Viscous hydrodynamics in different Israel-Stewart formalisms

Huichao Song
The Ohio State University
Ulrich Heinz
The Ohio State University & CERN

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References:

U. Heinz, H. Song and A. K. Chaudhuri, Phys. Rev. C73, 034904 (2006)
H. Song and U. Heinz, Phys. Lett. B658, 279 (2008)
H. Song and U. Heinz, arXiv:0712.3715[nucl-th] , PRC (in press)

04/22/2008

The QGP shear viscosity

- ideal hydro is a great success in describing RHIC data: spectra and v2
- quantum mechanics excludes the possibilities of a perfectly ideal fluid with zero viscosity-to-entropy ratio

- Weakly coupled QCD prediction: $T >> \Lambda_{QCD}$ P. Arnold, G.Moore & L.Yaffe '00,'03

$$\eta = \frac{T^3}{(\alpha_s)^2 \ln(1/\alpha_s)}$$

$$\eta/s = 0.15 \sim 1.5$$

However, to show liquid behavior the QGP must be a strongly coupled system.

- Strongly coupled AdS/CFT prediction:

AdS/CFT correspondence: gauge/gravity duality

4d gauge theory at strong coupling ⟷ 5d gravity at weak coupling

N=4 SYM ⟷ Type IIB superstring theory on $AdS_5 \times S^5$

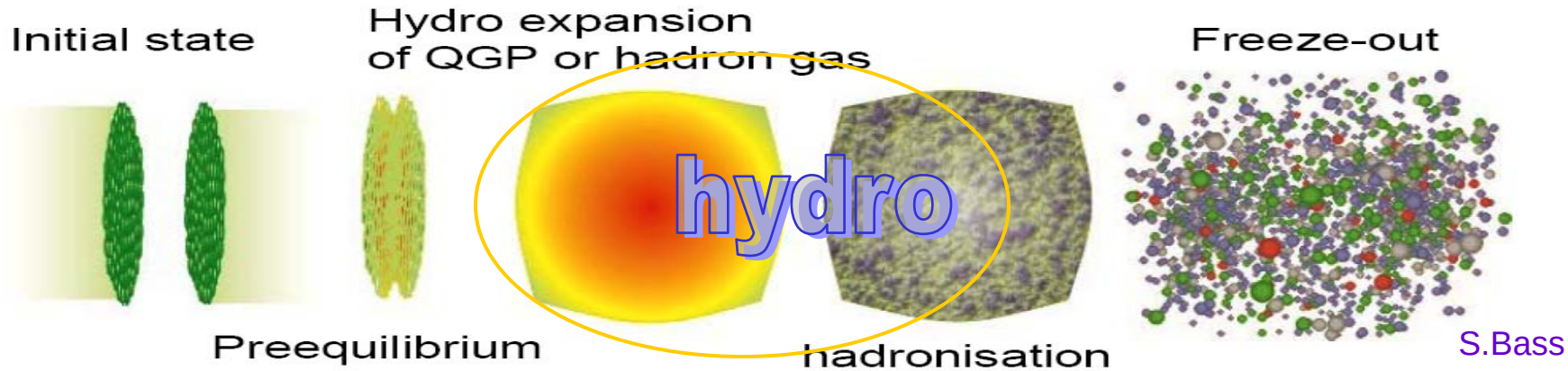
$$\eta/s \geq 1/4\pi \approx 0.08$$

D.T. Son et al. '01,'05

(not related to real QCD)

To extract the QGP viscosity from experimental data, we need viscous hydrodynamics

Viscous hydrodynamics



Conservation laws:

$$\partial_\mu T^{\mu\nu}(x) = 0 \qquad T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \pi^{\mu\nu}$$

Evolution equations for shear pressure tensor $\pi^{\mu\nu}$:

$$\tau_\pi \Delta^{\alpha\mu}\Delta^{\beta\nu}\dot{\pi}_{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$$

-simplified Israel-Stewart eqn.

Input: "EOS" $\varepsilon = \varepsilon(p,n)$ initial conditions and final conditions

With $\eta \to 0$ **viscous hydrodynamics** reduces to **ideal hydrodynamics**

A further simplification: Bjorken approximation $v_z = z/t$

Reduces (3+1)-d hydrodynamics to (2+1)-d hydrodynamics (τ, x, y, η)

(2+1)-d viscous hydrodynamics

-Romatschke & Romatschke: full I-S eqn. EOS I EOS L*

PRL'07 **Au+Au,** $T_{dec} = 150MeV$ (EOS L* here is the quasi-particle one based on lattice QCD)

-Song & Heinz: simplified I-S eqn. & full I-S eqn. EOS I SM-EOS Q EOS L

PLB'08 & arXiv:0712.3715[nucl-th] **Cu+Cu,** simplified I-S eqn., $T_{dec} = 130MeV$

(**Au+Au**, **Cu+Cu**, system size effects, full I-S eqn. vs. simplified I-S eqn., EOS L etc, in preparation)

-Dusling & Teaney: Öttinger-Grmela (O-G) eqn. EOS I

PRC'08 **Au+Au,** decoupling by scattering rate, arXiv:0803.1262 [nucl-th], (dilepton production)

-Huovinen & Molnar: full I-S eqn. EOS I

QM08 talk: comparing the results from viscous hydro and from transport model

-Chaudhuri: simplified I-S eqn. EOS I EOS Q

arXiv:0708.1252 [nucl-th], arXiv:0801.3180 [nucl-th], arXiv:0803.0643 [nucl-th] **Au+Au**

Issues:

- verification of the codes individually developed by different groups
- effects from different 2nd order formalisms

  simplified I-S eqn. vs. full I-S eqn., I-S eqn. vs. O-G eqn.
- effects from different EoS, systems sizes and freeze-out procedures

Will show numerical results for:

- ideal hydro vs. viscous hydro

Cu+Cu, simplified I-S eqn., Song & Heinz, PLB07 & arXiv:0712.3715[nucl-th]

- viscous hydro in different I-S formalisms

Simplified Israel-Stewart equation vs. full Israel-Stewart equation:

As $\tau_\pi \rightarrow 0$, simplified I-S eqn. and full I-S eqn. approach the same Navier-Stokes limit

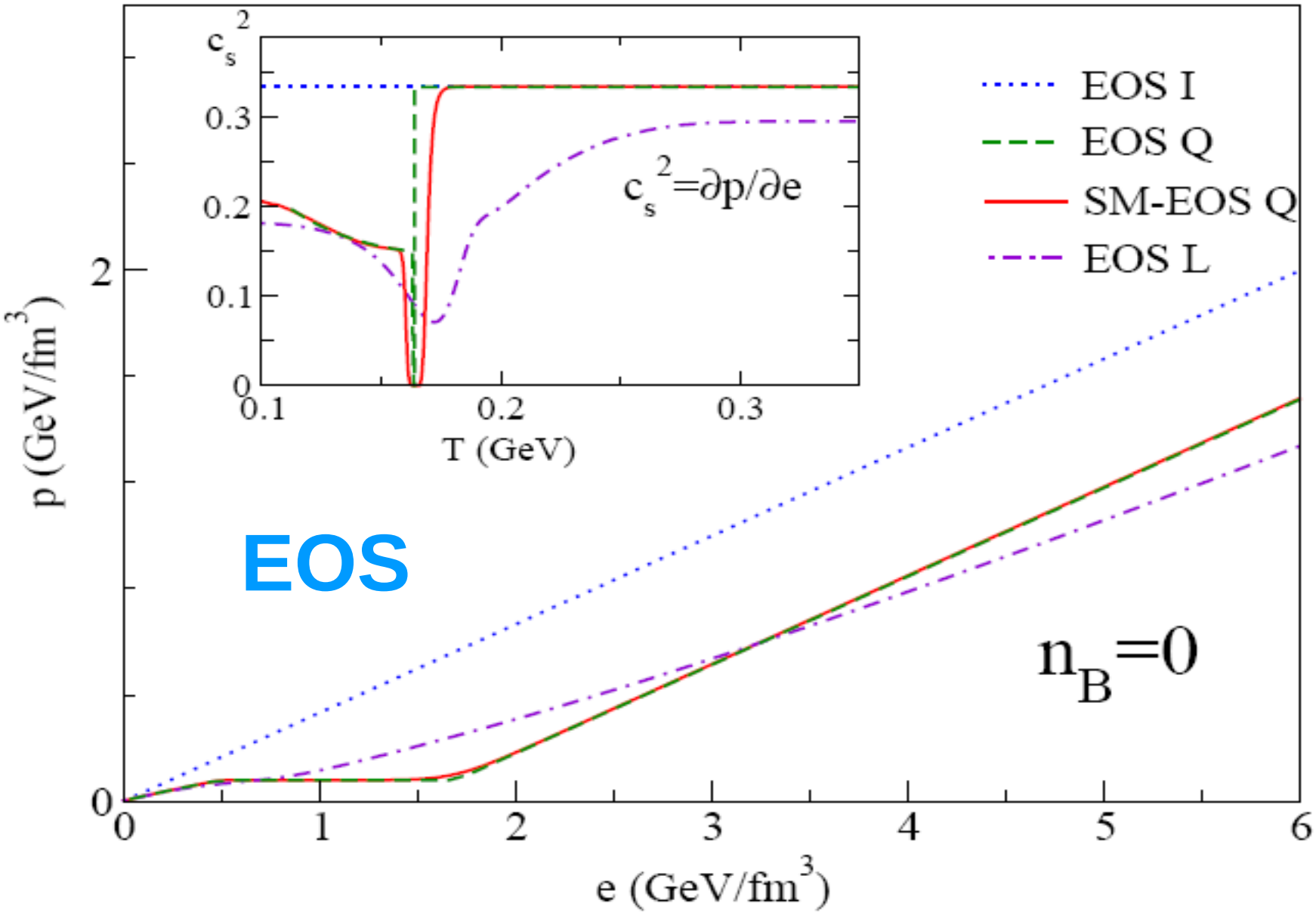
For EOS with a phase transition, the differences between simplified I-S eqn. and full I-S eqn. are small (but not for EOS I!)

- system size effects:

v_2 / ε multiplicity scaling: ideal vs. viscous, simplified vs. full I-S

- entropy production

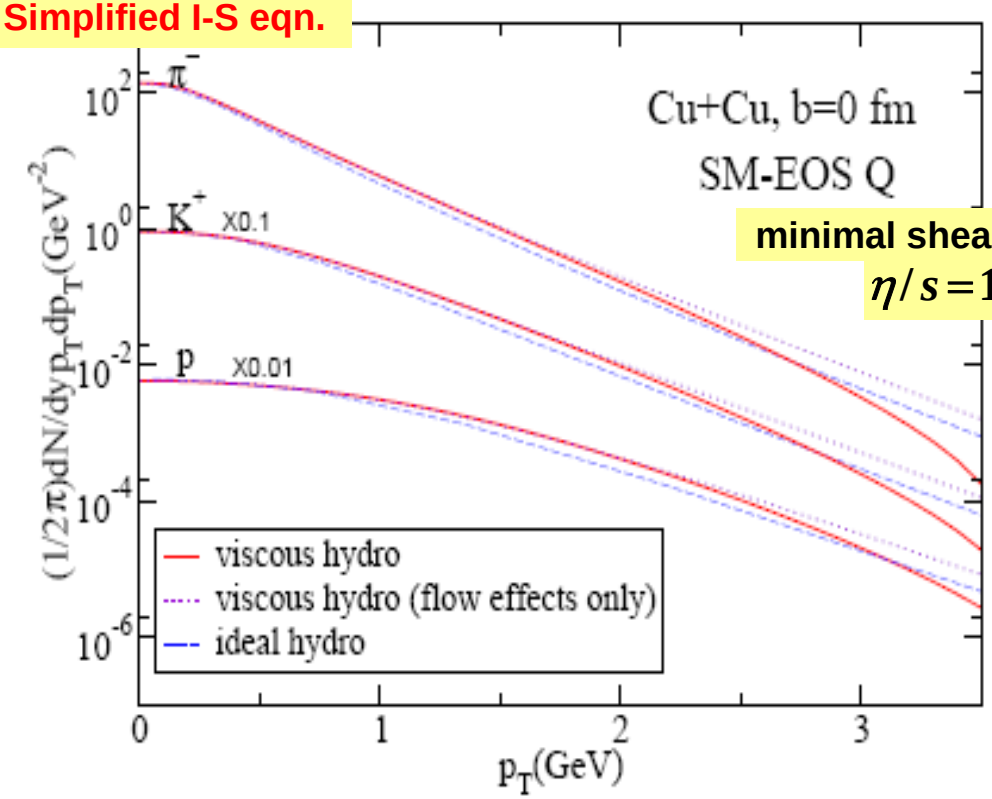
EOS


EOS
c_s^2
0.3
0.2
0.1
0
0.1
0.2
0.3
T (GeV)
$c_s^2=\partial p/\partial e$
EOS I
EOS Q
SM-EOS Q
EOS L
$n_B=0$
p (GeV/fm^3)
e (GeV/fm^3)
0
1
2
3
4
5
6

Viscous vs. ideal hydro – spectra & elliptic flow

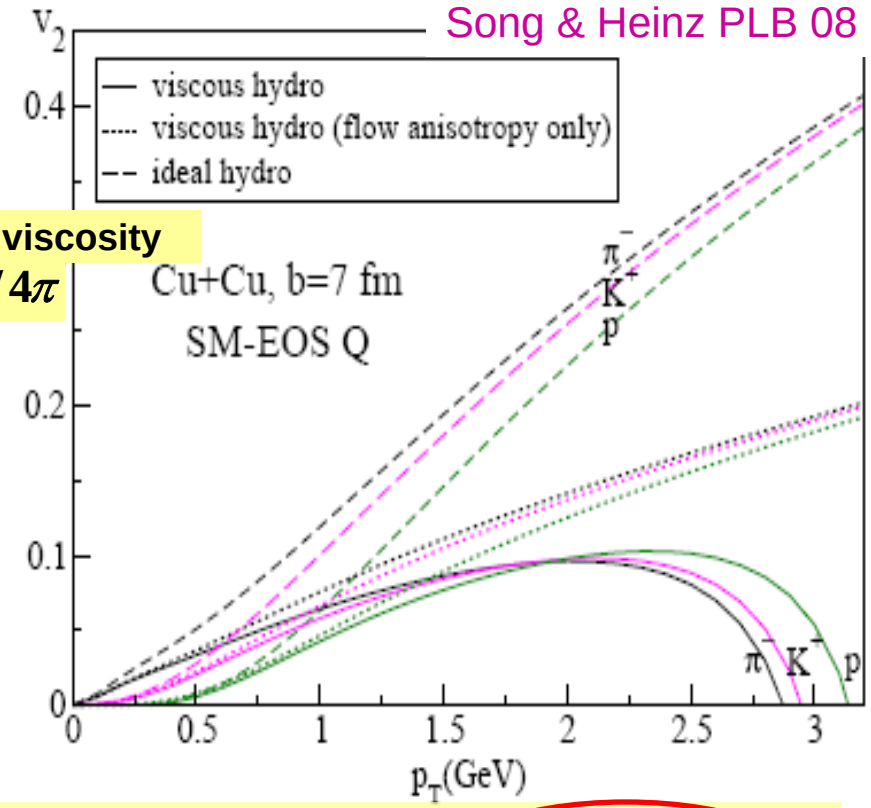
Simplified I-S eqn.

Song & Heinz PLB 08



minimal shear viscosity

$\eta/s = 1/4\pi$

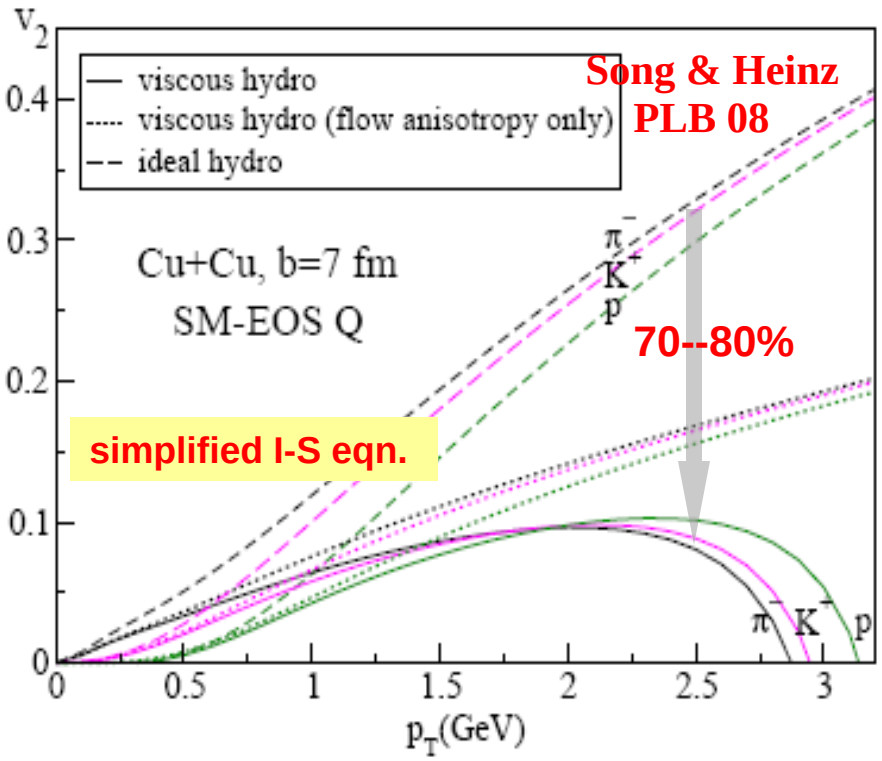


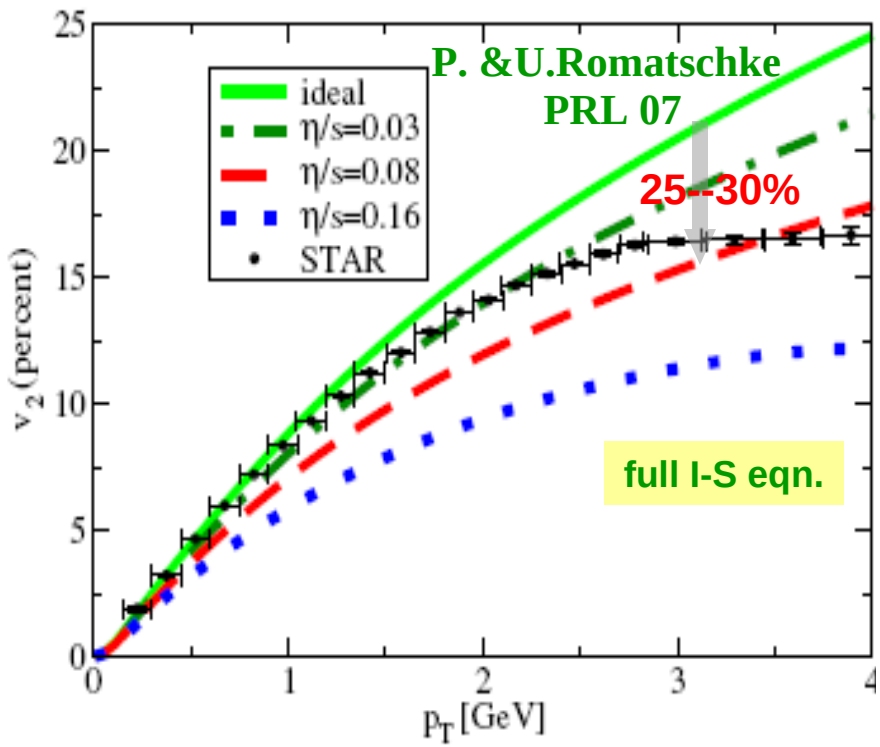
$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p\cdot d^3\sigma(x)}{2\pi^3}\left[\boxed{f_{eq}(x,p)} + \delta f(x,p)\right] = \int_\Sigma \frac{p\cdot d^3\sigma(x)}{(2\pi)^3} f_{eq}(x,p)\left(1+\frac{1}{2}\frac{p^\alpha p^\beta}{T^2(x)}\frac{\pi_{\alpha\beta}(x)}{(e+p)(x)}\right)$$

-More radial flow, flatter spectra; elliptic flow is very sensitive to shear viscosity

-Both the evolution correction (viscous correction to flow in f_0) and spectra correction (viscous correction to δf through $\pi^{\mu\nu}$) have significant effects on v_2.
For low p_T, the viscous correction to the flow is dominant.

Comparison with Romatschke 07 results





- different systems & EOS: CuCu, b=7, SM-EOS Q vs. Au+Au, min bias, EOS Lattice

- different Isreal-Stewart eqns. used: simplified I-S eqn. vs. full I-S eqn.

Effect of using different I-S eqns.?

Simplified I-S eqn. vs. full I-S eqn.:

simplified I-S eqn.:

$$\Delta^{\mu\alpha}\Delta^{\nu\beta} D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right]$$

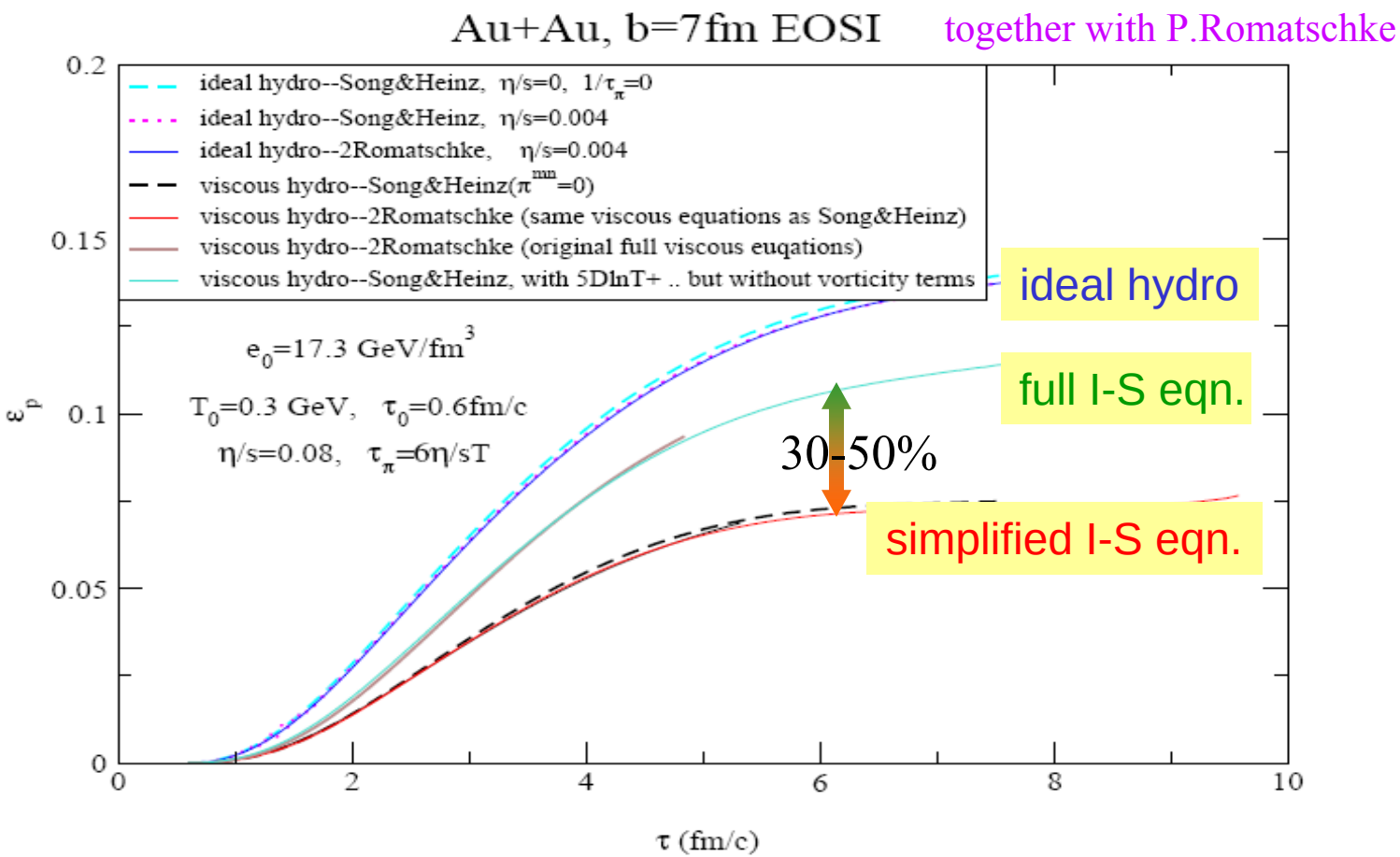
full I-S eqn.:

$$\Delta^{\mu\alpha}\Delta^{\nu\beta} D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right] + \frac{1}{2}\pi^{\mu\nu}\left[5D\ln T - \nabla_\alpha u^\alpha\right] - 2\pi^{\alpha(\mu}\omega_\alpha^{\nu)}$$

important for preserving the conformal symmetry

(Baier et al. ‘07)

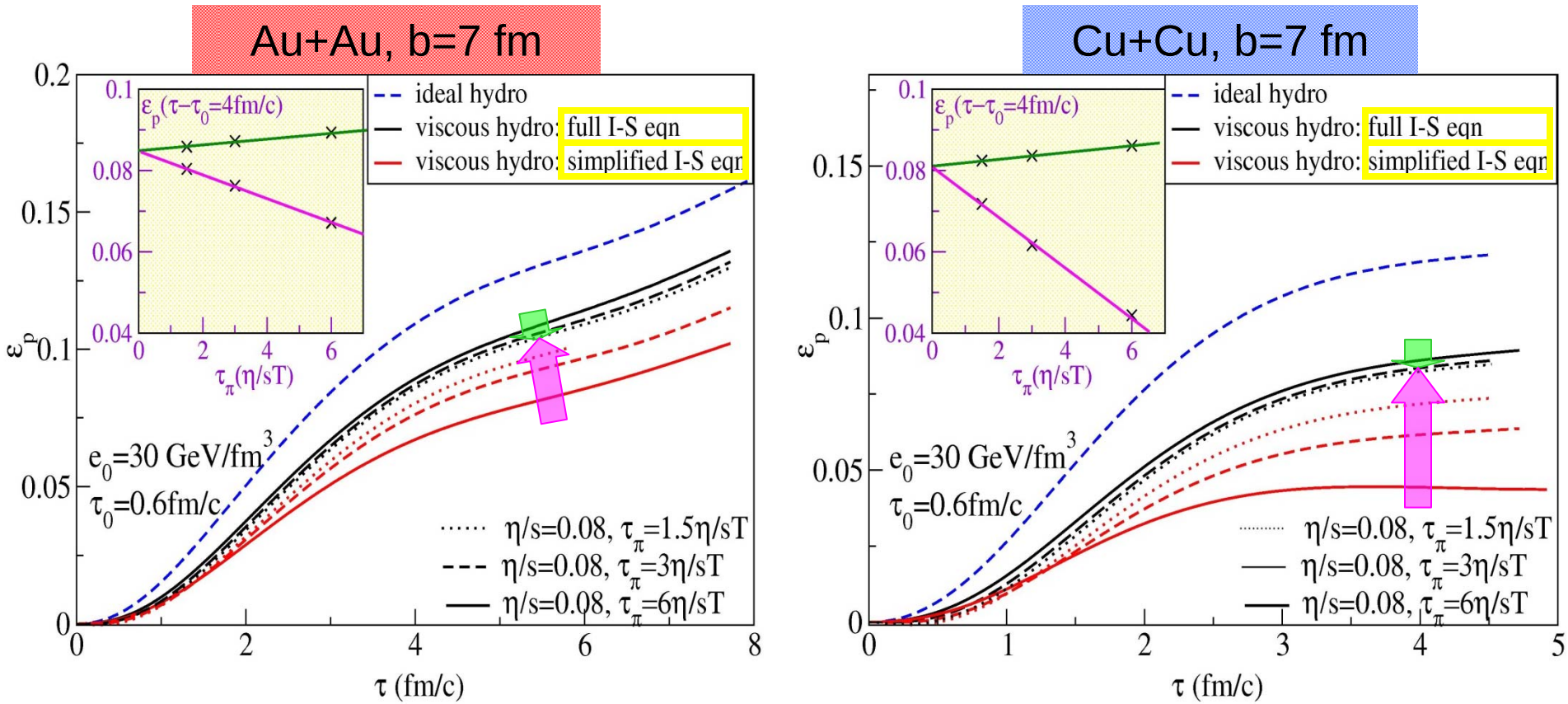
Code testing: VISH2+1 vs. Romatschke code



- for identical initial conditions, EOS (EOS I here), transport eqns. (full I-S eqn. or simplified I-S eqn.), the two codes agree well with each other

- But: the additional terms in the full I-S eqn. bring big differences for the late-stage momentum anisotropy and final elliptic flow ?!

simplified I-S eqn. vs. full I-S eqn. EOS I



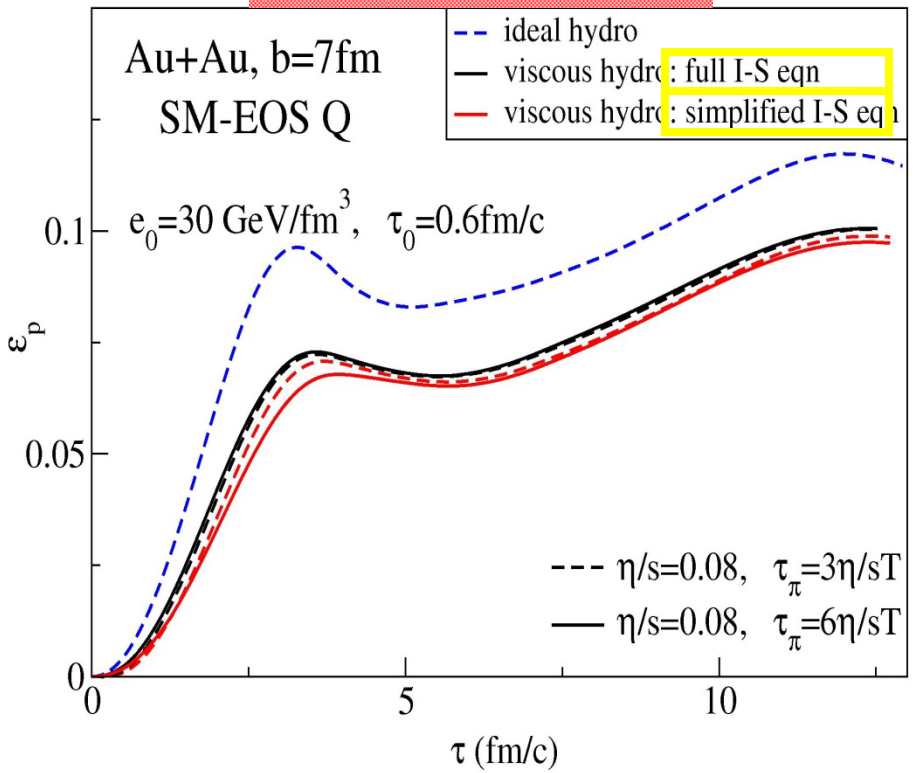
Momentum anisotropy evolution: simplified I-S eqn. vs. full I-S eqn. with different τ_π :

- for EOS I, the additional terms in full I-S eqn. bring 30-50% difference in the late-time momentum anisotropy and final v_2 suppression

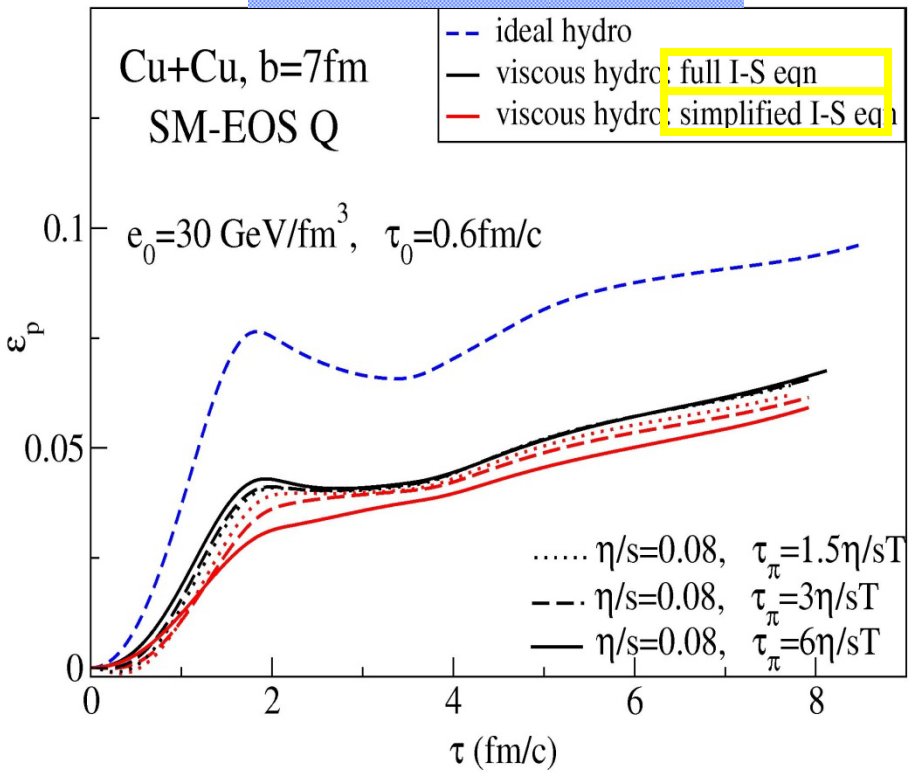
- numerical simulations also show that simplified I-S eqn. and full I-S eqn. approach the same Navier-Stokes limit as $\tau_\pi \rightarrow 0$, but the full I-S eqn. shows much weaker sensitivity to τ_π

simplified I-S eqn. vs. full I-S eqn.: EOS Q

Au+Au, b=7 fm



Cu+Cu, b=7 fm

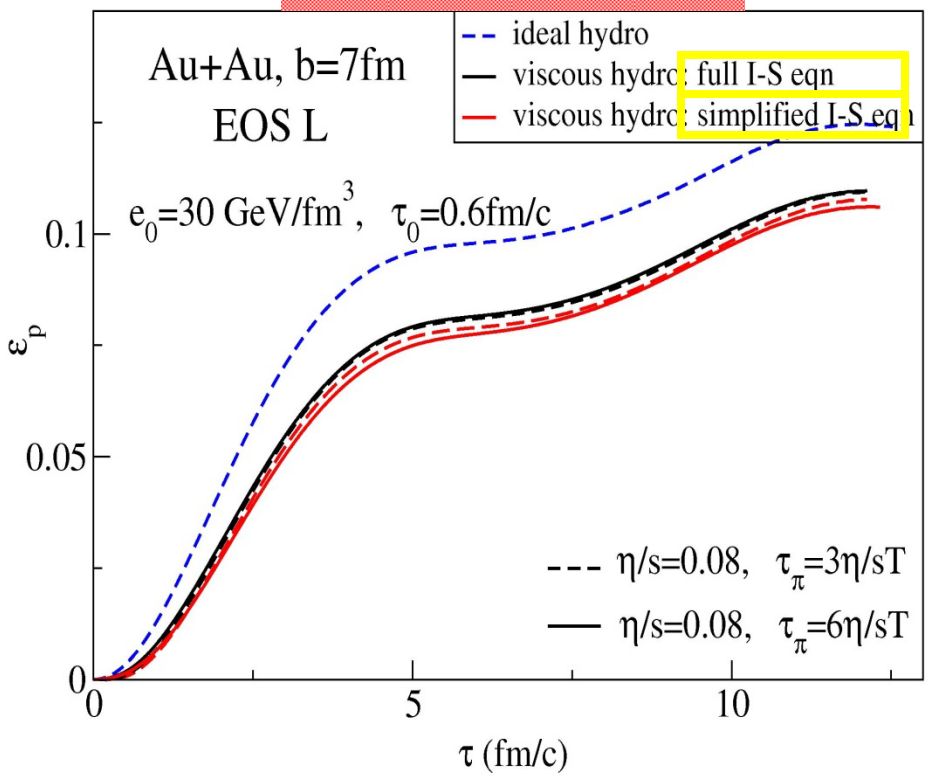


Momentum anisotropy evolution: simplified I-S eqn. vs. full I-S eqn. with different τ_π :

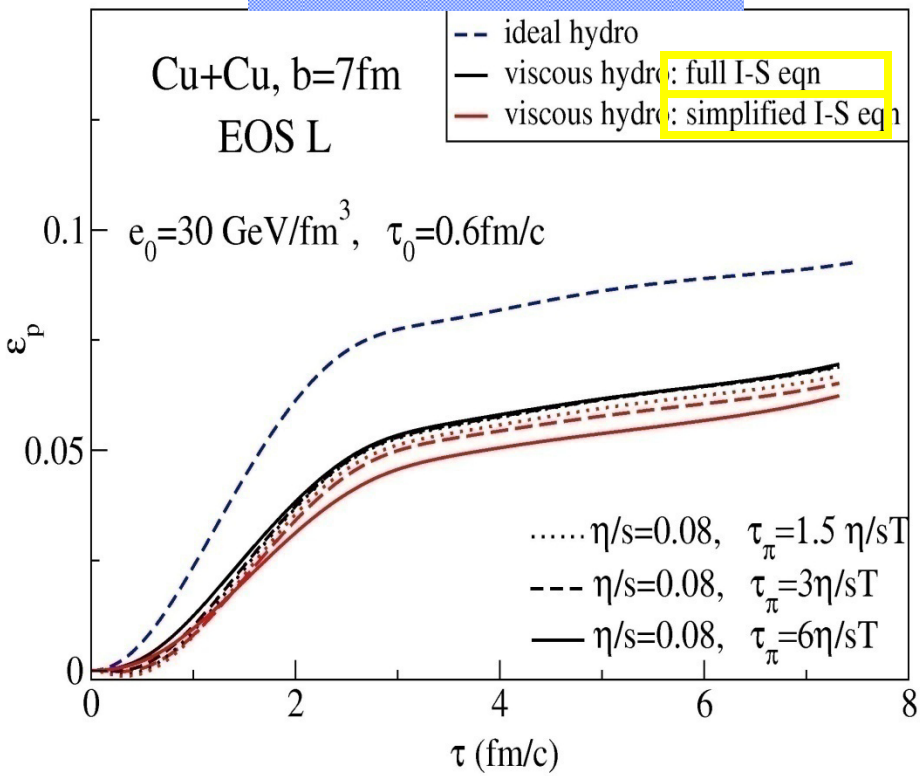
- simplified I-S eqn. vs. full I-S eqn. only: 5-10% differences (for SM-EOS Q)
- for smaller systems, the difference between full and simplified I-S eqns. increases
- for simplified I-S eqn. the sensitivity to the value of the relaxation time is stronger

simplified I-S eqn. vs. full I-S eqn.: EOS L

Au+Au, b=7 fm



Cu+Cu, b=7 fm

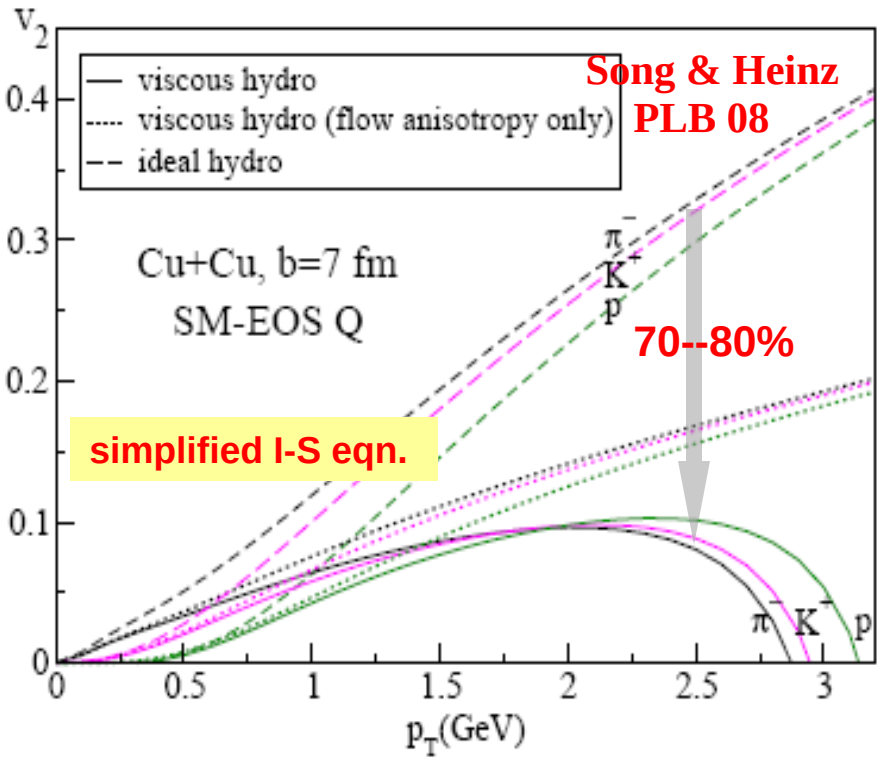


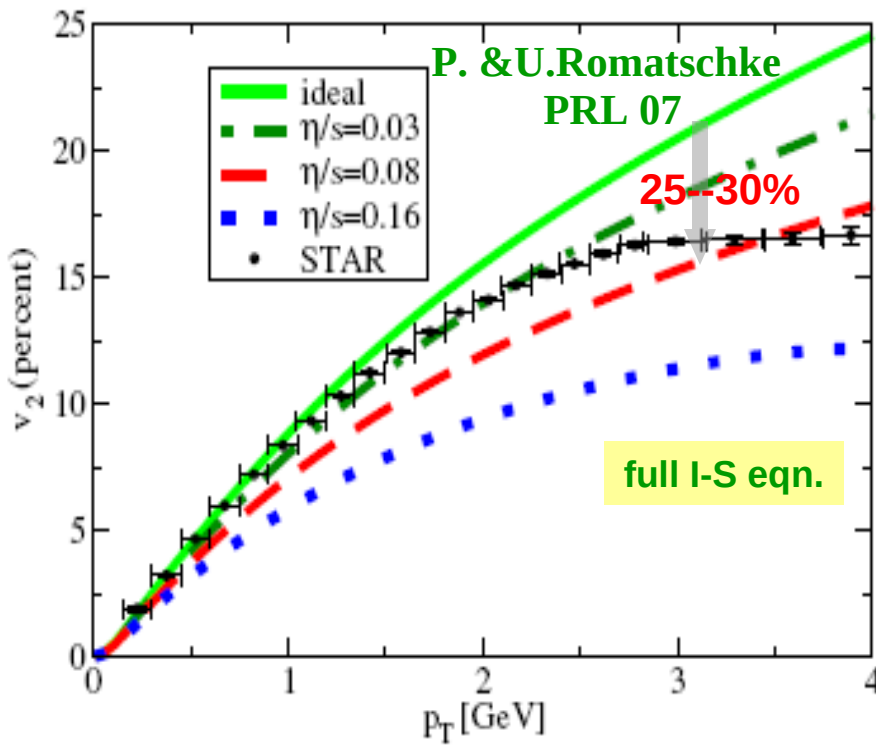
Momentum anisotropy evolution: simplified I-S eqn. vs. full I-S eqn. with different τ_π :

- for **EOS Q** and **EOS L**, viscous effects are largely similar

- for realistic EOS with a phase transition, the difference between simplified and full I-S eqns. for the viscous suppression of v_2 are small if the systems are not too small and the initial energy density is not too low

Comparison with Romatschke 07 results



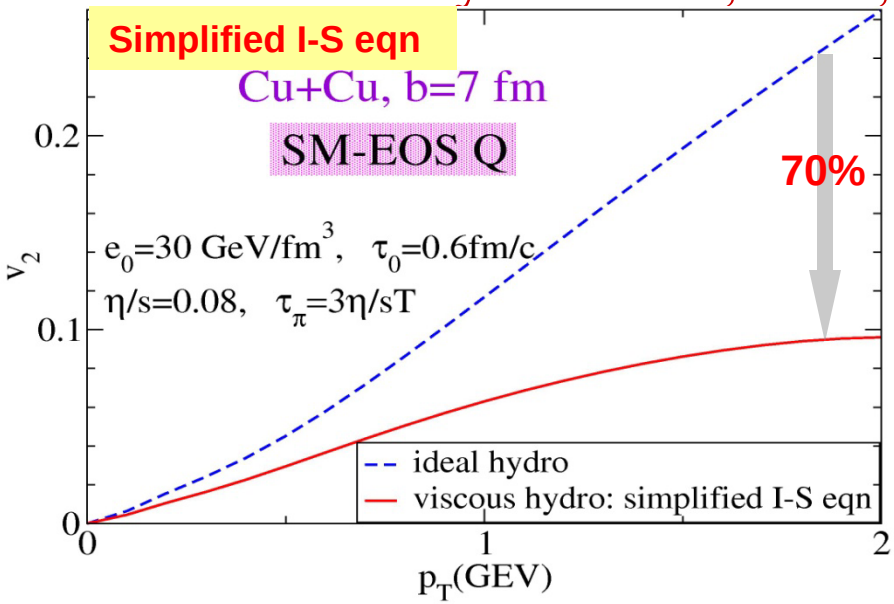


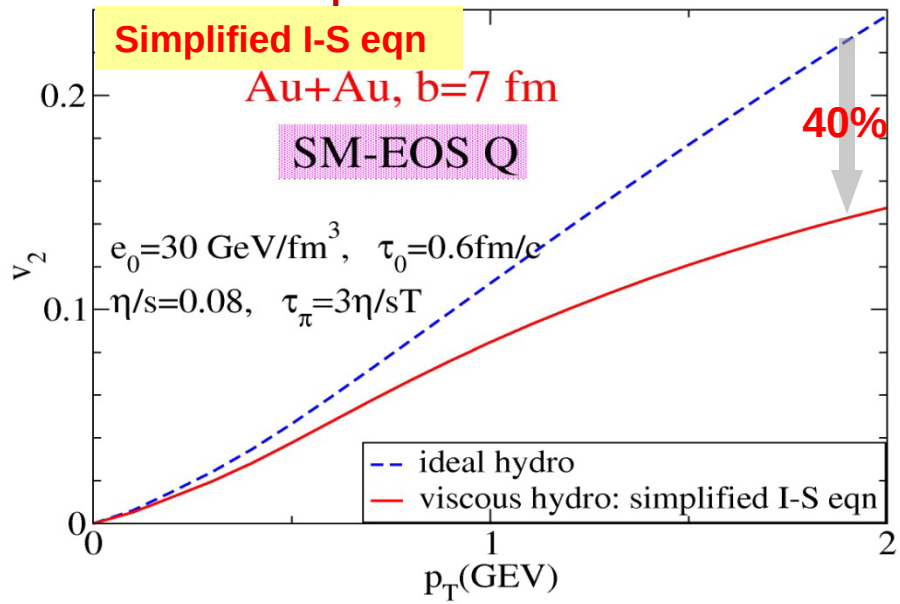
- different Isreal-Stewart eqns. used: simplified I-S eqn. vs. full I-S eqn.
- different systems & EoS: CuCu, b=7, SM-EOS Q vs. Au+Au, min bias, EOS Lattice

System size effects and EOS ?

Different effects contributing to v_2 suppression

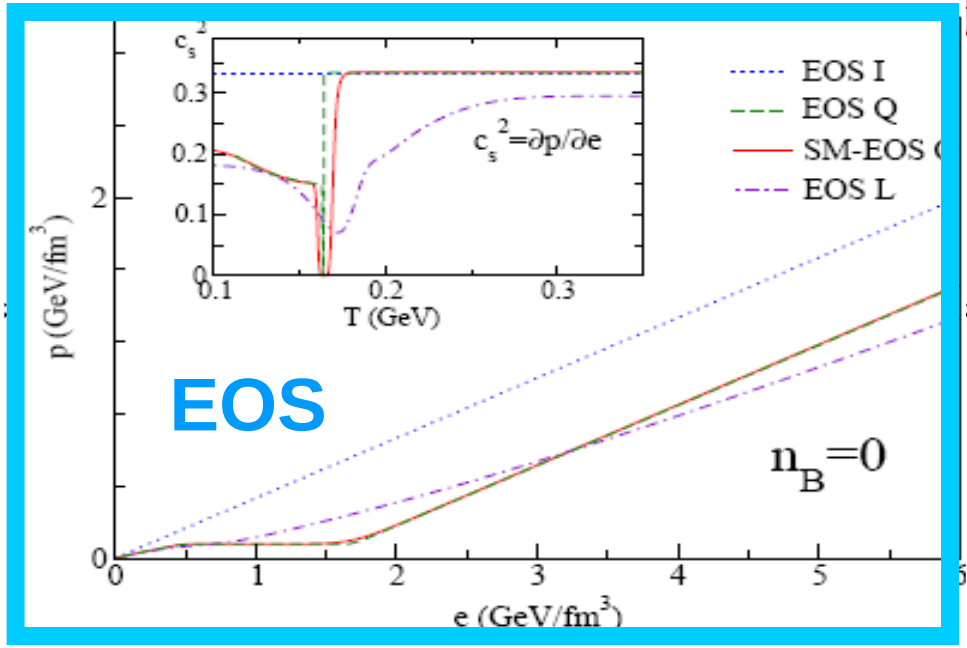
system size, EOS, different I-S equations:



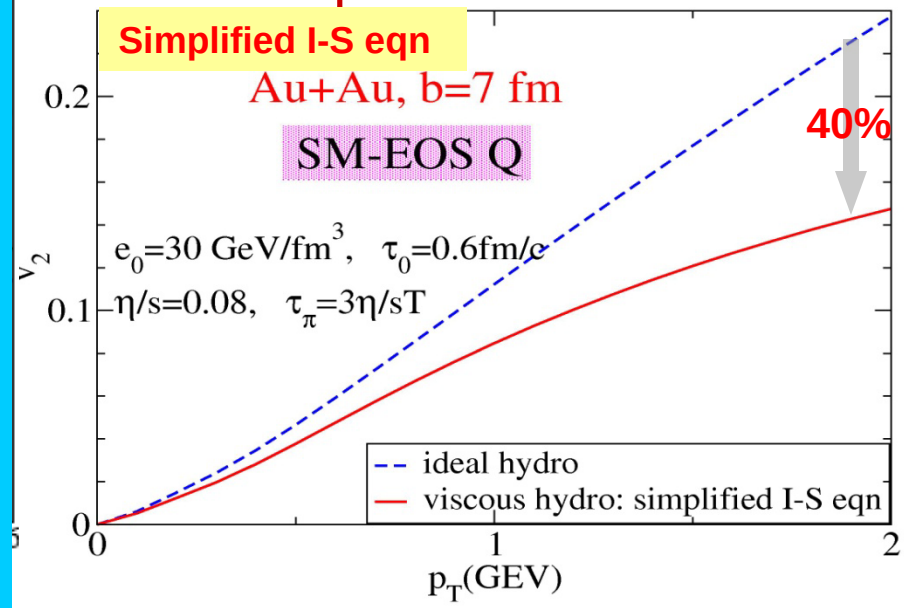


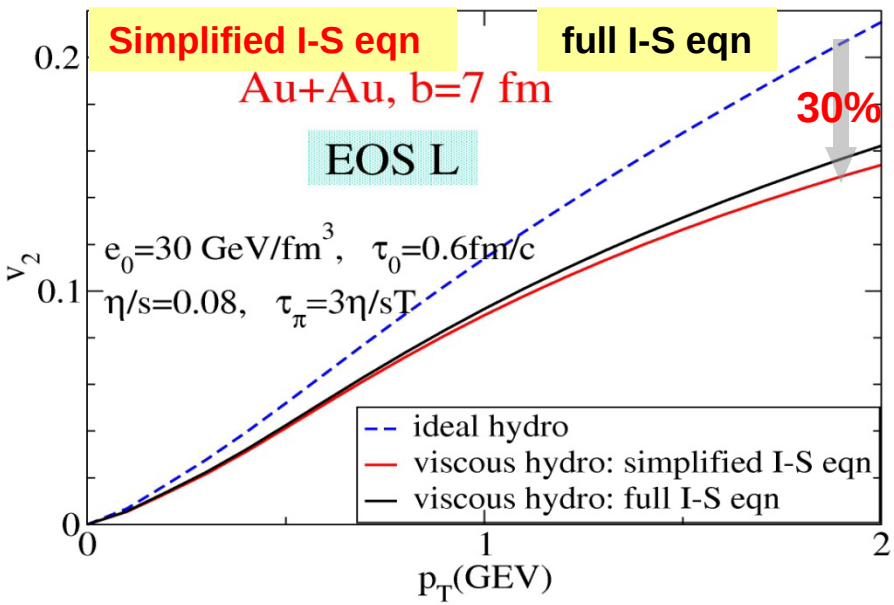
-system size: CuCu b=7fm vs. AuAu b=7fm: 20-30% effect

Different effects contributing to v_2 suppression



ifferent I-S equations:





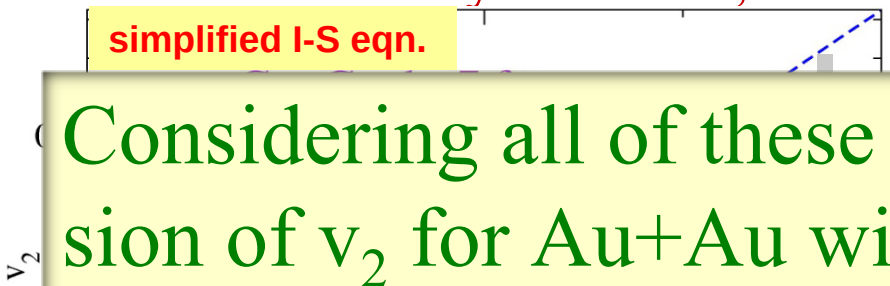
-system size: CuCu b=7fm vs. AuAu b=7fm: 20-30% effect

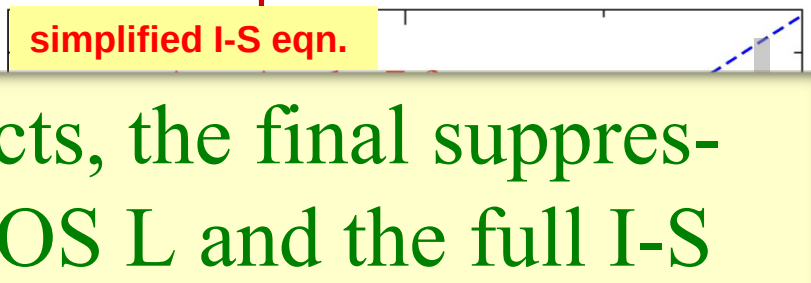
-EOS: SM-EOS Q vs. EOS L: ~10% effect

-different I-S eqns: simplified I-S eqn. vs. full I-S eqn.: ~5% effects (EOS Q and EOS L only)

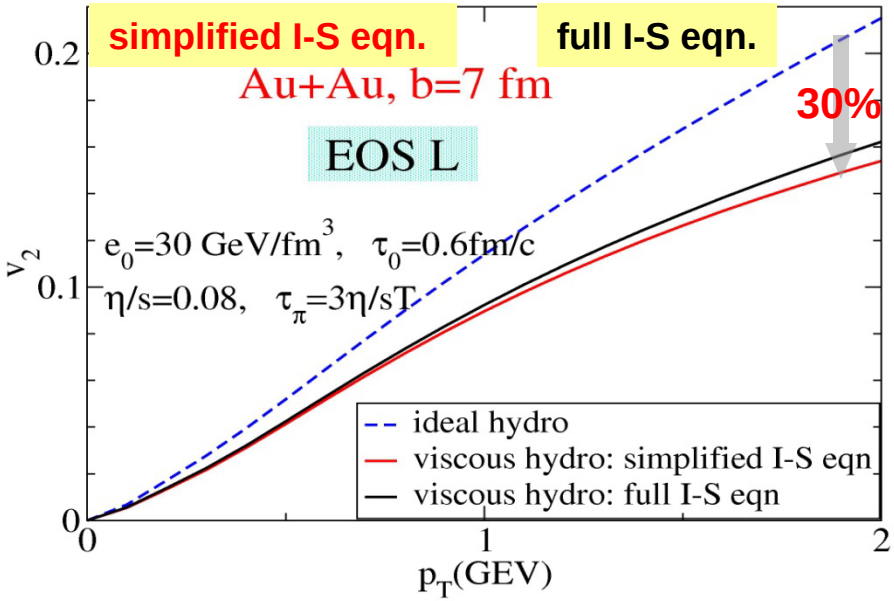
Different contributions to the suppression of v_2

System size, EOS, different I-S equations:





Considering all of these effects, the final suppression of v_2 for Au+Au with EOS L and the full I-S eqn., for minimal shear viscosity $\eta/s = 0.08$, is ~25%, approaching the results of P. & U. Romatschke (PRL 99, 172301 (2007)).



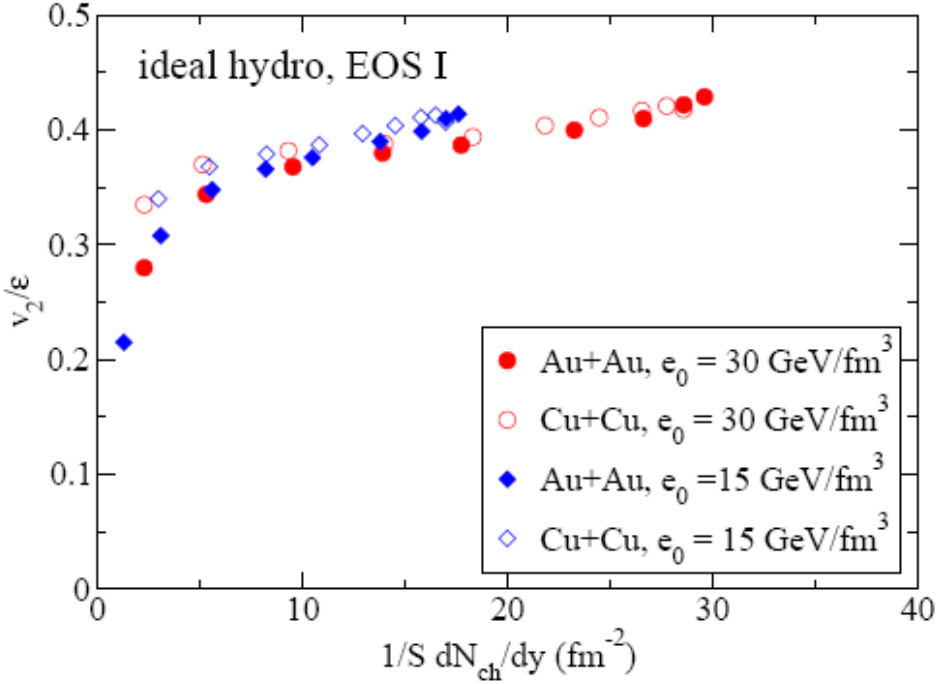
- system size: CuCu b=7fm vs. AuAu b=7fm: 20-30% effect
- EoS: SM-EOS Q vs. EOS L: ~10% effect
- different I-S eqn.: simplified vs. full I-S eqn.: ~5% effect (EOS Q and EOS L only)

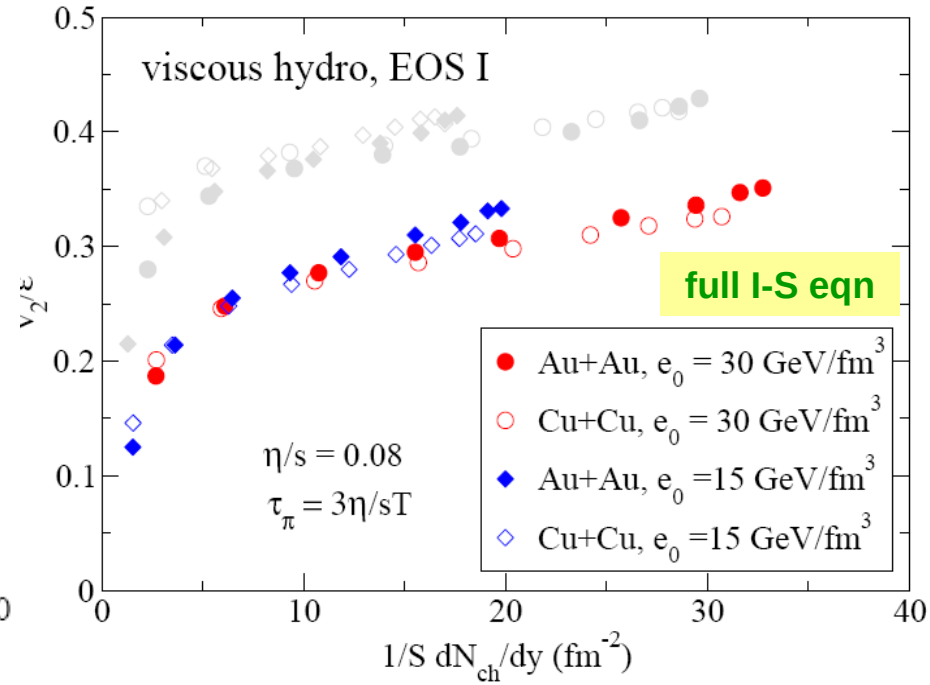
Comment: To extract QGP viscosity from exp. data by using viscous hydro, one needs a better description of EoS (Lattice EoS + chemical non-equil. HRG EoS)

System size effects

Multiplicity scaling of v_2/ε EOS I

Preliminary





Ideal hydrodynamics: multiplicity scaling of v_2/ε is weakly broken:

- freeze-out condition introduces time scale, breaking scale invariance of id. hydro eqns.
- Cu+Cu and Au+Au systems are not identical after a rescaling

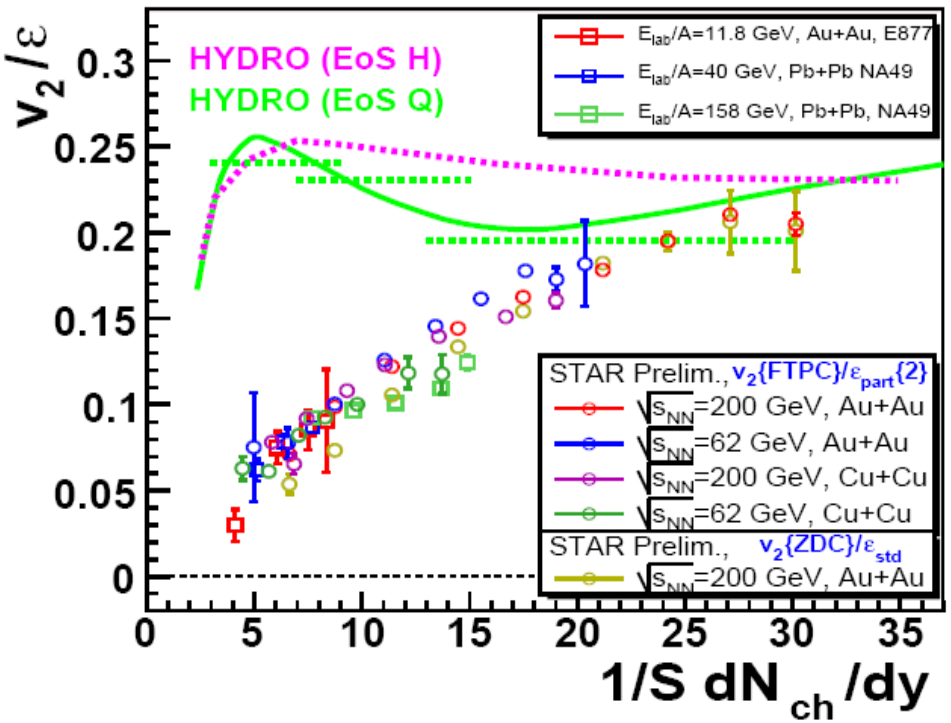
Viscous hydrodynamics: additional scale breaking by shear viscosity, resulting in fine structure of v_2/ε :

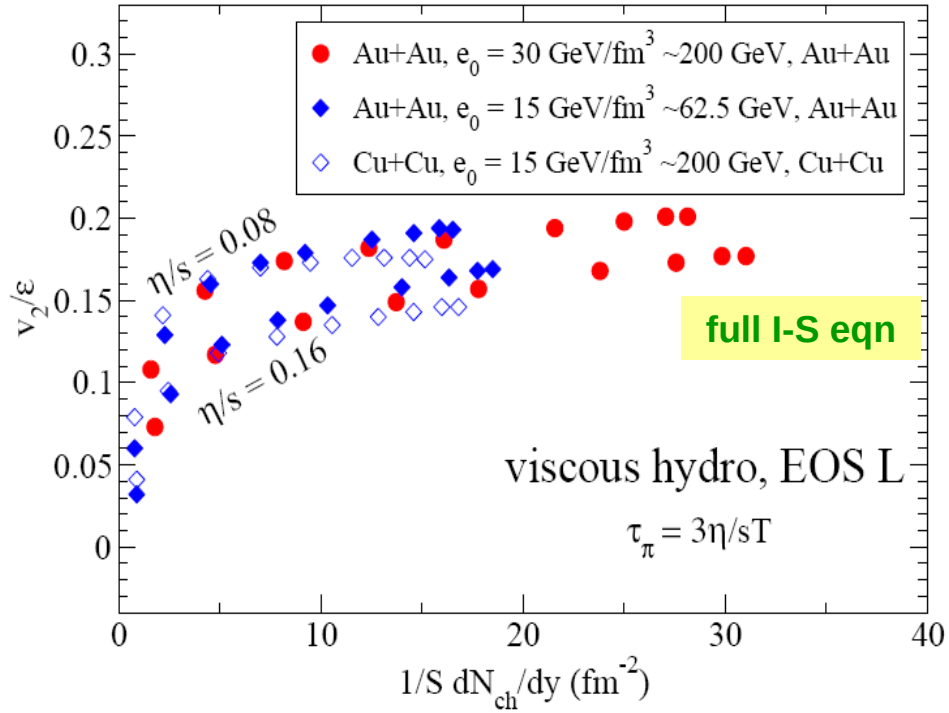
- for similar initial energy density, Cu+Cu curves are slightly below the Au+Au curves
- at fixed $\frac{1}{S}\frac{dN_{ch}}{dy}$, the $e_0 = 15\mathrm{GeV/fm}^3$ curves are slightly above the $e_0 = 30\mathrm{GeV/fm}^3$ ones

Viscous effects are larger for smaller systems and lower collision energies

Multiplicity scaling of v_2/ε EOS L

Preliminary

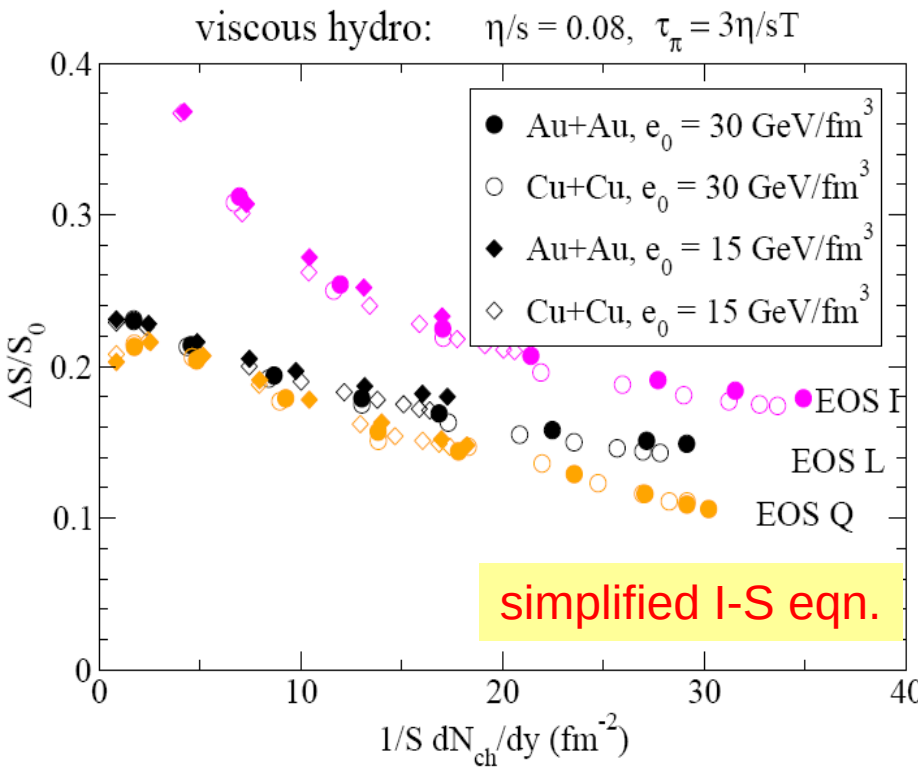


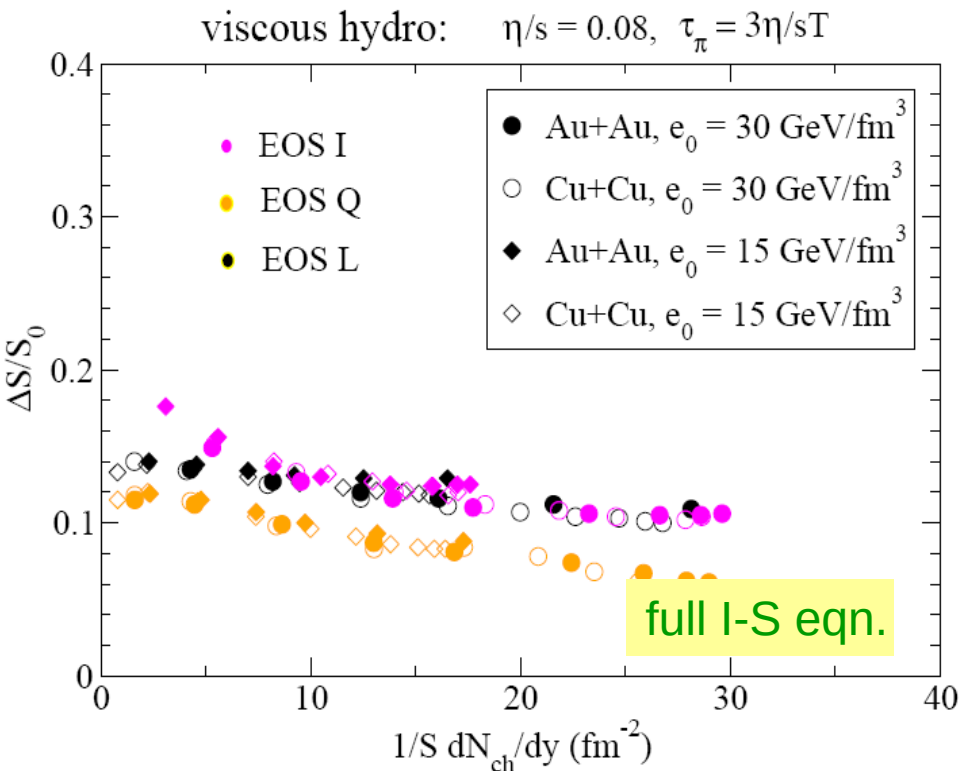


- experimental data show qualitatively similar fine ordering as viscous hydro prediction
- to reproduce slope of v_2/ε vs. (1/S)dN/dy, a better description of the highly viscous hadronic stage is needed: viscous hydro + hadron cascade
- the experimental v_2/ε vs. (1/S)dN/dy scaling (slope and fine structure) is another good candidate to constrain η/s (insensitive to Glauber-type vs. CGC initialization)
- this requires, however, experimental and theoretical improvements: reduced error bars, accounting for T-dependence of η/s, ζ/s near T_c, modeling hadronic phase with realistic cascade

Entropy production

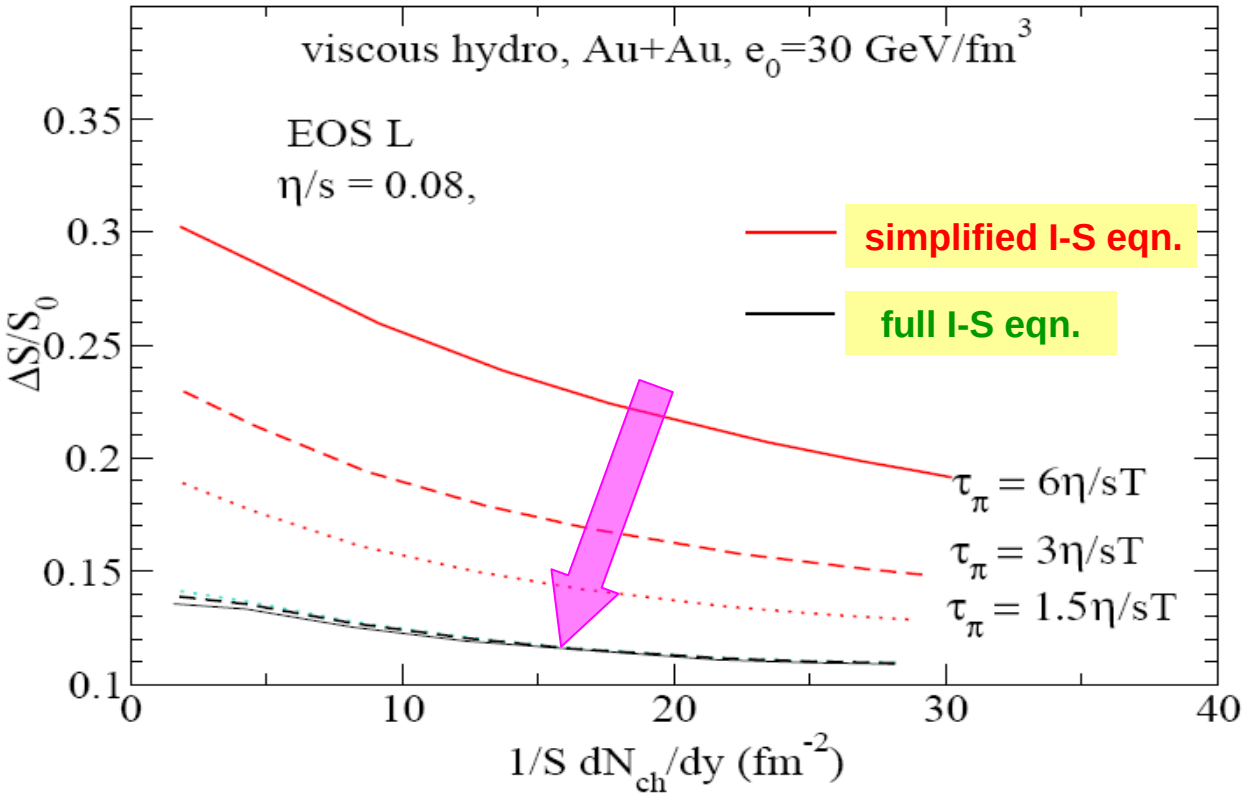
Entropy production (I)





- for each EoS, see good scaling of with (1/S)dN/dy (for both simplified and full I-S eqns)
- larger viscous effects in smaller systems and at lower collision energies: entropy production increases as $(1/S)dN_{ch}/dy$ decreases)
- for EOS L, the simplified I-S eqn. gives 5-10% more entropy production than the full I-S

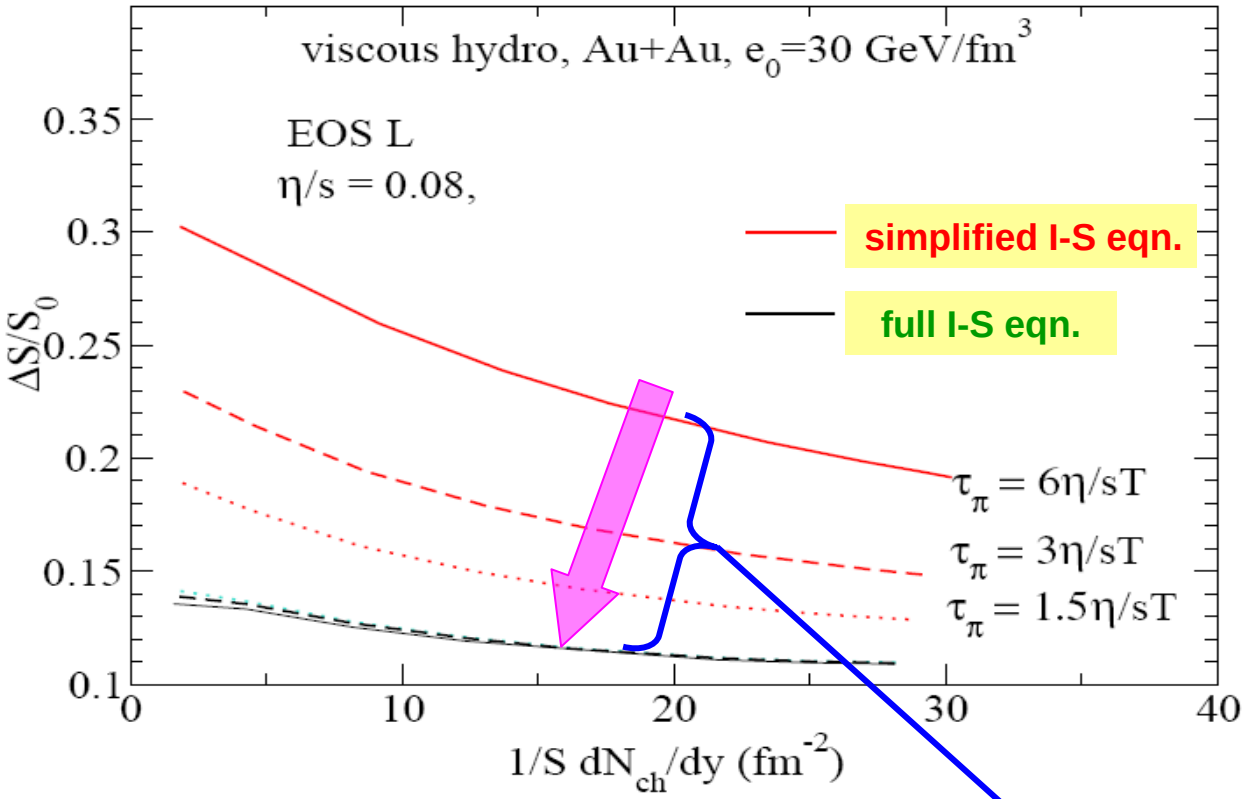
Entropy production (II)



- simplified vs. full I-S eqn.: entropy production approaches the same Navier-Stokes limit as we let $\tau_\pi \to 0$

- for the full I-S eqn., entropy production is very insensitive to τ_π

Entropy production (III)



$$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right] + \frac{1}{2}\pi^{\mu\nu}\left[5D\ln T - \nabla_\alpha u^\alpha\right] - 2\pi^{\alpha(\mu}\omega_\alpha^{\nu)}$$

- simplified vs. full I-S eqn.: entropy production approaches the same Navier-Stokes limit as we let $\tau_\pi \to 0$

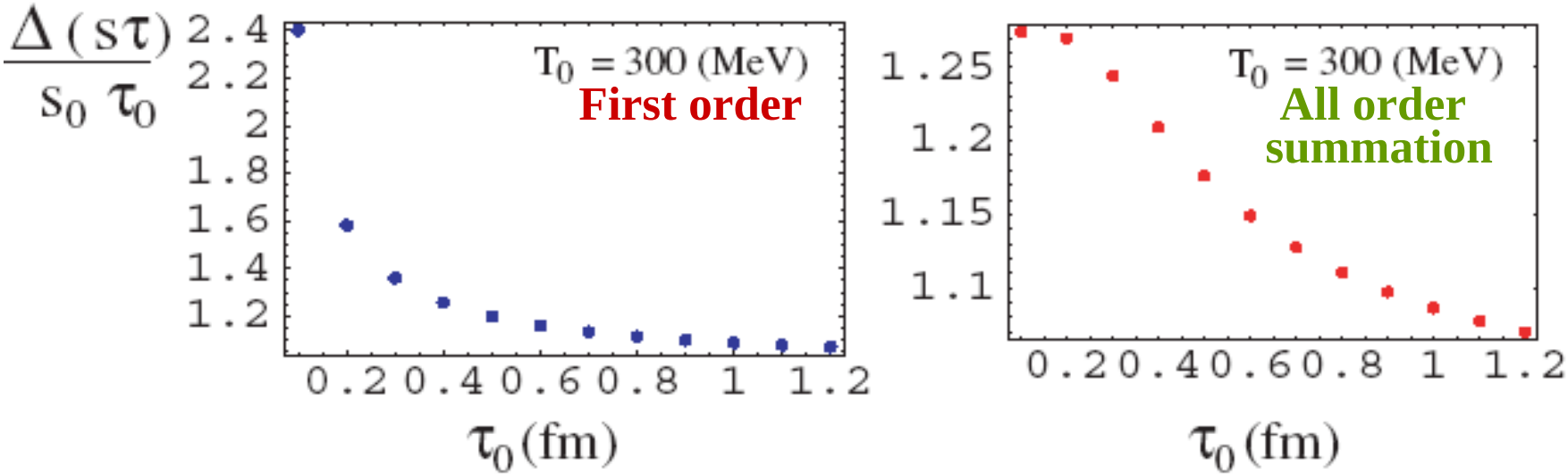
- for the full I-S eqn., entropy production is very insensitive to τ_π

Entropy production (IV)

- effects of higher order terms on entropy production

We argue that estimates of dissipative effects based on first-order hydrodynamics with shear viscosity are potentially misleading because higher order terms in the gradient expansion of the dissipative part of the stress tensor tend to reduce them. Using recently obtained sound dispersion relations in thermal $\mathcal{N} = 4$ supersymmetric plasma, we calculate the *resummed* effect of these high-order terms for Bjorken expansion appropriate to heavy ion collisions such as those performed at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC). A reduction of entropy production is found to be substantial, up to an order of magnitude.

--Lublinsky & Shuryak PRC2007

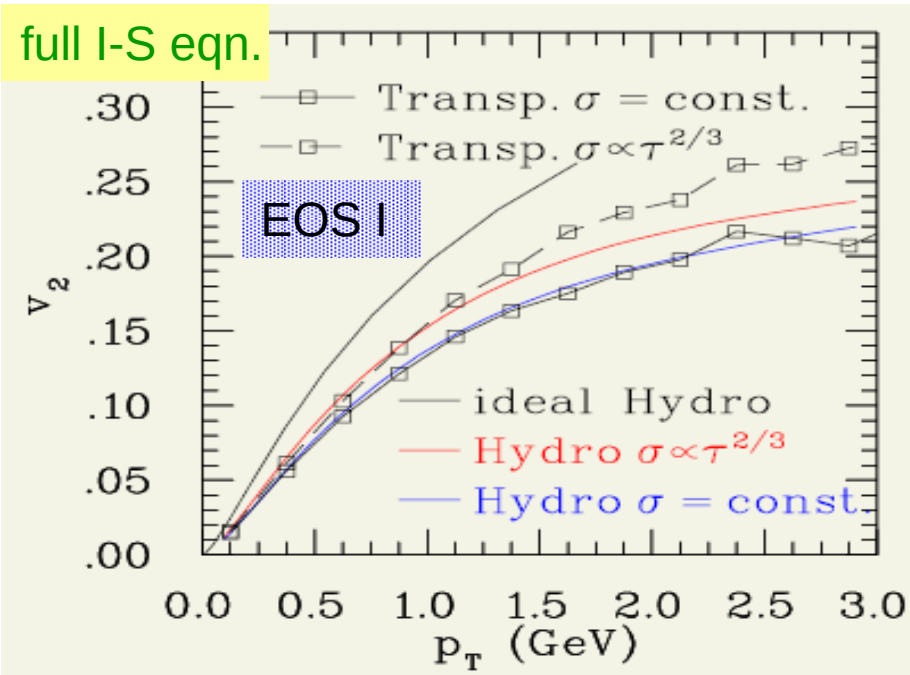


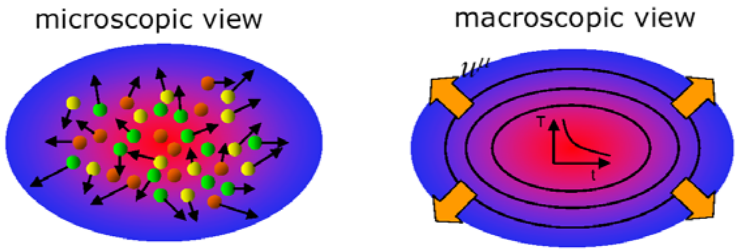
- Is the the 2nd order expansion formalism enough to describe the more viscous fluids created in smaller collision systems or at lower energies?

full I-S eqn. vs. parton cascade model

Important insight from comparing viscous hydro with parton cascade model:

Molnar & Huovinen QM08





massless Boltzmann particles $2 \leftrightarrow 2$ processes

$\sigma \propto \tau^{2/3}$ $\Rightarrow \eta/s \approx 1/(4\pi)$ $\eta = 4T/(5\sigma_{tr})$

- $\tau_0 = 0.6$ fm$/c$
- $b = 8$ fm
- $T_0 = 385$ MeV and $dN/d\eta|_{b=0} = 1000$
- freeze-out at constant $n = 0.365$ fm^{-3}

- parton cascade model: evolves directly $f(\vec{x}, t, \vec{p})$ with Boltzmann eqn.
- 2nd order viscous hydro: I-S eqn. is obtained from Boltzman eqn. by expanding to 2nd order in δf
- amazingly, the two approaches give results that agree well with each other

It appears that the 2nd order I-S equation is still sufficient for semi-central collisions starting with high enough initial energy density

Summary and discussion (I)

- Elliptic flow is very sensitive to even minimal shear viscosity

- Viscous effects are larger in smaller systems and at lower collision energies:
  - 20-30% more v_2 suppression in Cu+Cu b=7 fm than Au+Au b=7fm (similar initial ecc.)
  - more entropy production in smaller systems and at lower collision energies
  - fine structure in the multiplicity scaling curves for v_2/ε predicted by viscous hydro agrees qualitatively with experimental data and can be used to constrain the QGP shear viscosity

- Simplified I-S equation vs. full I-S equation:
  - approach same Navier-Stokes limit for $\tau_\pi \to 0$ (entropy prod., momentum anisotropy etc.)
  - for non-conformal fluids (EOS Q, EOS L) both eqns. are OK (~5-10% difference) if the system size and initial energy density are not too small (the approx. behind the simplified I-S eqn. are "OK" (~90-95% reliable) for EOS L and EOS Q)
  - for conformal fluids (EOS I) the full I-S eqn. (which preserves the conserve conformal symmetry) works but the simplified I-S eqn. fails
  - entropy production: the full I-S eqn. is much less sensitive to τ_π, leaving less ambiguity in the extraction of η/s due to the choice of τ_π

Summary and discussion (II)

- Extracting the QGP shear viscosity requires a better understanding of all sources of uncertainty from both sides, theory and experiment:

- For causal viscous hydro, one needs to consider at least the following aspects:

  - resolve the ambiguities among different 2nd order formalisms used by different groups when simulating causal viscous hydrodynamics

    a) simplified I-S (Song & Heinz 07-08) vs. full I-S eqn. (P.&U.Romatschke) ~5-10%
    b) I-S formalism vs. O-G formalism (Dusling & Teaney) ~ ? %

  - a realistic EOS: EOS L vs. SM-EOS Q ~10% (for v_2 and v_2/ε) (EoS L & EOS Q)

  - realistic initial conditions: CGC vs. Glauber-type initialization ~15-30% (for v_2)

  - bulk viscosity: with vs. without bulk viscosity ~?%

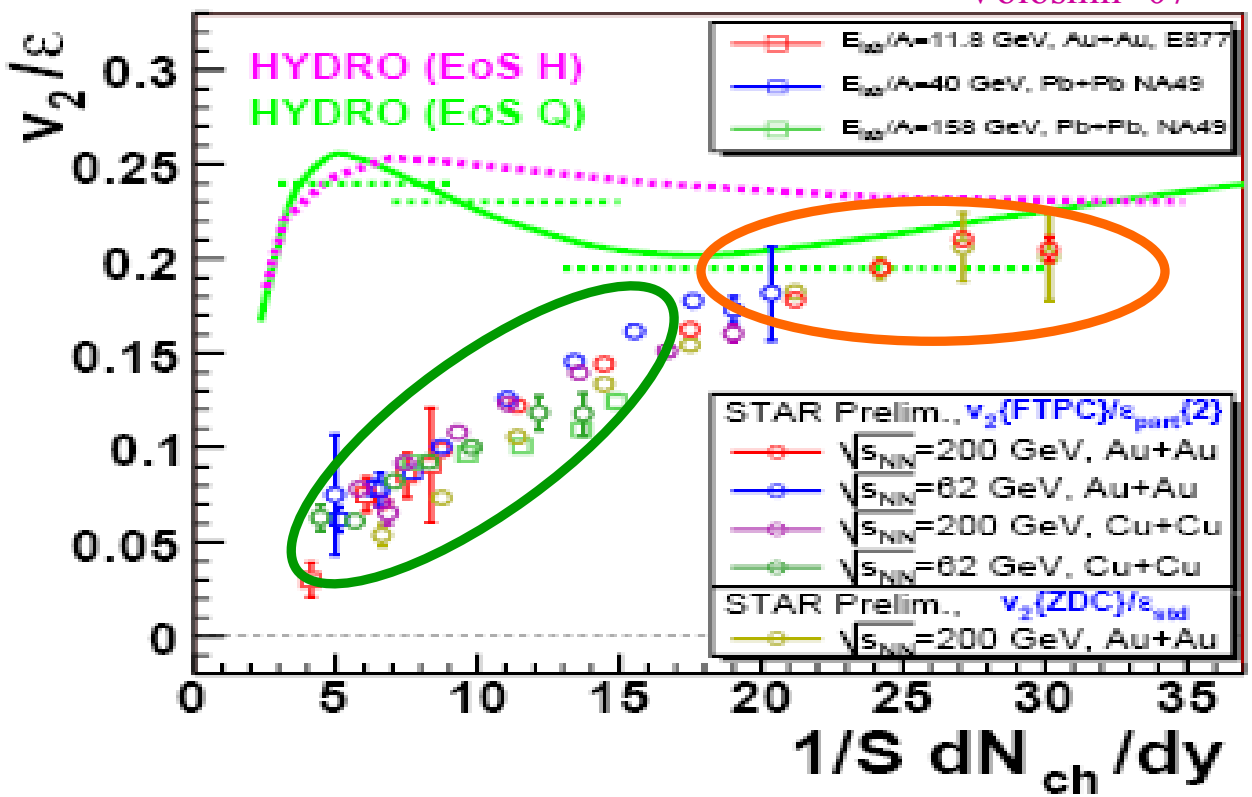
  - T-dependent η / s : ~?%

... ...

Thank You

Multiplicity scaling of v_2: Experiment

Voloshin '07



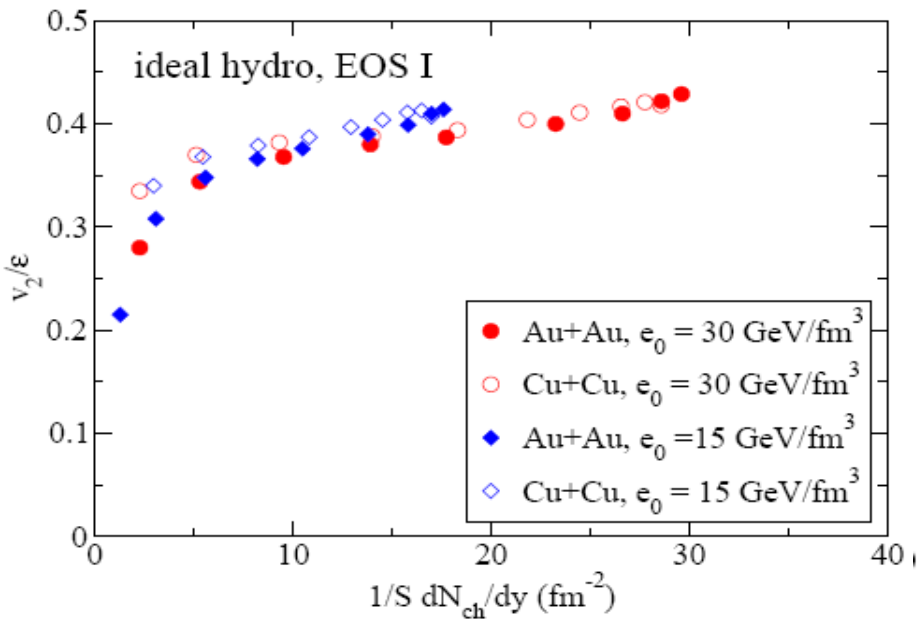
low energy limit: $v_2 / \varepsilon \propto (1/S)(dN_{ch}/dy)$ Voloshin PLB'00

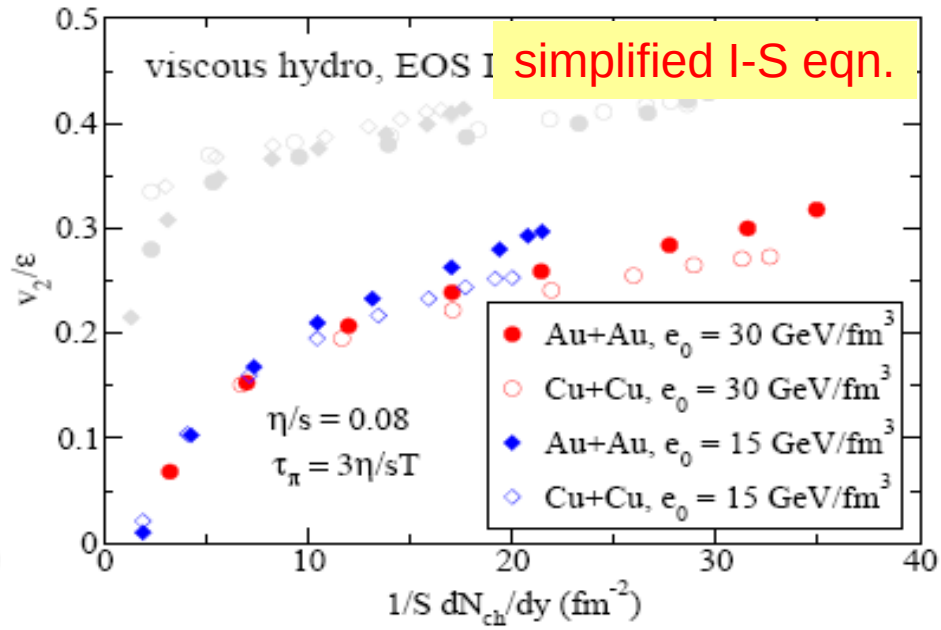
ideal hydro limit: v_2 / ε → a saturation value around 0.21-0.23 (EOS Q; depends on c_s)

- viscous hydro works for near equilibrium systems, which lie between the low energy and ideal hydro limits (how does shear viscosity change the scaling of ideal hydro?)
- v_2 / ε is insensitive to the type of initialization used (CGC or Glauber model)

Multiplicity scaling of v_2/ε – EOS I

Preliminary





Ideal hydrodynamics: multiplicity scaling of v_2/ε is weakly broken

- freeze-out condition introduces time scale, breaking scale invariance of id. hydro eqns.
- Cu+Cu and Au+Au systems are not identical after rescaling

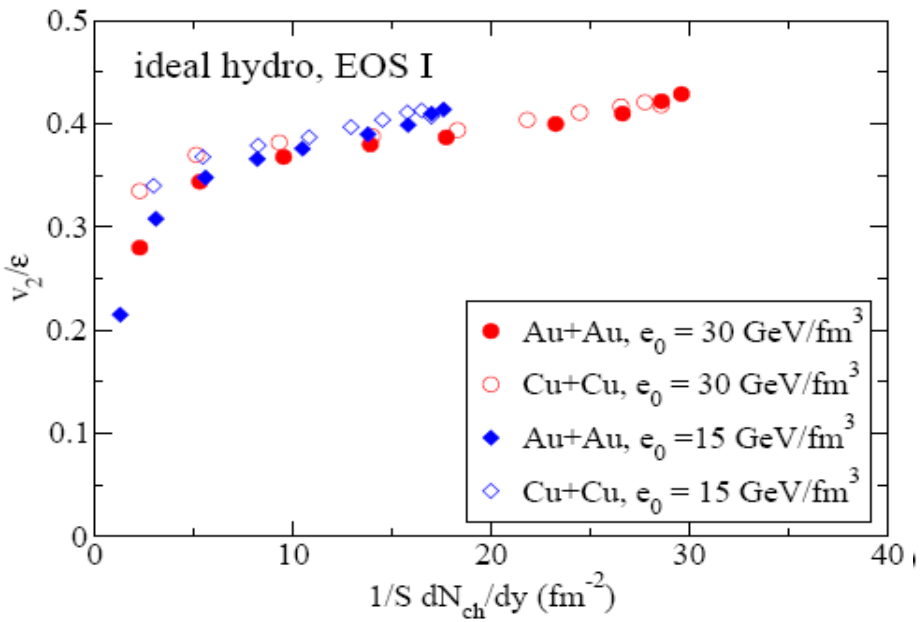
Viscous hydrodynamics: additional scale breaking by shear viscosity, resulting in fine structure of v_2/ε

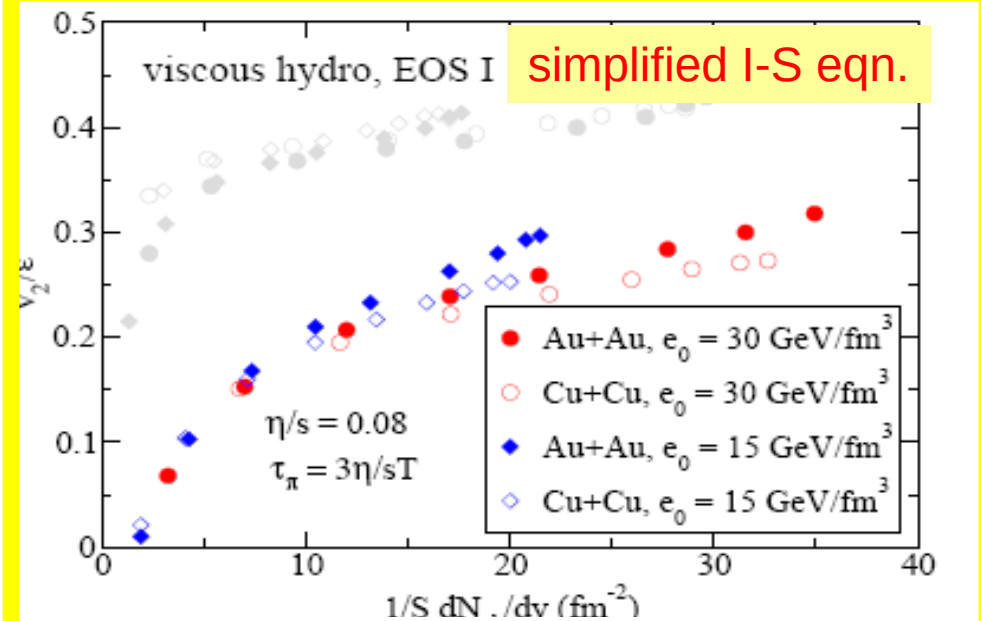
- for similar initial energy density, Cu+Cu curves lie slightly below the Au+Au curves
- at fixed $\frac{1}{S}\frac{dN_{ch}}{dy}$, the $e_0 = 15\text{GeV/fm}^3$ curves lie slightly above the $e_0 = 30\text{GeV/fm}^3$ ones

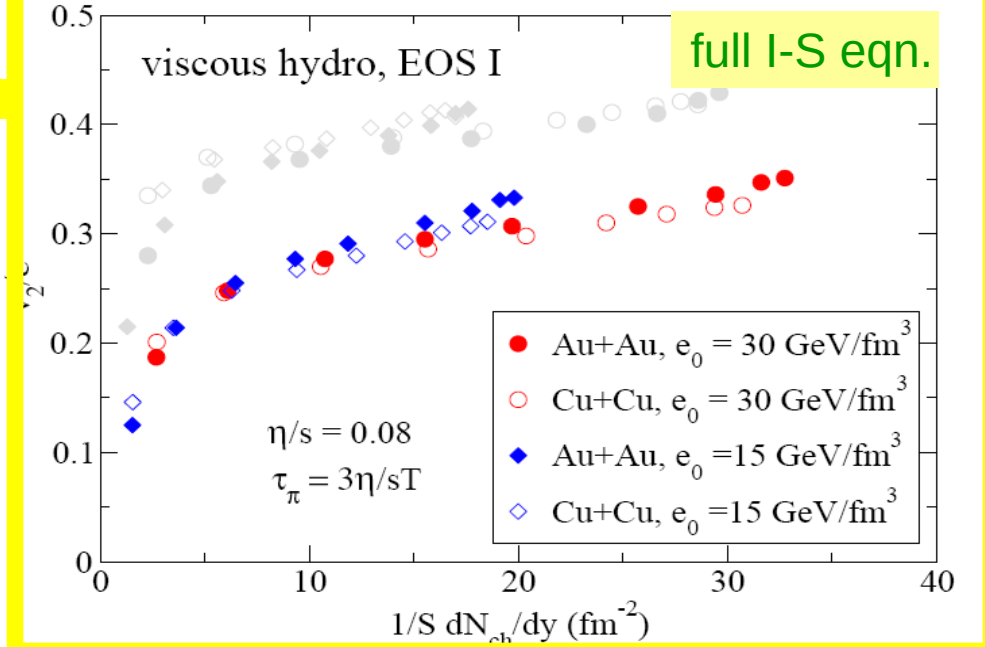
Viscous effects are larger for smaller systems and lower collision energies

Multiplicity scaling of v_2/ε – EOS I (b)

Preliminary







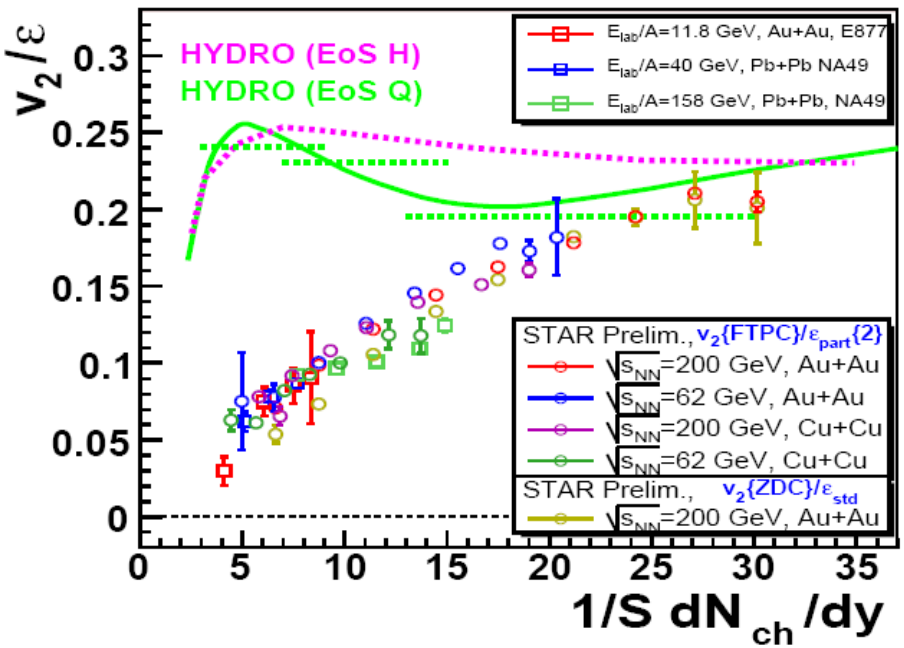
Compare the results from simplified I-S eqn. and full I-S eqn.

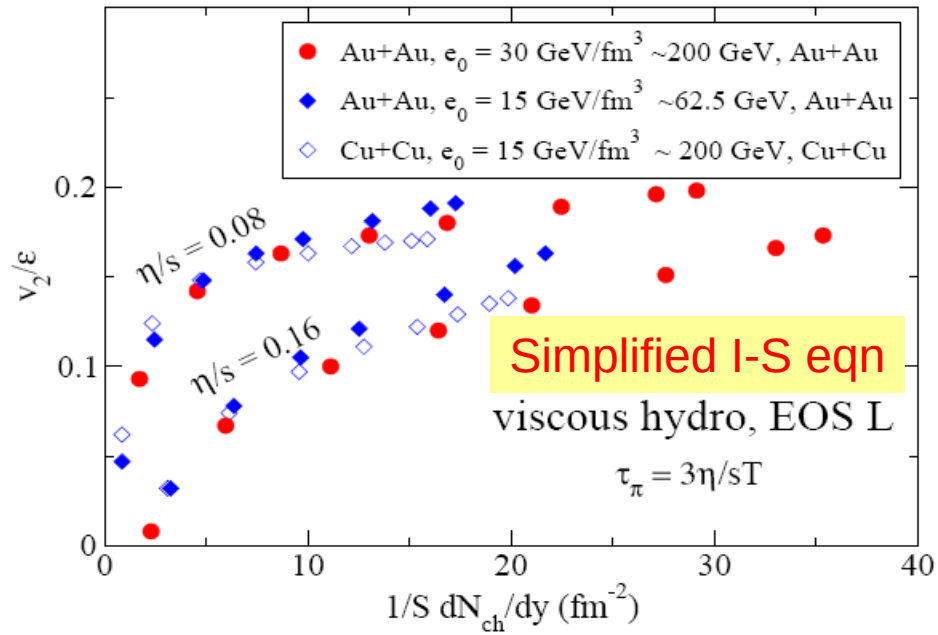
- they show the same fine structure
  Viscous effects are larger for smaller systems and lower collision energies

- for full I-S eqn. observe:
  - less v_2 suppression
  - the scaling violation is weaker

Multiplicity scaling of v_2/ε – EOS L

Preliminary

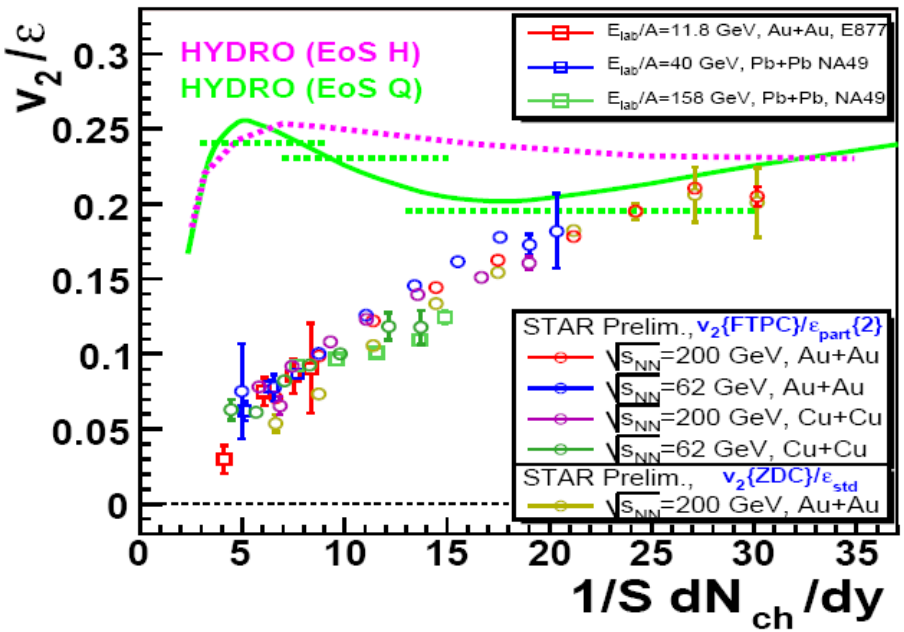


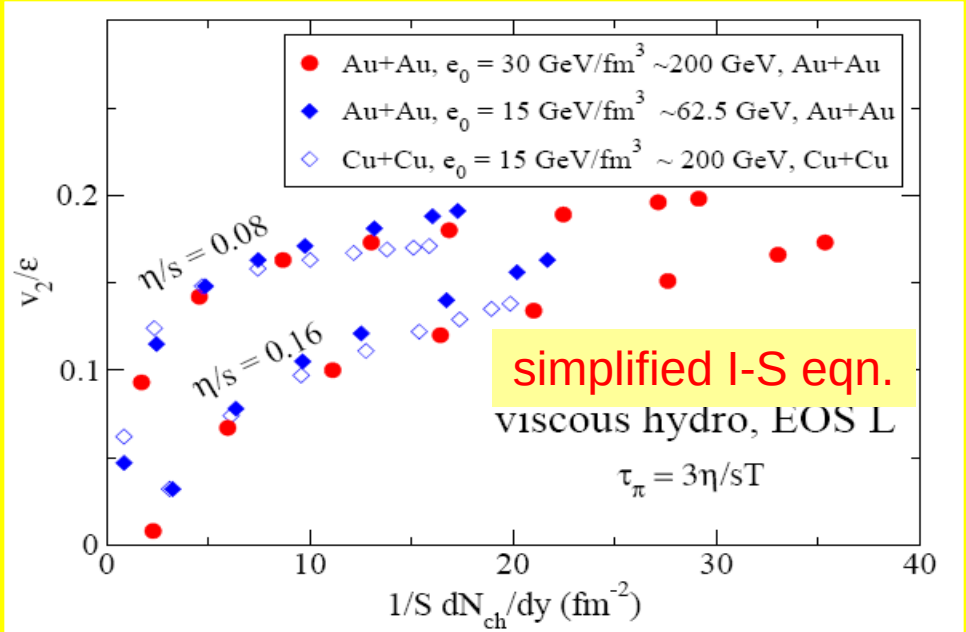


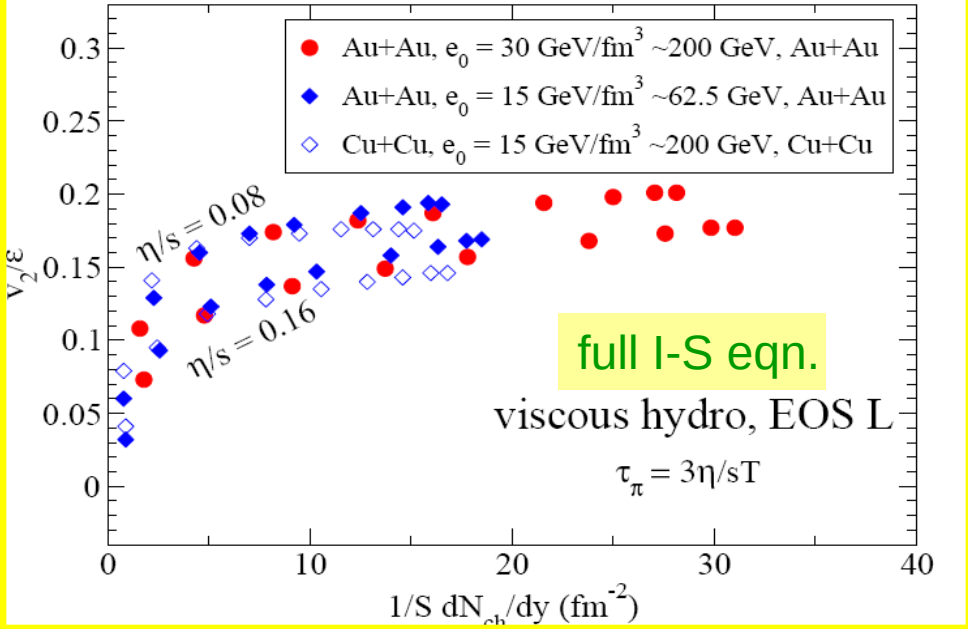
- experimental data show qualitatively similar fine structure as viscous hydro prediction
- to reproduce the slope of v_2/ε vs. (1/S)dN/dy, a better description of the highly viscous hadronic stage is needed: viscous hydro + hadron cascade
- the experimental v_2/ε vs. (1/S)dN/dy scaling (slope and fine structure) is another good candidate to constrain η/s (insensitive to Glauber-type vs. CGC initialization)
- this requires, however, experimental and theoretical improvements: reduced error bars, accounting for *T*-dependence of η/s, ζ/s near T_c, modeling hadronic phase with realistic cascade

Multiplicity scaling of v_2/ε – EOS L (b)

Preliminary





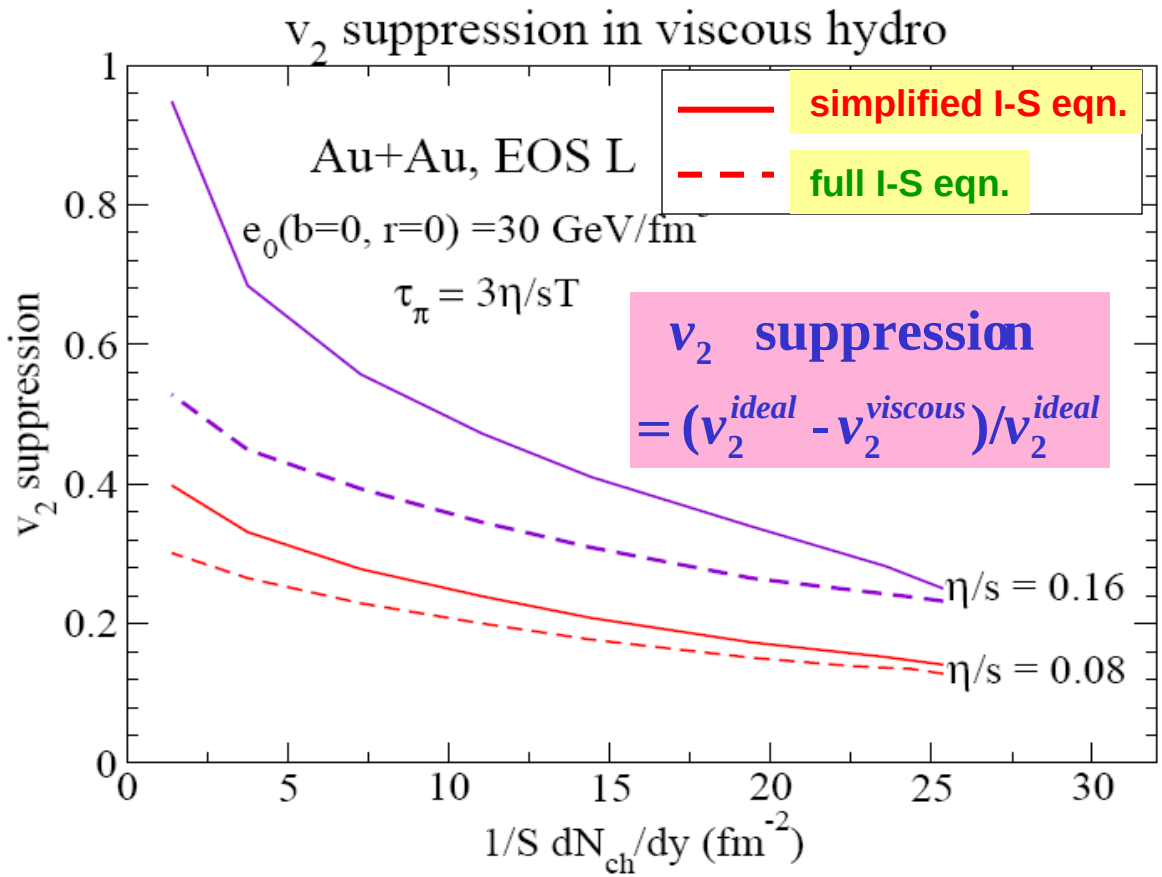


Compare the results from simplified I-S eqn. and full I-S eqn.

- they both show the same fine structure as experimental data

- for full I-S eqn. observe:
  - weaker scaling violations
  - weaker suppression of v_2/ε by η/s

Viscous suppression of v_2



Comment:
To better constrain the QGP shear viscosity from elliptic flow data, the ambiguities from different 2nd order formalisms (simplified vs. full I-S equations, Israel-Stewart vs. Öttinger-Grmela formalisms) need to be clarified.

- the difference between simplified and full I-S eqns. increases as the system size decreases and/or as η/s increases

- for b=7 fm, simplified I-S eqn. results in ~5% more v_2 suppression for $\eta/s = 1/4\pi$, but that increases to ~10% for $\eta/s = 2/4\pi$ (the effects are no longer negligible here)

simplified I-S eqn. vs. full I-S eqn. (I)

full I-S eqn.: $$\underbrace{\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right]}_{\text{exact 2nd order terms/ simplified I-S eqn}} + \frac{1}{2}\pi^{\mu\nu}\underbrace{[5D\ln T - \nabla_\alpha u^\alpha] - 2\pi^{\alpha(\mu}\omega_\alpha^{\nu)}}_{\text{higher order contributions}}$$

- obtained by expanding entropy current to 2nd order in δf $(\delta f << f_{eq})$
- the extra terms (in green) (partially third order terms) are necessary to preserve the conformal symmetry

In short, the extra terms in the full I-S equation result in:

- 30-50% less v_2 suppression for a conformal fluid (massless QGP, EOS I)
- 5-10% less v_2 suppression for a minimally viscous non-conformal fluid containing both QGP and hadronic phases (with 1st or 2nd order phase transition: EOS Q, EOS L)
- 5-10% less entropy production (minimally viscous fluid, non-conformal EOS L)
- most importantly, all these effects increase for smaller systems, lower collision energies, or more viscous fluids

- Is the the 2nd order expansion formalism sufficient to describe the more viscous fluid created in smaller collision systems or at lower energies?
- Is it OK to use a 2nd order formalism without conformal symmetry to describe a conformal fluid?