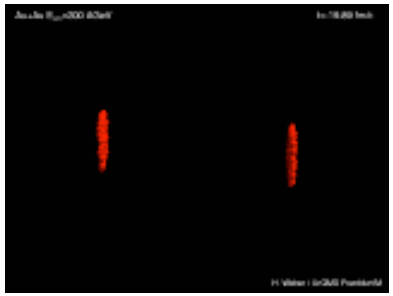
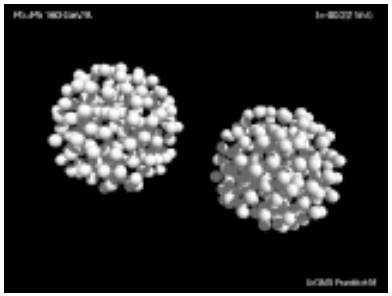
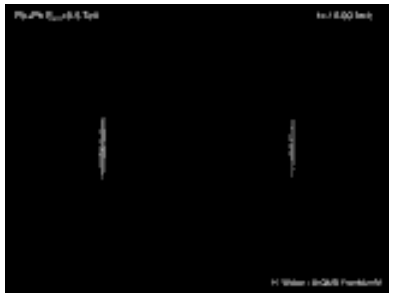




Viscous hydrodynamics and the QCD equation of state

Azwinndini Muronga [1,2]

[1] Centre for Theoretical Physics and Astrophysics

Department of Physics, University of Cape Town, South Africa

[2] UCT-CERN Research Centre

Department of Physics, University of Cape Town, South Africa

Workshop on "Hydrodynamics in Heavy Ion Collisions and QCD Equation of State"

April 21-22, 2008 : BNL, Long Island, NY, USA

References

- Literature: AM, AM and D.H. Rischke, D. Teaney; U. Heinz et al; T. Koide et al., P. Romatschke et al, …
- Recall talks by

P. Huovinen, M. Asakawa, C. Nonaka, T. Hirano, T. Csorgo,

- See also talks by

T. Koide, T. Kodama, K. Dusling, H. Song, R. Fries, P. Huovinen, N. Demir, A. I and Z. Xu

Or come to the 10 days workshop program on "Viscous Hydrodynamics and Transport Models"

Numerical and Conceptual Questions for Dissipative Relativistic Fluid Dynamics

- What is the underlying theory?
- What are the initial conditions?
- What are the boundary conditions?
- What is the underlying equation of state?
- What are the transport coefficients?
- Which numerical algorithm is implemented?
- Which tests are caried out?
- Numerical viscosity?
- What are the additional parameters?
- How is the freeze out implemented?

Non-Relativistic dissipative fluid dynamics equations

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} = 0$$

Where

$$U \equiv \begin{bmatrix} N \\ M^x \\ M^y \\ M^z \\ E \end{bmatrix}, \qquad F \equiv \begin{bmatrix} N v_x \\ M^x v_x + p + \prod + \pi^{xx} \\ M^y v_x + \pi^{xy} \\ M^z v_x + \pi^{xz} \\ \left(E + p + \prod + \pi^{xx}\right) v_x + \pi^{yx} v_y + \pi^{zx} v_z + q^x \end{bmatrix},$$

$$G \equiv \begin{bmatrix} N v_y \\ M^x v_y + \pi^{yx} \\ M^y v_y + p + \prod + \pi^{yy} \\ M^z v_y + \pi^{yz} \\ \left(E + p + \prod + \pi^{yy}\right) v_y + \pi^{xy} v_x + \pi^{zy} v_z + q^y \end{bmatrix}, \qquad H \equiv \begin{bmatrix} N v_z \\ M^x v_z + \pi^{zx} \\ M^y v_z + \pi^{zy} \\ M^z v_z + p + \prod + \pi^{zz} \\ \left(E + p + \prod + \pi^{zz}\right) v_z + \pi^{xz} v_x + \pi^{yz} v_y + q^z \end{bmatrix},$$

Relativistic dissipative fluid dynamics equations

- The equations of fluid dynamics

$$\partial_t U + \vec{\nabla} \bullet \vec{F}(U) = S$$

$$U = \begin{bmatrix} N \\ E \\ \vec{M} \\ \prod \\ \vec{q} \\ \overleftrightarrow{\pi} \end{bmatrix}, \quad \vec{F}(U) = \begin{bmatrix} \gamma \vec{v} \\ (E + p + \prod)\vec{v} + \gamma\left(\vec{q} - \left(\vec{q} \bullet \vec{v}\right)\vec{v}\right) + \overleftrightarrow{\pi} \bullet \vec{v} - \left(\left(\overleftrightarrow{\pi} \bullet \vec{v}\right) \bullet \vec{v}\right)\vec{v} \\ \vec{M} \otimes \vec{v} - (p + \prod)\overleftrightarrow{g} + \gamma\left(\vec{v} \otimes \vec{q} - \left(\vec{q} \bullet \vec{v}\right)\left(\vec{v} \otimes \vec{v}\right)\right) + \overleftrightarrow{\pi} - \left(\overleftrightarrow{\pi} \bullet \vec{v}\right) \otimes \vec{v} \\ \prod \vec{v} \\ \vec{q} \otimes \vec{v} \\ \overleftrightarrow{\pi} \otimes \vec{v} \end{bmatrix}$$

Relativistic dissipative fluid dynamics equations

$$S = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\gamma\tau_{\Pi}}\left(\Pi_E - \Pi\right) + \Pi\theta \\ \frac{1}{\gamma\tau_q}\left(\vec{q}_E - \vec{q}\right) + \vec{q}\theta \\ \frac{1}{\gamma\tau_\pi}\left(\overleftrightarrow{\pi}_E - \overleftrightarrow{\pi}\right) + \overleftrightarrow{\pi}\theta \end{bmatrix}$$

$$\Pi_E = -\zeta\Theta,$$

$$\vec{q}_E = -\kappa T\left(\gamma^2\vec{v}\partial_t \ln T - \overleftrightarrow{h}\bullet\vec{\nabla}\ln T + \vec{a}\right),$$

$$\overleftrightarrow{\pi}_E = 2\eta\overleftrightarrow{\sigma}$$

$$\Theta = \partial_t\gamma + \vec{\nabla}\bullet\left\{\gamma\vec{v}\right\}$$

$$\vec{a} = \gamma\partial_t\left\{\gamma\vec{v}\right\} + \gamma\vec{v}\bullet\vec{\nabla}\left\{\gamma\vec{v}\right\}$$

$$\overleftrightarrow{\sigma} = \frac{1}{2}\left\{\left(\vec{\nabla}\otimes\gamma\vec{v}\right) + \left(\vec{a}\otimes\gamma\vec{v}\right)\right\} - \frac{1}{3}\Theta\overleftrightarrow{h},$$

$$\theta = \vec{\nabla}\bullet\vec{v}, \quad \overleftrightarrow{h} \equiv \overleftrightarrow{g} - \gamma^2\vec{v}\otimes\vec{v}$$

Calculational frame vs local rest frame of the fluid

$$\vec{M} = \left(E + p + \prod\right)\vec{v} + \gamma\left(\vec{q} - \left(\vec{q}\bullet\vec{v}\right)\vec{v}\right) + \overleftrightarrow{\pi}\bullet\vec{v} - \left(\left(\overleftrightarrow{\pi}\bullet\vec{v}\right)\bullet\vec{v}\right),$$

$$\overleftrightarrow{P} = \vec{M}\otimes\vec{v} - \left(p + \prod\right)\overleftrightarrow{g} + \gamma\left(\vec{v}\otimes\vec{q} - \left(\vec{q}\bullet\vec{v}\right)\left(\vec{v}\otimes\vec{v}\right)\right) + \overleftrightarrow{\pi} - \left(\overleftrightarrow{\pi}\bullet\vec{v}\right)\otimes\vec{v},$$

$$\varepsilon = E - \left(\vec{M} + \sqrt{\left(1 - \vec{v}\bullet\vec{v}\right)}\vec{q}\right)\bullet\vec{v},$$

$$n = \sqrt{\left(1 - \vec{v}\bullet\vec{v}\right)}N.$$

$$\Lambda^{\mu}_{\nu} = \begin{pmatrix} \gamma & \gamma\vec{v}^{T} \\ \gamma\vec{v} & 1 + \left(\gamma - 1\right)v^{-2}\vec{v}\vec{v}^{T} \end{pmatrix}$$

$$\bar{q}^{\mu} = \left(0, Q^{x}, Q^{y}, Q^{z}\right)$$

$$\bar{\pi}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \tau^{xx} & \tau^{xy} & \tau^{xz} \\ 0 & \tau^{yx} & \tau^{yy} & \tau^{yz} \\ 0 & \tau^{zx} & \tau^{zy} & \tau^{zz} \end{pmatrix}$$

$$\pi^{\mu\nu} = \Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta}\bar{\pi}^{\alpha\beta},$$

$$q^{\mu} = \Lambda^{\mu}_{\alpha}\bar{q}^{\alpha}$$

Entropy 4-current, density and flux

Entropy 4-current

$$S^\mu = su^\mu + \beta q^\mu - \frac{1}{2}\beta(\beta_0\Pi^2 - \beta_1 q^\nu q_\nu + \beta_2 \pi^{\nu\lambda}\pi_{\nu\lambda})u^\mu - \beta(\alpha_0 \Pi q^\mu - \alpha_1 \pi^{\mu\nu} q_\nu)$$

Entropy density

$$s(\varepsilon, n, \Pi, q^\nu, \pi^{\nu\lambda}) = s_{eq}(\varepsilon, n) - \frac{1}{2}\beta(\beta_0\Pi^2 - \beta_1 q^\nu q_\nu + \beta_2 \pi^{\nu\lambda}\pi_{\nu\lambda})u^\mu$$

Entropy flux $\Phi^\nu = \beta(q^\nu - \alpha_0 \Pi q^\nu - \alpha_1 q_\lambda \pi^{\nu\lambda})$

Second Law of thermodynamics

$$\beta^{-1}\partial_\mu S^\mu = \zeta^{-1}\Pi^2 - \lambda^{-1} q^\nu q_\nu + (2\eta)^{-1}\pi^{\nu\lambda}\pi_{\nu\lambda} \geq 0$$

The 3 new coefficients in the entropy density are related to the relaxation times and are responsible for causality while the 2 new coefficients in the entropy flux are related to the relaxation length and are responsible for the coupling between heat flow and viscous stresses

Relaxation equations for dissipative fluxes

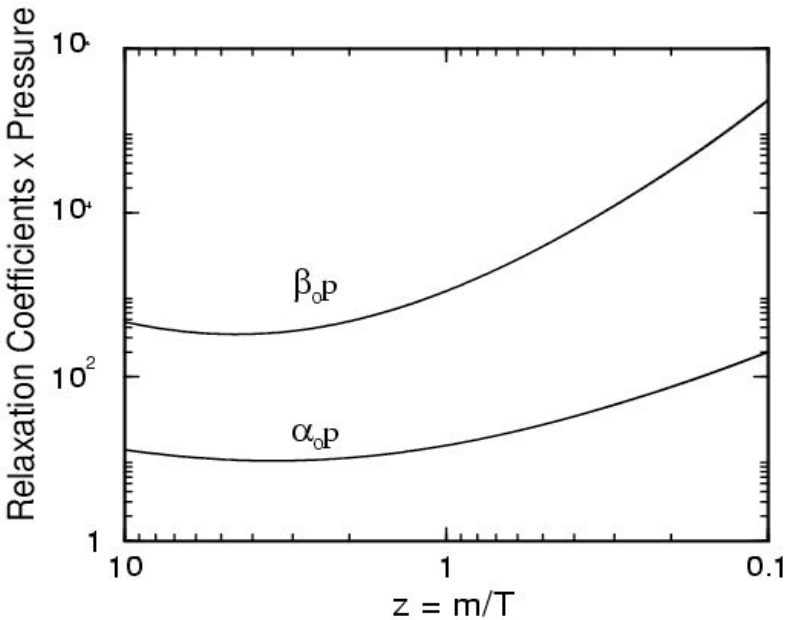
Relaxation equations for the dissipative fluxes

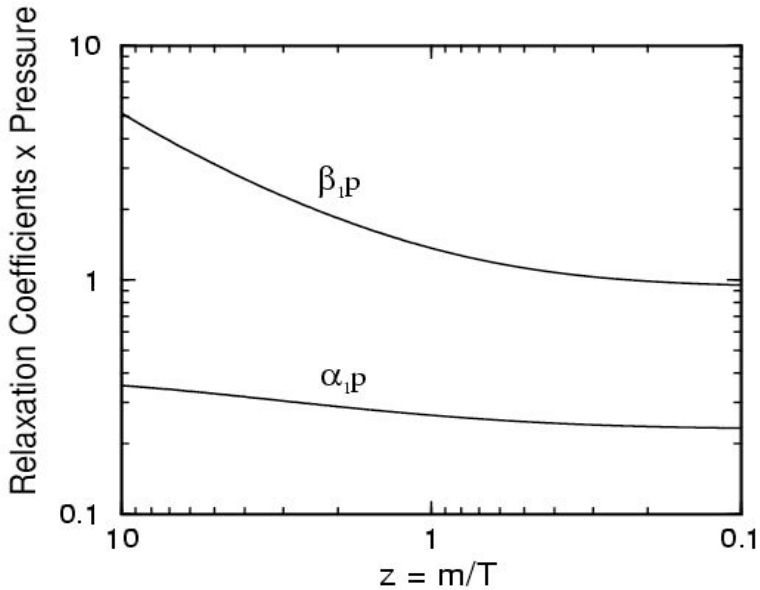
$$
\begin{aligned}
\tau_\Pi \dot{\Pi} + \Pi &= -\zeta\Theta + \lambda_{\Pi q}\nabla_q^\mu \\
\tau_q \dot{q}^\mu + q^\mu &= \kappa T\left(\frac{\nabla^\mu}{T} - a^\mu\right) - \lambda_{q\pi}\nabla_\nu \pi^{\nu\mu} - \lambda_{q\Pi}\nabla^\mu \Pi \\
\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + \lambda_{\pi q}\nabla^{\langle\mu} q^{\nu\rangle}
\end{aligned}
$$

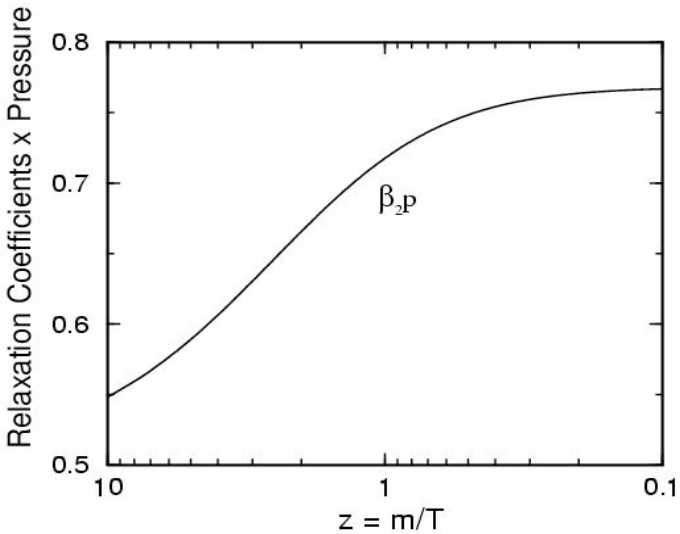
Transport and relaxation times/lengths

$$
\begin{aligned}
\tau_\Pi &= \zeta\beta_0, & \tau_q &= \kappa T\beta_1, & \tau_\pi &= 2\eta\beta_2 \\
\lambda_{\Pi q} &= \zeta\alpha_0, & \lambda_{q\Pi} &= \kappa T\alpha_0, & \lambda_{\pi q} &= 2\eta\alpha_1
\end{aligned}
$$

Relaxation Coefficients







Entropy density and entropy production

Entropy density

$$\mathrm{s}(\varepsilon, n, \Pi, q, \pi) = s_{eq}(\varepsilon, n) - \Psi(\varepsilon, n, \Pi, q, \pi)$$

with

$$\Psi(\varepsilon, n, \Pi, q, \pi) \equiv \tfrac{1}{2}\beta\left(\beta_o \Pi^2 - \beta_1 q^2 + \beta_2 \pi^2\right)$$

Entropy production

$$(\mathrm{D}\mathrm{s} + \mathrm{s}\Theta) = \Phi \geq 0$$

where

$$\Phi \equiv \beta(\zeta^{-1}\Pi^2 + \lambda^{-1}q^2 + (2\eta)^{-1}\pi^2)$$

Relevant ratios

$$\frac{s\left(\varepsilon, n, \Pi, q^{\nu}, \pi^{\nu\lambda}\right)}{s_{eq}\left(\varepsilon, n\right)} = 1 - \frac{\Psi\left(\varepsilon, n, \Pi, q, \pi\right)}{s_{eq}\left(\varepsilon, n\right)}$$

$$\mathrm{D}s = -\left(1 - \frac{\Phi}{s\Theta}\right)s\Theta$$

Shear viscosity in the scaling solution

Energy equation $\frac{\partial s}{\partial \tau} = \left(R^{-1} - 1\right)\frac{s}{\tau}$ where $R^{-1} = \frac{\Phi}{Ts}$

$$\Phi^{ideal} = 0$$

$$\Phi^{1st} = \frac{4}{3}\eta\frac{1}{\tau}$$

$$\frac{d\Phi^{2nd}}{d\tau} = -\frac{\Phi^{2nd}}{\tau_\pi} - \frac{1}{2}\Phi^{2nd}\left(\frac{1}{\tau} + \frac{T}{\beta_2}\frac{d}{d\tau}\left(\frac{\beta_2}{T}\right)\right) + \frac{2}{3}\frac{1}{\beta_2}\frac{1}{\tau}$$

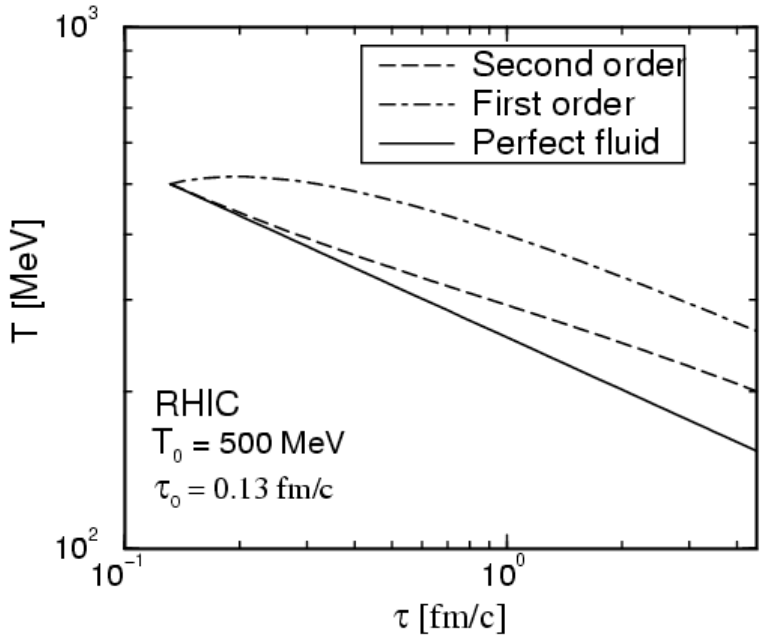
Reynold's number vs eta/s

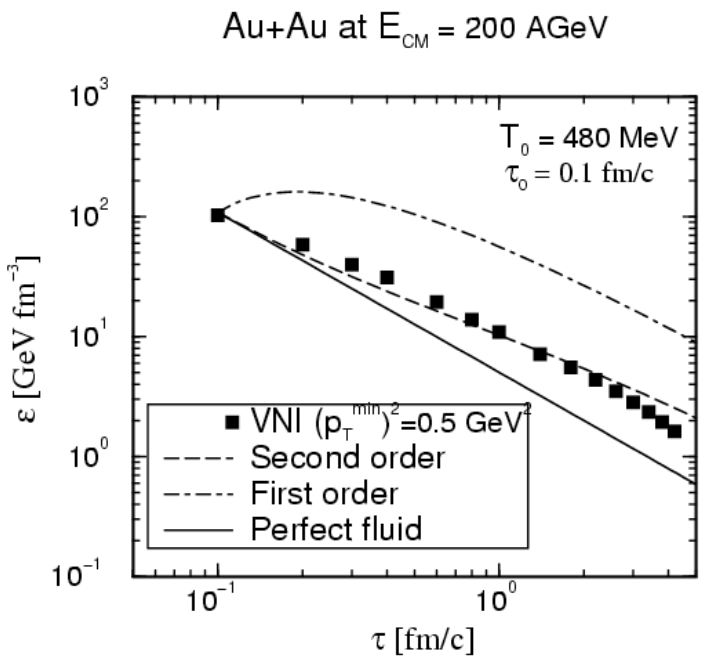
$$s = 4aT^3, \qquad \eta = bT^3, \qquad \beta_2 = \frac{3}{4aT^4}, \qquad \tau_\pi = \frac{3}{2}\frac{b}{a}\frac{1}{T}$$

$$\frac{\eta}{s} = \frac{1}{4}\frac{b}{a}$$

$$R^{-1} = \frac{1}{3}\frac{b}{a}\frac{1}{T\tau}$$

Time evolution of thermodynamic quantities





QCD EoS

Entropy equation of state

$$\frac{s}{s_c}(T) = \left[\frac{T}{T_c}\right]^3 \left(1 + \frac{d_Q - d_H}{d_Q + d_H} \tanh\left[\frac{T - T_c}{\Delta T}\right]\right)$$

$$\text{where} s_c = const. \times \frac{1}{2}\left(d_Q + d_H\right) T_c^3$$

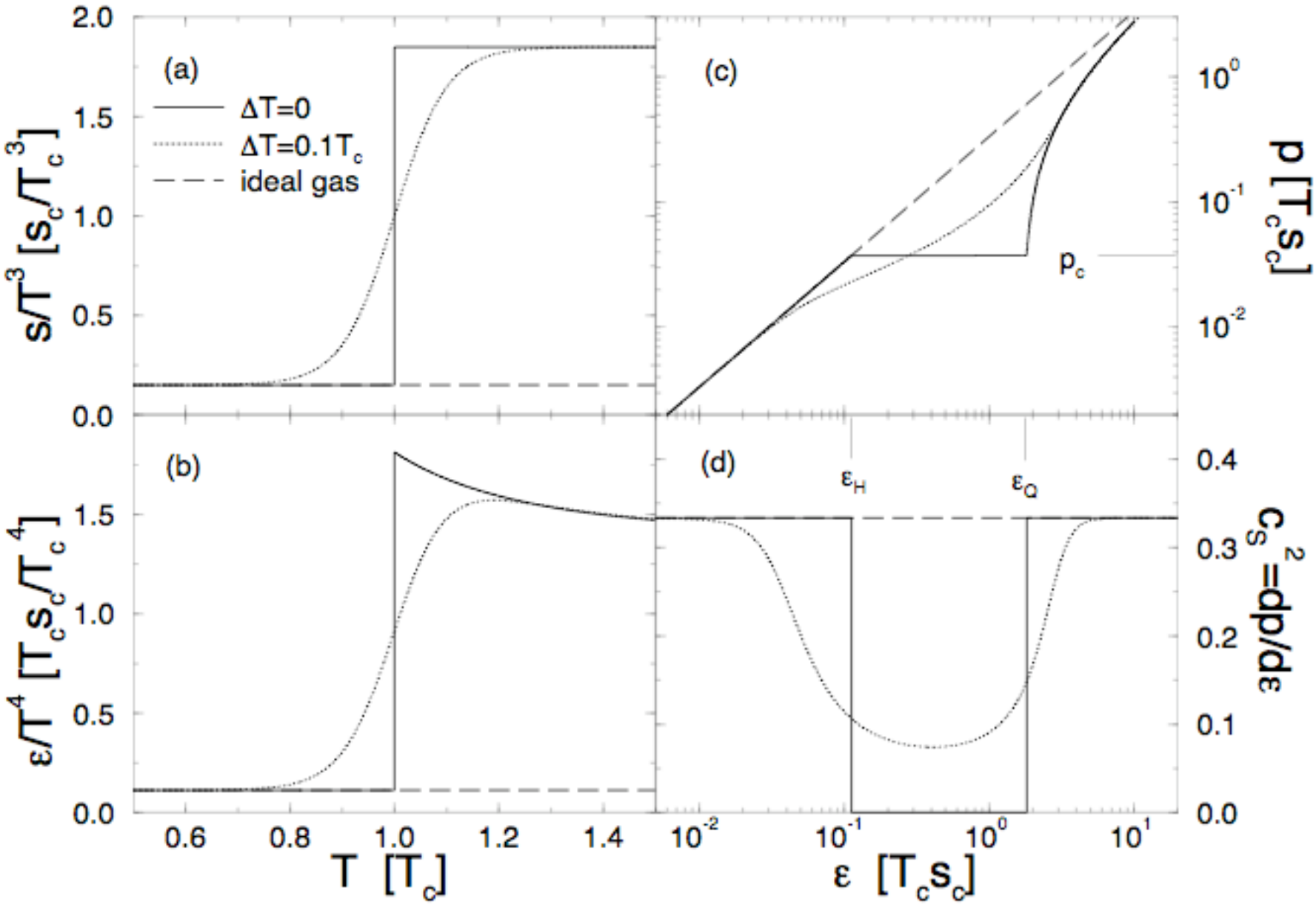
$$\text{For} \Delta T = 0, \quad \text{MIT bag EOS}$$

$$B = \frac{1}{2}\left[\frac{\frac{d_Q}{d_H} - 1}{\frac{d_Q}{d_H} + 1}\right] T_c s_c$$

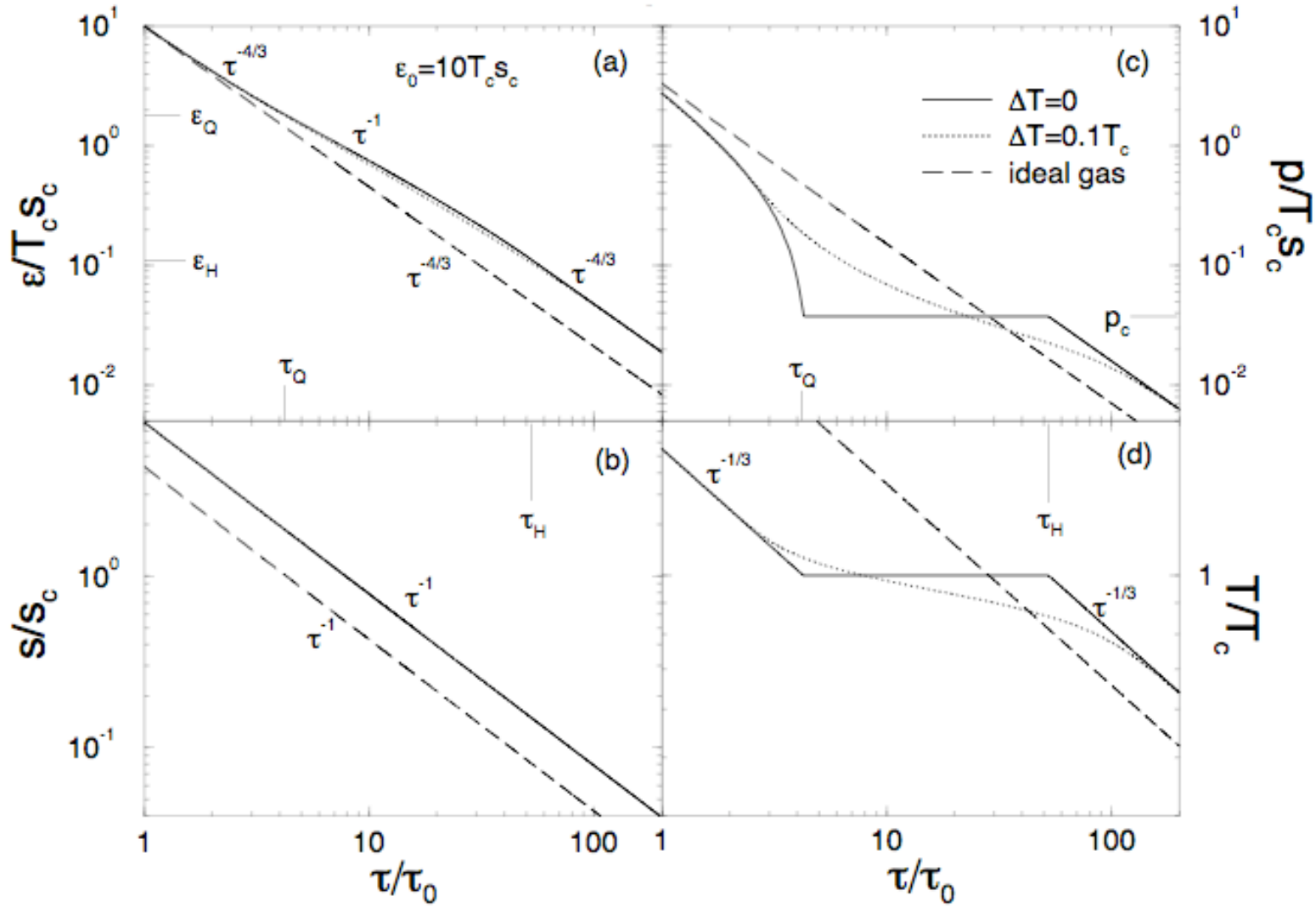
$$\varepsilon_Q - \varepsilon_H = 4B$$

$$\varepsilon_Q = \frac{1}{2}\left[\frac{4\frac{d_Q}{d_H} - 1}{\frac{d_Q}{d_H} + 1}\right] T_c s_c, \qquad \varepsilon_H = \frac{3}{2}\left[\frac{1}{\frac{d_Q}{d_H} + 1}\right] T_c s_c$$

Time evolution of thermodynamic quantities



Time evolution of thermodynamic quantities



Beyond the idealistic view of space-time evolution of relativistic heavy ion collisions

This talk discussed the intriguing questions of the non-equilibrium fluid dynamical description of the space-time evolution of the relativistic heavy ion collisions. This description is quite different from the equilibrium description that we have learned to accept as the one that works.

It would be interesting to explore new non-equilibrium phenomena by combining the knowledge of non-equilibrium relativistic fluid dynamics and relativistic kinetic/transport theory in the description of the space-time evolution of relativistic heavy ion collisions.