Heavy quark free energies and screening in lattice QCD

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RBC-Bielefeld collaboration

O. Kaczmarek, arXiv:0710.0498 [hep-lat]

RBC-Bielefeld, arXiv:0710.0354 [hep-lat]

Heavy quark bound states above deconfinement

Strong interactions in the deconfined phase $T \gtrsim T_c$

Possibility of heavy quark bound states?

Suppression patterns of charmonium/bottomonium

Charmonium ($\chi_c, J/\psi$) as thermometer above T_c

$\Longrightarrow$ Potential models

$\rightarrow$ heavy quark potential (T=0)

$$V_1(r) = -\frac{4}{3}\frac{\alpha(r)}{r} + \sigma\, r$$

$\rightarrow$ heavy quark free energies ($T > T_c$)

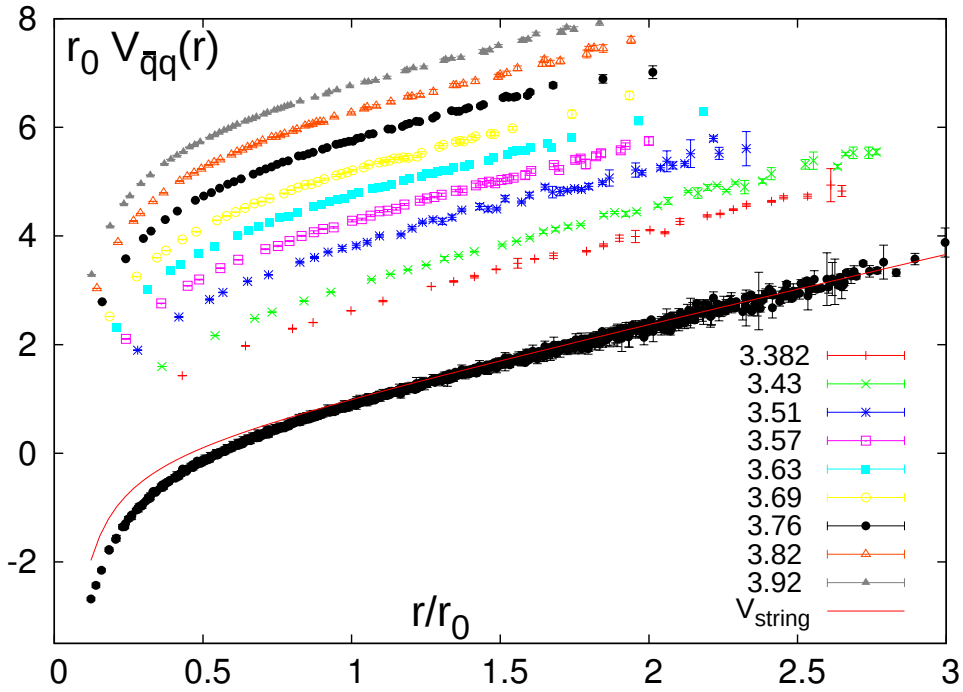
$$F_1(r,T) \simeq -\frac{4}{3}\frac{\alpha(r,T)}{r}\, e^{-m(T)r}$$

$\rightarrow$ heavy quark internal energies ($T \neq$0)

$$F_1(r,T) = U_1(r,T) - T\, S_1(r,T)$$

$\Longrightarrow$ Charmonium correlation functions/spectral functions

Zero temperature potential - n_f=2+1



2+1 flavor QCD

highly improved p4-staggered

almost realistic quark masses

$m_\pi \simeq 220$ MeV, physical m_s

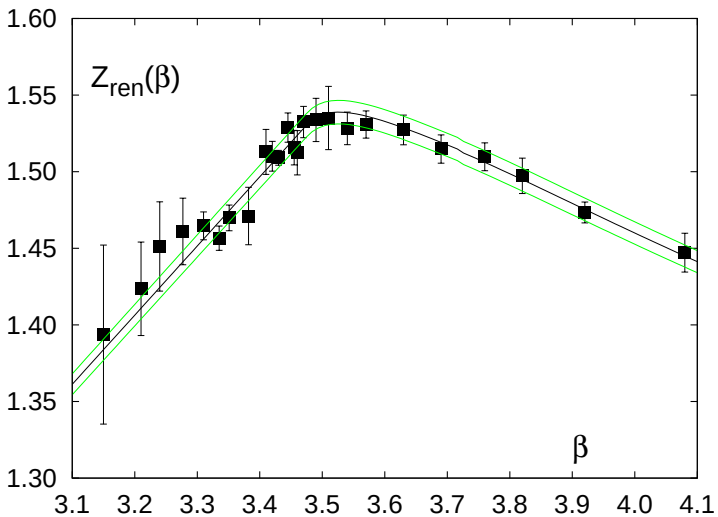
Large distance behaviour

consistent with string model prediction:

$$V(r) = -\frac{\pi}{12r} + \sigma\, r \quad , \text{ for large } r$$

$\longrightarrow$ used for renormalization

renormalization: $V_{T=0}(r) = -\log\left((Z_{ren}(\beta))^2 \frac{W(r,\tau)}{W(r,\tau+1)}\right)$

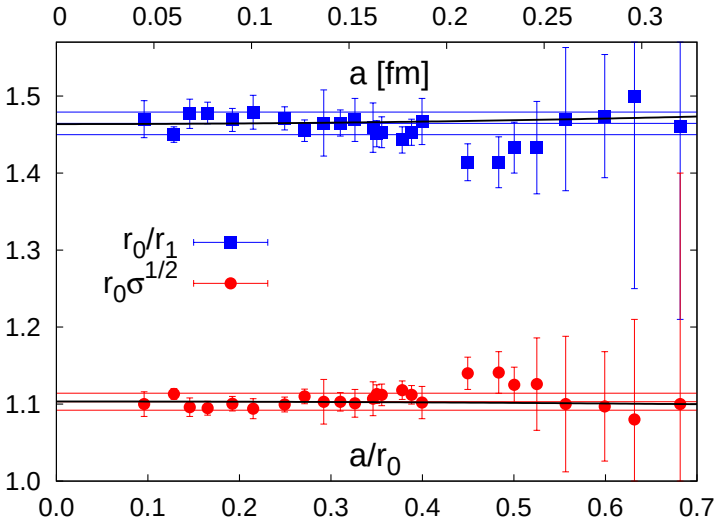


$$r^2 \frac{dV_{\bar{q}q}(r)}{dr}\bigg|_{r=r_0} = 1.65$$

$$r^2 \frac{dV_{\bar{q}q}(r)}{dr}\bigg|_{r=r_1} = 1.0$$

$$(r_0 = 0.469(7)\ \text{fm})$$

cut-off effects are small



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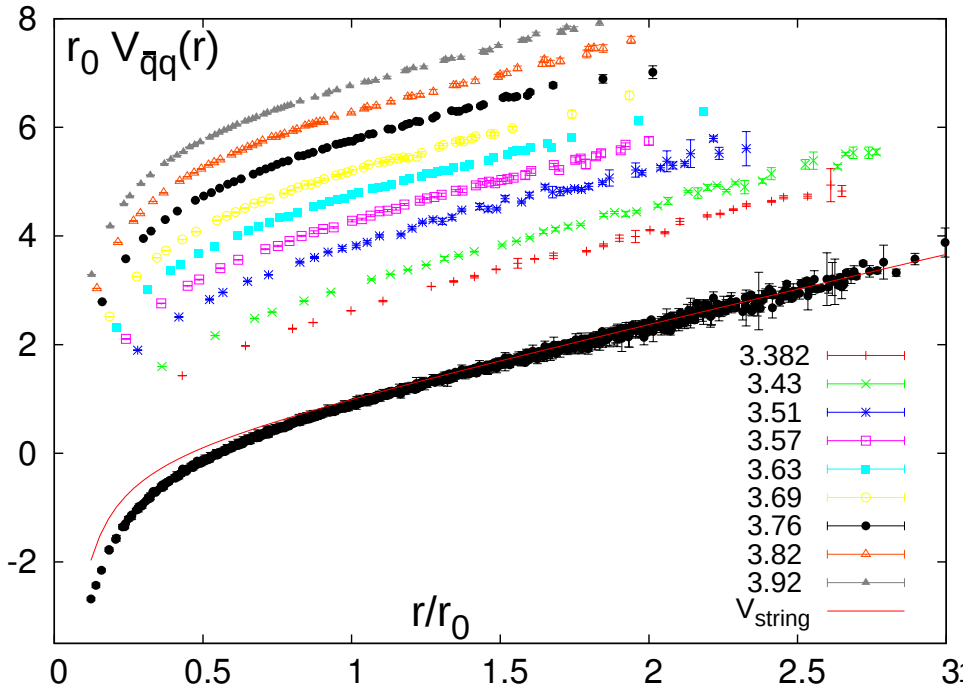

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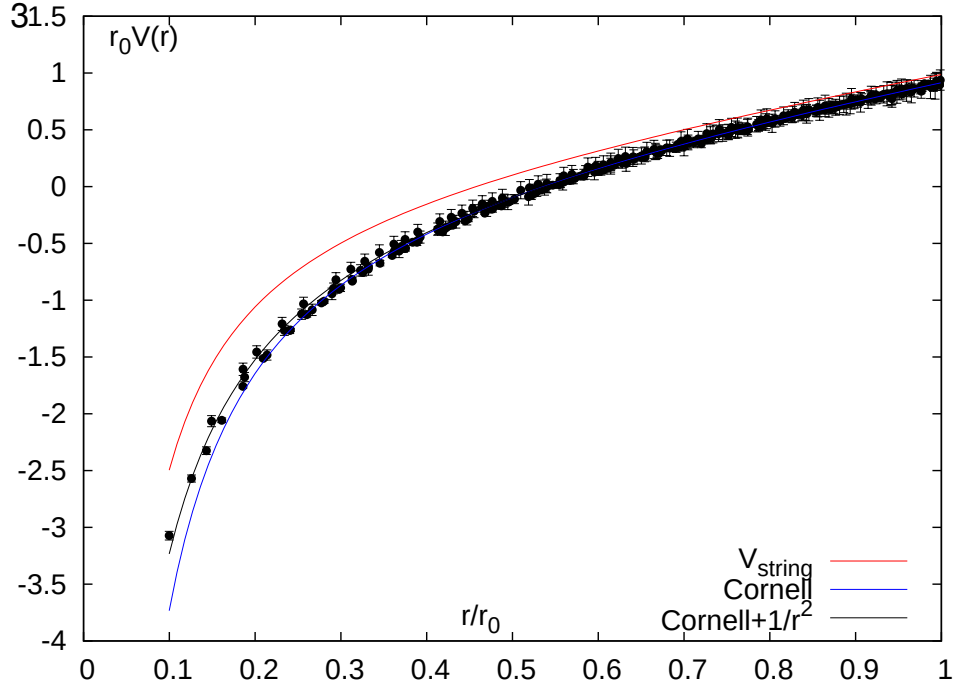


Short distance behaviour

deviations at small r

enhancement of the running coupling

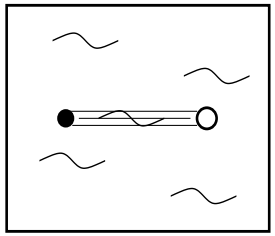
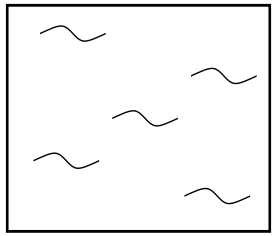
fit: $V(r) = -0.392(6)/r + \sigma\, r$

r-dependent running coupling $\alpha(r)$

Polyakov loop correlation function and free energy:

L. McLerran, B. Svetitsky (1981)

$$\frac{Z_{Q\bar{Q}}}{Z(T)} \simeq \frac{1}{Z(T)} \int \mathcal{D}A \ldots L(x)\, L^{\dagger}(y)\, \exp\left(- \int_0^{1/T} dt \int d^3x \mathcal{L}[A, \ldots] \right)$$

$\log() \Longrightarrow$  $-$  $=$ $-\dfrac{F_{Q\bar{Q}}(r,T)}{T}$

$Q\bar{Q} = 1,\ 8,\ \mathrm{av}$

Lattice data used in our analysis:

$N_f = 0$:	$N_f = 2$:	$N_f = 3$:	$N_f = 2+1$:
$32^3 \times 4, 8, 16$-lattices	$16^3 \times 4$-lattices	$16^3 \times 4$-lattices	$24^4 \times 6$-lattices
(*Symanzik*)	(*Symanzik, p4-stagg.*)	(*stagg., Asqtad*)	(*Symanzik, p4fat3*)
	hybrid-R	*hybrid-R*	*RHMC*
O. Kaczmarek, F. Karsch, P. Petreczky, F. Zantow (2002, 2004)	$m_\pi/m_\rho \simeq 0.7$ $(m/T = 0.4)$	$m_\pi/m_\rho \simeq 0.4$	$m_\pi \simeq 220$ **MeV, phys.** m_s
	O. Kaczmarek, F. Zantow (2005), O. Kaczmarek et al. (2003)	*P. Petreczky, K. Petrov (2004)*	*OK, RBC-Bielefeld arXiv:0710.0498,0710.0*

Polyakov loop correlation function and free energy:

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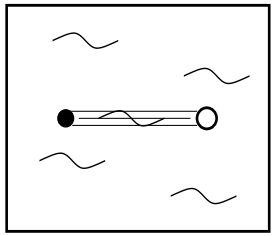
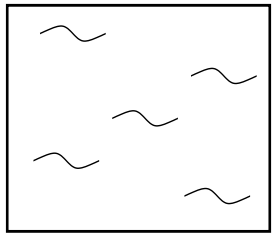
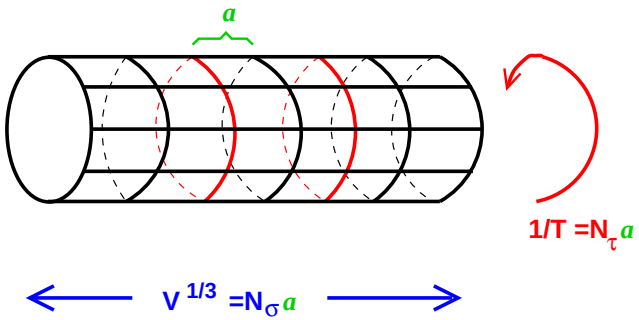
$$\frac{\mathcal{Z}_{\mathbf{Q}\bar{\mathbf{Q}}}}{\mathcal{Z}(\mathbf{T})} \simeq \frac{1}{\mathcal{Z}(\mathbf{T})}\int \mathcal{D}\mathbf{A}\ldots\, \mathbf{L}(\mathbf{x})\,\mathbf{L}^{\dagger}(\mathbf{y})\,\exp\left(-\int_{\mathbf{0}}^{\mathbf{1/T}}\mathbf{dt}\int \mathbf{d^3x}\mathcal{L}\left[\mathbf{A},\ldots\right]\right)$$

$\log() \Longrightarrow$  $-$  $=$ $-\dfrac{\mathbf{F_{Q\bar{Q}}(r,T)}}{\mathbf{T}}$

$\mathbf{Q\bar{Q}} = \mathbf{1},\ \mathbf{8},\ \mathbf{av}$

O. Philipsen (2002)
O. Jahn, O. Philipsen (2004)

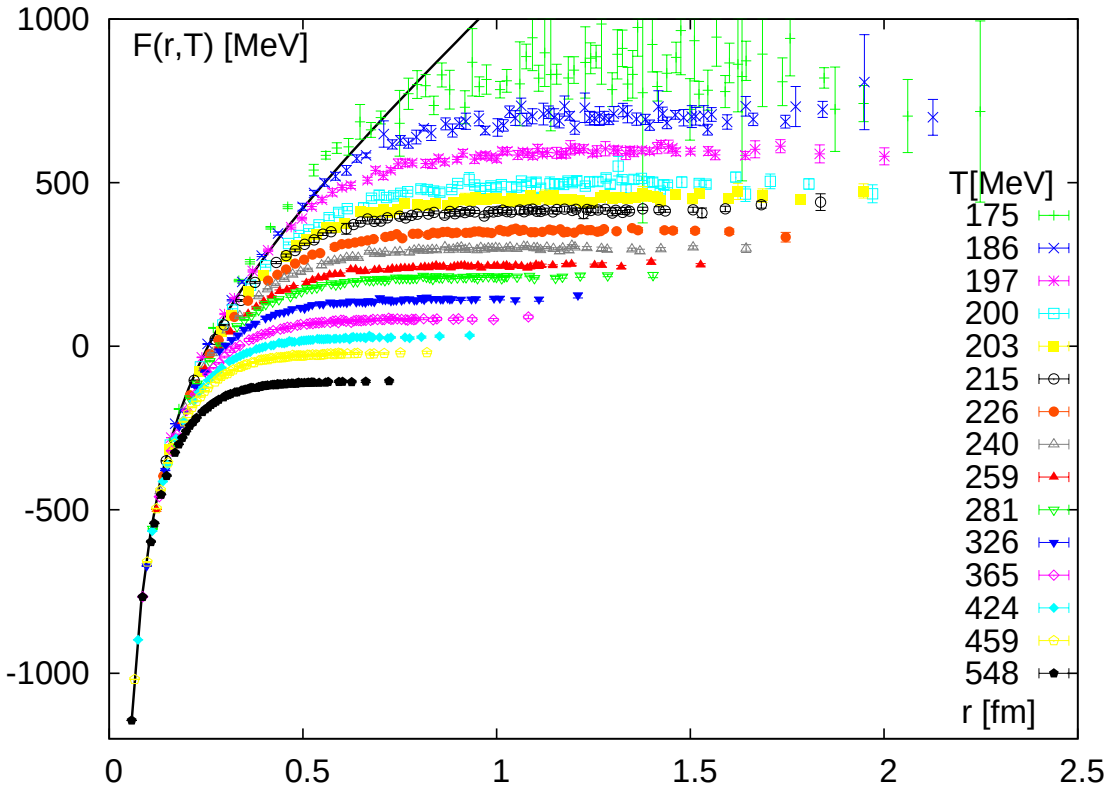


$$-\ln\left(\langle\tilde{\mathrm{Tr}}L(\mathbf{x})\tilde{\mathrm{Tr}}L^{\dagger}(\mathbf{y})\rangle\right) = \frac{F_{\bar{q}q}(r,T)}{T}$$

$$-\ln\left(\langle\tilde{\mathrm{Tr}}L(\mathbf{x})L^{\dagger}(\mathbf{y})\rangle\right)\Big|_{GF} = \frac{F_1(r,T)}{T}$$

$$-\ln\left(\frac{9}{8}\langle\tilde{\mathrm{Tr}}L(\mathbf{x})\tilde{\mathrm{Tr}}L^{\dagger}(\mathbf{y})\rangle - \frac{1}{8}\langle\tilde{\mathrm{Tr}}L(\mathbf{x})L^{\dagger}(\mathbf{y})\rangle\Big|_{GF}\right) = \frac{F_8(r,T)}{T}$$

Heavy quark free energy - 2+1-flavors



Renormalization of $F(r,T)$

using $Z_{ren}(g^2)$ obtained at T=0

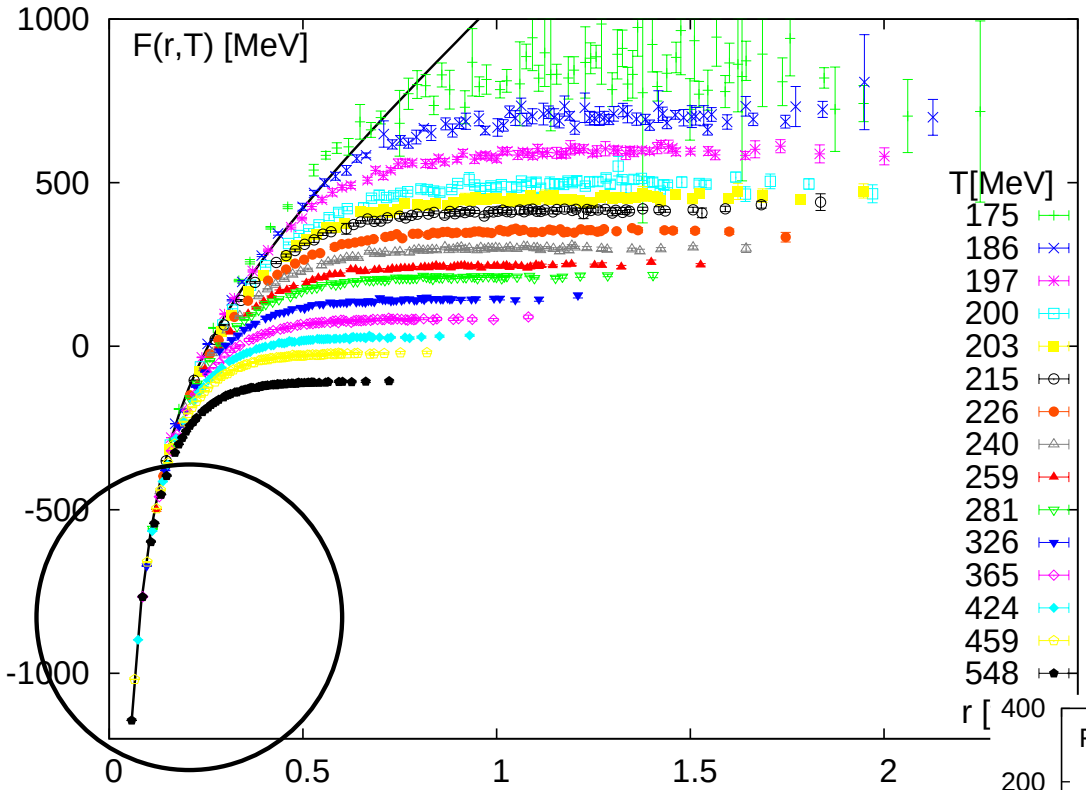
$$e^{-F_1(r,T)/T} = \left(Z_r(g^2)\right)^{2N_\tau} \langle \mathrm{Tr}\ (L_x L_y^\dagger) \rangle$$

alternative renormalization procedures

(PLB543(2002)41, PoS(Lattice2007)195)

all equivalent

Heavy quark free energy - 2+1-flavors



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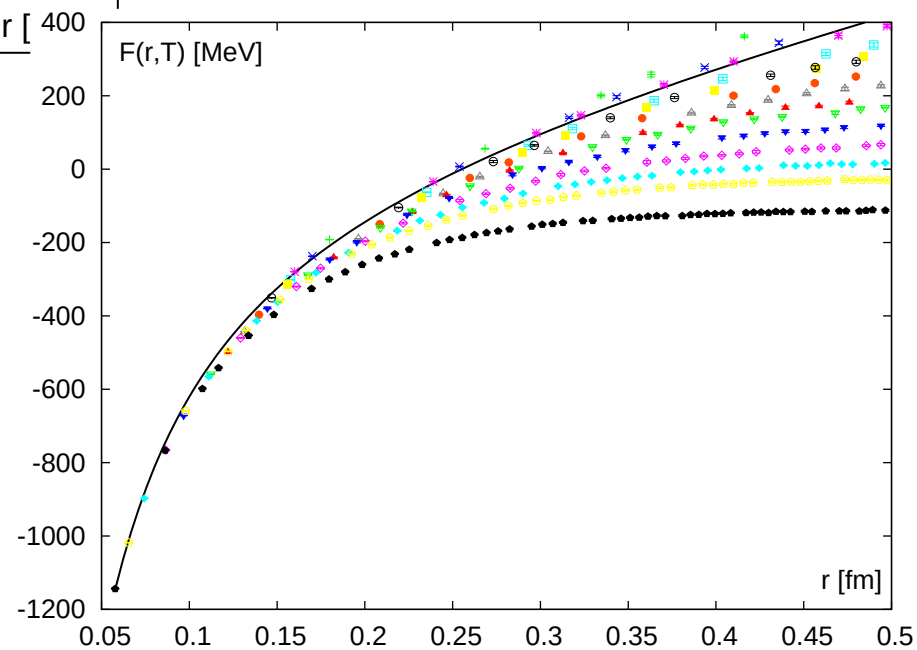
all equivalent

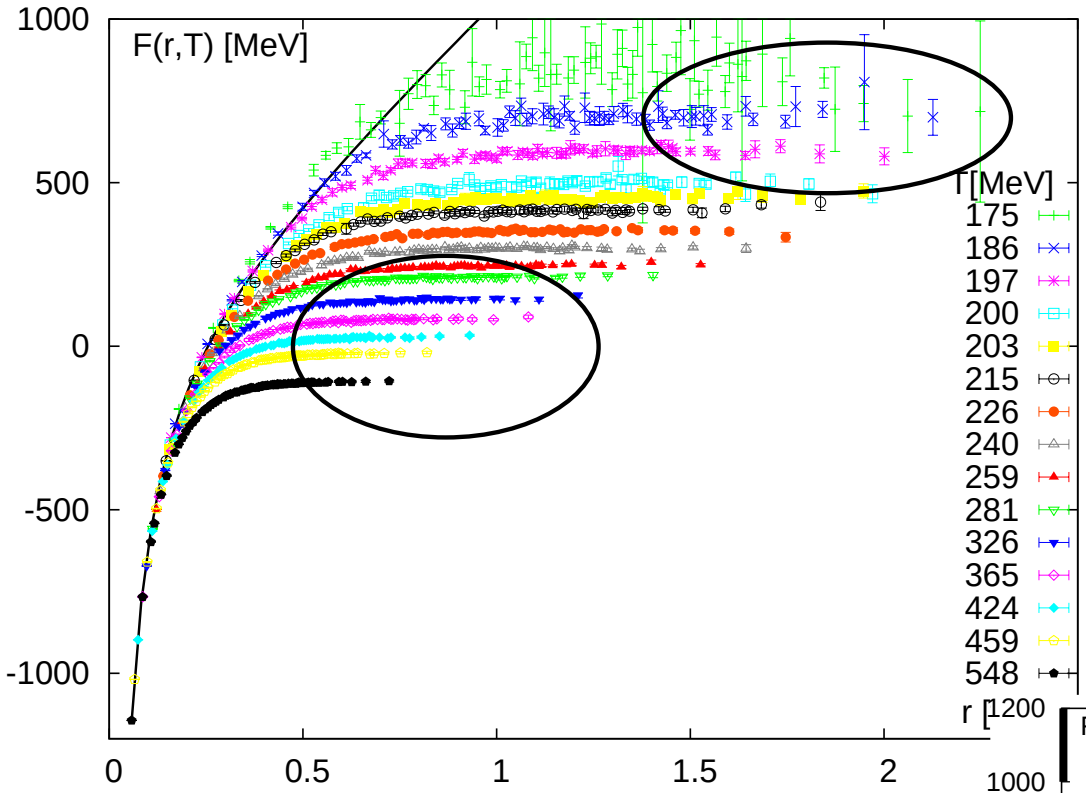
T-independent

$r \ll 1/\sqrt{\sigma}$

$F(r,T) \sim g^2(r)/r$



Heavy quark free energy - 2+1-flavors



String breaking

$T < T_c$

$F(r\sqrt{\sigma} \gg 1, T) < \infty$

high-T physics

$rT \gg 1$; screening

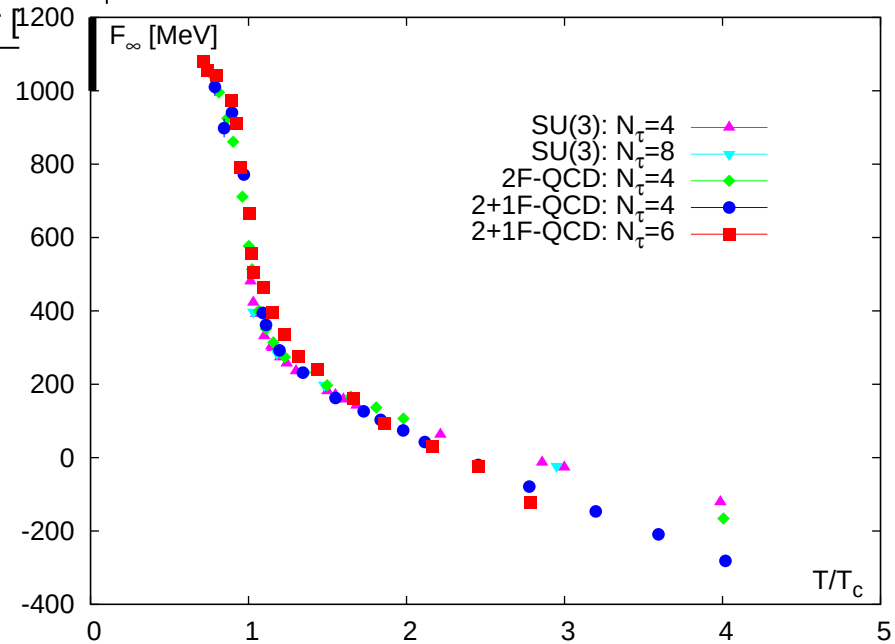
$\mu(T) \sim g(T)T$

$F(\infty, T) \sim -T$

T-independent

$r \ll 1/\sqrt{\sigma}$

$F(r,T) \sim g^2(r)/r$



Renormalized Polyakov loop

Using short distance behaviour of free energies

Renormalization of $F(r,T)$ at short distances

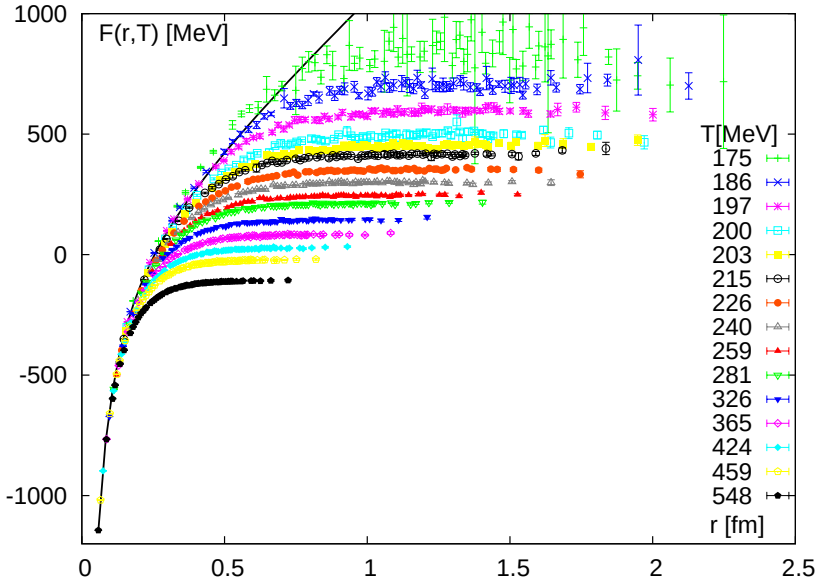
$$e^{-F_1(r,T)/T} = \left(Z_r(g^2)\right)^{2N_\tau} \langle \mathrm{Tr}\ (L_x L_y^\dagger) \rangle$$

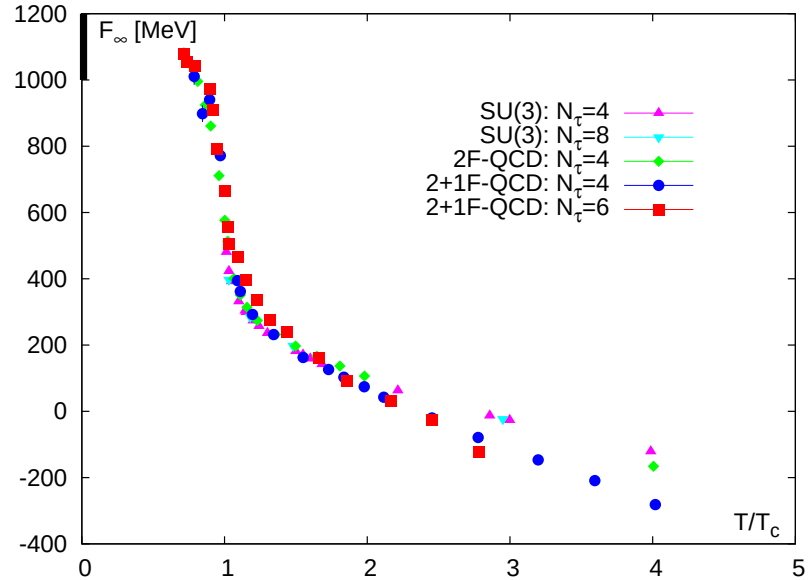
Renormalization of the Polyakov loop

$$L_{ren} = \left(Z_R(g^2)\right)^{N_t} L_{lattice}$$

L_{ren} defined by long distance behaviour of $F(r,T)$

$$L_{ren} = \exp\left(-\frac{F(r=\infty,T)}{2T}\right)$$





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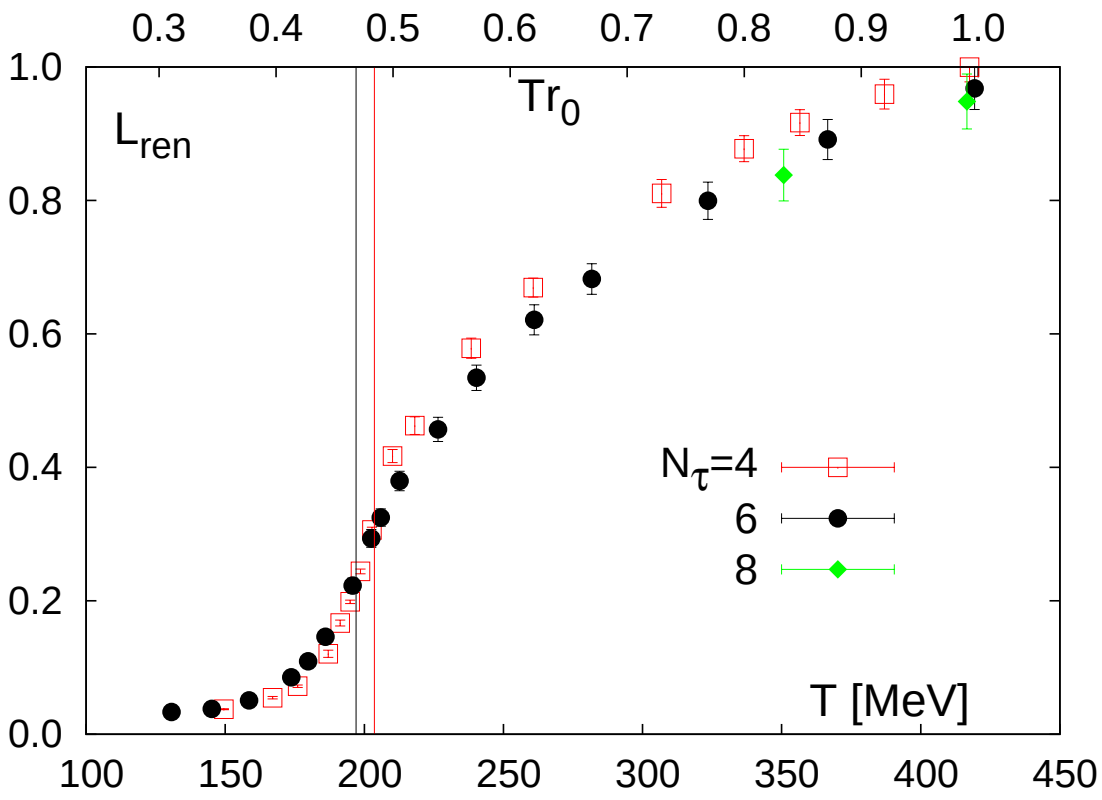
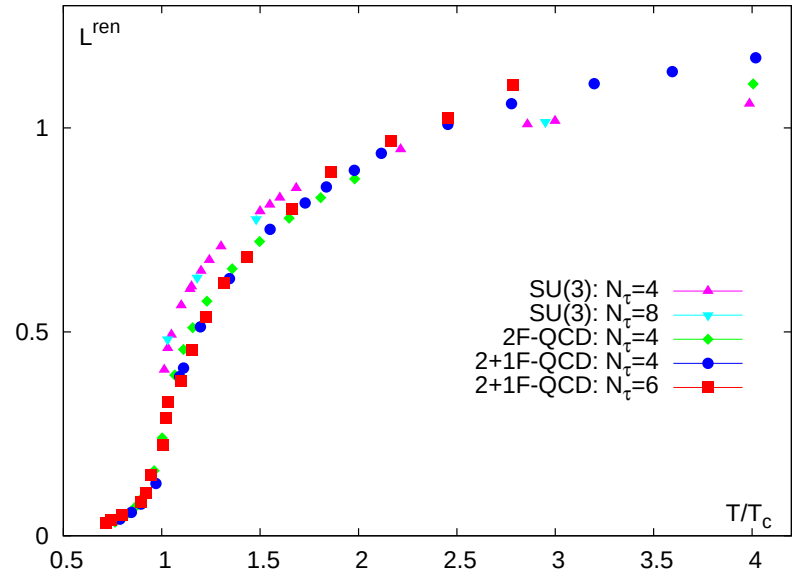
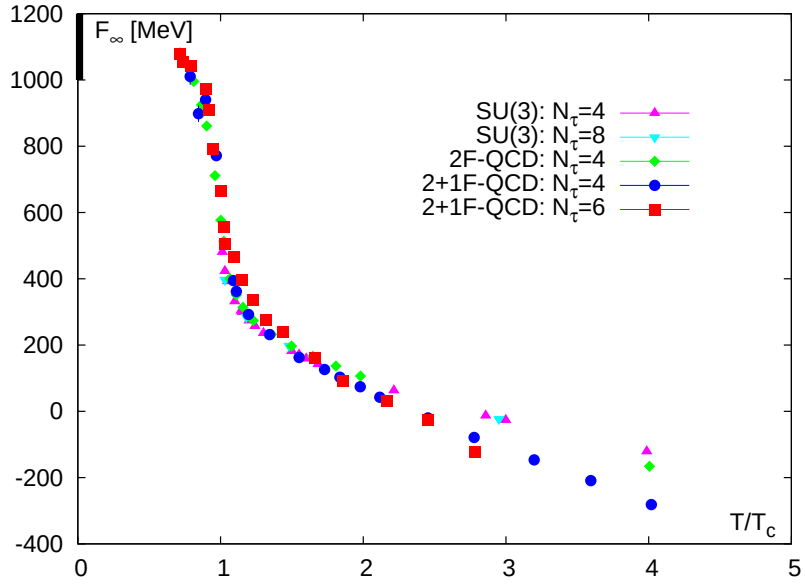
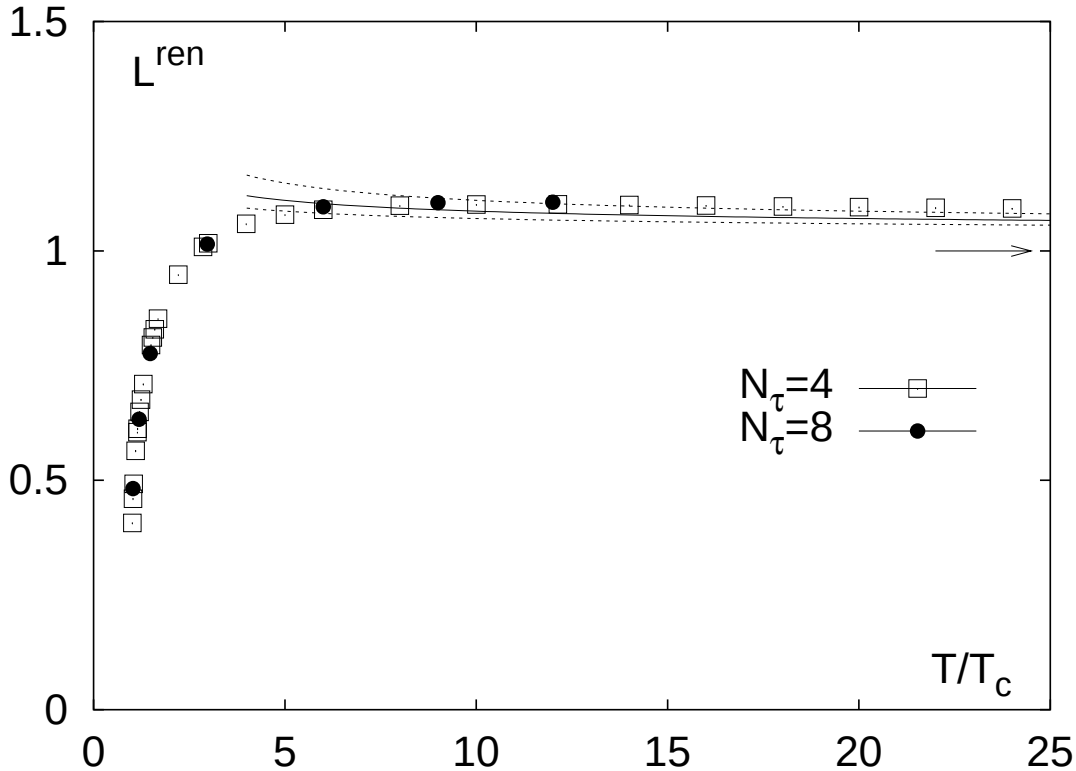






$$L_{ren} = \exp\left(-\frac{F(r=\infty,T)}{2T}\right)$$

SU(3) pure gauge results



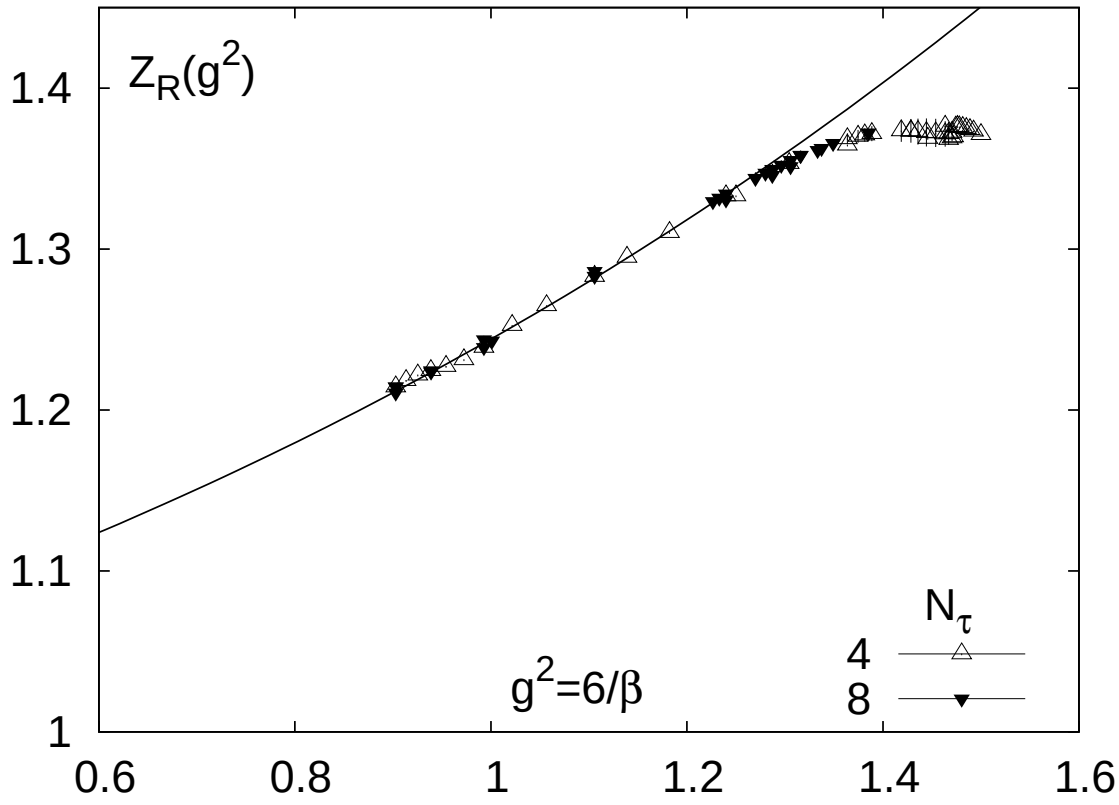
High temperature limit, $L^{ren} = 1$,
reached from above as expected from PT

Clearly non-perturbative effects below $5T_c$

$$L_{ren} = \exp\left(-\frac{F(r=\infty, T)}{2T}\right)$$

Renormalization constants

Renormalization constants obtained from heavy quark free energies

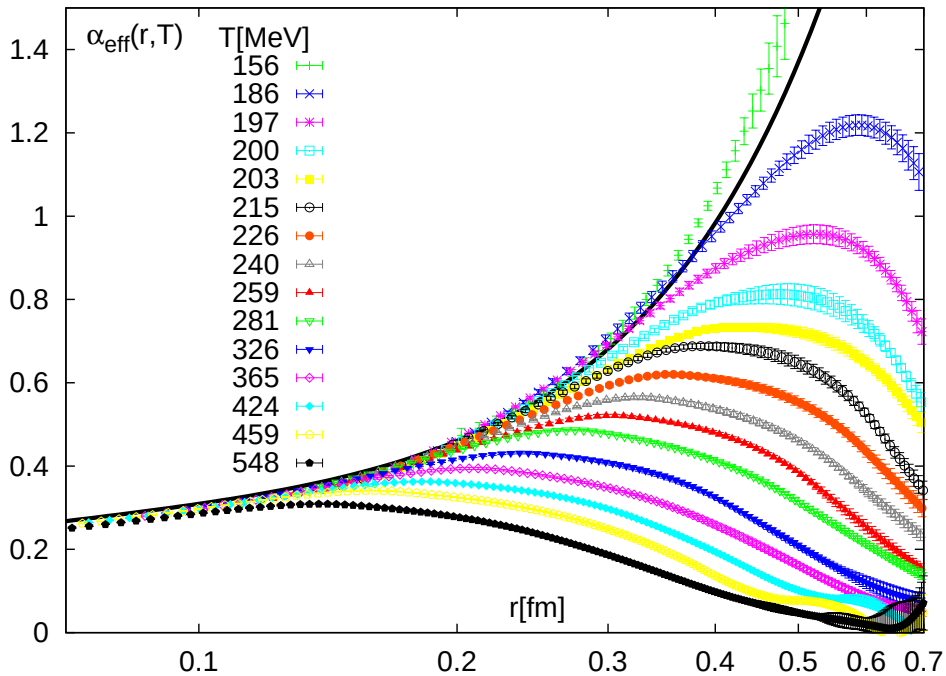


The renormalization constants depend on the bare coupling, i.e. $Z_R(g^2)$

$$Z_R(g^2) \simeq \exp\left(g^2(N^2-1)/NQ^{(2)} + g^4 Q^{(4)} + O(g^6)\right)$$

with $Q^{(2)} = 0.0597(13)$ consistent with lattice perturbation theory (Heller + Karsch, 1985)

Temperature depending running coupling



Free energy in perturbation theory:

$$F_1(r,T) \equiv V(r) \simeq -\frac{4}{3}\frac{\alpha(r)}{r} \quad \text{for} \quad r\Lambda_{\text{QCD}} \ll 1$$

$$F_1(r,T) \simeq -\frac{4}{3}\frac{\alpha(T)}{r}e^{-m_D(T)r} \quad \text{for} \quad rT \gg 1$$

QCD running coupling in the qq-scheme

$$\alpha_{qq}(r,T) = \frac{3}{4}r^2\frac{dF_1(r,T)}{dr}$$

non-perturbative confining part for $r \gtrsim 0.4$ fm

$$\alpha_{qq}(r) \simeq 3/4r^2\sigma$$

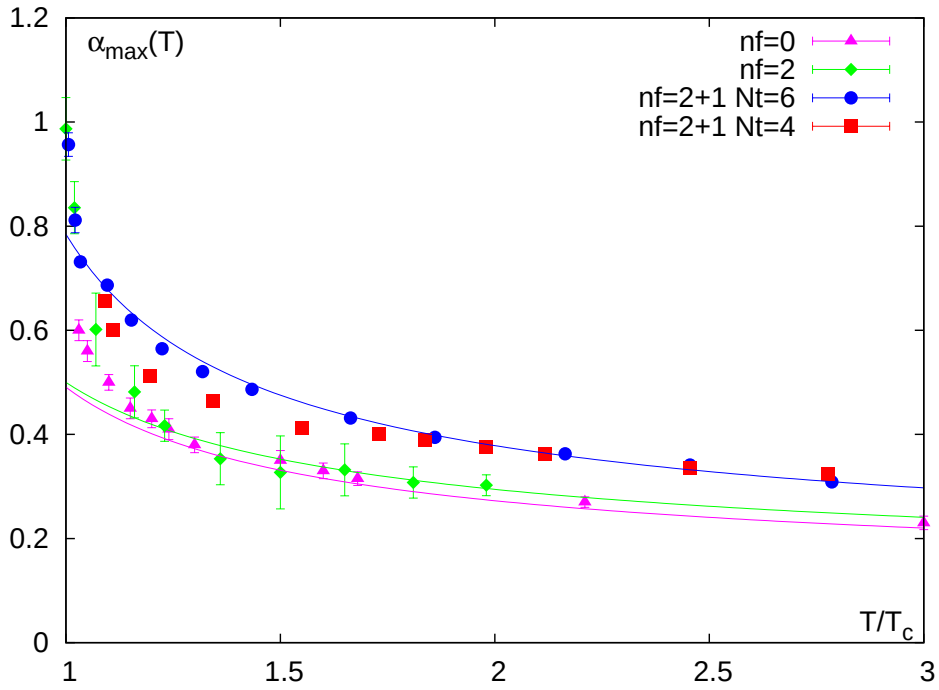
present below and just above T_c $\Longrightarrow$ At which distance do T-effects set in ?

remnants of confinement at $T \gtrsim T_c$ $\Longrightarrow$ definition of the screening radius/mass

temperature effects set in at smaller r with increasing T

maximum due to screening $\Longrightarrow$ definition of the T-dependent coupling

Temperature depending running coupling



define $\tilde{\alpha}_{qq}(T)$ by maximum of $\alpha_{qq}(r,T)$:

$$\tilde{\alpha}_{qq}(T) \equiv \alpha_{qq}(r_{max},T)$$

perturbative behaviour at high T:

$$g^{-2}_{\text{2-loop}}(T) = 2\beta_0 \ln\left(\frac{\mu T}{\Lambda_{\overline{MS}}}\right) + \frac{\beta_1}{\beta_0}\ln\left(2\ln\left(\frac{\mu T}{\Lambda_{\overline{MS}}}\right)\right),$$

non-perturbative large values near T_c

not a large Coulombic coupling

remnants of confinement at $T \gtrsim T_c$

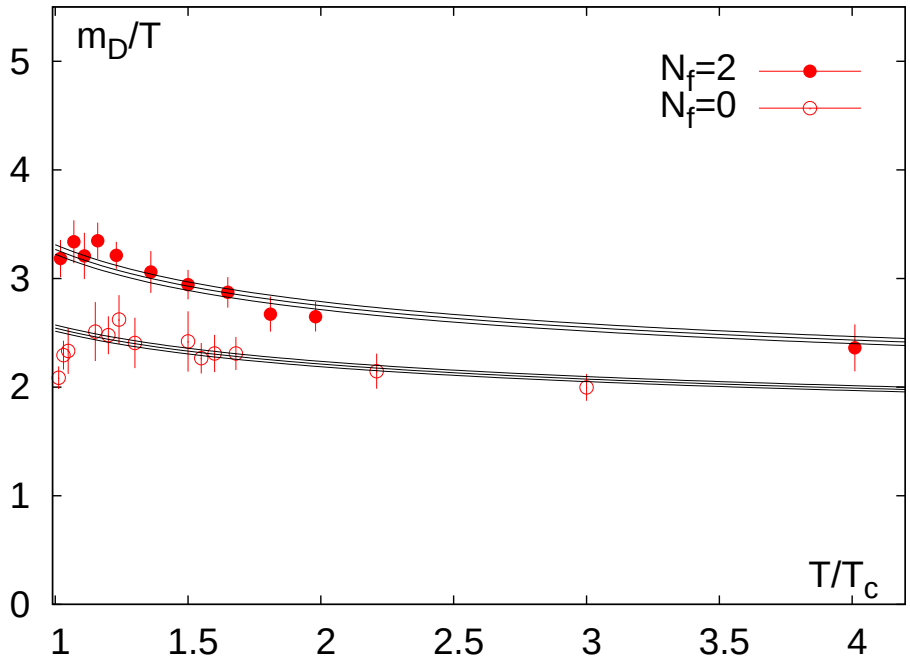
string breaking and screening difficult to separate

slope at high T well described by perturbation theory

$\Longrightarrow$ At which distance do T-effects set in ?

$\Longrightarrow$ calculation of the screening mass/radius

Screening mass - perturbative vs. non-perturbative effects



Screening masses obtained from fits to:

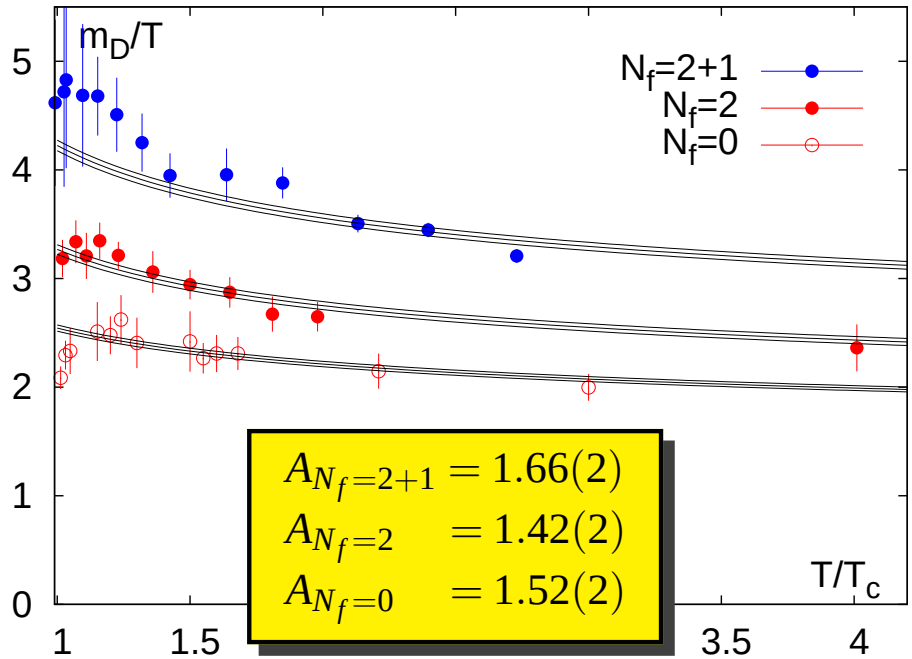
$$F_1(r,T) - F_1(r=\infty,T) = -\frac{4\alpha(T)}{3r} e^{-m_D(T)r}$$

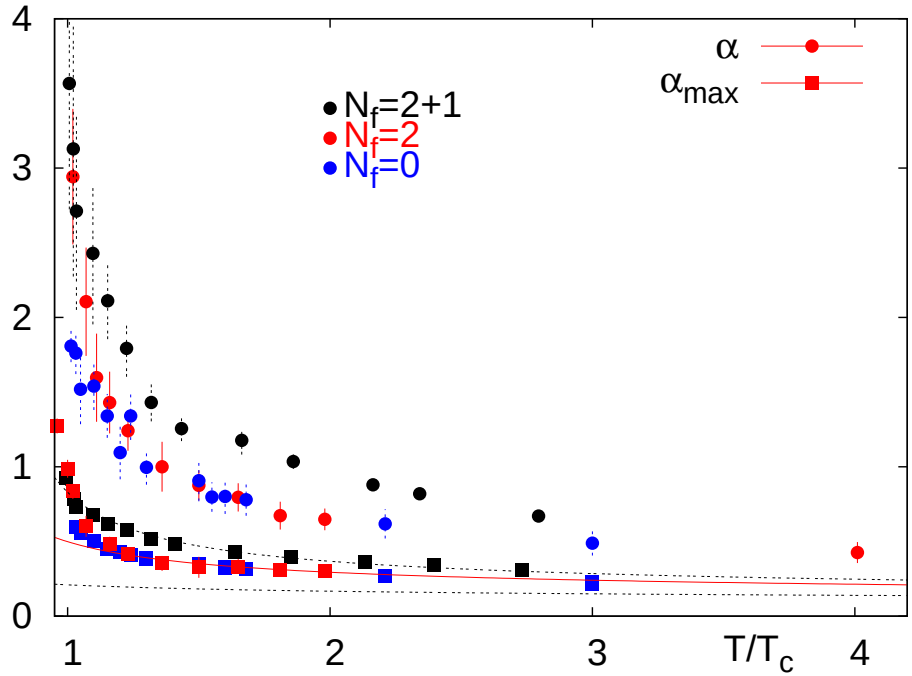
at large distances $rT \gtrsim 1$

leading order perturbation theory:

$$\frac{m_D(T)}{T} = \left(1 + \frac{N_f}{6}\right)^{1/2} g(T)$$

Screening mass - perturbative vs. non-perturbative effects





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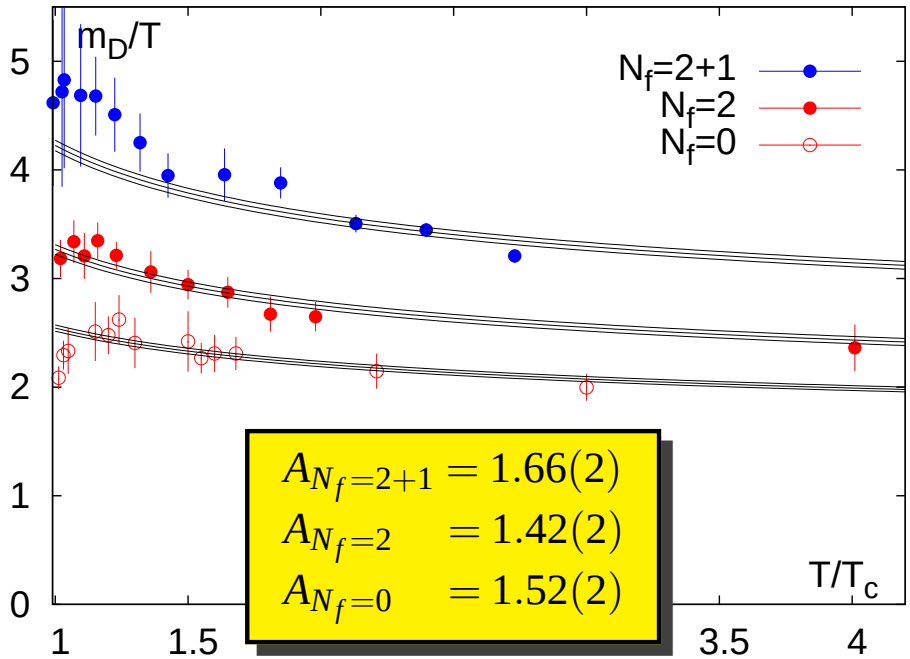
at large distances $rT \gtrsim 1$

leading order perturbation theory:

$$\frac{m_D(T)}{T} = A\left(1+\frac{N_f}{6}\right)^{1/2} g(T)$$

perturbative limit reached very slowly

Screening mass - perturbative vs. non-perturbative effects



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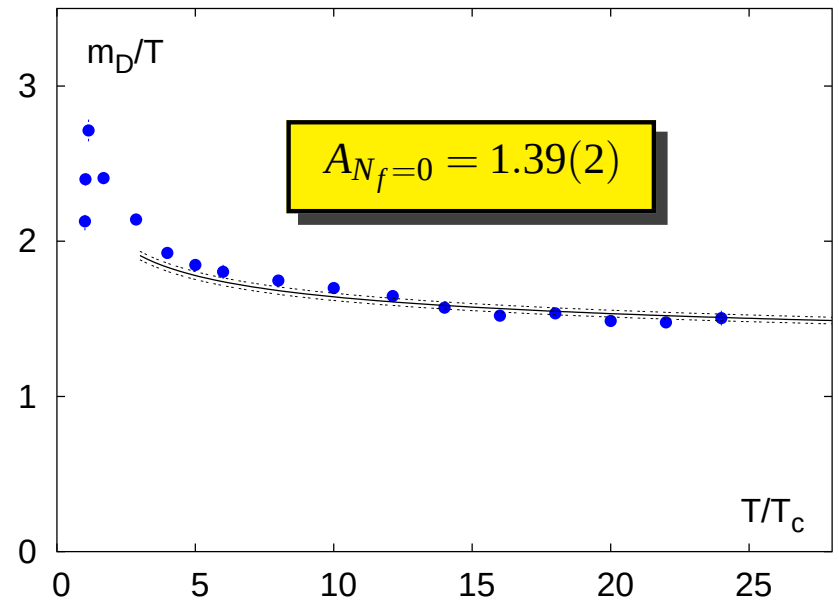


T depencence qualitatively described by perturbation theory

But $A \approx 1.4$ - 1.5 $\Longrightarrow$ non-perturbative effects

$A \to 1$ in the (very) high temperature limit

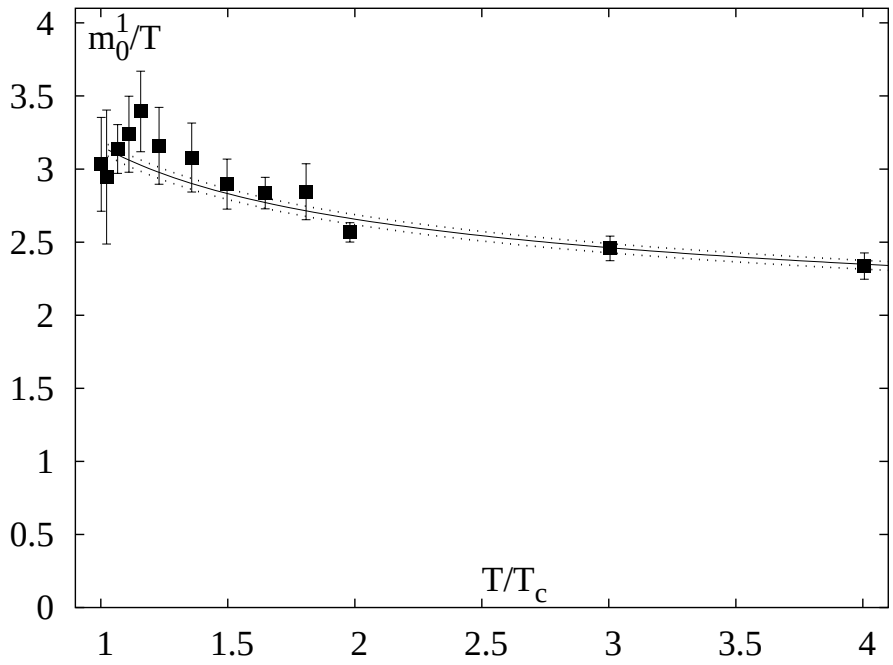
Screening mass - density dependence ($N_f = 2$) [M.Döring et al.]

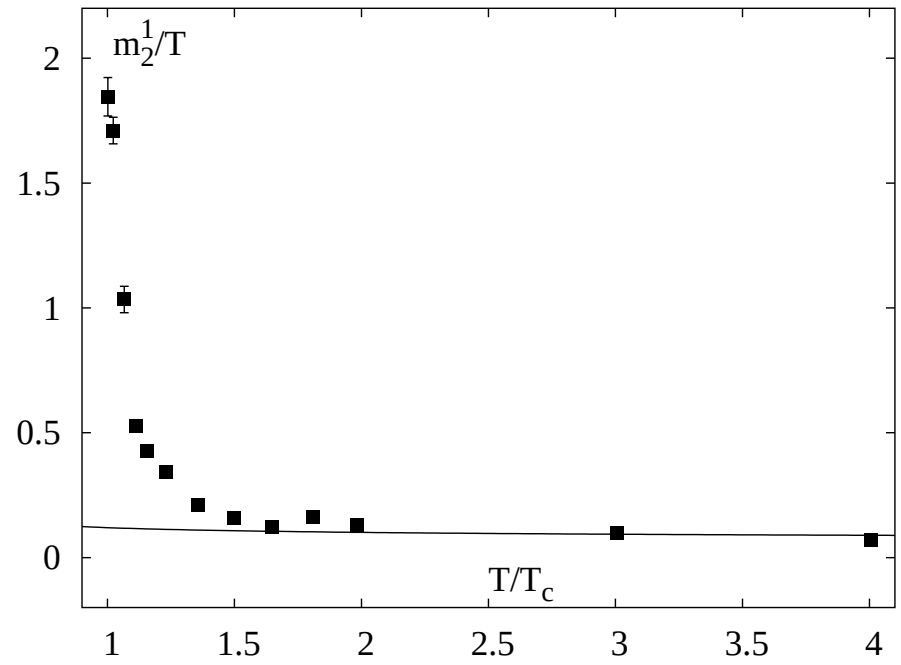
leading order perturbation theory:

$$\frac{m_D(T,\mu_q)}{T} = g(T)\sqrt{1+\frac{N_f}{6}+\frac{N_f}{2\pi^2}\left(\frac{\mu_q}{T}\right)^2}$$

Taylor expansion:

$$m_D(T) = m_0(T) + m_2(T)\left(\frac{\mu_q}{T}\right)^2 + O(\mu_q^4)$$

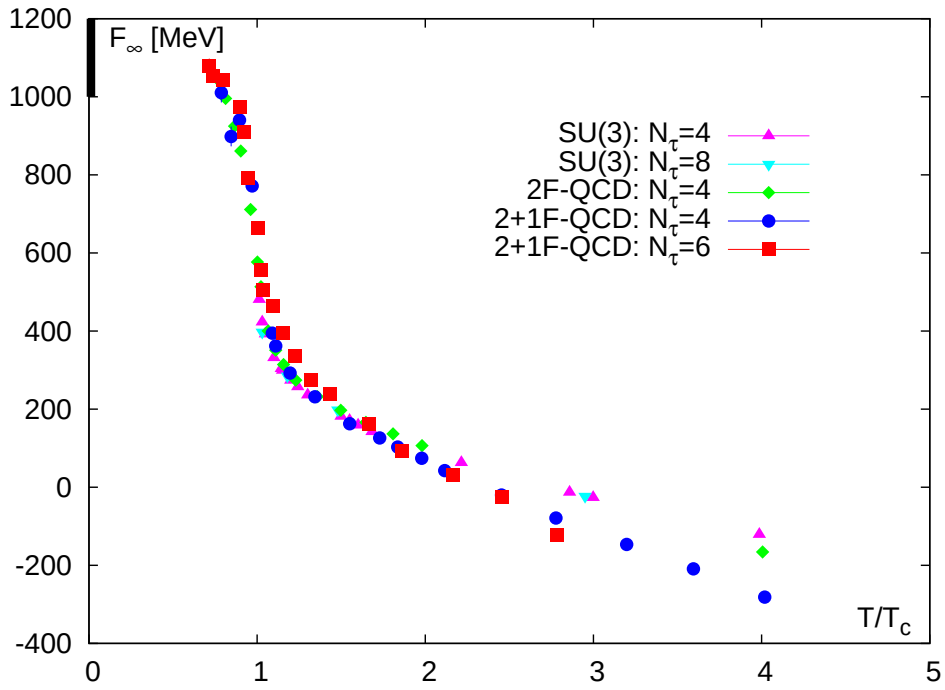




$m_2(T)$ agrees with perturbation theory for $T \gtrsim 1.5T_c$

non-perturbative effects dominated by gluonic sector

Free energy vs. Entropy at large separations



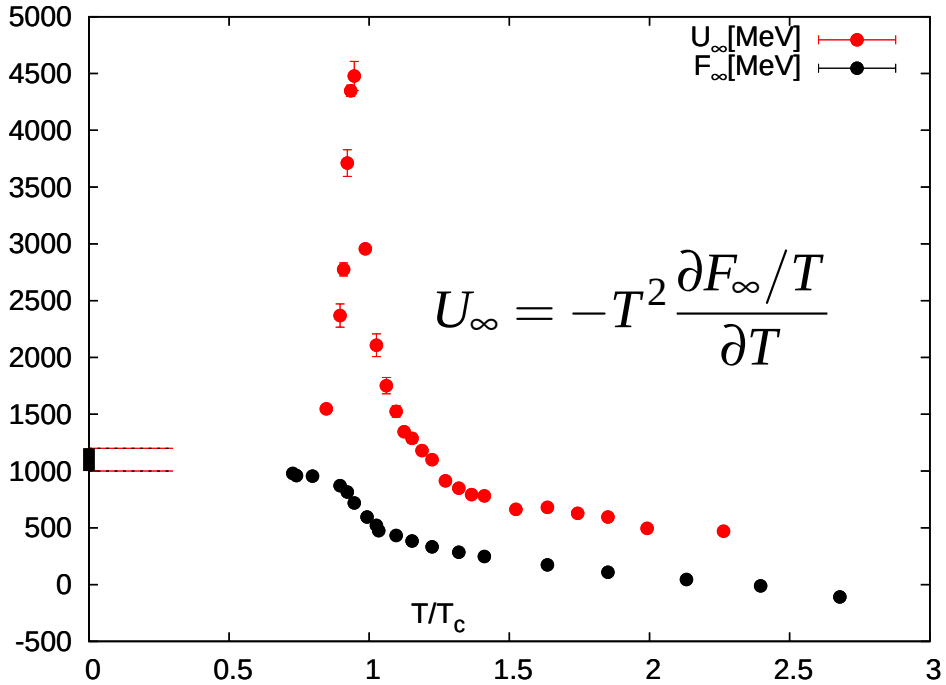
Free energies not only determined by potential energy

$$F_\infty = U_\infty - T S_\infty$$

Entropy contributions play a role at finite T

$$S_\infty = -\frac{\partial F_\infty}{\partial T}$$

Free energy vs. Entropy at large separations

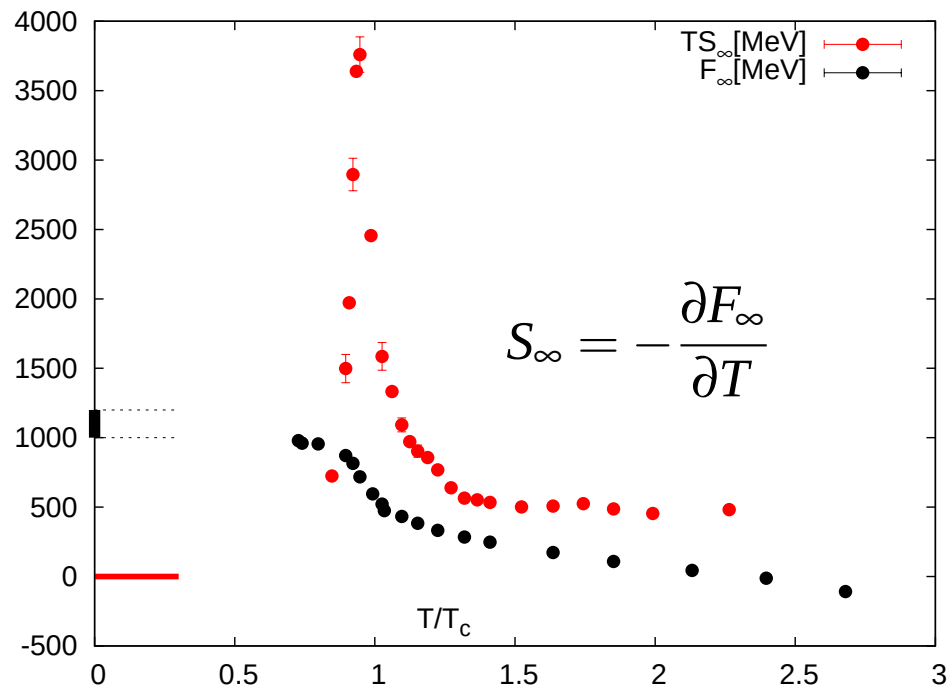


High temperatures:

$$\mathbf{F_\infty(T)} \simeq -\frac{4}{3}\mathbf{m_D(T)\alpha(T)} \simeq -o(g^3T)$$

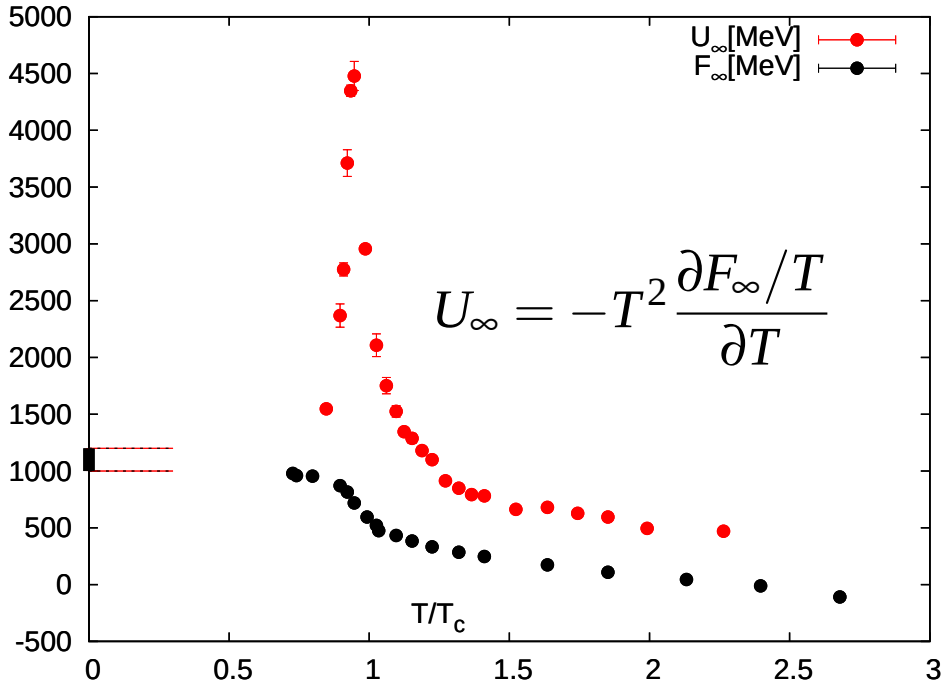
$$\mathbf{TS_\infty(T)} \simeq +\frac{4}{3}\mathbf{m_D(T)\alpha(T)}$$

$$\mathbf{U_\infty(T)} \simeq -\mathbf{4m_D(T)\alpha(T)}\frac{\beta(\mathbf{g})}{\mathbf{g}} \simeq -o(g^5T)$$



The large distance behavior of the finite temperature energies is rather related to screening than to t he temperature dependence of masses of corresponding heavy-light mesons!

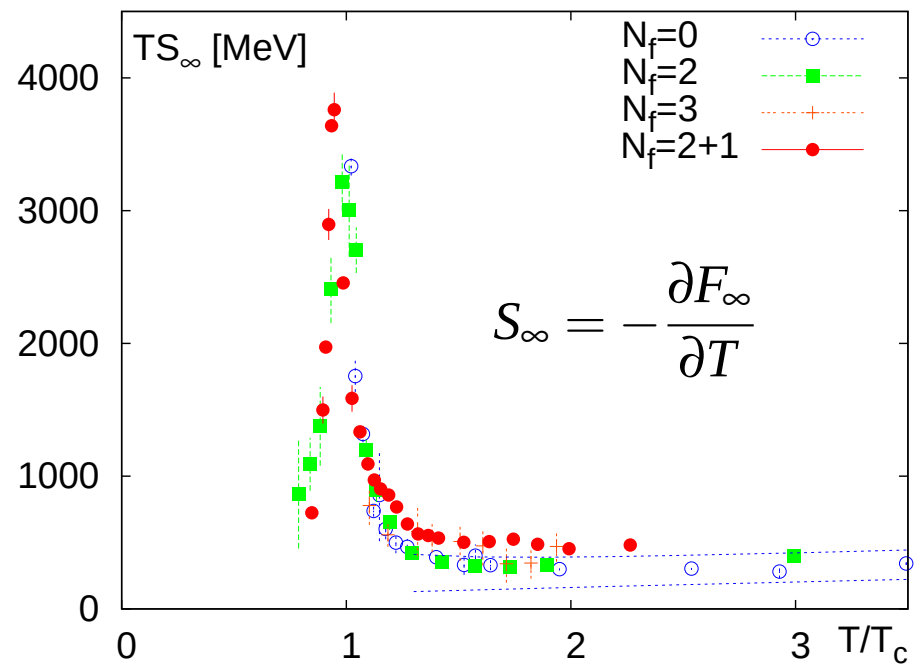
Free energy vs. Entropy at large separations



$$U_\infty = -T^2 \frac{\partial F_\infty / T}{\partial T}$$

High temperatures:

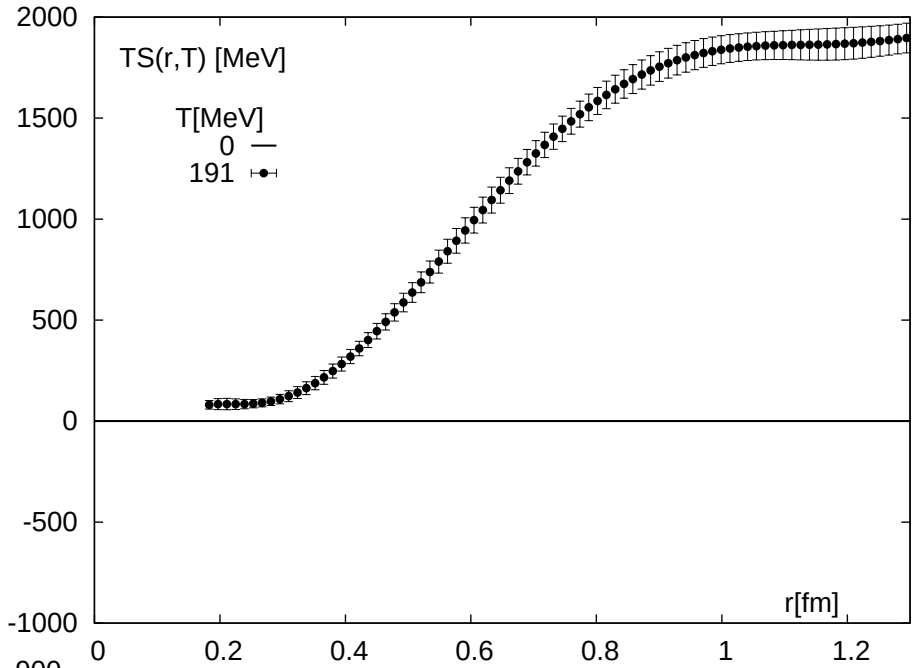
$$\begin{aligned}
\mathbf{F_\infty(T)} &\simeq -\frac{\mathbf{4}}{\mathbf{3}}\mathbf{m_D(T)}\alpha\mathbf{(T)} \simeq -o(g^3T) \\
\mathbf{TS_\infty(T)} &\simeq +\frac{\mathbf{4}}{\mathbf{3}}\mathbf{m_D(T)}\alpha\mathbf{(T)} \\
\mathbf{U_\infty(T)} &\simeq -\mathbf{4m_D(T)}\alpha\mathbf{(T)}\frac{\beta\mathbf{(g)}}{\mathbf{g}} \\
&\simeq -o(g^5T)
\end{aligned}$$

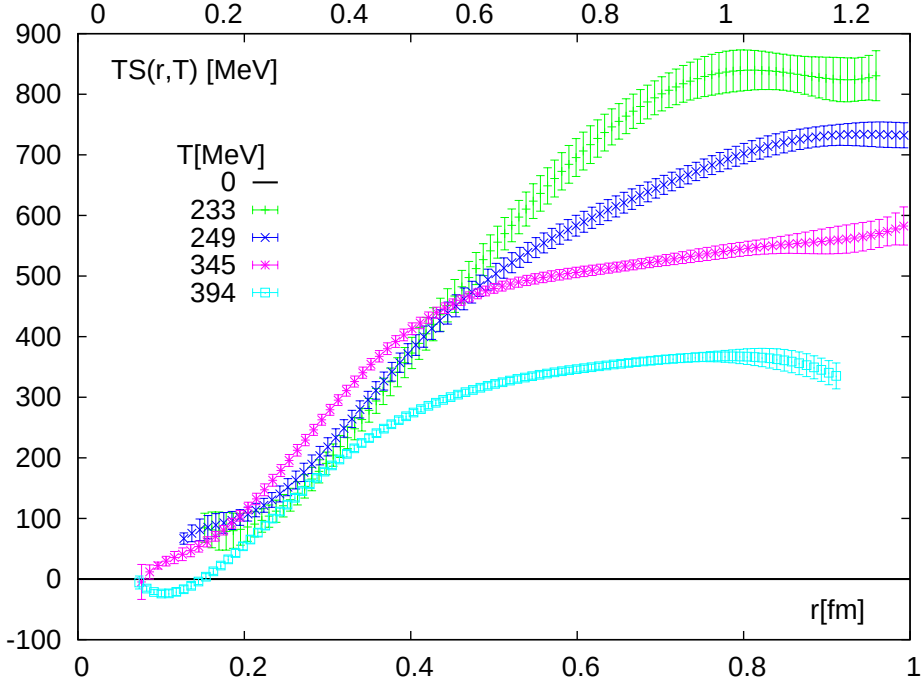


$$S_\infty = -\frac{\partial F_\infty}{\partial T}$$

The large distance behavior of the finite temperature energies is rather related to screening than to t he temperature dependence of masses of corresponding heavy-light mesons!

r-dependence of internal energies ($N_f = 2+1$)





$$F_1(r,T) = U_1(r,T) - TS_1(r,T)$$

$$S_1(r,T) = \frac{\partial F_1(r,T)}{\partial T}$$

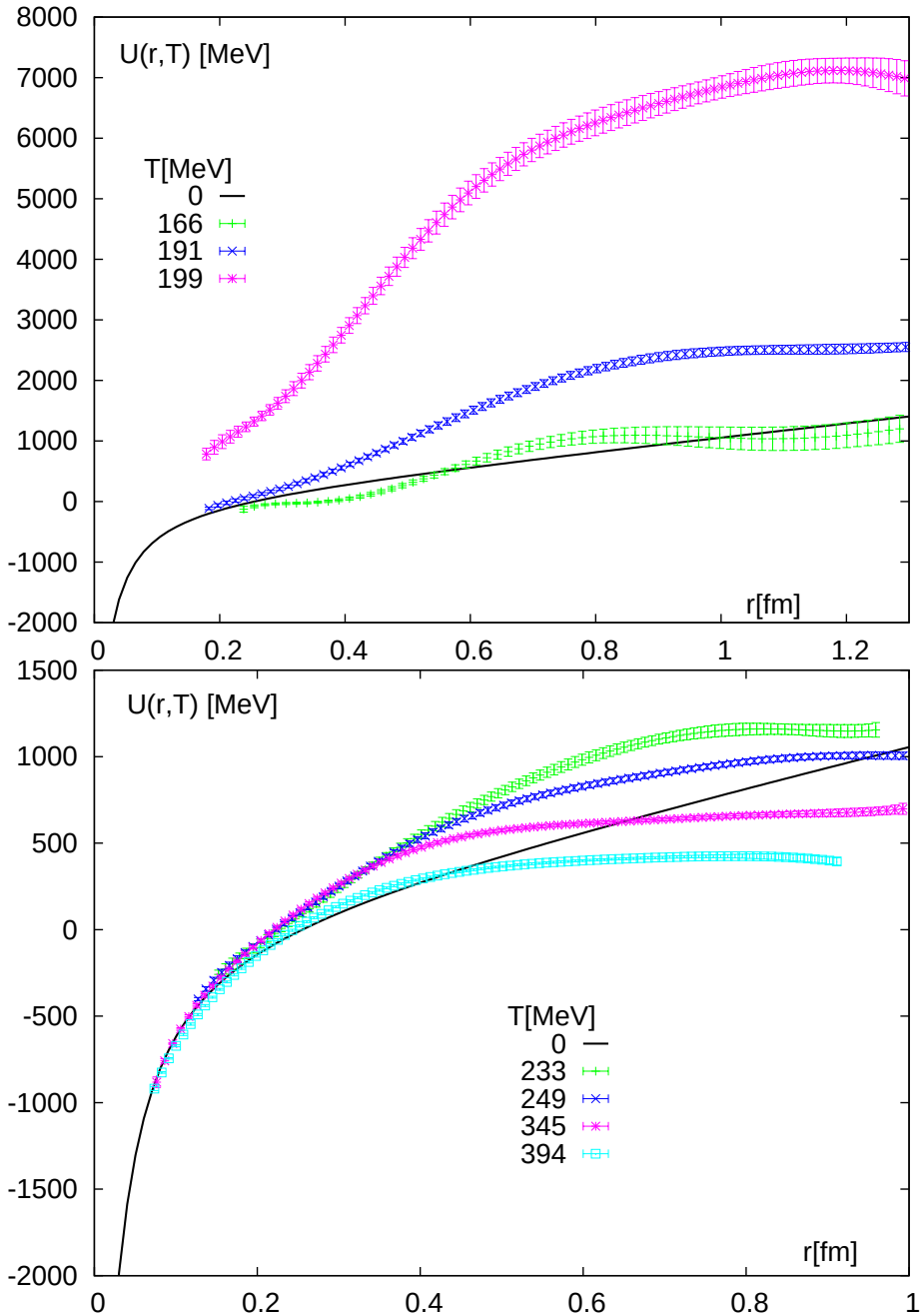
$$U_1(r,T) = -T^2 \frac{\partial F_1(r,T)/T}{\partial T}$$

Entropy contributions vanish in the limit $r \to 0$

$$F_1(r \ll 1, T) = U_1(r \ll 1, T) \equiv V_1(r)$$

important at intermediate/large distances

r-dependence of internal energies ($N_f = 2+1$)



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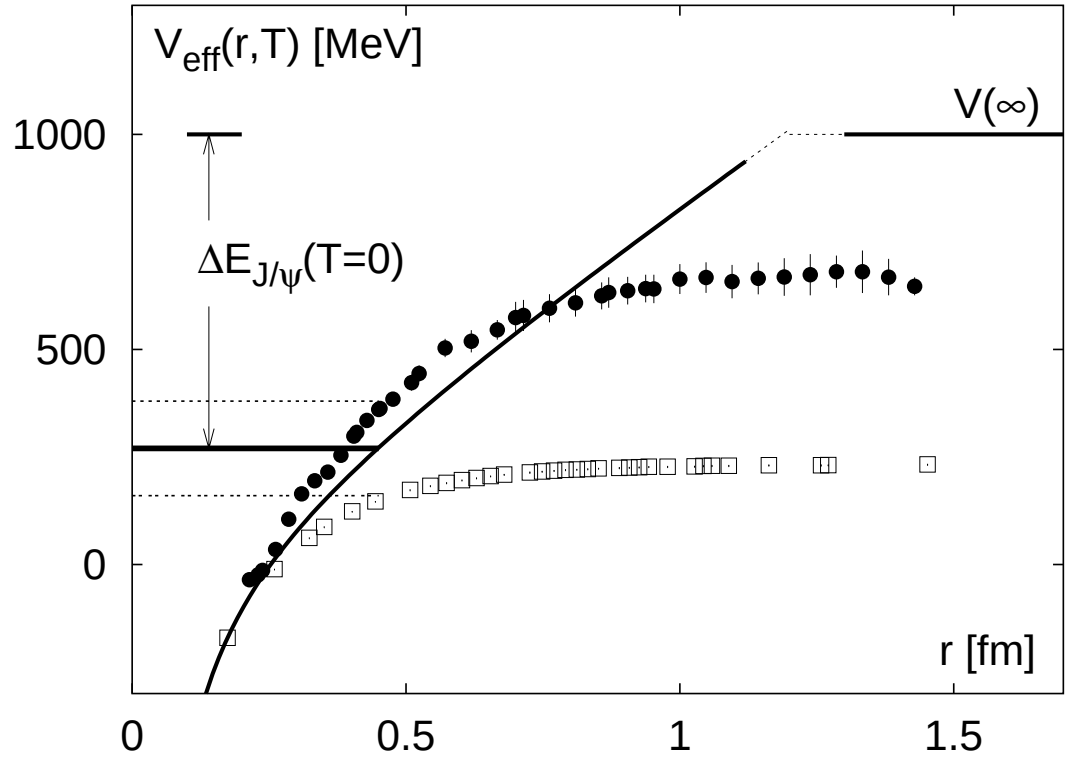
important at intermediate/large distances

$\Longrightarrow$ Implications on heavy quark bound states?

$\Longrightarrow$ What is the correct $V_{eff}(r,T)$?

Heavy quark bound states from potential models



steeper slope of $V_{eff}(r,T) = U_1(r,T)$

$\Longrightarrow$ J/ψ stronger bound using $V_{eff} = U_1(r,T)$

$\Longrightarrow$ dissociation at higher temperatures compared to $V_{eff}(r,T) = F_1(r,T)$

Conclusions

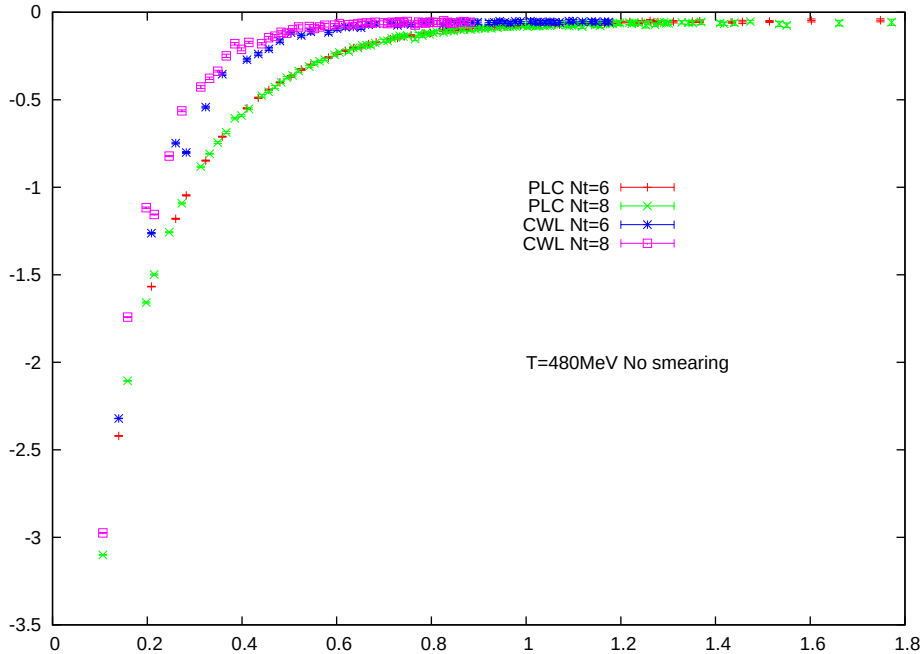
Heavy quark free energies, internal energies and entropy

- Results for almost physical quark masses, $n_f = 2+1$
- Complex r and T dependence
- Running coupling shows remnants of confinenement above T_c
- Entropy contributions play a role at finite T
- Non-perturbative effects in m_D up to high T
- Non-perturbative effects dominated by gluonic sector

Bound states in the quark gluon plasma

- Estimates from potential models?
- What is the correct potential for such models?
- Higher dissociation temperature using V_1
- (directly produced) J/ψ may exist well above T_c
- Full QCD calculations of correlation/spectral functions needed
- What are relevant processes for charmonium?

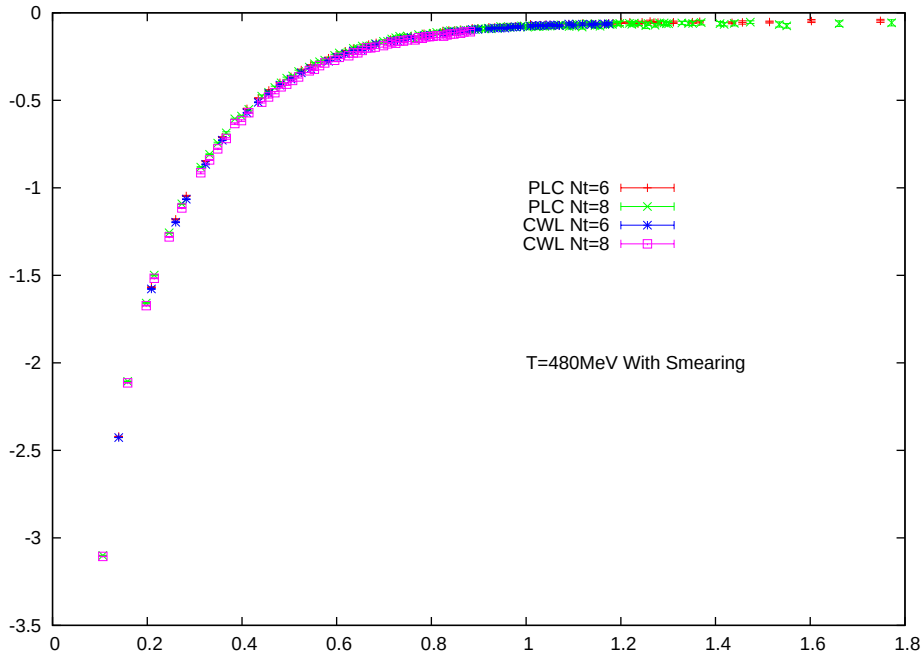
Polyakov loop correlators vs. cyclic Wilson loops



$$\frac{F_1(r,T)}{T} = -\ln\left(\langle \tilde{\mathrm{Tr}} L(\mathbf{x}) L^\dagger(\mathbf{y})\rangle\right)\Big|_{GF}$$

Gauge fixing needed, Polyakov loop correlator

- Coulomb gauge
- positive transfer matrix
- equivalent to gauge independent definition



$$\frac{F_1(r,T)}{T} = -\ln\left(\langle \tilde{\mathrm{Tr}} L(\mathbf{x}) U(\mathbf{x},\mathbf{y}) \mathbf{L}^\dagger(\mathbf{y}) \mathbf{U}^\dagger(\mathbf{x},\mathbf{y})\rangle\right)$$

Gauge independent definition, Cyclic Wilson loops

- Path dependence
- Smearing of spatial links