

LQCD at non-zero temperature and heavy ion phenomenology

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Study of the strongly interacting matter and its new “phases”

Theory:

Low T, low density:
Effective theories,
Virial expansion

Lattice
QCD
and
Super-
computing

High T, high density:
Weak coupling
methods

Experiment:

Past:

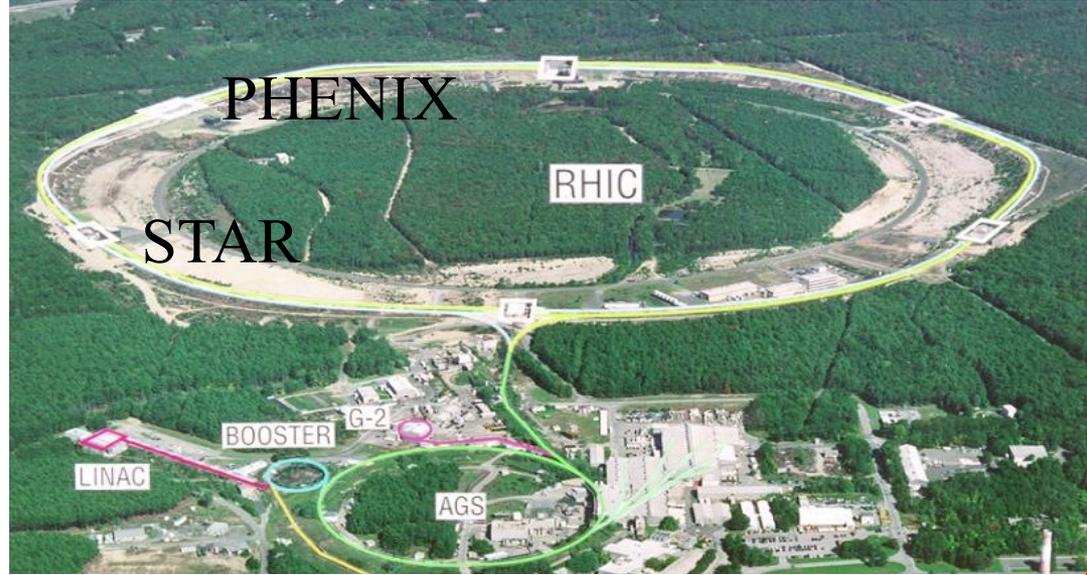
AGS, SPS, $E_{cm}=(1-17)$ GeV

Present :

RHIC, $E_{cm}=(5.5-200)$ GeV,
LHC, $E_{cm}=2.76$ TeV, 5.5 TeV

Future:

NICA, CBM@FAIR
 $E_{cm}=(1-17)$ GeV



Relativistic Heavy Ion Collisions

RHIC: Au+ Au, Cu+Cu,
Cu+Au, U+U
 $\sqrt{s} = 5.5 - 200\text{GeV}$
 $\epsilon \simeq 15 - 30\text{GeV}/\text{fm}^3$

LHC: ALICE, also
HI in CMS and ATLAS

Pb+Pb $\sqrt{s} = 2.76 - 5.5\text{TeV}$
 $\epsilon \simeq 100\text{GeV}/\text{fm}^3$

IBM BG/L (NYBlue) : 100Tflop

1 Tflop = 10^{12} floating point operation per second $\sim 10^3 \times$ PC performance

QCDOC Supercomputer : 20 Tflop



18 racks
1 rack = 2048 processors



24racks , 1 rack = 1024 processors

QCDOC at BNL
20 Teraflops

Strong interactions and QCD

Structure and Interaction of Hadrons

Quantum Chromo Dynamics (QCD) :



SU(3) non-Abelian gauge theory coupled to fermions

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \sum_i \bar{\psi}^{\alpha_i} (\gamma_\mu D^\mu - m) \psi_i^\alpha + m_i \bar{\psi}_i^\alpha \psi_i^\alpha$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + igf^{abc} A_\mu^b A_\nu^c$$

gluon self interactions

$i = u, d, s, c, b, t$
 3-5MeV 100 MeV

In medium :
 Chromo Magnetic screening,
 Gluon saturation

asymptotic freedom

$$\alpha_s(r \rightarrow 0) = \frac{g^2(r)}{4\pi} \sim 1 / \ln \frac{1}{r\Lambda_{\text{QCD}}} \rightarrow 0,$$

$\Lambda_{\text{QCD}} \sim 200\text{MeV}$

confinement

heavy
 $m_{c,b,t} \gg \Lambda_{\text{QCD}}$

Quarks and gluons cannot exist as free particles. Observed particles are color charge neutral



Nobel Prize 2004
 Gross, Politzer, Wilczek

quark masses make up only 2% of the mass of the proton or neutron (~940MeV) !
 98% of the visible mass in the Universe comes from gluon dynamics and confinement !

$\bar{q}q$ qqq

Mesons Baryons

Need lattice QCD to study their properties

New states of strongly interacting matter ?

I. Ya. Pomeranchuk, Doklady Akad. Nauk. SSSR 78 (1951) 889

Because of finite size of hadrons hadronic matter cannot exist up to arbitrarily high Temperature/density, hadron size has to be smaller than $1/T$

Hagedorn, Nuovo Cim. 35 (65) 395

Exponentially increasing density of hadronic states \Rightarrow limiting temperature

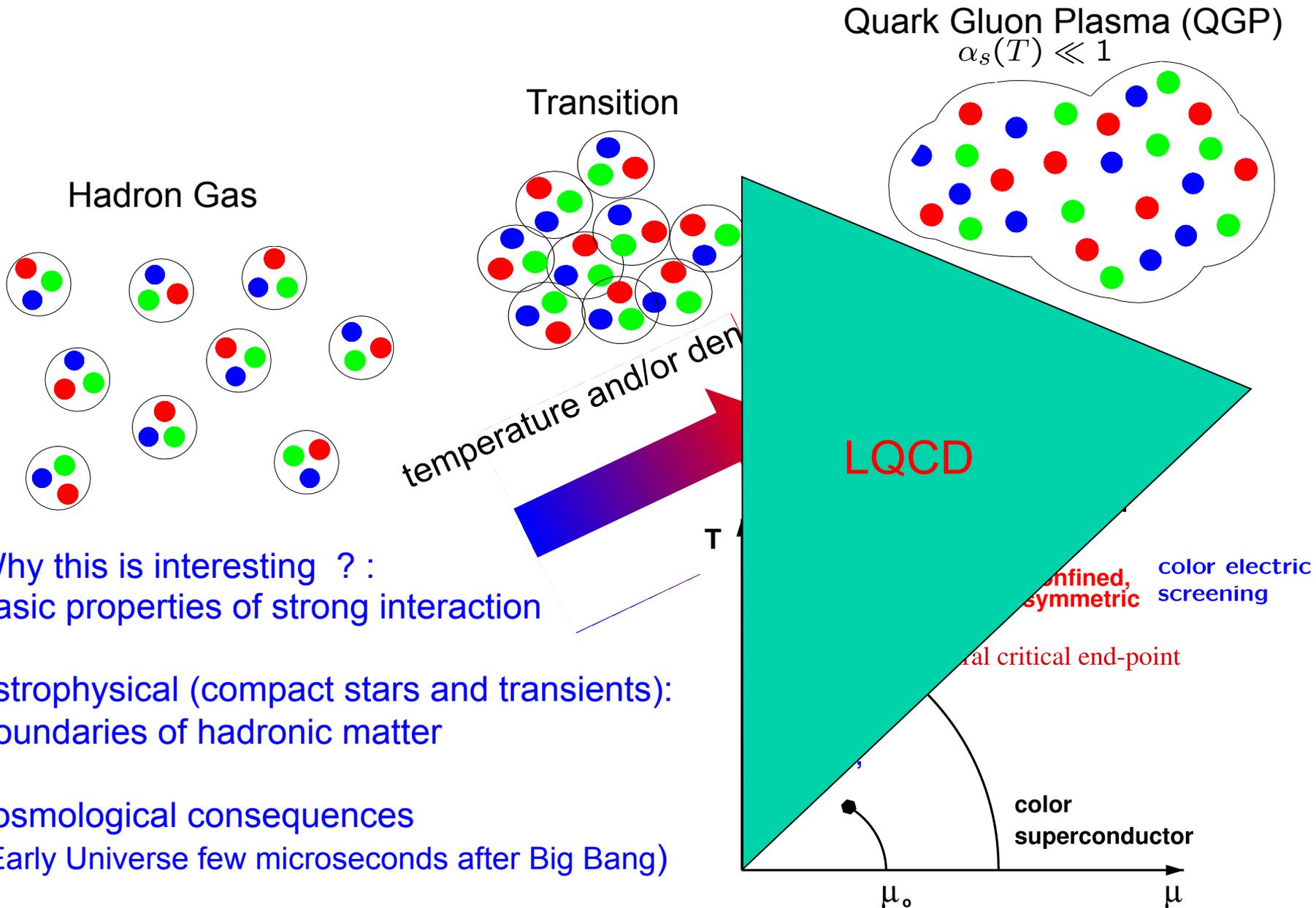
Cabbibo, Parisi, PLB 59 (75) 67

Realization that at high energy hadronic language is not appropriate and reinterpretation of the limiting temperature as the phase transition temperature
To medium consisting of quarks and gluons

Collins and Perry, PRL 34 (1975) 1353

At very high density strongly interacting matter should consist of quarks due to asymptotic freedom

Deconfinement at high temperature and density



Why this is interesting ? :
 basic properties of strong interaction

astrophysical (compact stars and transients):
 boundaries of hadronic matter

cosmological consequences
 (Early Universe few microseconds after Big Bang)

Symmetries of QCD in the vacuum at high T

$T \gg \Lambda_{QCD} :$

- **Chiral symmetry :** $m_{u,d} \ll \Lambda$

$SU_A(2)$ rotation $\psi \rightarrow e^{i\phi T^a \gamma_5 \psi}$ $\psi_{L,R} \rightarrow e^{i\phi_{L,R} T^a} \psi_{L,R}$

$\langle \bar{\psi} \psi \rangle = \langle \bar{\psi}_L \psi_R \rangle + \langle \bar{\psi}_R \psi_L \rangle \neq 0$

$\langle \bar{\psi} \psi \rangle = 0$

restored

spontaneous symmetry breaking or Nambu-Goldstone symmetry realization



hadrons with opposite parity have very different masses, interactions between hadrons are weak at low E

2008



- **Axial or $U_A(1)$ symmetry:** invariance $\psi \rightarrow e^{i\phi \gamma_5 \psi}$

is broken by anomaly (ABJ) : $\langle \partial^\mu j_\mu^a \rangle = -\frac{\alpha_s}{4\pi} \langle \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta}^a F_{\gamma\delta}^a \rangle$

Effectively restored ?



η' meson mass, π - a_0 mass difference

topology

- **Center (Z_3) symmetry :** invariance under global gauge transformation

$A_\mu(0, \mathbf{x}) = e^{i2\pi N/3} A_\mu(1/T, \mathbf{x}), \quad N = 1, 2, 3$

$\langle L \rangle \neq 0$

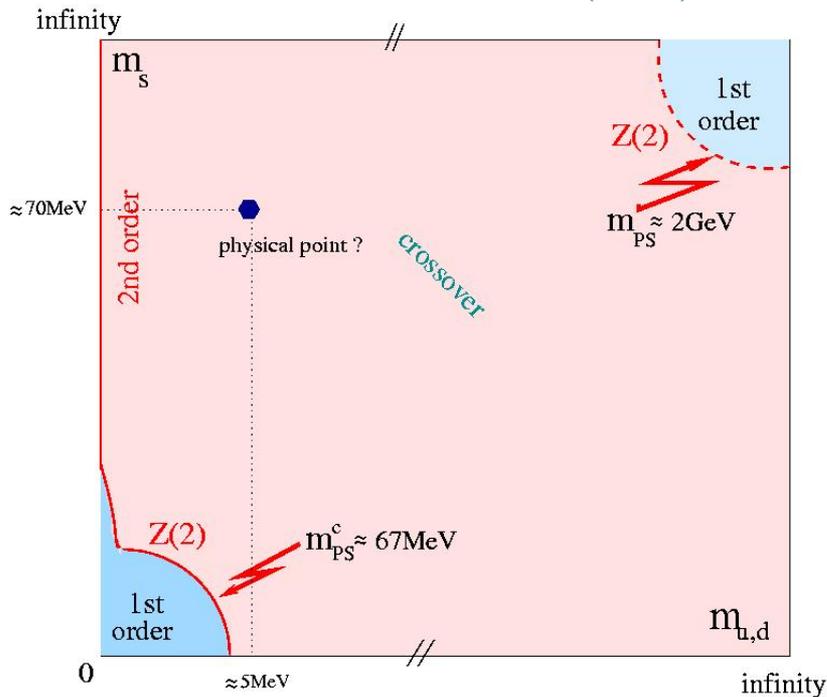
broken

Exact symmetry for infinitely heavy quarks and the order parameter is the Expectation value of the Polyakov loop:

$L = \text{tr} \mathcal{P} e^{ig \int_0^{1/T} d\tau A_0(\tau, \vec{x})}$ $\langle L \rangle = 0$

QCD phase diagram as function of the quark mass

Pisarski, Wilczek, PD29 (1984) 338



For very large quark masses there is a 1st order **deconfining phase transition**

Chiral transition:

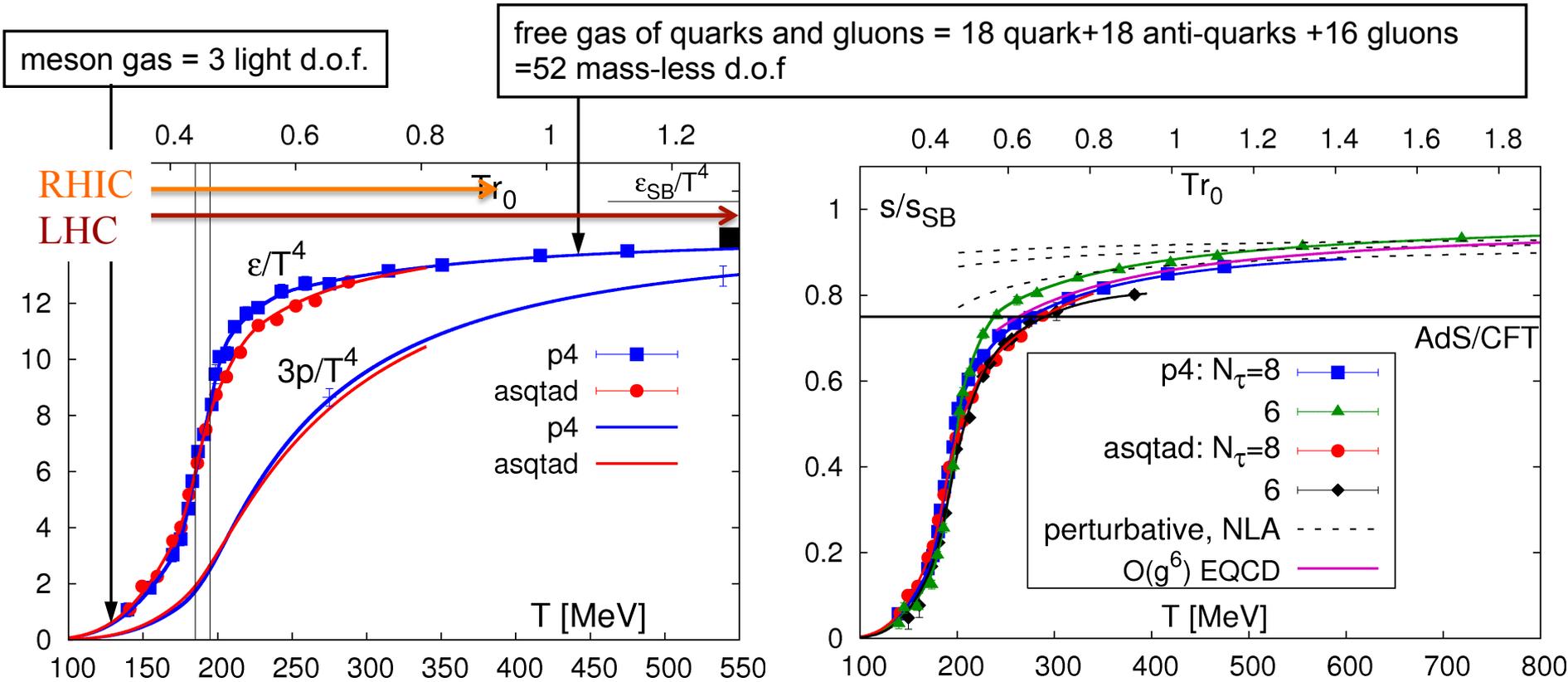
- For vanishing u, d -quark masses the Chiral transition is either 1st order or 2nd order phase transition
- For physical quark masses there could be a 1st order phase transition or crossover

Evidence for 2nd order transition in the chiral limit
 => universal properties of QCD transition:

$SU_A(2) \sim O(4)$
 relation to spin models

transition is a crossover
 for physical quark masses

Deconfinement : entropy, pressure and energy density

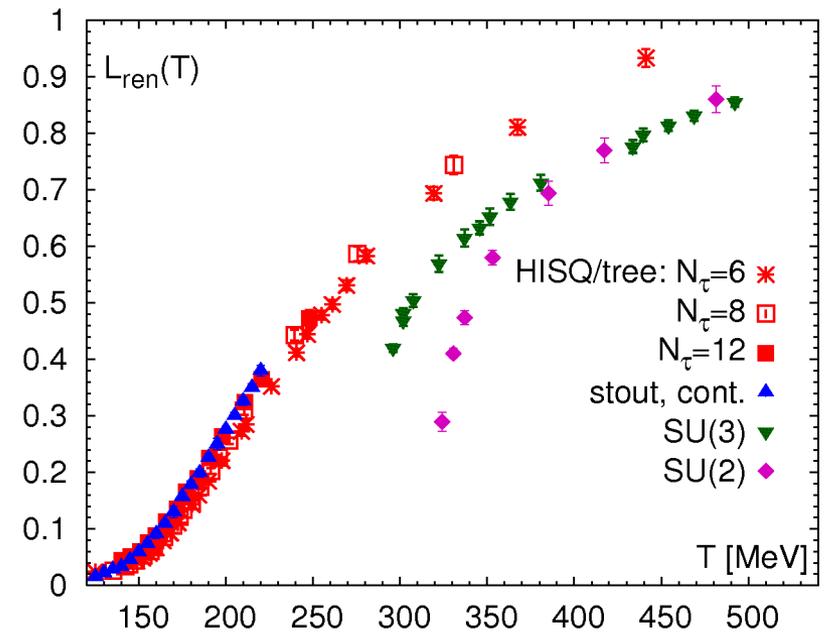
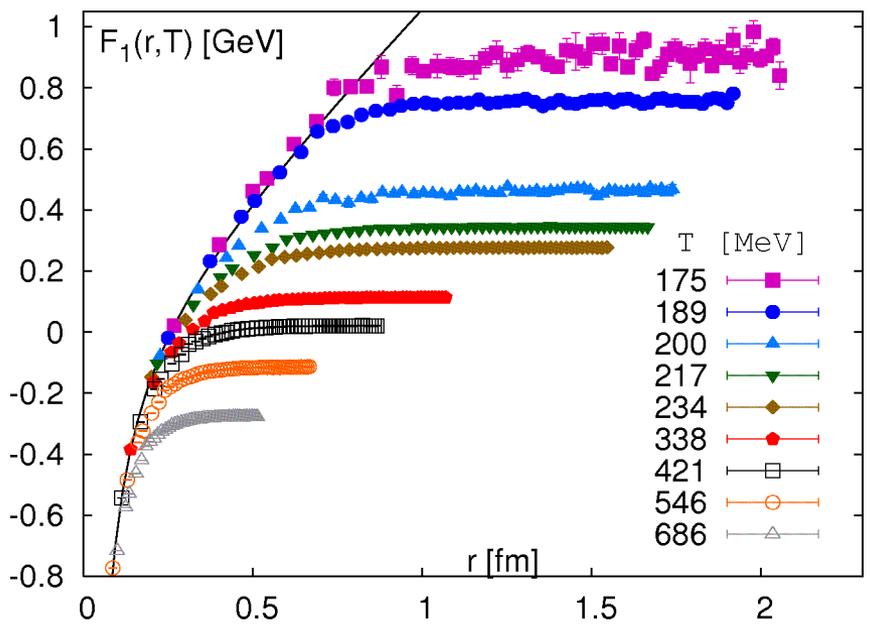


Bazavov et al (HotQCD), PRD 80 (09) 14504

Petreczky, NPA 830 (10) 11c

- rapid change in the number of degrees of freedom at $T=160-200\text{MeV}$: deconfinement
- deviation from ideal gas limit is about 10% at high T consistent with the perturbative result
- no obviously large discretization errors in the pressure and energy density at high T
- energy density at the chiral transition temperature $\epsilon(T_c=154\text{MeV})=240 \text{ MeV}/\text{fm}^3$:

Deconfinement and color screening



Singlet free energy of static quark anti-quark pair shows Debye screening at high temperatures

$$L = \text{tr} \mathcal{P} e^{ig \int_0^{1/T} d\tau A_0(\tau, \vec{x})} \Rightarrow L_{ren} = \exp(-F_Q(T)/T)$$

$$F_1(r) = -\frac{4\alpha_s}{3r} \exp(-m_D r) + 2F_Q(T), m_D \sim T$$

$$r_{bound} > 1/m_D$$

melting of bound states of heavy quarks => quarkonium suppression at RHIC

Polyakov loop

free energy of a static quark

infinite in the pure glue theory or large in the "hadronic" phase ~600MeV

decreases in the deconfined phase

$$F_Q(T) \simeq \Lambda_{QCD} - C_F \alpha_s m_D$$

Physics of heavy ion collisions and LQCD

high temperature QCD
weak coupling ?

Chiral transition, T_c , fluctuations of conserved charges

Initial State:
colliding nuclei

$Q_s \sim 1 \text{ GeV}$

Quark Gluon Plasma &
hydrodynamic expansion

EoS,
viscosity

hadronic rescattering
& freeze-out

Equilibration:
turbulent color fields

Hadronization

test of Hadron
Resonance Gas
(HRG)
using LQCD

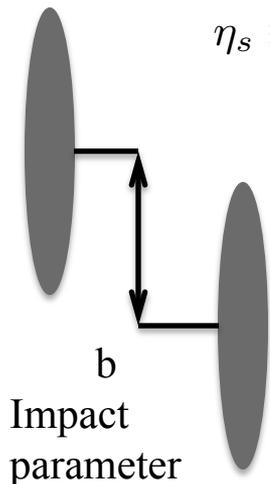
Control parameters: E_{cm} , rapidity η_s, y : energy density and baryon density

$$\eta_s \equiv \frac{1}{2} \log \left(\frac{t+z}{t-z} \right) \quad y \equiv \frac{1}{2} \log \frac{p_z + E}{E - p_z} \simeq \frac{1}{2} \log \frac{t+z}{t-z} \equiv \eta_s$$

Colliding species (Pb, Au, Cu, U), **centrality of the collisions**:
energy density, system geometry

Centrality is quantified by impact parameter b ,
number of participating nucleons N_p or centrality bins %
centrality $\sim (b/2R_A)$

More central collisions more Nucleons participate in the collisions

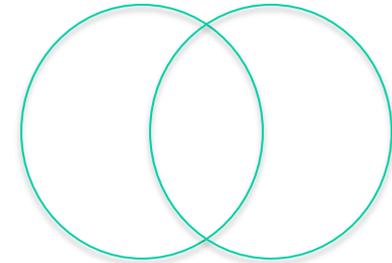


Physics of heavy ion collisions and LQCD (cont'd)

$$\frac{dN_p}{dx dy} = T_A(\mathbf{x}_\perp + \mathbf{b}/2) [1 - \exp(-\sigma_{NN} T_A(\mathbf{x}_\perp - \mathbf{b}/2))] \\ + T_A(\mathbf{x}_\perp - \mathbf{b}/2) [1 - \exp(-\sigma_{NN} T_A(\mathbf{x}_\perp + \mathbf{b}/2))]$$

\uparrow
 Nucleon density in the transverse plane

in-elastic nucleon-nucleon cross section $\sim 40\text{mb}$



Baryon density is the smallest at $y=0$ and almost zero to top RHIC and LHC energies and increases with y

Bulk observables :

yield of particles not containing heavy quarks

As function of azimuth and transverse momentum p_T , $p_T < 2 \text{ GeV}$

99% of all particle identified in HI Collisions

← LQCD:
 T_c , EoS, fluctuations
 of conserved charges

Penetrating probes :

high p_T particles (jets), quarkonia, photons

and dileptons, heavy flavor hadrons

Yield \sim # of binary collisions if no medium is formed

(incoherent superposition of nucleon-nucleon scatterings)

← LQCD:
 Meson and field
 Strength correlation
 Functions with
 exception of jets

Nuclear modification factor: $R_{AA} = \text{yield in AA} / (\text{yield in pp} \times \# \text{ of binary collisions})$

The perfect liquid created in RHIC

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

Monday, April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

Hydrodynamic models in heavy ion collisions

Ideal hydrodynamics:

$$T^{\mu\nu} = eu^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu}, \quad \partial_\mu T^{\mu\nu} = 0 \quad \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu,$$

$$D \equiv u^\mu \partial_\mu, \quad \nabla^\mu \equiv \Delta^{\mu\nu} \partial_\nu \quad De = -(e + \mathcal{P})\nabla_\mu u^\mu,$$

$$Du^\mu = -\frac{\nabla^\mu \mathcal{P}}{e + \mathcal{P}}.$$

1d hydro (Bjorken model):

$$e(t, \mathbf{x}) = e(\tau), \quad u^\mu(t, \mathbf{x}) = (u^0, u^x, u^y, u^z) = (\cosh(\eta_s), 0, 0, \sinh(\eta_s)) \quad \frac{de}{d\tau} = -\frac{e + \mathcal{P}}{\tau}$$

Viscous hydrodynamics:

$$T^{\mu\nu} = T_{\text{ideal}}^{\mu\nu} + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu} \quad \partial_\mu T^{\mu\nu} = 0, \quad \pi^{\mu\nu} = -\eta\sigma^{\mu\nu}, \quad \Pi = -\zeta\nabla_\mu u^\mu$$

$$\sigma^{\mu\nu} = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\lambda u^\lambda$$

$$1d \text{ hydro: } \frac{de}{d\tau} = -\frac{e + \mathcal{P} - \frac{4}{3}\eta/\tau}{\tau}$$

Hydrodynamics works when viscous corrections are small :

$$\underbrace{\frac{\eta}{s}}_{\text{medium parameter}} \times \underbrace{\frac{1}{\tau T}}_{\text{experimental parameter}} \ll 1$$

$$\frac{\eta}{e + \mathcal{P}} \frac{1}{\tau} \ll 1$$



Hydrodynamic models in heavy ion collisions (cont'd)

Hydro models:

1) Specify initial conditions, e.g.

$$s(\tau_0, \mathbf{x}_\perp) \propto \frac{dN_p}{dx dy} . \quad \text{fix constant to match the observed multiplicity} \quad \rightarrow \quad e(\tau, \mathbf{x}_\perp, \eta) \equiv e(\tau, \mathbf{x}_\perp)$$

$$u_x(\tau_0, \mathbf{x}_\perp) = u_y(\tau_0, \mathbf{x}_\perp) = 0 . \quad \pi^{\mu\nu}(\tau_0, \mathbf{x}_\perp) = \text{diag}(\pi^{\tau\tau}, \pi^{xx}, \pi^{yy}, \tau^2 \pi^{\eta\eta}) = \left(0, \frac{2}{3} \frac{\eta}{\tau}, \frac{2}{3} \frac{\eta}{\tau}, -\frac{4}{3} \frac{\eta}{\tau}\right)$$

2) Solve hydrodynamic equation

3) Stop hydro at the freeze-out temperature $T_f \sim 150$ MeV and calculate the freeze-out surface in space-time

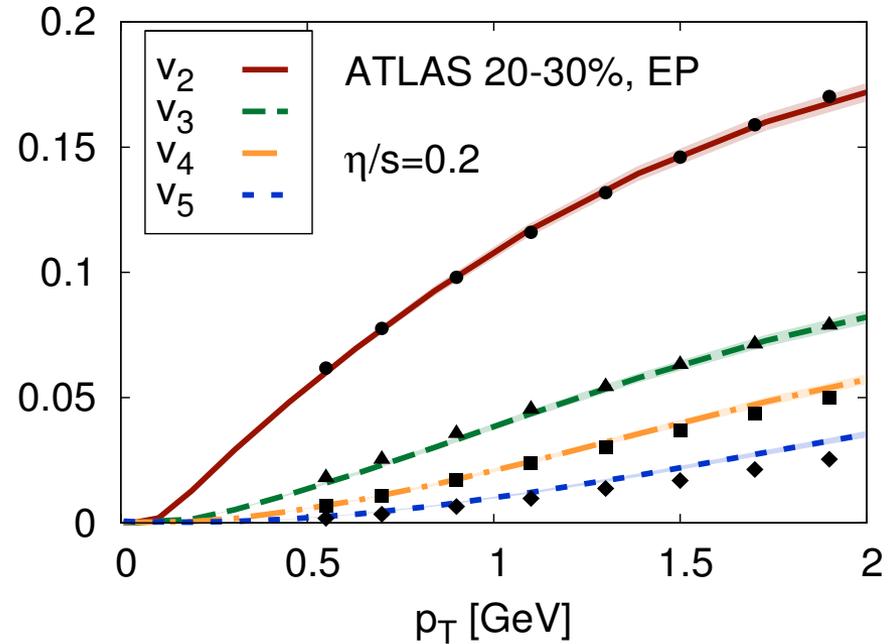
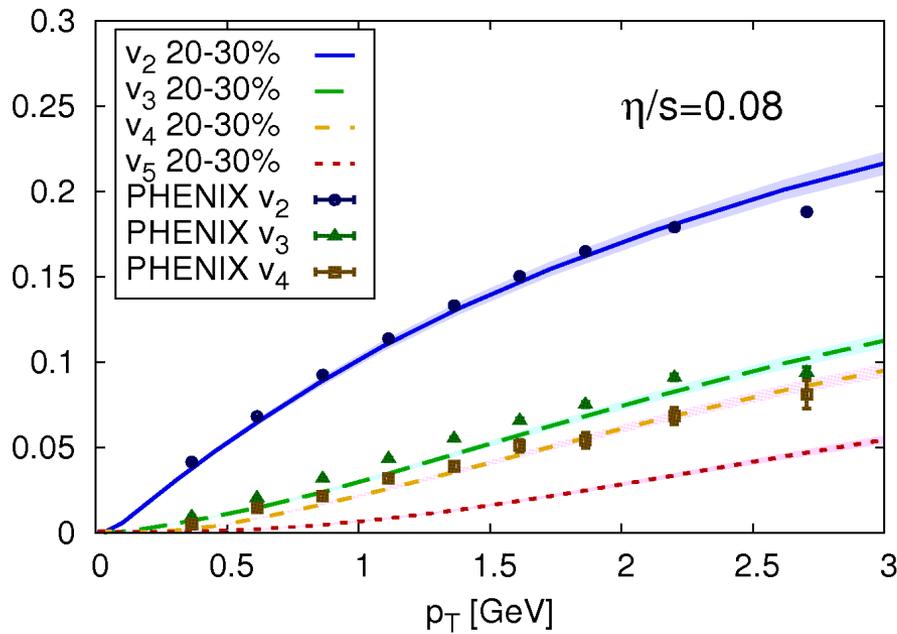
4) Calculate particle distributions by matching hydro calculations to kinetic theory and freeze-out surface

$$E \frac{dN^a}{d^3\mathbf{p}} = \frac{d_a}{(2\pi)^3} \int_\Sigma d\Sigma_\mu P^\mu f^a(-P \cdot u/T)$$

State of the art hydrodynamic model

- 1) Take into account higher order harmonics
- 2) More realistic initial conditions based on gluon saturation (IP-glasma)
- 3) 3D viscous hydrodynamics

Schenke et al



The matter is more viscous at higher collisions energy as the temperature is higher

How small is the shear viscosity ?

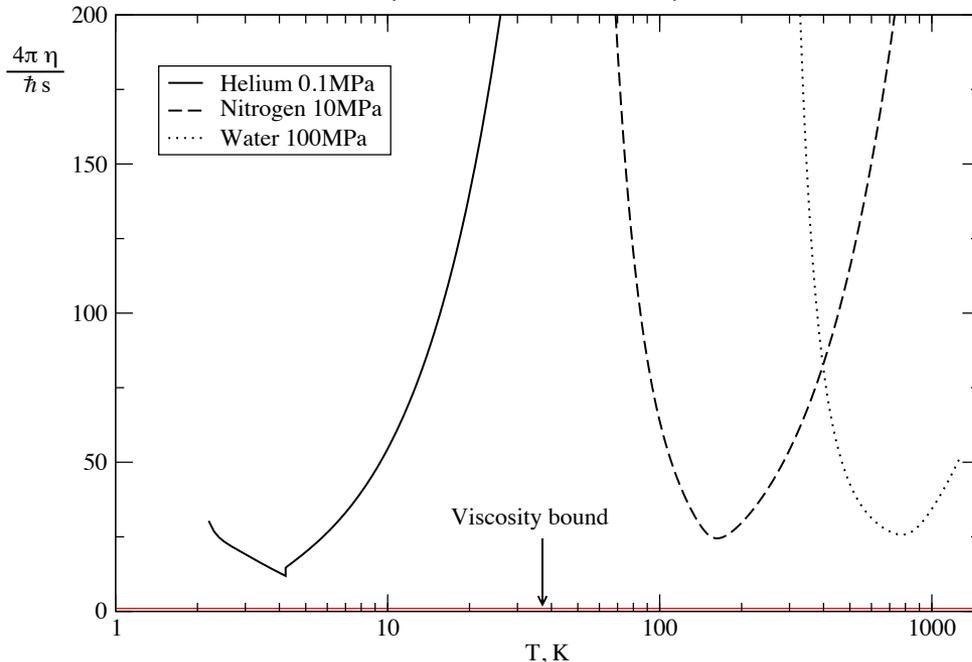
Validity of the hydrodynamics is governed by η/s

Hadron gas and QGP at very high temperature have large value η/s

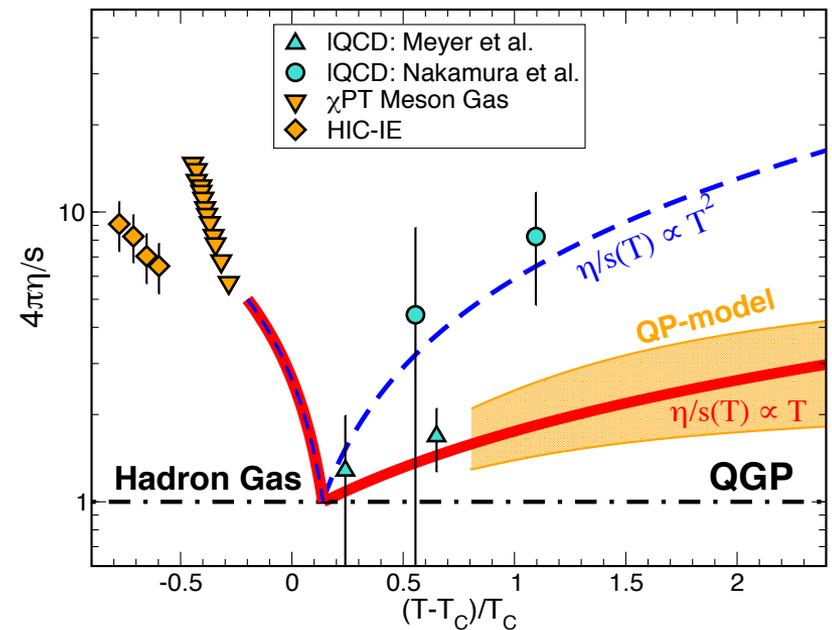
Super-symmetric gauge theories at strong coupling have small η/s with lower bound dictated by quantum mechanics $\eta/s > 1/(4\pi)$ (Kovtun, Son Starinets 2005)

\Rightarrow QGP near the transition temperature T_c has close to minimal η/s

Kovtun, Son Starinets, 2005



Csernai et al, 2013



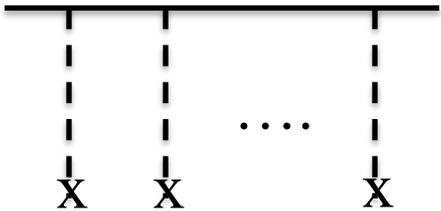
Extremely difficult to calculate in LQCD !

However, other transport coefficients are easier to calculate

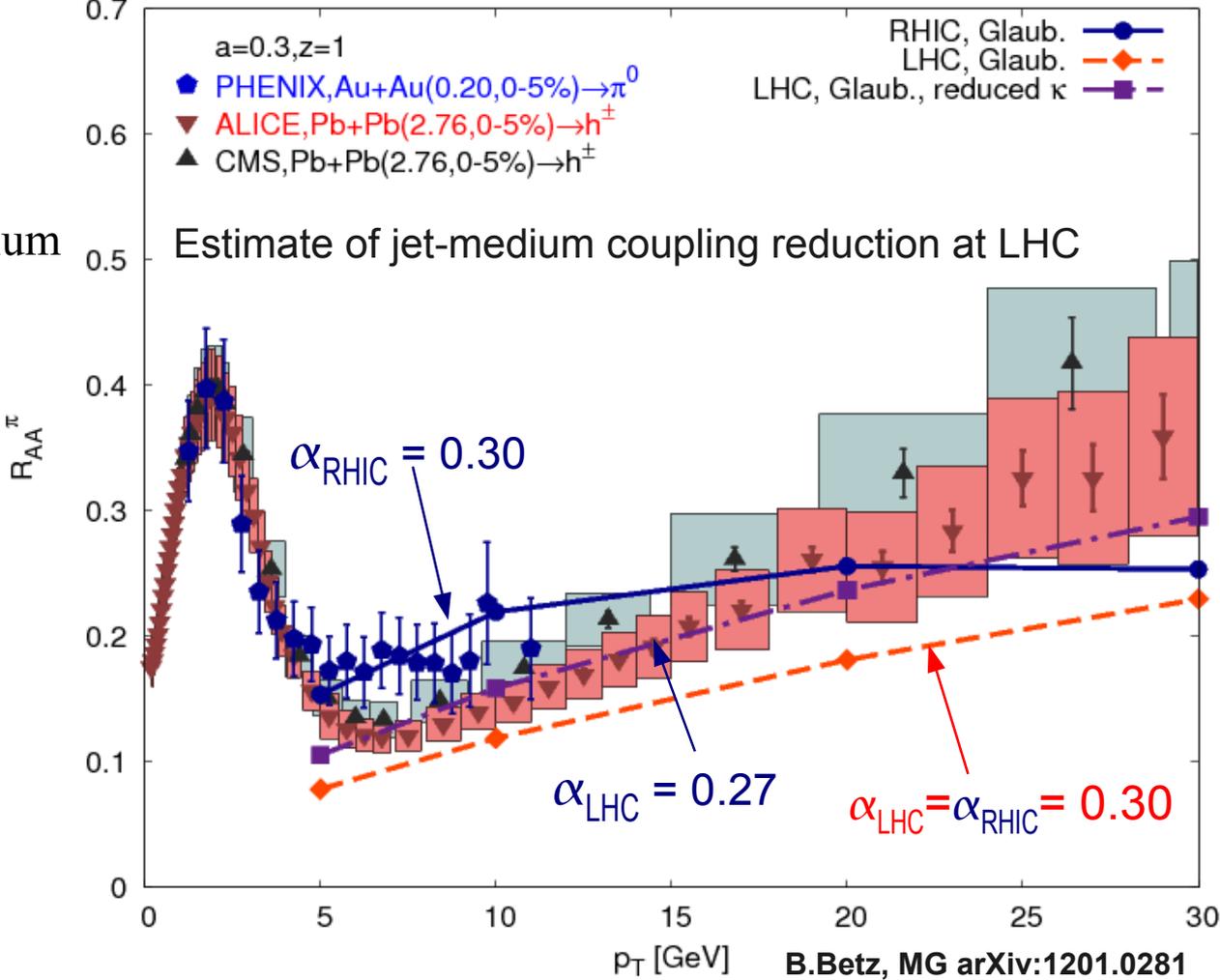
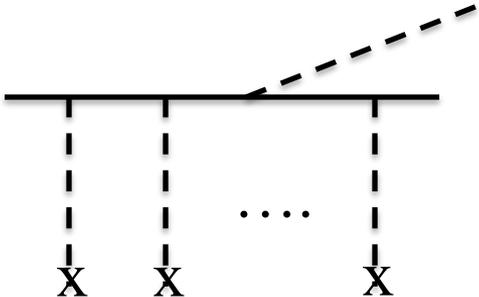
Energy loss and jet suppression

Energetic partons loose energy due to:

Elastic scattering with the constituents of the medium



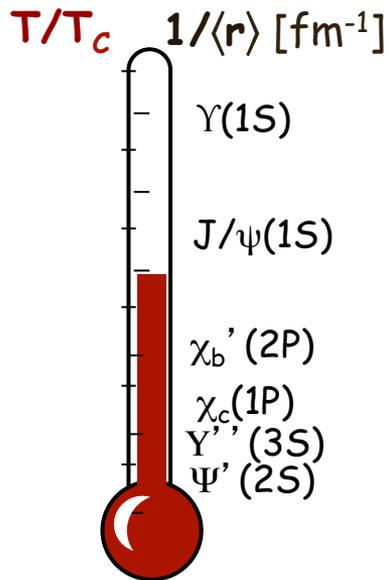
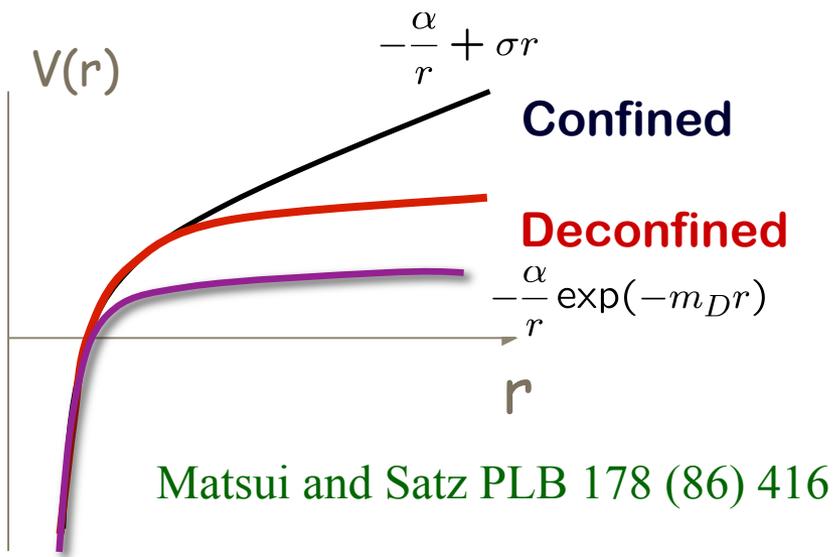
Medium induced radiation



➔ Fewer partons t large p_T suppression of RAA; The observed suppression implies Large number of scattering centers => deconfined medium

Physics at the light cone, not accessible to LQCD

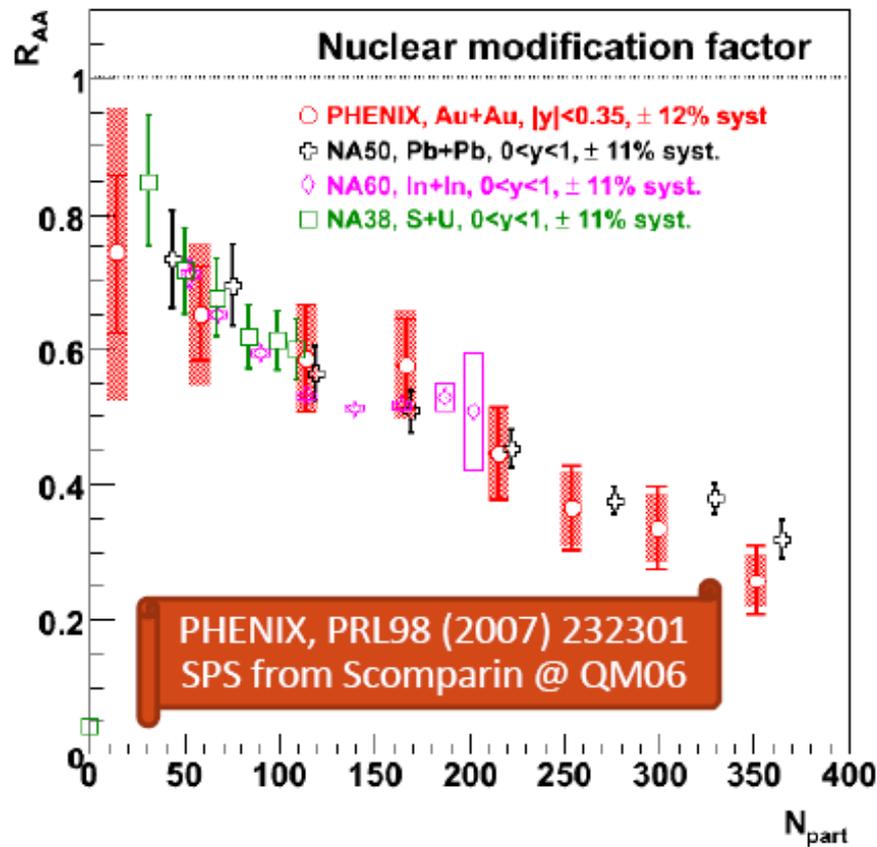
Quarkonium suppression



Melting depends on the binding energy



QGP thermometer



Strongly coupled QGP and heavy quarks

Heavy quarks ($M_c \sim 1.5 \text{ GeV}$) flow in the strongly coupled QGP

Analogy from Jamie Nagle

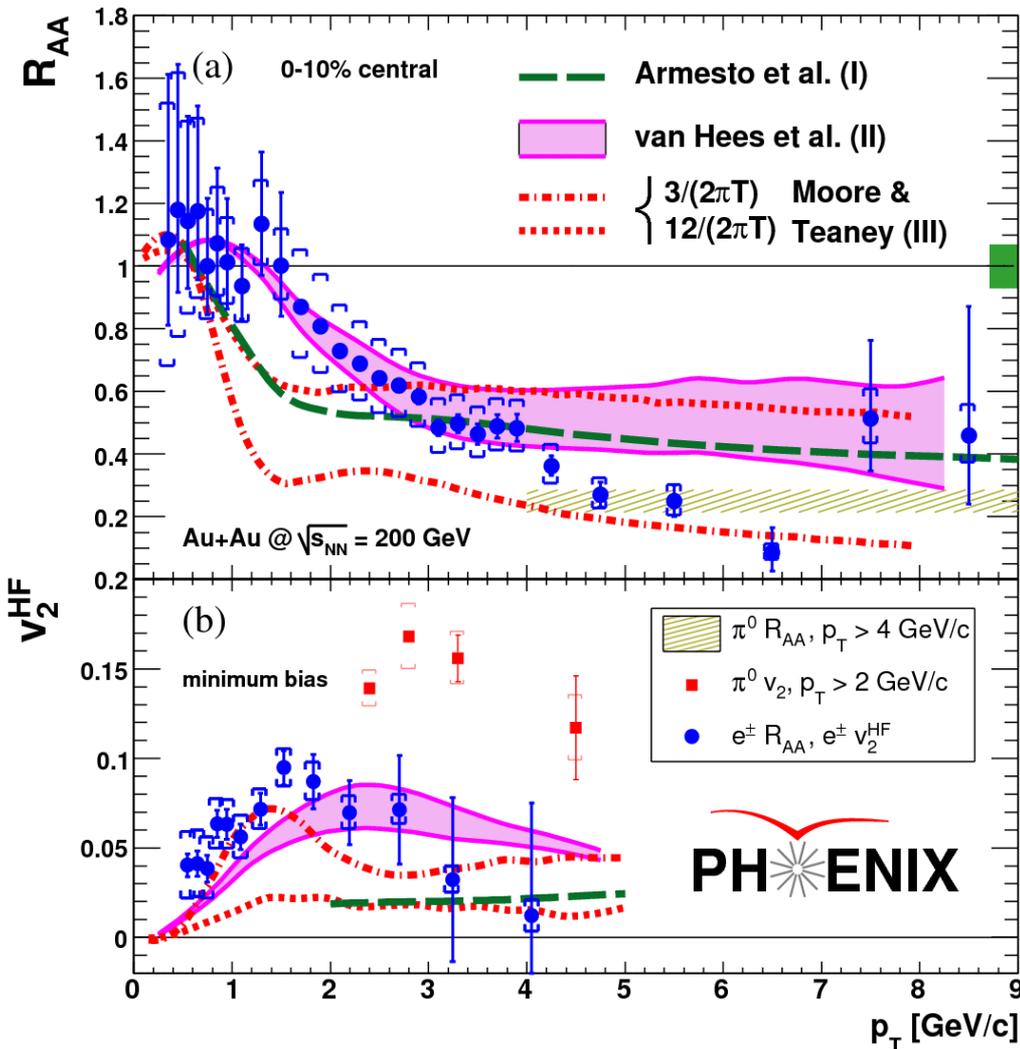


$t_{rel}^{heavy} \sim \frac{M_c}{T} t_{rel}^{light} \Rightarrow$ Langevin dynamics:

$$\frac{dx^i}{dt} = \frac{p^i}{M}, \quad \frac{dp^i}{dt} = \xi^i(t) - \eta p^i,$$

$$\langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t')$$

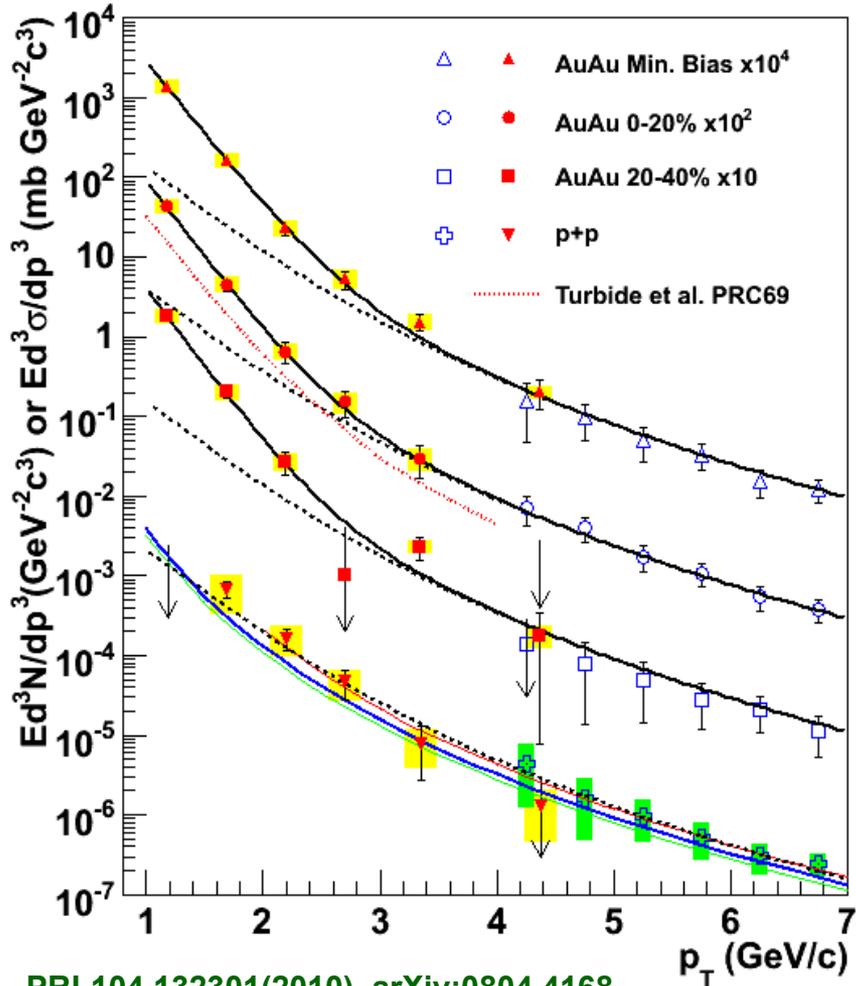
$$\eta = \frac{\kappa}{2MT}, \quad D = \frac{T}{M\eta}$$



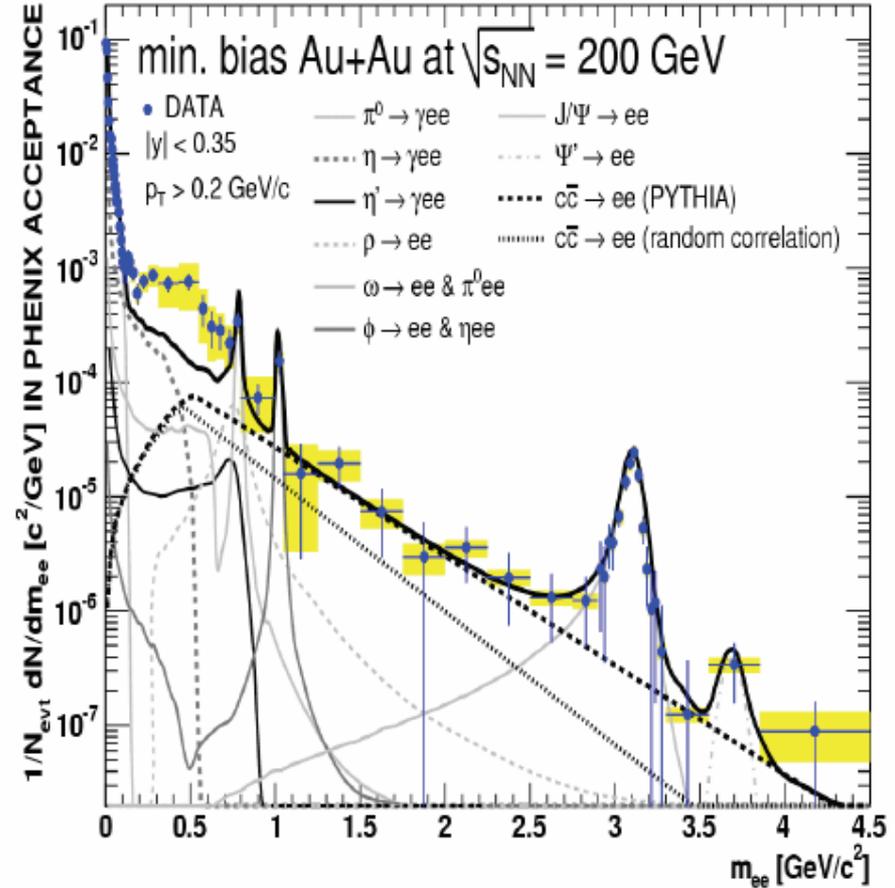
Thermal photons and dileptons

Thermal photons: analog of black body radiation

$T_{ave} = 221 \pm 19^{stat} \pm 19^{syst} \text{ MeV}$

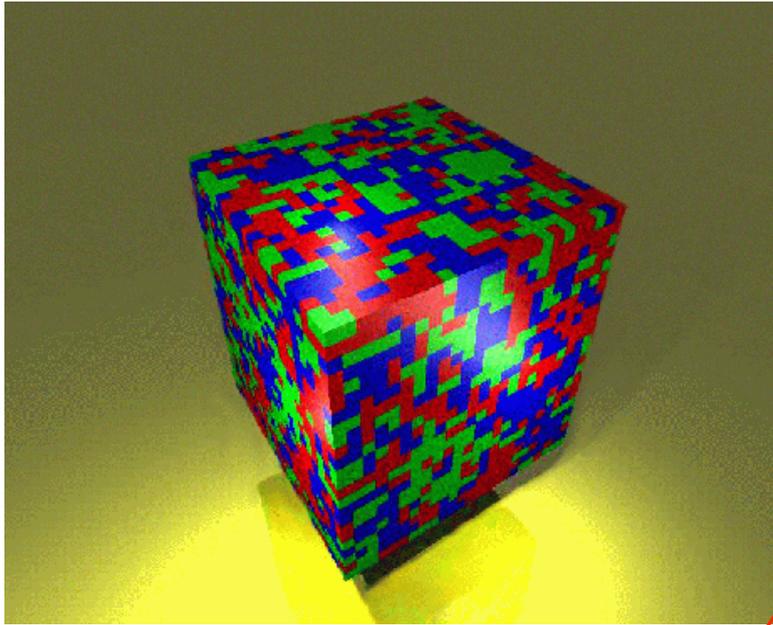


PRL104,132301(2010), arXiv:0804.4168



Thermal dileptons :
 direct measurement of the temperature of
 the produced matter, melting of the rho meson,
 test consequences of chiral
 symmetry restoration

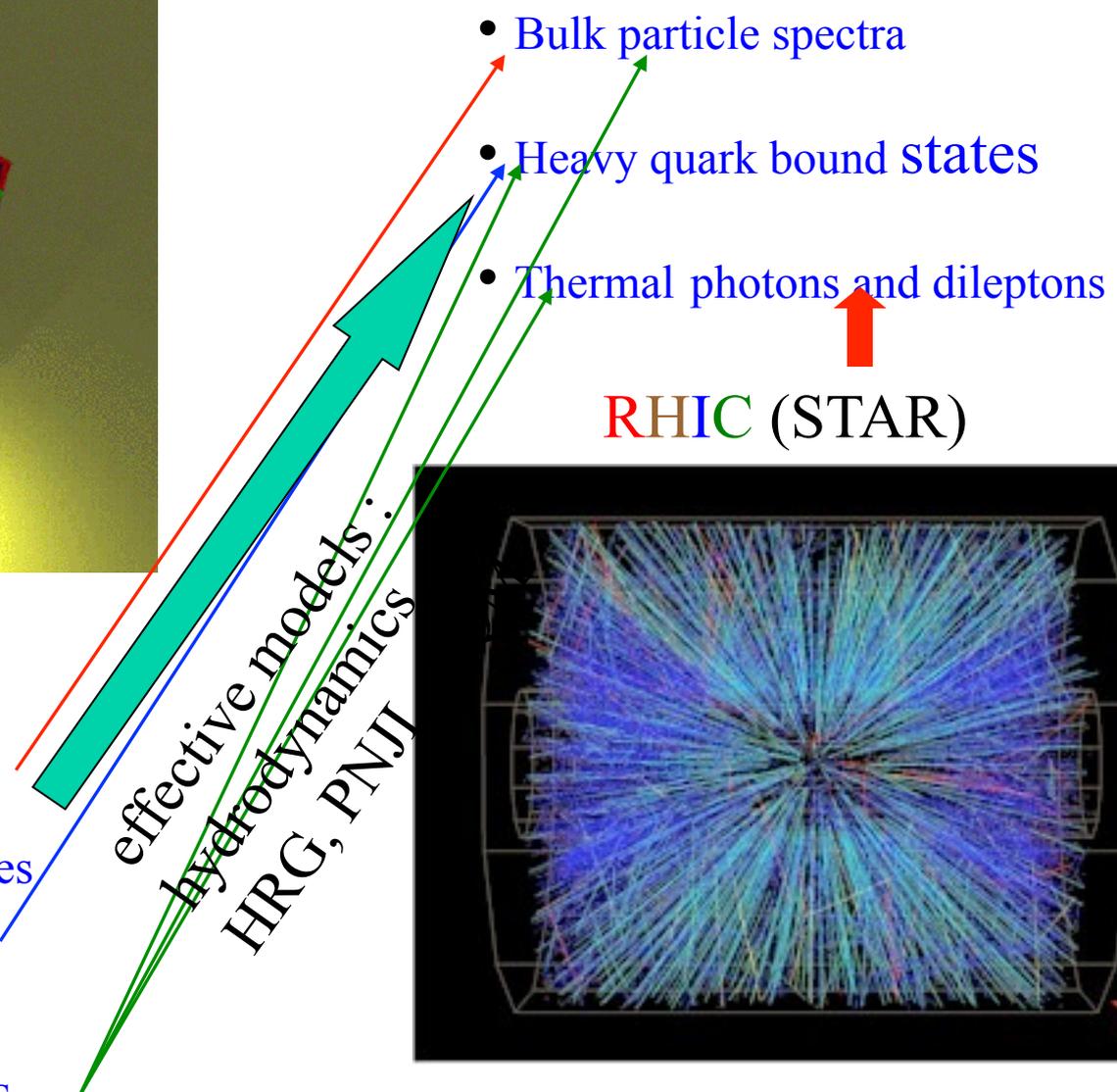
Lattice QCD at $T > 0$ and RHIC



LQCD

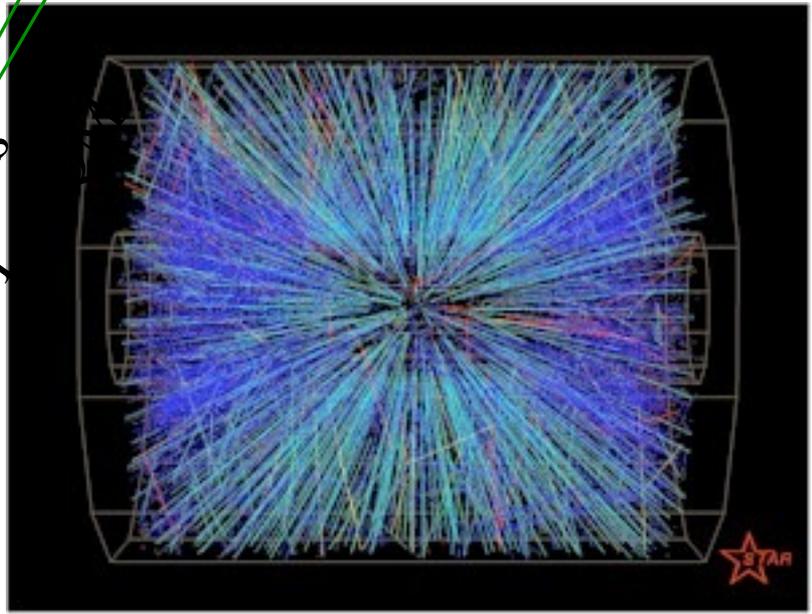


- Transition temperature, equation of state, susceptibilities
- Spatial correlation functions, heavy quark potential
- Temporal correlation functions, spectral function, transport coefficients



- Bulk particle spectra
- Heavy quark bound states
- Thermal photons and dileptons

RHIC (STAR)



In this lecture series:

1) Introduction

2) Basics of field theory at $T>0$

3) Basics of Lattice QCD: the Wilson actions, fermion formulations, Improved actions, HMC, meson correlation functions

4) Lattice QCD at $T>0$: EoS and the integral method, chiral transition, center symmetry and color screening, free energy of static quarks

5) Thermodynamics at low T , chiral perturbation theory and Hadron resonance gas model

6) Fluctuations of conserved charges in lattice QCD and experiment and the deconfinement Transition

7) Meson correlation functions, quarkonium properties at $T>0$ and quarkonium suppression, thermal photons and dileptons

Lectures posted at

http://quark.phy.bnl.gov/~petreczk/presentations/Bielefed_lectures/

For questions see me in my office, E6-124

Literature:

J. Kapusta, Finite-temperature field theory, Cambridge University Press 1989

M. LeBellac, Thermal Field Theory, Cambridge University Press 1996

H.J. Rothe, Lattice Gauge Theories, World Scientific 1997

D. Teaney, [arXiv:0905:2433](https://arxiv.org/abs/0905.2433)

P. Petreczky, [arXiv:1203:5320](https://arxiv.org/abs/1203.5320)