EoS with stout-link improved staggered fermions

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Thermodynamics with 2+1 flavor staggered fermions with stout link

- EoS (Equation of State)
  - Y.A., Z. Fodor, S. Katz, K. Szabo, JHEP 01(2006)089 [hep-lat/0510084].
  - N_t=4, 6. LCP1.
- Order of the transition
  - Y.A., G. Endrodi, Z. Fodor, S. Katz, K. Szabo, Nature 443 (2006) 675 [hep-lat/0611014].
  - N_t=4, 6, 8, 10. LCP2.
- T_c
  - Y.A., Z. Fodor, S. Katz, K. Szabo, PLB643 (2006) 46 [hep-lat/0609068].
  - N_t=4, 6, 8, 10. LCP2.

Outline

- EoS on the lattice
  - procedure
  - possible errors
- Staggered fermions with stout link improvement
  - what's good ?
- EoS results
- LCP discussion
  - LCP1, LCP2 with T=0 continuum spectrum
- Summary and Outlook

EoS procedure

- Integral (parameters $\beta,\ m_u,\ m_s$) along LCP $[\beta = 6/g^2]$

$$\frac{p}{T^4}\Big|_{(\beta_0,m_0)}^{(\beta,m)} = -\frac{f}{T^4}\Big|_{(\beta_0,m_0)}^{(\beta,m)} = -N_t^4 \int_{(\beta_0,m_0)}^{(\beta,m)} d(\beta, m_u, m_s) \begin{pmatrix} \langle -\mathrm{S_g}/\beta \rangle \\ 2\langle \bar{\psi}\psi_u \rangle \\ \langle \bar{\psi}\psi_s \rangle \end{pmatrix}$$

$$\frac{p}{T^4}\Big|^{(\beta,m)} = \frac{p}{T^4}\Big|_{(\beta_0,m_0)}^{(\beta,m)} (T \neq 0) - \frac{p}{T^4}\Big|_{(\beta_0,m_0)}^{(\beta,m)} (T = 0)$$

- where

$$\langle -S_g/\beta \rangle(\beta_0, m_0, T \neq 0) = \langle -S_g/\beta \rangle(\beta_0, m_0, T = 0)$$
$$\langle \bar{\psi}\psi \rangle(\beta_0, m_0, T \neq 0) = \langle \bar{\psi}\psi \rangle(\beta_0, m_0, T = 0)$$
$$N_t(T \neq 0) \ll N_t(T = 0)$$ temporal lattice size

- integral does not depend on the path, but just end point
- finite lattice spacing error does not depend on the path
- entropy, energy density obtained through thermodynamic eqs.

finite lattice spacing errors

- T=0 (staggered, improved Wilson, domain wall fermion)

$$X^{latt} = X^{cont} + c_a \cdot (\Lambda a)^2 + c_m \cdot (ma)^2 + \cdots$$

$$\Lambda, m \ll 1/a$$ to control errors

- T≠0 simulation, typically smaller 1/a

  1/a~0.7 GeV at $T_c(N_t=4)$

  - Λ~0.2 GeV, m_s~0.1 GeV, m_c~1.3 GeV

- another T≠0 error at fixed N_t

$$(Ta)^2 = 1/N_t^2$$

- all together at fixed m, T:

$$X^{latt} = X^{cont} + c \cdot a^2 + \cdots$$

- stout link staggered fermion will dramatically reduce c_a, c_m

discretization errors in staggered fermions

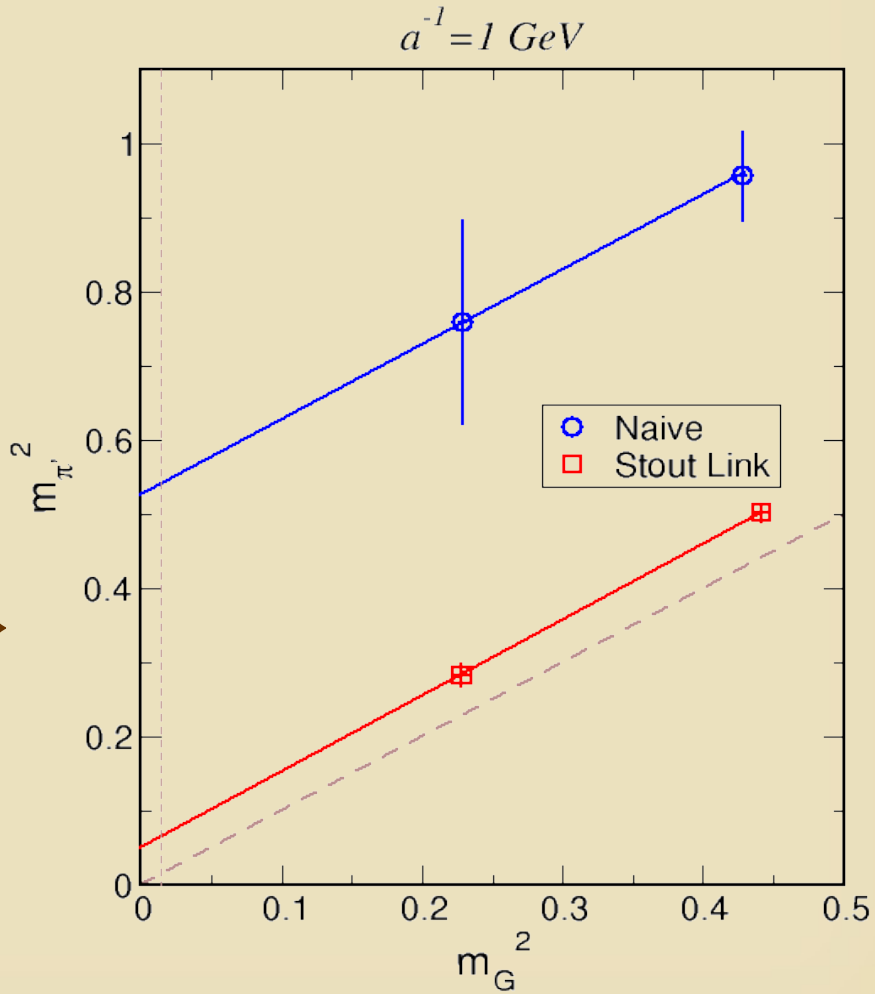
- due to taste symmetry breaking:
  - mostly related with bond state energies

    → affects low to intermediate T
  - reducing this will important to control non-locality problem of "rooted" staggered fermions.
- due to rotational symmetry breaking:
  - mostly related with dispersion relation

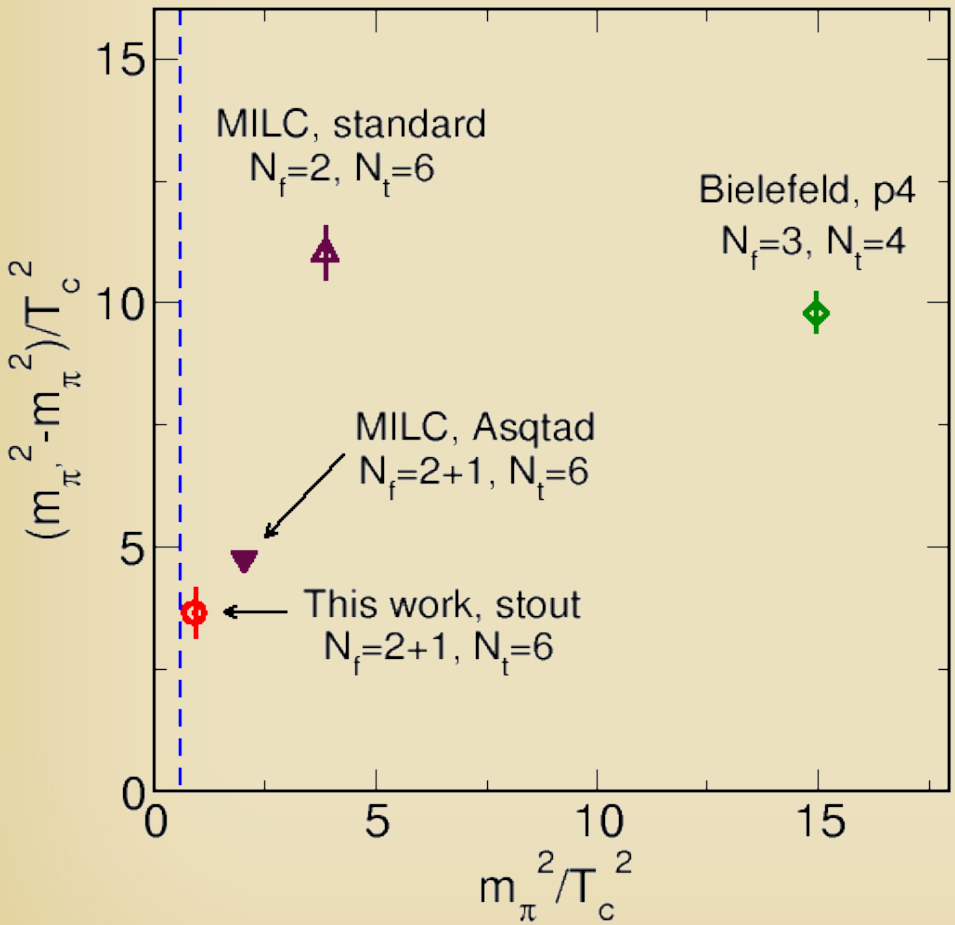
    → affects intermediate to high T

stout improves taste symmetry

- Taste symmetry breaking:
  - due to local fluctuation of the gauge field
  - reduced by stout-link smearing (Morningstar & Peardon)
  - test with quench (2 step)→
- stout link can be combined with the exact algorithm



a little outdated propaganda figure...



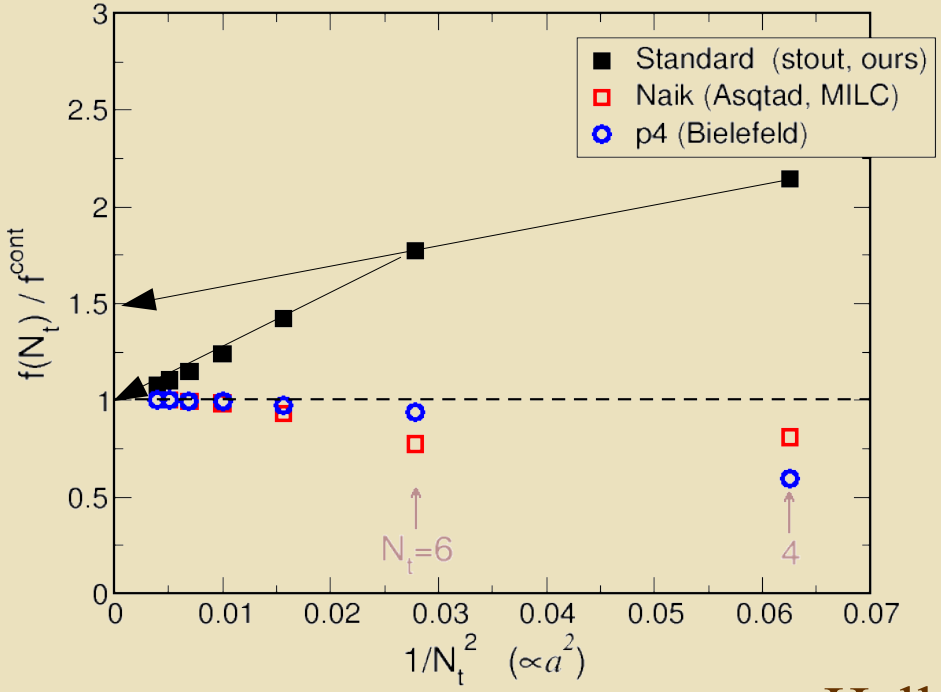
$$\Delta'_\pi = \frac{m^2_{\pi'} - m^2_\pi}{T_c^2} = (m^2_{\pi'} - m^2_\pi) N_t^2$$

$(T = 0$ masses, measured at $\beta_c(N_t)$

- Now other groups have N_t=8.

rotational symmetry

- our Symanzik improved gauge action → improves gauge sector
- stout link fermion → does not improve free energy at T=∞, but



Heller, Karsch, Sturm

- N_t=4→6, some higher oder effect, but N_t=6→8, $1/N_t^2$ very good
- a^2 extrapolation good for N_t=6→8.

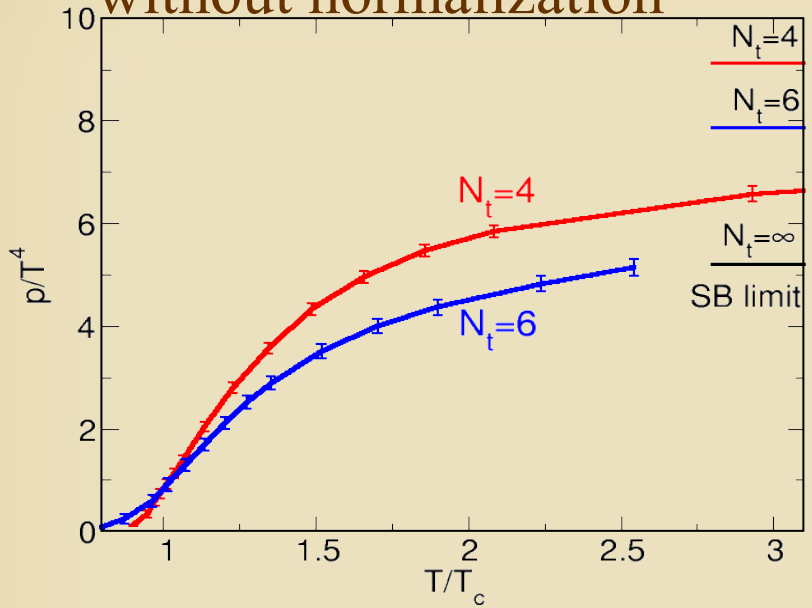
simulation parameters, technical notes

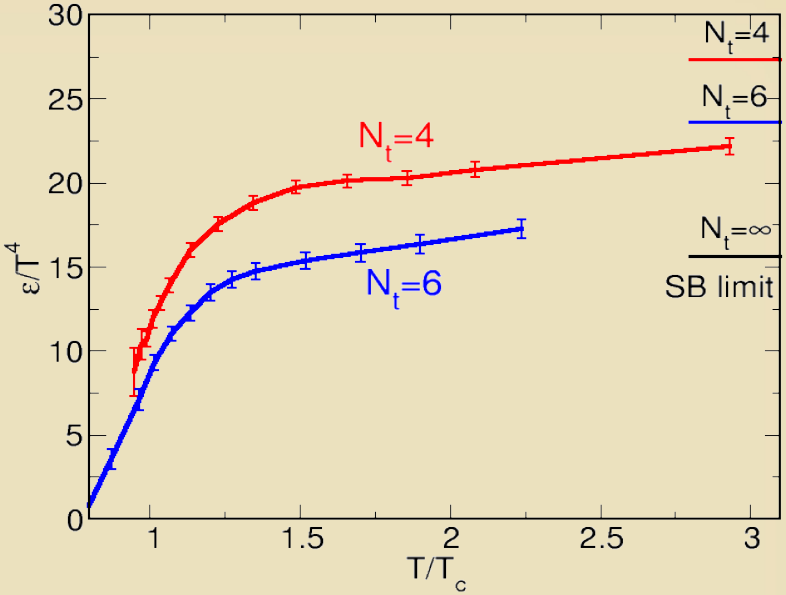
- N_t=4, 6
- β: 14-16 pts for each N_t
- m_s is fixed to physical (LCP1)
- T≠0:
  - $m_u=m_u^{phys}$
  - $N_s=3N_t$
- T=0:
  - $m_u=\{3,5,7,9\}\bullet m_u^{phys}$
  - $L_s m_\pi>3$
- finite size effect:
  - less than stat error for several β's checked
- Rational Hybrid Monte Carlo algorithm for fermions

  = exact algorithm
- stout link helps speeding up the Dirac operator inversion
- computational effort O(1/10) than asqtad, p4

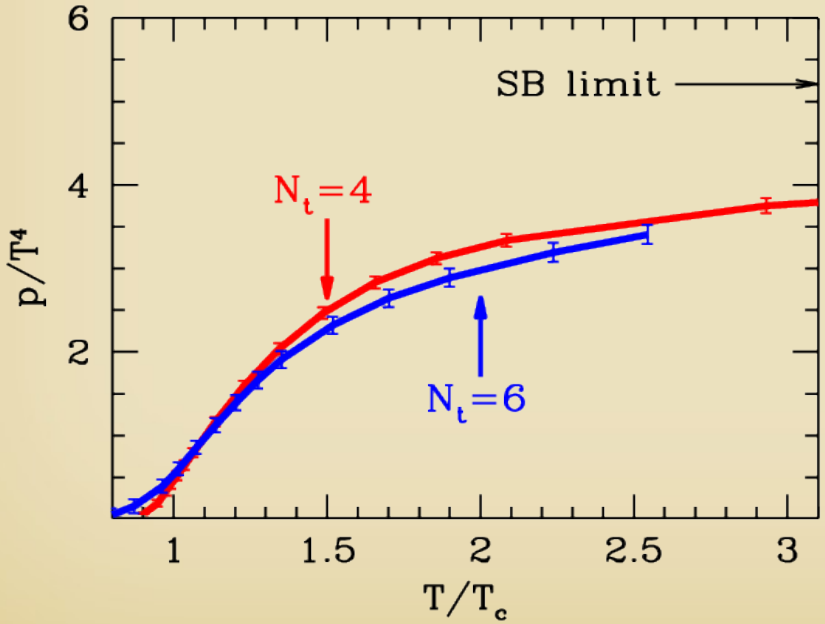
Equation of State

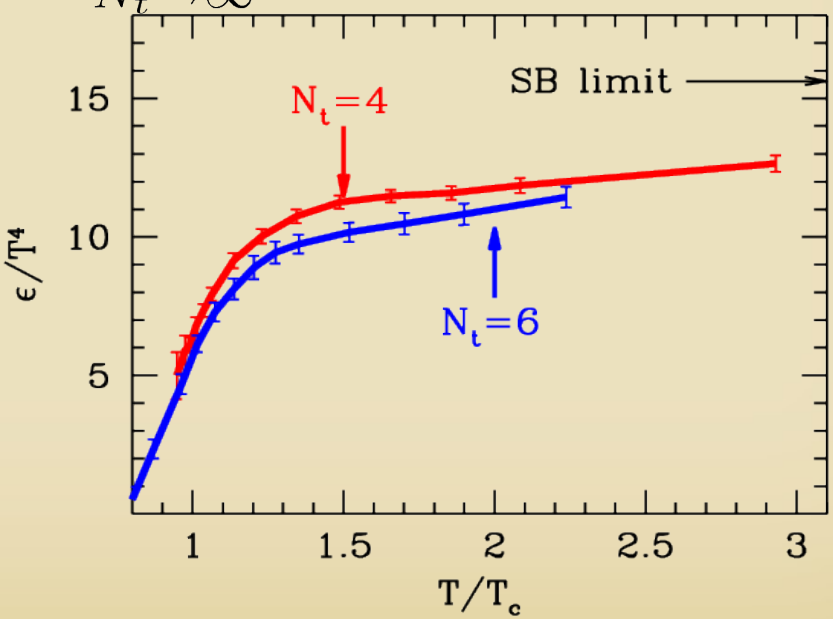
- without normalization





- multiplied with c_{SB}/c_{N_t} $(c_{SB} = \lim_{N_t \to \infty} c_{N_t})$, $N_t = 4,\ 6$





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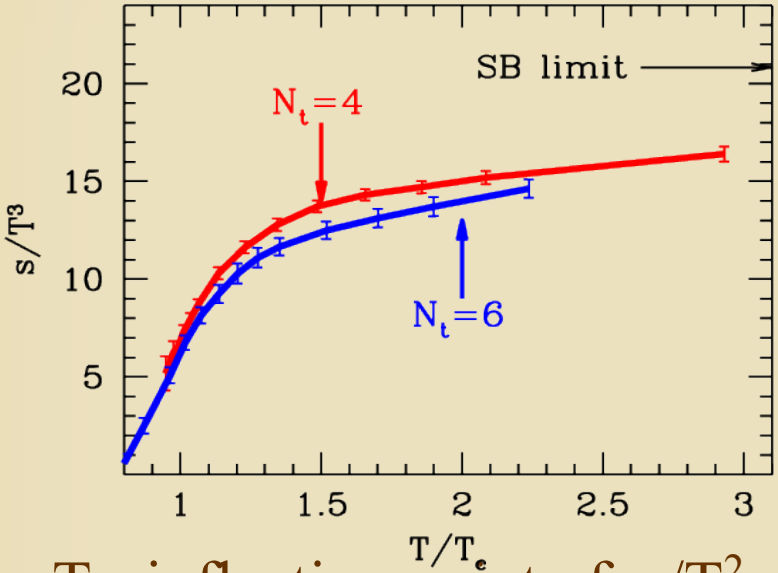
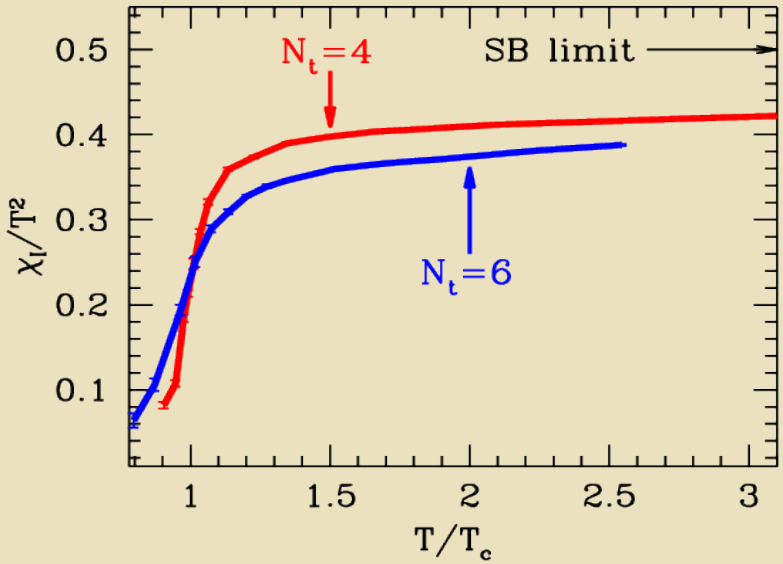
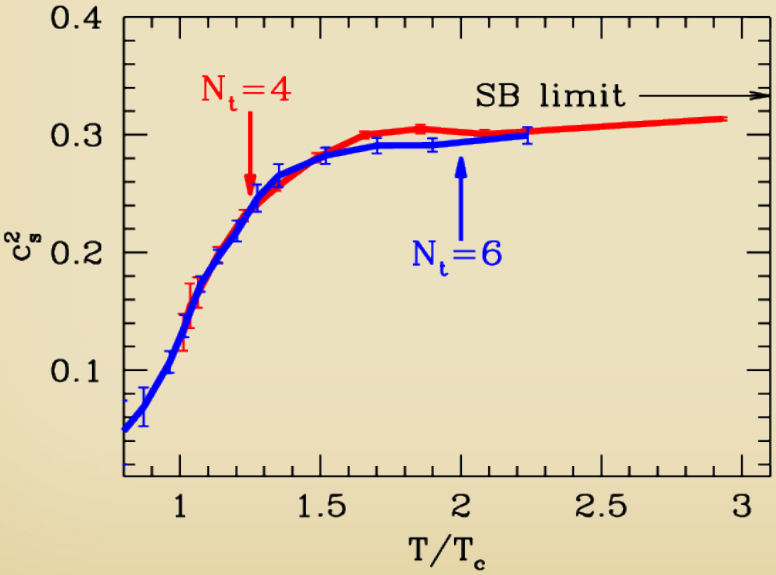
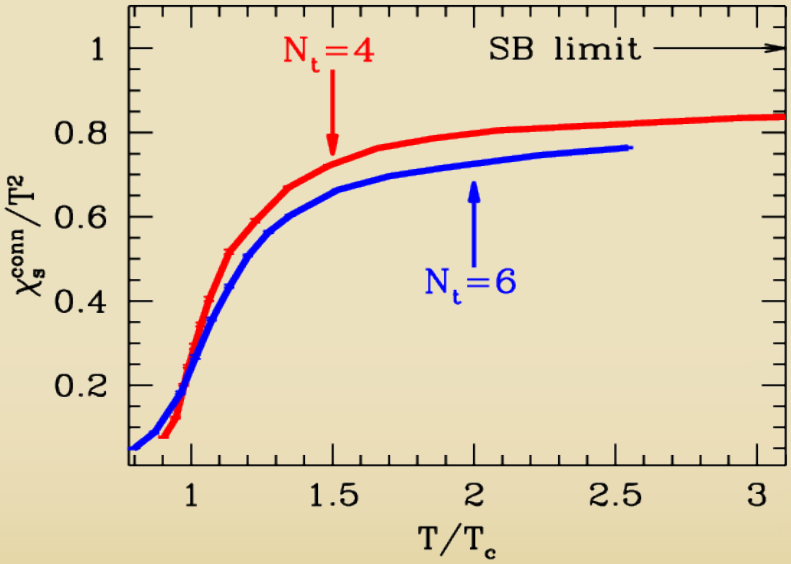




- T_c: inflection point of χ_l/T^2





Line of Constant Physics [LCP1]

- obtain $m_s(\beta)$

  with degenerate 3f simulations:

$$\frac{m_{PS}^2}{m_V^2}(m = m_s) = \frac{m_{\eta_s}^2}{m_\phi^2}$$

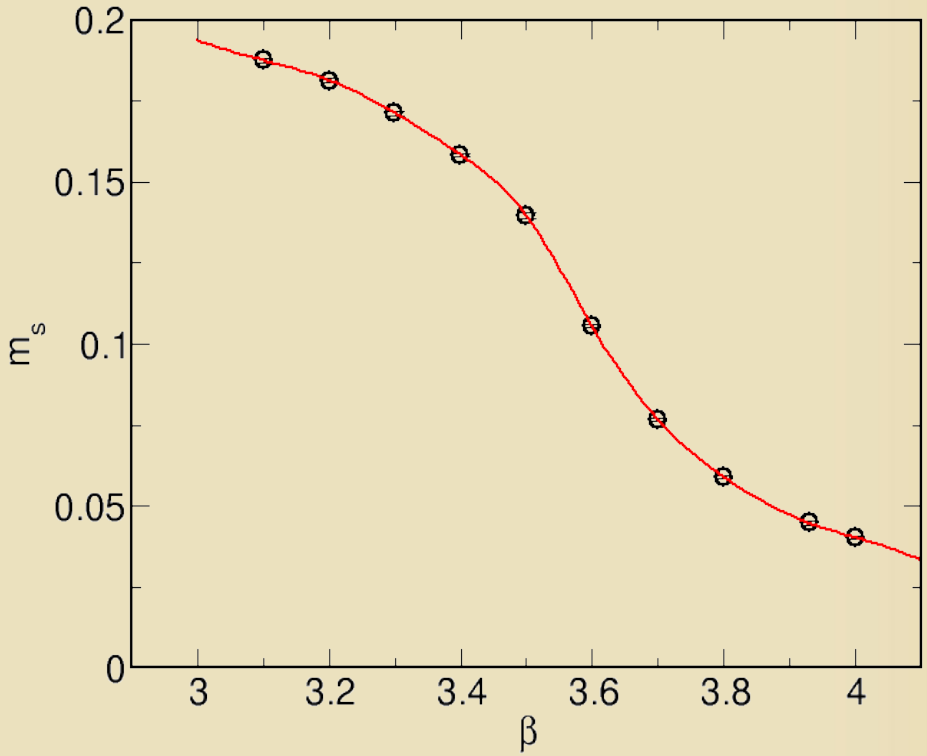
$$(m_{\eta_s}^2 = 2m_K^2 - m_\pi^2)$$

- = approx: beyond-leading term in ChPT negligible
- Isospin symm assumed

$$m_u = m_d$$

- fixed ratio (ball park)

$$m_s/m_u = 25$$



Problem of LCP1

- Sizable $O((ma)^2)$ effects
- LO ChPT to match 3f point to 2+1f
- ball park relation: $m_s/m_u = 25$

  → does not necessarily reproduce physical spectrum
- might be OK to some level for EoS normalized by T, T_c
- To establish order of transition and to obtain T_c[MeV] this might NOT be OK. And we could do better.

  → LCP2

LCP2

- Using 2+1f configurations with LCP1, find

  valence $m_{\mathsf{s}}(\beta)$, unitary $m_{\mathsf{u}}(\beta)$ points that satisfy

$$m_{PS}(m_u, m_u) = m_\pi = 135 MeV$$

$$m_{PS}(m_u, m_s) = m_K = 498 MeV$$

$$f_{PS}(m_u, m_s) = f_K = 160 MeV$$

- s quark is partially quenched.

  → this should be a good approximation.

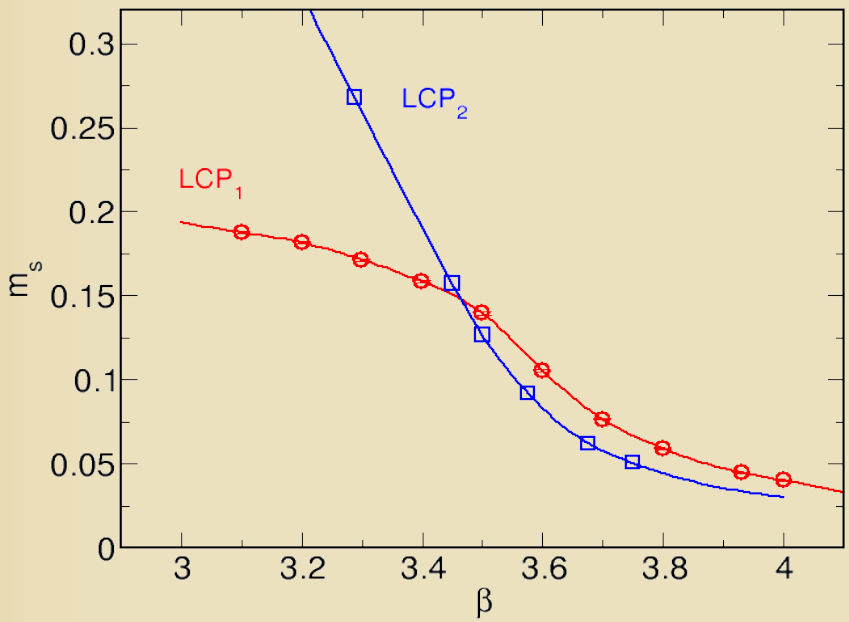
- chiral extrapolation is done with polynomial in mass.

  No chiral singularity in m_π , chiral log in f_K is weak in sChPT.

  → this should be a good approximation.

LCP2 & new simulation

- LCP2 obtained



- Gap 1→2 origin

  large β : higher oder ChPT

  small β : & $O((ma)^2)$

- $m_s/m_u = 27.3$

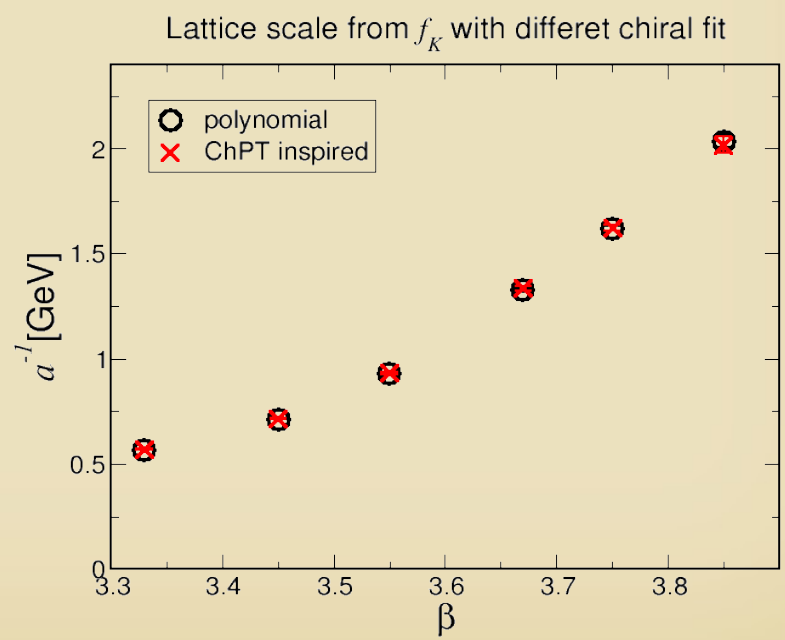
consistent with MILC (T=0), RBC(DWF)

- with new simulation on LCP2:

$$m_u^{sim\ u} = \{3, 5, 7, 9\} \times m_u^{phys} \quad \text{or}$$
$$m_u^{sim\ u} = \{4, 6, 8, 10\} m_u^{phys}$$

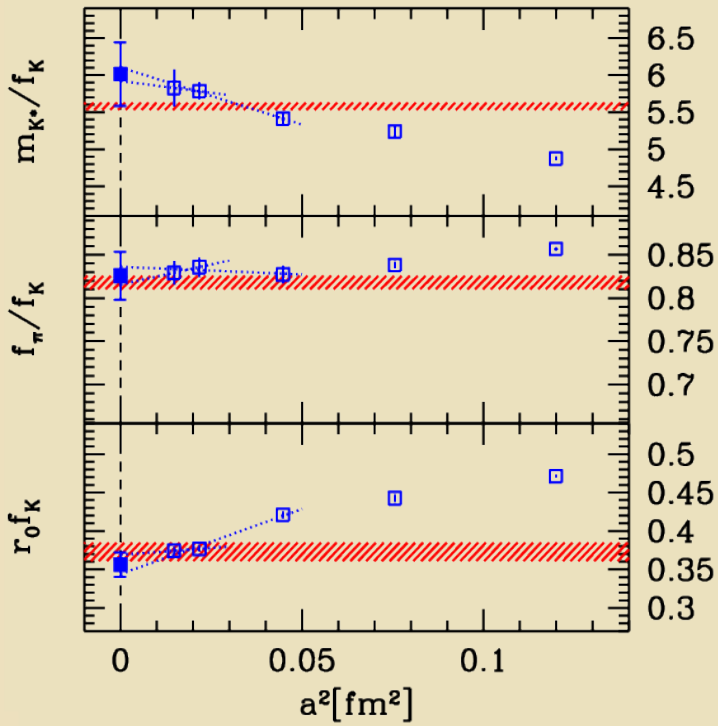
- polynomial chiral fit

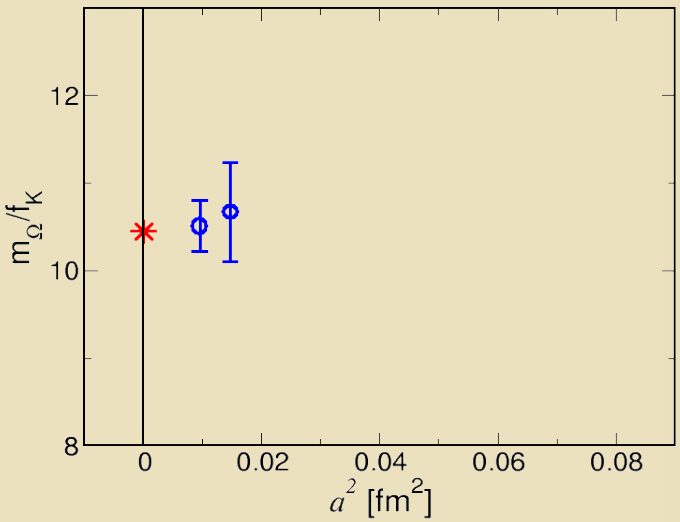
↔ChPT inspired chiral fit with log



- No difference!

T=0 continuum limit with LCP2





- T=0 spectrum OK

Summary and Outlook

- summary
  - EoS obtained with Nt=4, 6 stout-link improved staggered fermion using LCP1 (approximation).
  - There is better LCP2, where T=0 quantities in continuum limit reproduce experimetal values.
- outlook
  - Nt=8 or more with new LCP2 will be necessary for more reliable continuum limit of EoS.
  - Given the locality problem of staggered, different type of fermions like Wilson should be examined.