Lecture 2:
Recent results from LQCD: strtonly vs. weakly coupled QGP

Peter Petreczky

- Chiral transition in QCD
- QCD Equation of State
- Taylor expansion of the pressure, fluctuations of conserved charges, deconfinement
- Color screening, deconfinement and sQGP

QGP: state of strongly interacting matter for weakly interacting gas of quark and gluons ? $T \gg \Lambda_{QCD}, g \ll 1$

$$2\pi T \gg m_D \sim gT \gg g^2 T$$

EFT approach: EQCD

Magnetic screening scale: non-perturnative

Perturbative series is an expansion is in g and not α_s
Loop expansion breaks down at some order

Problem : $g(\mu = 10^2\text{GeV}) = \sqrt{4\pi\alpha_s(\mu = 10^2\text{GeV})} \simeq 1$ $\Rightarrow$ Lattice QCD
$g(\mu = 10^{16}\text{GeV}) \simeq 1/2$

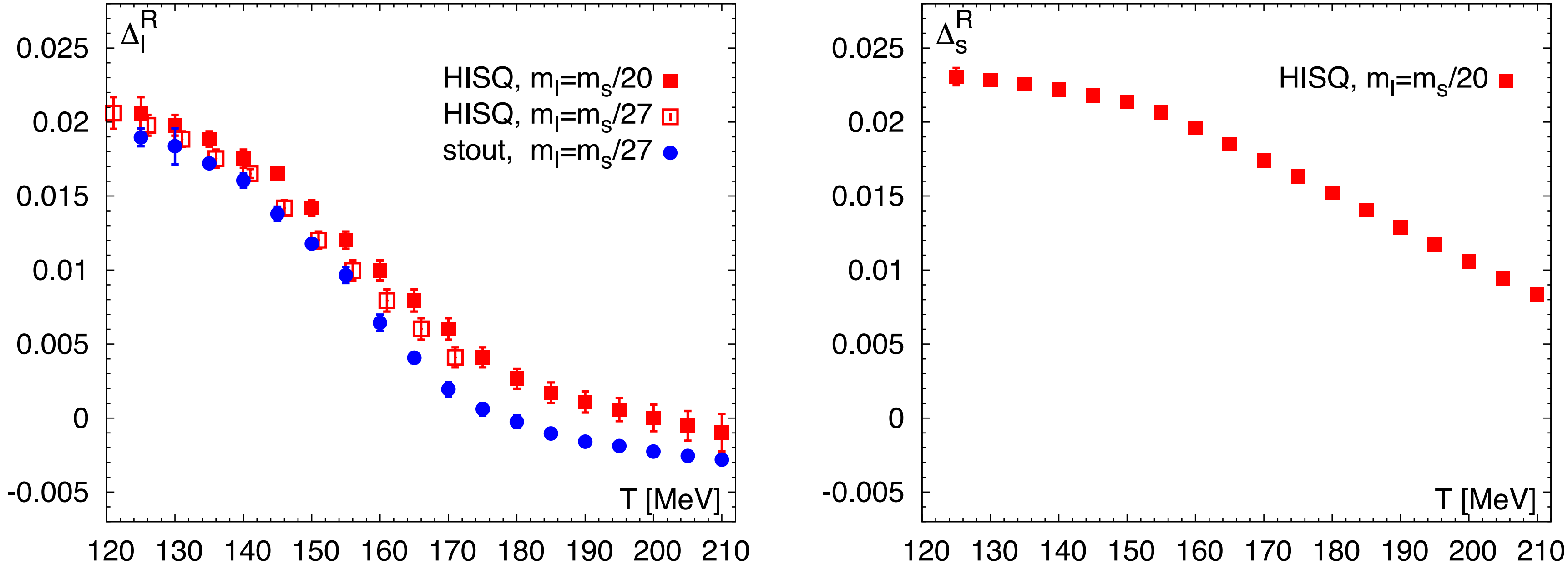
The temperature dependence of chiral condensate

Renormalized chiral condensate introduced by Budapest-Wuppertal collaboration

$$\langle\bar\psi\psi\rangle_q \Rightarrow \Delta_q^R(T) = m_s r_1^4\left(\langle\bar\psi\psi\rangle_{q,T} - \langle\bar\psi\psi\rangle_{q,T=0}\right) + d, \quad q = l, s$$

with our choice : $d = \langle\bar\psi\psi\rangle_{m_q=0}^{T=0}$

HotQCD : Phys. Rev. D85 (2012) 054503;
Bazavov, PP, RRD 87(2013) 094505

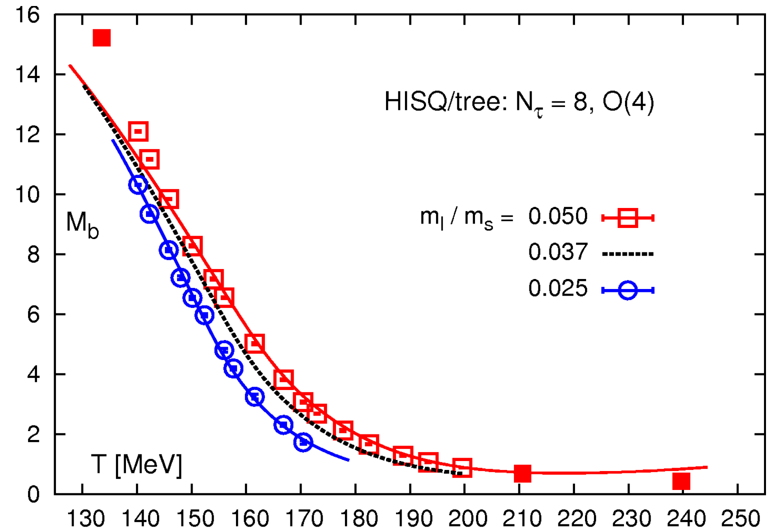


• after extrapolation to the continuum limit and physical quark mass HISQ/tree calculation agree with stout results

• strange quark condensate does not show a rapid change at the chiral crossover => strangequark do not play a role in the chiral transition

O(N) scaling and the transition temperature

The notion of the transition temperature is only useful if it can be related to the critical temperature in the chiral limit : fit the lattice data on the chiral condensate with scaling form + simple Ansatz for the regular part



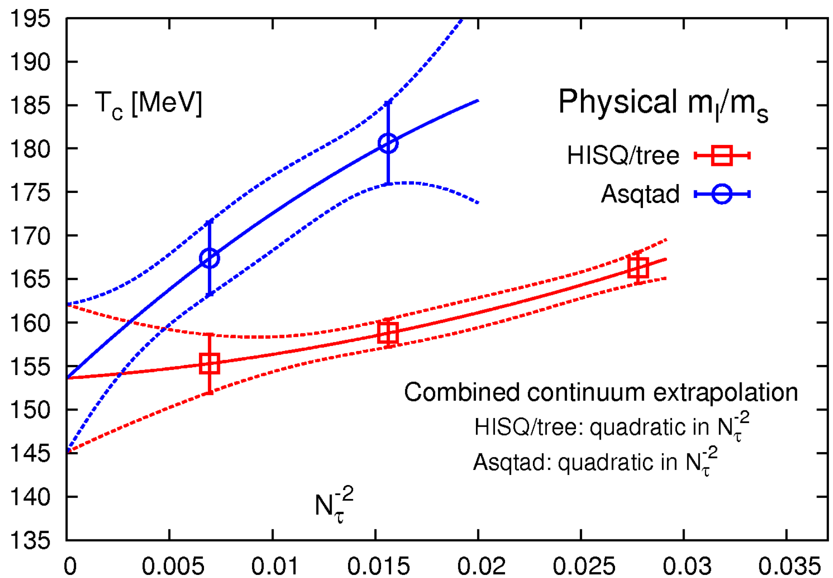
$$M_b = \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + f_{M,reg}(T,H)$$

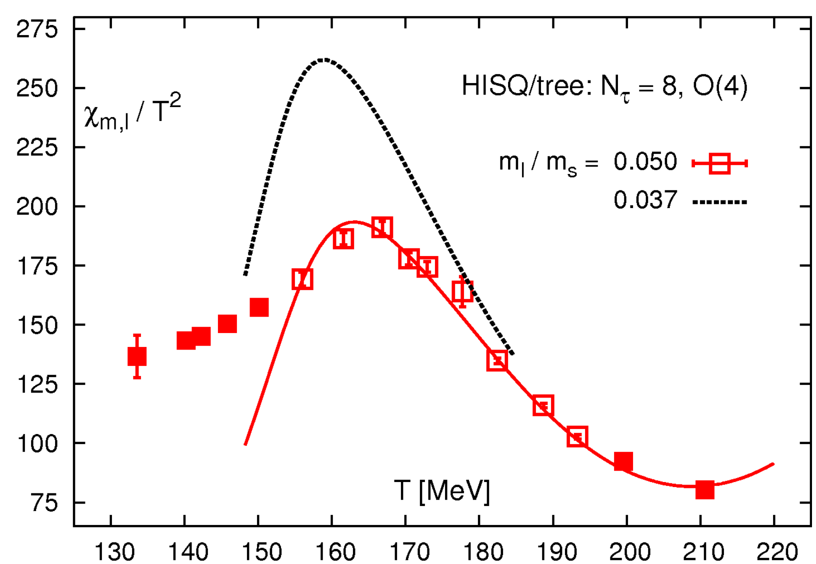
$$f_{reg}(T,H) = (a_1(T-T_c^0) + a_2(T-T_c^0)^2 + b_1)H$$

$$t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0} + \kappa \frac{\mu_q^2}{T^2}\right), \; H = \frac{m_l}{m_s}, h = \frac{H}{h_0}$$

6 parameter fit : $T_c^{\,0}$, t_0, h_0, a_1, a_2, b_1

$$T_c = (154 \pm 8 \pm 1(scale))\mathrm{MeV}$$

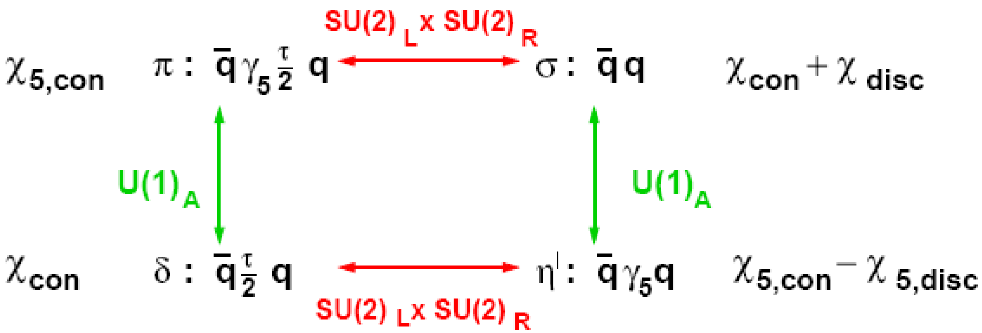




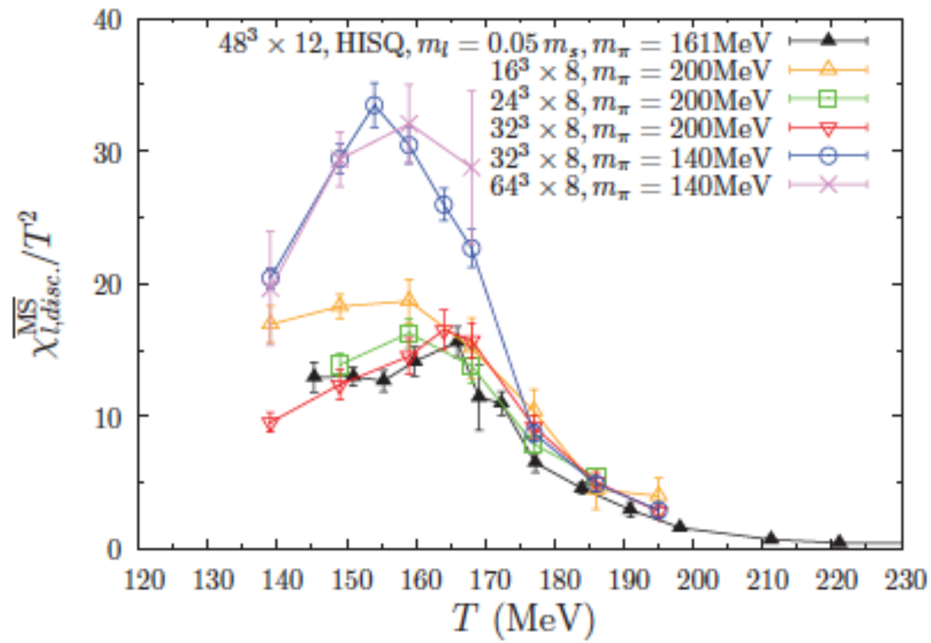
Chiral Fermions and chiral and $U_A(1)$ symmetry restoration

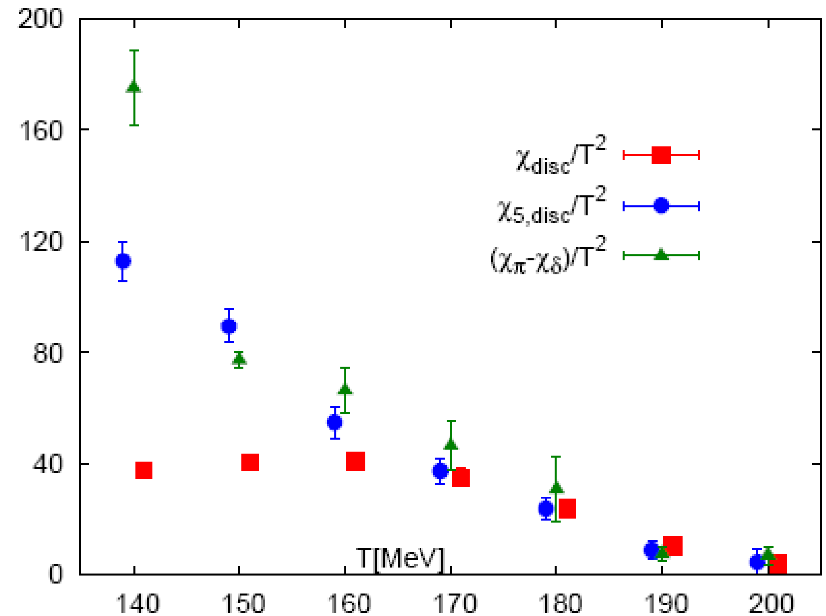
Domain Wall Fermions, Bazavov et al (HotQCD), PRD86 (2012) 094503

$$\chi_i = \int d^4x G_i(x)$$



chiral:	axial:
$\chi_\pi = \chi_\delta + \chi_{disc}$	$\chi_\pi = \chi_\delta$
$\chi_\delta = \chi_\pi - \chi_{5,disc}$	$\chi_\delta + \chi_{disc} = \chi_\pi - \chi_{5,disc}$
$\chi_{disc} = \chi_{5,disc}$	$\chi_{disc} = -\chi_{5,disc}$



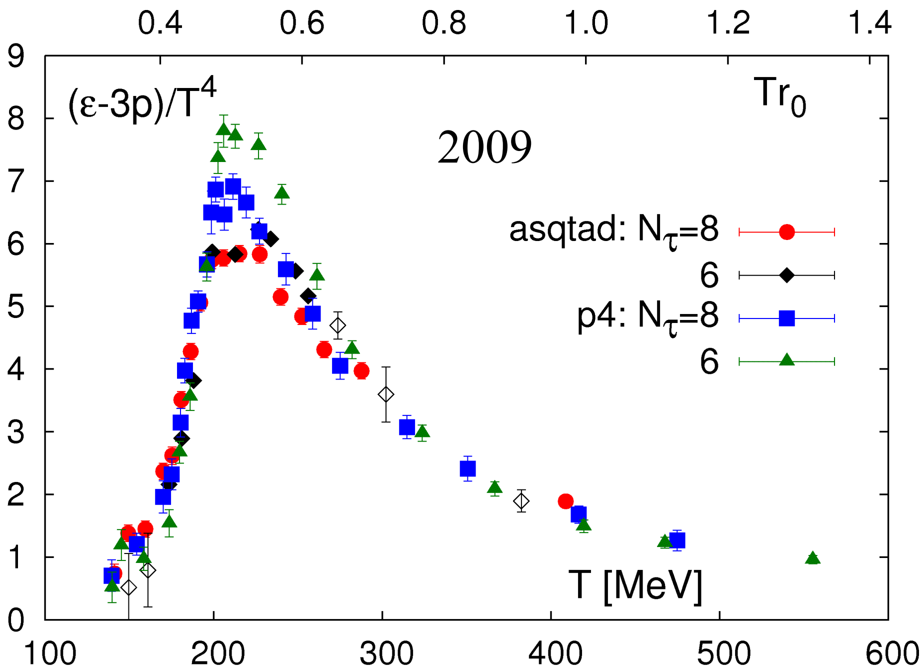


Peak position roughly agrees with previous staggered results

axial symmetry is effectively restored *T*>*200* MeV !

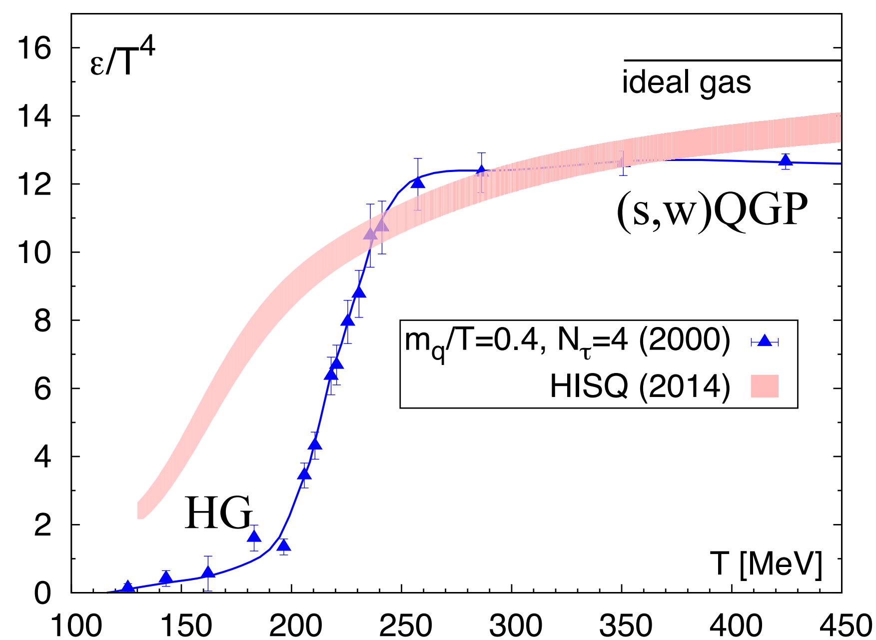
Lattice QCD EoS: now and then

Goal: calculate the equation of state (EoS) in the continuum limit for physical quark masses



Large cutoff effects in the trace anomaly: strong N_τ –dependence and strong Dependence on the discretization scheme (p4 vs. asqtad)

Here: calculations by HotQCD Collaboration using Highly improved staggered quark (HISQ) action



Much smoother transition to QGP
The energy density keeps increasing up to 450 MeV instead of flattening

Some lattice details

Temperature is varied by the lattice spacing a, continuum limit corresponds to $N_\tau \to \infty$

$$T = (1/N_\tau a)$$

HISQ/tree action, physical m_s , $m_l = m_s/20$: $m_K = 504$ MeV, $m_\pi = 161$ MeV

Lattice spacing set by the r_1 scale

$$\left(r^2 \frac{dV_{q\bar{q}}(r)}{dr} \right)_{r=r_1} = 1.0$$

$r_1 = 0.3106(14)(8)(4)$ fm

$$M_\pi^{RMS} = \sqrt{\frac{1}{16}\left(M_{\gamma_5}^2 + M_{\gamma_0\gamma_5}^2 + 3M_{\gamma_i\gamma_5}^2 + 3M_{\gamma_i\gamma_j}^2 + 3M_{\gamma_i\gamma_0}^2 + 3M_{\gamma_i}^2 + M_{\gamma_0}^2 + M_1^2\right)},$$

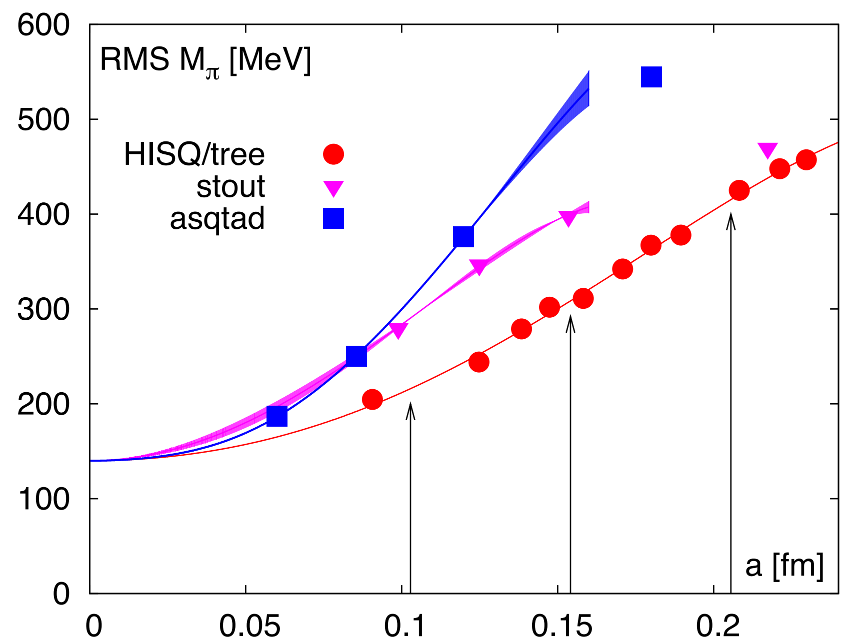
statistics for the $T>0$ runs:

24^3x6: 21-4K TU
32^3x8: 26-270K TU
40^3x10: 103-360K TU
48^3x12: 48-171K TU

statistics for the $T=0$ runs:

24^3x32: 4-8K TU
32^4, 32^3x64: 7-40K TU
48^4: 8-16K TU
48^3x64: 8-9K TU
64^4 : 9K

in molecular dynamic time units (TU)

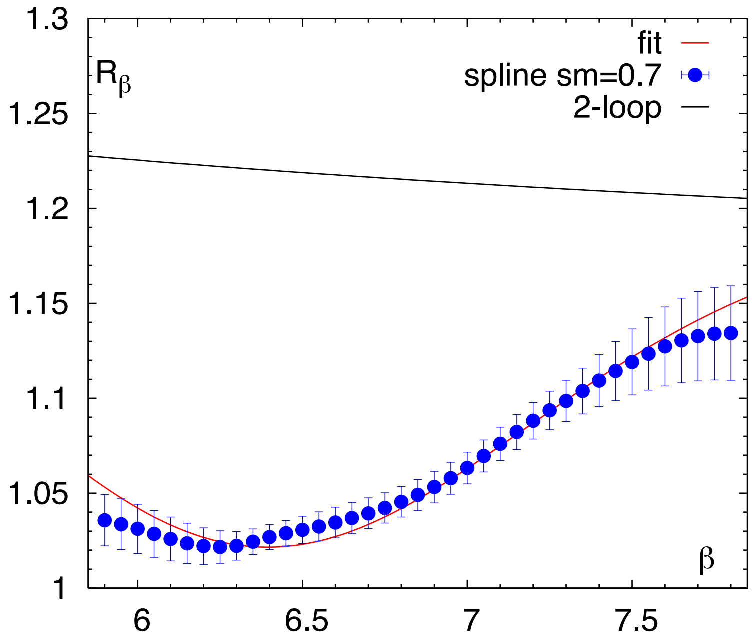


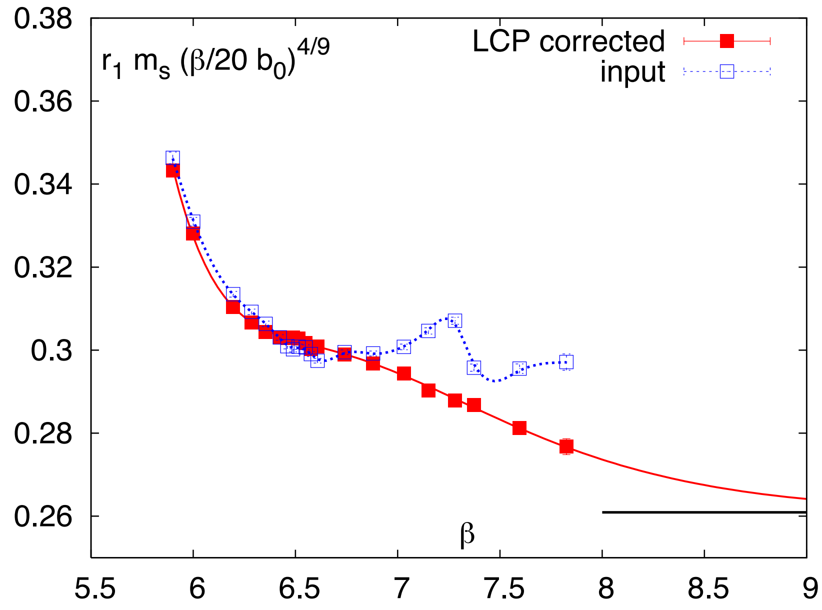
Trace anomaly and the integral method

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\varepsilon - 3p}{T^4} = T\frac{d}{dT}\left(\frac{p}{T^4}\right) \Rightarrow \frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Theta^{\mu\mu}(T')}{T'^5},$$

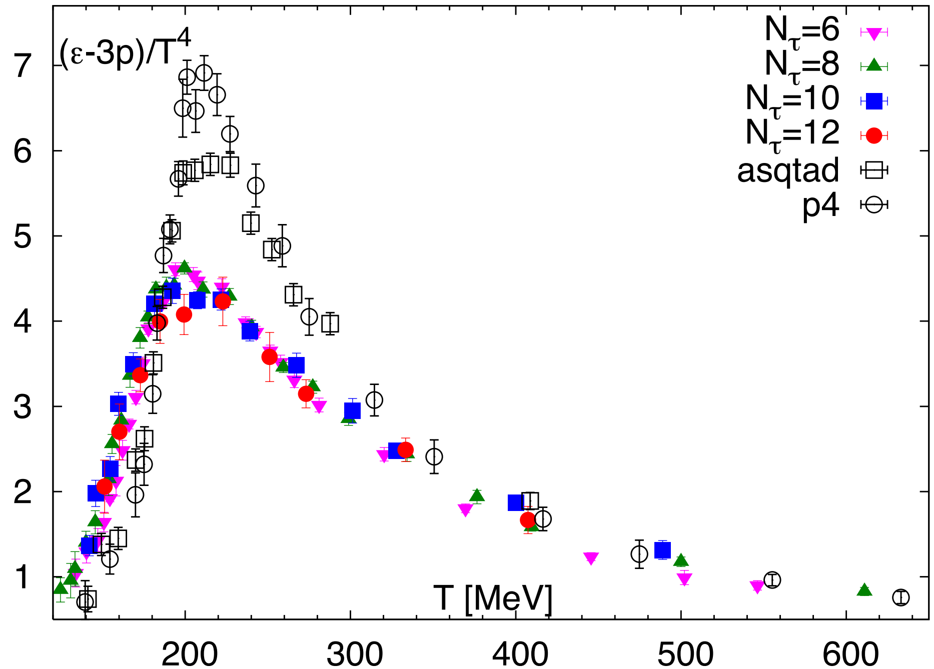
$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = \underbrace{R_\beta \{\langle S_G\rangle_0 - \langle S_G\rangle_T\}}_{\Theta_G^{\mu\mu}} - \underbrace{R_\beta R_m \{2m_l(\langle\bar{q}q\rangle_0 - \langle\bar{q}q\rangle) + m_s(\langle\bar{s}s\rangle_0 - \langle\bar{s}s\rangle_T)\}}_{\Theta_F^{\mu\mu}}$$

$$R_\beta(\beta) = -a\frac{d\beta}{da},\ R_m = \frac{1}{m_q(\beta)}\frac{dm_q(\beta)}{d\beta},\ \beta = 10/g^2$$





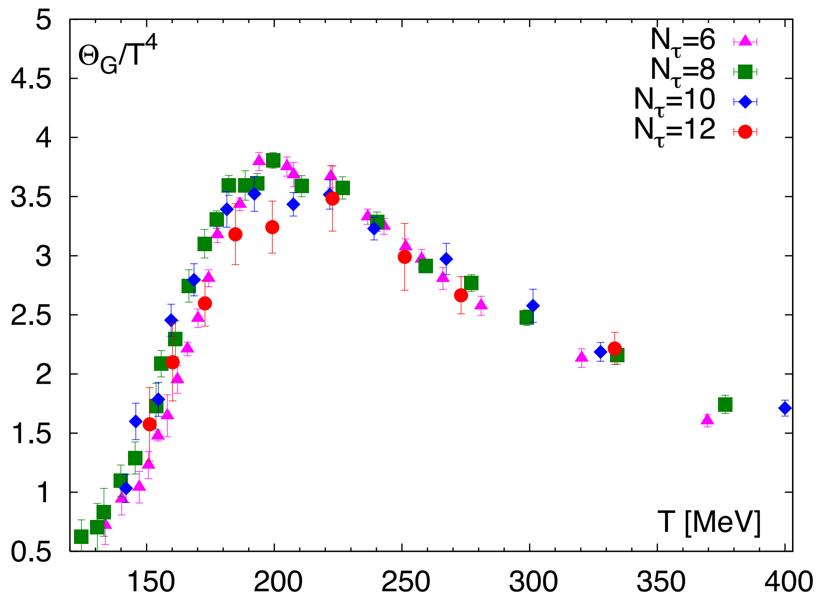
Lattice results on the trace anomaly

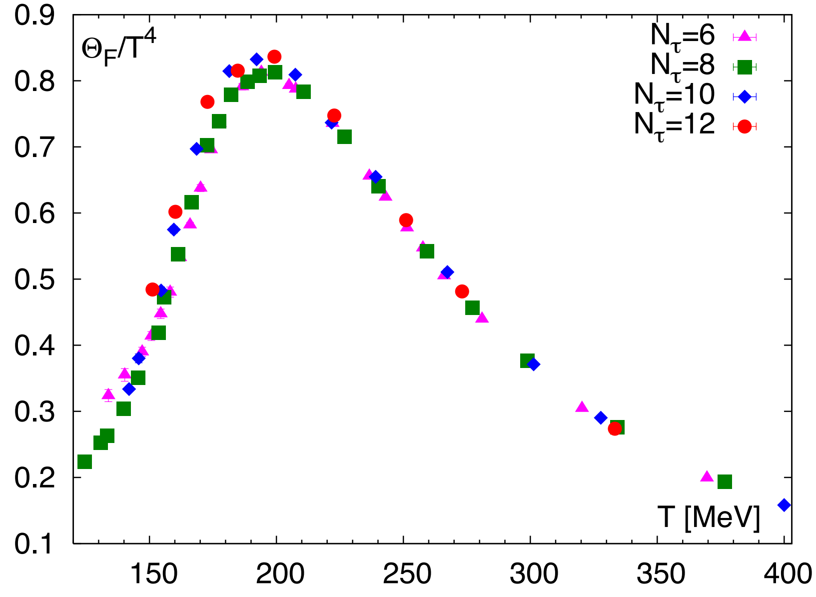


The peak height is much reduced compared to the asqtad and p4 $N_\tau=8$ calculations

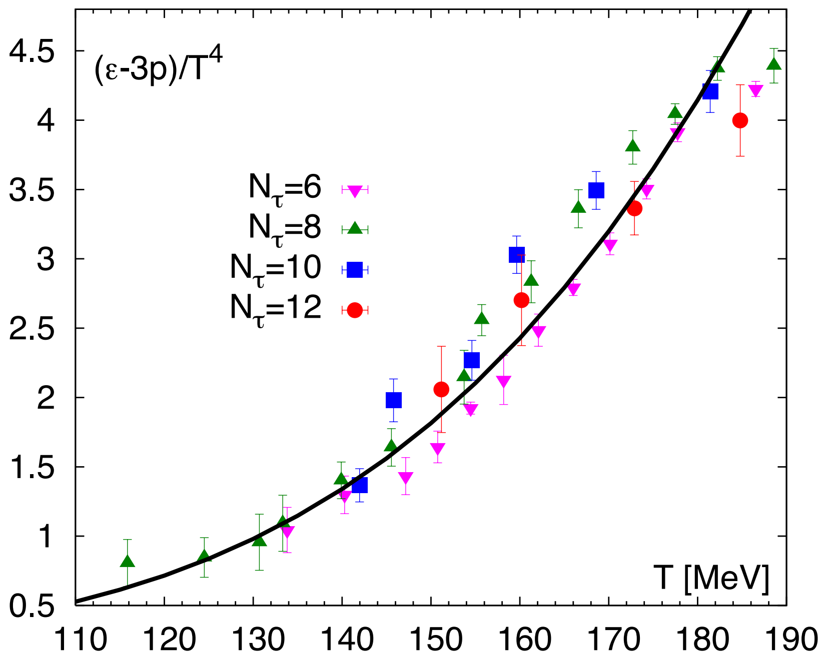
Agreement with p4 and asqtad calculations for $T>350$ MeV

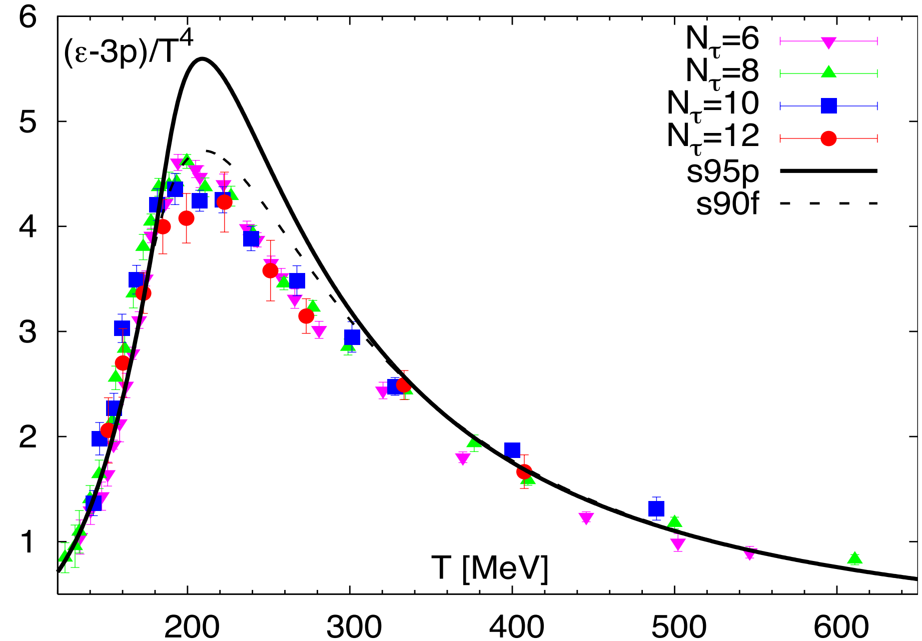
Small cutoff effects for HISQ except for $N_\tau=6$





Lattice results on the trace anomaly (cont'd)





Hadron resonance gas (HRG) model:

Interacting gas of hadrons = non-interacting gas of hadrons and hadronic resonances

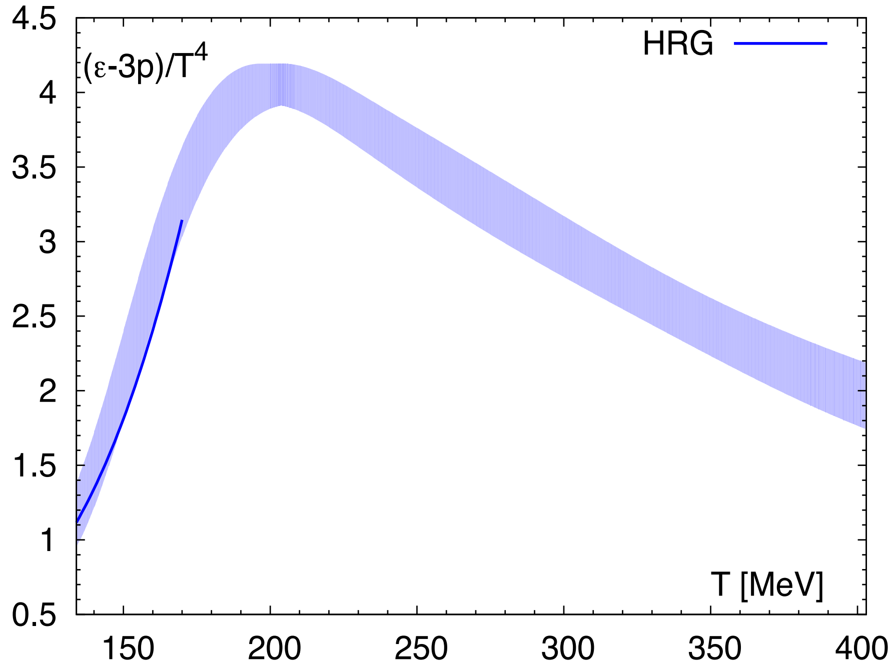
Good agreement with HRG with all PDG states for all N_τ if $T< 145$ MeV

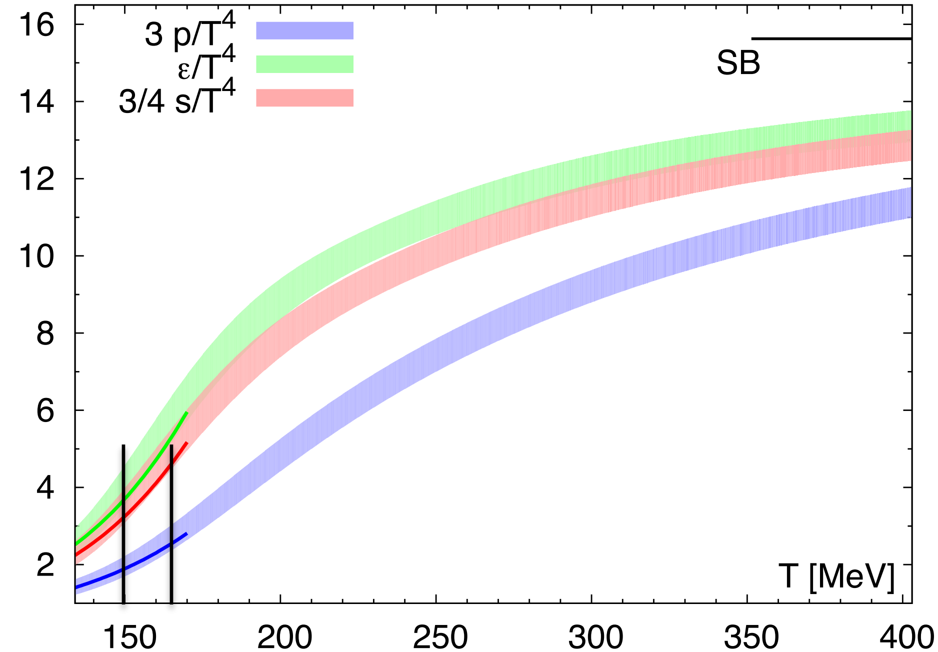
The lattice data are below the EoS parametrizations used in hydro models

Equation of state in the continuum limit

Perform spline interpolation of all the $N_\tau > 6$ data with spline coefficients of the form $a + b/N_\tau^2$, stabilize the spline demanding that $\varepsilon\text{-}3p$ is given by HRG at $T=130$ MeV

Set the lower integration limit to $T_0=130$ MeV and take $p_0=p^{HRG}(T=130\text{ MeV})$ ➡ $p(T)$





$$T_c = (154 \pm 9)\mathrm{MeV}$$

$$\epsilon_c \simeq 300\mathrm{MeV/fm}^3$$

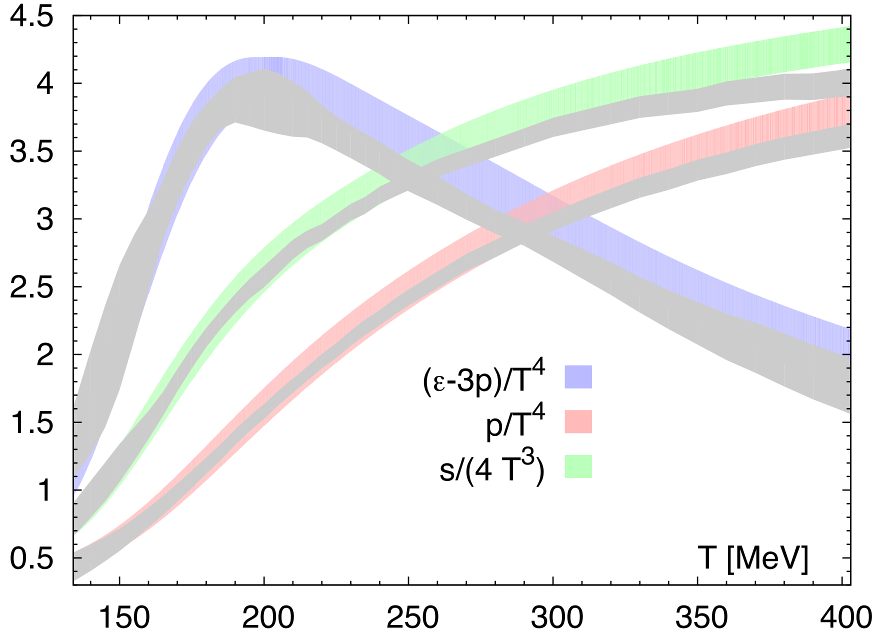
$$\epsilon_{low} \simeq 180\mathrm{MeV/fm}^3$$

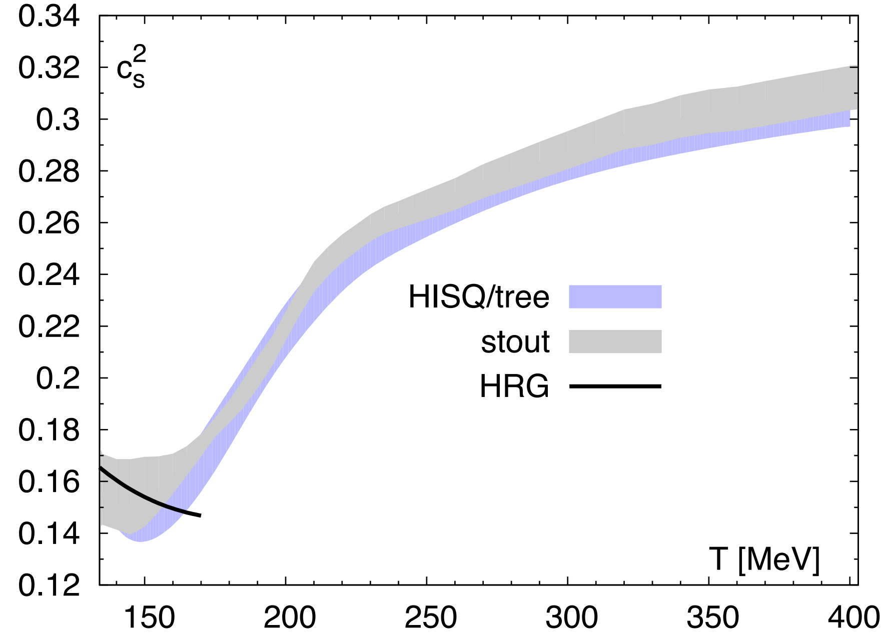
$$\epsilon_{high} \simeq 500\mathrm{MeV/fm}^3$$

$$\epsilon_{nucl} \simeq 150\mathrm{MeV/fm}^3$$

$$\epsilon_{proton} \simeq 450\mathrm{MeV/fm}^3$$

HISQ action vs. stout action

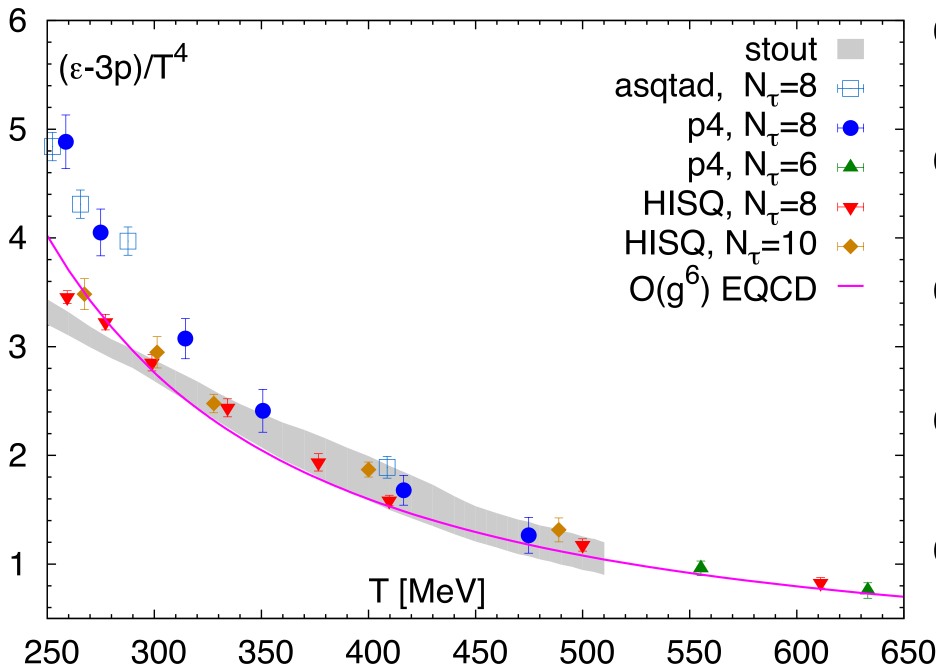


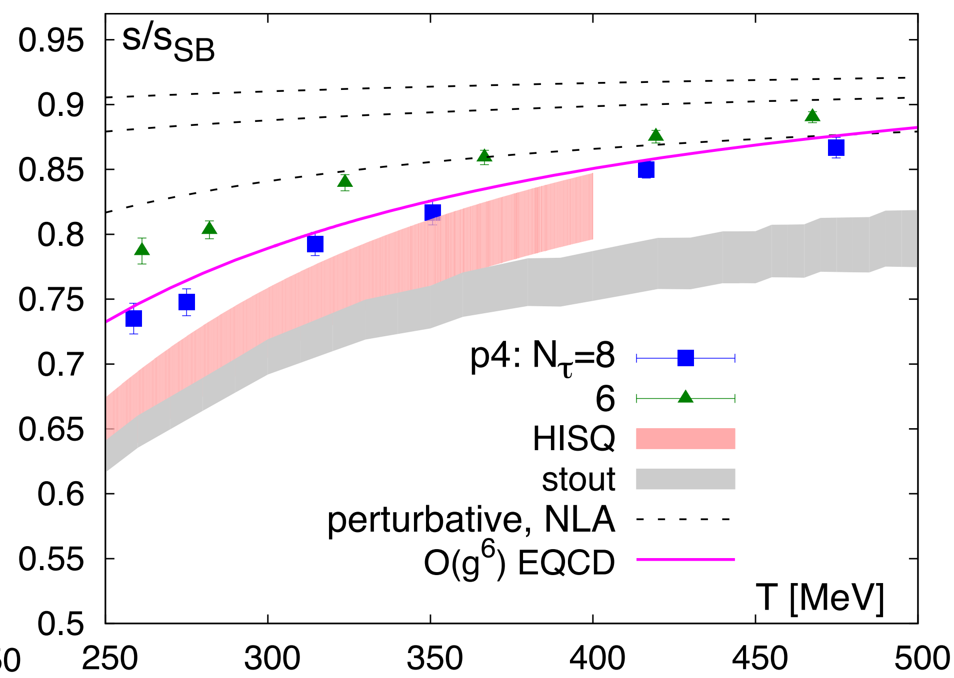


Continuum results obtained with stout and HISQ action agree reasonably well given their errors (some tension for the entropy density)

Even in the transition region the speed of sound is not much smaller than the HRG speed of sound (the EoS is never really soft)

Comparison with weak coupling results





The high temperature behavior of the trace anomaly is not inconsistent with weak coupling calculations (EQCD) for $T>350$ MeV

For the entropy density the continuum lattice results are below the weak coupling calculations For $T< 500$ MeV

At what temperature can one see good agreement between the lattice and the weak coupling results ?

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$ quark

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)\big|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2 \rangle - \langle X \rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$

$\Downarrow$

information about carriers of the conserved charges (hadrons or quarks)

$\Downarrow$

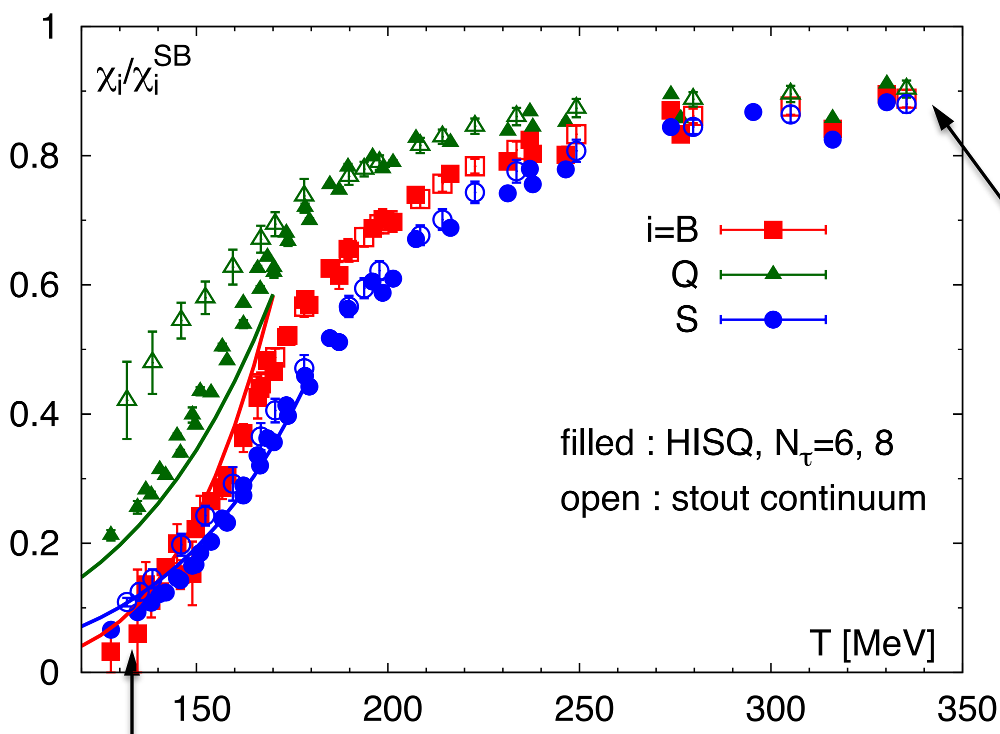
probes of deconfinement

Deconfinement : fluctuations of conserved charges

$$\chi_{\mathrm{B}}^{\mathrm{SB}} = \frac{1}{VT^3}(\langle B^2\rangle - \langle B\rangle^2)$$ baryon number

$$\chi_{\mathrm{Q}}^{\mathrm{SB}} = \frac{1}{VT^3}(\langle Q^2\rangle - \langle Q\rangle^2)$$ electric charge

$$\chi_{\mathrm{S}}^{\mathrm{SB}} = \frac{1}{VT^3}(\langle S^2\rangle - \langle S\rangle^2)$$ strangeness



conserved charges are carried by massive hadrons

Ideal gas of massless quarks :

$$\chi_B^{\mathrm{SB}} = \frac{1}{3} \qquad \chi_Q^{\mathrm{SB}} = \frac{2}{3}$$

$$\chi_S^{\mathrm{SB}} = 1$$

conserved charges carried by light quarks

HotQCD: PRD86 (2012) 034509

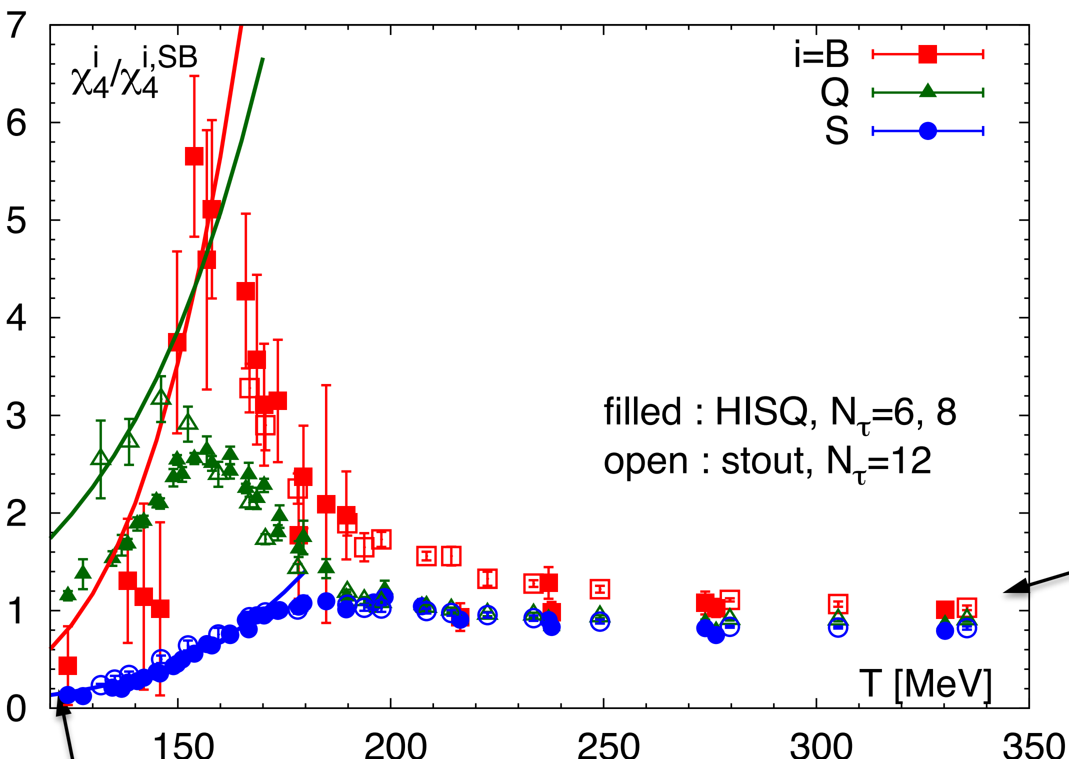
BW: JHEP 1201 (2012) 138,

Deconfinement : fluctuations of conserved charges

$$\chi_4^B = \frac{1}{VT^3}(\langle B^4\rangle - 3\langle B^2\rangle^2)$$ baryon number

$$\chi_4^Q = \frac{1}{VT^3}(\langle Q^4\rangle - 3\langle Q^2\rangle^2)$$ electric charge

$$\chi_4^S = \frac{1}{VT^3}(\langle S^4\rangle - 3\langle S^2\rangle^2)$$ strangeness



Ideal gas of massless quarks :

$$\chi_4^B{}_{\,SB} = \frac{2}{9\pi^2} \qquad \chi_4^Q{}_{\,SB} = \frac{4}{3\pi^2}$$

$$\chi_4^S{}_{\,SB} = \frac{6}{\pi^2}$$

conserved charges carried by light quarks

conserved charges are carried by massive hadrons

BNL-Bielefeld : talk by C. Schmidt
BW: talk by Borsanyi
@ Confinement X conference

Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

$$P_S = \frac{p(T) - p_{S=0}(T)}{T^4} = M(T)\cosh\left(\frac{\mu_S}{T}\right) +$$

$$B_{S=1}(T)\cosh\left(\frac{\mu_B - \mu_S}{T}\right) + B_{S=2}(T)\cosh\left(\frac{\mu_B - 2\mu_S}{T}\right) + B_{S=3}(T)\cosh\left(\frac{\mu_B - 3\mu_S}{T}\right)$$

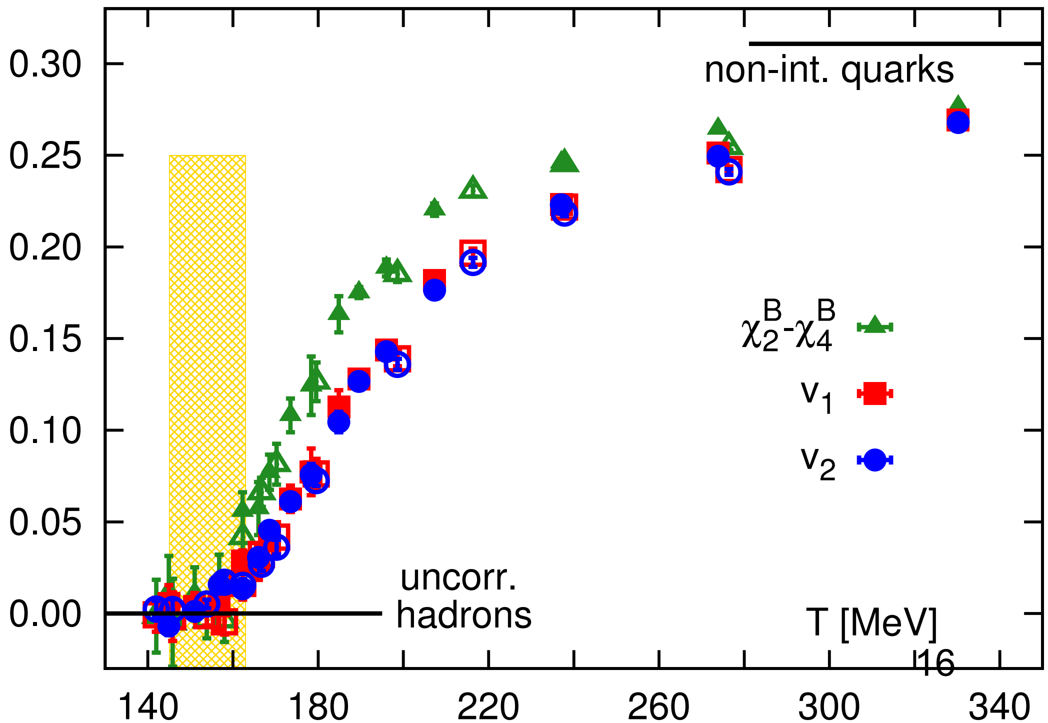
$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

$$v_2 = \frac{1}{3}\left(\chi_4^S - \chi_2^S\right) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

should vanish !

- v_1 and v_2 do vanish within errors at low T
- v_1 and v_2 rapidly increase above the transition region, eventually reaching non-interacting quark gas values

Bazavov et al, PRL 111 (2013) 082301



Deconfinement of strangeness (cont'd)

Using the six Taylor expansion coefficients related to strangeness

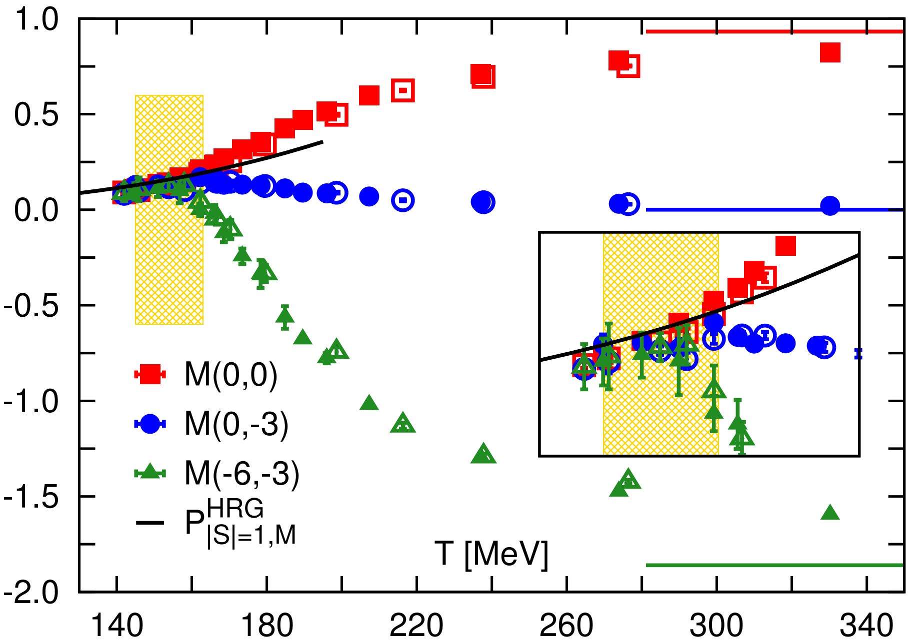
$$\chi_2^S,\ \chi_4^S,\ \chi_{13}^{BS},\ \chi_{22}^{BS},\ \chi_{31}^{BS}$$

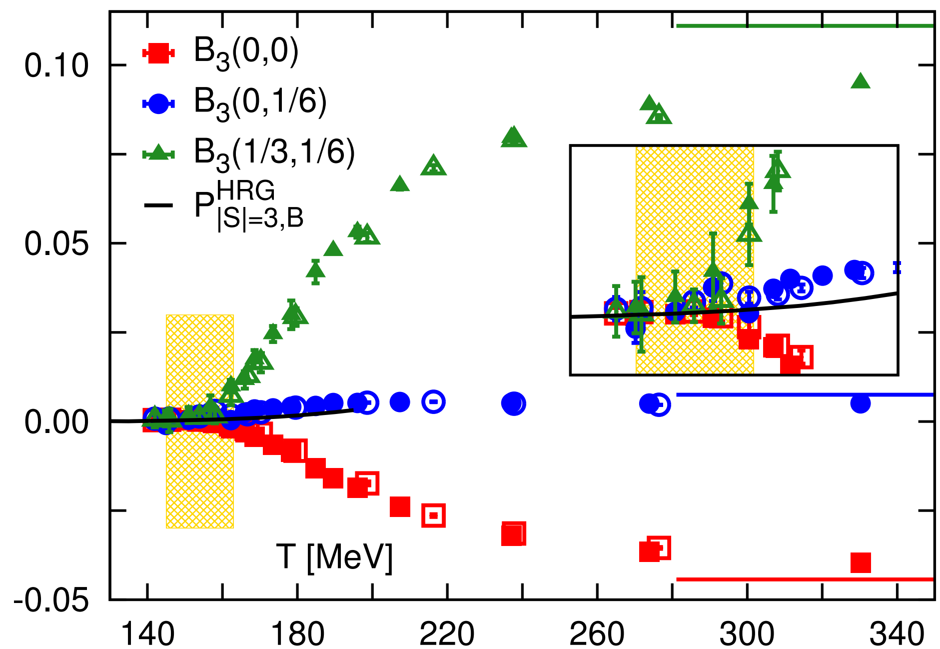
it is possible to construct combinations that give

$$M(T),\ B_{S=1}(T),\ B_{S=2}(T),\ B_{S=3}(T)$$

up to terms $c_1 v_1 + c_2 v_2$

Bazavov et al, PRL 111 (2013) 082301





Hadron resonance gas descriptions breaks down for all strangeness sectors above T_c
$\Rightarrow$ Strangeness deconfines at T_c

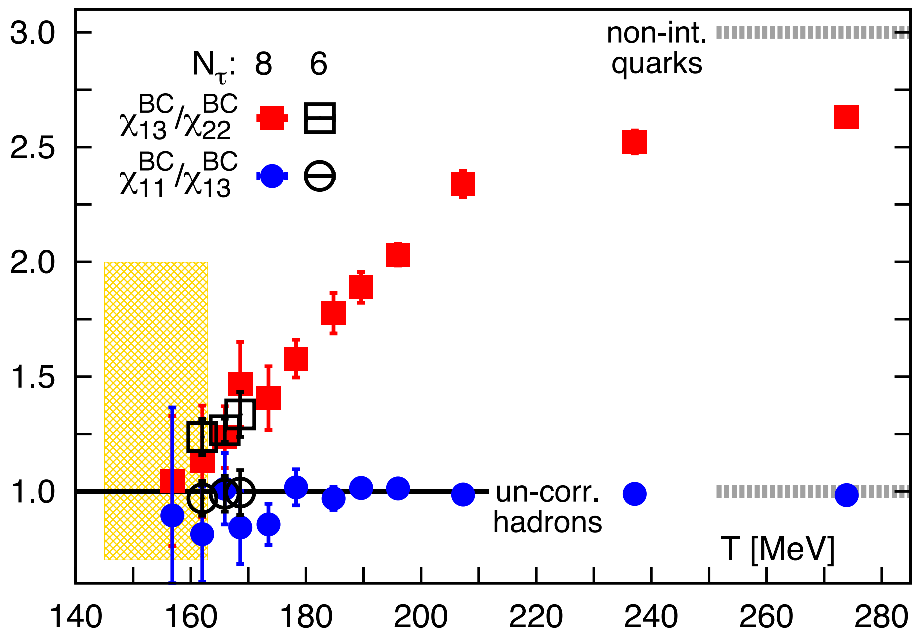
What about charm hadrons ?

We could introduce chemical potential for charm quarks and study the derivatives of the pressure with respect to the charm chemical potential

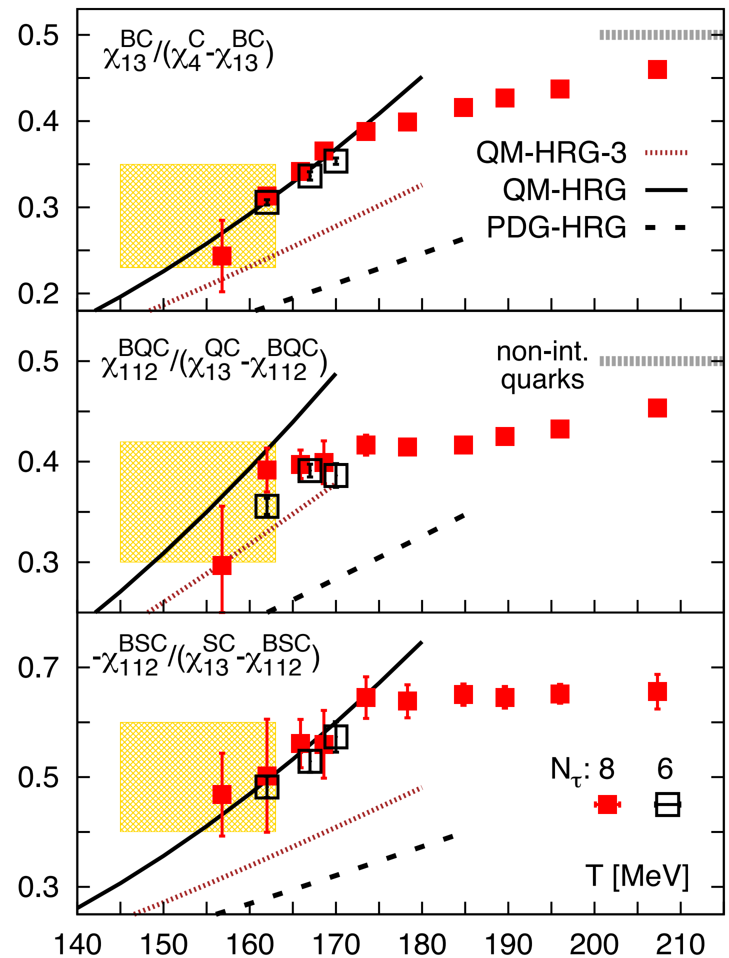
Bazavov et al, Phys.Lett. B737 (2014) 210

$m_c \gg T$ ⇨ only |*C*|=*1* sector contributes

In the hadronic phase all *BC*-correlations are the same !



Charm baryon to meson pressure



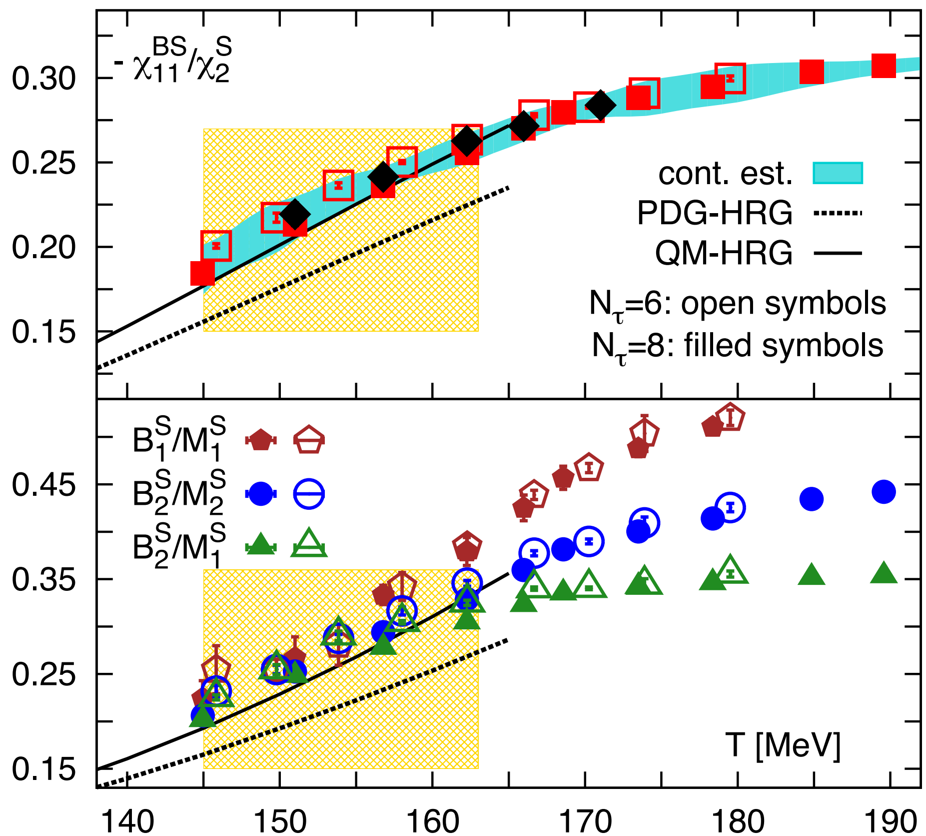
Hadronic description breaks down just above T_c

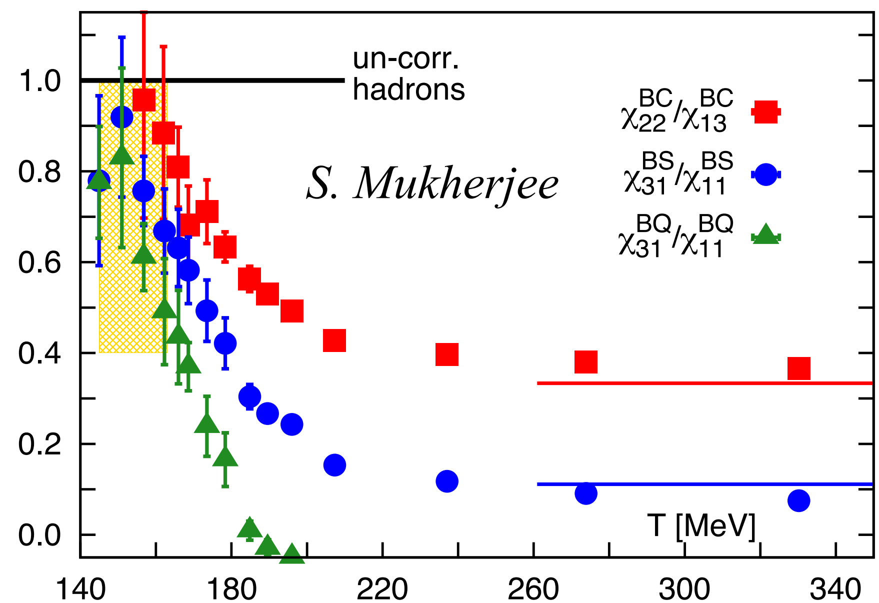
⇒ open charm deconfines above T_c

The charm baryon spectrum is not well known (only few states in PDG), HRG works only if the "missing" states are included

Strangeness baryon correlation revisited

Bazavov et al, PRL 113 (2014) 072001

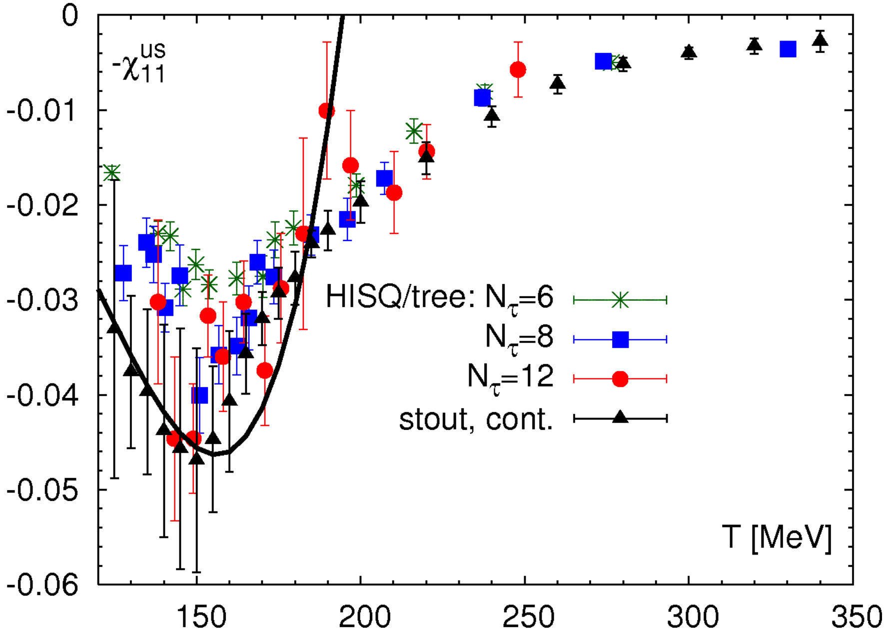




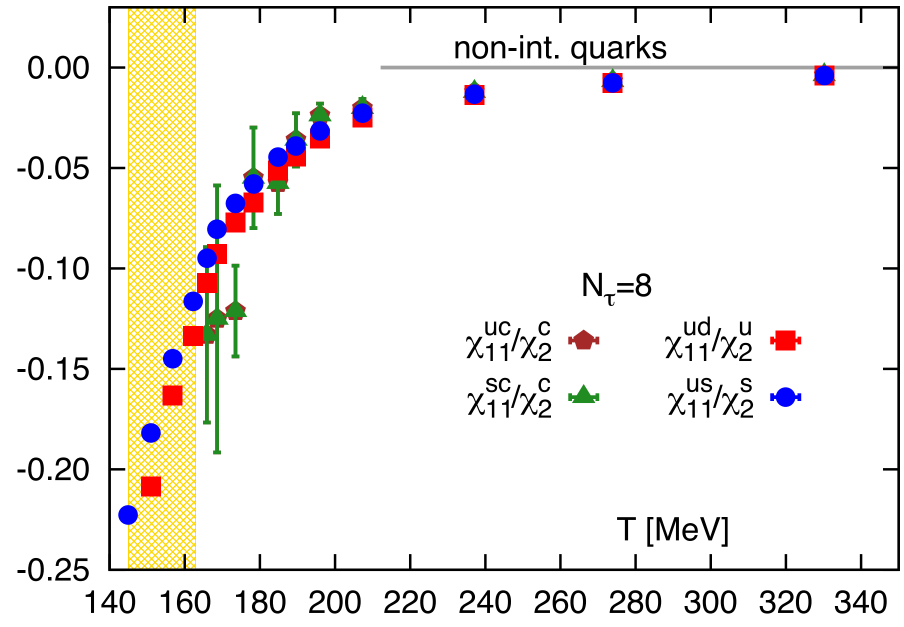
The description in terms of un-correlated gas of hadrons breaks down at T_c for all BX-correlations, $X=Q,S,C$

"Missing" strange baryons have to be included to obtain a good agreement between HRG and the lattice results

Quark number correlations



P.P. J.Phys. G39 (2012) 093002



Courtesy of S. Mukherjee

- Correlations between strange and light quarks at low *T* are due to the fact that strange hadrons contain both strange and light quarks but very small at high *T (>250* MeV*)* weakly interacting quark gas $\chi_{11}^{us} \sim \alpha_s^6$

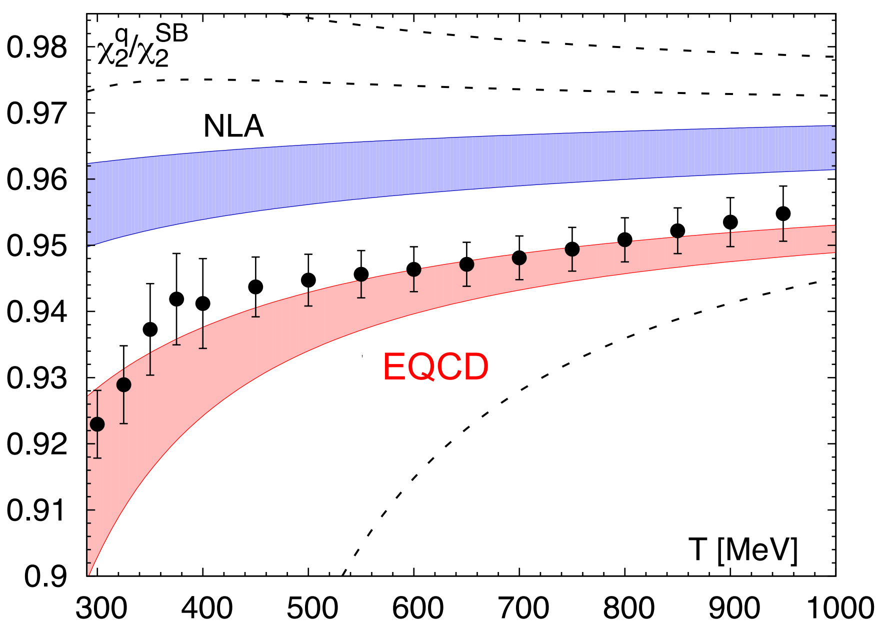
- Quark number correlations are flavor independent, correlations are unlikely due to bound states

- The transition region where degrees of freedom change from hadronic to quark-like is broad ~ *(100-150)* MeV

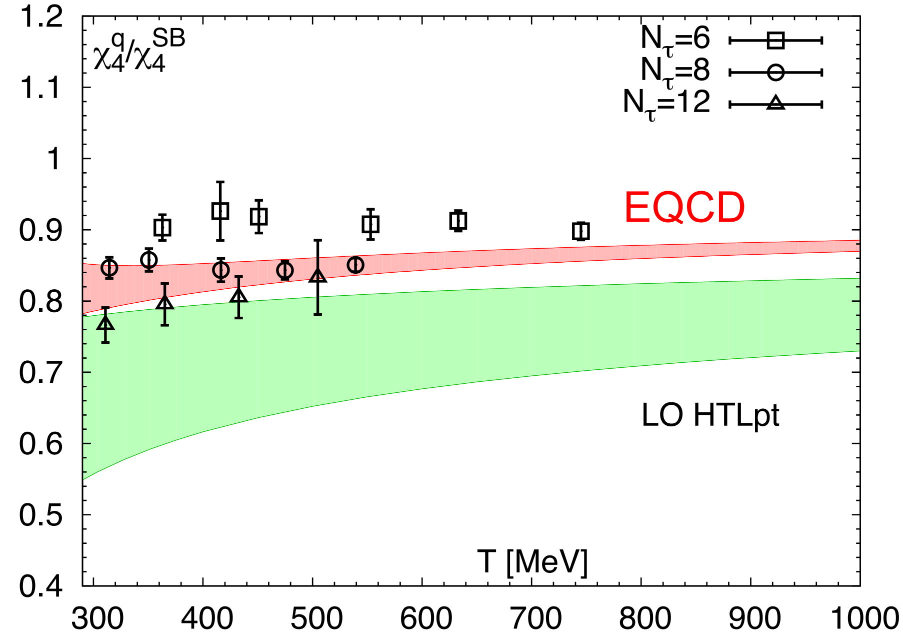
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

2nd order quark number fluctuations



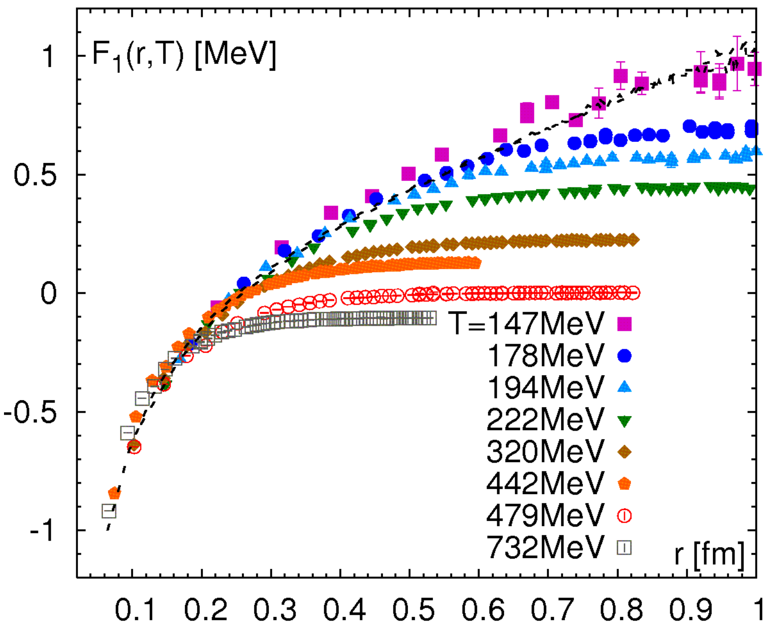
4th order quark number fluctuations

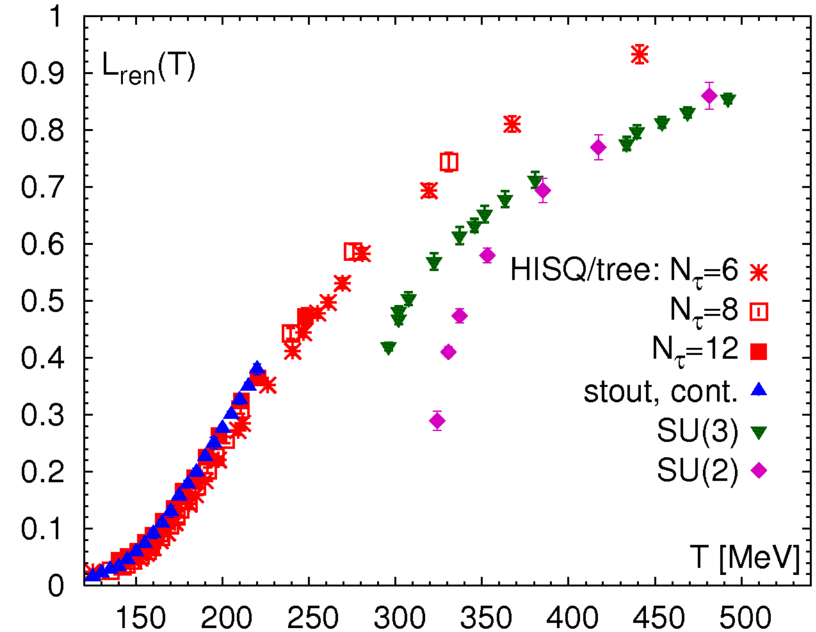


Bazavov et al, PRD88 (2013) 094021

- Lattice results converge as the continuum limit is approached
- Good agreement between lattice and the weak coupling approach for 2nd order quark number fluctuations
- For 4th order the weak coupling results are in reasonable agreement with lattice

Deconfinement and color screening





free energy of static quark anti-quark pair shows Debye screening at high temperatures

Pure glue ≠ QCD !

$$L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})} \Rightarrow L_{ren} = \exp(-F_Q(T)/T)$$

$$F_1(r) = -\frac{4}{3}\frac{\alpha_s}{r}\exp(-m_D r) + 2F_Q(T),\ m_D \sim T$$

$$F_Q(T) \simeq \Lambda_{QCD} - C_F\alpha_s m_D$$

infinite in the pure glue theory or large in the "hadronic" phase ~600MeV

Decreases in the deconfined phase

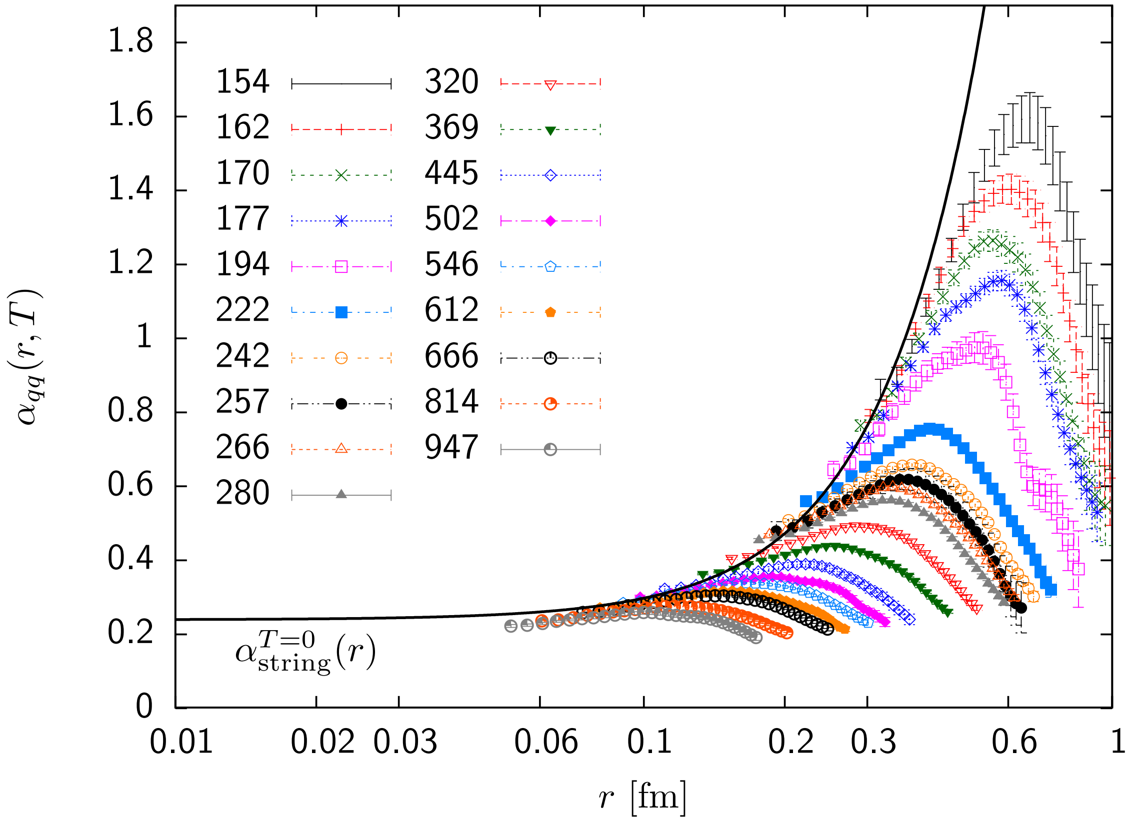
melting of bound states of heavy quarks => quarkonium suppression at RHIC: $r_{bound} > 1/m_D$

Color screening and strong coupling constant in QGP

Define a T-dependent effective running coupling constant

$$\alpha_{eff}(r,T) = \frac{3}{4} r^2 \frac{dF_1(r,T)}{dr}$$

In collaboration
with J. Webber



- For $r<0.5/T$ the coupling constant runs with the distance as at $T=0$
- It reaches a maximum at $r=r_{max} \approx 0.5/T$
- The maximal value of the α_{eff} is about *1* around $T= 200$ *MeV* and *0.5* at $T=300$ *MeV*

Summary

- The value chiral transition temperature is now well established in the continuum
  $T_c = 154(9)$ MeV
- Equation of state are known in the continuum limit up to $T=400$ MeV
- Hadron resonance gas can describe various thermodynamic quantities at low temperatures
- Deconfinement transition can be studied in terms of fluctuations and correlations of conserved charges, it manifest itself as a abrupt breakdown of hadronic description that occurs around the chiral transition temperature
- The approach to the weakly interacting quark gluon gas for $T>T_c$ is rather slow and the matter is strongly interacting for $T< 300$ MeV with no apparent quasi-particle composition
- For $T > (300\text{-}400)$ MeV weak coupling expansion works well for certain quantities (e.g. quark number susceptibilities), more work is needed to connect lattice and weak coupling results
- Comparison of lattice and HRG results for certain strangeness and charm correlations hints for existence of yet undiscovered excited baryons