"Quarkyonic" phases at large N_c & a deconfining CEP

1. Large N_c, small N_f:

   Quark-yonic matter - quark Fermi sea *plus* bar-yonic Fermi surface

   Deconfining Critical End Point (CEP) at *large* $\mu_{qk} \sim N_c^{1/2}$

2. "Purely pionic" effective Lagrangians and nuclear matter:

   The unbearable lightness of being (nuclear matter)?

3. Chiral symmetry and Skyrmions

4. Chiral density waves in quarkyonic matter

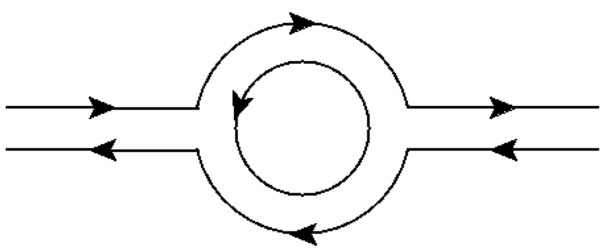
5. Large N_c, small N_f: exponentially many baryons

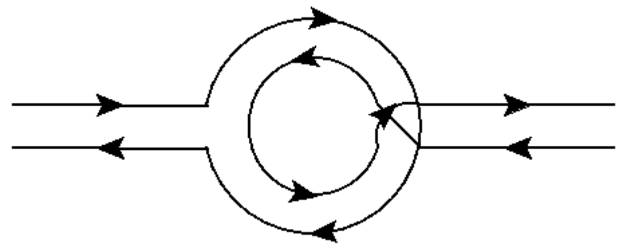
McLerran & RDP, 0706.2191. Hidaka, McLerran, & RDP 0803.0279
McLerran, Redlich & Sasaki 0812.3585
Hidaka, Kojo, McLerran, & RDP 09......
Blaizot, Nowak, McLerran & RDP 09......

Towards the phase diagram at $N_c = \infty$ $N_f =$

‘t Hooft ’74: let # colors, $N_c \rightarrow \infty$, with $\lambda = g^2 N_c$ fixed. Take # flavors N_f *finite*.

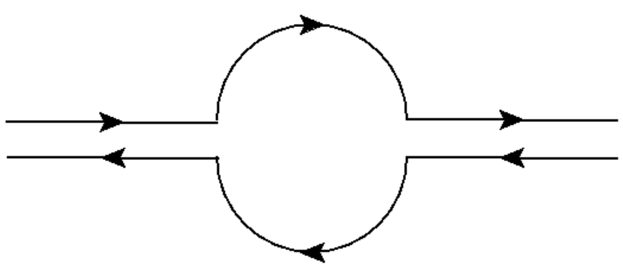
Gluons in gluon self energy at 1 loop, for *any* N_c: (Hidaka & RDP 0906.1751)





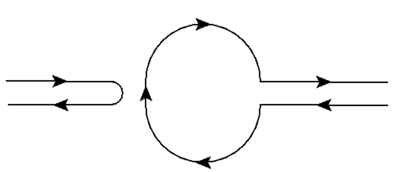
$$\sim g^2 N_c \sim \lambda$$

Quarks in gluon self energy at 1 loop, for any N_c:

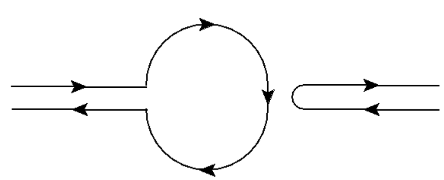


$$\sim g^2 N_f \sim \frac{N_f}{N_c}\,\lambda$$

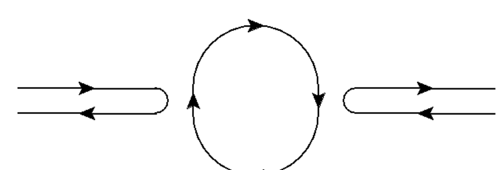
$-\frac{1}{N}$



$-\frac{1}{N}$



$+\frac{1}{N^2}$



If $N_f/N_c \rightarrow 0$ as $N_c \rightarrow \infty$, loops *dominated* by gluons, *blind* to quarks.

Phase diagram at $N_c = \infty$, I

Deconfining temperature T_d independent of $\mu \sim 1$!

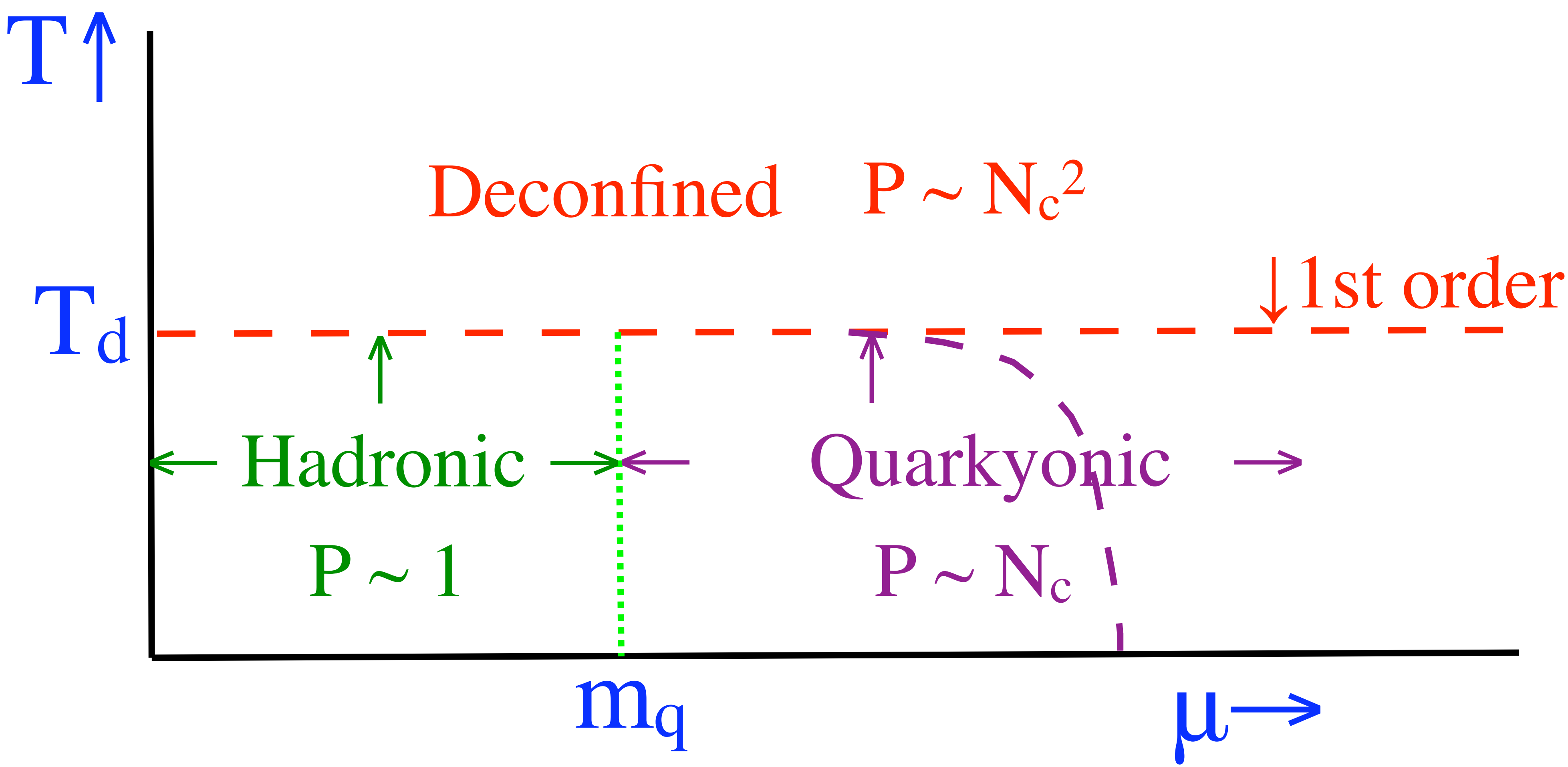
Transition 1st order for large N: usual hadronic and deconfined phases.

Also: nuclear matter, and "quarkyonic". Both *confined* as $T < T_d$.

Use pressure as order parameter. Red line: 1st order deconfining transition.

Green line: baryons condense, but 1st order transition, as pressure jumps?

Purple line: chiral transition, order weakly 1st order (at best)?



Quarkyonic phase at large N_c, large μ?

Let $\mu >> \Lambda_{QCD}$ but $\sim N_c^0$. Coupling runs with μ, so pressure $\sim N_c$ is close to perturbative! How can the pressure be (nearly) perturbative in a confined theory?

Pressure: dominated by quarks far from Fermi surf.: perturbative,

$p_{qk} \sim N_c\, \mu^4 \,(1 + g^2(\mu) + g^4(\mu) \log(\mu) + ...)$

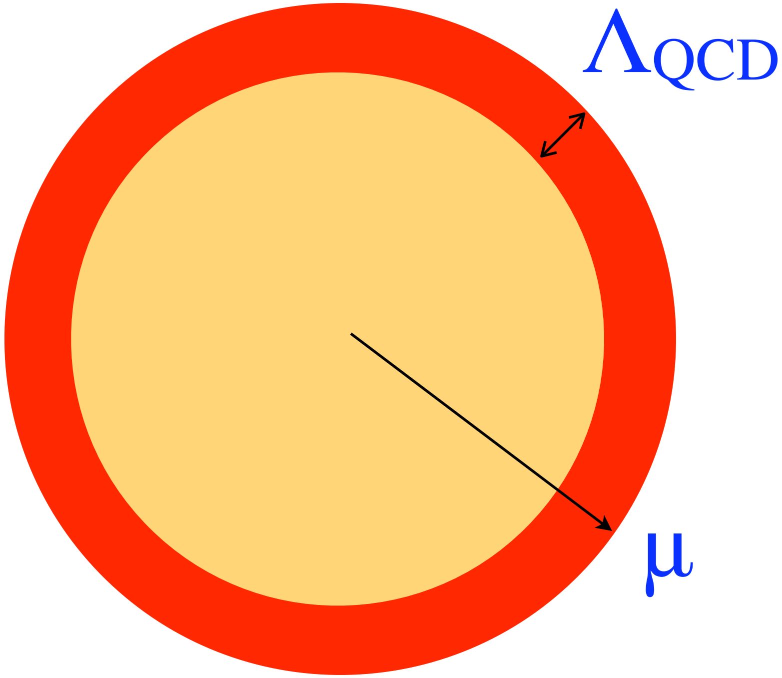
Within Λ_{QCD} of Fermi surface: *confined* states.

$p_{qk} \sim N_c\, \mu^4\, (\Lambda_{QCD}/\mu)^2$, *non*-perturbative.

Within skin, only confined states contribute.

Fermi sea of quarks + Fermi surface of bar-yons

= "quark-yonic". *N=3?*



Pressure dominated by quarks.

But transport properties *dominated* by confined states near Fermi surface!

For QCD: what is (cold) nuclear matter like at high density?

Just a quark NJL model?

Deconfining Critical End Point at large $\mu \sim N_c^{1/2}$

Semi-QGP theory of deconfinement: Hidaka & RDP 0803.0453

$$A_0 = \frac{T}{g}\, Q$$

For large μ: compute one loop determinant in background field.

Korthals-Altes, Sinkovics, & RDP hep-ph/9904305

$$S_{qk} = \mathrm{tr}\,(\mu + i\,T\,Q)^4\;,\; T^2\;\mathrm{tr}\,(\mu + i\,T\,Q)^2\;,\; N_c^2\,T^4\,V(Q)$$

Kharzeev & RDP '09: for large μ, expand:

$$S^{qk}_{\mu\sim\sqrt{N_c},T\sim 1} \sim N_c\,\mu^4 - 6\,\mu^2\,T^2\,tr\,Q^2 + \ldots \sim N_c^3\;,\; N_c^2\;(tr\,Q^2/N_c)$$

Consider $\mu \sim N_c^{1/2}$, $T \sim 1$: gluons *do* feel quarks.

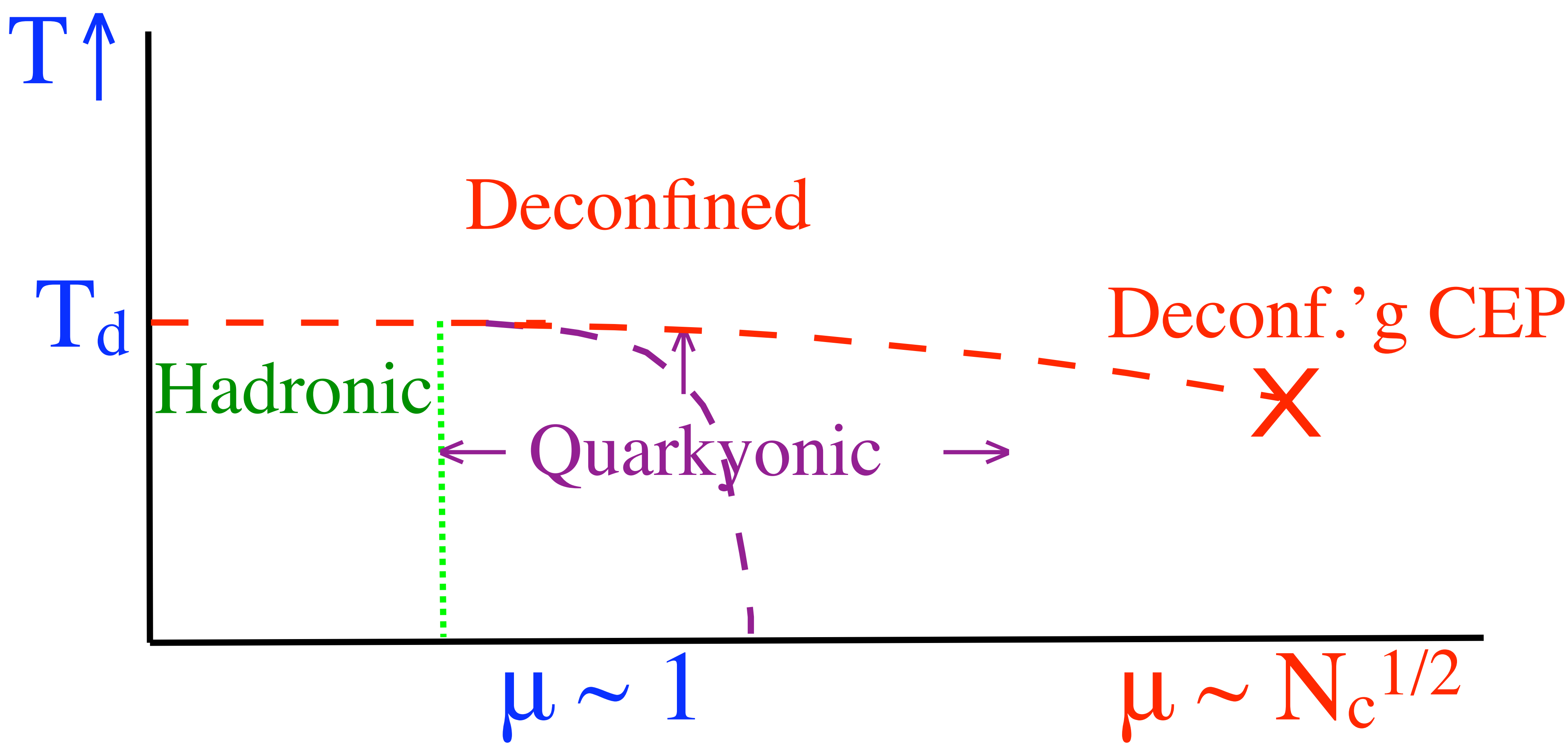
Term $\mu^4 \sim N_c^3$ dominates, but *independent* of Q *and* temperature.

Term $\mu^2 \sim N_c^2$ Q-dependent. Breaks $Z(N_c)$ symmetry, so washes out 1st order deconfining transition: Deconfining Critical End Point (CEP)

Phase diagram at $N_c = \infty$, II

About deconfining critical endpoint, smooth transition between deconfined and quarkyonic phases.

Since gluons are sensitive to quarks for such large μ, expect curvature in line.



Deconfin.'g CEP on a ($N = \infty$) femto-sphere

Sundborg, hep-th/9908001; Aharony, Marsano, Minwalla, Papadodimas, & Van Raamsdonk, hep-th/0310285 & 0502149; Schnitzer, hep-th/0402219; Dumitru, Lenaghan, & RDP, hep-ph/0410294.

Consider (pure gauge) SU(N) on a *very* small sphere: radius R, with $g^2(R) << 1$.

(Sphere because constant modes simple, spherically symmetric)

At $N = \infty$, can have a phase transition even in a *finite* volume.

At $g^2 = 0$: *precisely* defined Hagedorn temperature, T_H . Density of states:

$$\rho(E \to \infty) \sim \mathrm{e}^{E/T_H} \; , \; T_H = \frac{1}{\log(2+\sqrt{3})} \, \frac{1}{R}$$

$g^2 = 0$: 1st order deconfining transition at $T_d = T_H$. $T_d < T_H$ to $\sim g^4$.

At deconfining transition, tr L = exp(i Q)/N (only) becomes massless.

Add quarks. Since $\mu >> 1/R$, can use pressure in infinite volume.

Previous term dominates, acts like background field.

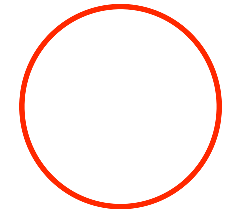
Washes out 1st order deconfining transition for any value of background field.

$N = \infty$ on a small sphere singular, deconf.'g CEP exists at any finite N.

Standard lore: nuclear matter at large N_c

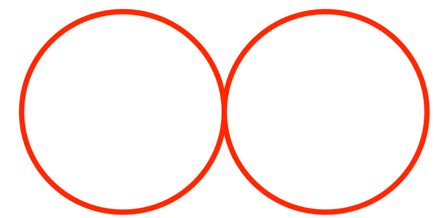
$\mu_{Baryon} = \sqrt{k_F^2 + M^2}$, k_F = baryon Fermi mom. Ideal pressure small, ~ $1/N_c$:

$$P_{\text{ideal baryons}} \sim n(k_F)\,\frac{k_F^2}{M} \sim \frac{1}{N_c}\;\frac{k_F^5}{\Lambda_{QCD}}$$



Usual large N counting: two body int.'s *big*, contribute ~ N_c to pressure:

$$\delta P_{\text{two body int.'s}} \sim N_c\,\frac{n(k_F)^2}{\Lambda_{QCD}^2} \sim N_c\,\frac{k_F^6}{\Lambda_{QCD}^2}$$



At large N_c , nuclear matter dominated by potential terms.
Two body, three body... interactions *all* contribute ~ N_c .
Pressure ideal ~ two body interactions for small momenta,

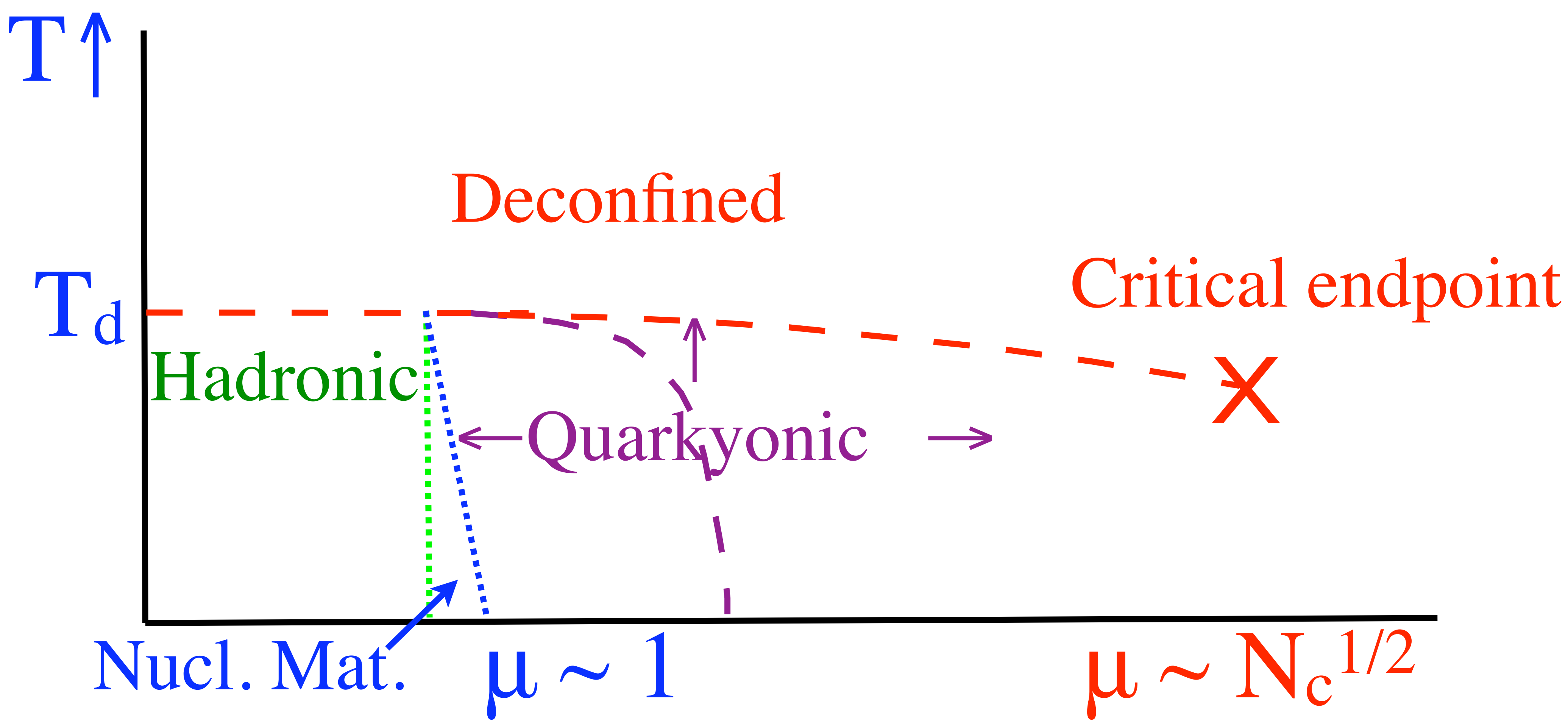
$$k_F \sim \frac{1}{N_c^2}\,\Lambda_{QCD}$$

$$\mu - m_q = \frac{\mu_B - M}{N_c} = \frac{k_F^2}{2MN_c} \sim \frac{1}{N_c^2}\,k_F^2$$

Hence "ordinary" nuclear matter only in a *very* narrow window.

Phase diagram at $N_c = \infty$, III

Standard lore: narrow window of nuclear matter.
Quarkyonic phases, with pressure ~ N_c, includes phases with chiral symmetry breaking and chiral symmetry restoration.
Red line: 1st order deconfining trans. Purple line chiral phase transition (order?)
Green line baryon condensation. Blue line: 1st order transition (pressure jumps)



Nucleon-nucleon potentials from the lattice

Ishii, Aoki & Hatsuda, PACS-CS, 0903.5497

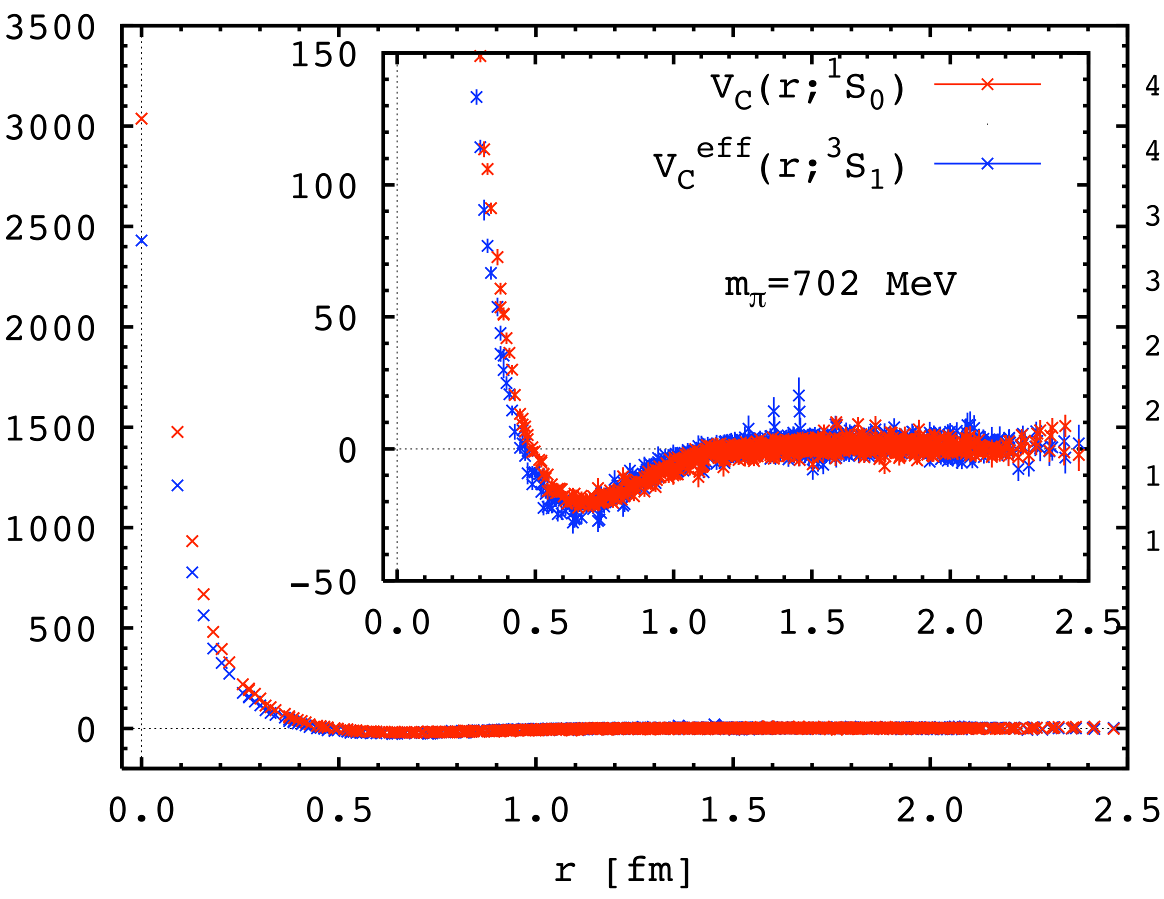
Nucleon-nucleon potentials from quenched and 2+1 flavors.

Pions heavy: 700 MeV (left) and 300 MeV (right)

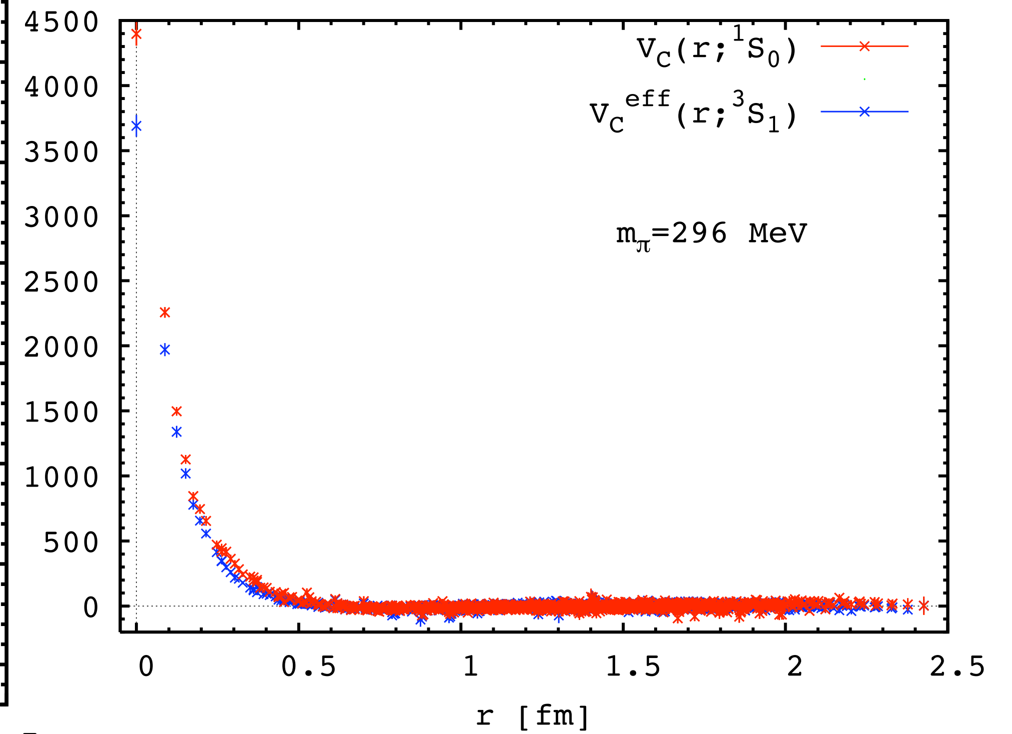
Standard lore: delicate cancellation. *So why independent of pion mass?*

Essentially *zero* potential plus strong hard core repulsion

$m_\pi = 702$ MeV



$m_\pi = 296$ MeV



Purely pionic nuclear matter

J.-P. Blaizot, L. McLerran, M. Nowak, & RDP '09....

At infinite N_c, integrate out *all* degrees of freedom *except* pions:

Lagrangian power series in $U = \mathrm{e}^{i\pi/f_\pi}$, $V_\mu = U^\dagger \partial_\mu U$

Infinite # couplings: Skyrme *plus* complete Gasser-Leutwyler expansion,

$$\mathcal{L}_\pi = f_\pi^2 \, V_\mu^2 + \kappa \, [V_\mu, V_\nu]^2 + c_1 \, (V_\mu^2)^2 + c_2 \, (V_\mu^2)^3 + \ldots$$

All couplings ~ N_c , every mass scale ~ typical hadronic.

Need *infinite* series, but nothing (special) depends upon exact values

Valid for momenta < f_π, masses of sigma, omega, rho...
Useful in (entire?) phase with chiral symmetry breaking?

Higher time derivatives, but no acausality at low momenta.

Purely pions give free baryons

From purely pionic Lagrangian, take baryon as stationary point.

Find baryon mass ~ N_c, some function of couplings.

Couplings of baryon dictated by chiral symmetry:

$$\overline{\psi}\left(i\partial\!\!\!/ + M_B\, \mathrm{e}^{i\tau\cdot\pi\gamma_5/f_\pi}\right)\psi$$

By chiral rotation, $W = \exp(-i\pi\gamma_5/2f_\pi)$

$$\mathcal{L}_B = \overline{\psi}\left(iW^\dagger\partial\!\!\!/ W + M_B\right)\psi \sim \frac{1}{f_\pi}\overline{\psi}\gamma_5\partial\!\!\!/\pi\psi + \ldots$$

At large ~ N_c , $f_\pi \sim N_c^{1/2}$ is *big*. Thus for momenta k < hadronic, interactions are *small*, ~ $1/f_\pi^2 \sim 1/N_c$.

Thus: baryons from chiral Lag. free at large N_c , down to distances $1/f_\pi$.

Manifestly special to chiral baryons. True for u, d, s, but *not* charm?

The Unbearable Lightness of Being (Nuclear Matter)

Nuclear matter: crystal of baryons.

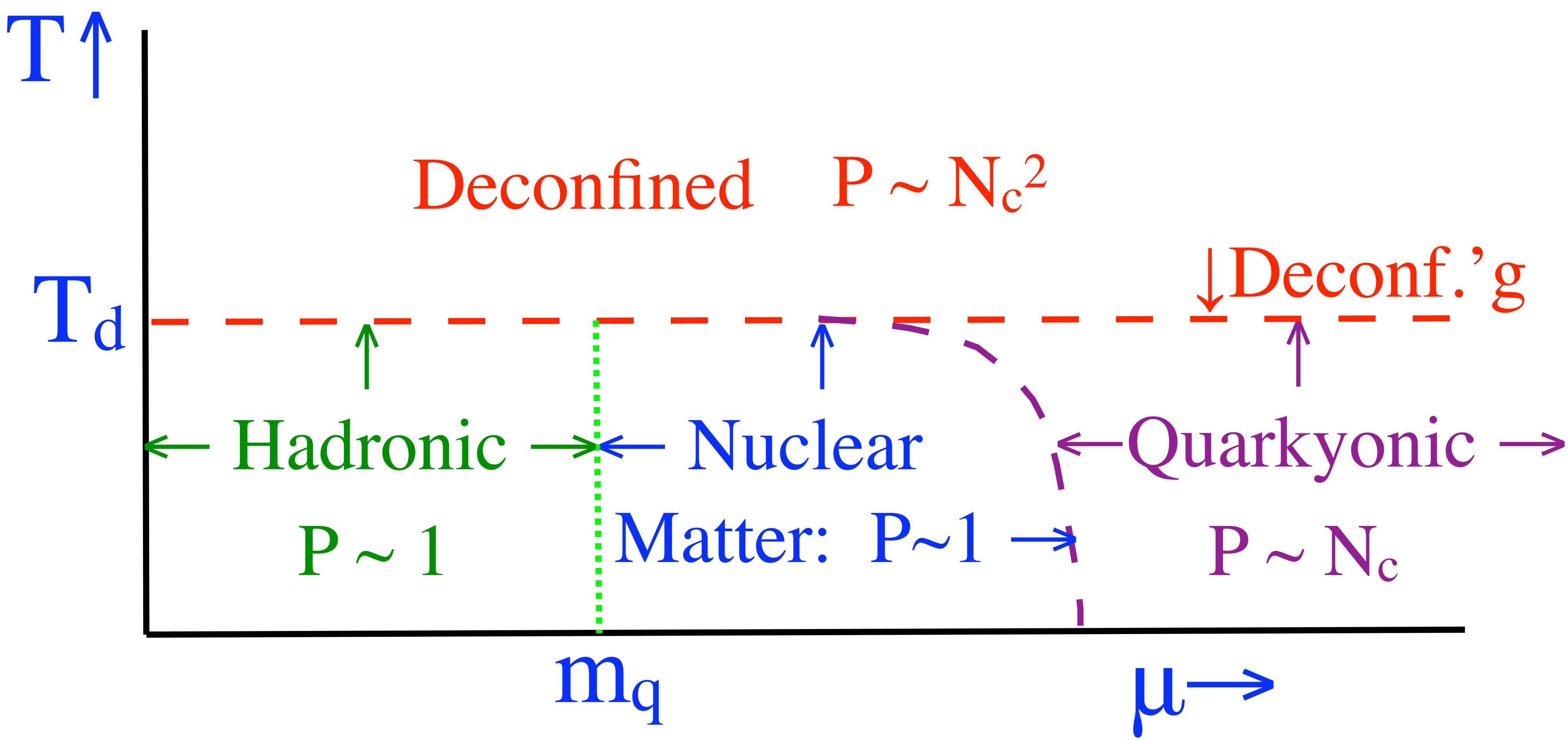
Use purely pionic Lagrangian for all of nuclear matter?

Then pressure ~ 1, and *not* N_c. Like hadronic phase, *not* quarkyonic.

*Un*like standard lore, where pressure(nucl mat) grows quickly, ~ N_c

Red line: 1st order. Green line: Baryons condense.

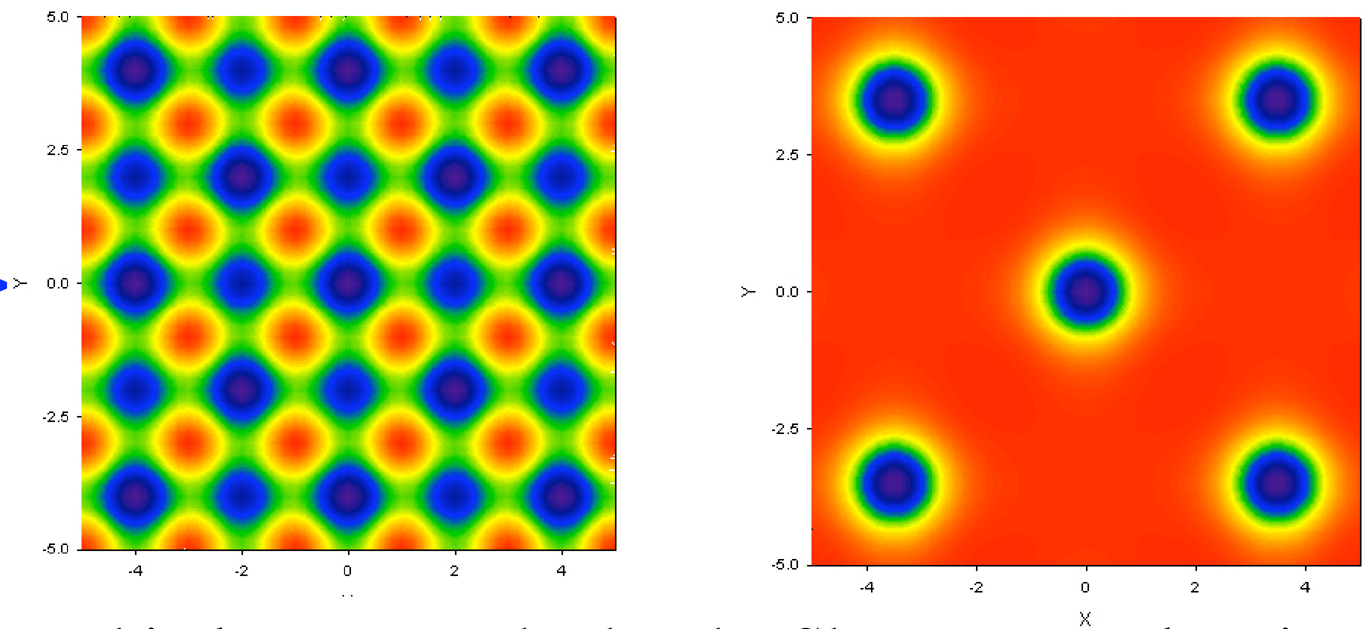
Purple: chiral trans. 1st order, as pressure jumps?



Skyrmion crystals

Kutschera, Pethick & Ravenhall (KPR) ’84; Klebanov ’85 + ...
Lee, Park, Min, Rho & Vento, hep-ph/0302019; Park, Lee, & Vento, 0811.3731:

At large N_c, baryons are heavy, so form a crystal.
Form Skyrmion crystal by taking periodic boundary conditions in a box.
Lee+... ‘03 : box of size L, units of length $1/(\sqrt{\kappa}\, f_\pi)$, plot baryon number density:

L=2.0→  ←L=3.5

At low density, chiral symmetry broken by Skyrme crystal, as in vacuum.

But chiral symmetry *restored* at nonzero L (density): $< U > = 0$ in *each* cell.

Skyrmion crystals as quarkyonic matter

Why chiral symmetry restoration in a Skyrmion crystal?

Goldhaber & Manton '87: due to "half" Skyrmion symmetry in each cell.

Easiest to understand with "spherical" crystal: sphere instead of cube...

KPR '84, Ruback & Manton '86, Manton '87. Consider the "trivial" map:

$$U(r) = \exp(i\, f(r)\, \hat{r} \cdot \tau) \;;\; f(r) = \pi \left(1 - \frac{r}{R}\right)$$

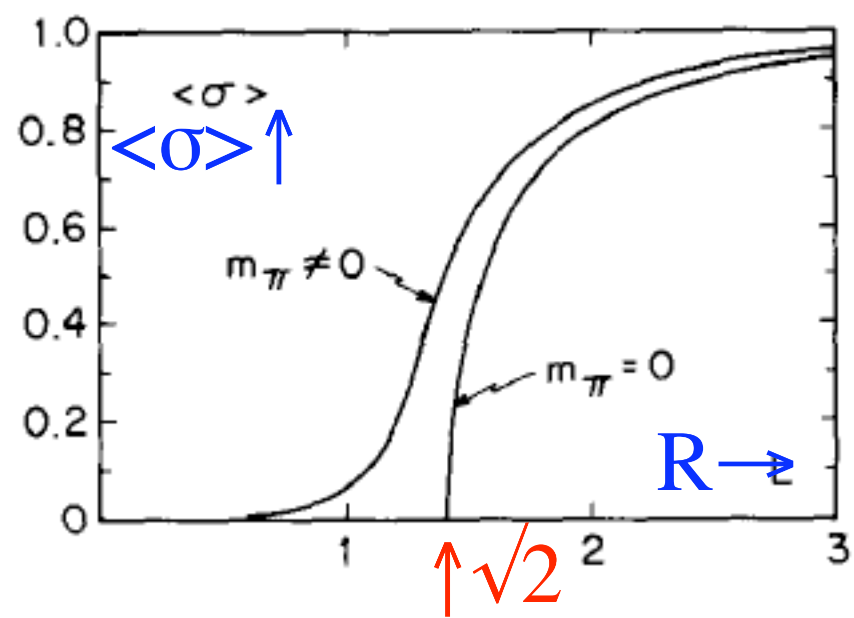
Solution has unit baryon number per unit sphere, and so is a crystal.

Solution is minimal when $R < \sqrt{2}$ (* $1/(\sqrt{\kappa}\, f_\pi)$).

Forkel, Jackson, Rho, & Weiss '89 =>

looks like standard chiral transition!

Excitations *are* chirally symmetric.



But Skyrmions are *not* deconfined.

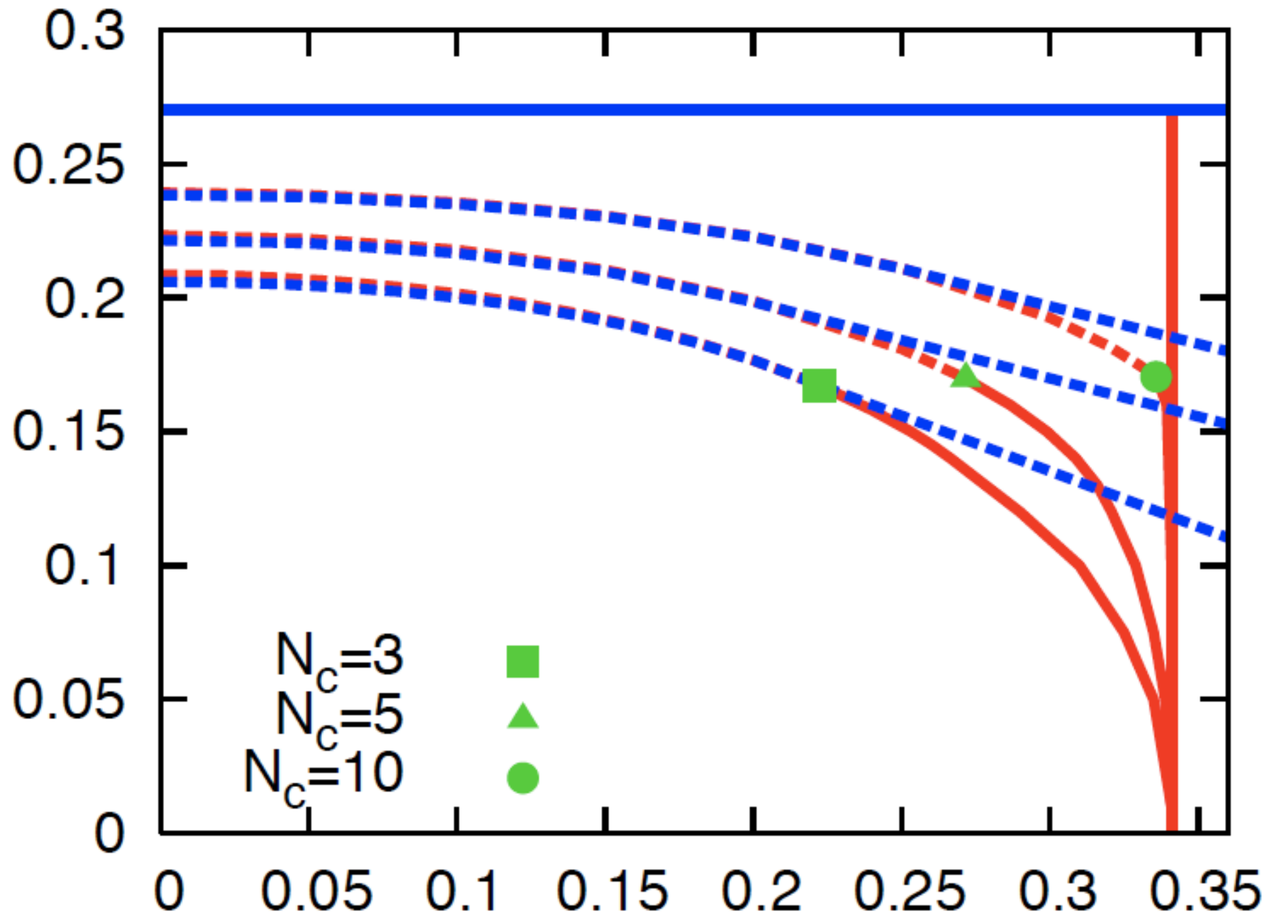
Example of quarkyonic matter,

chirally broken and chirally symmetric.

Quarkyonic in the Polyakov NJL model

McLerran, Redlich, & Sasaki 0812.3585 Use Polyakov NJL model:
K. Fukushima hep-ph/0303225, hep-ph/0310121, 0803.3318, 0809.3080, 0901.4821
Ratti, Thaler, & Weise ph/0506234, nucl-th/0604025, ph/0609281, 0712.3152, 0810.1099
Sasaki, Friman, & Redlich, hep-ph/0611143, hep-ph/0611147, 0806.4745, 0811.4708

"Straightening" of the line for deconfinement as N_c increases:

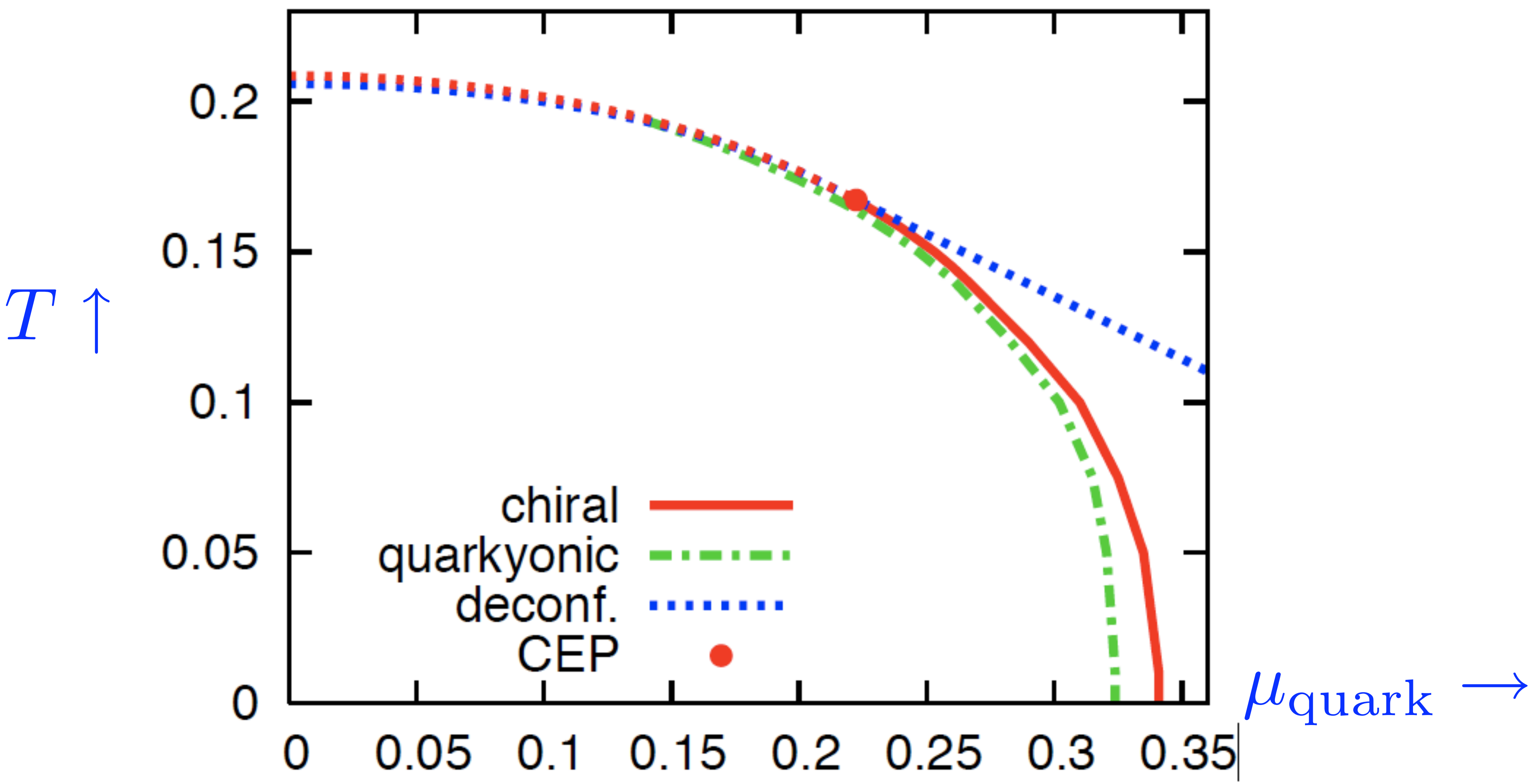


$T \uparrow$ $\mu_{\text{quark}} \longrightarrow$

Chiral vs quarkyonic in the P-NJL model

QCD: chiral Critical End Point (CEP), Shuryak, Stephanov, & Rajagopal '99 '00.

Chiral, quarkyonic, & deconfining transitions split at chiral CEP.
Deconfining CEP at larger densities?



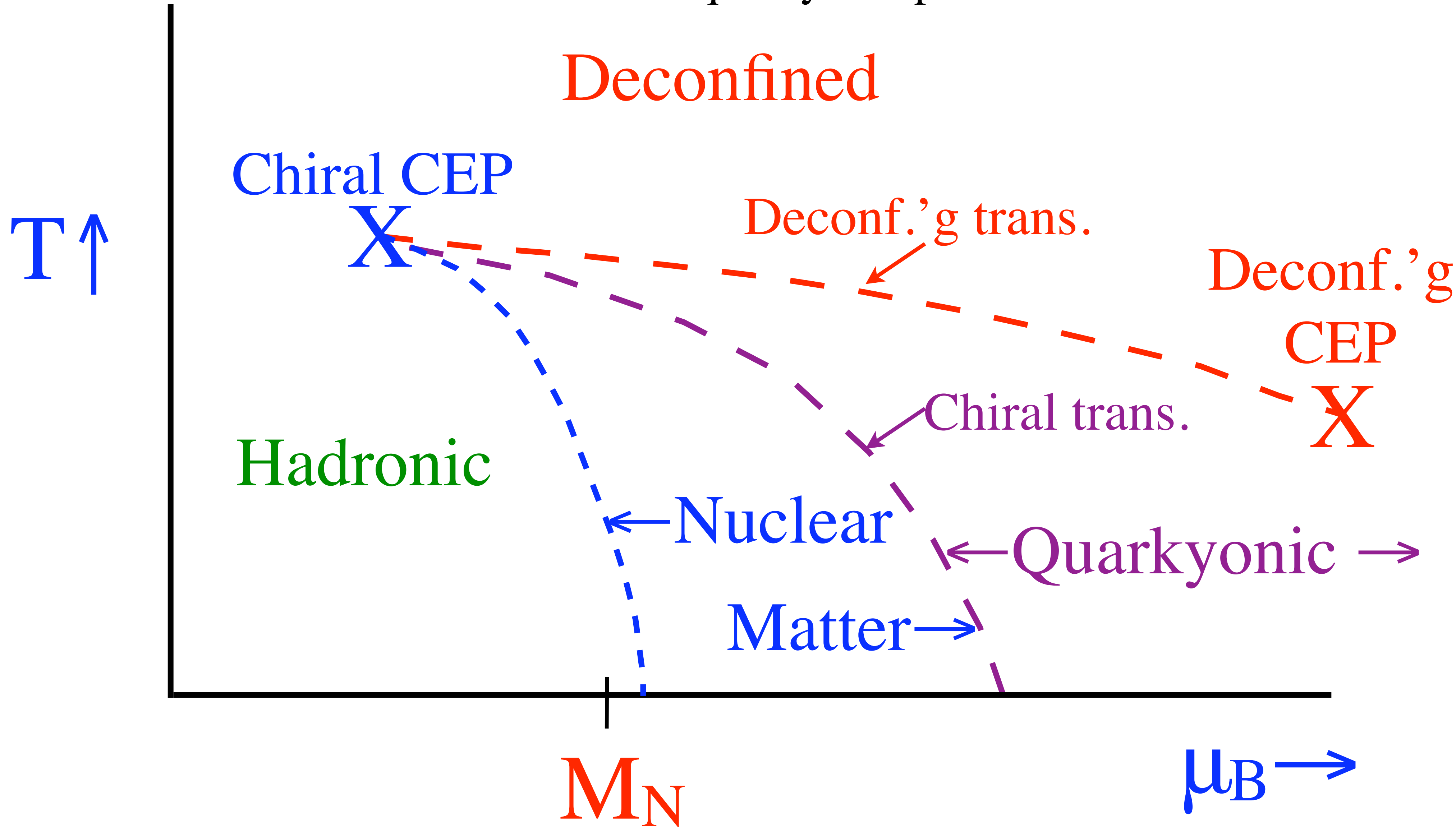
Possible phase diagram for QCD

Pure guesswork: deconf.g & chiral split at chiral CEP?

Two CEP's: one for chiral transition, one for deconfinement.

Moral: deconfining and chiral transitions need *not* be tied at nonzero density.

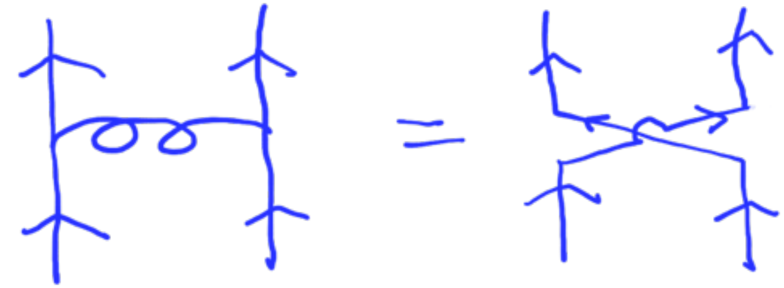
Effective models for nuclear matter and quarkyonic phases?



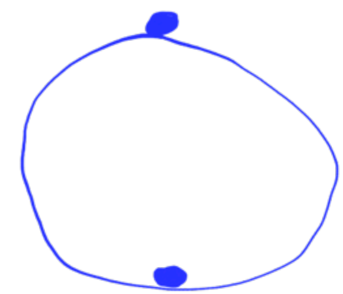
Chiral Density Waves (perturbative)

Excitations near the Fermi surface?

At large N_c, color superconductivity suppressed, ~ $1/N_c$: pairing into two-index state:



Also possible to have "chiral density waves", pairing of quark and anti-quark:
Deryagin, Grigoriev, & Rubakov '92. Shuster & Son, hep-ph/9905448.
Rapp, Shuryak, and Zahed, hep-ph/0008207.



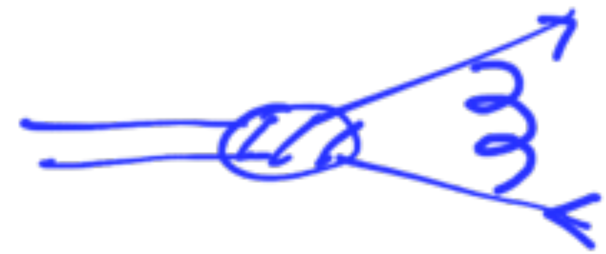
Order parameter $\langle \overline{\psi}(-\vec{p}_f)\, \psi(+\vec{p}_f) \rangle$
Sum over color, so *not* suppressed by $1/N_c$.

Pair quark at + p_f with anti-quark at - p_f : for a *fixed* direction.
Breaks chiral symmetry, with state varying ~ $\exp(-\, 2\, p_f\, z)$.

Wins over superconductivity in low dimensions. Loses in higher.
Shuster & Son '99: in perturbative regime, CDW only wins for $N_c > 1000\ N_f$

Quarkyonic chiral density waves

Consider meson wave function, with kernel:
Confining potential in 3+1 dimensions like
Coulomb potential in 1+1 dim.s:



$$\int dk_0\, dk_z \int d^2k_\perp \, \frac{1}{(k_0^2 + k_z^2 + k_\perp^2)^2} \sim \int dk_0\, dk_z \, \frac{1}{k_0^2 + k_z^2}$$

In 1+1 dim.'s, behavior of massless quarks near Fermi surface maps ~ $\mu = 0$!

Mesons in vacuum naturally map into CDW mesons.

Witten '84: in 1+1 dim.'s, use non-Abelian bosonization for QCD.

a, b= 1...N_c. i,j = 1... N_f.

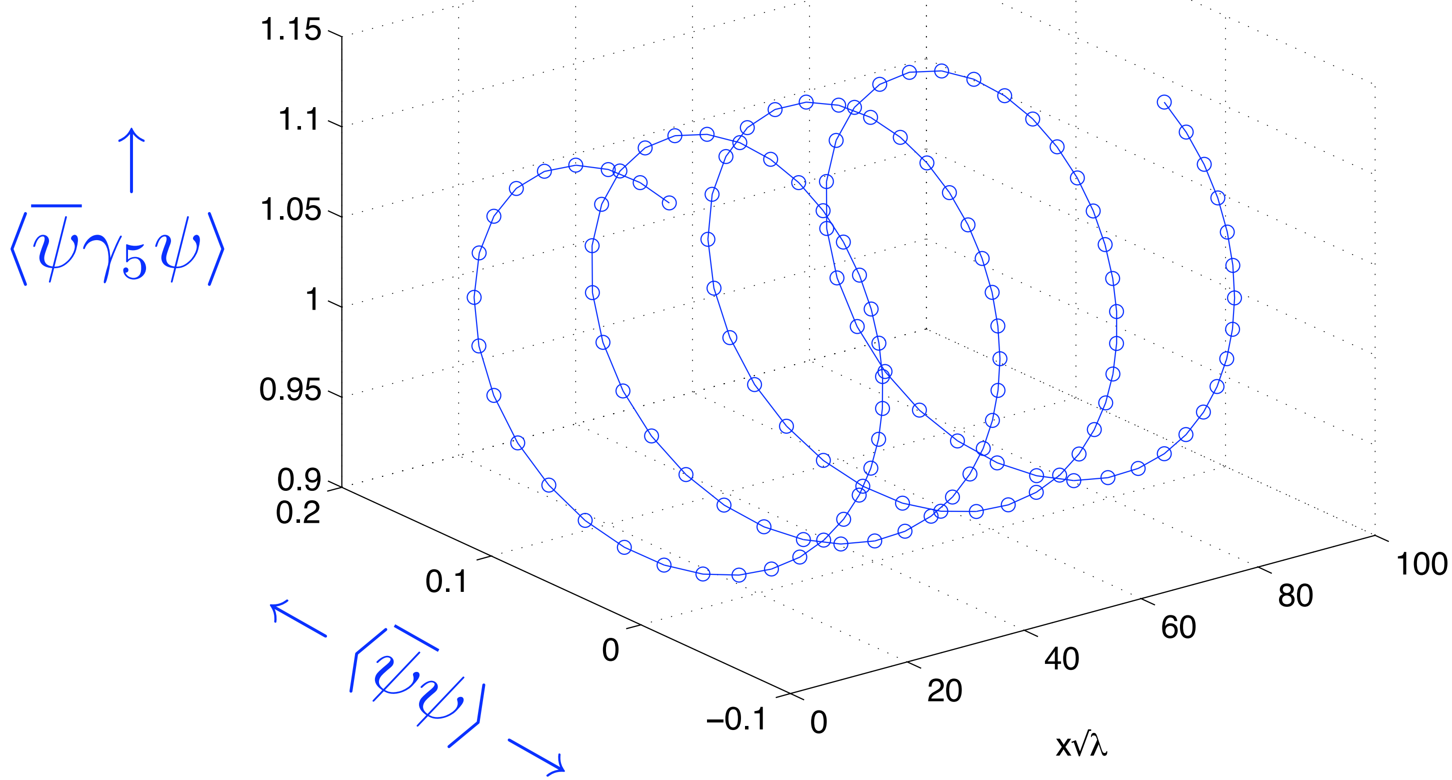
$$J_+^{ij} = \overline{\psi}^{a,i} \psi^{a,j} \sim g^{-1} \partial_+ g \; ; \; J_+^{ab} = \overline{\psi}^{a,i} \psi^{b,i} \sim h^{-1} \partial_+ h \; .$$

Steinhardt '80. Affleck '86. Frishman & Sonnenschein, hep-th/920717...
Armoni, Frishman, Sonnenschein & Trittman, hep-th/9805155; AFS, hep-th/0011043..
Bringoltz 0901.4035; Galvez, Hietanan, & Narayanan, 0812.3449.

Solution to dense QCD in 1+1 dimensions

Bringoltz, 0901.4035: ‘t Hooft model, with massive quarks.
Works in Coulomb gauge, in *canonical* ensemble: fixed baryon number.
Solves numerically equations of motion under constaint of nonzero baryon #
Finds chiral density wave.

N.B.: for massive quarks, should have massless excitations, but with energy $\sim 1/N_c$.

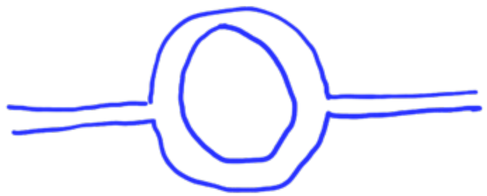


Baryons at Large N_f

Veneziano '78: take *both* N_c and N_f large. Mesons M^{ij} : $i,j = 1 \ldots N_f$.

Thus mesons interact weakly, but there are *many* mesons.

Thus in the hadronic phase, mesons interact *strongly*:



$$\Pi \sim N_f \, g_{3\pi}^2 \sim N_f / N_c$$

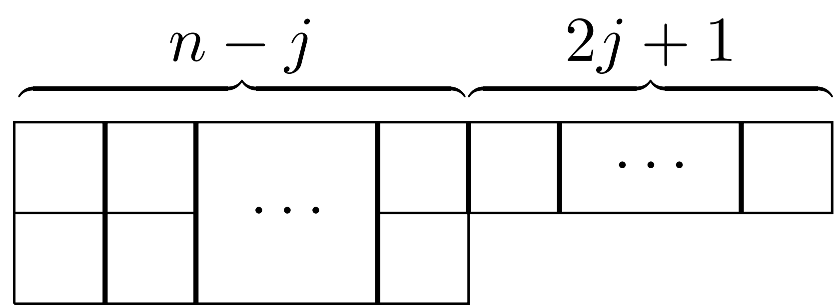
Pressure large in *both* phases:

$\sim N_f^2$ in hadronic phase, $\sim N_c^2$, $N_c \, N_f$ in "deconfined" phase.

Polyakov loop also nonzero in both phases.

Baryons: lowest state with spin j

has Young tableaux ($N_c = 2n + 1$) =>



$$d_j = \frac{(2j+2)\,(N_f+n+j)!\,(N_f+n-j-2)!}{(N_f-1)!\,(N_f-2)!\,(n+j+2)!\,(n-j)!}$$

Baryons at Large N_f: order parameters

Y. Hidaka, L. McLerran & RDP, 0803.0279: Use Sterling's formula,

$$d_j \sim \mathrm{e}^{+N_c\, f(N_f/n)}\ , \ \ f(x) = (1+x)\log(1+x) - x\log(x)$$

Degeneracy of baryons increases *exponentially*.

Argument is heuristic: baryons are strongly interacting.
Still, difficult to see how interactions can overwhelm exponentially growing spectrum, even for the lowest state.

Use *baryons* as order parameter. At T=0, fluctuations in baryon number,

$\langle B^2 \rangle \neq 0$ when $N_c\, f(N_c/n) = m_B/T$, or

$$T_{qk} = f(N_f/n)\, \frac{m_B}{N_c}$$

At $\mu \neq 0$, baryon number itself:

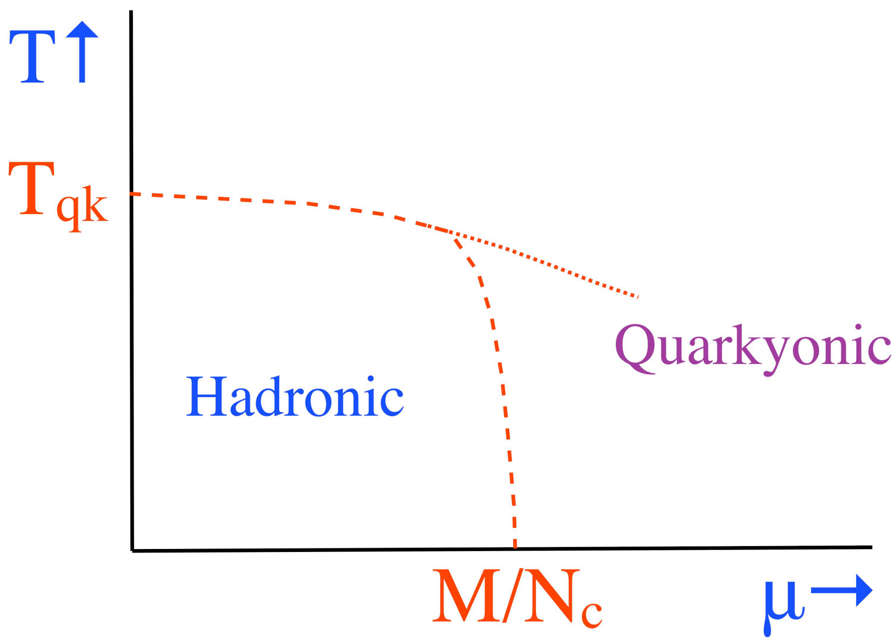
$\langle B \rangle \neq 0$ when $N_c\, f(N_c/n) = (m_B - Nc\ \mu)/T$:

$$T_{qk} = f(N_f/n) \left(\frac{m_B}{N_c} - \mu \right)$$

Possible phase diagrams at large N_f

The "rectangle" for small N_f becomes smoothed.
Eventually, maybe the quarkyonic line merges with that for baryon condensation.
All *VERY* qualitative. Clearly many possible phase diagrams!
With SUSY: condensation of Higgs fields as well.

Small N_f



Large N_f