Parton Cascade with soft Yang-Mills fields

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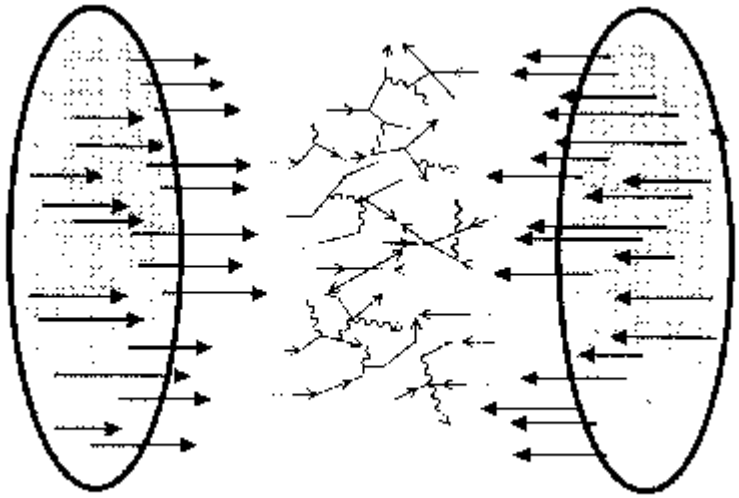
Boltzmann Transport - 'Parton Cascade'

Space-Time Parton Cascade in High-Energy Collisions :
(K. Geiger, B. Müller: 1992)



Boltzmann Equation :

$$p_\mu \partial^\mu f(x,p) = C[f]$$

$$C[f] = gg \to gg + \ldots$$

$$\frac{d\sigma}{dt} \sim \frac{\alpha_s^2}{t^2}$$

with <u>fixed</u> IR cutoff !

present state of the art incl. 2↔3 processes (Z. Xu, C. Greiner, PRC '05)

Boltzmann Eqn with self-consistent screening

- screening mass:

$$m^2 = \frac{3\alpha_s}{\pi^2} \int d^3p \, \frac{f(p)}{p^0} \sim \alpha_s \frac{n_g}{p}$$

- elastic cross-section:

$$\frac{d\sigma}{dt} \sim \frac{\alpha_s^2}{(t - m^2)^2} \quad , \quad \sigma_{\rm tr} \sim \frac{\alpha_s^2}{s} \log \frac{s}{m^2}$$

<u>BUT</u>:

- LL accuracy not good enough

- $(p^2 g^{\mu\nu} + p^\mu p^\nu + \Pi^{\mu\nu})$ exhibits unstable modes

- $p_\perp$ broadening of hard particle in thermal medium:

elastic 2→2 collisions, LL approximation:

$$\hat{q} = \frac{d\langle p_\perp^2 \rangle}{dt} \sim g^4 \, n_g \log\left(C \frac{p}{m}\right)$$

take p/m=10:

for C = 2, 1, 0.5: log() = 3.0, 2.3, 1.6

➔ expect strong sensitivity to cutoff !

Collision rates / transport coefficients in Boltzmann cascades may carry uncertainty by factors of 2 or more...

RESTRICTION TO HARD COLLISIONS NOT GOOD ENOUGH, CONSISTENT TREATMENT OF SOFT+HARD NEEDED !

sort of Braaten-Pisarski in a transport approach...

particles and classical gauge fields

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla_x f + g(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})\cdot\nabla_p f = C[f]$$

$$\dot{\boldsymbol{E}} = \nabla\times\boldsymbol{B} - \boldsymbol{J} \qquad \boldsymbol{J} = g\int\frac{d^3p}{(2\pi)^3}\boldsymbol{v}\,f$$

$$\dot{\boldsymbol{B}} = -\nabla\times\boldsymbol{E}$$

L.M.Lifshitz and L.P.Pitaevskii, Physical kinetics

$f(\boldsymbol{x},\boldsymbol{p})$ = Single-particle distribution function

Collisions only (E,B=0; C[f]≠0): Boltzmann equation

Soft field only (C[f]=0, E,B≠0): Vlasov equation.

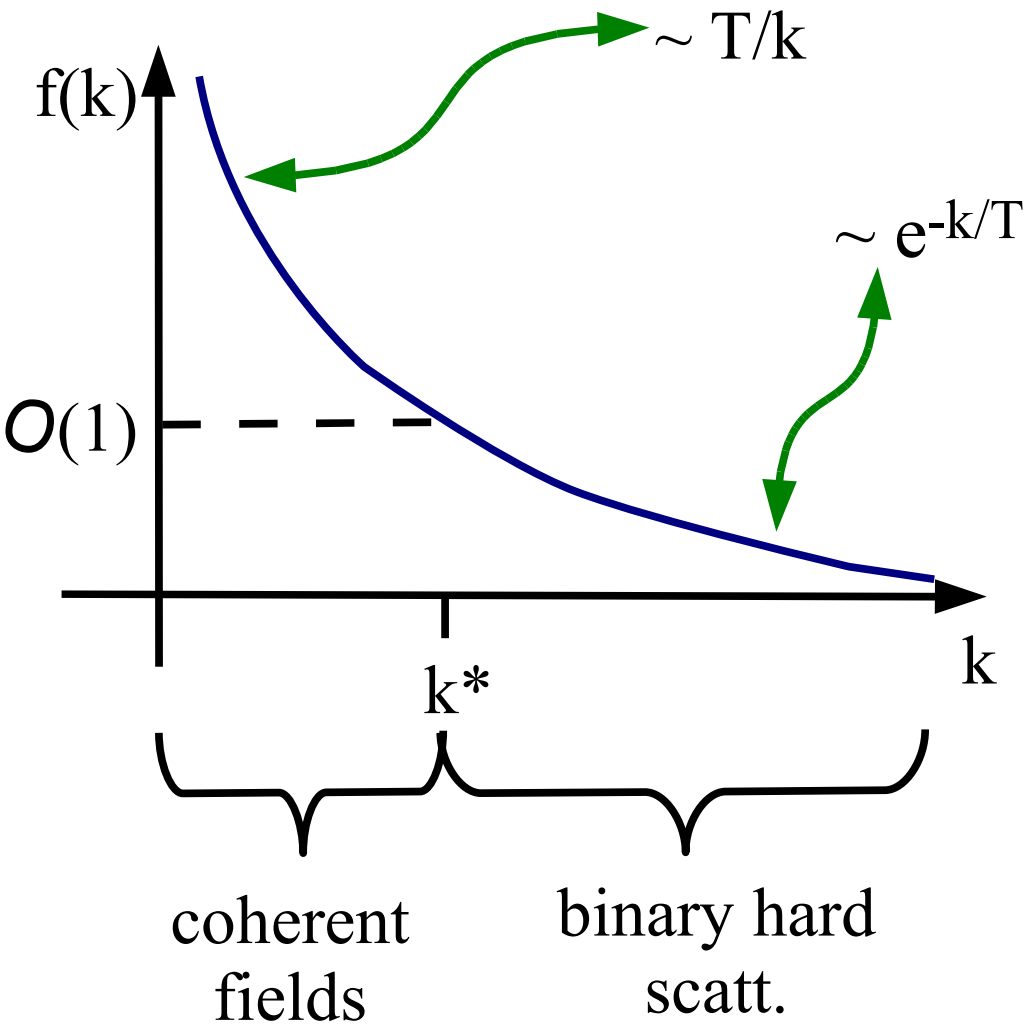
Boltzmann-Vlasov-Yang-Mills Theory

$$p^\mu \left[\partial_\mu - g Q^a F^a_{\mu\nu} \frac{\partial}{\partial p_\nu} + g f^{abc} A^b_\mu Q^c \frac{\partial}{\partial Q^a} \right] f = \mathcal{C}[f]$$

$$(D_\mu F^{\mu\nu})^a = J^{a\,\nu}$$

- Soft exchange (q < k*) via fields (Lorentz force)
- Hard exchange (q > k*) via collision term (2→2 elastic)
- NO CUTOFF

- soft/hard scale separation:



$\rightarrow k^* = \pi/a \sim T$

$$\mathcal{C}[f]: \quad \sigma(k^*) = \int_{k^{*\,2}}^{s/2} dq^2 \frac{d\sigma}{dq^2}$$

- Matching:

given: T and density of hard particles

$$n_g = 2(N_c^2 - 1) \int \frac{d^3k}{(2\pi)^3} \, e^{-k/T}$$

soft fields: $A_i \sim 1/k \quad , \quad E_i = 0$

energy density normalized according to:

$$\int \frac{d^3k}{(2\pi)^3} \, k \, f_{\rm Bose}(k) \, \Theta(k^* - k) \; \times \; \frac{n_g}{2T^3\zeta(3)}$$

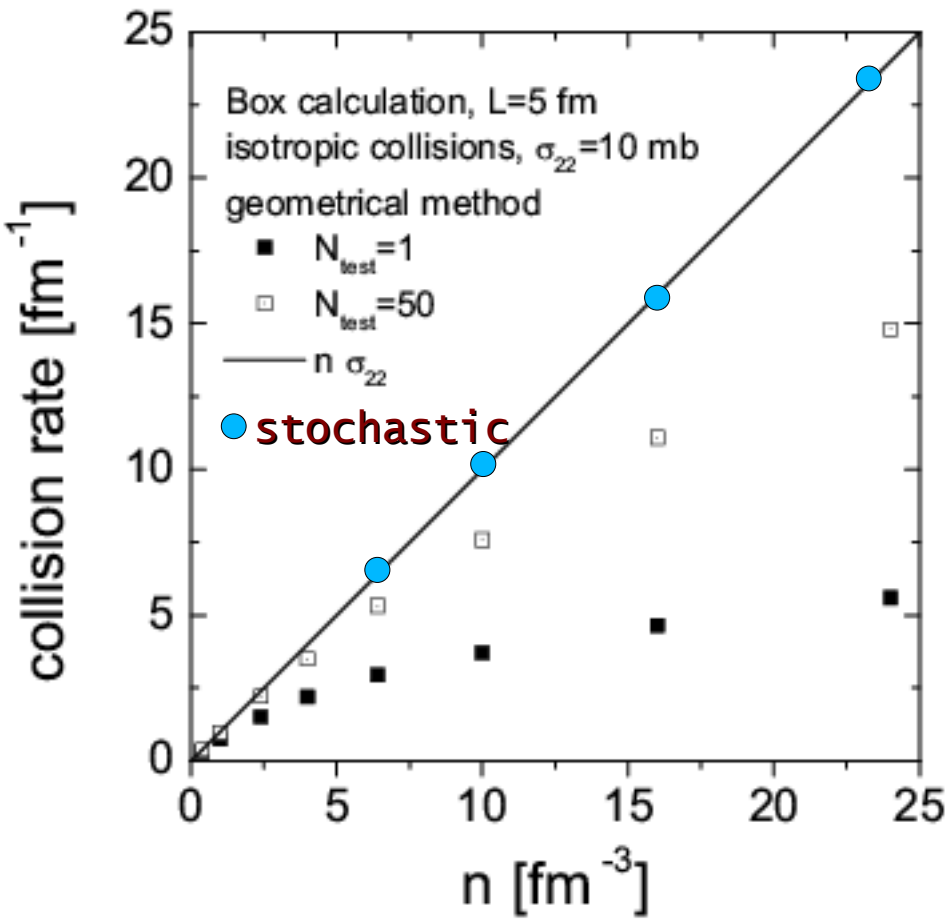
Binary Collision Rate Test:

- Geometric Method: $|\mathbf{r}_\perp(1) - \mathbf{r}_\perp(2)| < \sqrt{\sigma_{2\to2}/\pi N_{\rm test}}$

[Z. Xu, C. Greiner: PRC (2005)]

- Stochastic Method:

$$\frac{dP_{2\to2}}{dt} = \frac{s}{2E_1E_2}\frac{\sigma_{2\to2}}{a^3 N_{\rm test}}$$



➔ scales properly with density

Particle in cell simulation

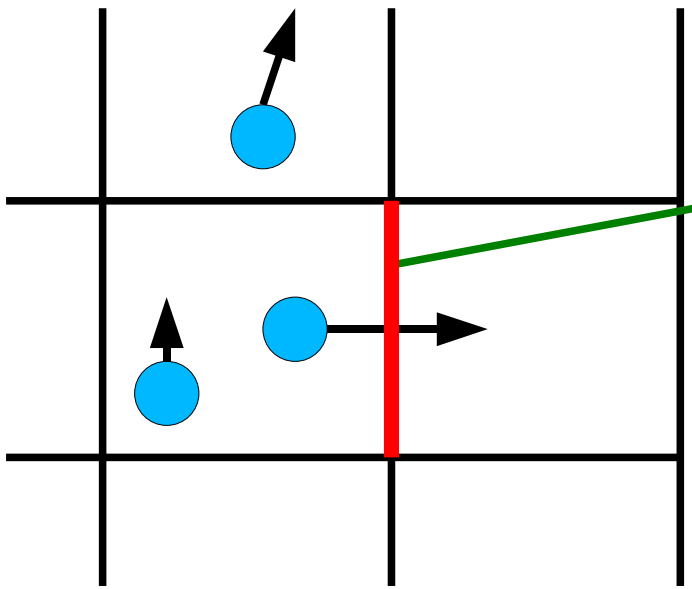
for colored particles

(Wong-YM)

$$\frac{d\,\boldsymbol{x}_i}{dt}=\boldsymbol{v}_i,\quad \frac{d\,\boldsymbol{p}_i}{dt}=gQ_i^a(\boldsymbol{E}_i^a+v_i\times\boldsymbol{B}_i^a),\quad \frac{d\,\boldsymbol{Q}_i}{dt}=igv_i^\mu[A_\mu,Q_i].$$

$$(D_\mu F^{\mu\nu})^a=J^{\nu a}=g\sum Q^a v^\nu \delta(x-x_i(t))$$

Nearest-Grid-Point (NGP) method for non-Abelian gauge theories



i i+1

- Density: Count the number of particles within a cell
- A current is generated only when a particle crosses a cell

$$J(i) = Q\,\delta(t - t_{cross}) / N_{test}$$

- Color charge must be parallel transported:

$$Q(i+1) = U_x(i)^\dagger Q(i) U_x(i)$$

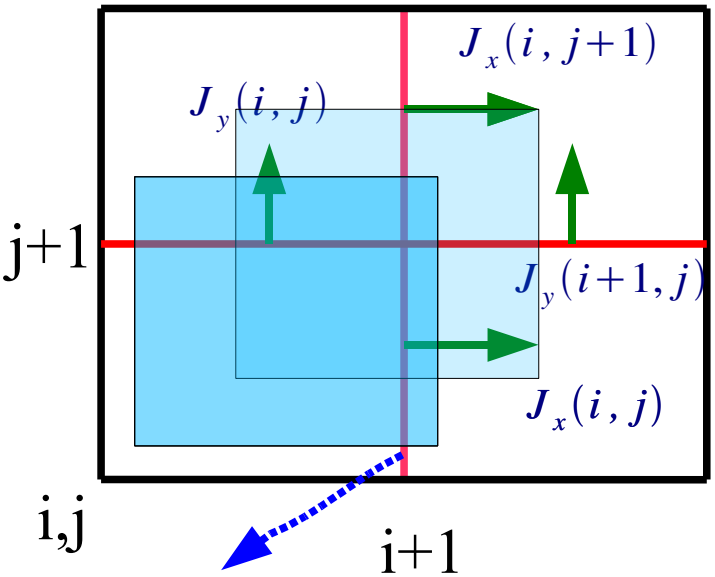
to satisfy the lattice covariant continuity equation,

$$\dot{\rho}(i) = \sum_x U_x(i-x)^\dagger J_x(i-x) U_x(i-x) - J_x(i)$$

in oder to conserve Gauss's law.

C. R. Hu and B. Muller, Phys. Lett. B409 (1997)377
G. D. Moore, C.R. Hu and B. Muller, Phys. Rev. D58 (1998)045001
A. Dumitru and Y. Nara, Phys. Lett. B 621 (2005) 89

Color-Particle-in-Cell (CPIC) method



Links experiencing current

$Tr(Q^2)$ conserved to $O(a^2)$

$\rho(i+1,j+1)$ depends on the path

We use current conservation to determine the increment of the color charge.

$$J_x(i,j)=Q\frac{x_{i+1}-x_i}{\delta t}(1-W_y),\quad J_x(i,j+1)=Q_y\frac{x_{i+1}-x_i}{\delta t}W_y,$$

$$J_y(i,j)=Q\frac{y_{i+1}-y_i}{\delta t}(1-W_x),\quad J_y(i+1,j)=Q_x\frac{y_{i+1}-y_i}{\delta t}W_x$$

Parallel transport of the charge:

$$Q_x=U_x(i)^\dagger Q U_x(i),\quad Q_y=U_y(i)^\dagger Q U_y(i)$$

Linear weighting function defined at the midpoint:

$$W_x=\frac{x_i+x_{i+1}}{2}-i,\quad W_y=\frac{y_j+y_{j+1}}{2}-j$$

satisfy the lattice covariant continuity equation

$$\dot{\rho}(i)=\sum_x U_x(i-x)^\dagger J_x(i-x)U_x(i-x)-J_x(i)$$

Sites (i,j), (i+1,j), (i,j+1) are consistent with

$$\rho(i,j)=Q(1-x)(1-y),\quad \rho(i,j+1)=Q_y(1-x)y$$
$$\rho(i+1,j)=Q_y x(1-y),\quad \rho(i+1,j+1)=Q_{xy}xy$$

Y. Nara (2006)

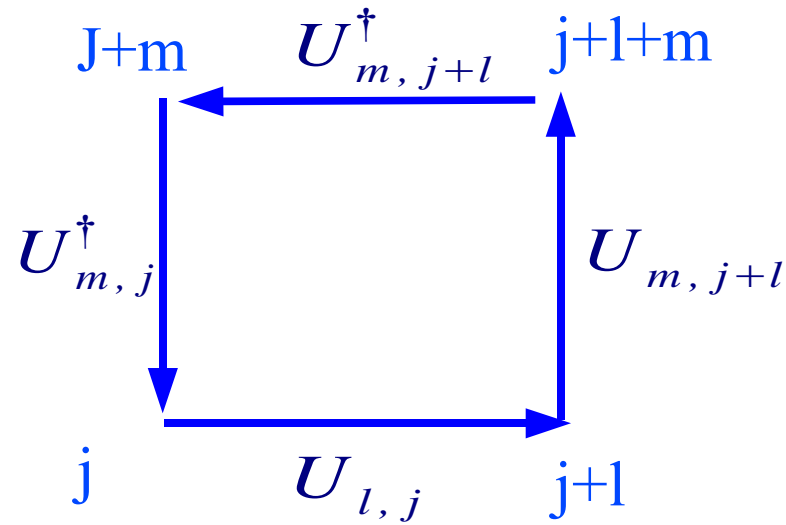
Yang-Mills Hamiltonian on the lattice

Numerical solution of real time classical YM:
Kogut-Susskind Hamiltonian:

$$H_{YM} = \frac{1}{2}\sum_{l} E_l^2 + \frac{1}{2}\sum_{\square}\left(N_C - \Re\, Tr\, U_{\square}\right)$$

Link $\quad U_{j,i} = \exp[-iga\, A_j(i)]$

Plaquette $\quad U_{\square} \equiv U_{l,j}\, U_{m,j+l}\, U^{\dagger}_{l,j+m}\, U^{\dagger}_{m,j}$



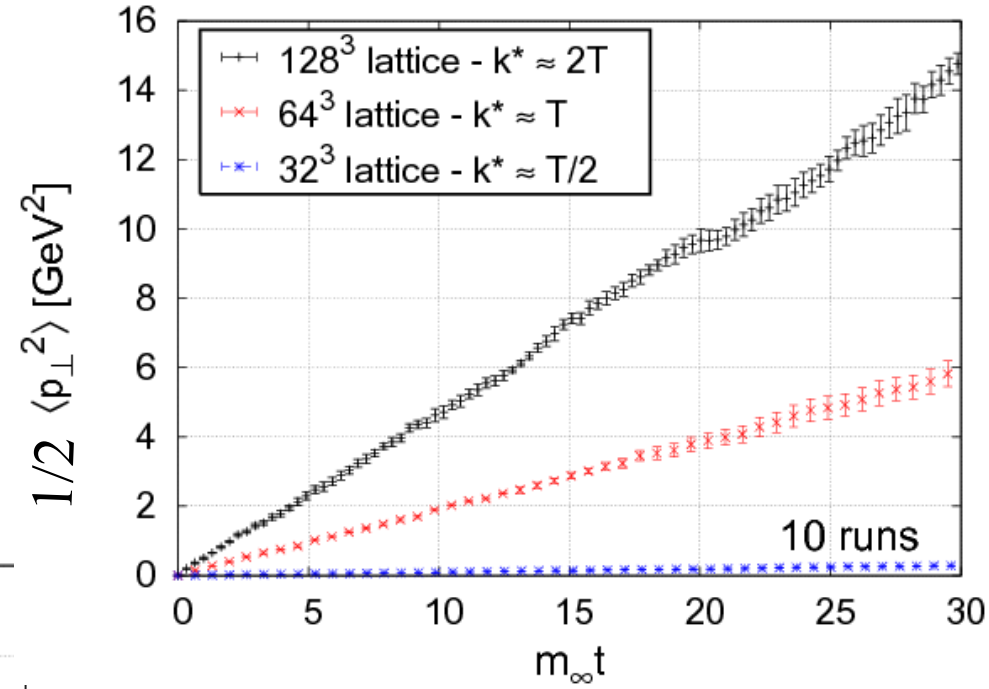
Equations of motion for dynamical variable X

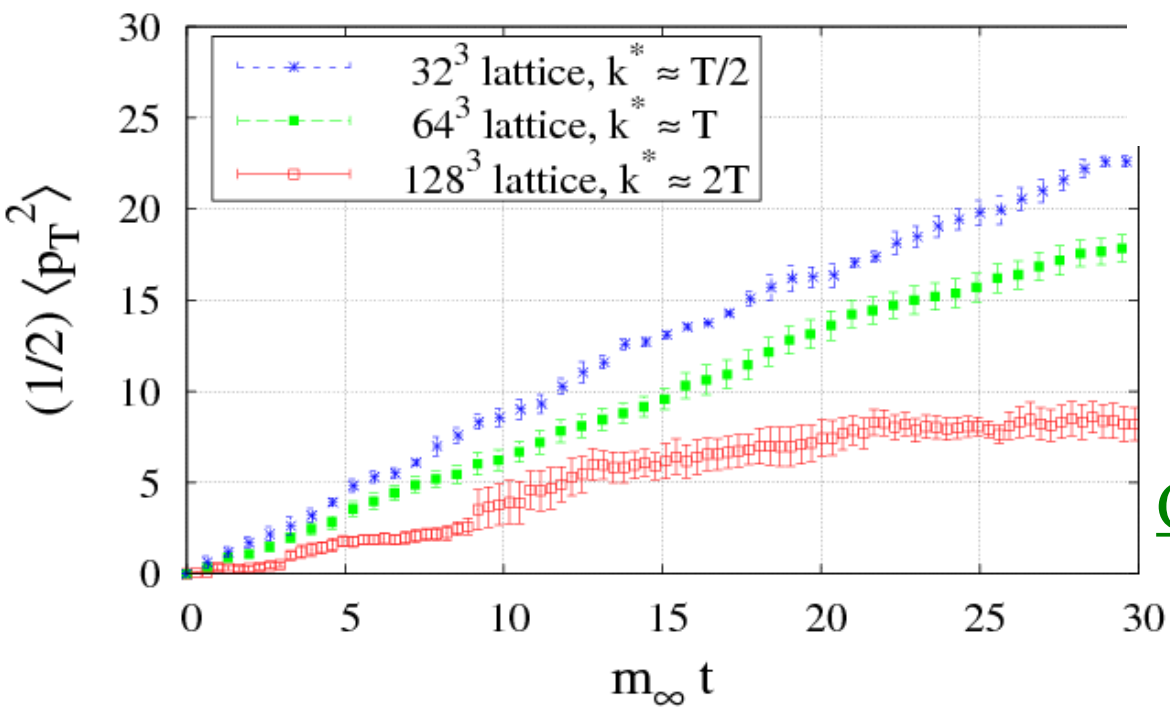
$$\frac{dX}{d\tau} = \{H_L, X\}$$

$p_\perp$ broadening in thermal medium, simulation results:

- Thermal (isotropic) SU(2) plasma
- 3T = 12 GeV
- g = 2
- $n_g = 10\ \mathrm{fm}^{-3}$
- jet: $p_z = 60$ GeV,
  $p/(3T) = 5$ (!)

Field only, no Coll.





Coll. only, no YM

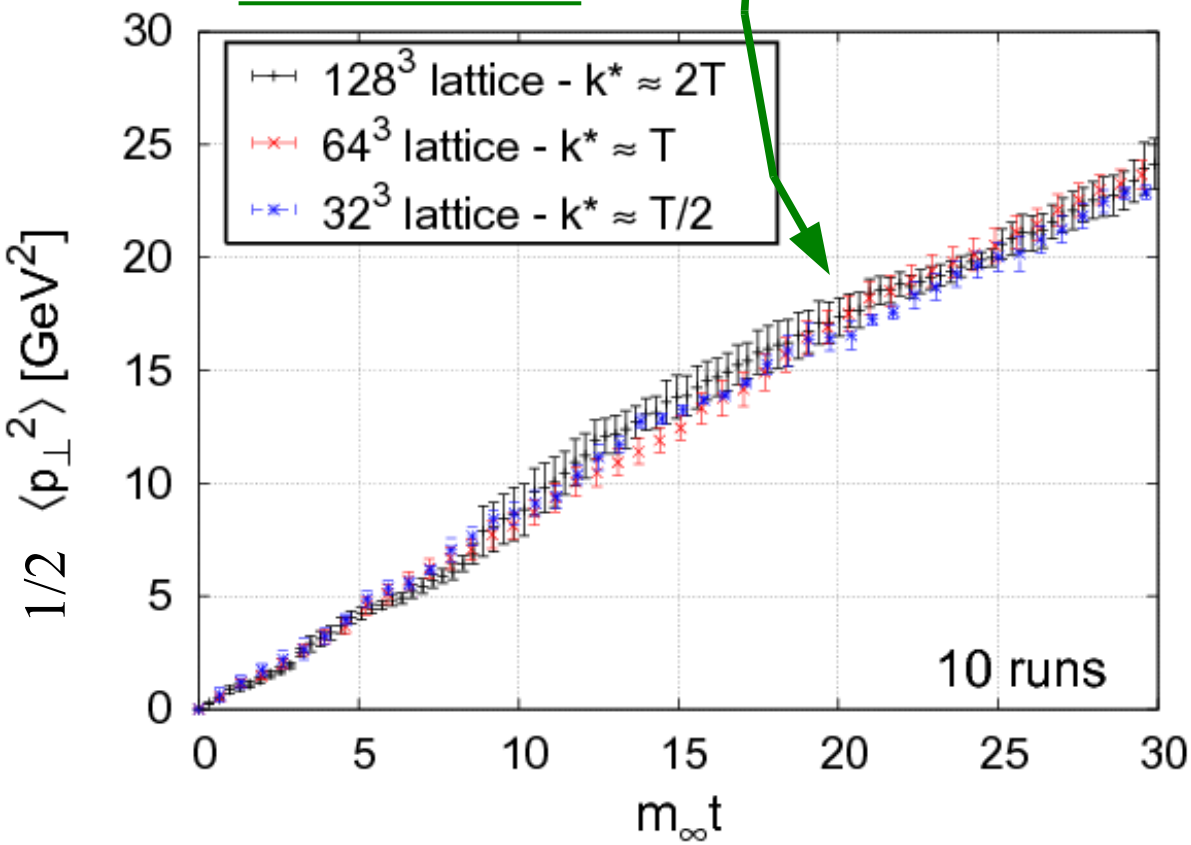
- Thermal (isotropic) SU(2) plasma
- same parameters, $m_\infty = 1.4\ \mathrm{fm}^{-1}$

arXiv:0710.1223

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{t} = 2.2\ \mathrm{GeV}^2/\mathrm{fm}$$

for $p_{\mathrm{hard}}/(3T) = 5,\ n_g = 10/\mathrm{fm}^3,\ N_c = 2$

Field+Coll.



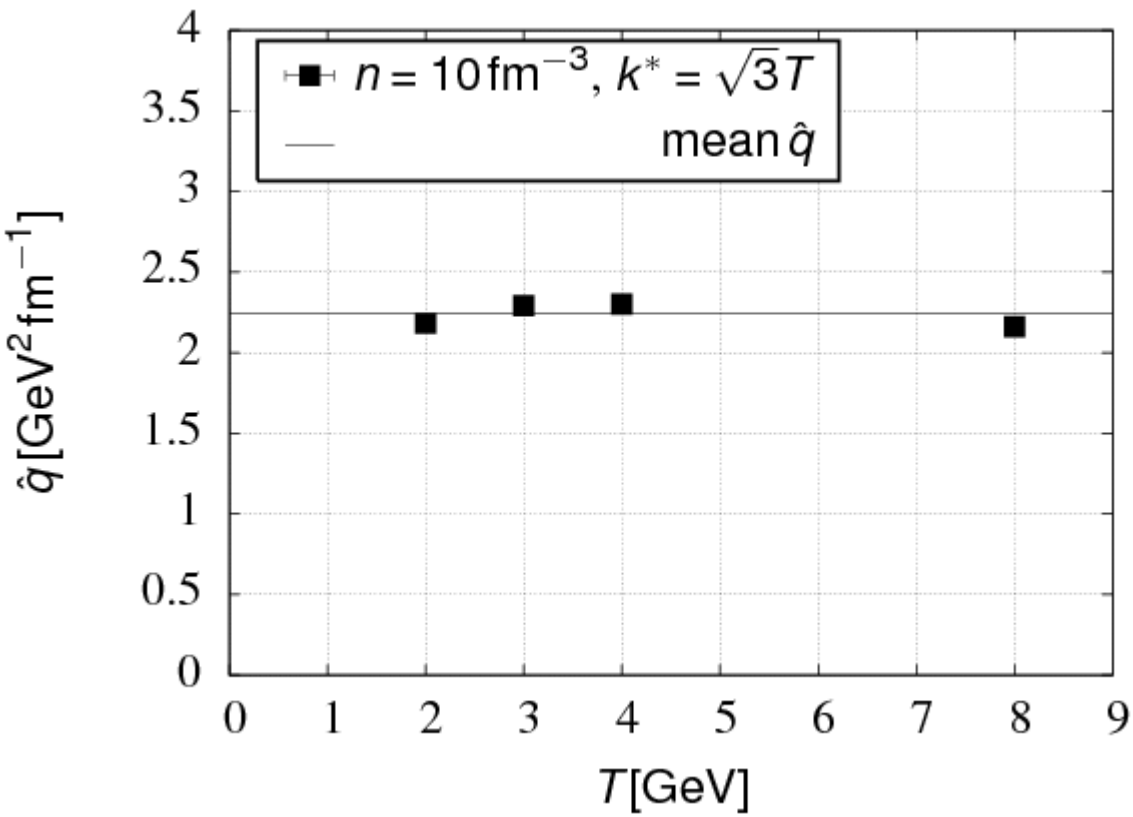
Independent of k* !

Good Thing®

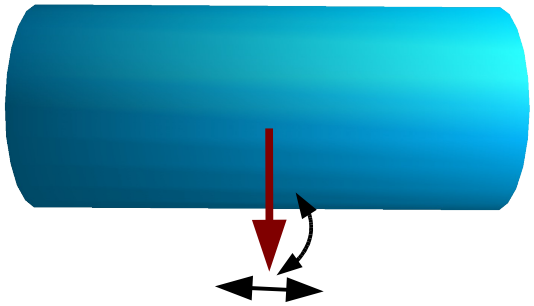
- Continuum limit: $\frac{\pi}{a} \sim T$

$$m_\infty a \ll 1 \quad \leftrightarrow \quad g^2 \frac{n_g}{T^3} \ll 1$$

for $g \sim 1$: $T \gg n_g^{1/3}$, should be ok for $\hat{q} \sim g^4 n_g \log\left(C \frac{p}{m}\right)$

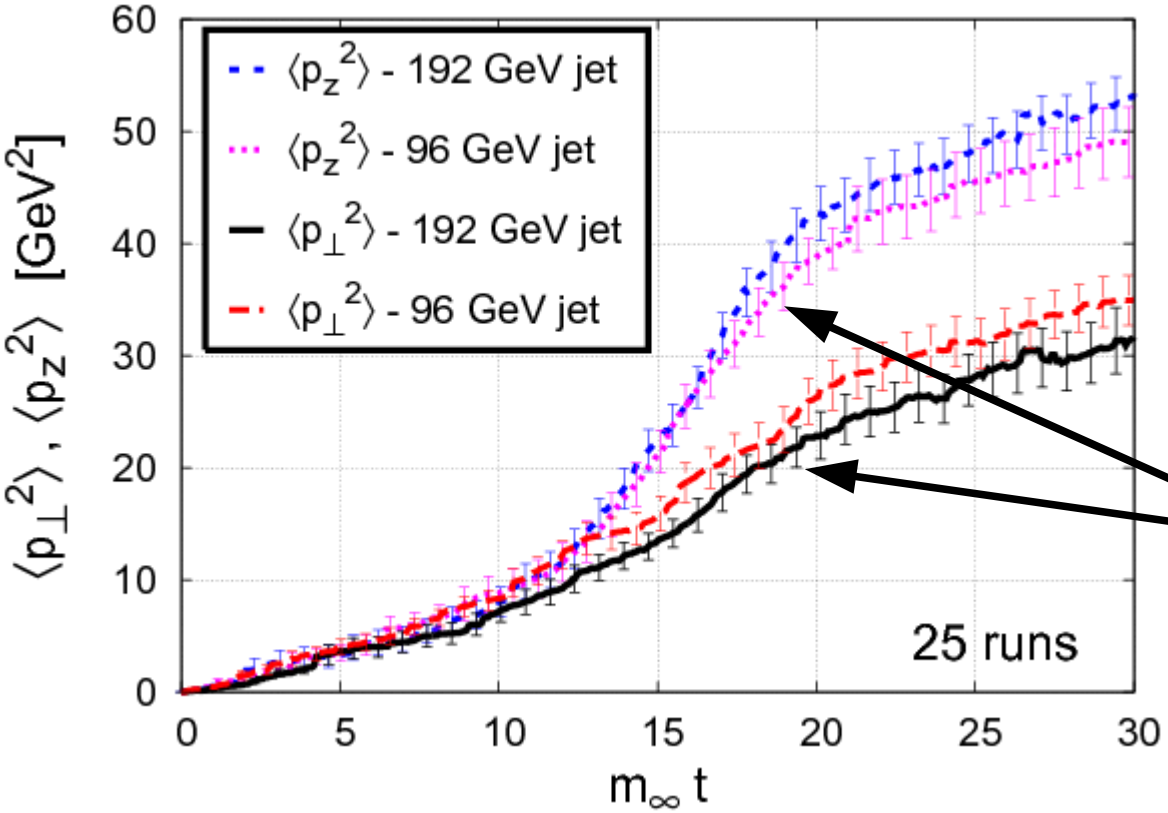


- anisotropic medium: instability...



medium: $f(\boldsymbol{p}) \sim \delta(p_z)\, e^{-p_\perp / p_{hard}}$

jet: $\boldsymbol{p} = (p_x, 0, 0)$, $p_x = 96, 192\ GeV$

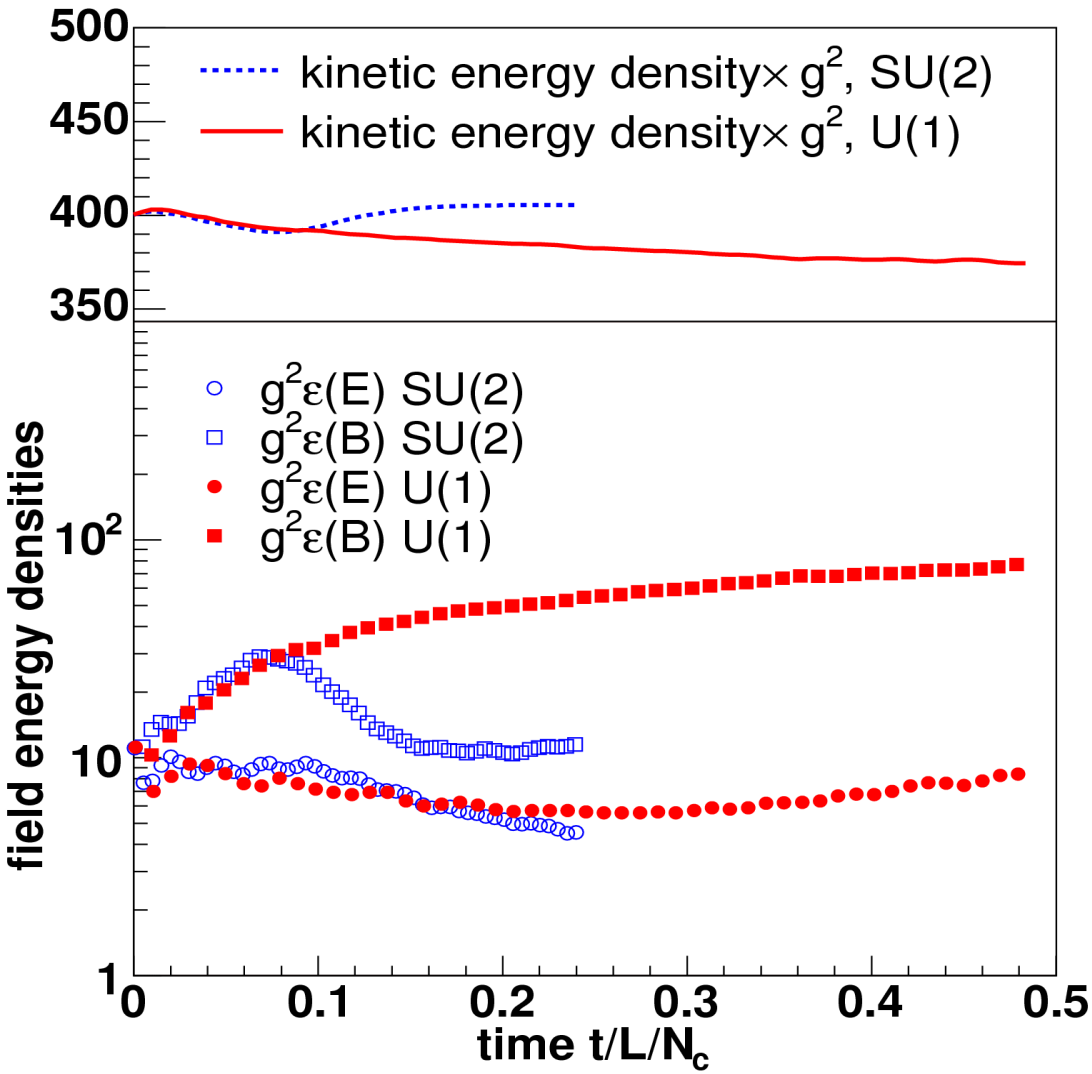


$$\kappa_z \equiv \frac{d\langle p_z^2 \rangle}{dt}$$

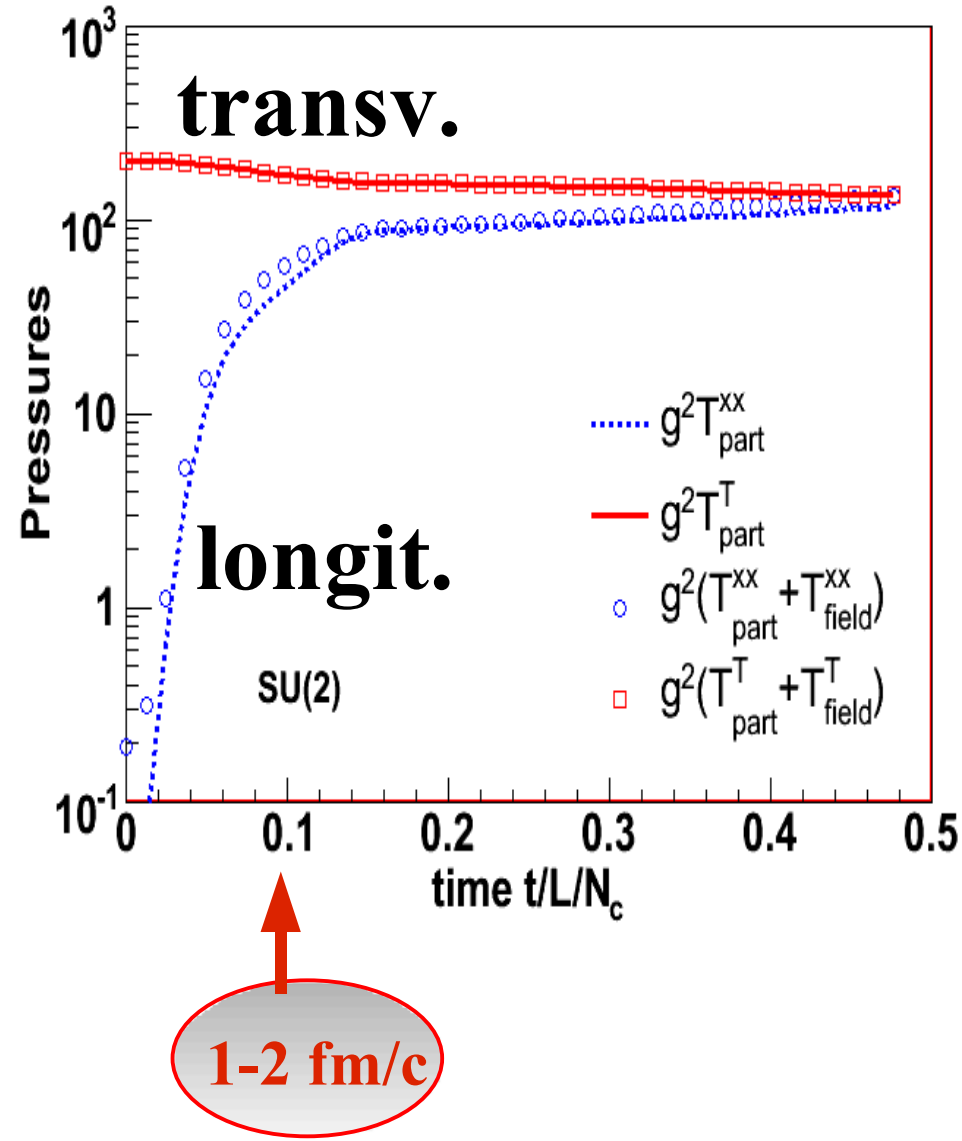
$$\kappa_\perp \equiv \frac{d\langle p_x^2 + p_y^2 \rangle}{dt}$$

$\kappa_z / \kappa_\perp = 2.3$

Large initial field fluctuations ($\rightarrow D_0$) will lead to rapid isotropization:



- Weak instability...



Outlook

Long way to go:

- Finite, expanding systems
- Other observables (η, dE/dx, ...)
- hard particle $1 \rightarrow 2$ (nearly) collinear splitting (LPM !)
- consistent treatment of $2 \leftrightarrow 3$ processes among hard particles