Axial anomaly and magnetism of nuclear and quark matter

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QCD vacuum in the magnetic field

- Typically, critical B which modifies QCD vacuum $eB \sim m_\rho^2,\ f_\pi^2$.
  (Shushpanov-Smilga, Kabat et al, Miransky et al, Cohen et al)
- In (or near) the chiral limit, the response is governed by chiral Lagrangian.

$$\mathcal{L} = \frac{f_\pi^2}{4}\,\mathrm{tr}\left[\partial_\mu\Sigma\partial^\mu\Sigma^\dagger + 2m_\pi^2\Sigma\right]; \qquad \Sigma = \exp\left(\frac{i\tau_a\varphi_a}{f_\pi}\right);$$

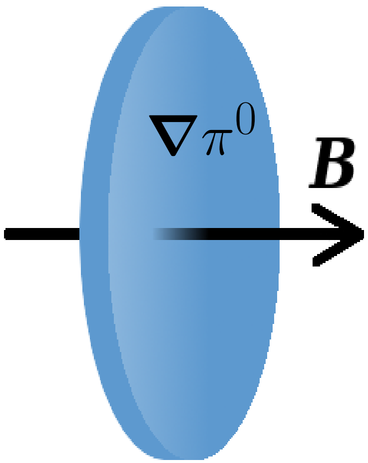
- We shall look at a nontrivial solution — π^0 domain wall:

$$\pi^0 \equiv \varphi_3 = 4f_\pi \arctan e^{m_\pi z}; \quad \varphi_1 = \varphi_2 = 0;$$

which is unstable ("unwinding"). The spectrum of excitations has tachyonic branch:

$$E^2 = k_x^2 + k_y^2 - 3m_\pi^2\,.$$

- This solution becomes *metastable* in the magnetic field $B > B_0$



$$E^2 = (2n+1)eB - 3m_\pi^2, \quad n = 0, 1, \ldots$$

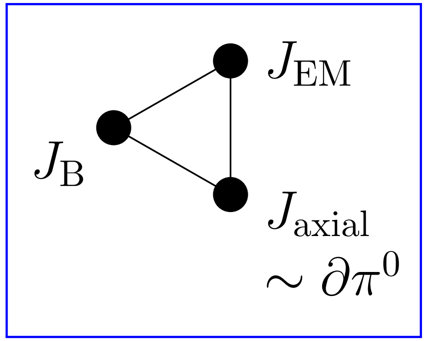
$$B_0 = \frac{3m_\pi^2}{e} \approx 1.0 \times 10^{19}\ \text{G},$$

- Can it become *globally* stable?

Global stability at finite μ_B

- The wall carries energy per unit area: $$\frac{\mathcal{E}}{S} = 8 f_\pi^2 m_\pi .$$
- But, for $\boldsymbol{B} \neq 0$, it also carries baryon number!
- The *gauge invariant* baryon current is given by
(Goldstone-Wilczek, Witten)

$$J_B^\mu = -\frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2}\,\mathrm{tr}\left\{(L_\nu L_\alpha L_\beta) - 3ie\partial_\nu\left[A_\alpha Q(L_\beta + R_\beta)\right]\right\},$$



$\pi^0 \to \gamma\gamma$

where $L_\mu = \Sigma\partial_\mu\Sigma^\dagger$, $R_\mu = \partial_\mu\Sigma^\dagger\Sigma$ and $Q = \tau_3/2 + 1/6$.

- For the wall, $\boldsymbol{\nabla}\pi^0 \neq 0$, in the magnetic field $\boldsymbol{B}$:

$$J_B^0 = \frac{e}{4\pi^2 f_\pi}\boldsymbol{B}\cdot\boldsymbol{\nabla}\pi^0; \quad \Rightarrow \qquad \frac{N_B}{S} = \frac{eB}{2\pi}.$$

- I.e., the wall is stable towards decay into vacuum when $\mu_B > \dfrac{\mathcal{E}}{N_B} = \dfrac{16\pi f_\pi^2 m_\pi}{eB}$.

And if

$$B > B_1 = \frac{16\pi f_\pi^2 m_\pi}{e m_N} \approx 1.1 \times 10^{19}\ \text{G}$$

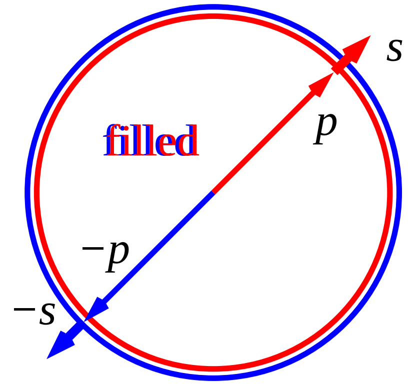
the wall wins over nuclear matter ($\mu_B \approx m_N$) in terms of $\mathcal{E}/N_B$.

- $B_1 \sim m_\pi \gg B_0 \sim m_\pi^2$ and both vanish in the chiral limit.

Large μ and color superconductivity

- Asymptotic freedom $\Rightarrow \alpha_s(\mu) \to 0$.

- Quarks of "different color" (color antisymmetric state) attract. Fermi sphere is unstable towards condensation of quark pairs (Cooper).



- For 2 flavors – 2SC: (Rapp et al, Alford et al, '97)

$$\langle u_R d_R \rangle = \langle u_L d_L \rangle \neq 0$$

— flavor singlet $\Rightarrow$ breaks U(1)$_A$ (not SU(2)$_A$).

- For 3 flavors — CFL: (Alford, Rajagopal, Wilczek)

$$\langle u_R d_R \rangle = \langle d_R s_R \rangle = \langle s_R u_R \rangle = (R \to L) \neq 0$$

— flavor triplet and color triplet

$$\left.\begin{array}{l} \mathrm{SU}(3)_R \times \mathrm{SU}(3)_{\mathrm{color}} \to \mathrm{SU}(3)_{R+\mathrm{color}} \\ \mathrm{SU}(3)_L \times \mathrm{SU}(3)_{\mathrm{color}} \to \mathrm{SU}(3)_{L+\mathrm{color}} \end{array}\right\} \quad \Rightarrow \quad \mathrm{SU}(3)_R \times \mathrm{SU}(3)_L \to \mathrm{SU}(3)_{L+R}$$

Breaks both U(1)$_A$ and SU(3)$_A$.

Domain walls in 2SC and CFL

- Spontaneously broken $U(1)_A$ and SU(3)$_A$ $\Rightarrow$ Goldstone bosons.
  - Note: U(1)$_A$ violation by QCD anomaly is suppressed at large μ.
- In 2SC and CFL: there are neutral axial Goldstone bosons.

The lightest is η in 2SC and $\eta - \eta'$ mixture ($s\bar{s}$) in CFL.

- Domain wall is energetically favorable state when $\mathcal{E}/N_B < \mu$.

$$\mathcal{E} \sim f_\eta^2 m_\eta \sim \mu^2 m_\eta, \quad N_B \sim eB;$$

$$B_c \sim \frac{\mu m_\eta}{e} \sim \underbrace{10^{17}}_{\text{CFL}} - \underbrace{10^{18}}_{\text{2SC}} \text{ G}$$

Magnetism of the wall

- Consider coupling of the Goldstone-Wilczek baryon current to the source $A_\nu = (\mu, \mathbf{0})$:

$$\mathcal{L}_{GW} = -A^B_\nu J^\nu_B = \frac{e}{4\pi^2 f_\pi} \mu\, \boldsymbol{B} \cdot \boldsymbol{\nabla} \pi^0$$

This means the wall is magnetized with magnetization density (Son, Zhitnitsky, ’04)

$$\boldsymbol{M} = \frac{e}{4\pi^2 f_\pi} \mu \boldsymbol{\nabla} \pi^0.$$

- If the wall is generated sponteneously, it will be ferromagnetic.

Goldstone gradient (supercurrent)

- Since it costs energy, how can a nonzero $\boldsymbol{\nabla}\pi$ be spontaneously generated?

- There is a coupling, $\boldsymbol{\nabla}\pi \cdot N^\dagger \gamma_5 \boldsymbol{\gamma} N$, between the Goldstone gradient (axial supercurrent) and nucleon axial current. If $N^\dagger \gamma_5 \boldsymbol{\gamma} N$ was present, this could offset the cost from $f_\pi^2 (\boldsymbol{\nabla}\pi)^2$.

- In vacuum we would have to pay m_N to create the requisite nucleons.

- In CFL quark excitations are also gapped. By Δ.

- But finite m_s lowers the energy cost of exciting a fermion.

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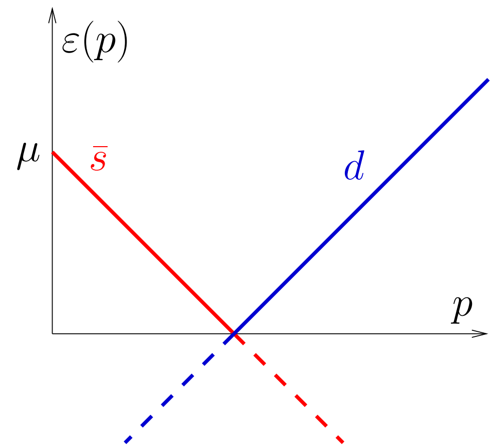

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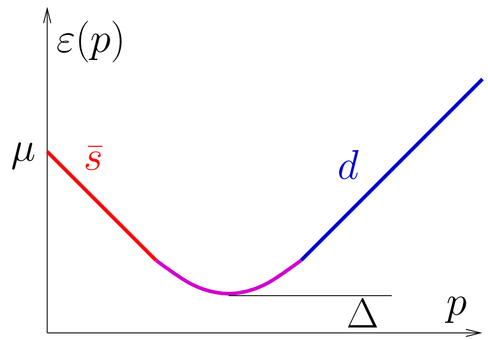

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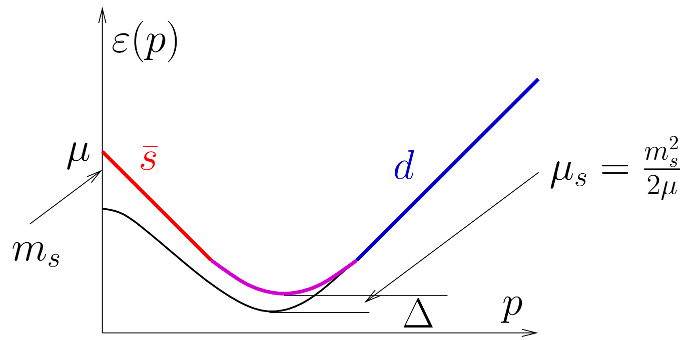

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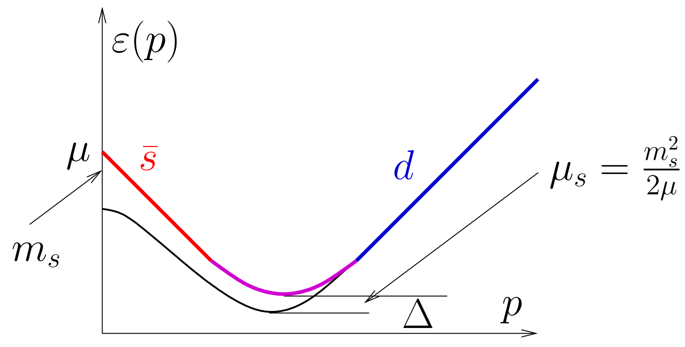
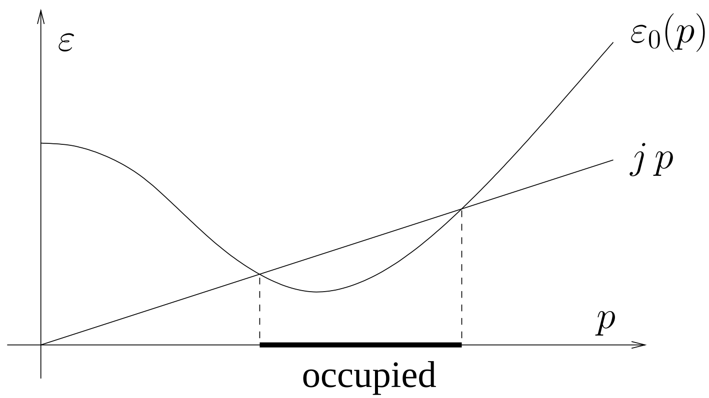


- When the excitation is close to being gapless, one can lower the energy by creating supercurrent $\boldsymbol{j} \sim \boldsymbol{\nabla}\phi$:
  - Modes with

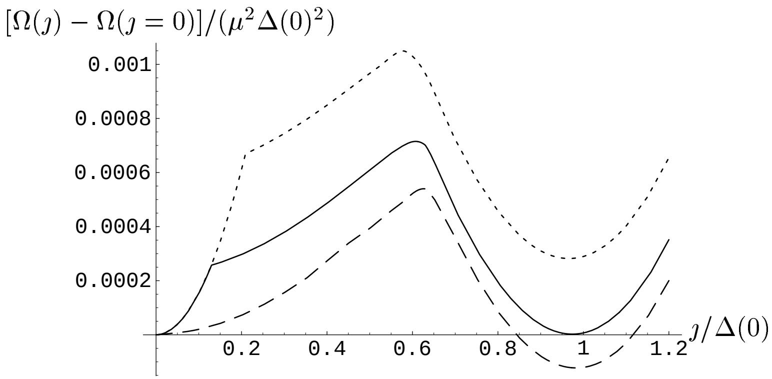
$$\varepsilon(p) = \varepsilon_0(p) - \boldsymbol{j} \cdot \boldsymbol{p} < 0$$



are occupied and contribute negatively to the energy due to the supercurrent–normal current coupling. (In cold atoms: cond-mat/0507586).

Narrow window of μ

- As a function of j, the effective potential develops the second minimum:



from Gerhold-Schafer-Kryjevski hep-ph/0612181

- It is lower than $j = 0$ minimum only in a small interval of μ_s:

$$1.605\Delta < \mu_s < 1.615\Delta,$$

But in a neutron star μ changes and all one needs is this small interval to be present somewhere in the full range of μ from surface to center.

Ferromagnetism of CFL quark matter

- If $\boldsymbol{\nabla}\phi$ is spontaneously generated at finite μ, the term $e\mu\boldsymbol{B}\cdot\boldsymbol{\nabla}\phi$ means there is spontaneous magnetization.

$$\boldsymbol{M} \sim e\mu\boldsymbol{\nabla}\phi$$

- Such a $\boldsymbol{\nabla}\phi \sim \Delta$ does occur in the "Goldstone current" (or "meson supercurrent") state in CFL. I.e., such a state is *ferromagnetic*.

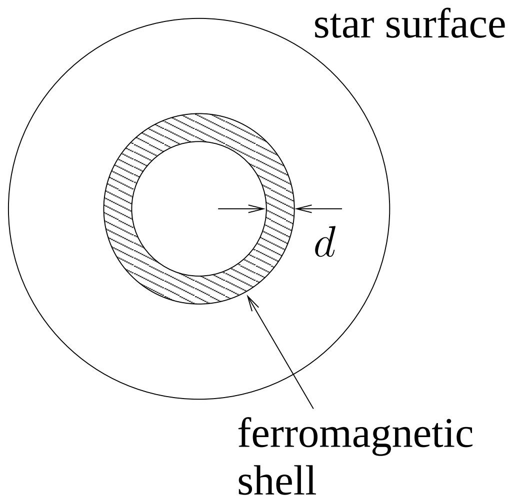
$$M \sim \frac{e}{3\pi^2}\mu\,\Delta \approx 2.4\cdot 10^{16}\ \text{G} \times \left(\frac{\mu}{1.5\ \text{GeV}}\right)\left(\frac{\Delta}{30\ \text{MeV}}\right)$$

- Only a narrow window of μ (Gerhold-Schäfer-Kryjevski):

$$\frac{m_s^2}{2\Delta}(1.615)^{-1} < \frac{\mu}{3} < \frac{m_s^2}{2\Delta}(1.605)^{-1},$$

i.e., a shell in a star of width $d \sim 2\% R$. Then

$$B \sim M\frac{d}{R} \sim 10^{14} - 10^{15}\ \text{G}.$$



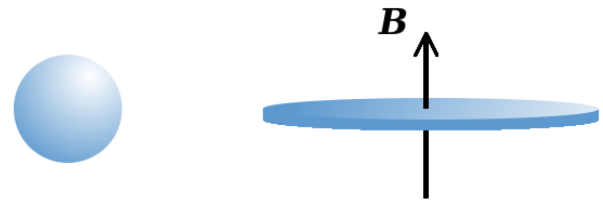
- Could account for the field of a magnetar.

Summary

- At $B_0 = 3m_\pi^2/e$ the π^0 domain wall becomes metastable.

- The wall carries baryon number $N_B/S = eB/(2\pi)$ and competes with the nuclear matter.

The wall wins at $B > B_1 \approx \dfrac{16\pi f_\pi^2 m_\pi}{e m_N} \sim 10^{19}$ G.



- Both B_1 and B_0 vanish in the chiral limit.

- In color superconducting quark matter η/η' domain wall wins for $B \sim 10^{17} - 10^{18}$ G.

- The “meson supercurrent” state in CFL is ferromagnetic and capable of producing $B \sim 10^{14} - 10^{15}$ G in a typical compact star.