Fluctuation of Electric Dipole Moment in the Chirally Broken Phase

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and
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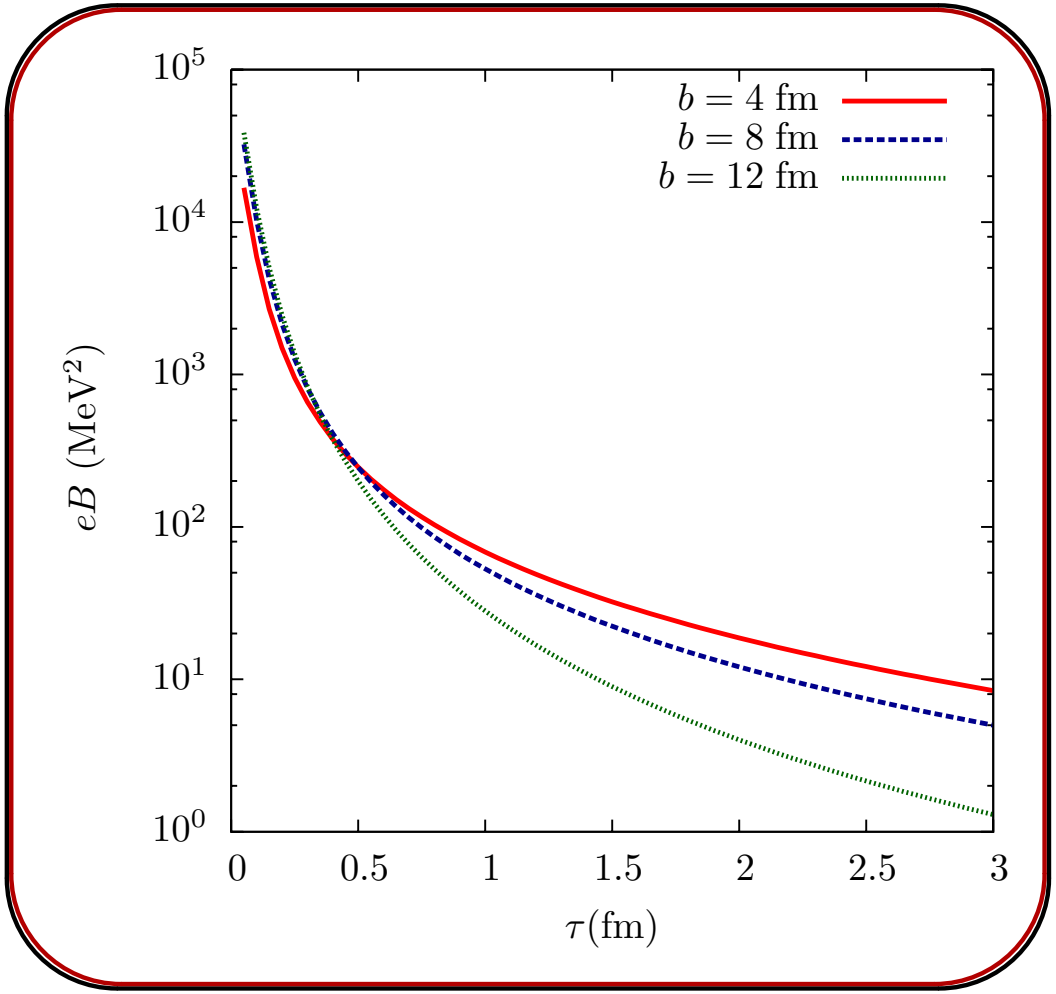
Chiral magnetic effect

$eB \sim 10^{19}$ G



Kharzeev, McLerran, Warringa

Chiral magnetic effect

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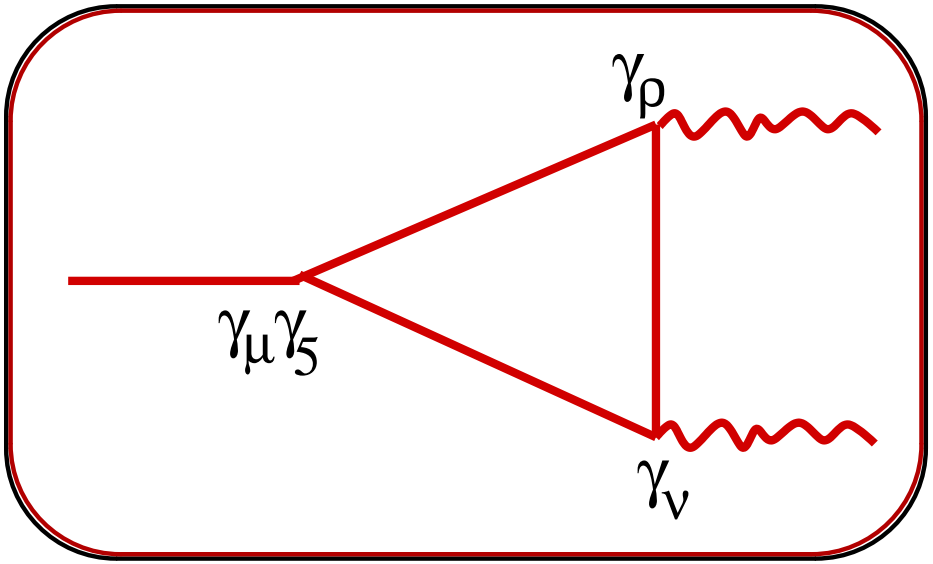

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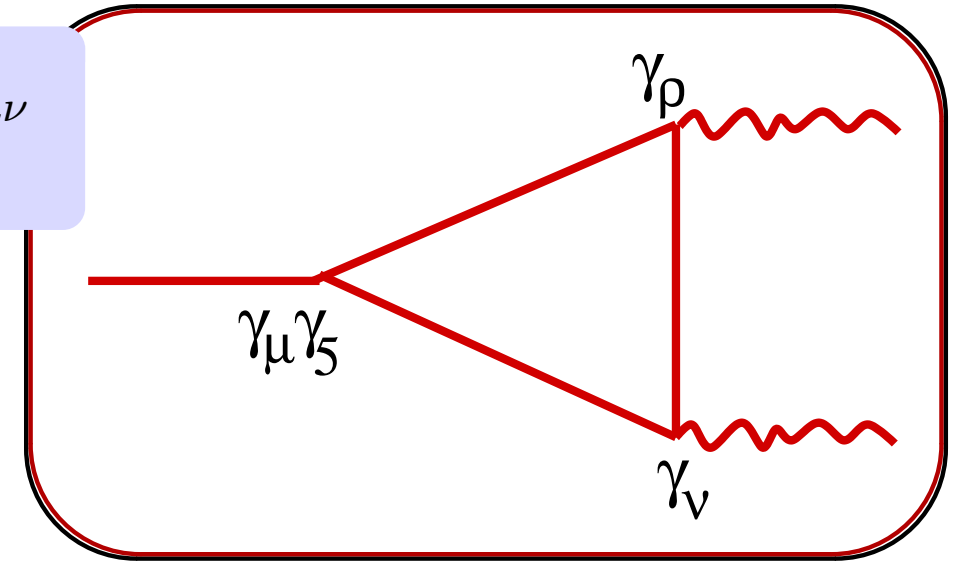

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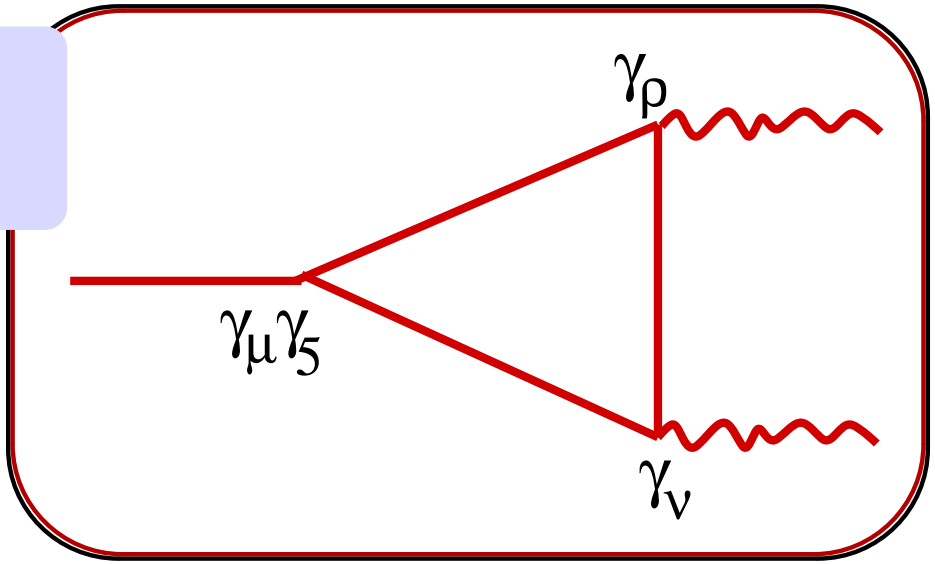

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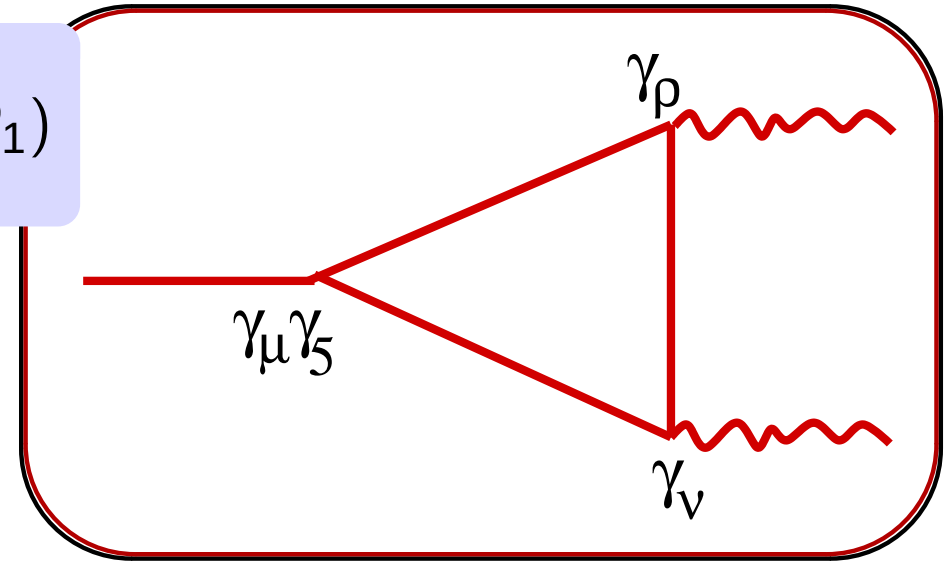

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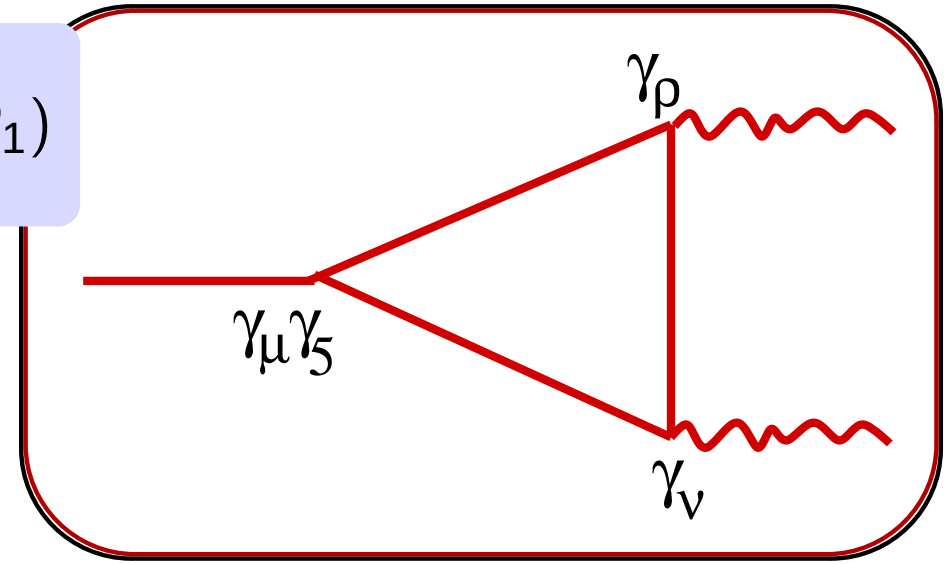

$$J_z = \frac{e^2}{8\pi^2}\left(q_u^2 + q_d^2\right) B\left(\partial_t \theta_1\right)$$

$$J_0 = \frac{e^2}{8\pi^2}\left(q_u^2 + q_d^2\right) B\left(\partial_z \theta_1\right)$$

High T: sphalerons

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QCD inequality in presence of EM

$$\mathcal{D} = \gamma_\mu \left(\partial_\mu + \imath g A_{\alpha\mu} t_\alpha + \imath e \mathcal{Q} A_\mu\right) + \mathcal{M}$$

- $\mathcal{Q} = \mathrm{diag}\,(2/3, -1/3)$
- $\mathcal{M} = \mathrm{diag}\,(m, m)$
- $\mathcal{Q} = \mathcal{I}_3 + \mathcal{B}/2,\ \mathcal{I}_3 = \tau_3/2,\ \mathcal{B} = \mathbb{1}/3$
- $(\gamma_5 \tau_{1,2})\, \mathcal{D}\, (\gamma_5 \tau_{1,2})^\dagger \neq \mathcal{D}^\dagger$

$$(\gamma_5 \tau_3)\, \mathcal{D}\, (\gamma_5 \tau_3)^\dagger = \mathcal{D}^\dagger$$

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- $\underline{I = 1}$: $\qquad M(\boldsymbol{x}) = \bar{\Psi}(\boldsymbol{x}) \Gamma \Psi(\boldsymbol{x})$

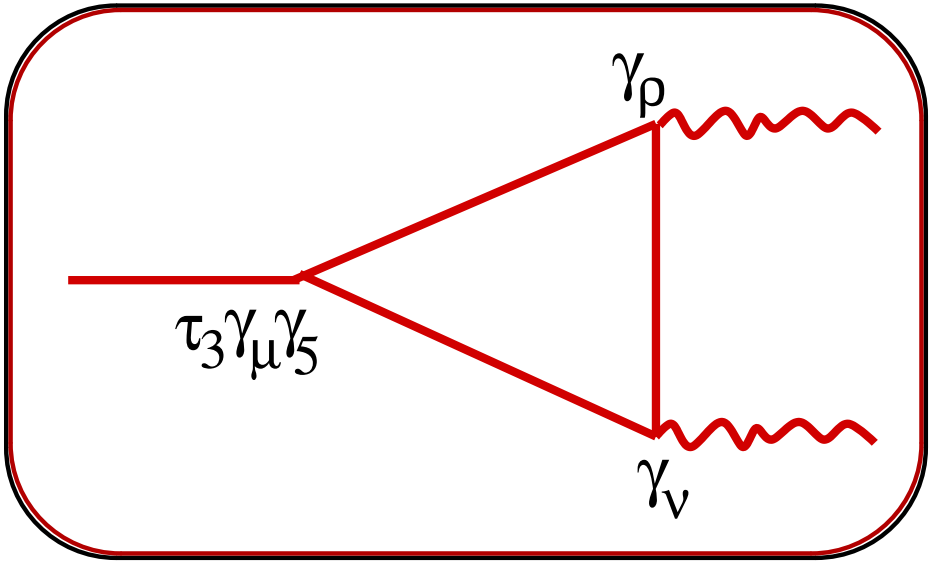
$$\left\langle M(\boldsymbol{x}) M^\dagger(\boldsymbol{0}) \right\rangle = \left\langle \mathrm{Tr}\, S(\boldsymbol{x}, \boldsymbol{0}) \Gamma\, (\gamma_5 \tau_3)\, S^\dagger(\boldsymbol{x}, \boldsymbol{0})\, (\gamma_5 \tau_3)\, \bar{\Gamma} \right\rangle \leq \left\langle \mathrm{Tr}\, S(\boldsymbol{x}, \boldsymbol{0}) S^\dagger(\boldsymbol{x}, \boldsymbol{0}) \right\rangle$$

- $S(\boldsymbol{x}, \boldsymbol{0}) = \mathcal{D}^{-1}_{\boldsymbol{x}, \boldsymbol{0}}, \qquad \bar{\Gamma} = \gamma_0 \Gamma \gamma_0$

$$\boldsymbol{\Gamma = \gamma_5 \tau_3}$$

Low T & π^0

$$\partial_\mu J^{\mu 5,3} = -\frac{3e^2}{16\pi^2}\left(q_u^2 - q_d^2\right) F_{\mu\nu}\widetilde{F}^{\mu\nu}$$



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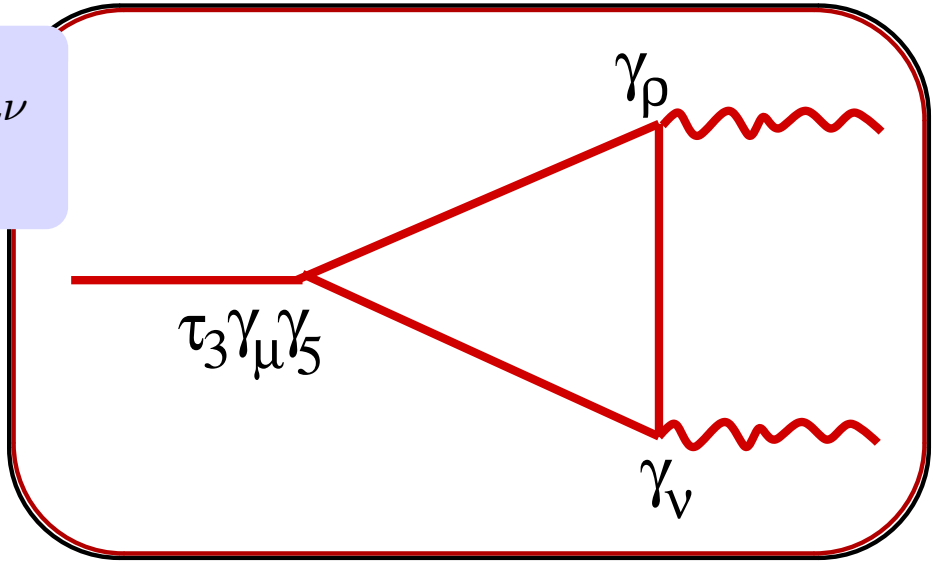

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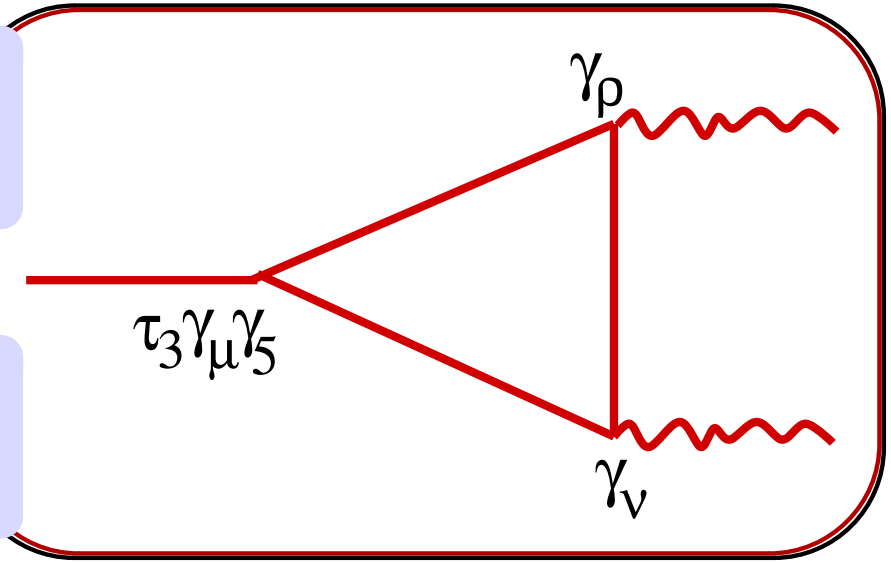

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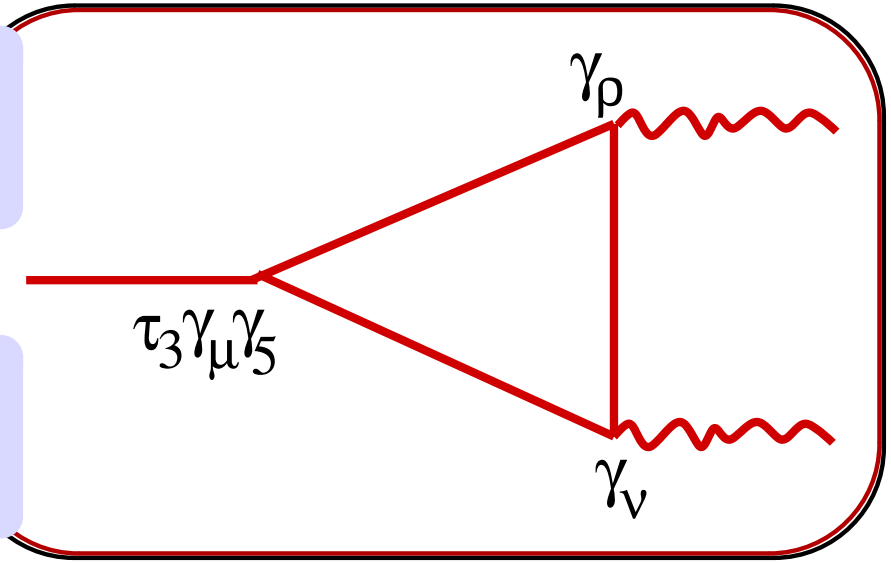

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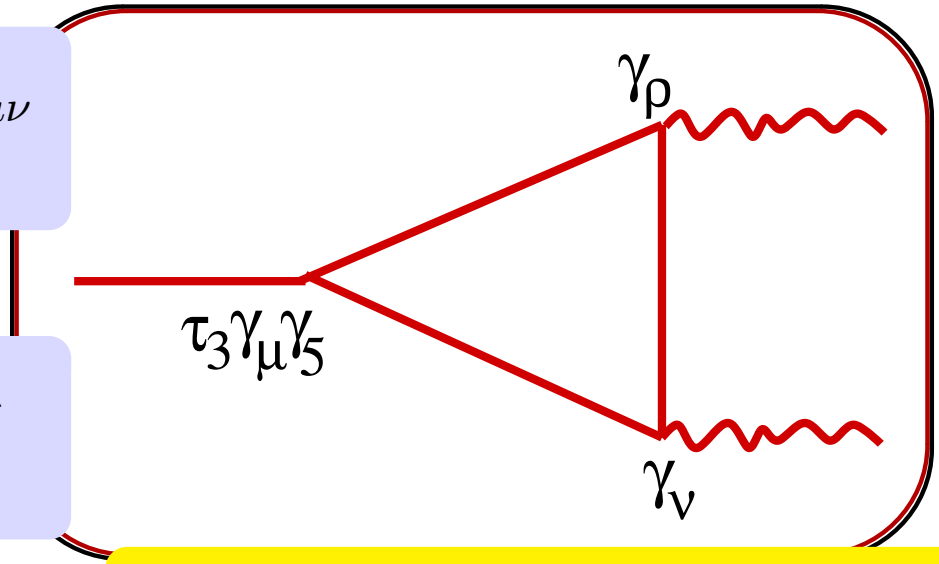

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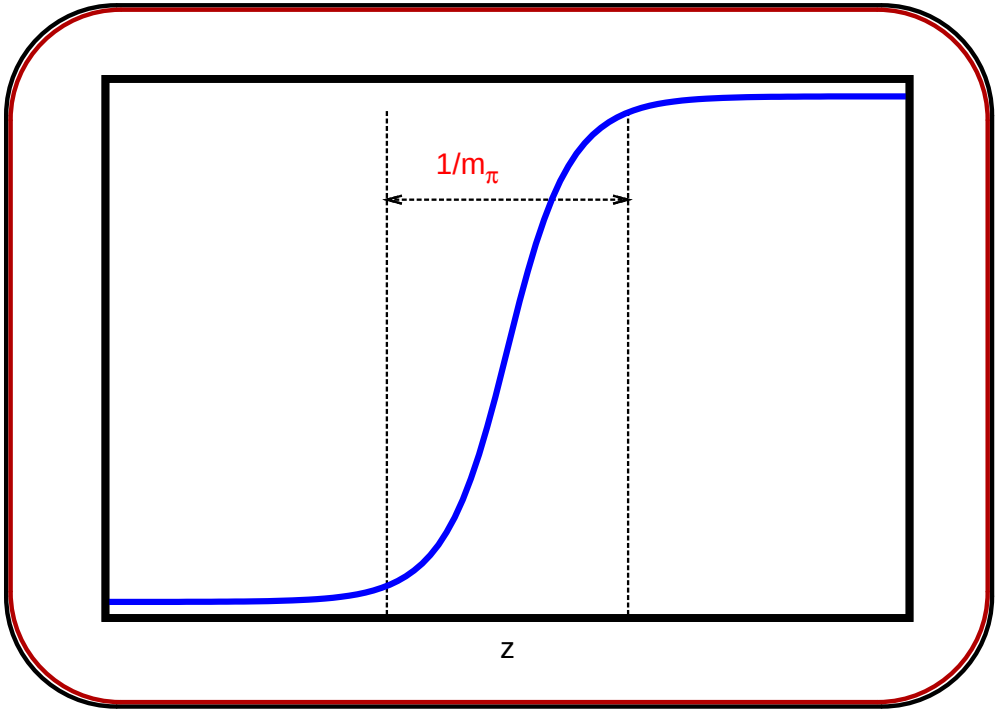
π^0 domain-wall

$$\pi^0 = 4\tan^{-1}\exp\left[m_\pi z\right]$$

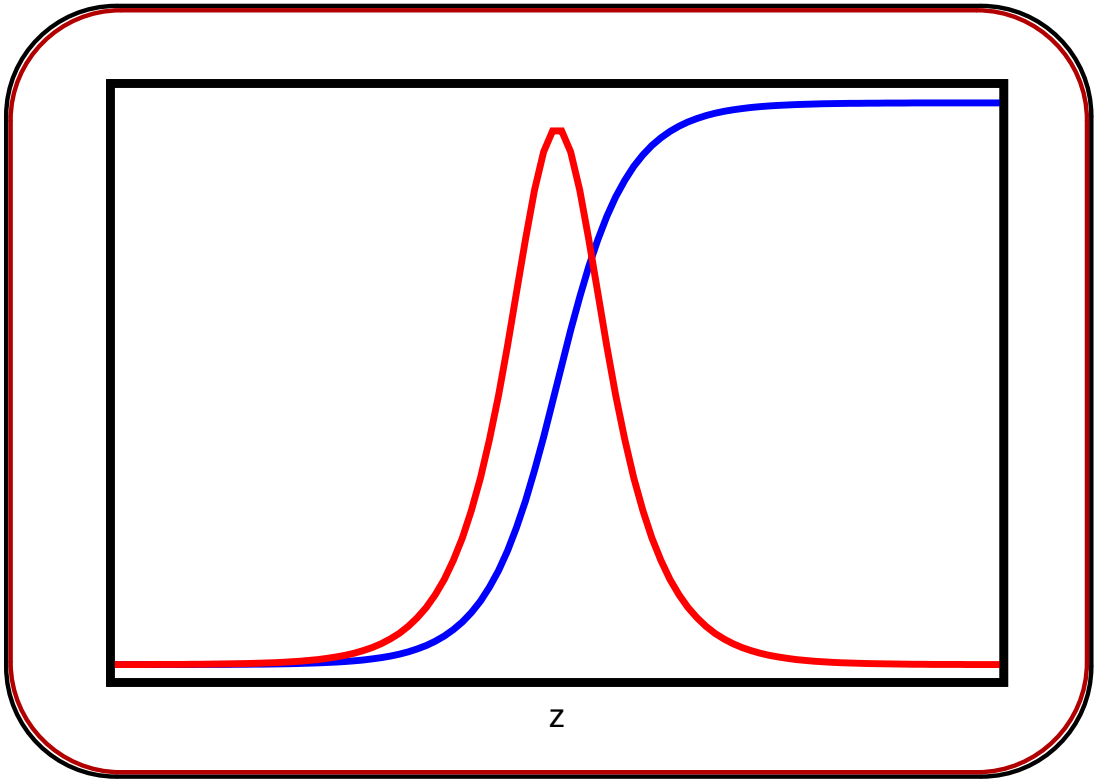


- Metastable: $eB \gtrsim 3m_\pi^2$
- $E \sim f_\pi^2 m_\pi$

Son & Stephanov, PRD 77, 014021

π^0 domain-wall

$$J_0 = \frac{3e^2 B}{2\pi^2}\left(q_u^2 - q_d^2\right)\frac{\boldsymbol{m_\pi}\, e^{m_\pi z}}{1 + e^{2m_\pi z}}$$



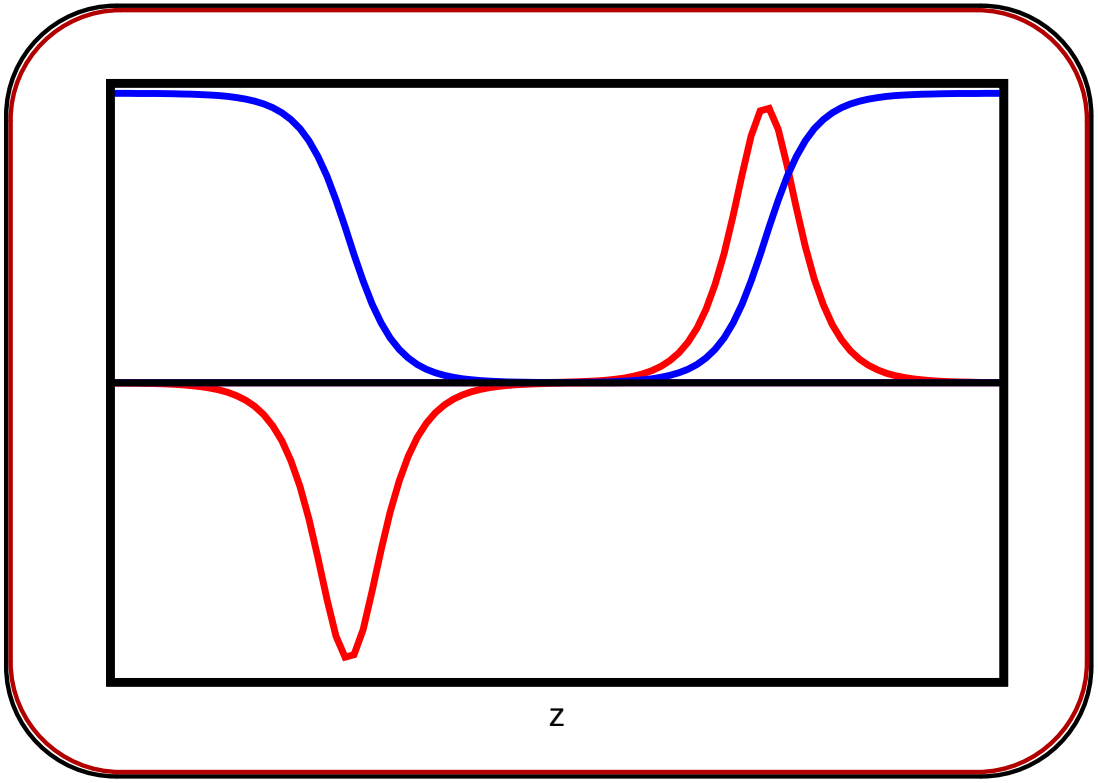
- $J_0 \to 0$ as $m_\pi \to 0$

π^0 domain-wall

- very large B $\Longrightarrow$ effectively (1+1)-d system
- (1+1)-d $\Longrightarrow$ $J_z^5 = J_0$
- $J_z^5 \sim \partial_z \pi^0$ $\Longrightarrow$ $J_0 \sim \partial_z \pi^0$

Fluctuation of edm at low T

domain-wall — anti-domain-wall configuration



- $edm = d\ J_0$
- $\left\langle (edm)^2 \right\rangle \neq 0$
- vanishes in the chiral limit

Possible test on the lattice

- Chiral magnetic effect:

$$\left\langle \left(\bar{\Psi}\gamma_5\Psi\right)\left(\bar{\Psi}\gamma_0\Psi\right)\right\rangle$$

  - enhanced at high T, suppressed at low T
  - enhanced as $m \to 0$

- This scenario:

$$\left\langle \left(\bar{\Psi}\tau_3\gamma_5\Psi\right)\left(\bar{\Psi}\gamma_0\Psi\right)\right\rangle$$

  - gluons do not couple to $J^{\mu 5,3} \Longrightarrow$ suppressed at high T
  - vanishes as $m \to 0$

Electric current fluctuation

$$J_z = \frac{3e^2}{8\pi^2 f_\pi}\left(q_u^2 - q_d^2\right) B \left(\partial_t \pi^0\right)$$

- initial stages of the collision
- largest B
- if there is a DCC like scenario

Summary

- **mechanism of $\mathcal{P}$-odd fluctuations in chirally broken phase ??**
- QCD inequality in presence of B $\Longrightarrow$ π^0
- iso-triplet anomaly & inhomogeneous π^0 condensate $\Longrightarrow$ charge separation
- large B $\Longrightarrow$ metastable π^0 DW
- π^0 DW $\Longrightarrow$ electric charge
- π^0 DW–ADW $\Longrightarrow$ fluctuation of edm
- suppressed for high T & low m
- lattice simulations $\Longrightarrow$ distinguish between cme and this scenario