \quad \text{with} \quad \tilde{r}_\pm = \left[\left(y \mp \frac{b}{2}\right)^2 + z^2 + \gamma^2 (x \mp vt)^2\right]^{1/2}$$

The combined field is highly peaked at $x = t = 0$, with width $\tau_{mag} = R/\gamma$, and has a power law tail, which is enhanced by baryon number stopping.

Processes that cannot resolve times shorter than τ_{mag} are only sensitive to the integral

$$eB_{\text{int}}(y,z) = \int_{-\infty}^{\infty} dt\, eB_z(x=0,y,z,t) \approx 2.32\, Z\alpha \frac{b}{R^2}$$

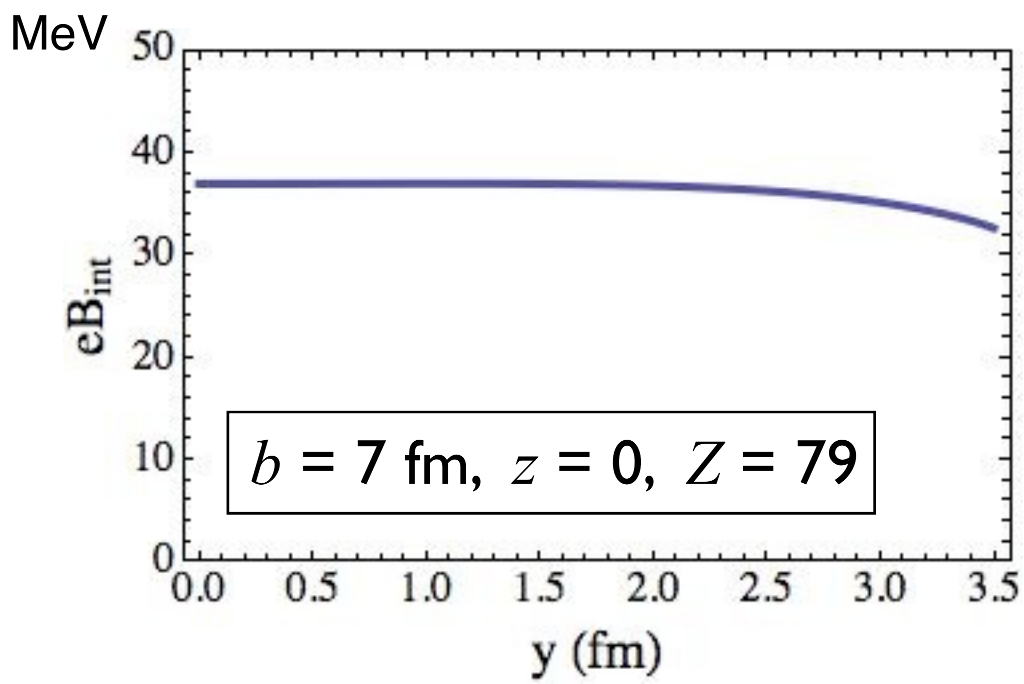
which is independent of the beam energy. At top RHIC energy $\tau_{mag} \approx 0.1$ fm; thus no soft QCD process can resolve the magnetic pulse. Conclusion: Dynamic charge asymmetry fluctuations may not be larger at LHC than at RHIC.

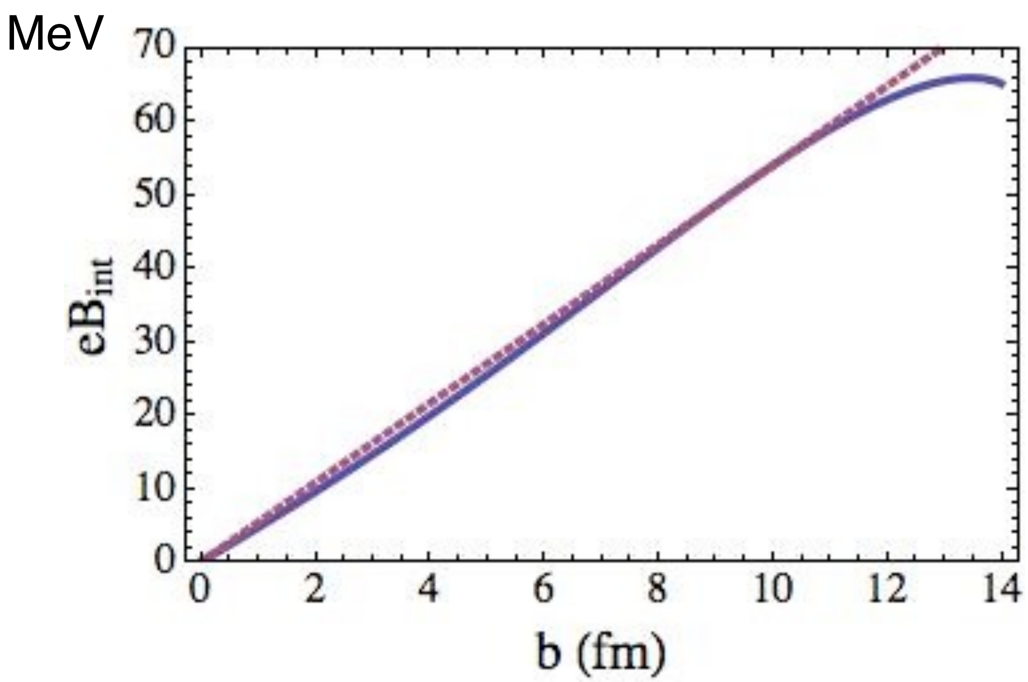
Magnetic field (II)

Quantitatively, the integrated magnetic field pulse is not in the "strong field" domain when compared to QCD momentum scales, such as Λ_{QCD}, m_π, T, etc.

$$eB_{\rm int}(y,z) = \int_{-\infty}^{\infty} dt\, eB_z(x=0,y,z,t) \approx 2.32\, Z\alpha\, \frac{b}{R^2}$$

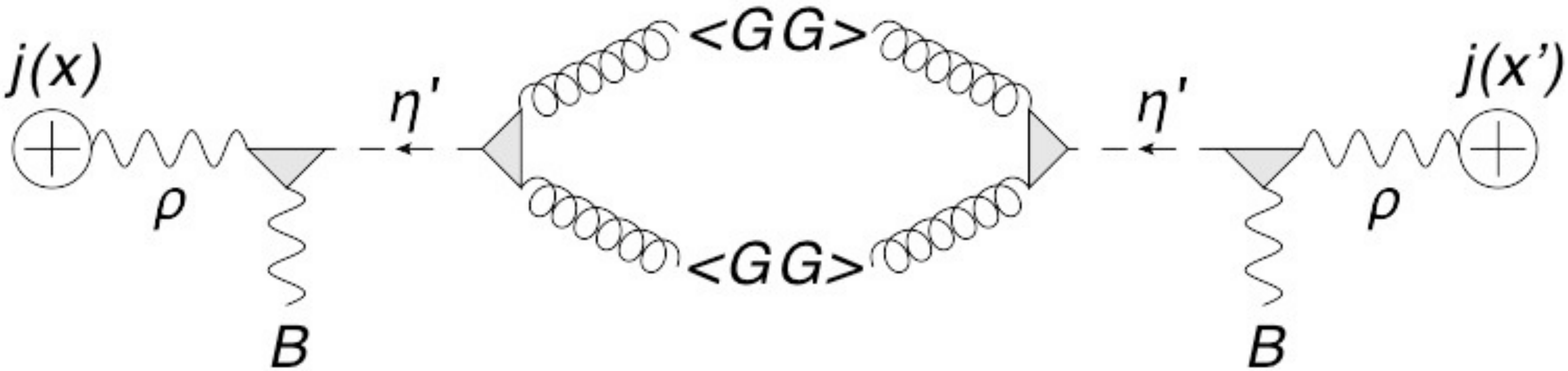
Calculations based on the lowest Landau level approximation are not adequate.





Results

CGC mechanism



$$\boxed{\langle j^3(x) j^3(x') \rangle \approx e^2\, C \big\langle (\mathbf{E}^a \cdot \mathbf{B}^a)(x) (\mathbf{E}^b \cdot \mathbf{B}^b)(x') \big\rangle}$$

with $C = \frac{g^2_{\rho\eta'\gamma}}{g^2_{\rho\pi\pi}} \frac{3(Z\alpha)^2 \alpha_s^2 \cos^2\theta}{(2\pi f_\eta)^2 m^2_{\eta'} m^2_\rho} \frac{b^2\gamma^2}{R^6}$

Because CGC color fields are nearly transverse, the fields in $\mathbf{E}^a \cdot \mathbf{B}^a$ must come from different nuclei. The gluon matrix element can be expressed in terms of the nuclear gluon distribution:

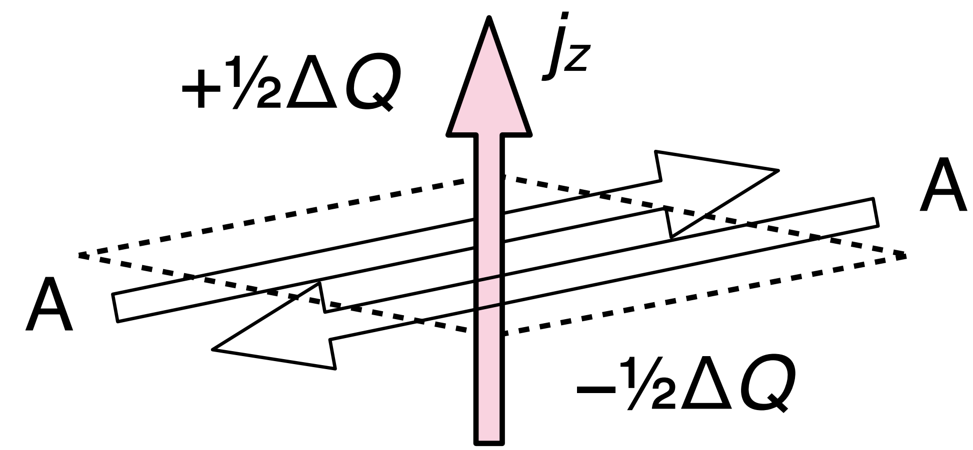
$$\big\langle (\mathbf{E}^a \cdot \mathbf{B}^a)(\mathbf{E}^b \cdot \mathbf{B}^b) \big\rangle \propto [x_0 G(x_0)]^2\, T_{AA}(\bar{\mathbf{x}}_\perp; \mathbf{b})$$

This mechanism can be enhanced if the η′ mass is lowered by medium interactions.

CGC mechanism (II)

The nuclear gluon density can be related to the CGC saturation scale Q_s by:

$$A\,[\xi_0 G(\xi_0)] = \frac{(N_c^2-1)\,R^2 Q_s^2}{8\pi^2\alpha_s}$$



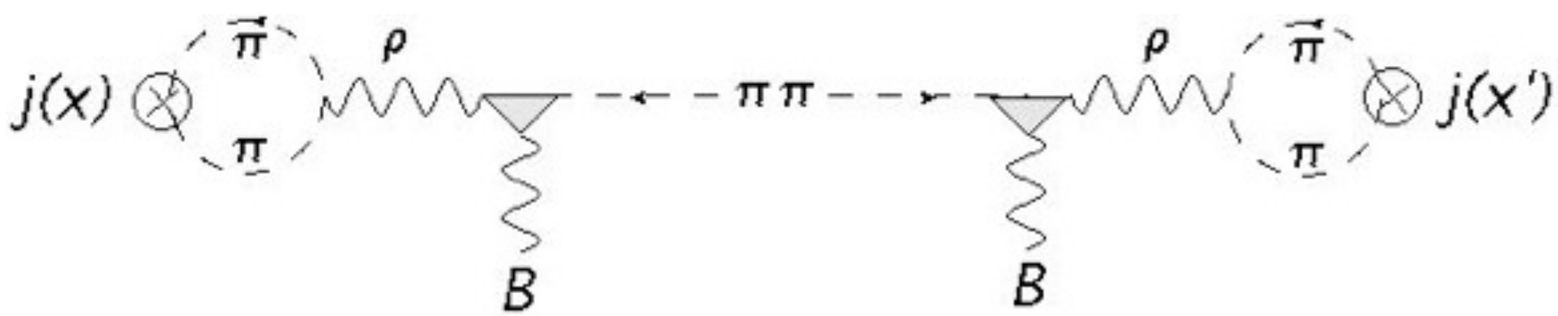
To calculate the up-down charge asymmetry fluctuations, we calculate the current through the reaction plane and assume that all charges above are emitted upwards and all charges below are emitted downwards (an overestimate!). After a lengthy calculation involving various additional "reasonable" approximations one finds:

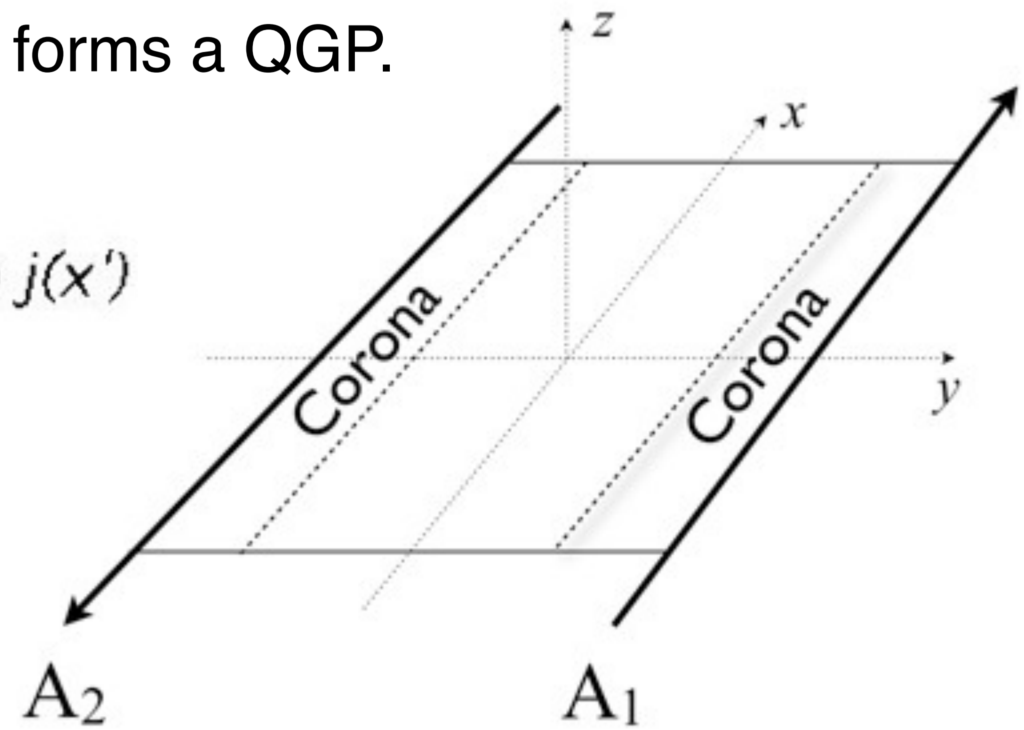
$$\langle(\Delta N_{\mathrm{ch}})^2\rangle = \begin{cases} C\,\frac{9(N_c^2-1)v_f^2}{(8\pi)^3\alpha_s^2}\,Q_s^2\,f(b) \approx 5\times 10^{-5}\,v_f^2\,\frac{b^2}{R^2}\,f(b) & \text{(weak flow)} \\ C\,\frac{3(N_c^2-1)}{32\pi^4\alpha_s^2}\,Q_s^3 R\,f(b) \approx 1.7\times 10^{-3}\,\frac{b^2}{R^2}\,f(b) & \text{(geometric)} \end{cases}$$

with $f(b) \approx 1-(b^2/R^2)(1-b/4R)^2$

Corona mechanism

We now consider the hadronic corona which never forms a QGP.





Assuming a thermal pion gas with T = 150 MeV and integrating over the entire B-field history, one finds in the geometric approximation:

$$\langle(\Delta N_{\mathrm{ch}})^2\rangle \approx \frac{(\pi Z\alpha g_{\rho\pi\gamma}g_{\rho\pi\pi})^2}{768\, e^2}\left(\frac{T}{m_\rho}\right)^6 \frac{b^2}{R^2} f(b) \approx 3\times 10^{-5}\,\frac{b^2}{R^2} f(b)$$

Both results are far too small to explain the STAR data, because this would require:

$$\frac{\langle(\Delta N_{\mathrm{ch}})^2\rangle}{(N_{\mathrm{ch}})^2} \approx 10^{-3}\,\frac{b^2}{R^2}$$

Summary & Outlook (I)

- Charge asymmetry fluctuations caused by the Chiral Magnetic Effect occur in both, the partonic and the hadronic, phases of QCD. In the partonic phase they are dominated by winding number carrying gauge field configurations; in the hadronic phase they are dominated by the process $\pi^0 + \gamma \to \rho^0 \to \pi^+\pi^-$.
- Charge asymmetry fluctuations do not require parity violation.
- The effects are rigorously calculable, at least in a near vacuum environment.
- Medium modifications of the interactions are, in principle, known and understood for $T < T_c$, e.g. pion loop corrections to the ρ-meson propagator. They can and should be included in future calculations. Qualitatively, they will lead to enhancements.

Summary & Outlook (II)

- Our estimates are much smaller than the effect seen by STAR, this suggests that the data may be due to some other effect (see, e.g. Hannah Petersen's talk on Wednesday!).
- The effect of charge separation can be strongly diluted by
  - the need to convert a position-space effect into a momentum space observable;
  - dissipative transport effects such as charge diffusion.
  - Uncertainties are factors 1/10 - 1/100.
- Rigorous future calculations of the effects discussed here and of the chiral magnetic effect must track the charge transport from the creation of the anomalous current to the freeze-out hypersurface, e.g. in the framework of relativistic hydrodynamics.

The END

“Local” symmetry violation ?

In QFT, symmetries can be violated at the (effective) Lagrangian level, e.g.

$$\text{CP symmetry}: \quad \mathcal{L}_\theta = \frac{\theta}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \qquad \text{Chiral symmetry}: \quad \mathcal{L}_m = m\bar{\psi}\psi$$

or by the state of the system (spontaneous symmetry breaking), which does not respect the symmetry of the Lagrangian. Examples: Breaking of rotational symmetry by a ferromagnet; breaking of chiral symmetry by the QCD vacuum.

Generally this requires a degeneracy of states and a lack of ergodicity of the system due to vanishingly small transition probabilities between states.

Example: chiral symmetry breaking [BM & S. Schramm, PRC 43 (1991) 2791].
NJL-type model interaction:

$$H_{\rm int} = \frac{g_0^2}{2}\int d^3x\, d^3x' \bar{\psi}(x)\gamma^\mu\psi(x)V(x-x')\bar{\psi}(x')\gamma_\mu\psi(x')$$

with $\quad V(\mathbf{q}) = [\mathbf{q}^2(1+\mathbf{q}^2/\Lambda^2)]^{-1}$

"Local" SSB (II)

Chiral symmetry is spontaneously broken for $g_0 > g_{\text{crit}}$. The field can be expanded in modes of massless fermions

$$\psi(\mathbf{x}) = V^{-1/2} \sum_{\mathbf{k},s} \left(b_{\mathbf{k}s} u^{(0)}_{\mathbf{k}s} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}} + d^\dagger_{\mathbf{k}s} v^{(0)}_{\mathbf{k}s} \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}} \right)$$

or in modes of massive fermions

$$\psi(\mathbf{x}) = V^{-1/2} \sum_{\mathbf{k},s} \left(B_{\mathbf{k}s} u^{(m)}_{\mathbf{k}s} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}} + D^\dagger_{\mathbf{k}s} v^{(m)}_{\mathbf{k}s} \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}} \right)$$

The creation / annihilation operators are related by a Bogoliubov transformation

$$B_{\mathbf{k}s} = \cos\theta_k \, b_{\mathbf{k}s} - s \sin\theta_k \, d^\dagger_{-\mathbf{k}s} \qquad D_{\mathbf{k}s} = \cos\theta_k \, d_{\mathbf{k}s} + s \sin\theta_k \, b^\dagger_{-\mathbf{k}s}$$

The broken symmetry vacuum can be expressed in terms of the symmetric one:

$$|\Phi_0^{(m)}\rangle = \prod_{\mathbf{k},s} (\cos\theta_k + s \sin\theta_k b^\dagger_{\mathbf{k}s} d^\dagger_{-\mathbf{k}s})|0\rangle$$

"Local" SSB (III)

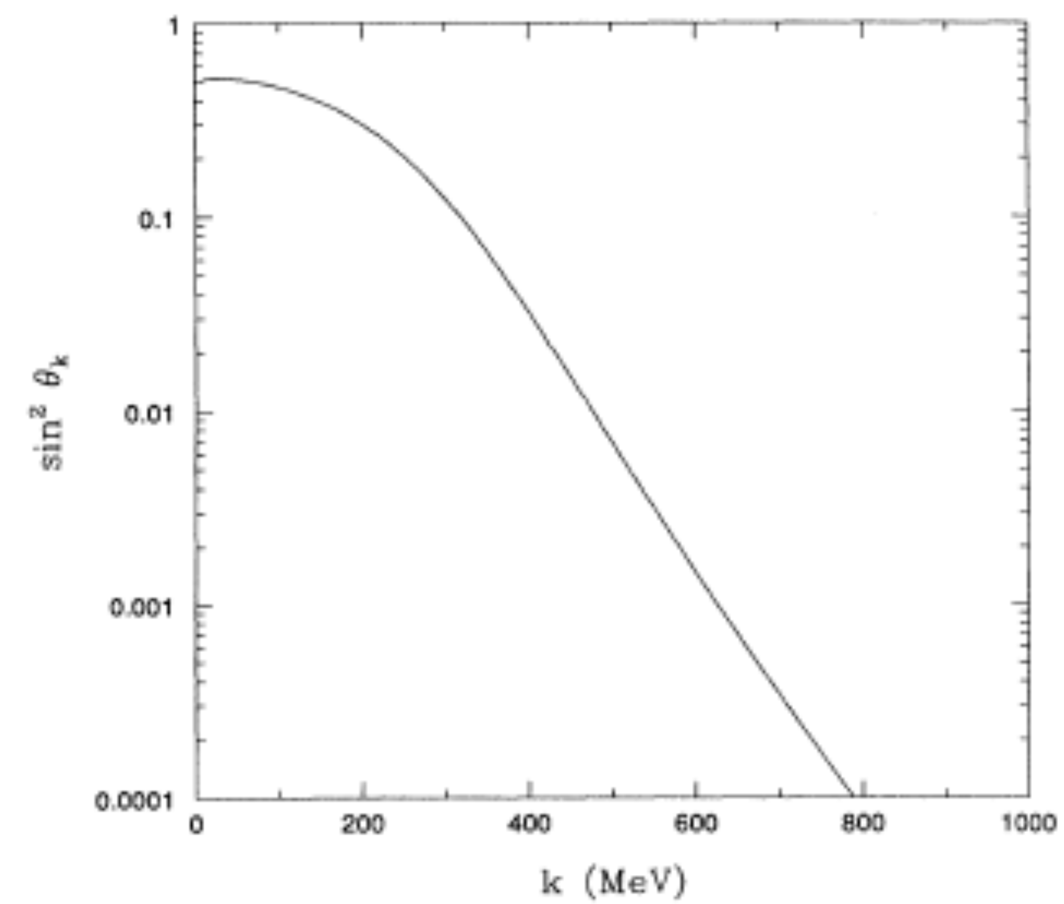
The relevant quantity is the overlap between the broken symmetry vacuum $|\Phi_0\rangle$ and the symmetric vacuum state $|0\rangle$:

$$|\langle\Phi_0|0\rangle|^2 = \exp\left[-\frac{N_f V}{\pi^2}\int_0^\infty k^2 dk\ \left|\ln(1-\sin^2\theta_k)\right|\right]$$

When the overlap vanishes, ergodicity is broken and the excitations of the two vacua do not communicate. This can occur for two reasons:

1. $V \to \infty$ (thermodynamic limit)
2. $\displaystyle\int_0^\infty k^2 dk|\ln(1-\sin^2\theta_k)| \ \to\ \infty$

Case 2 would be "local" spontaneous symmetry breaking. It requires the chiral symmetry to be broken on all scales.



Is the analogue of $$|\langle\Phi_0|0\rangle|^2 = \exp\left[-\frac{N_f V}{\pi^2}\int_0^\infty k^2 dk\ \Big|\ln(1-\sin^2\theta_k)\Big|\right] = 0$$

for the gauge field configurations attributed with “local parity violation” ?

In practice, the term “spontaneous symmetry violation” is used when exp(- c N_A) is just very, very small, not exactly zero. Is this the case here? In what sense?

Prima facie, one would consider any configuration occurring in a lattice simulation is not violating ergodicity, and thus not appropriately called “local symmetry violation”, but just a local fluctuation of the gauge field.

Widely accepted agreement on the meaning of the term would be desirable, so that it can be defended against skeptics....