Deconfinement and properties of QGP: lattice QCD vs. weak coupling

Charm correlations and fluctuations and charmed hadrons above $T_c$

Lattice determination of heavy quark diffusion in quenched approximation:
   a) electric field correlator method
   b) comments current-current correlators

Summary
Deconfinement and color screening

Onset of color screening is described by Polyakov loop (order parameter in SU(N) gauge theory)

\[ L = \text{tr} \mathcal{P} e^{ig \int_0^{1/T} d\tau A_0(\tau, \bar{x})} \]

\[ L_{\text{ren}} = \exp(-F_Q(T)/T) \]

The screening properties of SU(3) gauge and QCD are similar

Only for \( T > 300 \) MeV

The onset of screening corresponds to peak is \( S_Q \) and its position coincides with \( T_c \)

Bazavov et al, PRD 93 (2016) 114502

\[ S_Q = -\frac{\partial F_Q}{\partial T} \]
QCD thermodynamics at non-zero chemical potential

Taylor expansion:

\[
\frac{p(T, \mu_B, \mu_Q, \mu_S, \mu_C)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl} BQSC \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k \left( \frac{\mu_C}{T} \right)^l
\]

\[
\frac{p(T, \mu_u, \mu_d, \mu_s, \mu_c)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl} udsc \left( \frac{\mu_u}{T} \right)^i \left( \frac{\mu_d}{T} \right)^j \left( \frac{\mu_s}{T} \right)^k \left( \frac{\mu_c}{T} \right)^l
\]

\[
\chi_{ijkl}^{abcd} = T^{i+j+k+l} \frac{\partial^i}{\partial \mu_b^i} \frac{\partial^j}{\partial \mu_c^j} \frac{\partial^k}{\partial \mu_d^k} \frac{\partial^l}{\partial \mu_d^l} \ln Z(T, V, \mu_{a}, \mu_{b}, \mu_{c}, \mu_{d}) \bigg|_{\mu_{a}=\mu_{b}=\mu_{c}=\mu_{d}=0}
\]

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

\[
\chi_X^2 = \chi_X = \frac{1}{VT^3} (\langle X^2 \rangle - \langle X \rangle^2)
\]

\[
\chi_{11}^{XY} = \frac{1}{VT^3} (\langle XY \rangle - \langle X \rangle \langle Y \rangle)
\]

information about carriers of the conserved charges ( hadrons or quarks )

probes of deconfinement
Quark number fluctuations at high T

Good agreement between lattice and the weak coupling approach for 2\textsuperscript{nd} and 4\textsuperscript{th} order quark number fluctuations


Correlations are large for \( T<200 \) MeV but agree with weak coupling expectations for \( T>300 \) MeV, e.g.
Deconfinement of charm

\[ \chi_{nml}^{XYC} = T^{m+n+l} \frac{\partial^{n+m+l} p(T, \mu_X, \mu_Y, \mu_C)}{\partial \mu_X^n \partial \mu_Y^m \partial \mu_C^l} / T^4 \]

\[ m_c \gg T \quad \text{only } |C|=1 \text{ sector contributes} \]

In the hadronic phase all BC-correlations are the same!

Hadronic description breaks down just above \( T_c \)

\( \Rightarrow \) open charm deconfines above \( T_c \)

The charm baryon spectrum is not well known (only few states in PDG), HRG works only if the “missing” states are included.
Charm fluctuations at high temperatures

\[ \chi_2^c \]

\[ m_c(T) \text{ [GeV]} \]
Quasi-particle model for charm degrees of freedom

Charm dof are good quasi-particles at all $T$ because $M_c >> T$ and Boltzmann approximation holds

$$p^C(T, \mu_B, \mu_c) = p^C_q(T) \cosh(\hat{\mu}_C + \hat{\mu}_B/3) + p^C_B(T) \cosh(\hat{\mu}_C + \hat{\mu}_B) + p^C_M(T) \cosh(\hat{\mu}_C)$$

$$\chi^C_2, \chi^{BC}_{13}, \chi^{BC}_{22} \Rightarrow p^C_q(T), p^C_M(T), p^C_B(T)$$

Partial meson and baryon pressures described by HRG at $T_c$ and dominate the charm pressure then drop gradually, charm quark only dominant dof at $T > 200$ MeV or $\varepsilon > 6$ GeV/fm$^3$

Partial pressures drop because hadronic excitations become broad at high temperatures (bound state peaks merge with the continuum)

See

Jakovác, PRD88 (2013), 065012

Biró, Jakovác, PRD(2014)065012

Vice versa for quarks

Mukherjee, PP, Sharma, PRD 93 (2016) 014502
Current-current correlators and heavy quark diffusion

\[ \rho_{ij}^\mu (\omega) \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x \left\langle \hat{J}^\mu (t, \vec{x}), \hat{J}^\nu (0, \vec{0}) \right\rangle \]

\[ \sum_i \frac{\rho_{ij}^{ii} (\omega)}{\omega} \approx 3 \chi_2^q D \frac{\eta^2}{\eta^2 + \omega^2}, \quad \omega < \omega_{UV}, \quad \eta = \frac{T}{M} \frac{1}{D} \]

Spatial diffusion constant \( \sim \) mean free path (weak coupling)

\[ D \sim \frac{1}{g^4 T} \]

Momentum diffusion coefficient

\[ \kappa^{(M)} = \frac{M^2 \omega^2}{3T \chi_2^q} \sum_i \left. \frac{2T \rho_{ij}^{ii} (\omega)}{\omega} \right|_{\eta \ll \omega < \omega_{UV}} \]

\[ \kappa^{(M)} = 2T^2 / D \]

For large quark mass the transport peak is very narrow even for strong coupling and its difficult to reconstruct it accurately from Euclidean correlator calculated on the lattice.

area under the peak \( \sim \chi_2^q \)

heavy quark coefficient \( \sim \) width of the peak
Current-current correlators in the heavy quark limit

\[ \kappa = \frac{1}{3T} \sum_{i=1}^{3} \lim_{\omega \to 0} \left[ \lim_{M \to \infty} \frac{M^2}{\chi_2^q} \int_{-\infty}^{\infty} dt \ e^{i\omega(t-t')} \int d^3 \vec{x} \left\langle \frac{1}{2} \left\{ \frac{d\hat{J}^i(t, \vec{x})}{dt}, \frac{d\hat{J}^i(t', \vec{0})}{dt'} \right\} \right\rangle \right] \]

\[ \frac{d\hat{J}^i}{dt} = \frac{1}{M} \left\{ \hat{\phi}^\dagger gE^i \hat{\phi} - \hat{\theta}^\dagger gE^i \hat{\theta} \right\} + O\left( \frac{1}{M^2} \right) \quad t \to i\tau \]

\[ G_E(\tau) = \frac{1}{3\chi_2^q T} \sum_{i} \int d^3 \vec{x} \left\langle \left[ \phi^\dagger gE_i \phi - \theta^\dagger gE_i \theta \right] (\tau, \vec{x}) \left[ \phi^\dagger gE_i \phi - \theta^\dagger gE_i \theta \right] (0, \vec{0}) \right\rangle \]

Integrate out \( \phi, \theta \)

\[ G_E(\tau) = \]

\[ -\frac{1}{3} \sum_{i=1}^{3} \left\langle \text{ReTr} \left[ U(\beta, \tau) gE_i(\tau, \vec{0}) U(\tau, 0) gE_i(0, \vec{0}) \right] \right\rangle \]

\[ \left\langle \text{ReTr}[U(\beta, 0)] \right\rangle \]

\[ G_E(\tau) = \int_{0}^{\infty} \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh \left( \tau - \frac{1}{2T} \right) \omega}{\sinh \frac{\omega}{2T}} \]

Transport coefficient \( \sim \) intercept of the spectral function not its width

\[ \kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega) \]

Caron-Huot, Laine, Moore, JHEP 0904 (2009) 053
Calculating the electric field strength correlator on the lattice

Straightforward to discretize by deforming the path of the Wilson lines to spatial direction

Challenge: MC noise

- multilevel algorithm + link integration (only works for pure glue theory)
  Luscher, Weisz, JHEP 0109 (2010), 010; Parisi, Petronzio, Rapuano, PLB 128 (1983) 418


\[ G_{\text{norm}}(\tau) = g^2 C_F \pi^2 T^4 \left[ \frac{\cos^2(\pi \tau T)}{\sin^4(\pi \tau T)} + \frac{1}{3 \sin^2(\pi \tau T)} \right] \]
Extracting the spectral function and the diffusion constant

Fit the lattice using a forms of the spectral function constrained by low and high energy asymptotic behavior + corrections

\[ \rho^{\text{low}}(\omega) = \frac{\kappa \omega}{2T} \]

\[ \rho^{\text{high}}(\omega) = \frac{g^2(\mu_\omega) C_F}{6\pi} \omega^3, \mu_\omega = \max(\omega, \pi T) \]

T \sim 1.5 T_c, T_c = 1.24 \Lambda_{\overline{\text{MS}}}


\[ \kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega) = (1.8 - 3.4) \]
Diffusion constant as function of the temperature

See talk by Viljami Leino
Summary

- Chiral symmetry restoration and deconfinement in terms of appearance of quark dof and screening happen at the same temperature.

- QCD with dynamical quarks and quenched QCD behave similarly for $T > 300$ MeV in terms of color screening => quenched can be used for qualitative guidance.

- Heavy quark diffusion coefficient can be determined in quenched approximation and at present is the best known transport coefficient on the lattice: $2 \pi T = 1.8 - 3.4$.

- It is not known how to do the calculations of heavy quark diffusion coefficient in QCD with dynamical quarks.

- It would be of interest to extend the lattice study of heavy diffusion coefficient to higher temperatures to make contact with weak coupling calculations.

- For $T < 200$ MeV the deconfined matter is strongly interacting and charm hadrons may exist. Nothing is known about transport coefficients in this region.
lattice cut-off effects visible at small separations (left figure)

$\rightarrow$ **tree-level improvement** (right figure) to reduce discretization effects

$$ G_{\text{cont}}^{\text{LO}}(\tau T) = G_{\text{lat}}^{\text{LO}}(\tau T) $$

From Kaczmarek
Does the quasi-particle model makes sense ?

4 non-trivial constraints on the model provided by: $\chi_{31}^{BC}$, $\chi_{31}^{SC}$, $\chi_{121}^{BSC}$, $\chi_{211}^{BSC}$

$c_1 \equiv \chi_{13}^{BC} - 4\chi_{22}^{BC} + 3\chi_{31}^{BC} = 0,$
$c_2 \equiv 2\chi_{121}^{BSC} + 4\chi_{112}^{BSC} + \chi_{22}^{SC} + 2\chi_{13}^{SC} - \chi_{31}^{SC} = 0,
$c_3 \equiv 6\chi_{121}^{BSC} + 6\chi_{112}^{BSC} + \chi_{13}^{SC} - \chi_{31}^{SC},$
$c_4 \equiv \chi_{211}^{BSC} - \chi_{112}^{BSC}.$

Diquark pressure is zero !

Models with charm quark only: correlations from an effective mass

$m_c = m_c(T, \mu_C, \mu_S, \mu_B)$

Taylor expand the effective mass in chemical potential

$c_n \quad \Rightarrow$ Un-natural fine tuning of the expansion coefficients
Lattice calculations of the vector spectral functions:

Ding et al, PRD 83 (11) 034504

Isotropic Wilson gauge action, quenched non-perturbatively improved clover fermion action on $128^3 \times N_\tau$ lattices, $T = 1.45T_c$, $m_q^{MS}(2\text{GeV}) = 0.1/T$, $N_\tau = 24, 32, 48$ ($a^{-1} = 9.4 - 18.8\text{GeV}$)

$$\sigma_{ii}(\omega) = \chi c_{BW} \frac{\omega \Gamma/2}{2\pi \omega^2 + (\Gamma/2)^2} + \frac{3}{4\pi^2} (1 + k) \omega^2 \tanh(\omega/4T) \Theta(\omega_0, \Delta_\omega),$$

$$\Theta(\omega_0, \Delta_\omega) = \left(1 + e^{(\omega_0^2 - \omega^2)/\omega \Delta_\omega}\right)^{-1}$$

Fit parameters: $c_{BW}$, $\Gamma$, $k$

Different choices of: $\omega_0$, $\Delta_\omega$
Comparison with other lattice approaches

Determination from vector charmonium correlators
Ding et al, PRD 86 (2012) 014509

The width of the transport peak is dominated by Systematic effects, it too broad because of limited resolution => $D$ is too small

Electric correlator method with multi-level algorithm but no continuum limit
Banerjee et al, PRD 85 (2012) 014510

$D$ is slightly smaller than the pQCD result