Extra dimensions of space: are they going to be found any time soon?

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Extra dimensions of space: old idea

T. Kaluza (1921) and O. Klein (1926):

attempt to unify the only known forces at the time, gravity and electromagnetism

Since then, unification aspect is still at the heart of constructions with extra dimensions

1980's: strongest motivation.
Theories that pretend to be most fundamental (superstring theory, M-theory) are consistent in (9+1)- or (10+1)-dimensional space-time. They either cannot be defined at all, or have pathologies in other dimensions.

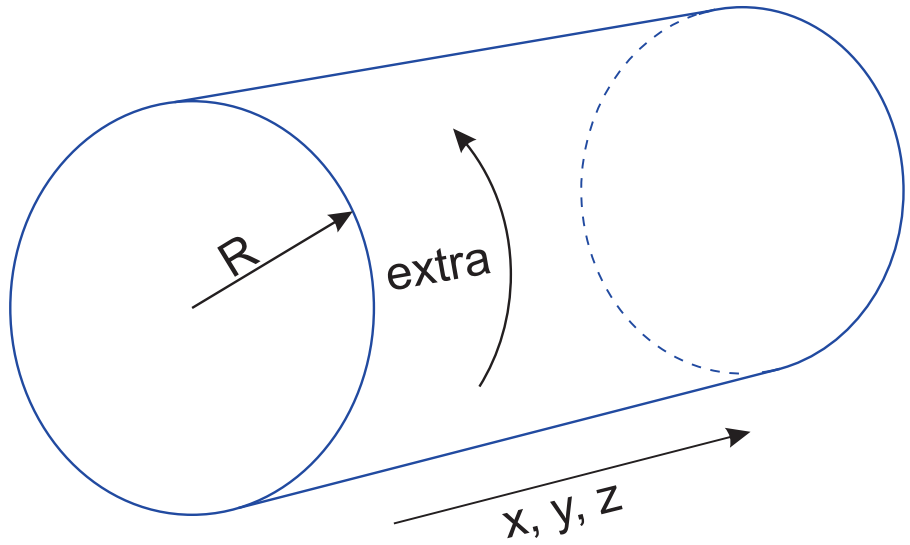
Despite long history, theories with extra dimensions is still a very lively area

No consensus on how extra dimensions look like.
This talk: several popular scenarios

Why don't we see extra dimensions?

Two proposals (or combination thereof)

- Original Kaluza–Klein: extra dimenions are compact



Quantum mechanics: angular momentum of rotation along extra dimension(s) is quantized,

$$L = 0,\ \ \pm\hbar,\ \ \pm 2\hbar, \ldots$$

$\Longrightarrow$ Energy of this rotation

$$E = 0,\ \ \frac{\hbar c}{R},\ \ 2 \cdot \frac{\hbar c}{R}, \ldots$$

$\hbar = c = 1$ in what follows

Small $R \Longleftrightarrow$ large E.

Except for $L = 0 \Longleftrightarrow$ no motion along extra dimensions, no feel of them.

We have not yet reached energy threshold of motion along extra dimensions

Extra dimensions will open up at $E > 1/R$

Signature: Kaluza–Klein partners of known particles

We do not see the motion along extra dimensions. For us, states of different angular momenta l look like different particles.

KK particle at rest w.r.t. us: no motion along our 3 dimensions, rotation along extra dimensions
$\Longrightarrow$ energy at rest in 3d sense $= l/R \Longrightarrow$

$$M_l = \frac{l}{R}$$

Heavy electrons, photons, quarks, ...

Regular pattern of their masses

Kaluza–Klein partners are characteristic of most of theories with extra dimensions. Their properties vary from model to model.

Conservation of angular momentum along extra dimensions

$\Longrightarrow$ lightest Kaluza–Klein particles stable

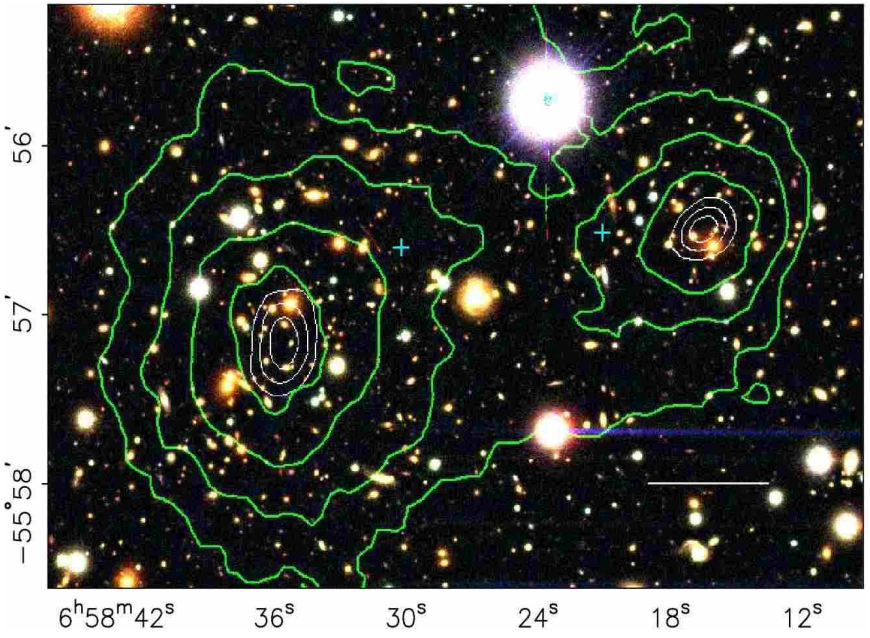
Maybe we have already discovered them?

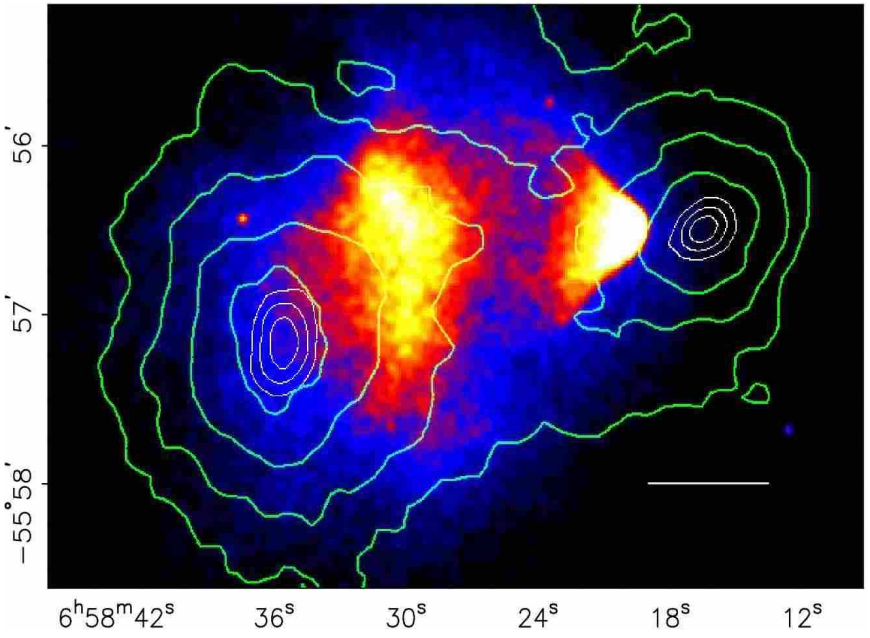
Dark matter in the Universe

Unknown form of matter filling the Universe, including this hall.

Mass density 5 times greater than that of usual matter.

Has the same gravitational interactions as ordinary matter. Clumps in galaxies, clusters of galaxies. (Almost?) no other interactions.





Mass and usual matter separated in space!

Dark matter: electrically neutral, heavy, stable particles.

No candidates in the zoo of known particles:

stable are:

electron, proton, but electrically charged

photon, neutrinos: electrically neutral, but too light

Dark matter particles: Kaluza–Klein partners of photon or neutrino?

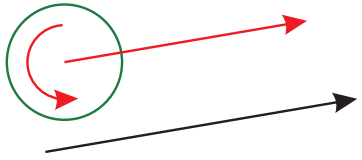
Crazy enough ...

NB: the lowest Kaluza–Klein partner is not necessarily stable, as extra dimensions need not be symmetric.

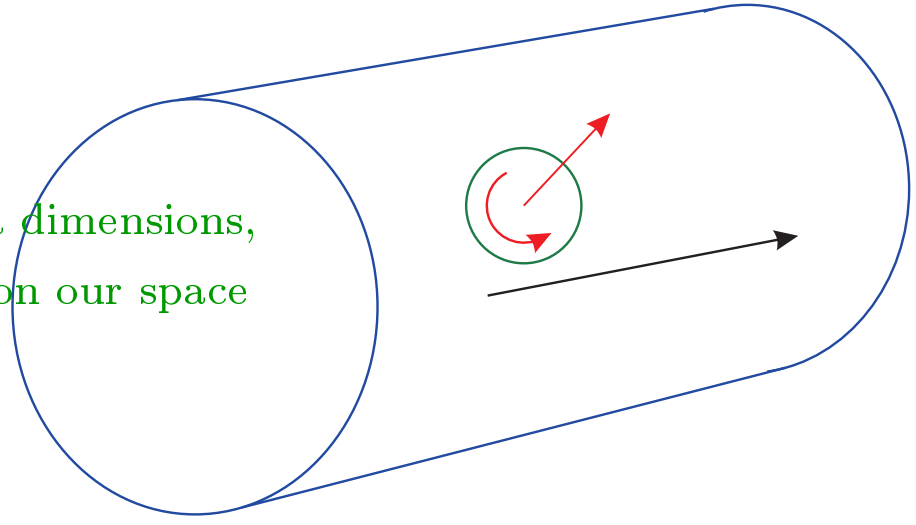
Unification

Photon = graviton, but with spin partially oriented along extra dimensions

graviton in 3 dimensions,
spin 2,
along or opposite direction of motion



graviton in extra dimensions,
spin projection on our space
$2 \times 1/2 = 1$
$\Longrightarrow$ photon



Probably too naive ...

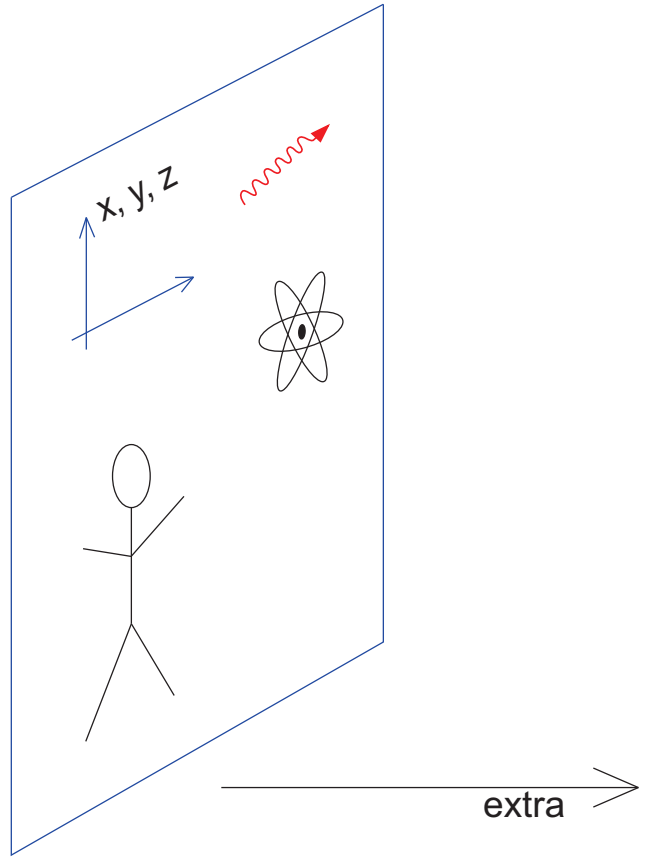
Why don't we see extra dimensions?

- Brane-world proposal

V.R., Shaposhnikov

Akama

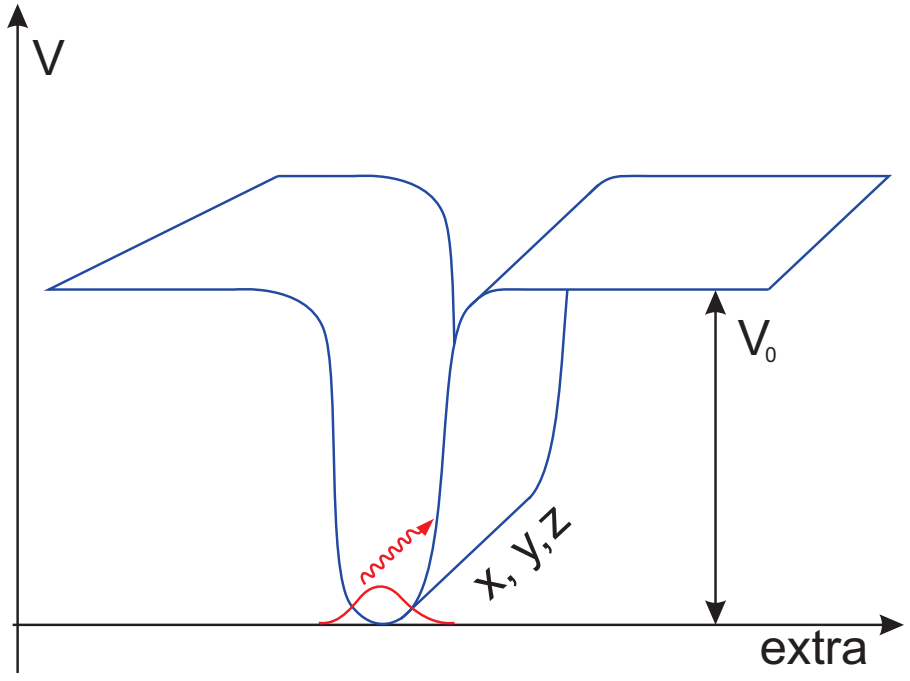
Our matter resides on a 3-dimensional manifold ("brane", from membrane) embedded in higher-dimensional space



Is this conceivable?

Dynamical mechanism:
potential well stretched along our 3 dimensions

similar to quantum wire



Lowest states often have zero or small energy (mass),
as compared to V_0. These are our particles. They freely propagate along 3-dimensional bottom of the well $\Longrightarrow$ world is effectively 3-dimensional at low energies.

Extra dimensions open up at $E > V_0$:

particles escape into extra dimensions.

Depending on a scenario, escape into extra dimensions may be possible also at $E < V_0$ due to tunneling. This happens if effective potential V vanishes at large distance from the brane

Dubovsky, V.R., Tinyakov

Effect of extra dimenions on low energy physics

Brane world: support from superstring theory

Superstring theory: fundamental objects are strings rather than particles. They live in 9-dimensional space.

Sizes of strings are tiny $\Longrightarrow$ we cannot resolve them (yet?) and perceive them as point-like particles.
Lowest string states = known particles, graviton included.

However, strings are soft at short distances/high energies $\Longrightarrow$ no ultraviolet divergencies, unlike in Quantum Field Theory

Consistent Quantum Gravity

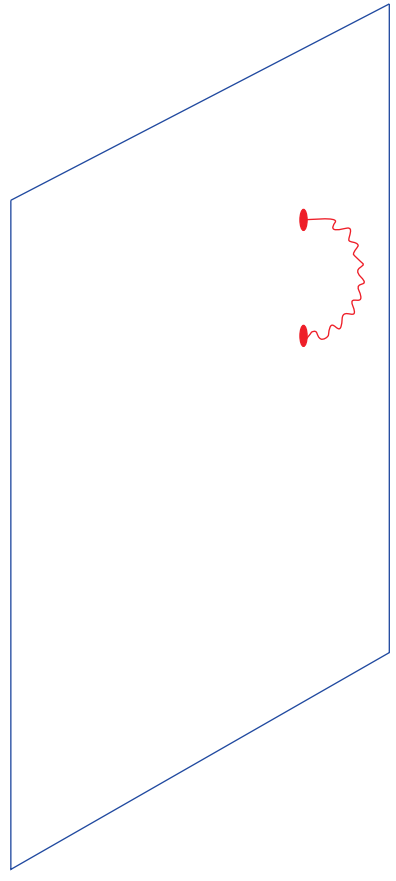
Closed strings = gravitons

Open strings = other particles

Open strings have their ends attached to D-branes,
manifolds of dimension generally smaller than 9.

Polchinksi

Our world may be one of D-branes



photon

graviton

Brane-world picture emerges naturally

Key issue: energy scale of extra dimensions

Energy scales thought to be fundamental:

- Strong interactions: $E_{QCD} \sim 100$ MeV $- 1$ GeV

  Under study for a long time, presently at RHIC

- Weak interactions: $E_W \sim 100$ GeV $- 1$ TeV

  Study began at LEP and Tevatron. Next step: LHC

- Gravitational interactions: $E_G \sim M_{Pl} = 10^{19}$ GeV

  Hopeless for colliders

If the gravitational scale M_{Pl} is really fundamental, the energy scale of extra dimensions is most probably roughly comparable to M_{Pl}

$\Longrightarrow$ No way to probe extra dimensions directly.

However,

if there are extra dimensions, the scale M_{Pl} is not necessarily fundamental.

Planck scale comes from Newton's gravity constant,

$$M_{Pl} = 1/\sqrt{G_4}$$

Recall that $\hbar = c = 1$

G_4 is not a fundamental parameter in theories with extra dimensions.

Can the fundamental energy scale of gravity
be $E_W \sim$ TeV rather than M_{Pl}?

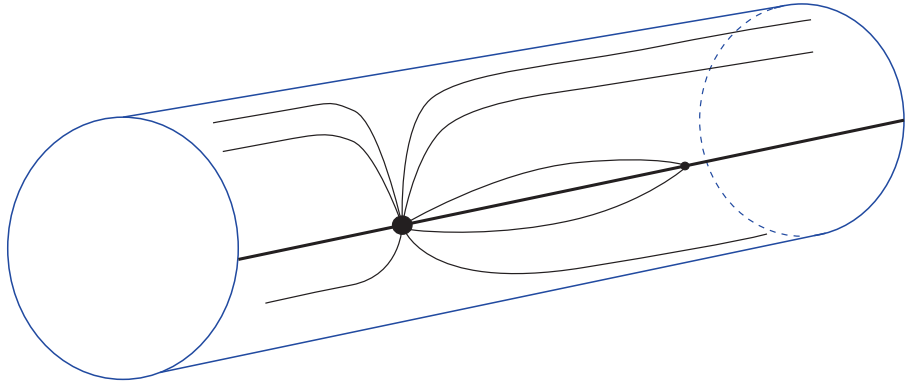
This would be a solution to the gauge hierarchy problem:

Why E_W is so much smaller than M_{Pl}?

Example # 1:

Large extra dimensions

Arkani-Hamed, Dimopoulos, Dvali



Gravitational lines of force emanating from mass on our brane spread over extra dimensions $\Longrightarrow$ gravity gets weak,

$$G_4 = \frac{G_{4+n}}{V_n}\ , \qquad V_n \simeq R^n = \text{volume of X-tra dim's}$$

$n =$ number of extra dimensions, $R =$ their size.

$M = 1/G_{4+n}^{1/(2+n)}$: fundamental gravity scale

$M_{Pl} = \sqrt{V_n} \cdot M^{(2+n)/2}$: effective gravity scale of 4-dimensional low energy theory

Large volume of extra dimensions $V_n \iff$ large M_{Pl}

M may be of order $E_W \sim 1$ TeV

Size of extra dimensions must be really large, e.g.,

$$R \sim 10^{-6} \text{ cm} \quad \text{for} \quad n = 3 \quad \text{and} \quad M \sim 1 \text{ TeV}$$

But only gravitons know about that.

NB: Gauge hierarchy problem reshuffled, not solved: why R is large?

How does fundamental gravity scale $M \sim$ TeV show up?

Kaluza–Klein gravitons of mass

$$m = \frac{l}{R}$$

Some are very light:
$l \sim 1$, $R \sim 10^{-6}$ cm $\Longrightarrow$ $m \sim 5$ eV

Each of them is harmless: interacts with our matter at gravitational strength G_4.

However, number of KK graviton species of mass $m < E$ rapidly increases with energy E:

$$N(E) \simeq (ER)^n$$

$R \sim 10^{-6}$ cm, $E \sim 100$ GeV, $n = 3$ $\Longrightarrow$ $N \sim 10^{31}$ (!)

KK graviton production in collisions of usual paricles rapidly increases with energy $\Longrightarrow$ becomes dominant process at $E \sim M$.

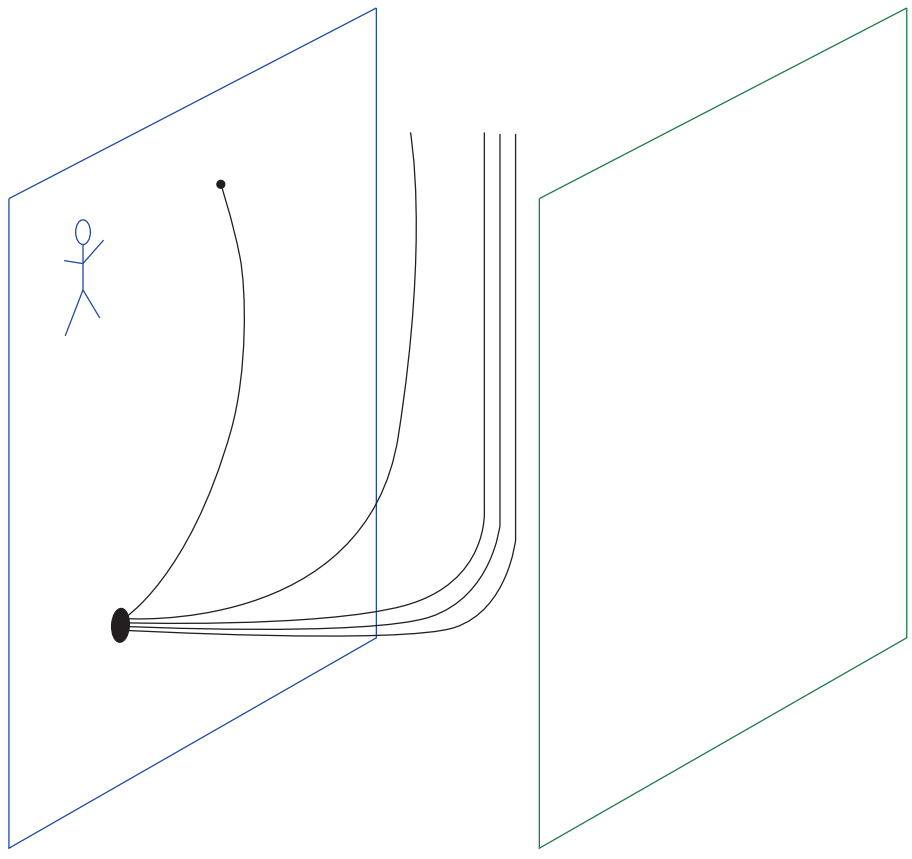
$E > M$: Quantum Gravity regime,
whatever this means.

NB: single KK graviton production possible. Because of brane, rotational symmetry along extra dimensions is broken, angular momentum is not conserved.

Example # 2:

warped extra dimensions, space is curved along them.

Randall, Sundrum



Gravitational lines of force get attracted to another brane due to curvature of extra dimensions $\Longrightarrow$ gravity gets weak

Gravitational force on our brane exponentially depends on interbrane distance $l \Longrightarrow$

$$G_4 \propto \mathrm{e}^{-Ml} G_5 \quad \Longrightarrow \quad M_{Pl} = \mathrm{e}^{Ml/2} M$$

Very large M_{Pl} with $M \sim 1$ TeV

and without particular tuning of parameters

How does the fundamental gravity scale M show up?

Graviton has KK partners with $m \sim M$. Their interaction with our matter is unsuppressed.

Again Quantum Gravity regime at $E > M$

One more reason to think about extra dimensions at energy scale $M \sim$ TeV: adS/CFT

- Physics at weak scale $E_W \sim 1$ TeV is something we have not encountered so far in particle physics

Electroweak symmetry breaking, origin of elementary particles' masses

F. Wilczek's BSA lecture

- Simple picture: elementary Higgs field that condenses and breaks symmetry

- However, many examples in condensed matter theory (and QCD!) show that symmetry breaking may be a complex phenomenon

  - E.g., symmetry breaking in superconductor due to condensation of Cooper pairs rather than elementary field

curiously, first described by effective Ginzburg–Landau theory with elementary condensate

Quest for electroweak symmetry breaking in a strongly coupled theory

- Strongly coupled theory we know of:
  Quantum Chromodynamics, theory of quarks and gluons.

Handful of elementary particles, but complex dynamics: color confinement, hundreds of composite particles, hadrons.
Quark–gluon "fluid", discovered at RHIC.

Symmetry breaking is there, but dynamics is not quite right to think of similar theory at TeV scale (hard to satisfy existing experimental constraints)

- Quite different strongly coupled theory: CFT

  Tractable via adS/CFT correspondence

adS/CFT

Original adS/CFT conjecture

Maldacena;
Gubser, Klebanov, Polyakov;
Witten

(no proof, but overwhelming supporting evidence):

Duality between a certain 4d conformal field theory* and entirely different higher-dimensional theory**

Duality = set of rules relating the two theories

Most amazing property:

Strong coupling in 4d $\Longleftrightarrow$ Weak coupling in higher dimensions

At first sight, purely academic result

*maximal supersymmetric extension of the Yang–Mills theory

*superstring theory in $\mathrm{adS}_5 \times \mathrm{S}_5$

Conformal Field Theories in 4d

Purely academic constructs at first glance.

- Conformal symmetry includes dilatations, $x^\mu \to \lambda x^\mu$.
  Physics at all length scales is all the same
  $\Longrightarrow$ No energy scale whatsoever.

Nothing like real world

- strongly coupled CFT:
  coupling constants are large at all energies.

- No interpretation in terms of particles.

Response to excitation by external probe dissipates along light cone.

Conformal symmetry cannot hold in our world

What does one expect from strongly coupled CFT
with conformal symmetry broken at energy scale Λ?

- Particle physics interpretation is back.
  Resonances = "bound states" = composite states of CFT fields.
  Tower of states whose masses start from Λ

Quite similar to theories with extra dimensions
and their Kaluza–Klein towers

In fact, this is probably more than similarity.

Extended adS/CFT conjecture

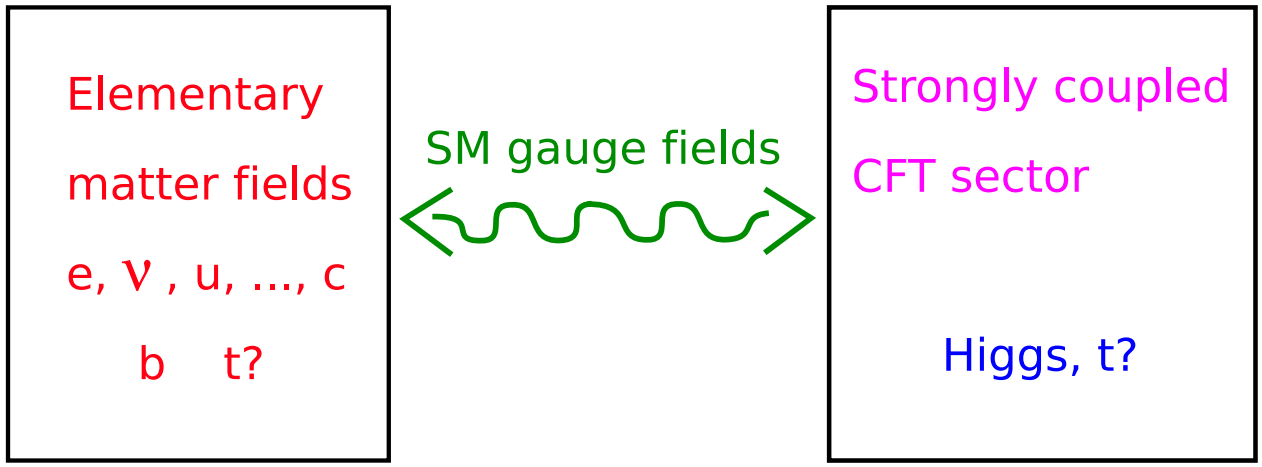
Maldacena; Verlinde; Witten; Gubser;
Arkani-Hamed, Porrati, Randall; Rattazzi, Zaffaroni; ...

(less understood, but still a lot of supporting evidence

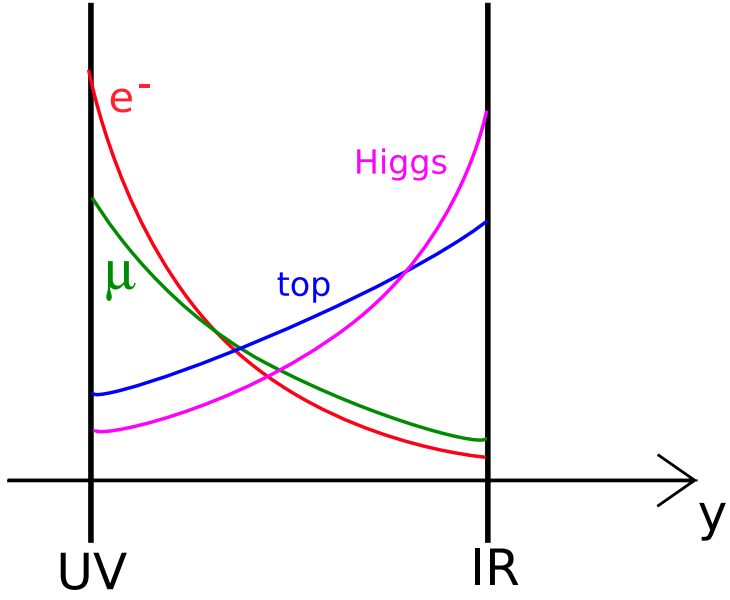
Gregory, V.R., Sibiryakov; Giddings, Katz; ...)

There is an entire class of 4-dimensional theories in strong coupling regime that are equivalent to weakly coupled theories in higher dimensions

Emerging picture



and its dual in extra dimensions



- Message:

Some fully consistent and possibly realistic theories in 4d describe the same physics as theories in higher dimensions.

- Spin-off: adS/QCD

Understanding strong interactions, including heavy ion collisions at RHIC, in terms of theories with extra dimension(s)

- Physics at the electroweak scale may well be best described by a theory with extra dimensions.

The energy scale is certainly $E_W \sim 1$ TeV

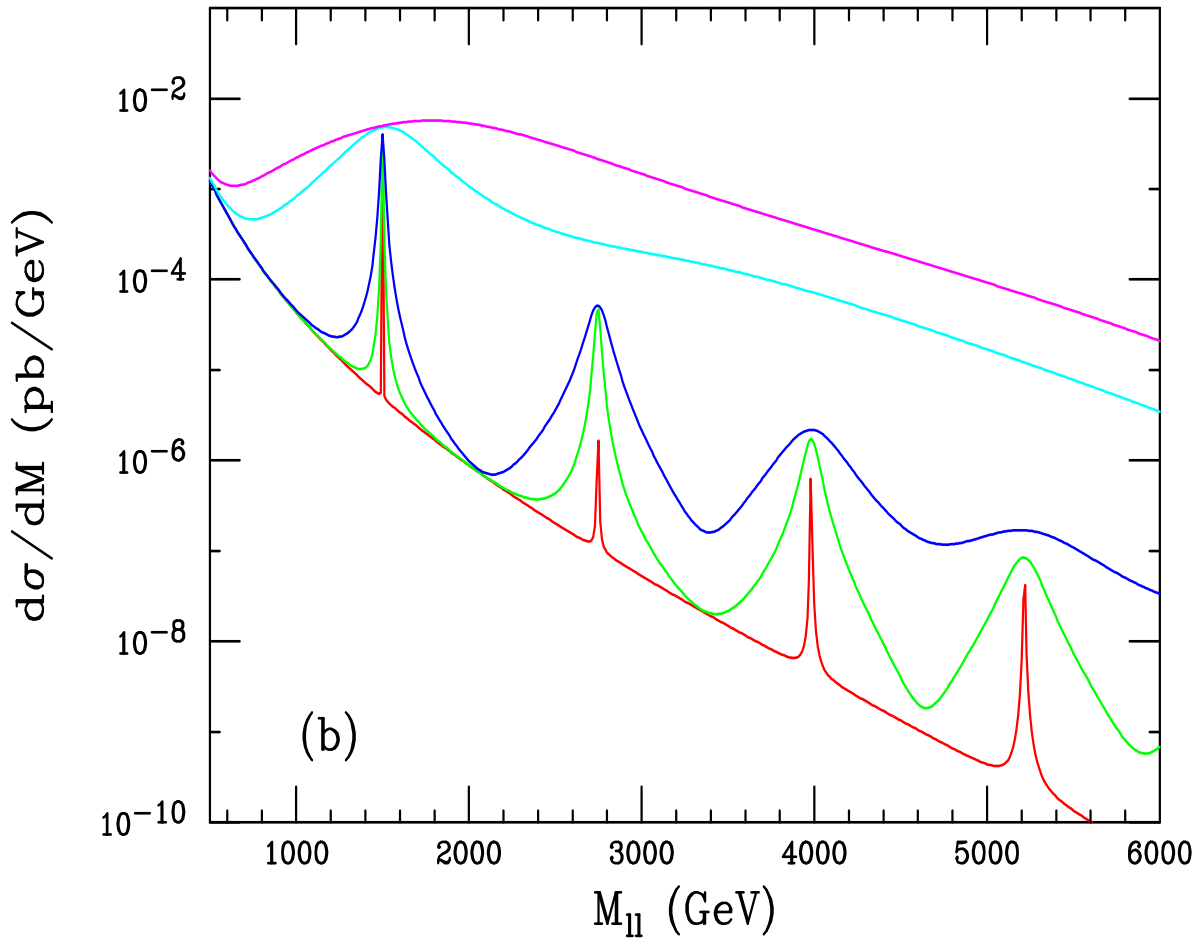
In any case, if the energy scale of extra dimensions is $E_W \sim 1$ TeV, they will be probed at the Large Hadron Collider at CERN.

Signatures:

Production and decay of Kaluza–Klein resonances,

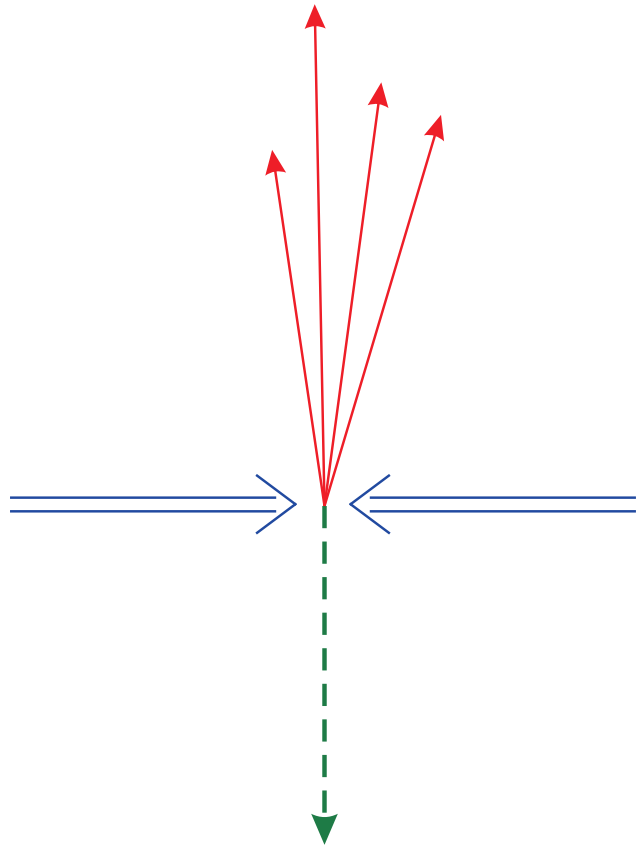
$$q\bar{q}, gg \longrightarrow \text{KK state} \longrightarrow \mu^+\mu^-$$

Davoudiasl, Hewett, Rizzo



Escape of particles into extra dimensions:

$q\bar{q}, gg \rightarrow g +$ escaping particle



unbalanced transverse energy

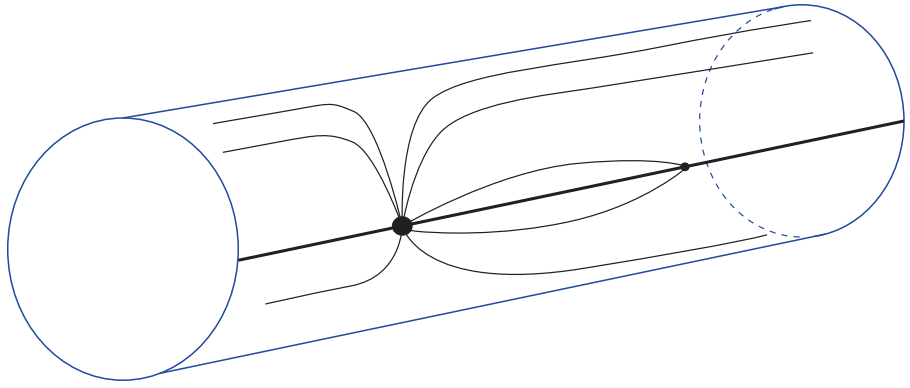
Search for these and similar phenomena is on the LHC agenda, with detailed studies of signals and backgrounds.

Bottom line: effects due to extra dimensions will be discovered, if their energy scale is below $2-3$ TeV

This covers the most interesting range.

Where else extra dimensions may show up?

- Table-top: Gravity at short, but macroscopic distances



Gravity is $(4+n)$-dimensional at $r < R$.

$$F(r) = -G_4 \frac{mM}{r^2} \quad \Longrightarrow \quad F(r) = -G_{(4+n)} \frac{mM}{r^{2+n}}$$

Square law checked down to $r = 50\ \mu\text{m} = 5 \cdot 10^{-3}$ cm (!)

Eöt-Wash experiment,
Kapner et.al.

- Low energy particle physics

Brane world: escape from the brane at low probabilty

Invisible decay of positronium, Z-boson

Decay of electron

Dubovsky, V.R., Tinyakov

- NB: theoretical tool to understand consistency of apparent energy and charge non-conservation

Work in progress: cosmological constant problems

L. Krauss' BSA lecture

- Vacuum weighs. Its energy density curves space-time.
  This should show up as accelerated expansion of the Universe.

- Problem # 1: The Universe expands slowly
  $\Longrightarrow$ vacuum energy density is essentially zero.

Off hand, one would think on dimensional grounds that

$$\rho_{vac} \sim (1 \text{ GeV})^4, \quad \text{QCD}$$
$$\rho_{vac} \sim (100 \text{ GeV})^4, \quad \text{Electroweak}$$

This is at least 50 orders of magnitude too large.

- Proposal: Vacuum energy warps extra dimensions instead of accelerating the Universe

V.R., Shaposhnikov;
Arkani-Hamed, Dimopouplos, Kaloper, Sundrum;
Kachru, Schulz, Silverstein

Attempts failed. Probably, does not work, at least in cosmological context

- Problem # 2: Universe does undergo accelerated expansion.

If attributed to vacuum energy density, then

$$\rho_{vac} \sim (0.002 \text{ eV})^4$$

- Proposal:

  (i) $\rho_{vac} = 0$ for some reason

  (ii) Expansion accelerates because gravity gets modified at ultra-large distances/times

  Gravity slowly leaks into extra dimensions

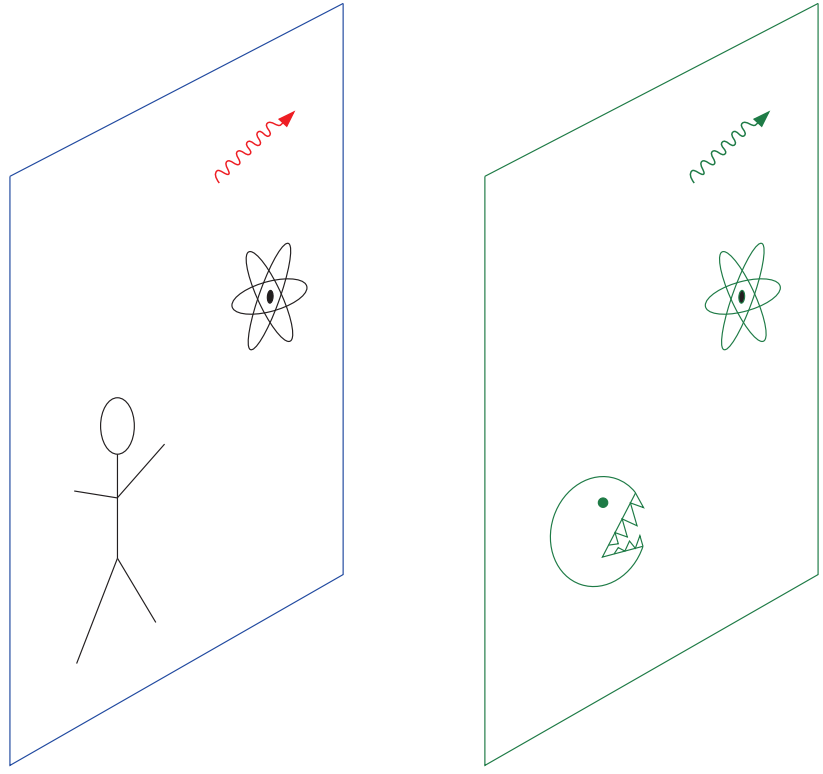
  Kogan, Mouslopoulos, Papazoglou, Ross;
  Gregory, V.R., Sibiryakov;
  Dvali, Gabadadze, Porrati

No consistent model so far.

No deep reason for obstruction known.

Braneworld picture suggests:

- Existence of other branes, possibly with some forms of matter



Once we reach the energy scale, we will start exploring our surroundings in extra dimensions.

- Picture of Big Bang gets twisted.

This may be spontaneous nucleation of our brane

Gorsky, Selivanov

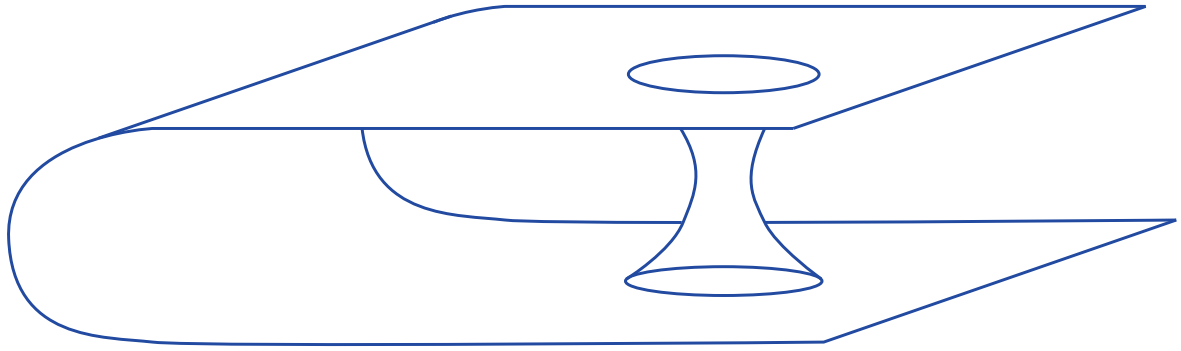
or collision of branes

Khouri, Ovrut, Steinhardt, Turok

- Geometry of our brane may be non-trivial

  One of solutions:

Dubovsky, Sibiryakov



- A lot of room for speculations

To summarize:

- There is a chance that energy scale of extra dimensions is TeV.

If so, extra dimensions will start to open up at LHC.

Quantum Gravity will become experimentally accessible.

- With even more luck, effects of extra dimensions may show up in table top or low energy particle physics experiments.

- There is fascinating connection between some theories in 4 and higher dimensions.

New way to approach complex dynamics
of strongly coupled theories.

It may well be that electroweak symmetry breaking
is best understood in terms of extra dimensions.

- There is a lot of room for crazy ideas. If any of them is true, revolution guaranteed.