

Holographic Chiral Magnetic Conductivity, Chiral Shear Wave and Chiral Magnetic Spiral

Ho-Ung Yee

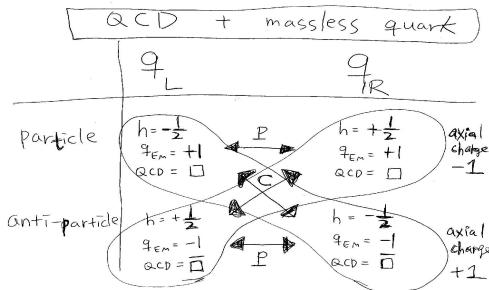
ICTP, Trieste

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BNL, "P and CP odd phenomena in hot and dense matter"

References : [arXiv:0908.4189 \[hep-th\]](#)
[arXiv:0910.5915 \[hep-th\]](#) (with B.Sahoo)
A work in progress with K.-Y.Kim and B.Sahoo

P and C basics for me



- P and C are good symmetries of QCD with massless quarks
- $U(1)_A$ suffers from anomaly in QCD : it is **not** a true symmetry

$$\partial_\mu J_A^\mu \sim \epsilon^{\mu\nu\alpha\beta} \text{Tr} (G_{\mu\nu} G_{\alpha\beta}) \quad (1)$$

One fact : this anomaly is $\frac{1}{N_c^2}$ -suppressed in large N_c limit

- There are also **triangle anomaly** of AVV and AAA : issues when we try to **gauge** these symmetries : **If we work only inside QCD, these are ok**

A table of P and CP

	$q_{EM}(\mu_B)$	μ_A	\vec{B}	\vec{E}	\vec{J}_{EM}	\vec{J}_A	θ_{QCD}	\vec{S}	MDM $\vec{\mu}_M$	EDM $\vec{\mu}_E$
P	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1
CP	-1	-1	-1	+1	+1	+1	-1	+1	-1	+1

	P	CP
$(q_{EM}\vec{S}, \vec{B}, \vec{\mu}_M)$	+1	-1
$(\vec{E}, \vec{J}_{EM}, \vec{\mu}_E)$	-1	+1
(θ_{QCD}, μ_A)	-1	-1
(q_{EM}, μ_B)	+1	-1
\vec{J}_A	+1	+1

Some examples :

- $\vec{\mu}_E = \theta_{QCD} (q_{EM}\vec{S})$: Neutron EDM from θ_{QCD}
- $\vec{J}_A = \mu_B \vec{B}$ (Son,Zhitnitsky,Newman,Metlitski,Gorbar,Miransky,Shovkovy)
- $\vec{J}_{EM} = \mu_A \vec{B}$ (Fukushima,Kharzeev,Warringa)
- $\vec{\mu}_E = \mu_A \vec{B}$ (Ioffe,Smilga,Millo,Faccioli)
- Can we try others ?

Three phenomena we will discuss

All these relations have their roots in [triangle anomaly](#)

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[AdS/CFT or Gauge/Gravity correspondence](#)

- Chiral magnetic conductivity ([Fukushima-Kharzeev-Warringa](#))
- Chiral shear waves
([Matsuo-Sin-Takeuchi-Tsukioka,Sahoo-Yee,Nakamura-Ooguri-Park](#))
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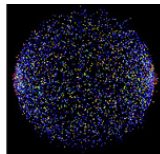
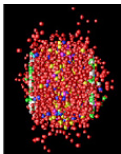
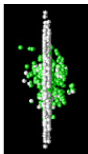
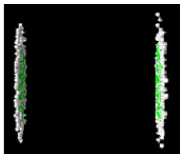
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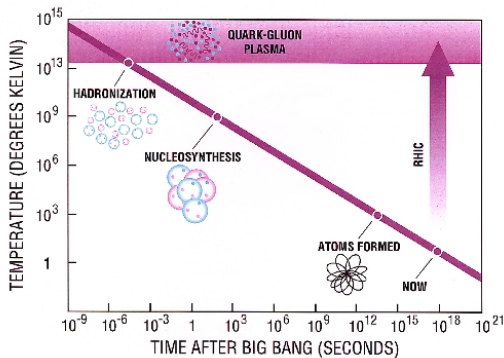
[Note](#) : the last example is for the χ SB phase at a late stage of RHIC

My naive understanding of RHIC...



- Gold-Gold collision with several hundreds of GeV's
- Plasma lasts less than 10^{-23} seconds
- Temperature can reach 10^{12} K

The quark-gluon plasma...



- Some kind of plasma of **quarks** and **gluons**
- Naive perturbative QCD seems to fail in describing it, eg. jet quenching, small $\frac{\eta}{s}$ (D.Teaney), etc
- More like a **strongly coupled hydrodynamic liquid** (E.Shuryak, I.Zahed)

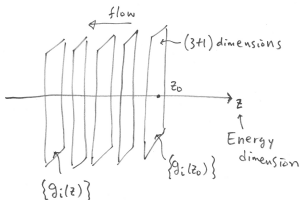
Holography or AdS/CFT approach

Very brief words about AdS/CFT ideas

- We have an **effective theory** for large N_c and strong t'Hooft coupling limit of a gauge theory living in **five dimensions**
- The additional dimension is roughly energy-scale of the theory. Recall RG still works in large N_c limit. 5D theory involves **gravity** :

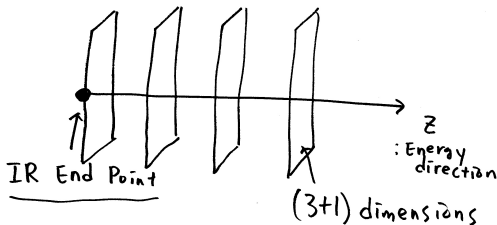
Quantum RG invariance \longleftrightarrow **Classical** general covariance

- Large N_c factorization implies **classical or statistical** description of **master variables** while RG still makes sense : **holography is one way of combining these two**



Finite temperature plasma is described by Black Hole

In the 5D gravity description, finite temperature plasma is described by **Black Hole** or more precisely, **Black Brane**



- There is no rigorous understanding of this
- It has been more like **“empirical”**
- Hawking temperature is identified with plasma temperature naturally

How to study physics in AdS/CFT ?

Just two facts we need from AdS/CFT dictionary

- Global symmetry in gauge theory \longleftrightarrow Gauge symmetry in 5D

How it works?

$$A_\mu(x^\mu, Z) \sim A_\mu^{(0)} + \sum_n f_n(Z) A_\mu^{(n)} \quad (2)$$

where $A_\mu^{(0)}$ is interpreted as a source for the current J^μ in gauge theory, and $A_\mu^{(n)}$ are massive vector mesons that can be created by J^μ

It is a **unified** description of external sources and dynamical excitations

- An important ingredient for our purposes

Triangle anomaly in 4D \longleftrightarrow 5D Chern-Simons term $A \wedge F \wedge F$

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Holographic chiral magnetic conductivity

Chiral magnetic effect:

In the presence of axial charge density, an applied **EM** magnetic field induces a net electromagnetic current along the same direction

- In linear response framework,

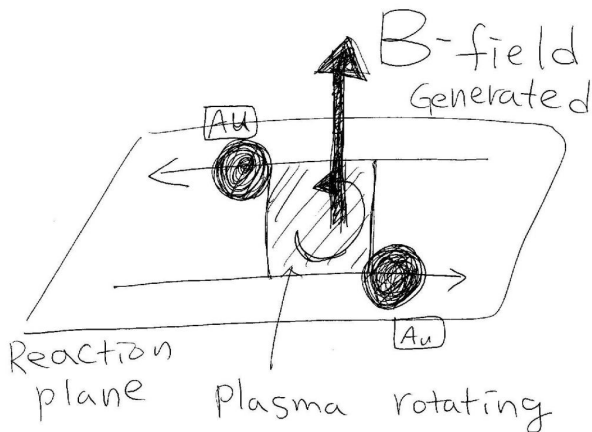
$$j(\omega) = \sigma(\omega)B(\omega) \quad \omega = \text{frequency} \quad (3)$$

and $\sigma(\omega)$ is called **chiral magnetic conductivity**

- It has been estimated that the relevant scales in RHIC experiments justify linear response approximation
- As the RHIC plasma is time-dependent, chiral magnetic conductivity of general ω seems interesting
- In the field theory side, only 1-loop perturbative computation by Kharzeev-Warringa is available
- Let's provide a strong-coupling prediction using AdS/CFT technique
!!! (Rebhan-Schmitt-Stricker, Yee)

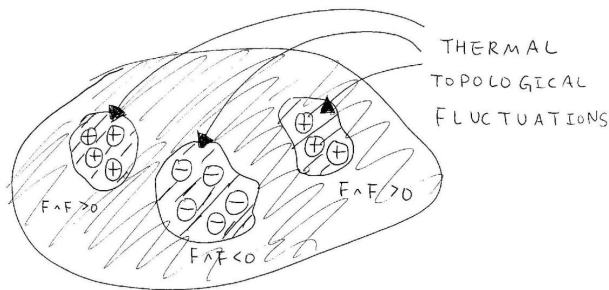
Some pictorial understanding

Some cartoon understanding of how chiral magnetic effects may play a role in RHIC experiments



We need an axial charge ...

WARNING : For these to be effective, we need a **local non zero axial charge density** generated by thermal sphaleron-like fluctuations



Therefore, we need a **CHARGED** AdS black-hole background :

YES, there exists a known **Reisner-Nordstrom** black-hole background we can use !!!

Bottom-line is

As a model computation, let's take the following 5D theory that has all the necessary symmetries $U(1)_B \times U(1)_A$, and importantly, **correct axial anomaly**

$$(16\pi G_5) \mathcal{L} = R + 12 - \frac{1}{2}(F_B)_{MN}(F_B)^{MN} - \frac{1}{2}(F_A)_{MN}(F_A)^{MN} \quad (4)$$
$$- \frac{\kappa}{2\sqrt{-g_5}} \epsilon^{MNPQR} \left(3(A_A)_M (F_B)_{NP} (F_B)_{QR} + (A_A)_M (F_A)_{NP} (F_A)_{QR} \right) ,$$

with $\kappa = -\frac{2G_5}{3\pi} (N_F^{eff} N_c)$.

For an **axial-charged** plasma background, we have the following **charged** black-hole solution we can use

$$ds^2 = -r^2 V(r) dt^2 + 2drdt + r^2 \sum_{i=1}^3 (dx^i)^2 ,$$
$$A_A = \left(\frac{Q}{r_H^2} - \frac{Q}{r^2} \right) dt , \quad A_B = 0 ,$$
$$V(r) = 1 - \frac{m}{r^4} + \frac{2Q^2}{3r^6} \quad (5)$$

Technical details

- First obtain linearized equations of motion for small fluctuations from the plasma background
- EM magnetic field will be encoded as an **external, UV non-normalizable** mode of 5D bulk gauge field A_B
- Imposing physically sensible **in-coming** boundary condition on the black-hole horizon, this EM magnetic field induces a non-trivial profile of A_B in the bulk by equations of motion
- One can see that this is possible only with the CS term
- The induced current can be read off from A_B by applying AdS/CFT dictionary

The upshot is

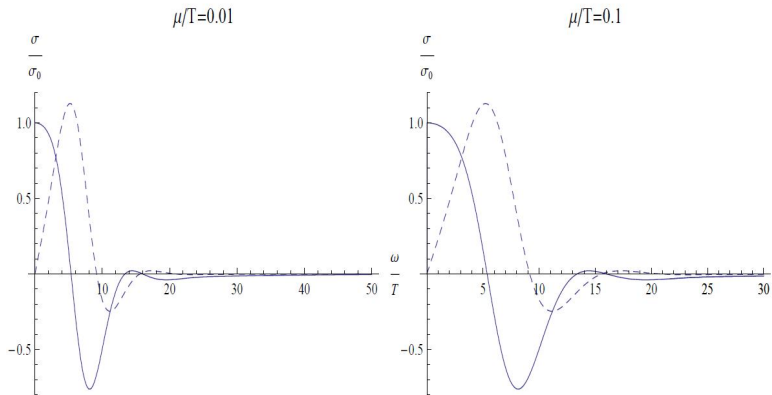
$$\frac{\sigma(\omega)}{\sigma_0} = \frac{2r_H^2}{(f_0(\infty))^2} \int_{r_H}^{\infty} dr' \frac{(f_0(r'))^2}{(r')^3} e^{-2i\omega \int_{\infty}^{r'} \frac{dr''}{(r'')^2 V(r'')}} \quad (6)$$

where f_0 is a solution of

$$\partial_r \left(-i\omega r f_0 + r^3 V(r) \partial_r f_0 \right) - i\omega r (\partial_r f_0) = 0 \quad (7)$$

$$\text{and } \sigma_0 = -\frac{3\kappa Q e^2}{4\pi G_5 r_H^2}$$

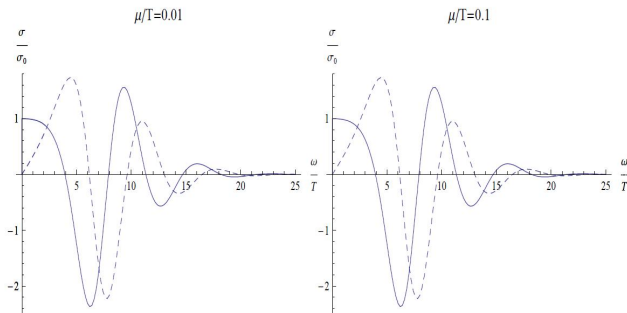
Numerical results



Solid line: $\text{Re}\sigma(\omega)$, Dashed line : $\text{Im}\sigma(\omega)$

In the case of Sakai-Sugimoto model

In the case of [Sakai-Sugimoto model](#), the necessary CS term is already present, and we can compute things more reliably
(Cf. [Rebhan-Schmitt-Stricker](#) also computed it in this model)



Solid line: $\text{Re}\sigma(\omega)$, Dashed line : $\text{Im}\sigma(\omega)$

Issue : canonical or grand canonical ???

WARNING : There is an issue in these computations !!

- Previous plots are computed from a wrong formula for the current missing **additional contribution from CS term** which looks as (Rebhan-Schmitt-Stricker)

$$\Delta j_{EM}^{\mu} = \frac{e^2 N_F^{\text{eff}} N_c}{12\pi^2} \epsilon^{\mu\nu\rho\sigma} ((A_a)_{\nu} (F_{EM})_{\rho\sigma} + (A_{EM})_{\nu} (F_a)_{\rho\sigma}) \quad , \quad (8)$$

- Indeed, a rigorous holographic renormalization in 1004.3541[hep-th] (Sahoo-Yee) confirms this mistake
- Two curious facts about it : this additional term is **frequency-independent** and it depends on **renormalization scheme** such as adding Bardeen counter-term.
- More importantly, this term exists in **grand canonical ensemble** of having external A_0 , but is absent in **canonical ensemble** !!!

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Canonical or grand canonical, continued ...

In AdS/CFT correspondence, the choice of canonical or grand-canonical ensemble is encoded in the profile of 5D A_0 field

$$A_0 = \mu \left(1 - \frac{r_H^2}{r^2} \right) \quad \text{or} \quad A_0 = \mu \left(0 - \frac{r_H^2}{r^2} \right) \quad (9)$$

This choice does not matter at the equations of motion level because only field-strengths enter

$$\nabla_N (F)^{MN} - \frac{3\kappa}{4\sqrt{-g_5}} \epsilon^{MNPQR} (F)_{NP} (F)_{QR} = 0 \quad (10)$$

- Usual argument for preferring grand canonical ensemble by finiteness of $A_M A^M$ at the horizon is in fact **not** correct
- It is **not** a gauge-invariant statement
- The coordinate $ds^2 = -r^2 V(r) dt^2 + \frac{dr^2}{r^2 V(r)} + r^2 (dx^i)^2$ at the horizon $V(r_H) = 0$ covers only the bifurcation point on the horizon
- Physically more sensible coordinate is **Eddington-Finkelstein coordinate** $ds^2 = -r^2 V(r) dt^2 + 2dr dt + r^2 (dx^i)^2$, which covers future horizon correctly
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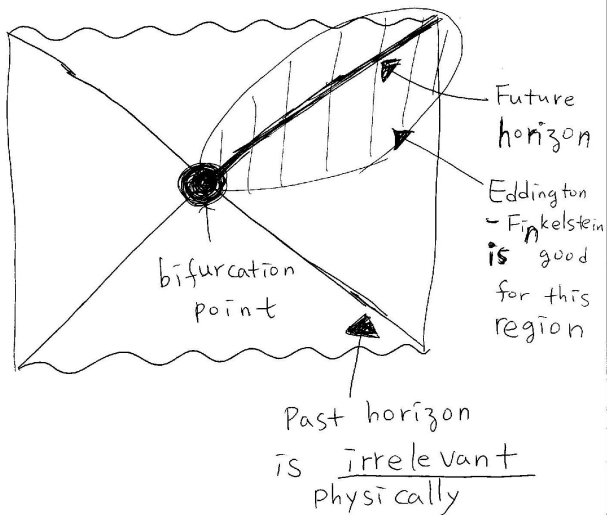
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Penrose diagram of AdS black-hole



Canonical or grand canonical, continued again...

If one works in **canonical** ensemble, $A_0 = \mu \left(0 - \frac{r_H^2}{r^2}\right)$, this ambiguity of additional contribution to the current is simply absent, and the result is insensitive to renormalization scheme choice such as adding Bardeen counter-term

The main point is that

- For **anomalous** U(1) symmetry, holographic computations give **different** results for canonical ensemble and grand canonical ensemble
- We propose that this may be in fact true in the field theory side too. Grand canonical ensemble is computing $\text{Tr} (e^{-\beta H - \mu N})$, but for anomalous U(1),

$$[H, N] \neq 0 \quad (11)$$

so that there is an ordering ambiguity in **defining** $e^{-\beta H - \mu N}$ itself !!!
This parallels to the non-gauge invariance of 5D Chern-Simons term

- Question is : For anomalous U(1), which is physically more correct, canonical or grand canonical ???
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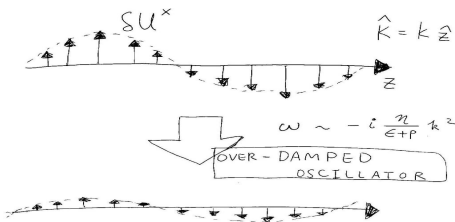
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Our next subject : Chiral shear waves

Some pictorial **cartoon** of **damping shear waves**



- **Shear modes** are waves of transverse velocity field δu^i where $i = x$ or $i = y$ when the wave-vector is $\vec{k} = k\hat{z}$
- It is **not** a propagating mode, but exponentially damping
- From the constitutive relation of $T^{\mu\nu}$, the **leading order** dispersion relation starts as

$$\omega \approx -\frac{i\eta}{\epsilon + p}k^2 + \dots \quad (12)$$

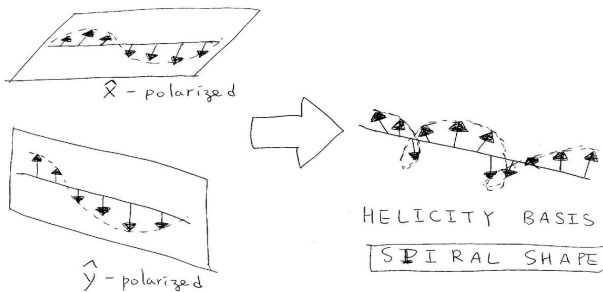
where η is the shear viscosity

Why anomaly affects this and how ?

Q : How come anomaly could affect these transverse shear modes ?

The basic reason : In the **charged** plasma, transverse velocity fluctuations δu^i induce **charge current flow** too, i.e. they are coupled with each other dynamically

Net effect : δu^x and δu^y mix with each other, and the correct eigenmodes are **definite helicity modes**



The two opposite helicity modes turn out to have different **sub-leading** dispersion relations from **anomaly**

AdS/CFT computation of chiral shear waves

- As a model theory, we take

$$(16\pi G_5)\mathcal{L} = R + 12 - \frac{1}{4}F_{MN}F^{MN} - \frac{\kappa}{4\sqrt{-g_5}}\epsilon^{MNPQR}A_MF_{NP}F_{QR} \quad , \quad (13)$$

and consider a charged black-hole plasma background

- Study linear fluctuations, especially helicity ± 1 transverse shear modes. One finds that Chern-Simons effect (anomaly effect) appears only on these modes
- From analyzing these modes in AdS/CFT, one can obtain their dispersion relations. The result depends on **the sign of helicity**, which maybe called **chiral shear waves**
- Our result is

$$\omega \approx -i\frac{\eta}{\epsilon + p}k^2 \pm i\frac{\kappa Q^3}{8m^2r_H^3}k^3 + \mathcal{O}(k^4) \quad : \quad \text{helicity } \pm 1 \quad , \quad (14)$$

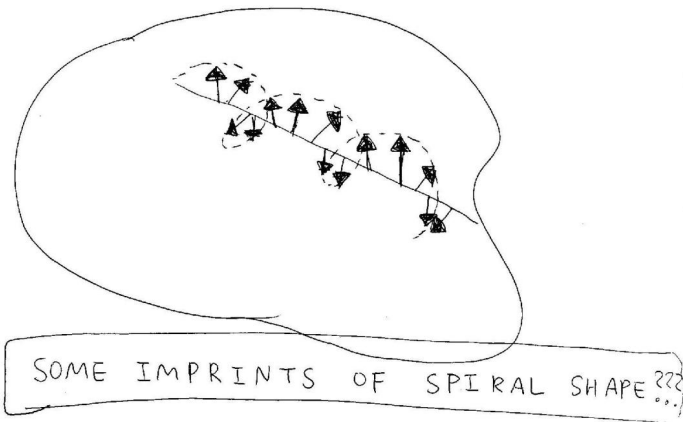
where $\mathcal{O}(k^3)$ is the chiral term from anomaly. In fact, any term with **odd** powers of k comes from anomaly

- Nakamura-Ooguri-Park observed that for a sufficiently large κ , the anomaly-induced term can be big enough to overcome the leading piece, to induce an **instability** toward forming chiral shear waves

How to observe experimentally ?

- One helicity mode has a larger imaginary frequency than the other, so it would decay faster than the other
- After some time, only one helicity modes should dominate.

It will look like a SPIRAL SHAPE transverse fluctuation



The last subject : Chiral magnetic spirals

- Consider the situation of magnetic field with an axial/baryonic chemical potential as in chiral magnetic effects, but in a $T = 0$ χ SB vacuum rather than a deconfined plasma
- The situation with purely baryonic case was studied by Son-Stephanov. Note that if $\mu_B \ll M_N \sim 1\text{GeV}$, real baryons can not be created, and possibly physics of μ_B could be empty
- But, triangle anomaly helps !!! In χ SB phase, chiral effective theory in terms of pion field $U = e^{\frac{2i}{f_\pi} \pi}$ is the right language for low energy physics. Triangle anomaly is encoded in WZW-term, from which the baryon number current which is both gauge invariant and conserved is found by Witten

$$\begin{aligned} B^\mu &= \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{tr} \left(U^{-1} \partial_\nu U U^{-1} \partial_\alpha U U^{-1} \partial_\beta U \right) \\ &\quad - \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu \left[3A_\alpha^{EM} \text{tr} \left(Q \left(U^{-1} \partial_\beta U + \partial_\beta U U^{-1} \right) \right) \right] \end{aligned}$$

Note the additional term to the first topological Skyrmion number.

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- In fact, this can be re-derived in holographic QCD very easily from

$$B^\mu = \frac{1}{8\pi^2} \int dZ \epsilon^{\mu\nu\alpha\beta} \text{tr} (F_{\nu\alpha} F_{\beta Z})$$

with (Hong-Lee-Park-Yee)

$$A_\mu(x, Z) = [(U^{-1}QU)\psi_+(Z) + Q\psi_-(Z)]A_\mu^{EM} + \psi_+(Z)U^{-1}\partial_\mu U + \text{higher modes}$$

Note that the above is automatically gauge-invariant and conserved !!

- This additional term

$$\frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu \left[3A_\alpha^{EM} \text{tr} \left(Q \left(U^{-1} \partial_\beta U + \partial_\beta U U^{-1} \right) \right) \right] \quad (15)$$

can give a net baryon number with $F_{12} = B$, $A_0 = \mu_B$ and pion gradient $(\partial_3 U)U^{-1} \sim \partial_3 \pi^0 \neq 0$

- It has been argued that this phase is indeed favorable compared to other phases. This phase is called **chiral spiral phase** because phase of U is rotating along x^3
- Pion gradient can be thought of as **axial current**, so it is indeed similar to current generation by anomaly, but in a χ SB phase.
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Issue again : Non-gauge invariance of 5D Chern-Simons term

Interestingly, there is again an ambiguity in these works in identifying baryon chemical potential μ_B from the solution !!!

$$\begin{aligned} F_{0Z} &= \frac{1}{1+Z^2} \left(C_B \sinh \left[\tilde{B} \tan^{-1}(Z) \right] \right) , \\ F_{3Z} &= -\frac{1}{1+Z^2} \left(C_B \cosh \left[\tilde{B} \tan^{-1}(Z) \right] \right) \end{aligned} \quad (16)$$

- The constant C_B is presumably related to the baryon chemical potential μ_B . But, the precise identification by the three groups differ from each other
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A proposal of chiral magnetic spirals

It is easy to think of a situation with **axial chemical potential** as well, in addition to baryonic chemical potential.

Basar-Dunne-Kharzeev's proposal :

The true ground state is not a homogeneous phase as before, but the spiral waves of **transverse axial/baryonic currents** may be formed in the true ground state

- We will hear details of the proposal from prof.Dunne shortly
- The idea is effective dimensional reduction to $(1+1)$ -dimensions in the presence of strong magnetic field
- It will be very convincing to have a **strong-coupling proof** from the holographic QCD, especially Sakai-Sugimoto model
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Basic idea

It is not difficult to extend previous works to include axial chemical potential μ_A in the holographic QCD solutions which are **homogeneous**

$$\begin{aligned} F_{0Z} &= \frac{1}{1+Z^2} \left(C_A \cosh \left[\tilde{B} \tan^{-1}(Z) \right] + C_B \sinh \left[\tilde{B} \tan^{-1}(Z) \right] \right) , \\ F_{3Z} &= -\frac{1}{1+Z^2} \left(C_B \cosh \left[\tilde{B} \tan^{-1}(Z) \right] + C_A \sinh \left[\tilde{B} \tan^{-1}(Z) \right] \right) \end{aligned} \quad (17)$$

Although the relation between C_B and μ_B is **not** crystal-clear, the relation between C_A and μ_A is **crystal-clear** :

$$\begin{aligned} \mu_A &= -\frac{1}{2} (\mu_L - \mu_R) = -\frac{1}{2} (A_0(+\infty) - A_0(-\infty)) = -\frac{1}{2} \int dZ F_{Z0} \\ &= \left[\frac{\sinh \left(\frac{\pi}{2} \tilde{B} \right)}{\tilde{B}} \right] C_A \end{aligned} \quad (18)$$

Therefore, to be conservative we will work in the parameter space of (C_B, μ_A) , instead of (μ_B, μ_A)

BASIC IDEA :

Study linearized instability toward chiral magnetic spirals from the above **homogeneous** solutions

Synopsis of analysis

- It is straightforward to obtain the equation of motion for linearized fluctuations of **transverse currents**
- In holographic QCD, they correspond to transverse modes of 5D gauge fields (A_1, A_2)
- 5D Chern-Simons term in fact mixes them together, so the natural basis is **helicity basis** $A^{(\pm)} = A_1 \pm iA_2$
- Study modes with frequency ω and **real** momentum p along x^3 by assuming $e^{-i\omega t + ipx^3}$. Given a **real number** p , the problem is an eigenvalue problem for ω
- If ω becomes a **complex** number with non-zero imaginary part for a range of p ,

it signals the tachyonic instability toward forming chiral magnetic spirals

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MASTER EQUATION

$$\left(1 + Z^2\right)^{-\frac{1}{3}} \left(\omega^2 - p^2\right) A^{(\pm)} + M_{KK}^2 \partial_Z \left[\left(1 + Z^2\right) \partial_Z A^{(\pm)} \right] \\ \pm \frac{N_c}{8\pi^2 \kappa} (p F_{0Z} + \omega F_{3Z}) A^{(\pm)} = 0$$

- (F_{0Z}, F_{3Z}) are previous background solutions, and (C_B, C_A) enter here
- Solve eigenvalue problem for ω given (C_B, C_A, p) , and search for a region where ω becomes **complex**
- Numerical analysis is necessary for precise results

Rough estimate of the phase diagram

Preliminary rough estimate of the phase diagram seems possible

- One can replace $M_{KK}^2 \partial_Z \left[(1 + Z^2) \partial_Z A^{(\pm)} \right]$ with $-\lambda^2 A^{(\pm)}$ with $\lambda^2 \geq \lambda_{min}^2 = \text{mass}^2$ of lowest vector meson
- Near IR region of $Z \sim 0$, one can replace $F_{0Z} \sim C_A$ and $F_{3Z} \sim -C_B$ because $\cosh \left[\tilde{B} \tan^{-1}(Z) \right] \sim 1$ and $\sinh \left[\tilde{B} \tan^{-1}(Z) \right] \sim 0$
- The eigenvalue problem becomes algebraic in this approximation

$$\left(\omega^2 - p^2 \right) - \lambda^2 \pm \frac{N_c}{8\pi^2 \kappa} (C_A p - C_B \omega) = 0$$

or

$$\left(\omega \mp \frac{N_c C_B}{16\pi^2 \kappa} \right)^2 = \left(p \mp \frac{N_c C_A}{16\pi^2 \kappa} \right)^2 - \left(\frac{N_c}{16\pi^2 \kappa} \right)^2 (C_A^2 - C_B^2) + \lambda^2$$

- **Complex** ω happens for a range of p if

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$$\left(\omega^2 - p^2 \right) - \lambda^2 \pm \frac{N_c}{8\pi^2 \kappa} (C_A p - C_B \omega) = 0$$

or

$$\left(\omega \mp \frac{N_c C_B}{16\pi^2 \kappa} \right)^2 = \left(p \mp \frac{N_c C_A}{16\pi^2 \kappa} \right)^2 - \left(\frac{N_c}{16\pi^2 \kappa} \right)^2 (C_A^2 - C_B^2) + \lambda^2$$

- **Complex** ω happens for a range of p if

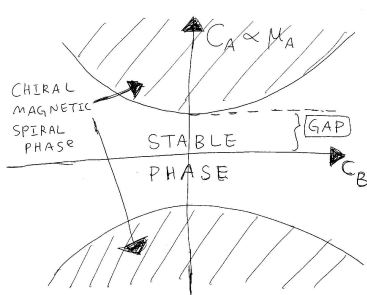
$$- \left(\frac{N_c}{16\pi^2 \kappa} \right)^2 (C_A^2 - C_B^2) + \lambda^2 < 0$$

Rough phase diagram for chiral magnetic spirals

Chiral magnetic spirals would exist in the region

$$C_A^2 - C_B^2 > \left(\frac{16\pi^2 \kappa}{N_c} \right)^2 \lambda_{\text{eff}}^2$$

with some finite number λ_{eff}^2



- Pure baryonic branch seems **stable** against chiral magnetic spirals
- Pure axionic branch seems to develop chiral magnetic spirals **above a finite critical gap**

Thank you very much

Thank you very much for listening