QCD Thermodynamics at High Temperature

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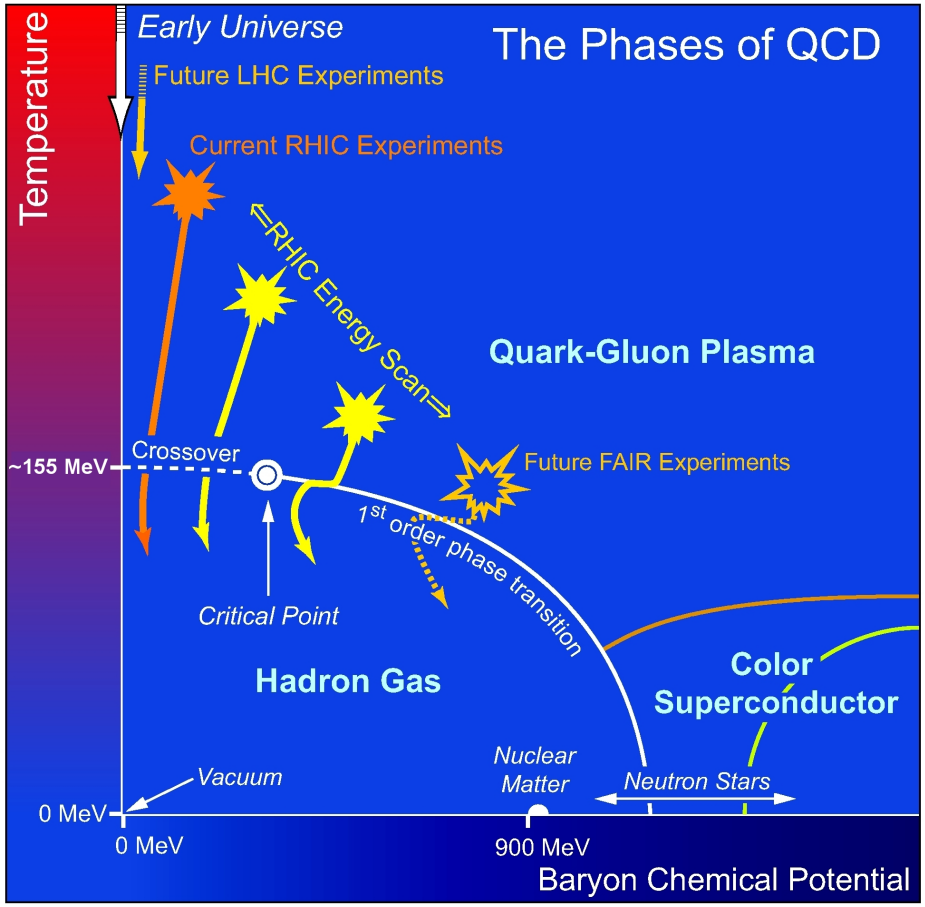
Defining questions of nuclear physics research in US:

Nuclear Science Advisory Committee (NSAC) "The Frontiers of Nuclear Science", 2007 Long Range Plan

"What are the phases of strongly interacting matter and what roles do they play in the cosmos ?"

"What does QCD predict for the properties of strongly interaction matter ?"



Challenges for LQCD:

1) Equation of state

2) Phase diagram and the transition temperature

3) Fluctuations of conserved Charges

4) In-medium hadron spectral functions

5) Transport properties

Deconfinement at high temperature and density

Quark Gluon Plasma (QGP)

Transition

$\alpha_s(T) \ll 1$

Hadron Gas

Color screening

magnetic screening, non-perturbative

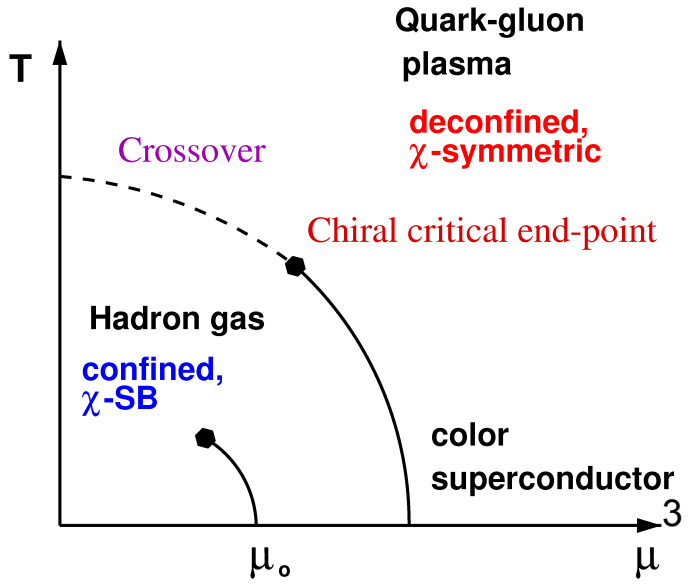
temperature and/or density

$\mathbf{T_c = (154 \pm 9)\ MeV}$

Chiral symmetry is broken in the low temperature (hadronic) "phase"

but is restored at high *T*

QCD analog of feromagnet-paramagnet transition

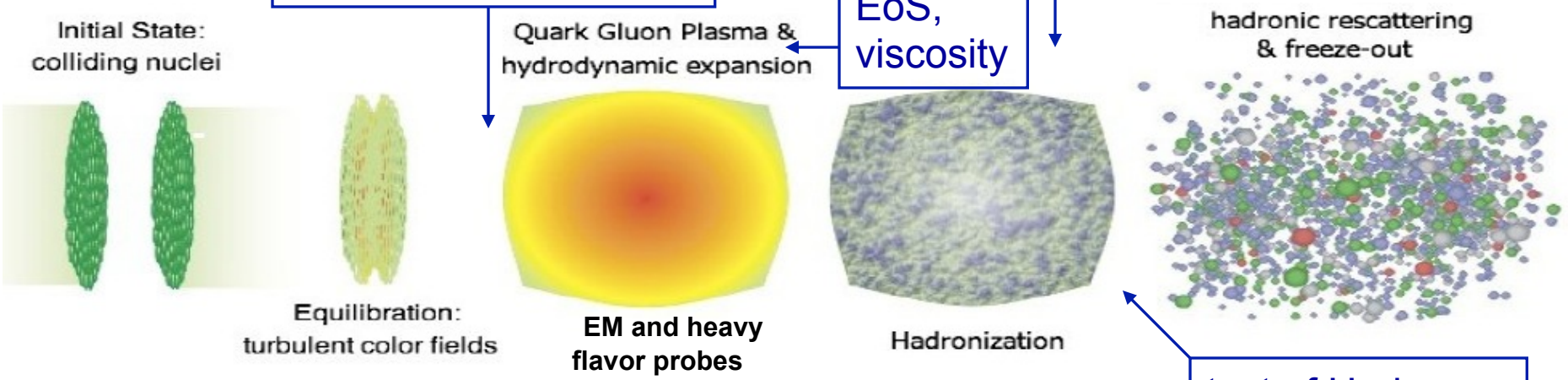


Physics of heavy ion collisions and LQCD

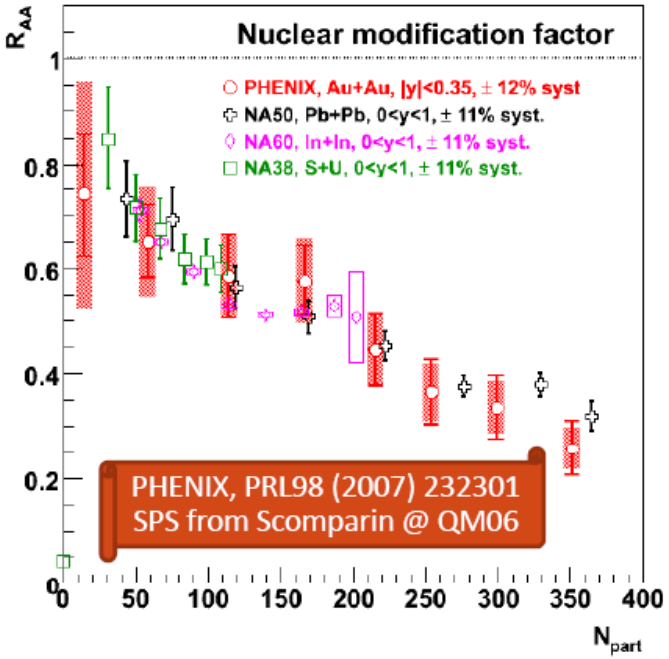
high temperature QCD
weak coupling ?

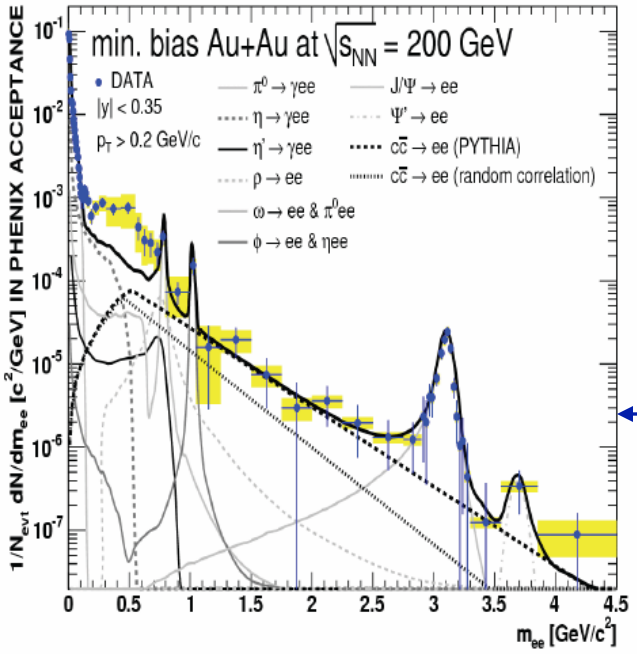
EoS,
viscosity

Chiral transition, T_c fluctu-
ations of conserved charges



test of Hadron
Resonance Gas
(HRG)
using LQCD





quarkonium spectral
functions,
heavy quark diffusion,
thermal dileptons

Finite Temperature QCD and its Lattice Formulation

$$\langle O \rangle = \mathrm{Tr} O e^{-\beta H - \mu N} \qquad \beta = 1/T$$

$$\Downarrow$$

$$\langle O \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} O e^{-\int_0^\beta d\tau d^3x \mathcal{L}_{QCD}}$$

$$A_\mu(0,\mathbf{x}) = A_\mu(\beta,\mathbf{x}) \;\; \psi(0,\mathbf{x}) = -\psi(\beta,\mathbf{x})$$

Lattice

$$\langle O \rangle = \int \prod_x dU_\mu(x) O(detD_q[U,m,\mu]) e^{-\sum_x S_G[U(x)]}, U_\mu(x) = e^{igaA_\mu(x)}$$

Generation of gauge configuration (Markov chain) $\mu = 0$ ➡ Hybrid Monte-Carlo

Most of cycles : D_q^{-1} *A*

$\mu \neq 0$: $detD_q(U,m,\mu)$ complex ➡ sign problem ➡ Taylor expansion for not too large μ

Most of cycles : D_q^{-1} *A*

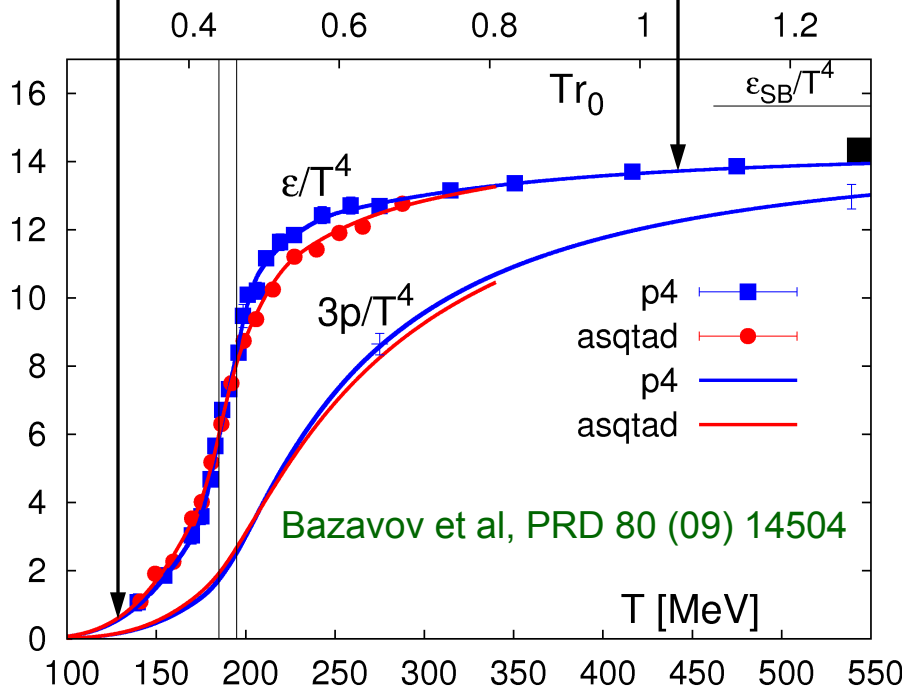
- Highly Improved Staggered Quark (HISQ) relatively inexpensive numerical but does not preserve all the symmetries of QCD (except for zero *a*)
- Domain Wall Fermion (DWF) formulation: preserves all the symmetries but *costs ~ 100x of staggered formulation*

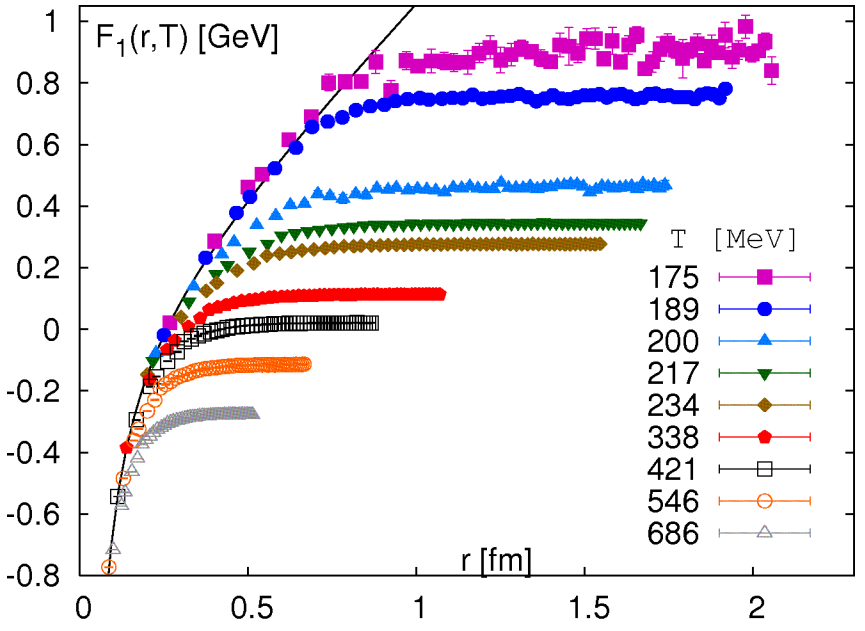
Need to take the limit of zero *a* : cost ~ $1/a^7$, $T=1/(a\,N_\tau)$

Deconfinement : pressure, energy density and color screening

meson gas = 3 light d.o.f.

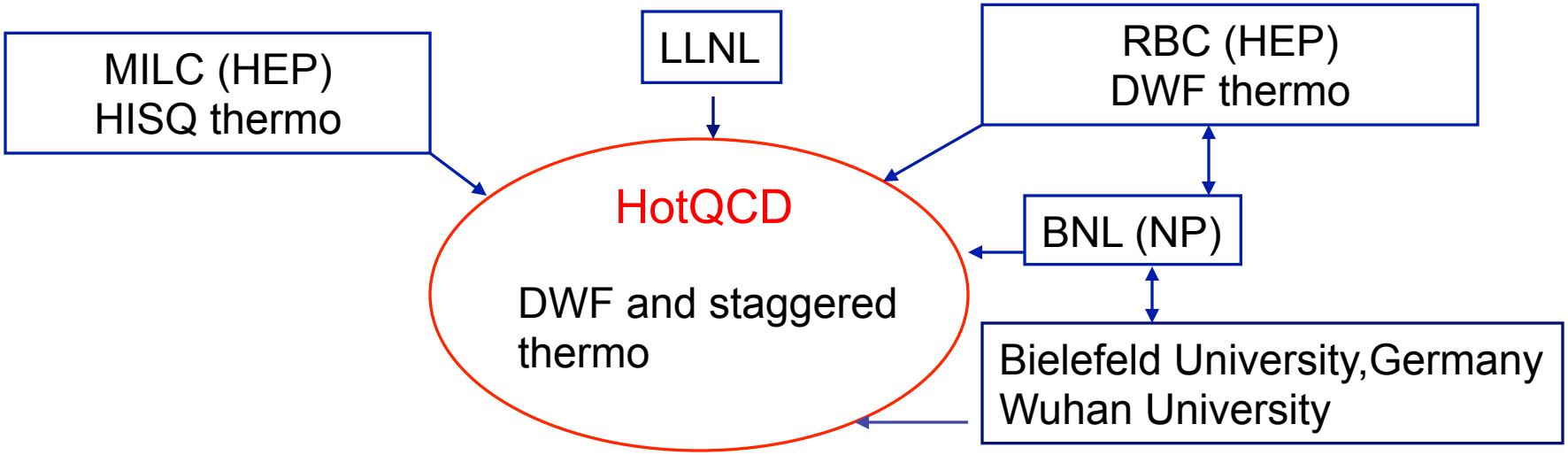
free gas of quarks and gluons = 18 quark+18 anti-quarks +16 gluons =52 mass-less d.o.f





- rapid change in the number of degrees of freedom at *T=160-200*MeV: deconfinement
- deviation from ideal gas limit is about 10% at high *T* consistent with the weakly coupled quark gluon gas
- free energy of static quark anti-quark pair shows Debye screening => quarkonium suppression @RHIC

HotQCD : a collaborative effort



Software (USQCD):

MILC code (MIMD C code + platform dependent optimization at t lower level) a lso for GPU (CUDA)

Columbia Physics System, CPS (C++, optimized for BG/Q, DWF only)

Optimized CUDA GPU code from Bielefeld U.

Resource in 2013-2014 (in core h):

1) CPU clusters of USQCD: 40M
2) GPU clusters of USQCD: 2.5 M
3) INCITE allocation of USQCD: 120M
4) BNL, BG/Q: 18M, BG/L: 50M
5) GPU cluster at LLNL and Bielefeld U.: 3.1M
6) BG/Q in Europe: 15M
7) NERSC allocation (PI Bazavov): 10M
8) BG/Q (LLNL) : 100 M (DWF thermo only)

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi^{BQS}_{ijk} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_Q}{T}\right)^k$$

LQCD : Taylor expansion coefficients ⇒ fluctuations of conserved charges: *X*= *B, S, Q* ⇒ Beam energy scan @ RHIC → parameters of the distribution

$$\chi_1^X = \frac{1}{VT^3}\langle N_X\rangle \qquad M_X = \langle N_X\rangle$$ mean

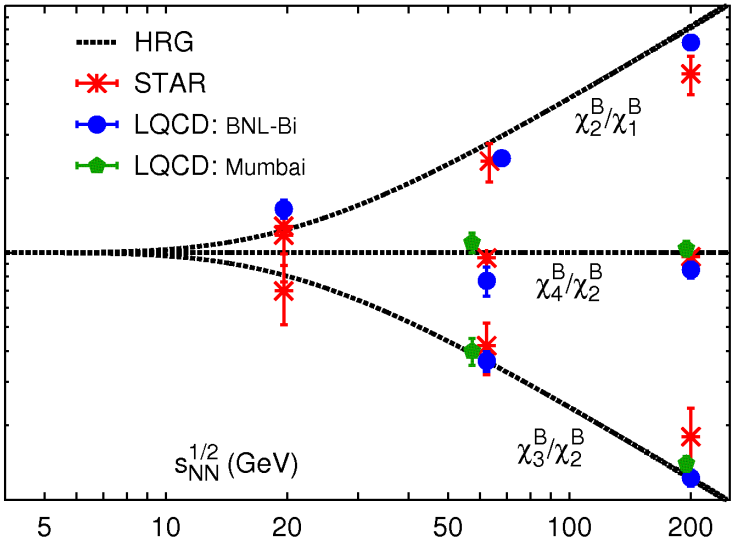
$$\chi_2^X = \frac{1}{VT^3}\langle (\delta N_X)^2\rangle \qquad \sigma_X = \langle (\delta N_X)^2\rangle$$ variance

$$\chi_3^X = \frac{1}{VT^3}\langle (\delta N_X)^3\rangle \qquad S_X = \langle (\delta N_X)^3\rangle/\sigma_X^3$$ Skewness

$$\chi_4^X = \frac{1}{VT^3}[\langle (\delta N_X)^4\rangle - 3\langle (\delta N_X)^2\rangle^2] \qquad K_X = \langle (\delta N_X)^4\rangle/\sigma_X^4 - 3$$ Kurtosis

$$N_X = X - \bar{X}, \quad \delta N_X = N_X - \langle N_X\rangle$$

can calculated very effectively on single GPUs



Volume independent combinations:

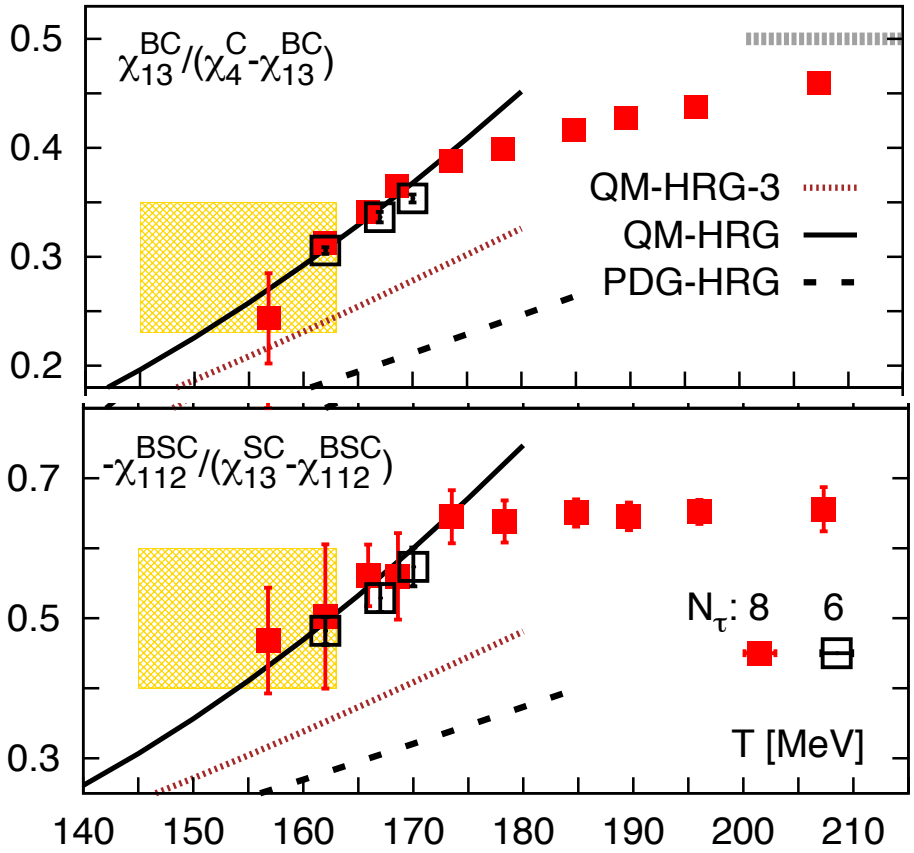
$$M_X/\sigma_X = \chi_1^X/\chi_2^X$$

$$S_X \cdot \sigma_X = \chi_3^X/\chi_2^X$$

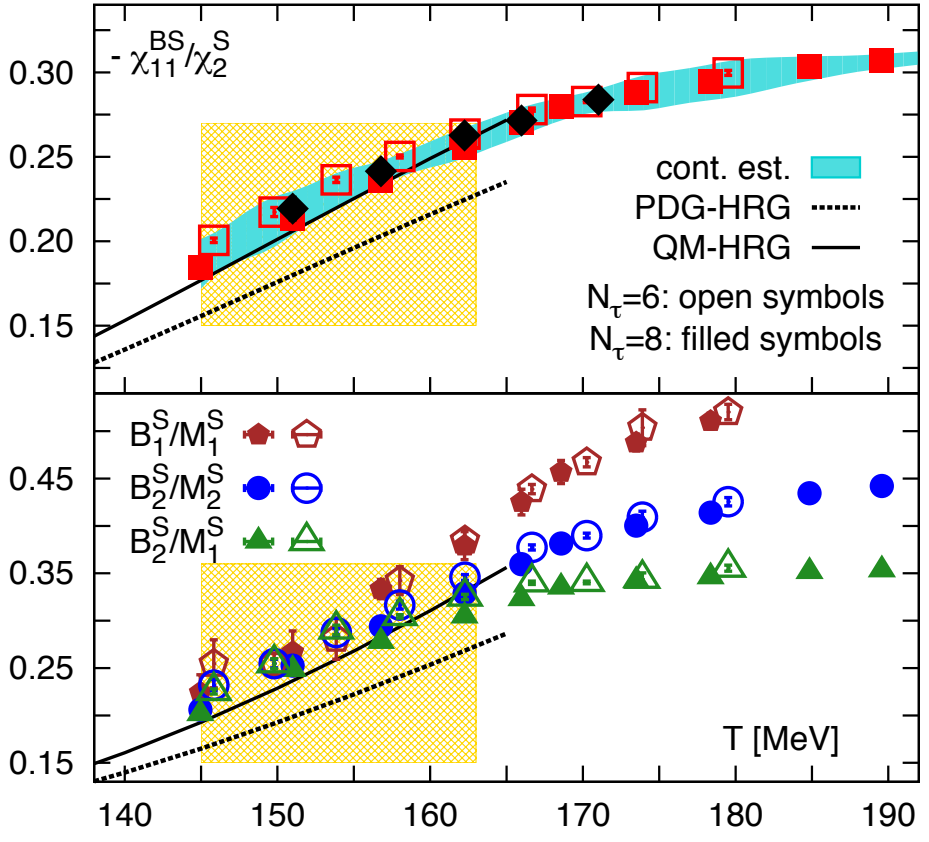
$$K_X \cdot \sigma_X^2 = \chi_4^X/\chi_2^X$$

Charge and strangeness fluctuations and missing hadrons

Bazavov et al, arXiv:1404.4043v1



Bazavov et al, arXiv:1404.6511

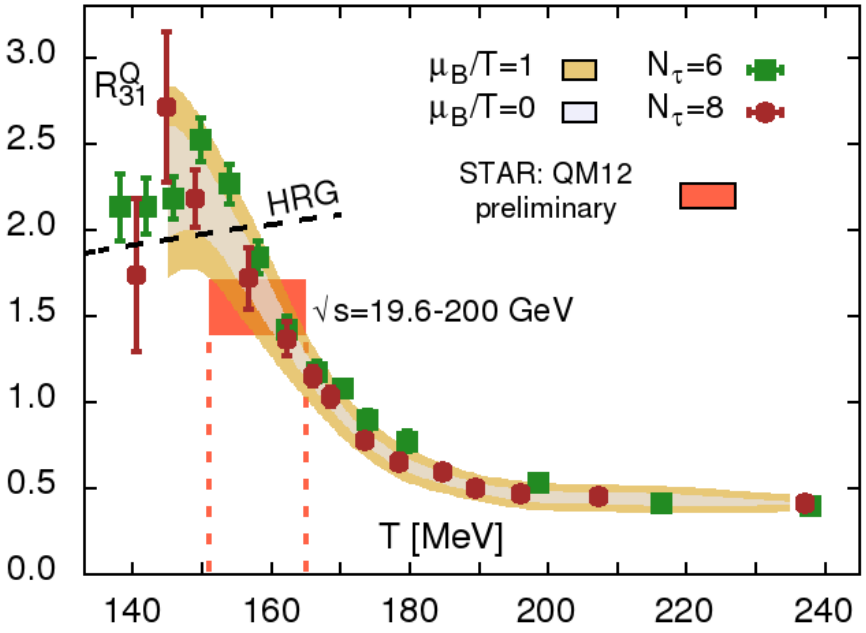


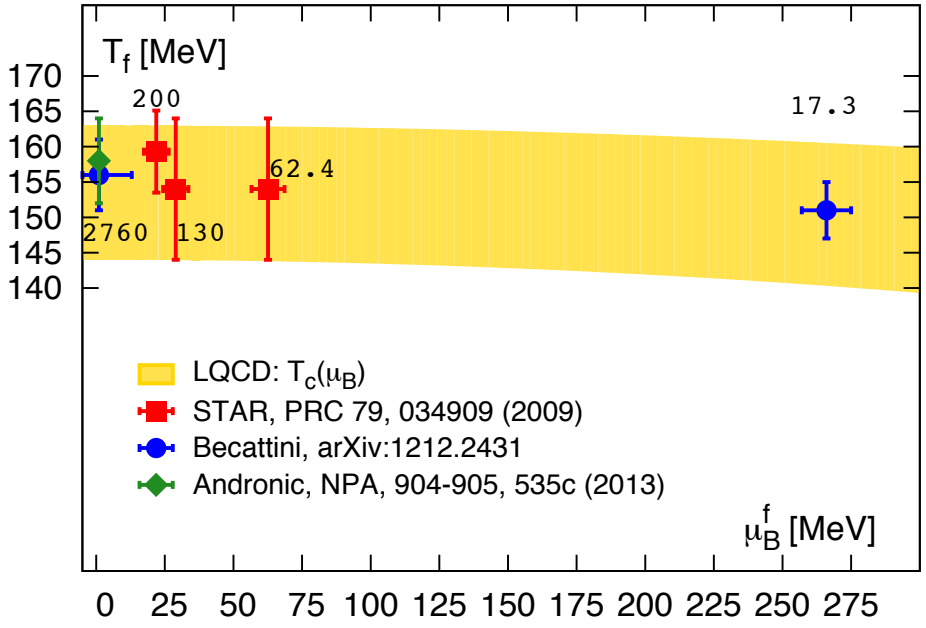
Experimental knowledge of strange and charm hadron spectrum is rather incomplete
Future experiments @ Jlab and FAIR (Germany) will address this problem

HRG that includes hadron states predicted by by quark model (also LQCD) agrees better with lattice results that HRG with PDG states only !

QCD thermodynamics at non-zero chemical potential

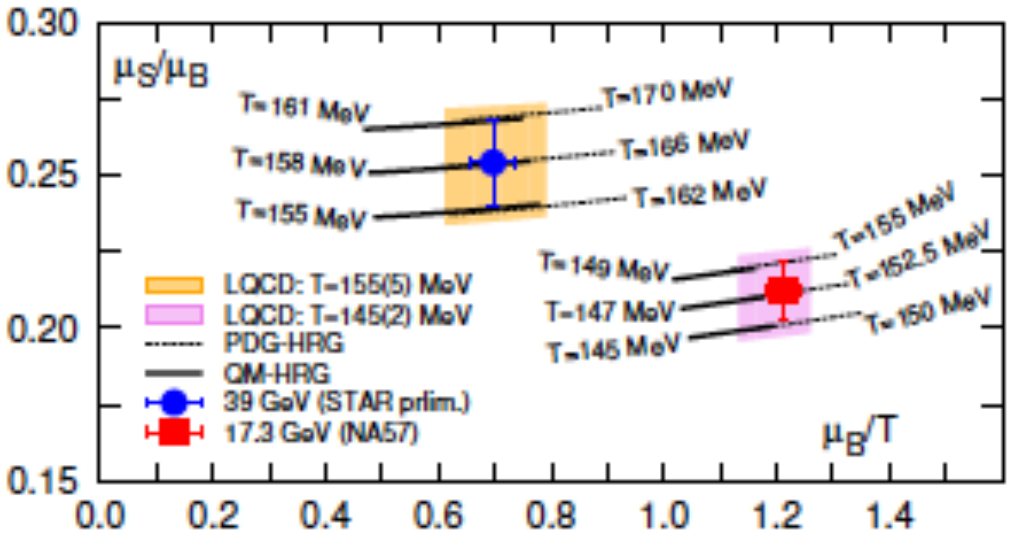
Bazavov et al, PRL109 (2012) 192302, Mukherjee, Wagner, arXiv:1307.6255





For consistent description of
The freeze-out of strange hadrons
Need to include the contribution of
"missing states"

Bazavov et al, arXiv:1404.6511



Spectral functions at *T>0* and physical observables

$$G(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T))}{\sinh(\omega/(2T))}$$

Heavy meson spectral functions:

$$J_H=\bar{\psi}\Gamma_H\psi$$

Heavy flavor probes at RHIC

quarkonia properties at *T>0*
heavy quark diffusion in QGP: *D*

Light vector meson spectral functions:

$$J_\mu=\bar{\psi}\ \gamma_\mu\psi$$

Thermal photons and dileptons provide information about the temperature of the

thermal dilepton production rate

$$\frac{dW}{d\omega d^3p}=\frac{5\alpha_{em}^2}{27\pi^2}\frac{1}{e^{\omega/T}-1}\frac{\sigma_{\mu\mu}(\omega,p,T)}{\omega^2-p^2}$$

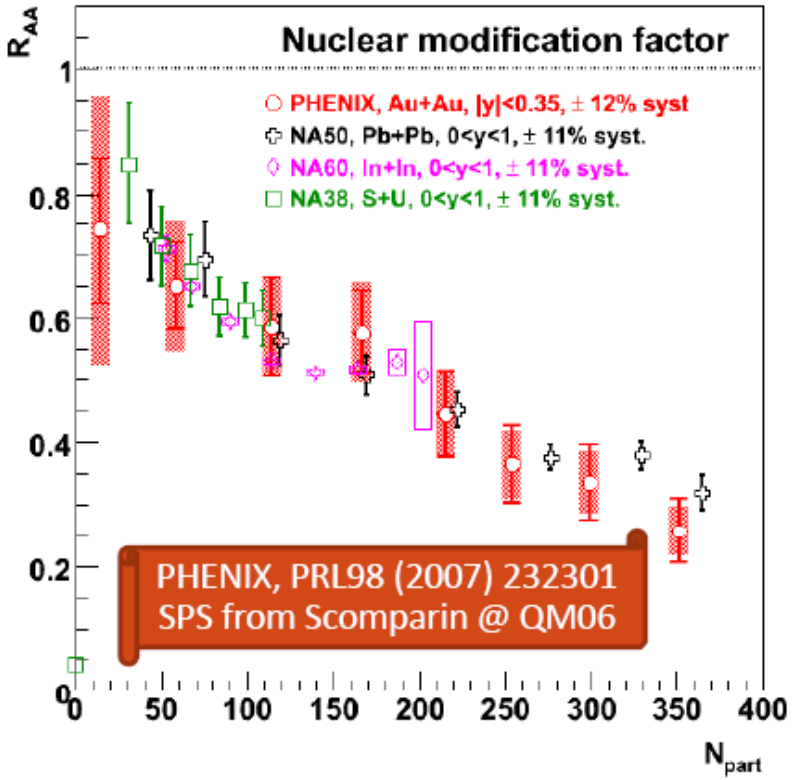
thermal photon production rate :

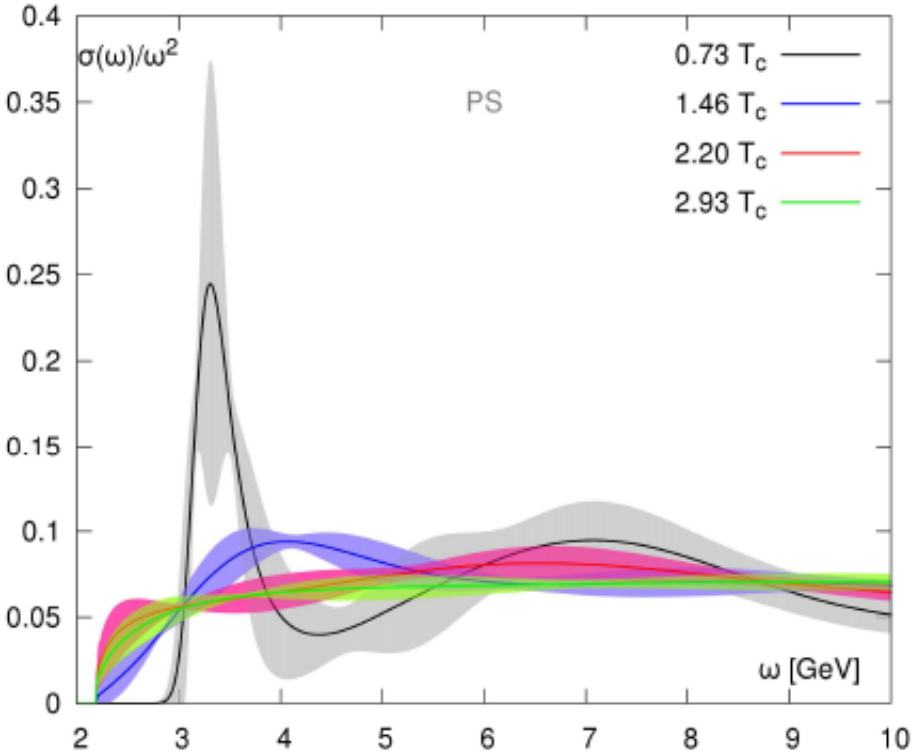
$$p\frac{dW}{d^3p}=\frac{5\alpha_{em}}{9\pi}\frac{1}{e^{p/T}-1}\sigma_{\mu\mu}(\omega=p,p,T)$$

electric conductivity ζ :

Quarkonium spectral spectral functions

Charmonium spectral functions on isotropic lattice in quenched approximation with Wilson quarks:
H.-T. Ding et al, arXiv:1204.4945
N_τ=24-96, a^{-1}=18.97GeV





No clear evidence for charmonium bound state peaks above T_c in spectral functions !

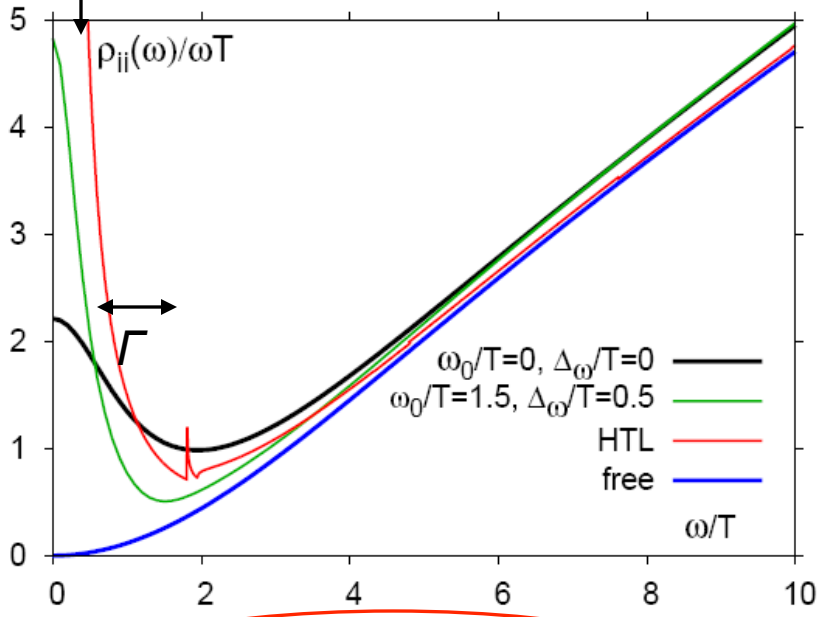
Lattice calculations of transport coefficients

Electric condictivity:

Ding et al, PRD 83 (11) 034504

peak at $\omega \approx 0$ = transport peak

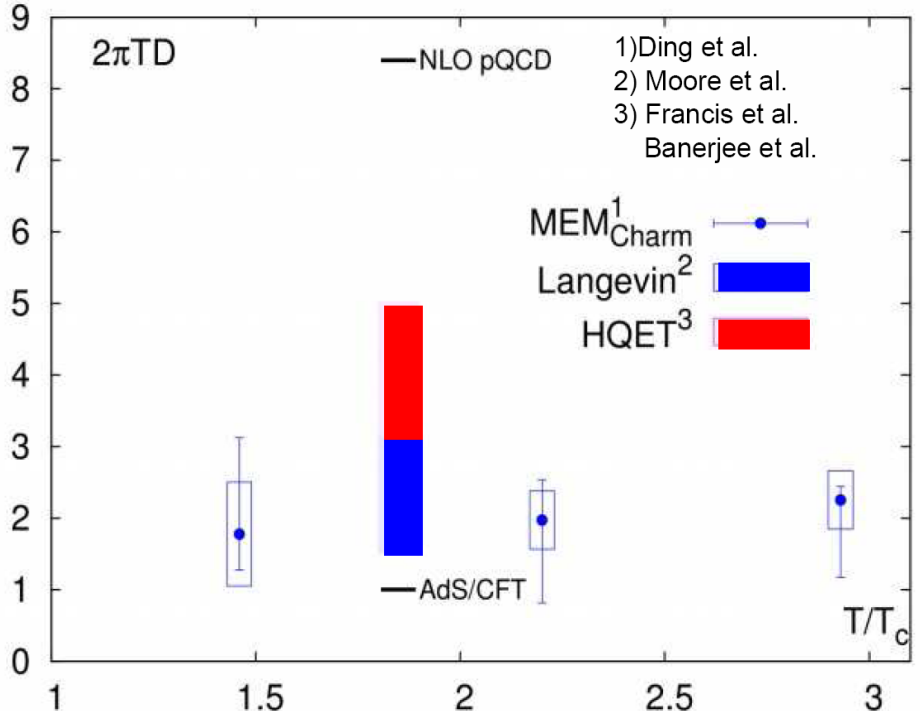
$$\Gamma \sim 1/\tau_{relax}, \quad \sigma_{el} \sim \chi_Q/\Gamma$$



$$1/3 < \frac{1}{C_em}\frac{\sigma_{el}}{T} < 1, \quad C_{em} = \sum_f Q_f^2$$

Heavy quark diffusion constant:

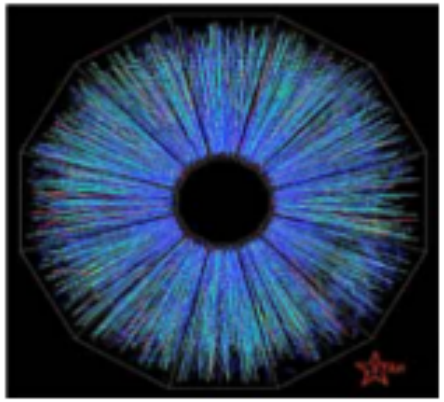
Ding et al, arXiv:1204:4954
Banarjee et al, arXiv:1109.5738
Kaczmarek et al, arXiv:1109:3941

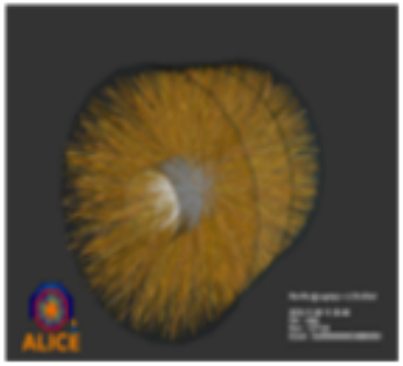


Quenched QCD calculations up to 128^3x32 lattice

Perfect liquid: $\eta/s = 1/(4\pi)$ ⇔ no transport peak !

Summary





nature of QCD transition

transport coefficients

QCD critical point

di-lepton, photon rate

di-lepton, quarkonia w/o dynamical fermions

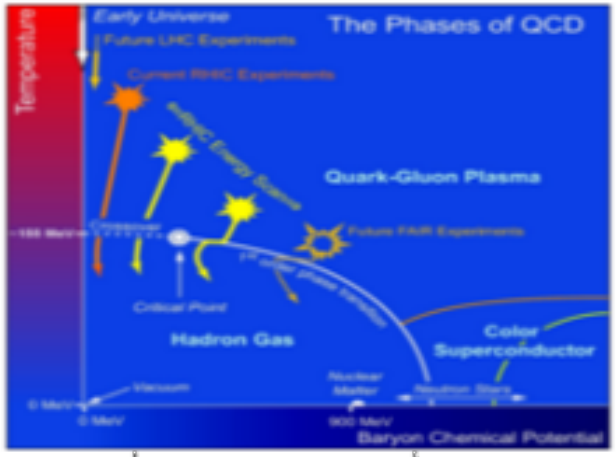
quarkonia melting

QCD at non-zero density on coarse lattices

freeze-out conditions

EOS with dynamical charm quark

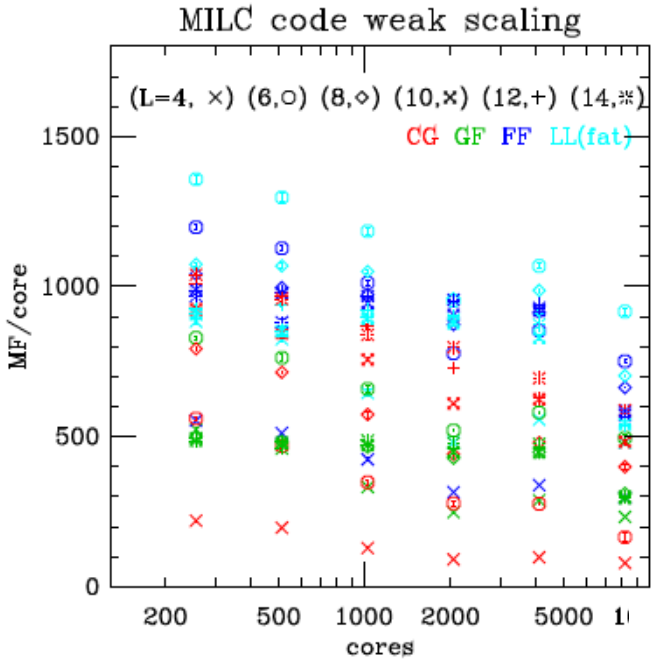
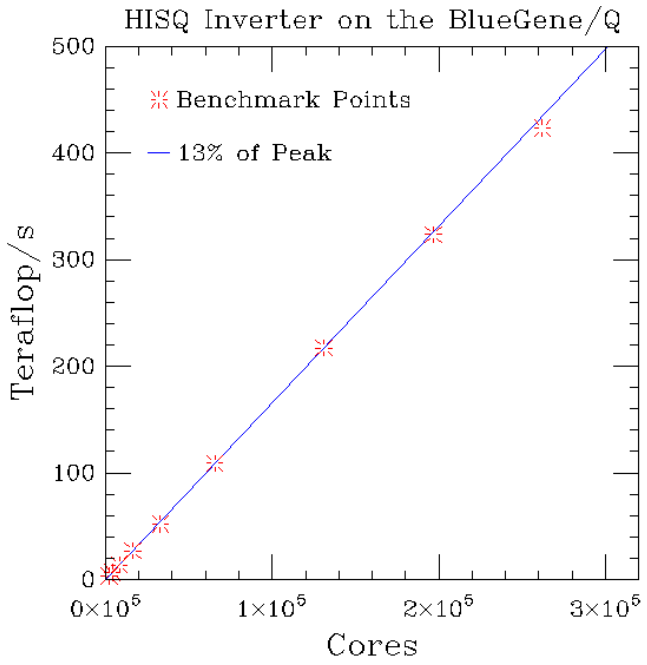
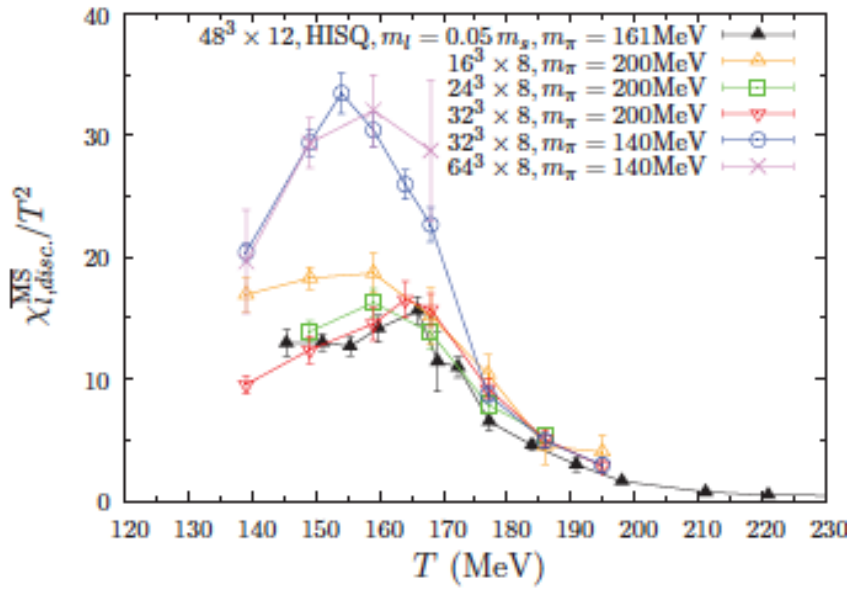
EOS w/o dynamical charm quark



~2012 ~2017

0.01 0.1 1 10 100 1000

Sustained Petaflop-Years

~2022

Back-up slide