# Issue : canonical or grand canonical ???

**WARNING :** There is an issue in these computations !!

- Previous plots are computed from a wrong formula for the current missing additional contribution from CS term which looks as (Rebhan-Schmitt-Stricker)

$$\Delta j^{\mu}_{EM} = \frac{e^2 N_F^{eff} N_c}{12\pi^2} \epsilon^{\mu\nu\rho\sigma} \left( (A_a)_\nu (F_{EM})_{\rho\sigma} + (A_{EM})_\nu (F_a)_{\rho\sigma} \right) \quad , \tag{8}$$

- Indeed, a rigorous holographic renormalization in 1004.3541[hep-th] (Sahoo-Yee) confirms this mistake
- Two curious facts about it : this additional term is frequency-independent and it depends on renormalization scheme such as adding Bardeen counter-term.
- More importantly, this term exists in grand canonical ensemble of having external $A_0$, but is absent in canonical ensemble !!!

# Canonical or grand canonical, continued ...

In AdS/CFT correspondence, the choice of canonical or grand-canonical ensemble is encoded in the profile of 5D $A_0$ field

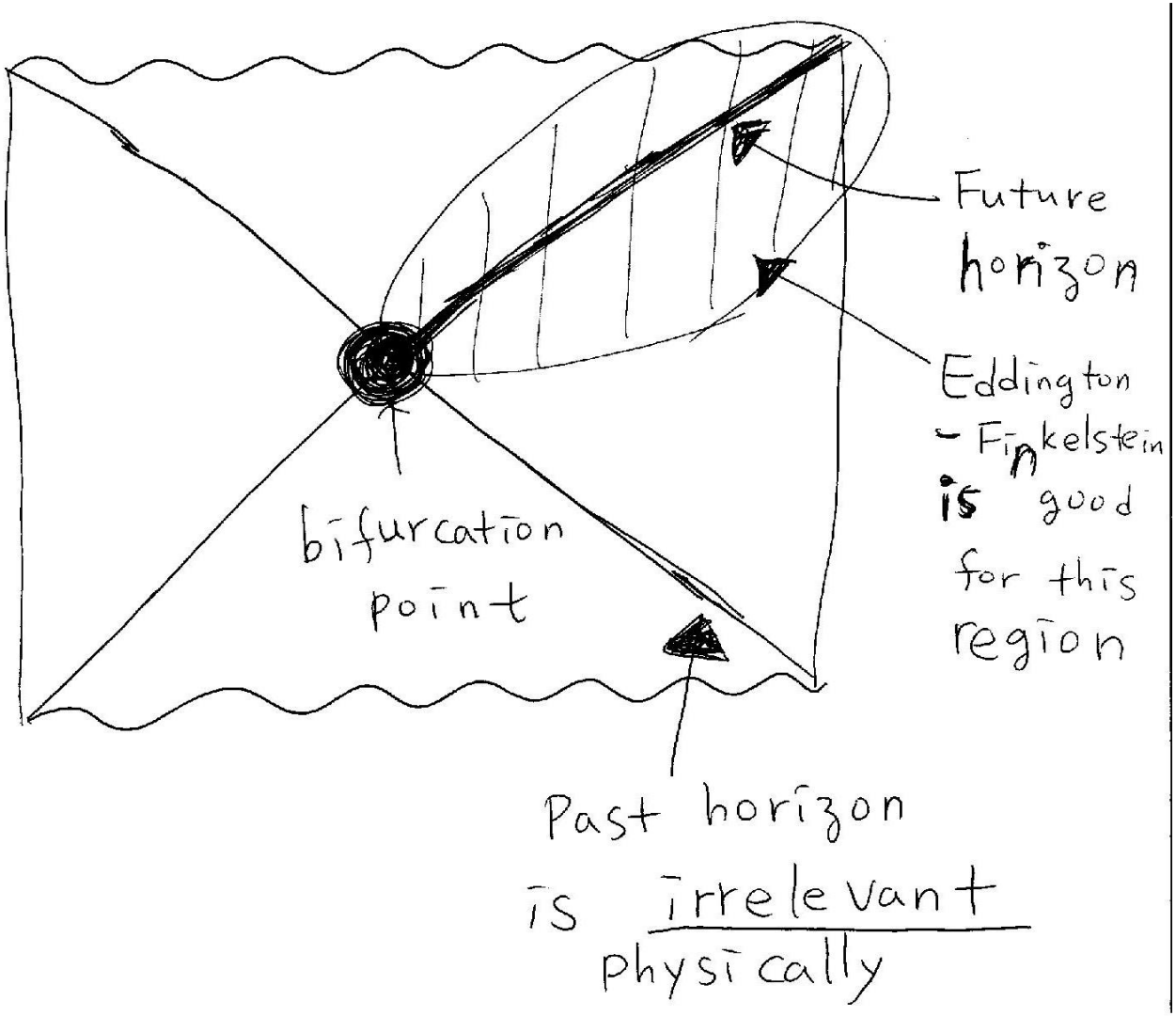
$$A_0 = \mu\left(1 - \frac{r_H^2}{r^2}\right) \quad \text{or} \quad A_0 = \mu\left(0 - \frac{r_H^2}{r^2}\right) \tag{9}$$

**This choice does not matter at the equations of motion level because only field-strengths enter**

$$\nabla_N (F)^{MN} - \frac{3\kappa}{4\sqrt{-g_5}}\epsilon^{MNPQR}(F)_{NP}(F)_{QR} = 0 \tag{10}$$

- Usual argument for preferring grand canonical ensemble by finiteness of $A_M A^M$ at the horizon is in fact not correct
- It is not a gauge-invariant statement
- The coordinate $ds^2 = -r^2 V(r) dt^2 + \frac{dr^2}{r^2 V(r)} + r^2 (dx^i)^2$ at the horizon $V(r_H) = 0$ covers only the bifurcation point on the horizon
- Physically more sensible coordinate is Eddington-Finkelstein coordinate $ds^2 = -r^2 V(r) dt^2 + 2drdt + r^2 (dx^i)^2$, which covers future horizon correctly
- In this coordinate, $A_M A^M$ for both canonical and grand canonical ensembles are finite, and well-behaved

# Penrose diagram of AdS black-hole



# Canonical or grand canonical, continued again...

**If one works in canonical ensemble, $A_0 = \mu\left(0 - \frac{r_H^2}{r^2}\right)$, this ambiguity of additional contribution to the current is simply absent, and the result is insensitive to renormalization scheme choice such as adding Bardeen counter-term**

The main point is that

- For anomalous U(1) symmetry, holographic computations give different results for canonical ensemble and grand canonical ensemble
- We propose that this may be in fact true in the field theory side too. Grand canonical ensemble is computing Tr $\left(e^{-\beta H - \mu N}\right)$, but for anomalous U(1),

$$[H, N] \neq 0 \tag{11}$$

  so that there is an ordering ambiguity in defining $e^{-\beta H - \mu N}$ itself !!! **This parallels to the non-gauge invariance of 5D Chern-Simons term**
- Question is : For anomalous U(1), which is physically more correct, canonical or grand canonical ???
- In real plasma, there is no $A_0$ really exists, and we propose that canonical ensemble maybe physically more correct