Things that confuse/excite me about hot/dense QCD

The $U(1)_A$ anomaly:

Where does it show up in the mass spectrum in vacuum?

Does it get small by $T\chi (\mu = 0)$?

Lattice & $\mu \neq 0$: tensor networks

A critical endpoint: is that all there is?

Lifshitz regime (not point)

Quarkyonic:

Taking the shell seriously
What happens with the anomaly at T=0: is it just the $\eta'$?

Standard lore: the U(1)$_A$ anomaly only matters for the $\eta'$

Dudek, Edwards, Guo & Thomas, 1309.2609, $m_\pi \sim 400$ MeV

”...little mixing...in most JPC channels...except the $\eta$ and $\eta'$”

Koenigstein & Giacosa 1608.08777: $\eta_2(1645)$ & $\eta_2(1870)$ mix like $\eta$ & $\eta'$, $\theta \sim -42^\circ$

$\eta_2(1645) \rightarrow a_2(1320) \pi$, , $\eta_2(1870) \rightarrow \eta + 2\pi$

Koenigstein, Giacosa & rdp: 1709.07454:

Infinite number of heterochiral mesons:

$\Phi_{\mu\nu}... = \bar{q}_L \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu \ldots q_R$

Like the $\eta$ and $\eta'$: anomaly gives determinental interactions

Homochiral mesons:

Like the $\rho$, $\omega$, & $\phi$:

$\chi^L_{\mu\nu}... = \bar{q}_L \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu \ldots q_L$

anomaly only matters through Wess-Zumino-Witten terms.
What happens with the anomaly at $T \neq 0$?

Old song: determinental term breaks $U(1)_A$, not $SU(3)_L \times SU(3)_R$,

$$\mathcal{V}_\Phi = m^2 \text{tr}\Phi^\dagger\Phi - \kappa (\text{det}\Phi + \text{c.c.}) + \lambda \text{tr}(\Phi^\dagger\Phi)^2 + ...$$

For three flavors, det $\Phi$ is cubic, must have 1st order $\chi$ transition. *Must.*

The coefficient of det $\Phi$ is big: the $\eta'$ is heavy.

Consider 3-flavor symmetric case.

For sufficiently light pions, there must be a 1st order $\chi$ transition if the coefficient of det $\Phi$ remains large. *If.*
Lattice: anomaly large at $T\chi$

Bhattacharya et al, 1402.5175: Domain wall q’s, $U(1)_A$ violating susceptibilities:

$T\chi \sim 150 \text{ MeV} \ll T_{U(1)_A} \sim 200$. \hspace{1cm} $\chi_\pi - \chi_{a_0} \sim \int d^3x \langle \pi(x)\pi(0) \rangle - (\pi \to a_0)$

Magnitude of signal?

In $\chi$-matrix model, left is too large by $\sim 10$

rdp & Skokov, 1604.00022
So where is the 1st order transition at small $m_\pi$?

If the anomaly is big at $T_\chi$, easy to see at 1st order $\chi$ trans. for small $m_\pi$.

**Lattice:** Ding, Hegde, Karsch, Lahiri, Li, Mukherjee & Petreczky, 1807.05727:

HISQ q’s. *NO* 1st order $\chi$ trans. for $m_\pi > 80$ MeV. *Is this consistent?*


Due to quark fluctuations, $m_\pi^{\text{crit}} \sim 20$ MeV

Change in gluonic fluctuations? rdp & rennecke, 1812....

**Lattice:** Brandt, Francis, Meyer, Philipsen, Robaina & Wittig, 1608.06882

Wilson fermions, $m_\pi$: 200 $\rightarrow$ 540 MeV: at $T_\chi$, $m_\eta$, - $m_\pi$ $\sim$ 100 MeV!

**Lattice:** Fukaya + JLQCD, 1712.05536. Use Mobious & domain wall q’s

*Strong* $U(1)_A$ suppression at small $m_\pi$ (c DWF: “bad” eigenvalues)
Maybe: the $U(1)_A$ is effectively restored by $T\chi$?

If so, very interesting...

Lattice is the bedrock
Solving the sign problem

With a Hamiltonian formulation, the partition function is real:

$$Z = \text{Tr} e^{-H/T + \mu N}$$

But how does one solve, practically, with a Hamiltonian? Use tensor networks, Matrix Product States, Projected Entangled Pairs States...

Review: Orus, 1306.2164

Thirring model in 1+1 D
Banuls, Cichy, Kao, Lin, Lin & Tan, 1810.12038

Works great in 1+1 D
Competitive in 2+1 D
Efficiency in 3+1 D?

$$|\psi\rangle = \sum_{\sigma_1...\sigma_N} A_{\sigma_1...\sigma_N} |\sigma_1...\sigma_N\rangle$$

$$\sigma_n = 1, ..., d \text{ local dimension}$$

$$i = 1, ..., D \text{ bond dimension}$$

Matrix product state (MPS)
$\Leftrightarrow$ store $d \cdot N$ matrices of dimension up to $D \times D$

Numerical cost:
$$O(D^3)$$
The critical endpoint:

Is that all there is...
Phase diagram for QCD in T & $\mu$: usual picture

Two phases, one Critical End Point (CEP) between crossover and line of 1$^{st}$ order transitions
Ising fixed point, dominated by *massless* fluctuations at CEP
Lifshitz phase diagram for QCD

Possibly: **Unbroken 1st order line** to “chiral spirals” *Could be CEP as well...*

Lifshitz regime: strongly coupled, *large* fluctuations. Like spin liquid...
Lifshitz phase diagram (in mean field theory)

\[ \mathcal{L}_{\text{Lifshitz}} = (\partial_0 \phi)^2 + Z (\partial_i \phi)^2 + \frac{1}{M^2} (\partial_i^2 \phi)^2 + m^2 \phi^2 + \lambda \phi^4 \]

Negative kinetic term, \( Z < 0 \), generates spatially inhomogeneous phase, CS. Three phases.

\[ \langle \phi \rangle \neq 0 \quad \langle \phi \rangle = 0 \]

\[ m^2 \rightarrow \]

\[ X = \text{Lifshitz point}, \ m^2 = Z = 0 \]

\[ \langle \phi \rangle_{CS} \neq 0 \]
No massless modes in too few dimensions

No massless modes in \( d \leq 2 \) dimensions:

\[
\int d^2 k \frac{1}{k^2} \sim \log \Lambda_{\text{IR}}
\]

Cannot break a continuous symmetry in \( d \leq 2 \) dimensions: instead of Goldstone bosons, generate a mass non-perturbatively.

**Lifshitz point:** \( Z = m^2 = 0 \), so propagator just \( \sim 1/k^4 \):

\[
\int d^4 k \frac{1}{k^4} \sim \log \Lambda_{\text{IR}}
\]

Hence *no* Lifshitz point in \( d \leq 4 \) (spatial) dimensions.

*Must* generate either a mass \( m^2 \), or term \( \sim Z \ p^2 \neq 0 \), non-perturbatively
Lifshitz regime

Lifshitz regime (shaded):
\( Z \) and/or \( m^2 \) are \( \neq 0 \) everywhere
strongly coupled, non-perturbative

\[ \langle \phi \rangle \neq 0 \]

\[ \langle \phi \rangle_{CS} \neq 0 \]

\[ Z \uparrow \]

\[ 2^{nd} \rightarrow \]

\[ m^2 \rightarrow \]

Brazovski 1st →
Fluctuations in the Lifshitz regime

Near at critical endpoint,
large fluctuations from a massless sigma:
Affects $\pi$’s more than $K$’s

In the Lifshitz regime,
large fluctuations from a massive field with higher derivatives:

$$\Delta_{\sigma} = \frac{1}{p^2}$$

$$\Delta_{\pi} = \frac{1}{(p^2)^2 + m_{\pi}^2}$$

Not tied to chiral symmetry, can affect both $K$’s as well as $\pi$’s

Exp’y, need to measure fluctuations not just in (net) protons, but $\pi$’s & $K$’s
Fluctuations at 7 GeV

Beam Energy Scan, down to 7 GeV.
Fluctuations *MUCH* larger when up to 2 GeV than to 0.8 GeV
Trivial multiplicity scaling? ... or Chiral Spiral?
But fluctuations in net protons, not pions.

X. Luo & N. Xu, 1701.02105, fig. 37; Jowazee, 1708.03364
Do frozen dense quarks ever deconfine?

McLerran & rdp, 0706.2191.
Cold, dense quarks are “Quarkyonic” at large $N_c$:
If quark $\mu >> \Lambda_{QCD}$, consider excitations within $\sim \Lambda_{QCD}$ of the edge of the Fermi surface.

Quarks which move radially have $\sim$ zero energy and interact with (unscreened) static magnetic gluons: confined into baryons.

McLerran & Reddy 1812....: gives very “stiff” equation of state for neutron stars.

At $N_c = 3$, $\mu_{qk} \sim 100 \text{ GeV}$: aren’t quarks near the Fermi surface still confined?
“The Great Wave” of High Energy Heavy Ion Physics

“The Great Wave off Kanagawa”, by Hokusai
...WITH GHOSTS FROM THE PAST

I always thought Italy was screwed-up for voting Berlusconi

→ then the US made me appreciate my country again!
→ but it also feels like ghosts from past are following me (SAD)

- nuclear codes
- corruption
- friend with Putin
- tax evasion, fraud
- 50% racist
- mafia

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