Lecture 7-8: Correlation functions and spectral functions

Relation of spectral function and Euclidean correlation functions

Maximum Entropy Method

Quarkonium in QGP

pNRQCD and potential models

Dilepton rate and electric conductivity

Euclidean correlators and spectral functions

Lattice QCD is formulated in imaginary time

$$G(\tau,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(\tau,\vec{x})\,J_H^{\,+}(0,0)\right\rangle,$$

$$J_H(\tau,\vec{x})=\overline{\psi}(\tau,\vec{x})\Gamma_H\psi(\tau,\vec{x})$$

$$\Gamma_H=1,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\cdot\gamma_\mu$$

Physical processes take place in real time

$$D^>(t,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(t,\vec{x})\,J_H\ (0,0)\right\rangle,$$

$$D^<(t,\vec{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(0,\vec{0})\,J_H\ (t,\vec{x})\right\rangle$$

$$\frac{D^>(\omega)-D^<(\omega)}{2\pi}=\frac{1}{\pi}\mathrm{Im}\,D_R(\omega)=\sigma(\omega)$$

$$R(\omega)=\frac{\sigma_{e^+e^-\to hadrons}}{\sigma_{e^+e^-\to\mu^+\mu^-}}=\frac{\sigma(\omega)}{\omega^2}$$

$$\boxed{G(\tau,T)=D^>(-i\tau)}$$

$$\boxed{G(\tau,T)=\int_0^\infty d\omega\,\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T)))}{\sinh(\omega/(2T))}}$$

if $T=0$ and $\sigma(\omega)=\sum_n A_n\delta(\omega-E_n)$ $\Rightarrow$ $G(\tau)=A_0e^{-E_0\tau}+A_1e^{-E_1\tau}+\ldots$

fit the large distance behavior of the lattice correlation functions

This is not possible for $T>0$, $\tau_{max}=1/T$ $\Rightarrow$ Maximum Entropy Method (MEM)

Spectral functions at T>0 and physical observables

Heavy meson spectral functions:

$$\boxed{J_H = \bar{\psi}\,\Gamma_H\,\psi}$$

→ quarkonia properties at T>0
heavy quark diffusion in QGP: D

Quarkonium suppression (R_{AA})
Open charm/beauty suppression (R_{AA})

thermal dilepton production rate
(# of dileptons/photons per unit 4-volume)

Light vector meson spectral functions:

$$\boxed{J_\mu = \bar{\psi}\,\gamma_\mu\,\psi}$$

→

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha_{em}^2}{27\pi^2}\frac{1}{e^{\omega/T}-1}\frac{\sigma_{\mu\mu}(\omega,p,T)}{\omega^2-p^2}$$

thermal photon production rate :

$$p\frac{dW}{d^3p} = \frac{5\alpha_{em}}{9\pi}\frac{1}{e^{p/T}-1}\sigma_{\mu\mu}(\omega=p,p,T)$$

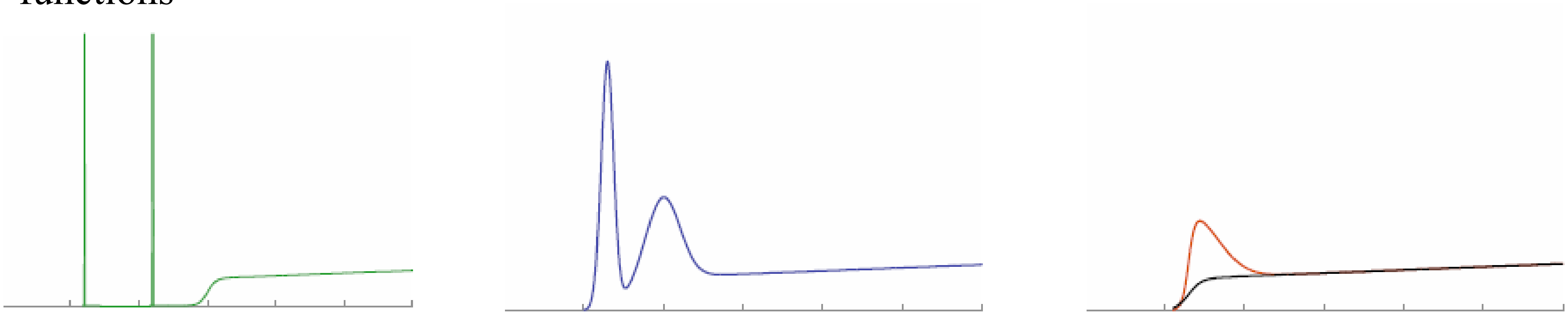
2 massless quark (u and d) flavors are assumed; for arbitrary number of flavors $5/9 \rightarrow \Sigma_f Q_f^2$

electric conductivity ζ :

Thermal photons and dileptons provide information about the temperature of the medium produced in heavy ion collisions
Low mass dileptons are sensitive probes of chiral symmetry restoration at $T>0$

Meson spectral functions and lattice QCD

In-medium properties and/or dissolution of quarkonium states are encoded in the spectral functions

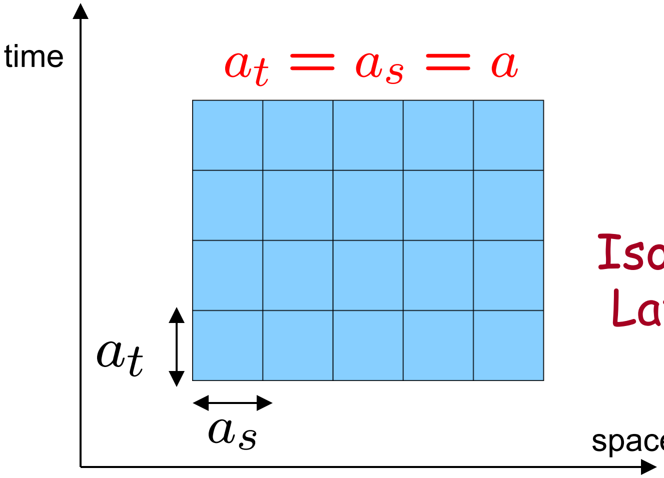


Melting is seen as progressive broadening and disappearance of the bound state peaks

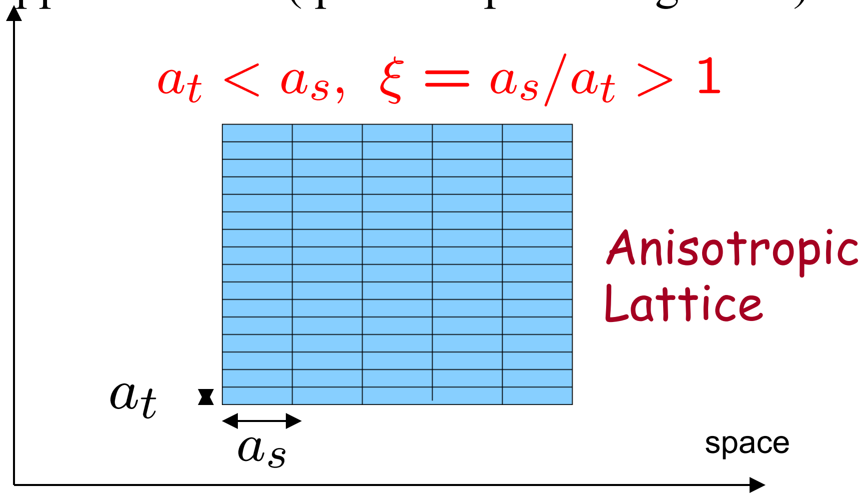
Need to have detailed information on meson correlation functions → large temporal extent N_τ

Good control of discretization effects → small lattice spacing a

Computationally very demanding → use quenched approximation (quark loops are neglected)



$$T = \frac{1}{N_t a_t}$$



Meson spectral functions in the free theory

At high energy ω and high T the meson spectral functions can be calculated using perturbation theory

LO (free theory) :

$$\sigma_i(\omega) = \theta(\omega^2 - 4M^2)\frac{1}{4\pi^2}\omega^2\sqrt{1-\frac{4M^2}{\omega^2}}\left(A_i + B_i\frac{4M^2}{\omega^2}\right)\tanh(\omega/4T) + \chi_i\omega\delta(\omega)$$

$$\chi^i(T) = \frac{6}{\pi^2}\int_0^\infty dp p^2 \left(a_i + b_i\frac{M^2}{E_p^2} + c_i\frac{p^2}{E_p^2}\right)\left(-\frac{\partial n_F}{\partial E_p}\right)$$

zero mode contribution
$\rightarrow$ transport coefficients

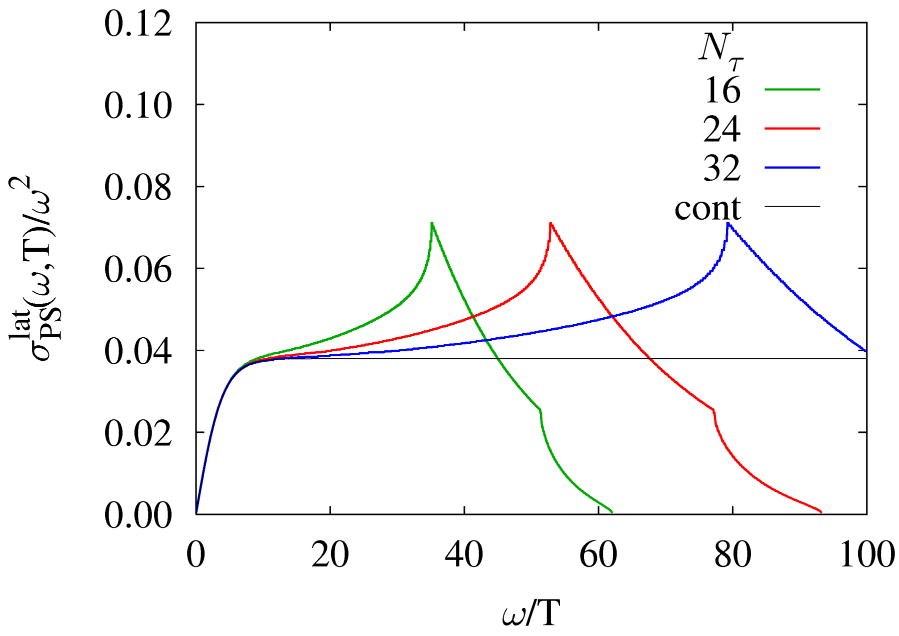
$a_{sc} = 0,\ a_{ax} = 1,\ a_{vc} = 0,\ a_{ps} = 0;$

$b_{sc} = 1,\ b_{ax} = 2,\ b_{vc} = 0,\ b_{ps} = 0;$

$c_{sc} = 0,\ c_{ax} = 0,\ c_{vc} = 1,\ c_{ps} = 0;$

Karsch et al, PRD 68 (03) 014504
Aarts, Martinez Resco NPB 726 (05) 93

The free spectral functions can also be calculated on the lattice using Wilson type fermions

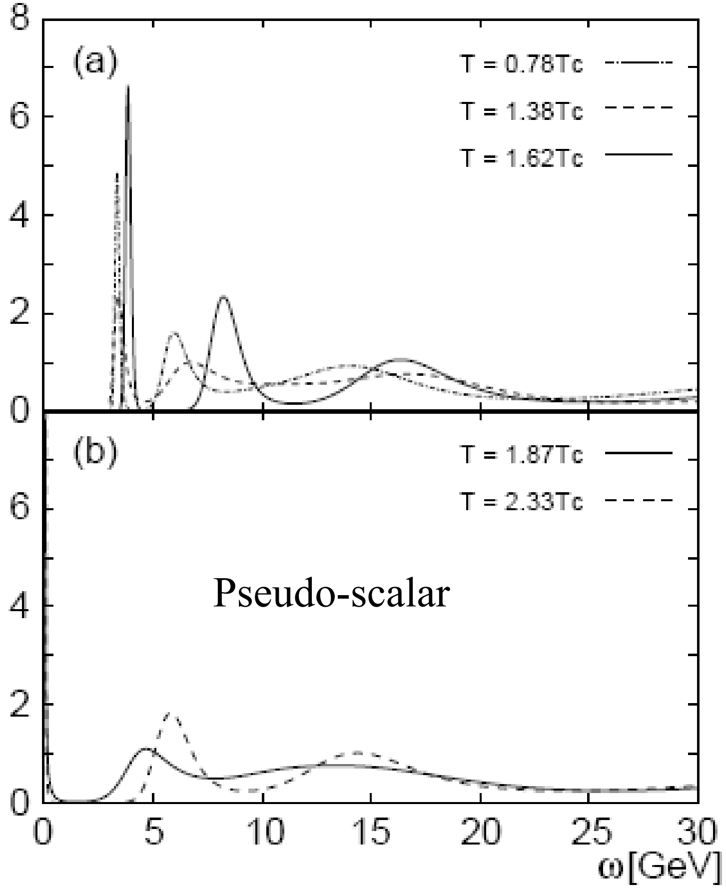


An example: charmonium spectral functions in lattice QCD

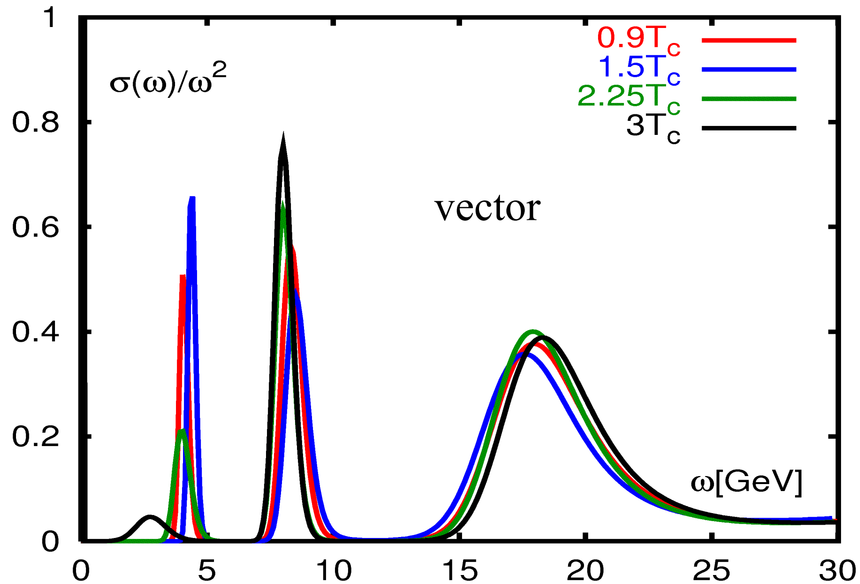
γ_5 : Pseudo – scalar(PS) $\rightarrow$ η_c $(^1S_0)$ 1 : Scalar(SC) $\rightarrow$ χ_{c0} $(^3P_0)$

γ_μ : Vector(VC) $\rightarrow$ J/ψ $(^3S_1)$ $\gamma_5\gamma_\mu$: Axial – Vector(AX) $\rightarrow$ χ_{c1} $(^3P_1)$

Asakawa, Hatsuda, PRL 92 (2004) 01200



Datta, et al, PRD 69 (04) 094507



1S state charmonia may survive at lease up to *$1.6T_c$??*

see also

Umeda et al, EPJ C39S1 (05) 9, Iida et al, PRD 74 (2006) 074502

Reconstruction of the spectral functions : MEM

$$G(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T)\cdot K(\omega,T)$$

$\mathcal{O}(10)$ data and $\mathcal{O}(100)$ degrees of freedom to reconstruct

Bayes' theorem :
P[X|Y]=P[Y|X]P[X]/P[Y]

P[XY]=P[X|Y]P[Y]
=P[Y|X]P[X]

Bayesian techniques: find $\sigma(\omega,T)$ which maximizes

$$P[\sigma|DH]\sim P[D|\sigma H]P[\sigma|H]$$

data

Prior knowledge

prior probability

H : $\sigma(\omega,T)>0$ Maximum Entropy Method (MEM): $P[\sigma|H]=e^{\alpha S}$

Asakawa, Hatsuda, Nakahara, PRD 60 (99) 091503, Prog. Part. Nucl. Phys. 46 (01) 459

$$P[\sigma|DH]=P[\sigma|D\alpha m]=\exp(-\frac{1}{2}\chi^2+\alpha S)$$

Likelihood function

Shannon-Janes entropy:

$$S=\int_0^\infty d\omega[\sigma(\omega)-m(\omega)-\sigma(\omega)\ln\frac{\sigma(\omega)}{m(\omega)}]$$

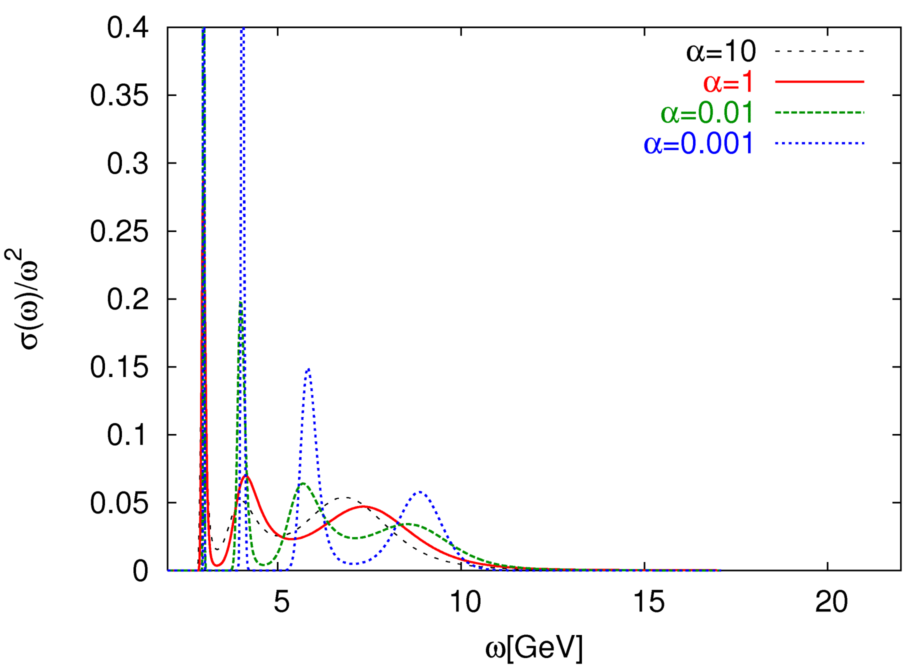
$m(\omega)$ - default model $m(\omega\gg\Lambda_{QCD})=m_0\omega^2$ -perturbation theory

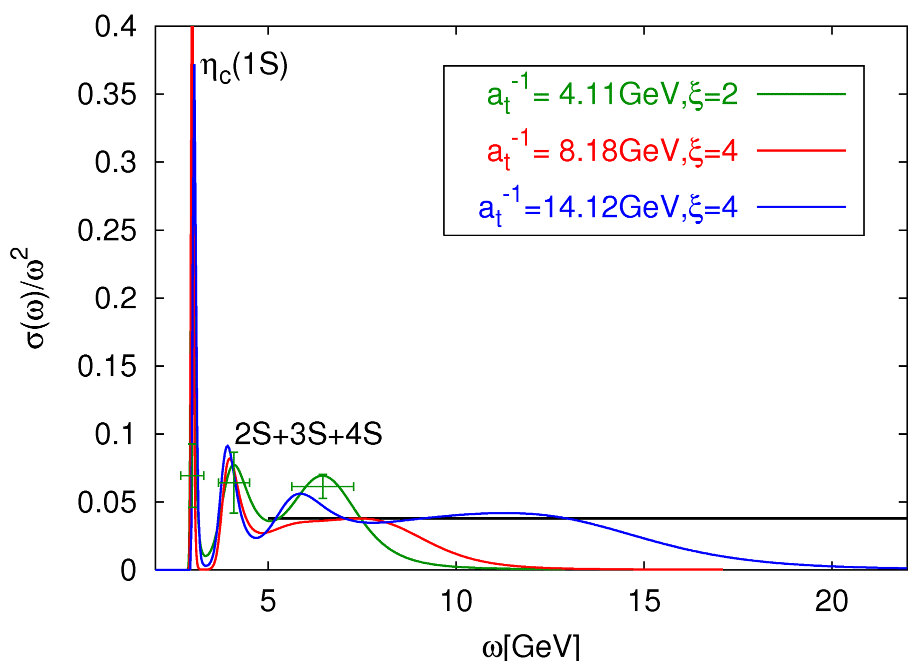
Charmonium spectral functions at T=0

Anisotropic lattices: $16^3 \times 64, \xi = 2$ $16^3 \times 96, \xi = 4$, $24^3 \times 160, \xi = 4$
$L_s = 1.35 - 1.54$fm, #configs=500-930;
Wilson gauge action and Fermilab heavy quark action

Pseudo-scalar (PS) $\rightarrow$ S-states

Jakovác, P.P. , Petrov, Velytsky, PRD 75 (2007) 014506





For $\omega > 5$ GeV the spectral function is sensitive to lattice cut-off ;
good agreement with 2-exponential fit for peak position and amplitude

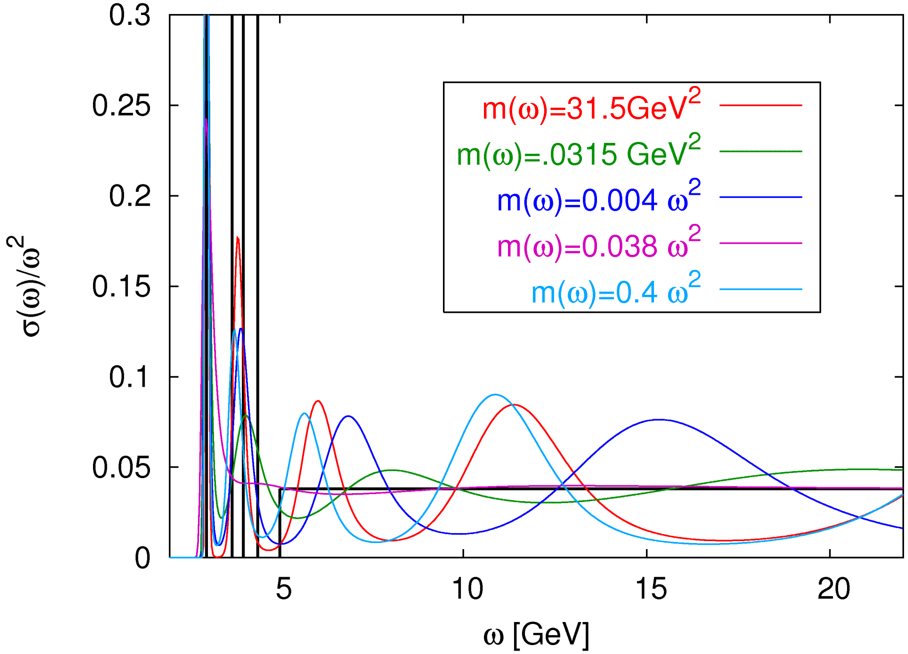
Charmonium spectral functions at T=0 (cont' d)

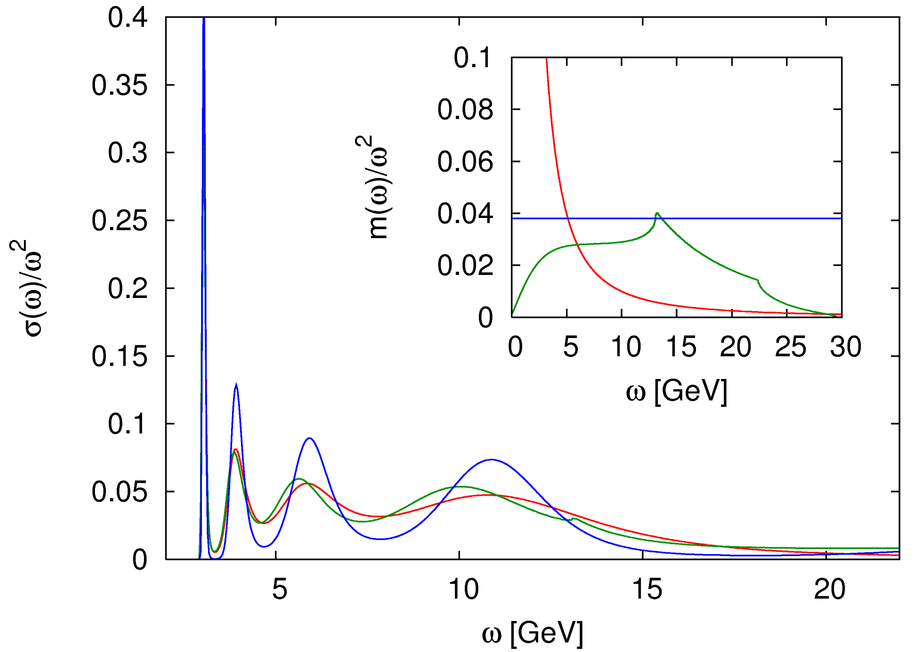
What states can be resolved and what is the dependence on the default model ?

Reconstruction of an input spectral function :

Lattice data in PS channel for:

$$a_t^{-1} = 14.12\text{GeV},\ N_t = 160$$





Ground states is well resolved, no default model dependence;
Excited states are not resolved individually, moderate dependence on the default model;
Strong default model dependence in the continuum region, $\omega > 5$ GeV

Charmonia correlators at T>0

temperature dependene of $G(\tau,T)$

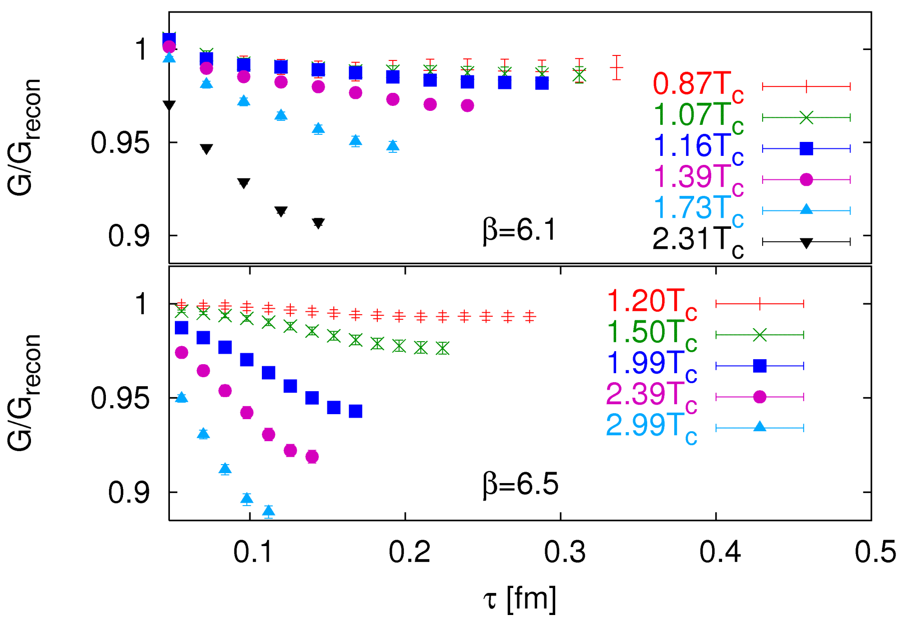
$$G(\tau,T) = \int_0^\infty d\omega\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T))}{\sinh(\omega/(2T))}$$

If there is no T-dependence in the spectral function, $G(\tau,T)/G_{recon}(\tau,T) = 1$

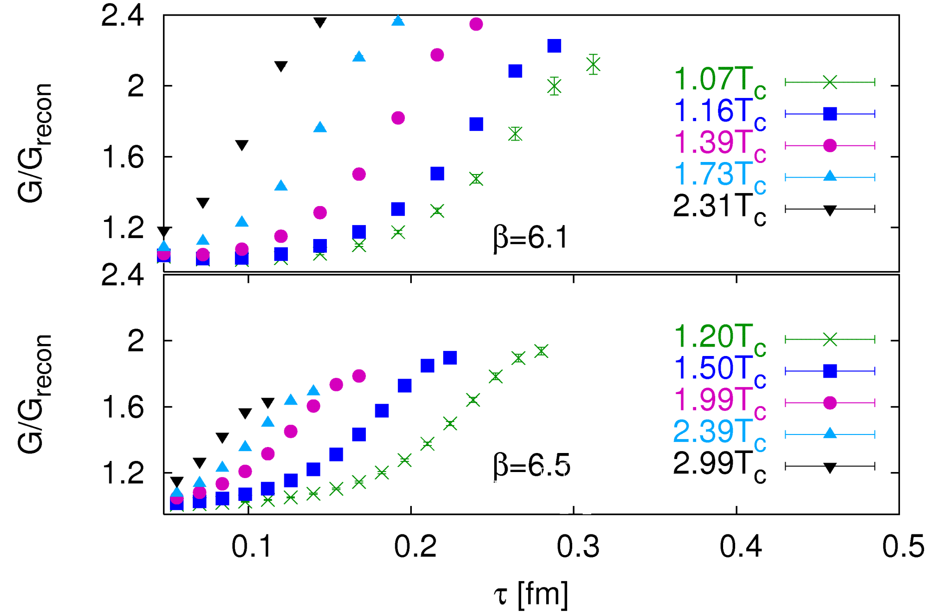
$$G_{recon}(\tau,T) = \int_0^\infty d\omega\sigma(\omega,T=0)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))}$$

Jakovác, P.P. , Petrov, Velytsky, PRD 75 (2007) 014506

PS, $\Gamma_H = \gamma_5$

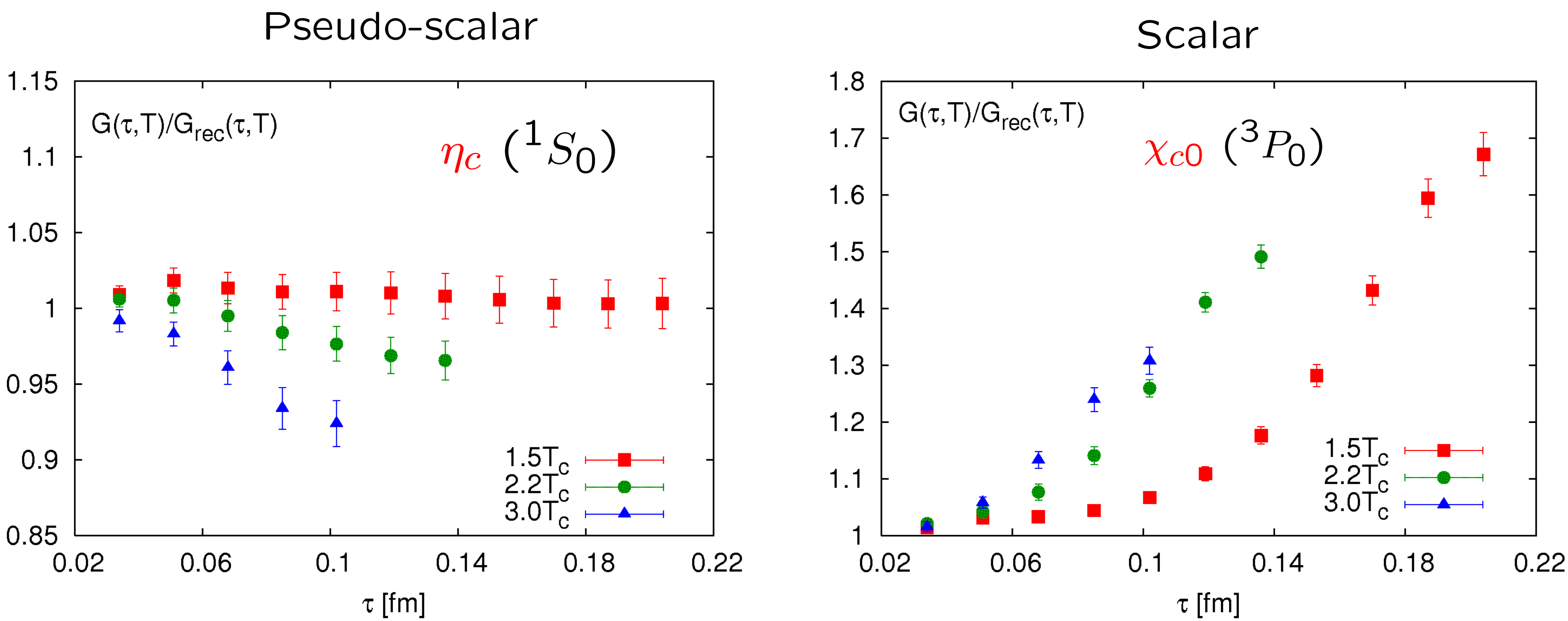


SC, $\Gamma_H = 1$



in agreement with calculations on isotropic lattice: Datta et al, PRD 69 (04) 094507

Temperature dependence of quarkonium



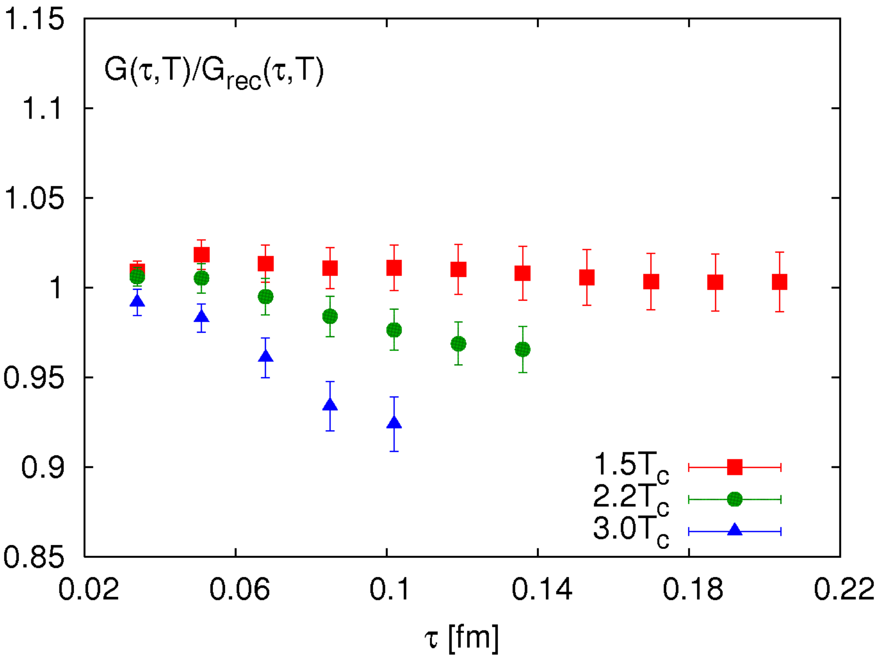
Datta, Karsch, P.P , Wetzorke, PRD 69 (2004) 094507

zero mode contribution is not present in the time derivative of the correlator

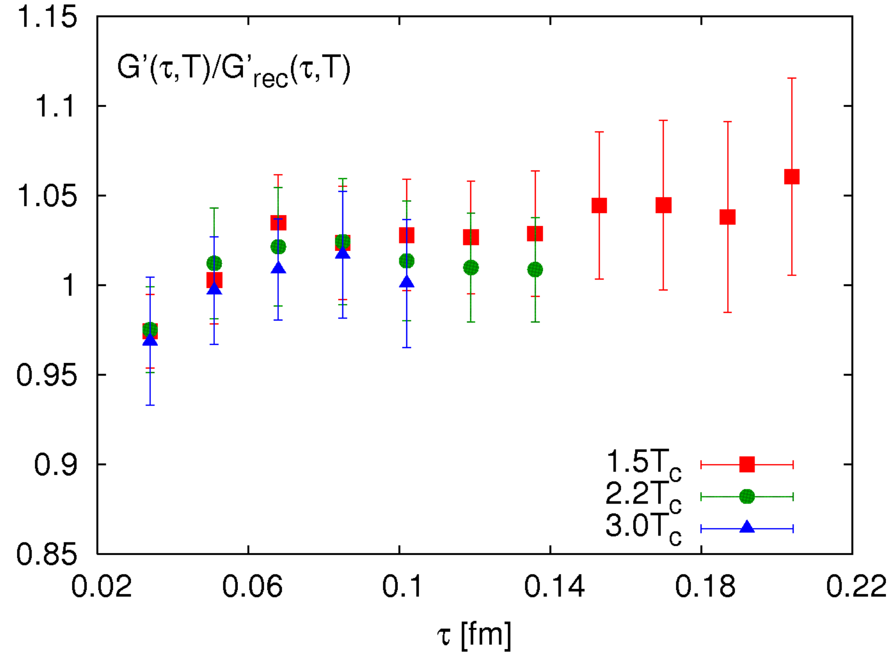
Umeda, PRD 75 (2007) 094502

Temperature dependence of quarkonium

Pseudo-scalar



Scalar

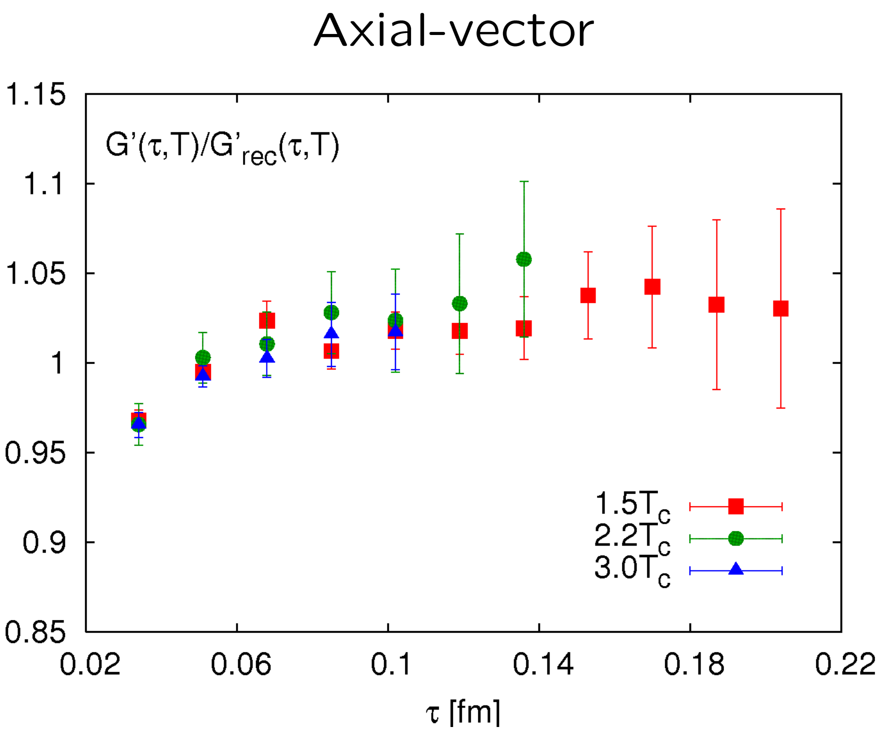


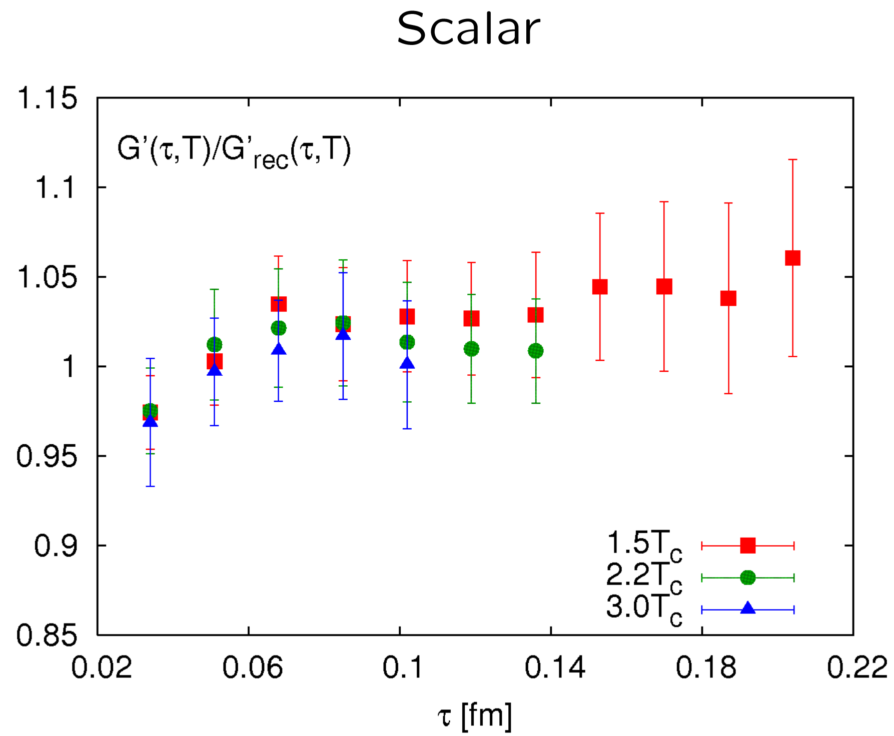
No change in the derivative of the scalar quarkonium correlator up to $3T_c$!

Almost the entire temperature dependence of the scalar correlators is given by the zero mode contribution !

In agreement with previous findings: Mócsy, P.P, PRD 77 (2008) 014501
Umeda, PRD 75 (2007) 094502

Temperature dependence of quarkonium

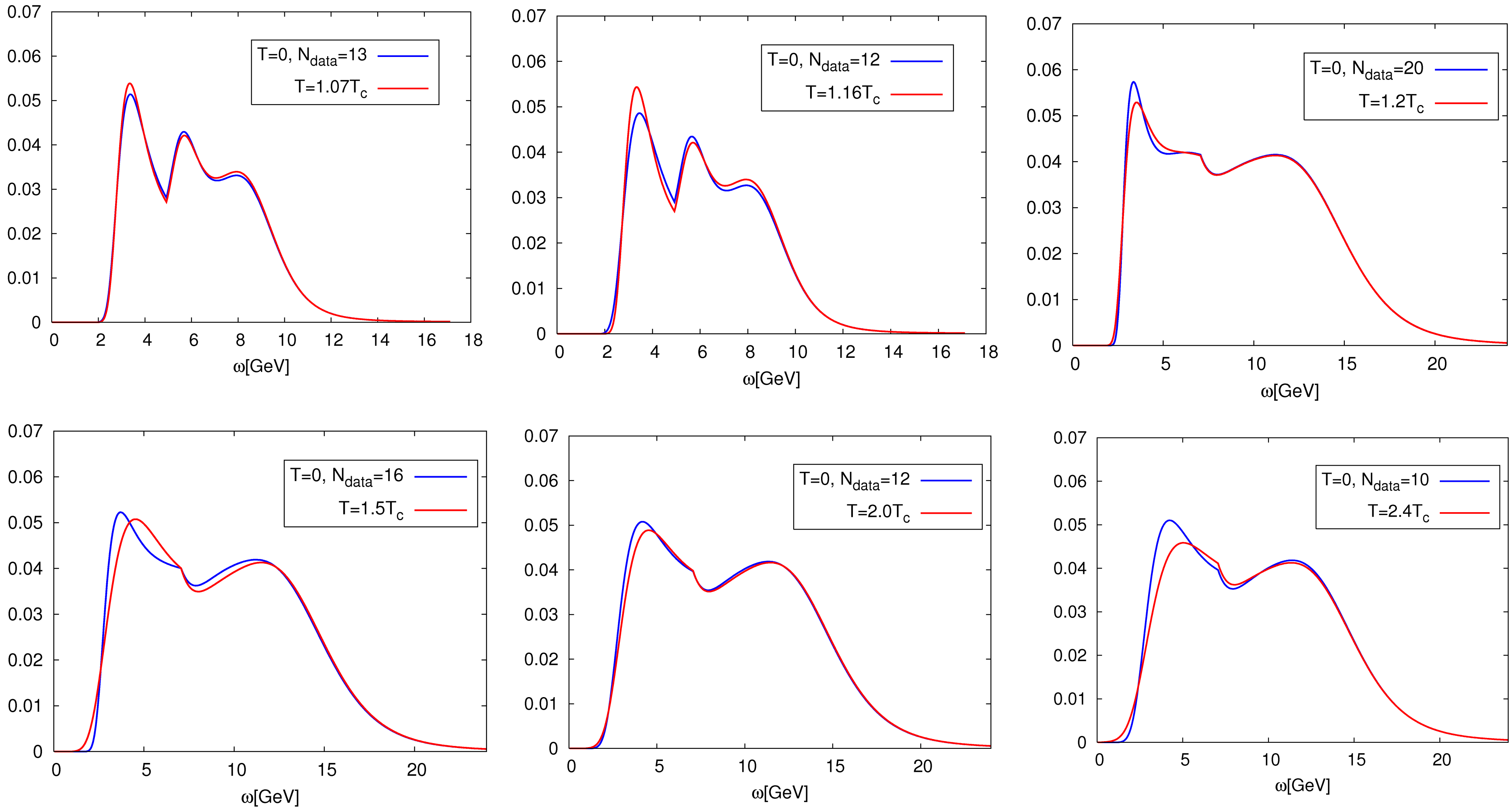




The situation is the same in the axial-vector correlator

In agreement with previous findings: Mócsy, P.P, PRD 77 (2008) 014501
Umeda, PRD 75 (2007) 094502

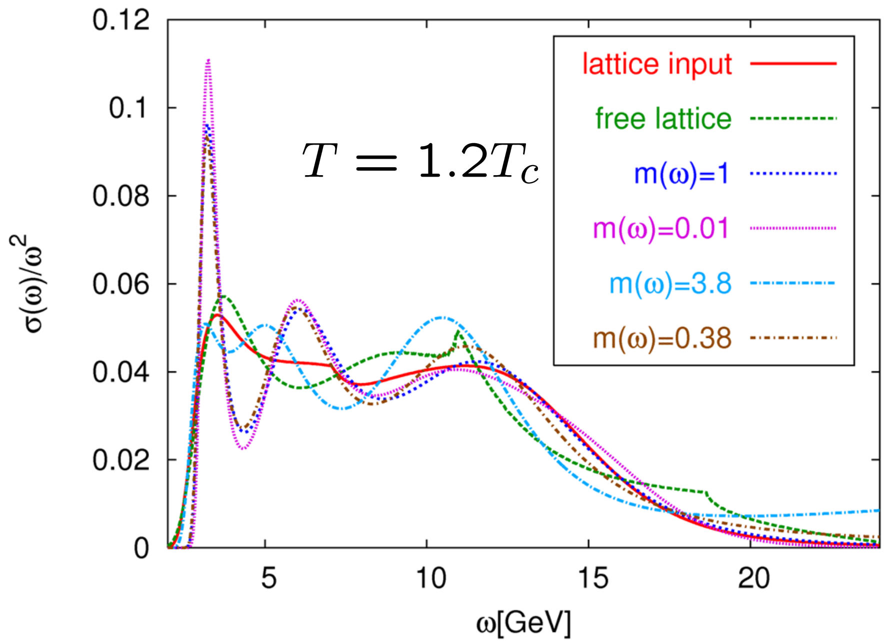
Using default model from the high energy part of the T=0 spectral functions :
resonances appears as small structures on top of the continuum,
little T-dependence in the PS spectral functions till $T \simeq 2.4T_c$
but no clear peak structrure



Charmonia spectral functions at T>0 (cont' d)

Jakovác, P.P. , Petrov, Velytsky, PRD 75 (2007) 014506

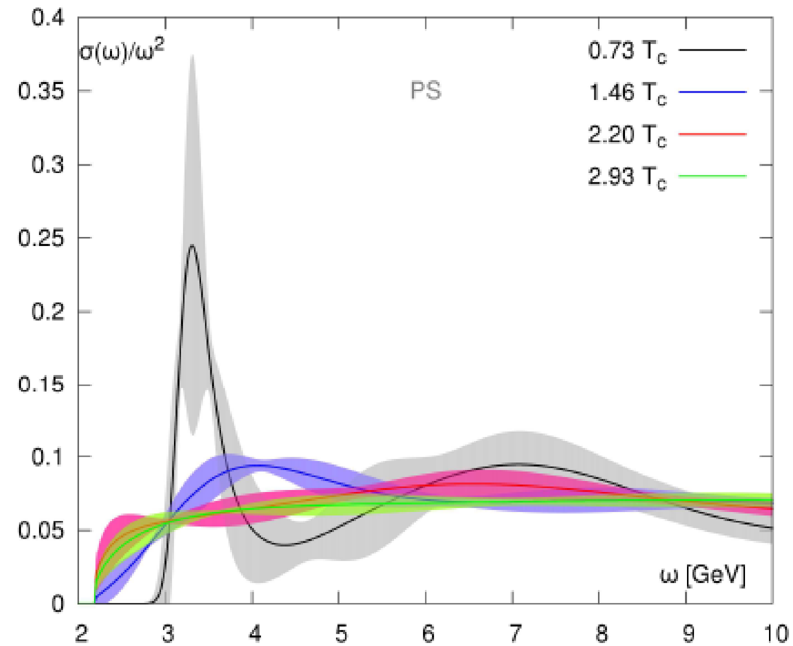
PS, $24^3 \times 40$, $a_t^{-1} = 14.12$ GeV, $\xi = 4$,



there is a strong dependence on the default model $m(\omega)$ at finite temperature

Ding et al arXiv:1011.0695 [hep-lat]

$128^3 \times N_\tau$, $N_\tau = 96 - 24$
$a_t^{-1} = a_s^{-1} = a^{-1} = 18.97$ GeV

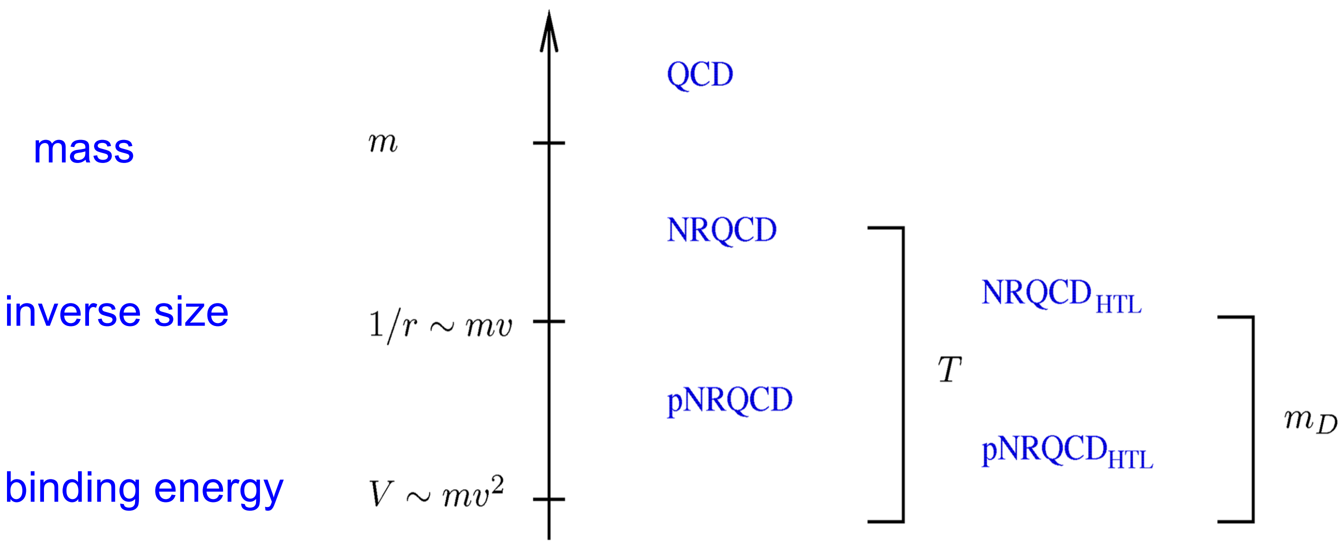


$m(\omega)$:
free lattice spectral functions

with realistic choices of the default model no peaks can be seen in the spectral functions in the deconfined phase !

Effective field theory approach for heavy quark bound states and potential models

The heavy quark mass provides a hierarchy of different energy scales



The scale separation allows to construct sequence of effective field theories: NRQCD, pNRQCD

Potential model appears as the tree level approximation of the EFT and can be systematically improved

Brambilla, Ghiglieri, P.P., Vairo, PRD 78 (08) 014017

pNRQCD at finite temperature for static quarks

EFT for energy scale : $E_{bind} \sim \Delta V = (V_o - V_s) \sim mv^2$

Ultrasoft quark and gluons

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{i=1}^{n_f} \bar{q}_i \, i \not{D} q_i$$

Singlet $Q\bar{Q}$ field Octet $Q\bar{Q}$ field

$$+ \int d^3r \, \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left[i\partial_0 - \frac{-\nabla^2}{m} - V_s(r,T) \right] \mathrm{S} + \mathrm{O}^\dagger \left[iD_0 - \frac{-\nabla^2}{m} - V_o(r,T) \right] \mathrm{O} \right\}$$

$$+ V_A \, \mathrm{Tr}\left\{ \mathrm{O}^\dagger \vec{r} \cdot g\vec{E} \, \mathrm{S} + \mathrm{S}^\dagger \vec{r} \cdot g\vec{E} \, \mathrm{O} \right\} + \frac{V_B}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \vec{r} \cdot g\vec{E} \, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \vec{r} \cdot g\vec{E} \right\} + \ldots$$

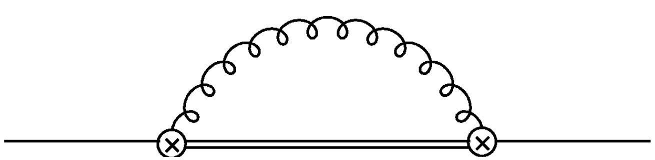
potential is the matching parameter of EFT !

Free field limit => Schrödinger equation

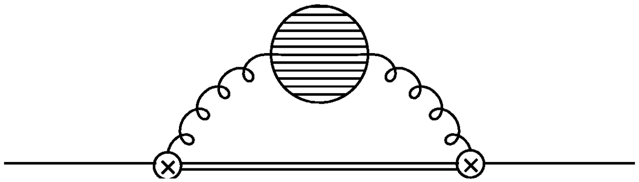
$$\left[i\partial_0 - \frac{-\nabla^2}{m} - V_s(r,T) \right] S(r,t) = 0$$

$E_{bind} \sim \Delta V \sim \alpha_s / r \ll T, \; m_D$ there are thermal contribution to the potentials

Singlet-octet transition :



Landau damping :



Brambilla, Ghiglieri, P.P., Vairo, PRD 78 (08) 014017

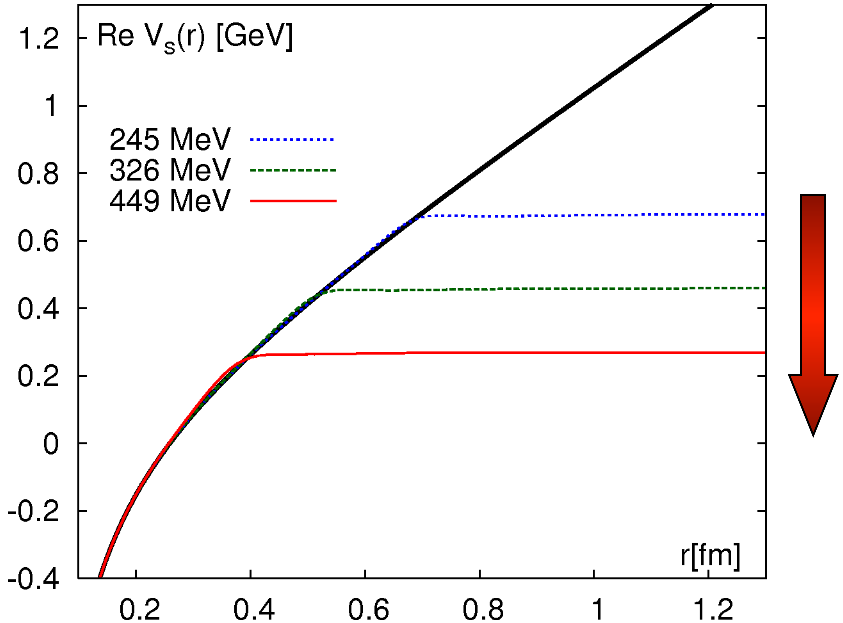
pNRQCD beyond weak coupling and potential models

Above deconfinement the binding energy is reduced and eventually $E_{bind} \sim mv^2$ is the smallest scale in the problem (zero binding) $mv^2 >> \Lambda_{QCD}, 2\pi T, m_D$ => most of medium effects can be described by a T-dependent potential

Determine the potential by non-perturbative matching to static quark anti-quark potential calculated on the lattice

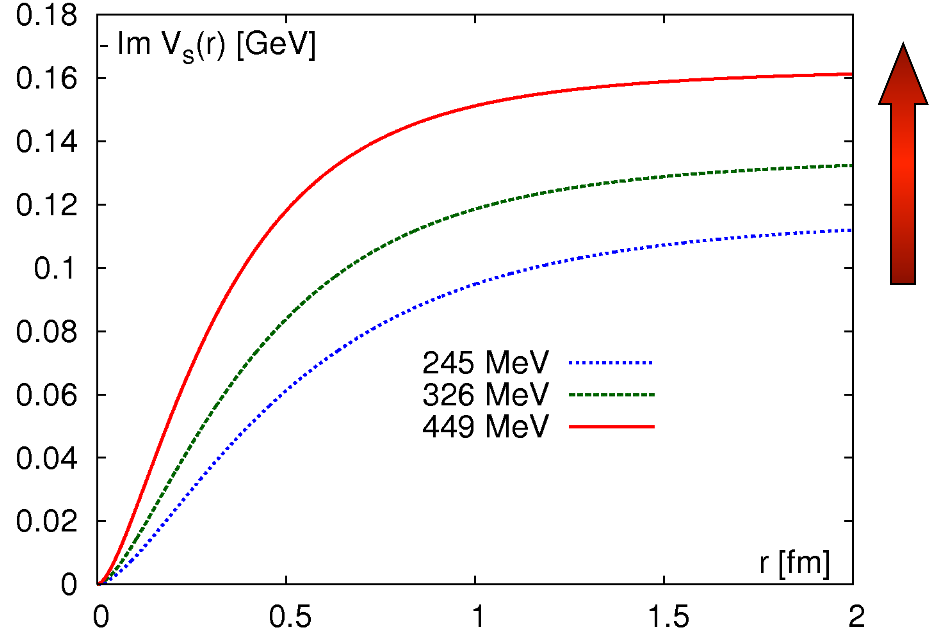
Caveat : it is difficult to extract static quark anti-quark energies from lattice correlators => constrain $\mathrm{Re}V_s(r)$ by lattice QCD data on the singlet free energy, take $\mathrm{Im}V_s(r)$ from pQCD calculations

"Maximal" value for the real part



Mócsy, P.P., PRL 99 (07) 211602

Minimal (perturbative) value for imaginary part



Laine et al, JHEP0703 (07) 054,
Beraudo, arXiv:0812.1130

Lattice QCD based potential model

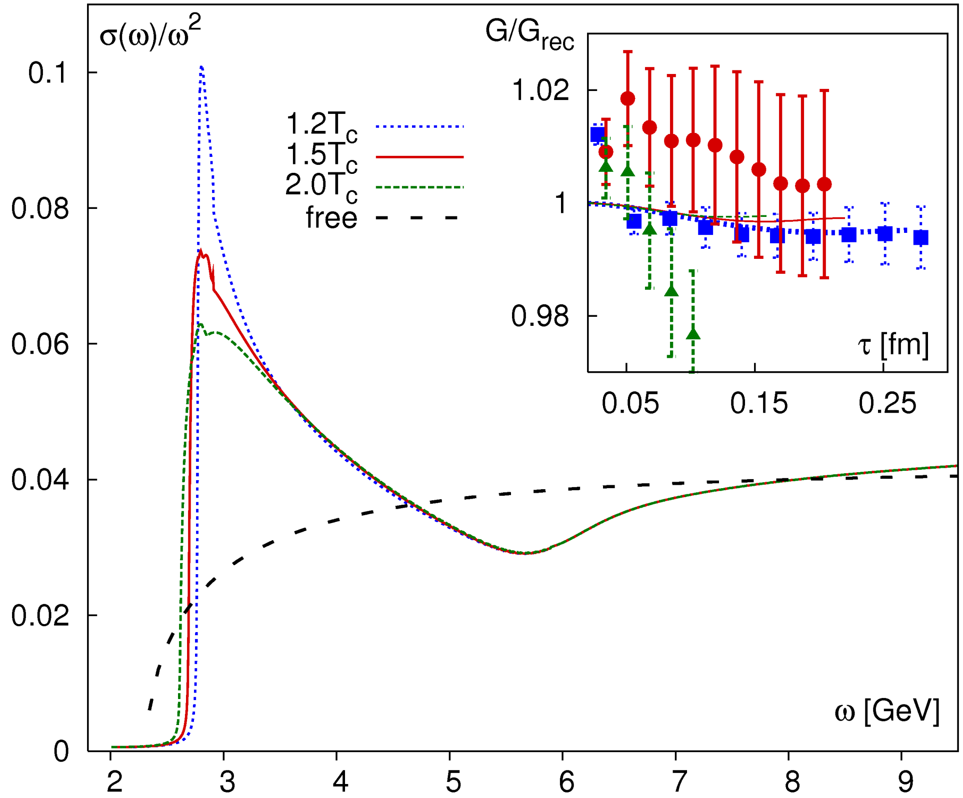
If the octet-singlet interactions due to ultra-soft gluons are neglected :

$$\left[i\partial_0 - \frac{-\nabla^2}{m} - V_s(r,T)\right] S(r,t) = 0 \quad \Rightarrow \quad \sigma(\omega, T)$$

potential model is not a model but the tree level approximation of corresponding EFT that can be systematically improved

Test the approach vs. LQCD : quenched approximation, $F_1(r,T) < \mathrm{Re}V_s(r,T) < U_1(r,T)$, $\mathrm{Im}V(r,T) \approx 0$

Mócsy, P.P., PRL 99 (07) 211602, PRD77 (08) 014501, EPJC ST 155 (08) 101



- resonance-like structures disappear already by $1.2T_c$
- strong threshold enhancement above free case

  => indication of correlations
- height of bump in lattice and model are similar
- The correlators do not change significantly despite the melting of the bound states => it is difficult to distinguish bound state from threshold enhancement in lattice QCD

The role of the imaginary part for charmonium

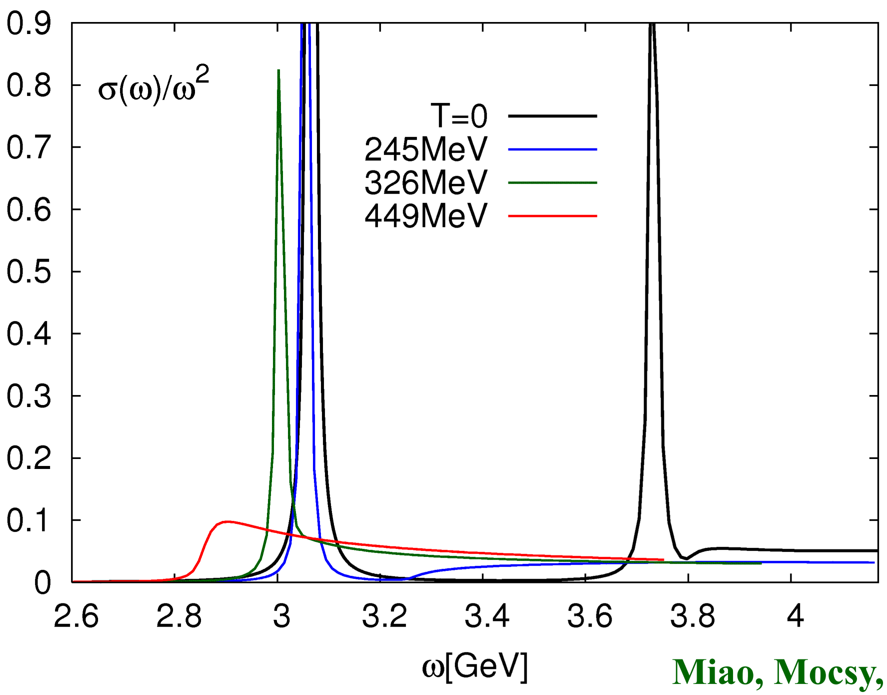
Take the upper limit for the real part of the potential allowed by lattice calculations

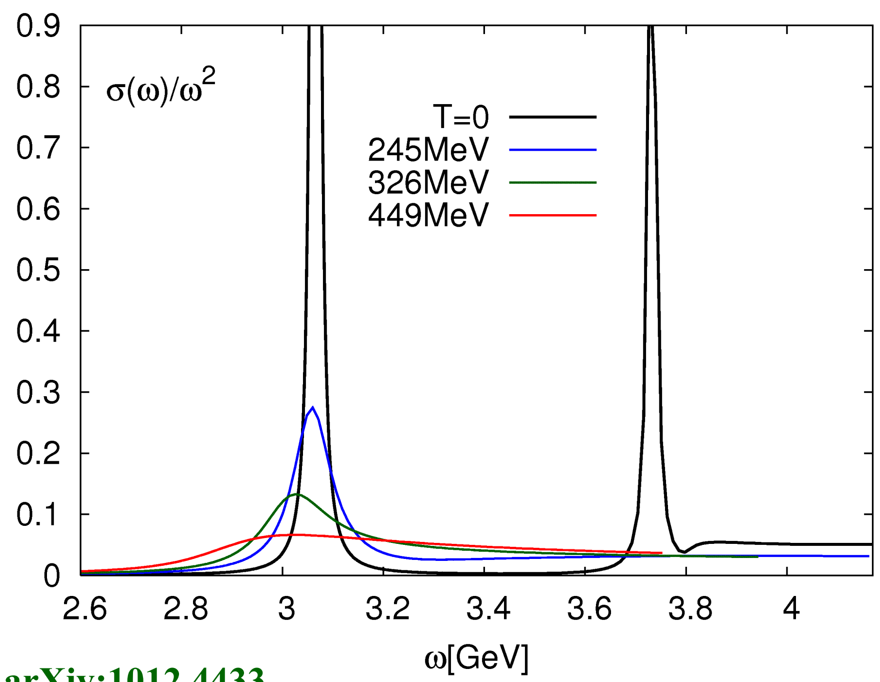
Mócsy, P.P., PRL 99 (07) 211602,

Take the perturbative imaginary part
Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

Im $V_s(r) = 0$:
1S state survives for $T = 330$ MeV

imaginary part of $V_s(r)$ is included :
all states dissolves for $T > 250$ MeV





Miao, Mocsy, P.P., arXiv:1012.4433

no charmonium state could survive for $T > 250$ MeV

this is consistent with our earlier analysis of **Mócsy, P.P., PRL 99 (07) 211602** ($T_{dec} \sim 204 MeV$)

as well as with **Riek and Rapp, arXiv:1012.0019 [nucl-th]**

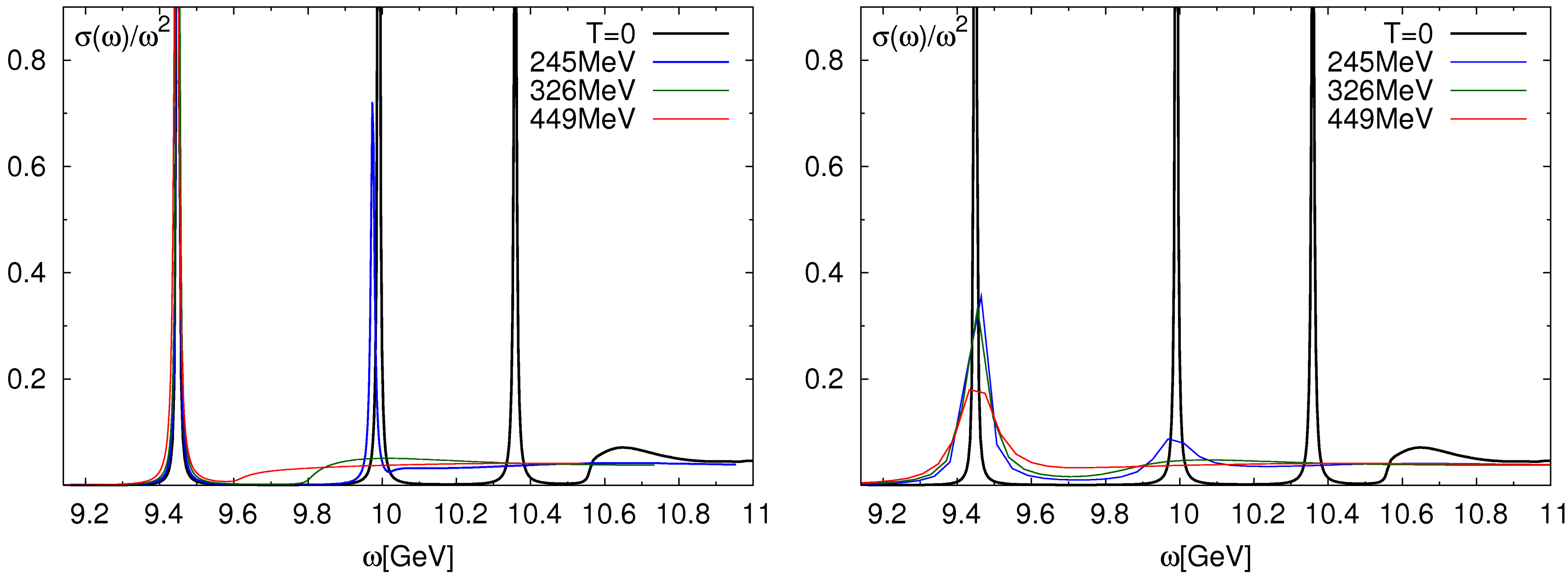
The role of the imaginary part for bottomonium

Take the upper limit for the real part of the potential allowed by lattice calculations
Mócsy, P.P., PRL 99 (07) 211602,

Take the perturbative imaginary part
Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

Im $V_s(r)$ =0:
2S state survives for $T > 250$ MeV
1S state could survive for $T>450$ MeV

with imaginary part:
2S state dissolves for $T>250$ MeV
1S states dissolves for $T>450$ MeV



Miao, Mocsy, P.P., arXiv:1012.4433

Excited bottomonium states melt for $T \approx 250$ MeV ; 1S state melts for $T \approx 450$ MeV
this is consistent with our earlier analysis of **Mócsy, P.P., PRL 99 (07) 211602** *($T_{dec} \sim 204 MeV$)*
as well as with **Riek and Rapp, arXiv:1012.0019 [nucl-th]**

Spatial charmonium correlators

Spatial correlation functions can be calculated for arbitrarily large separations $z \to \infty$

$$G(z,T) = \int_0^{1/T} d\tau \int dxdy \langle J(\mathbf{x}, -i\tau), J(\mathbf{x}, 0)\rangle_T, \quad G(z \to \infty, T) \simeq Ae^{-m_{scr}(T)z}$$

but related to the same spectral functions $G(z,T) = \int_{-\infty}^{\infty} e^{ipz} \int_0^{\infty} d\omega \frac{\sigma(\omega, p, T)}{\omega}$

Low T limit :

$$\sigma(\omega, p, T) \simeq A_{mes}\delta(\omega^2 - p^2 - M_{mes}^2)$$

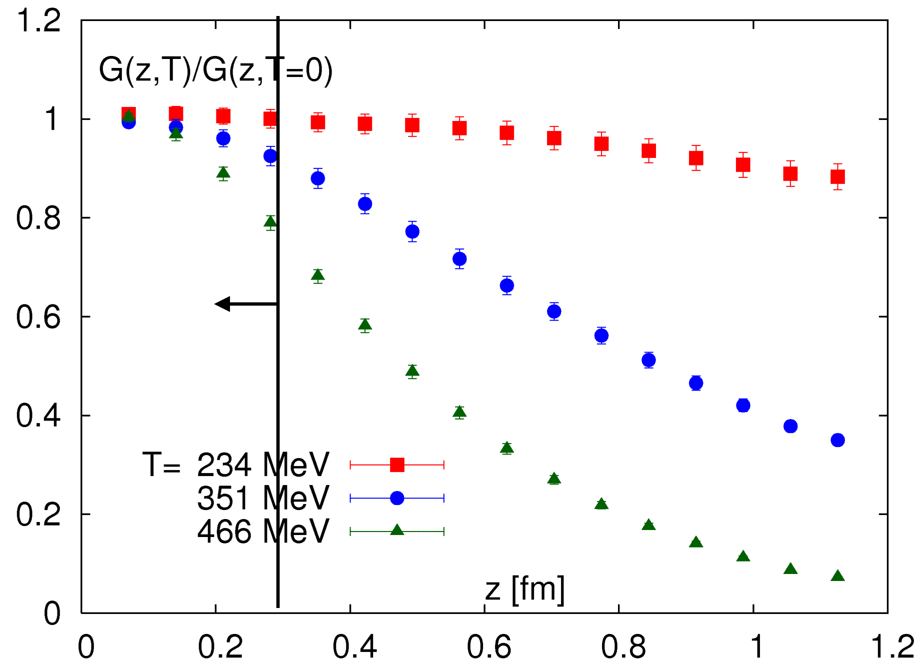
$$A_{mes} \sim |\psi(0)|^2 \to m_{scr}(T) = M_{mes}$$

$$G(z,T) \simeq |\psi(0)|^2 e^{-M_{mes}(T)z}$$

High T limit :

$$m_{scr}(T) \simeq 2\sqrt{m_c^2 + (\pi T)^2}$$

p4 action, dynamical *(2+1)-f* 32^3x*8* and 32^3x*12* lattices

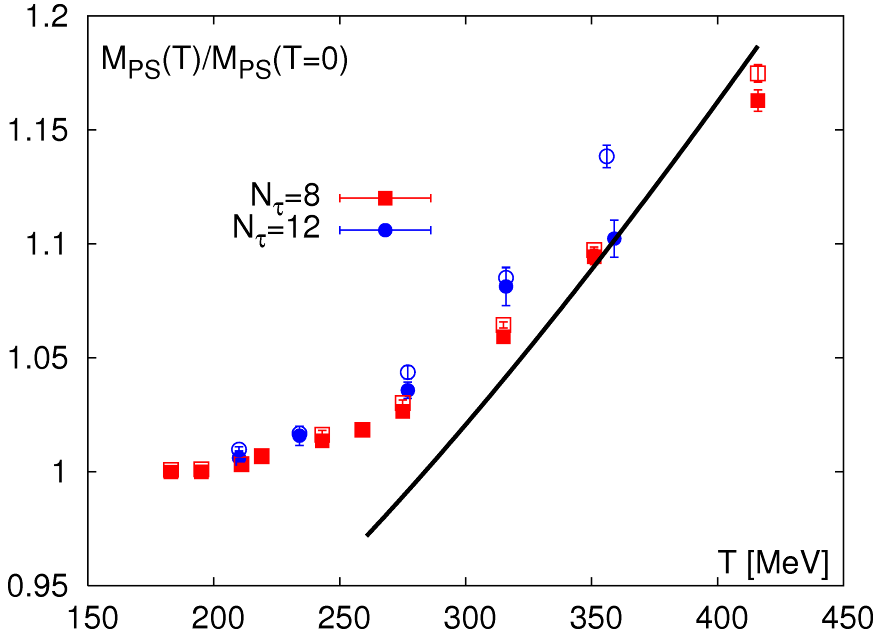


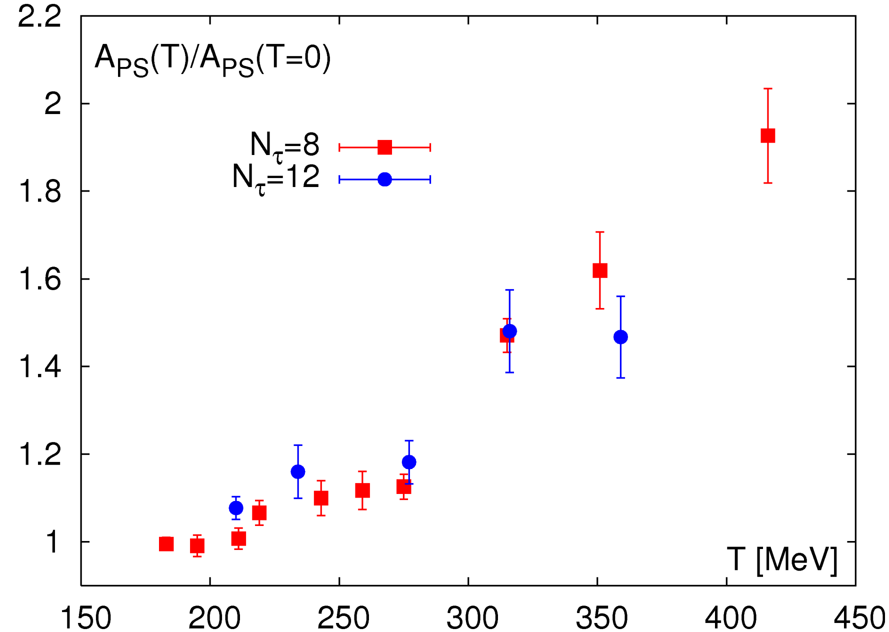
Significant temperature dependence already for *T=234* MeV, large *T*-dependence in the deconfined phase

For small separations *(z T<1/2)* significant *T*-dependence is seen

Spatial charmonium correlators

pseudo-scalar channel => 1S state ,   point sources: filled; wall sources: open





• no T-dependence in the screening masses and amplitudes (wave functions) for ***T<200*** **MeV**

• moderate T-dependence for ***200<T<275*** **MeV** => medium modification of the ground state

• Strong T-dependence of the screening masses and amplitudes for ***T>300*** **MeV**, compatible with free quark behavior assuming $\boldsymbol{m_c=1.2}$ **GeV** => dissolution of 1S charmonium !

Heavy quark diffusion, linear response and Euclidean correlatots

Linear response :

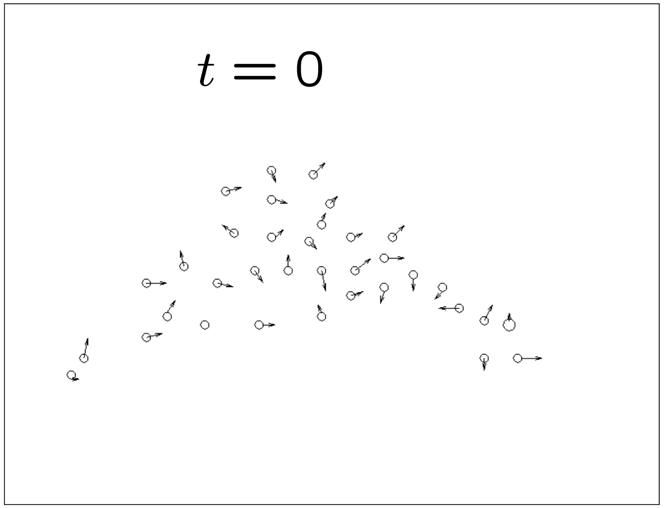
$$H = H_0 - \int d^3x \mu(x,t) N(x,t), \;\; N(x,t) = \bar{q}(x,t)\gamma_0 q(x,t), \; \mu(x,t) = e^{\epsilon t}\theta(-t)\mu(x)$$

$$\langle \delta N(x,t) \rangle = \int_{-\infty}^{\infty} dt' D^R_{NN}(x,t'-t)\mu(x,t')$$

$$\sigma_{NN}(k,\omega) = \frac{1}{\pi} \mathrm{Im} D^R_{NN}(k,\omega)$$

$$D^{Rij}_{JJ}(k,\omega) = (\delta_{ij} - \frac{k_i k_j}{k^2}) D^R_{JJ,T}(k,\omega) + \frac{k_i k_j}{k^2} D^R_{JJ,L}(k,\omega)$$

$$\frac{\omega^2}{k^2} D^R_{NN}(k,\omega) = D^R_{JJ,L}(k,\omega)$$



P.P., Teaney, PRD 73 (06) 014508

Euclidean correlators:

$$G^{00}(k,\tau) = \int d^3x e^{i\mathbf{kx}} \langle J^0_E(x,\tau) J^0_E(0,0) \rangle = -D_{NN}(k,-i\tau) = -\int_0^{\infty} d\omega \sigma_{NN}(k,\omega) K(\tau,\omega,T)$$

$$G^{ij}(k,\tau) = \int d^3x e^{i\mathbf{kx}} \langle J^i_E(x,\tau) J^j_E(0,0) \rangle = D^{ij}_{JJ}(k,-i\tau) = \int_0^{\infty} d\omega \sigma^{ij}_{JJ}(k,\omega) K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

Correlators and diffusion

$$t_{tran} \sim M/T^2 \gg 1/T$$

Moore, Teaney, PRC 71 (05) 064904

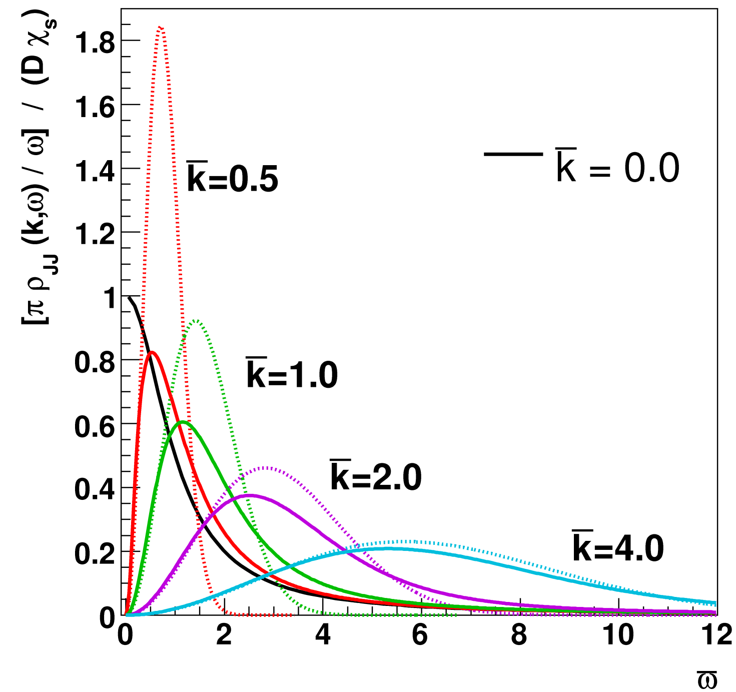
$$\frac{dx^i}{dt} = \frac{p^i}{M}, \frac{dp^i}{dt} = \xi^i(t) - \eta p^i,$$

$$\langle \xi^i(t)\xi^j(t')\rangle = \kappa\delta^{ij}\delta(t-t')$$

$$\eta = \frac{\kappa}{2MT}, \;\; D = \frac{T}{M\eta}$$

$$t \gg 1/\eta : \partial_t N(x,t) + D\nabla^2 N(x,t) = 0$$

$\bar{k} = kD\sqrt{M/T}, \;\; \bar{\omega} = \omega D(M/T)$



$k \ll \eta\sqrt{M/T}$:

$$D^R_{NN}(k,\omega) = \frac{\chi D k^2}{-i\omega + k^2 D} - \frac{\chi D k^2}{-i\omega + \eta}$$

$$\sigma_{NN}(k=0,\omega) = \chi\omega\delta(\omega)$$

$$\sigma_{JJ}(k=0,\omega) = \chi\omega\frac{1}{\pi}\frac{T}{M}\frac{\eta}{\omega^2+\eta^2}$$

P.P., Teaney, PRD 73 (06) 014508

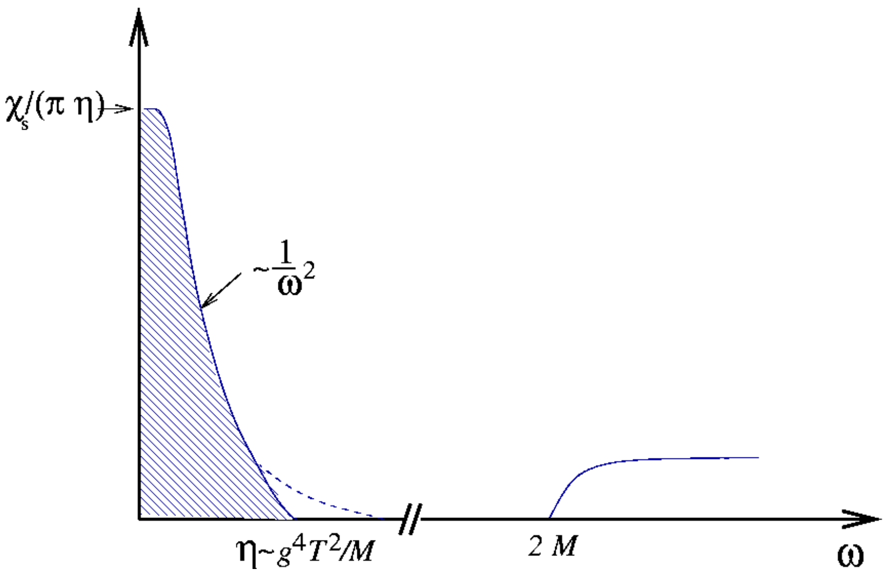
Transport contribution to the Euclidean correlators

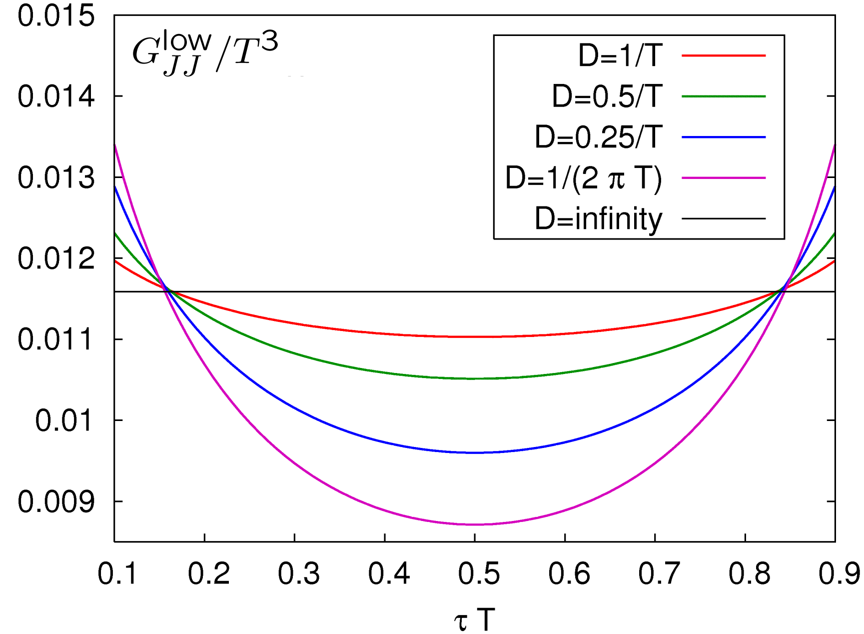
Interactions smear out the $\chi\omega\delta(\omega)$ term
$width \sim \eta$

valid only for $\omega < \eta \ll T$ but
$K(\omega,\tau,T) - \frac{2T}{\omega}$ has support for $\omega \sim T$

$G^{\rm low}_{JJ}(\tau) \neq const$ but has a small curvature around $\tau = 1/(2T)$

$$\sigma^{\rm low}_{JJ}(\omega) = \chi\omega \frac{1}{\pi}\frac{T}{M}\frac{\eta}{\omega^2+\eta^2}$$





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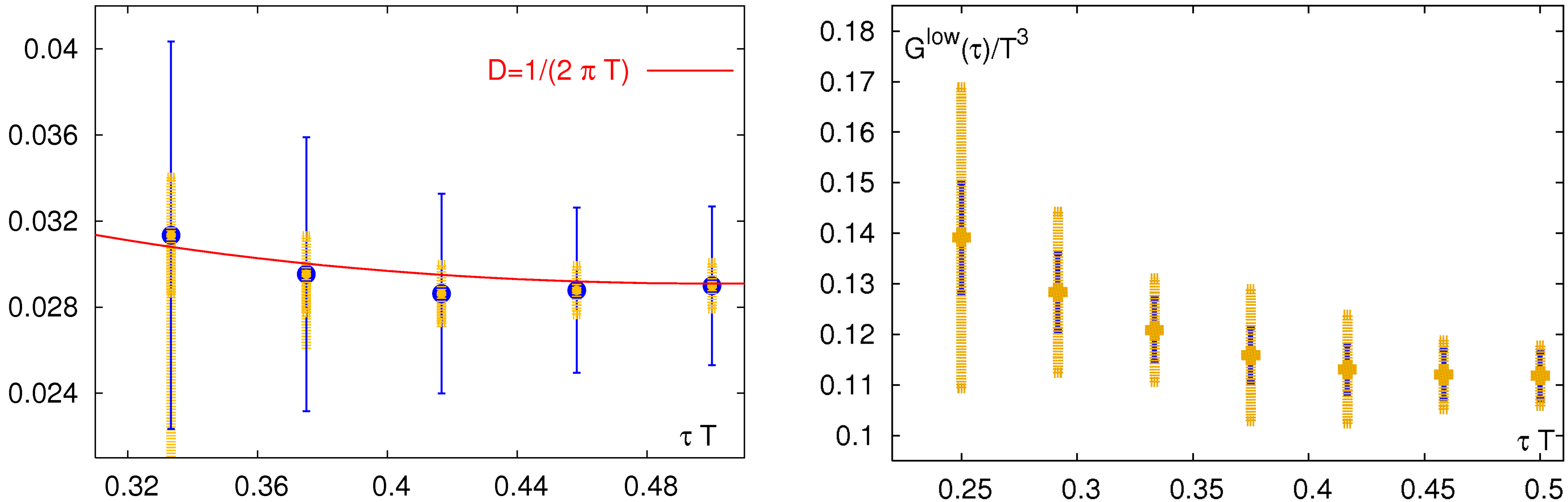
Estimating the zero mode contribution

$$G^{\mathrm{low}}(\tau,T)=G(\tau,T)-G_{rec}(\tau,T)$$

vector $1.5T_c$ axial-vector



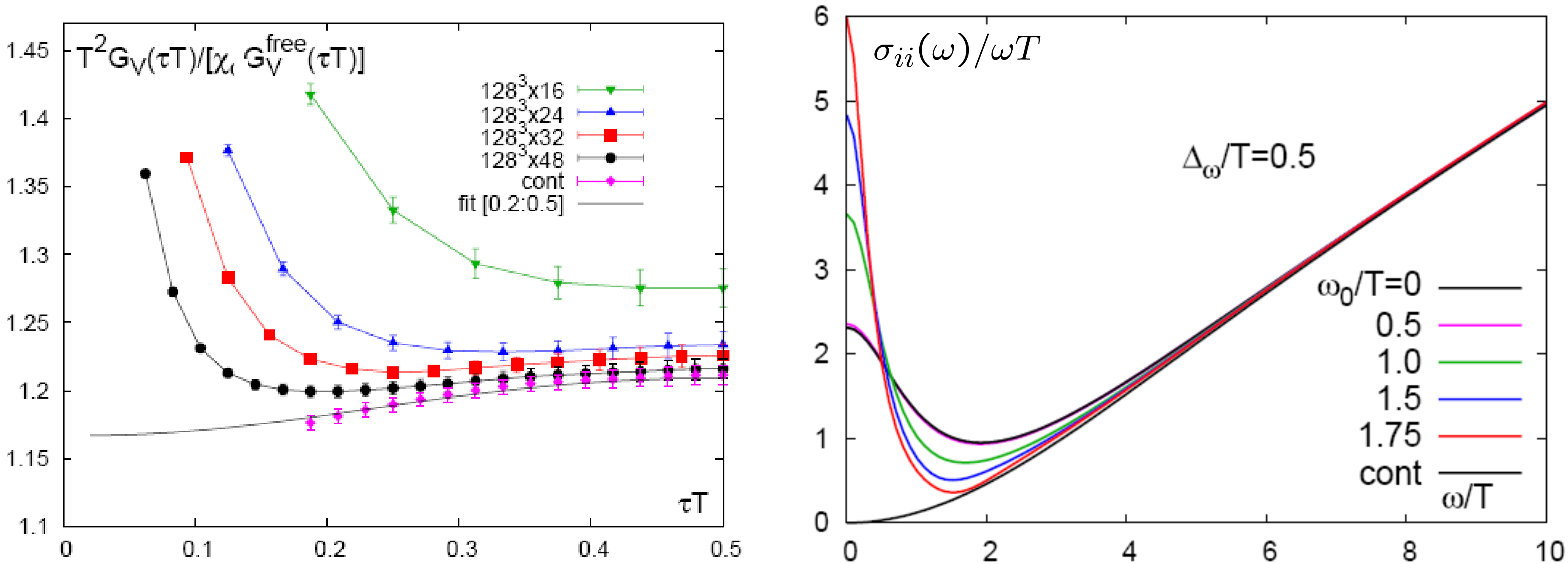
The curvature of $G_i^{\mathrm{low}}(\tau,T)$ is governed by heavy quark diffusion
No diffusion ($D=\infty\leftrightarrow\eta=0$): $G_i^{\mathrm{low}}=const=T\chi_i(T)$

$G_i^{\mathrm{low}}(\tau,T)$ is τ-idependent within errors and
$\chi_i(T)\simeq G_i^{\mathrm{low}}(\tau=1/(2T),T)$

Lattice calculations of the vector spectral functions

Ding et al, PRD 83 (11) 034504

Isotropic Wilson gauge action, quenched non-perturbatively improved clover fermion action on $128^3 \times N_\tau$ lattices, $T = 1.45T_c$, $m_q^{\overline{MS}}(2\text{GeV}) = 0.1/T$,
$N_\tau = 24,\ 32, 48\ (a^{-1} = 9.4 - 18.8\text{GeV})$



$$\sigma_{ii}(\omega) = \chi c_{BW} \frac{1}{\pi} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{4\pi^2}(1+k)\omega^2 \tanh(\omega/4T)\Theta(\omega_0, \Delta_\omega),$$

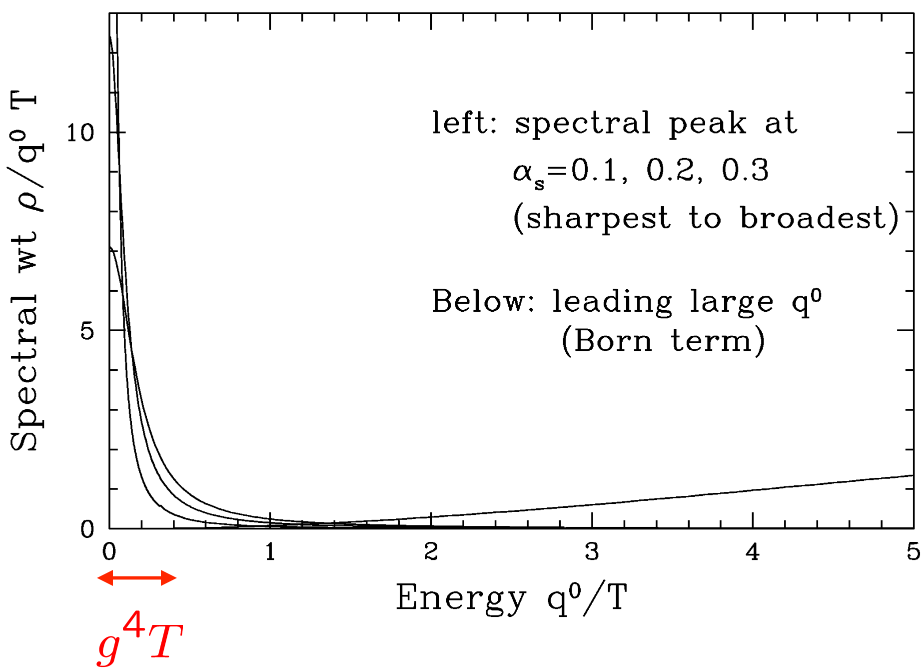
$$\Theta(\omega_0, \Delta_\omega) = (1 + e^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega})^{-1}$$

Fit parameters : $c_{BW},\ \Gamma,\ k$ Different choices of : ω_0, Δ_ω

Strongly coupled or weakly coupled QGP ?

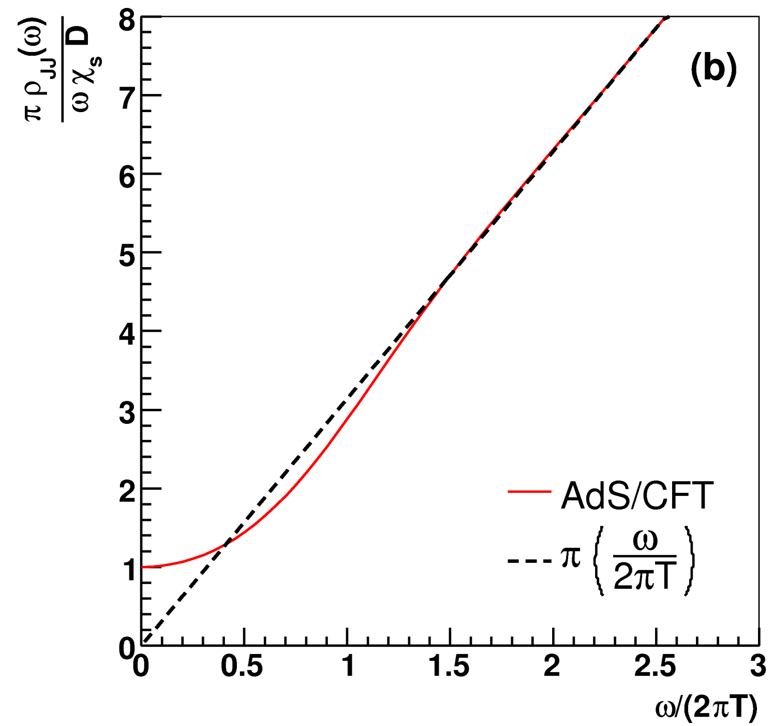
Weak coupling caculation of the vector current spectral function in QCD

Moore, Robert, hep-ph/0607172



vector current correlator in N=4 SUSY at strong coupling

Teaney, PRD74 (06) 045025



lattice results are closer to the weakly coupled QGP

Lattice calculations of the vector spectral functions

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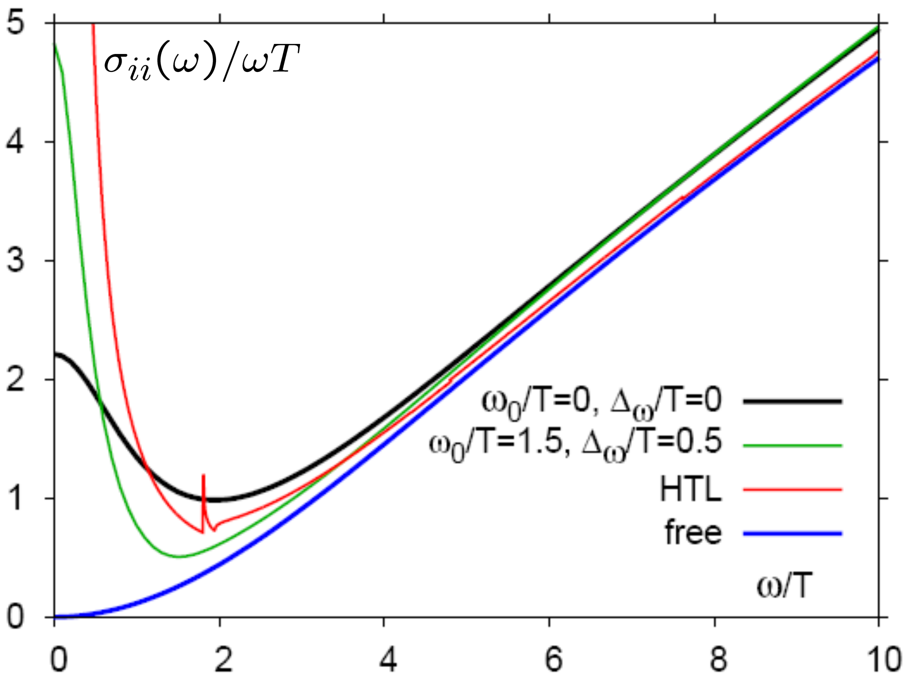

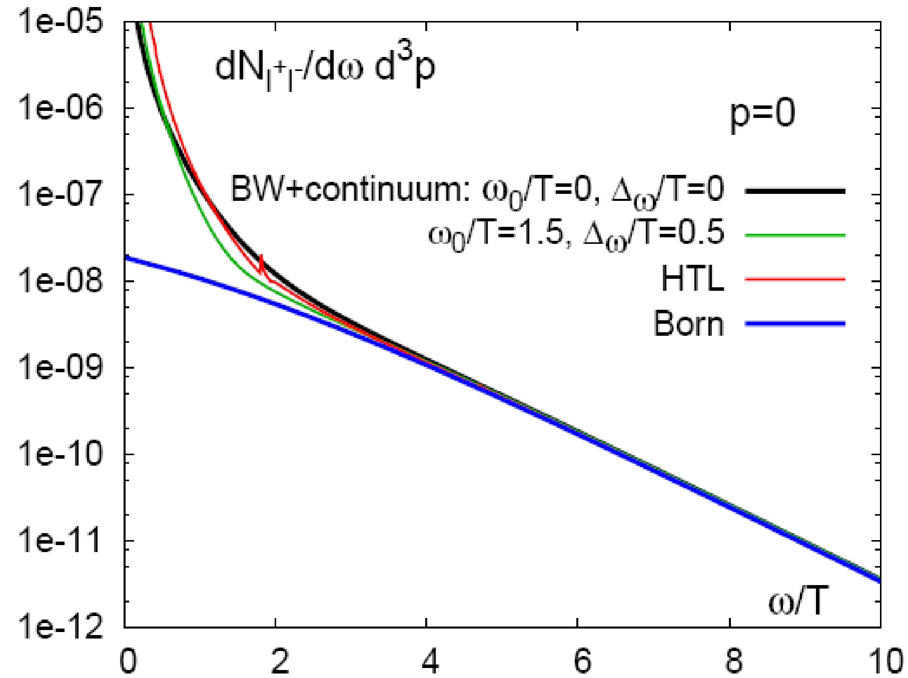

- The HTL resummed perturbative result diverges for *ω→0* limit
- The lattice results show significant enhancement over the LO (Born) result for small *ω*
- The lattice result is HTL result for *2<ω/T<4* but is much smaller for *ω/T<2*