Comparing viscous hydrodynamics to a parton cascade

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Hydrodynamics in Heavy Ion Collisions and QCD Equation of State

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based on work in collaboration with Denes Molnar

Viscous hydrodynamics

relativistic Navier-Stokes hydro: small corrections linear in gradients

$$
\begin{aligned}
T^{\mu\nu}_{NS} &= T^{\mu\nu}_{ideal} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha) + \zeta\Delta^{\mu\nu}\partial^\alpha u_\alpha \\
N^\mu_{NS} &= N^\mu_{ideal} - \frac{n}{e+p}\kappa\nabla^\mu T
\end{aligned}
$$

where $\Delta^{\mu\nu} \equiv u^\mu u^\nu - g^{\mu\nu}$, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$

η, ζ shear and bulk viscosities, κ heat conductivity

two problems:

- parabolic equations $\rightarrow$ acausal Müller ('76), Israel & Stewart ('79) ...
- instabilities Hiscock & Lindblom, PRD31, 725 (1985) ...

Causal viscous hydro

Müller, Israel & Stewart...

$$\Delta T^{\mu\nu} \equiv \pi^{\mu\nu} + \Pi\Delta^{\mu\nu} \quad , \qquad \Delta N^{\mu} = -\tfrac{n}{e+p} q^{\mu}$$

bulk pressure Π, shear stress $\pi^{\mu\nu}$ heat flow q^{μ} treated as independent dynamical quantities that relax to their Navier-Stokes value on time scales $\tau_{\Pi}(e,n)$, $\tau_{\pi}(e,n)$, $\tau_{q}(e,n)$ - corresponds to keeping not only first but (certain) second derivatives.

Entropy four-flow including terms second order in dissipative fluxes:

$$S^{\mu} = \quad su^{\mu} + \tfrac{\mu}{T}\tfrac{q^{\mu}}{h} \quad - \left(\beta_0\Pi^2 - \beta_1 q_{\nu}q^{\nu} + \beta_2\pi_{\lambda\nu}\pi^{\lambda\nu}\right)\frac{u^{\mu}}{2T} - \frac{\alpha_0 q^{\mu}\Pi}{T} + \frac{\alpha_1 q_{\nu}\pi^{\nu\mu}}{T}$$

Require non-decrease of entropy:

$$0 \geq \partial_{\mu}S^{\mu} = \Pi X + q_{\mu}X^{\mu} + \pi_{\mu\nu}X^{\mu\nu}$$

Which terms to keep?

$$\text{Muronga, Romatschke :} \quad u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - \left(u^\mu \pi^{\nu\alpha} + u^\nu \pi^{\mu\alpha}\right) u^\lambda \partial_\lambda u_\alpha - \frac{1}{2}\pi^{\mu\nu}\left(u^\lambda \partial_\lambda \ln\frac{\beta_2}{T} + \partial_\lambda u^\lambda\right)$$

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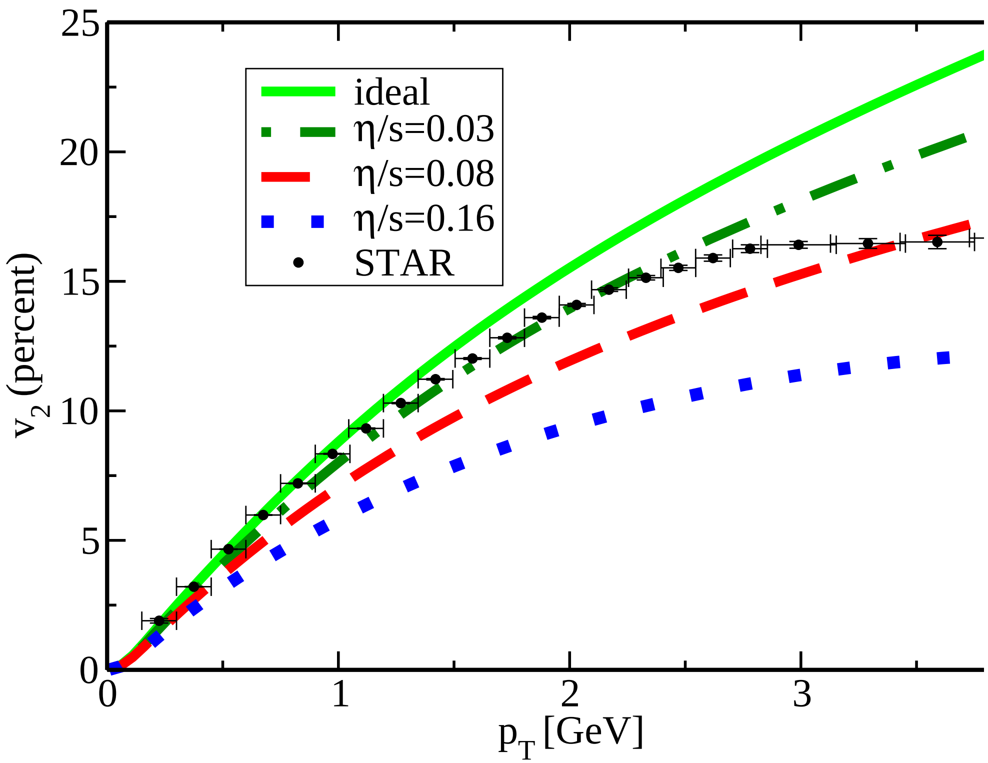
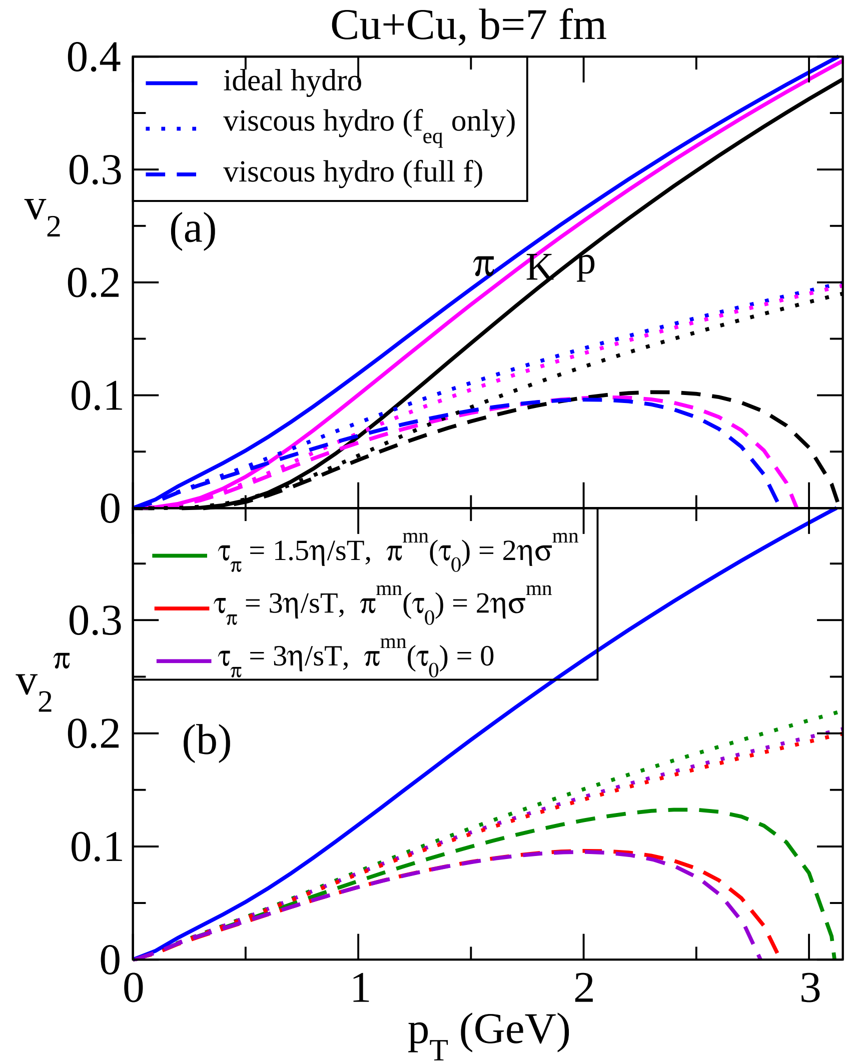
$$\text{Chaudhuri}: \quad u^\alpha \partial_\alpha \pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right)$$

$\eta/s = 1/(4\pi)$ corrections from IS hydro





Romatschke & Romatschke, arxiv:0706.1522 **Song & Heinz, arxiv:0709.0742**

10-20% **or** **50+% reduction??**

Potential caveat

Whereas Navier-Stokes is an expansion in λ/R (keeps only first derivatives), Israel-Stewart hydro is NOT a controlled approximation (retains certain second derivatives). For example in kinetic theory, it corresponds to Grad's 14-moment approximation

$$f(x,p) \approx [1 + C_{\alpha\beta} p^\alpha p^\beta] e^{(\mu - p^\mu u_\mu)/T}$$

while NS comes from the Chapman-Enskog expansion in small gradients

$$E\partial t f + \varepsilon \cdot \vec{p}\vec{\nabla} f = C[f] \quad , \qquad f = f_0 + \epsilon f_1 + \epsilon^2 f_2 + \ldots$$

If relaxation effects important, NS and IS are different

$\Rightarrow$ control against a nonequilibrium theory is crucial

Covariant transport

Boltzmann ..., Israel, Stewart, de Groot, ... Pang, Zhang, Gyulassy, DM, Vance, Csizmadia, Pratt, Cheng, Xu, Greiner ...

Covariant, causal, nonequil. approach - formulated in terms of local rates.

$$\Gamma_{2\to 2}(x) \equiv \frac{dN_{scattering}}{d^4x} = \sigma v_{rel} \frac{n^2(x)}{2}$$

This theory has a hydrodynamic limit (i.e., it equilibrates) Boltzmann

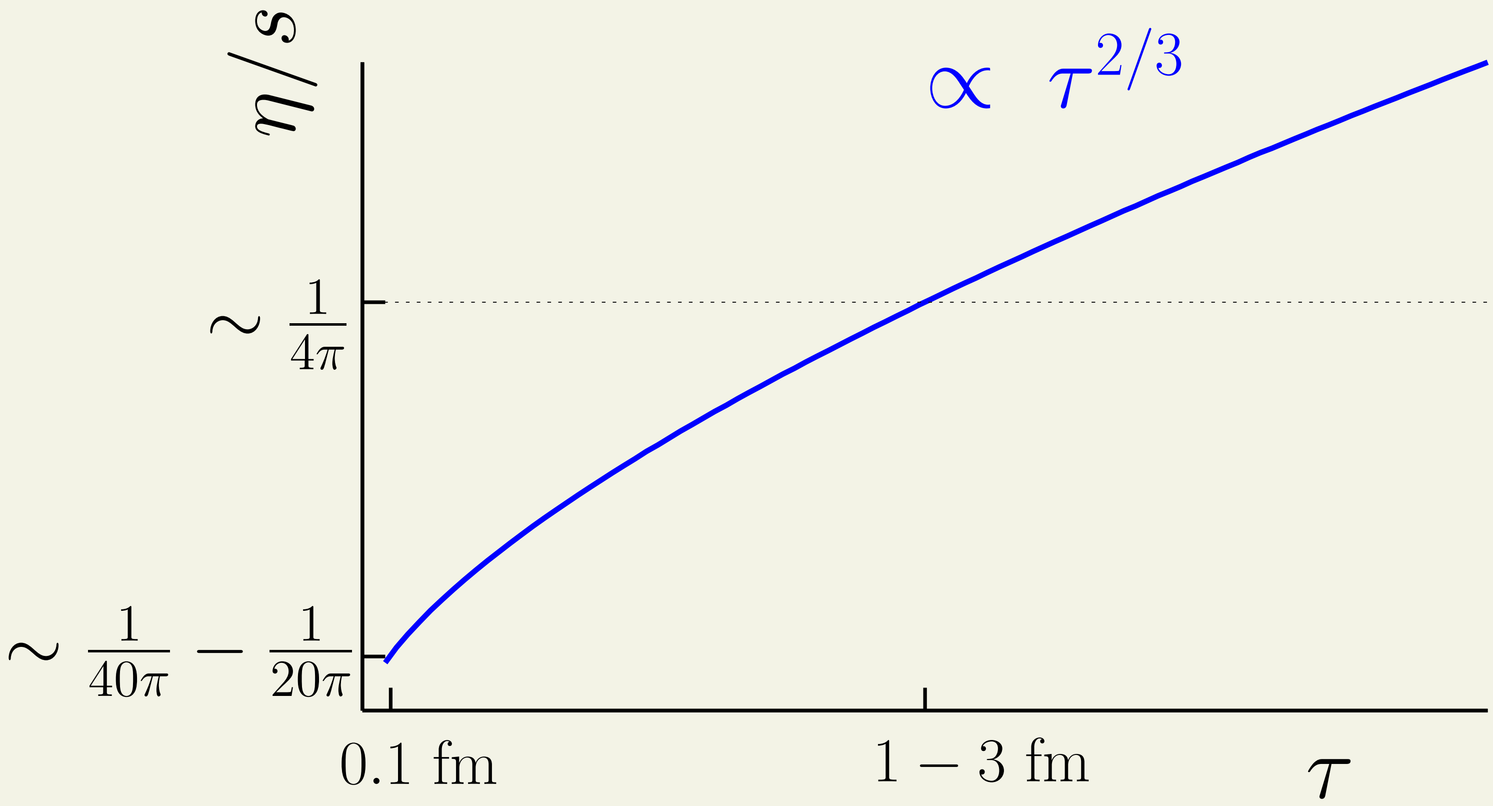
Parameter σ controls transport coefficients and relaxation:

$$\eta \approx 0.8 \frac{T}{\sigma_{tr}} \qquad \tau_\pi = 1.2\lambda_{tr}$$

solvable numerically: HERE, utilize MPC algorithm DM, NPA 697 ('02)

In transport $\eta/s \sim \lambda_{tr} T \sim 1/(\sigma T^2)$

e.g., for $\sigma \approx 50$ mb ($\sigma_{tr} \approx 14$ mb)



$\Rightarrow$ $\eta/s = const$ **needs <u>growing</u>** $\sigma(\tau) \propto 1/T^2 \propto \tau^{2/3}$

in perturbative QCD: $\sigma_{tr} \sim \frac{\alpha_s^2}{s} \ln \frac{s}{\mu_D^2} \sim \frac{g^4}{T^2} \ln \frac{1}{g^2}$

Viscous hydro vs transport

We solve the full Israel-Stewart-Muronga equations, including vorticity terms from kinetic theory, in a 2+1D boost-invariant scenario. Shear stress only.

Mimic a known reliable transport model:

- **massless Boltzmann particles** $\Rightarrow \epsilon = 3P$
- **only** $2 \leftrightarrow 2$ **processes, i.e. conserved particle number**
- $\eta = 4T/(5\sigma_{\rm tr})$
- **either** $\sigma_{\rm tr} =$ **const.** $= 47$ **mb** $(\sigma_{tr} = 14$ **mb)** $\leftarrow$ **the simplest in transport**

  or $\sigma_{\rm tr} \propto \tau^{2/3}$ **close to** $\eta/s = 1/(4\pi)$

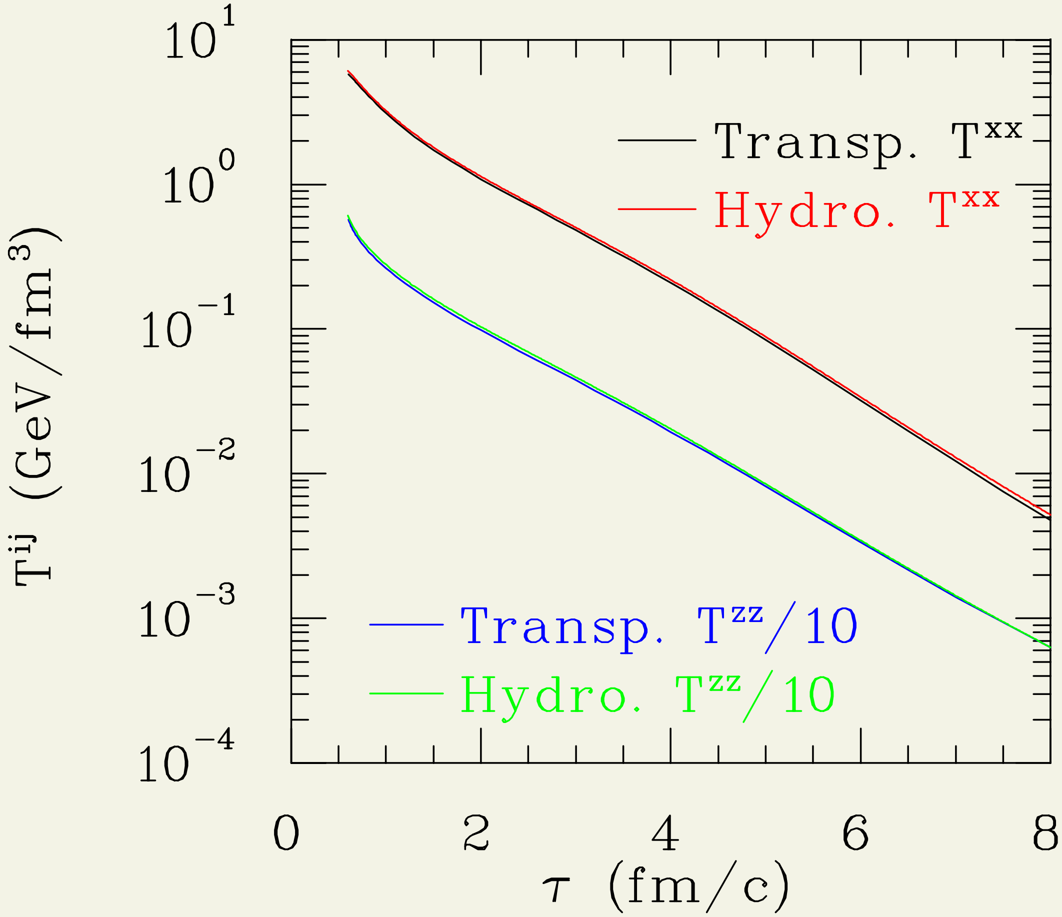
Our “RHIC-like” initialization:

- $\tau_0 = 0.6$ **fm/**c
- $b = 8$ **fm**
- $T_0 = 385$ **MeV and** $dN/d\eta|_{b=0} = 1000$
- **freeze-out at constant** $n = 0.365$ **fm**$^{-3}$

Pressure evolution in the core

T^{xx} **and** T^{zz} **averaged over the core of the system, $r < 1$ fm:**

$\eta/s \approx 1/(4\pi)$ ($\sigma_{\rm tr} \propto \tau^{2/3}$)



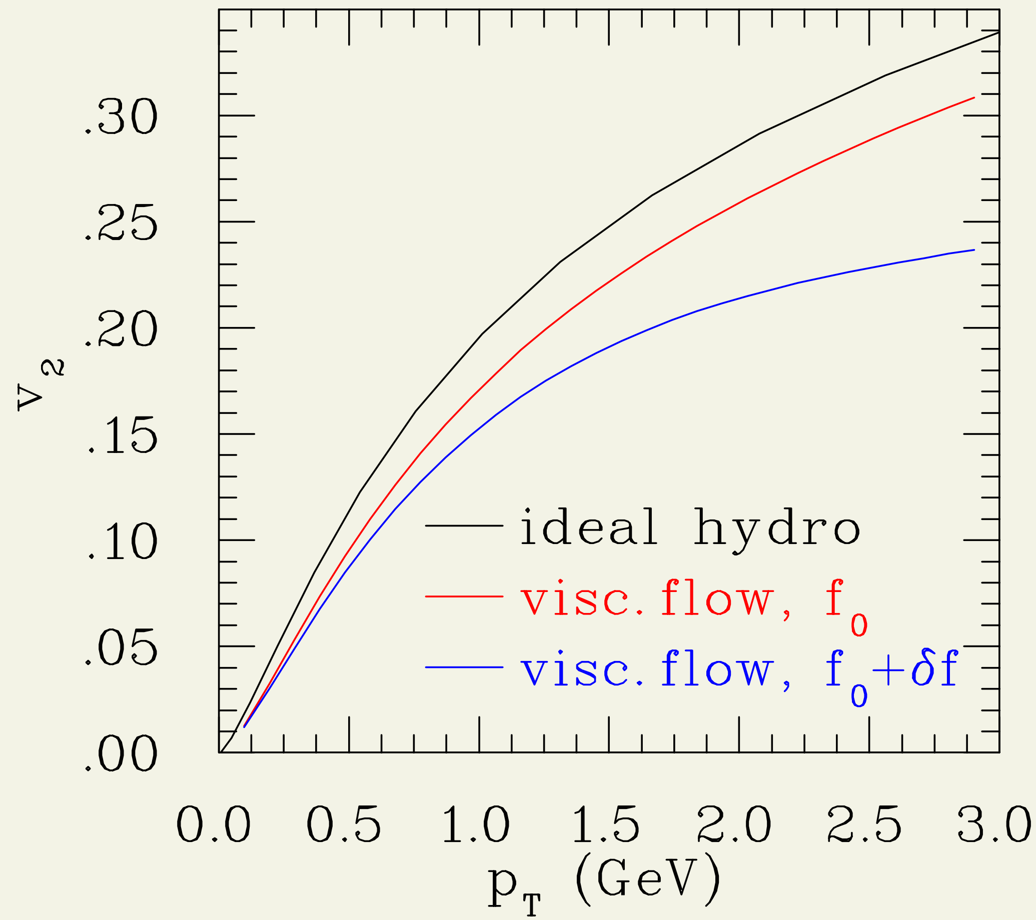
remarkable similarity!

Viscous hydro elliptic flow

TWO effects: - **dissipative corrections to hydro fields** u^μ, T, n
- **dissipative corrections to thermal distributions** $f \to f_0 + \delta f$

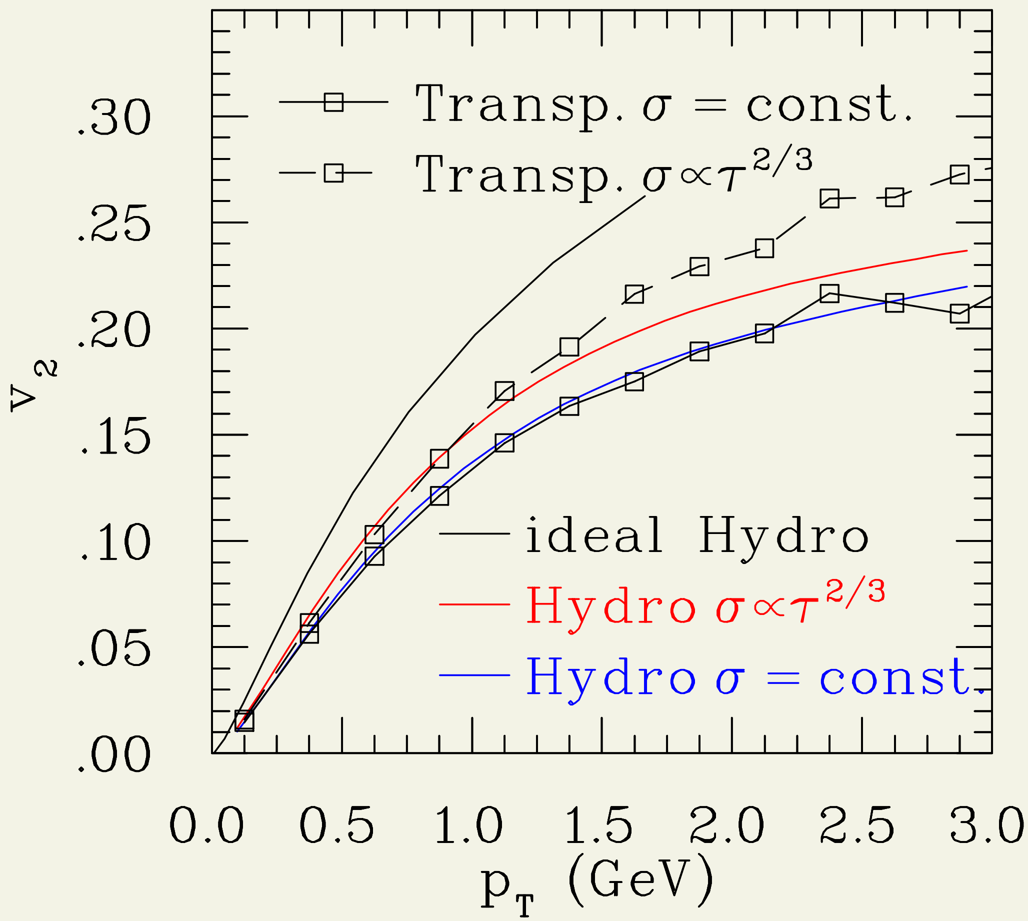
$\eta/s \approx 1/(4\pi)$ $(\sigma_{\mathrm{tr}} \propto \tau^{2/3})$

$$\delta f = f_0 \left[1 + \frac{p^\mu p^\nu \pi_{\mu\nu}}{8nT^6}\right]$$



Calculation for $\sigma_{\mathrm{tr}} =$ **const.** ~ 15 **mb shows similar behaviour**

Viscous hydro vs transport v_2

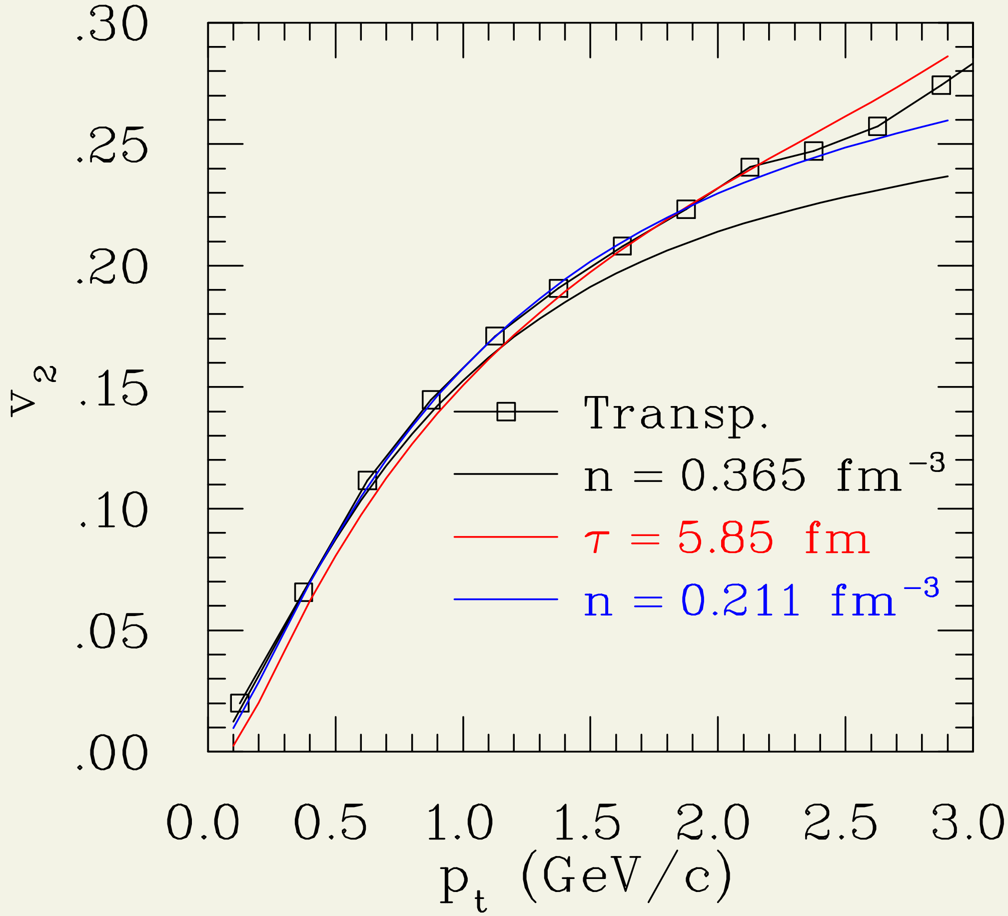


- **excellent agreement when $\sigma =$ const ~ 47 mb**
- **good agreement fo $\eta/s \approx 1/(4\pi)$, i.e. $\sigma \propto \tau^{2/3}$**

- **BUT results sensitive to freeze-out criterion, especially at high p_T**

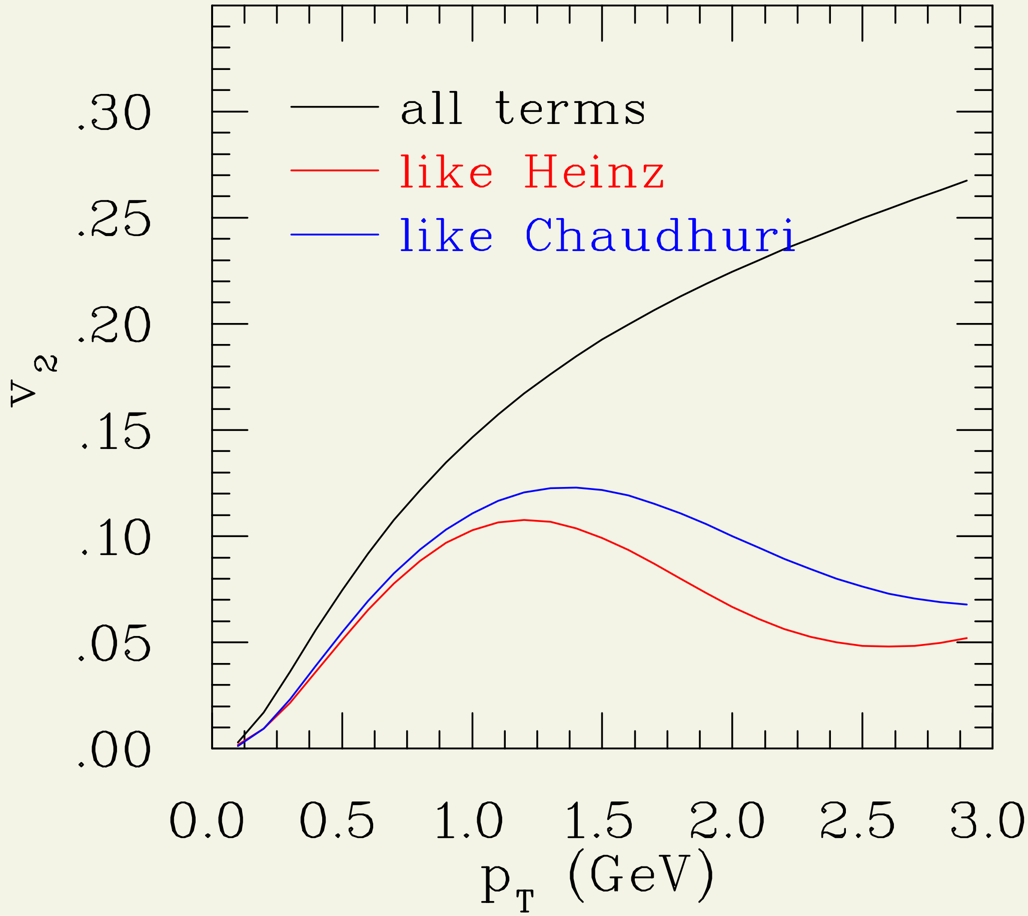
Effect of freeze-out criterion

$\eta/s \approx 1/(4\pi)$ $(\sigma_{\rm tr} \propto \tau^{2/3})$



- **some sensitivity to the freeze-out criterion**
- **not crucial for the results**

Which terms to keep?



- **Important to keep all terms!**

Conclusions

Prospects for applicability of Israel-Stewart causal hydrodynamics at RHIC look promising, based on comparisons with covariant $2 \rightarrow 2$ transport in 2+1D Bjorken scenario

Dissipative effects change both flow and distributions

Dissipation reduces $v_2(p_T)$ by $20 - 30$% for $\eta/s = 1/(4\pi)$ and conditions expected at RHIC

Hydrodynamical results are sensitive to the freeze-out procedure

$\longrightarrow$ being investigated