Chiral symmetry and phase transitions in QCD

1. Flavor and chiral symmetries

2. Two flavors: second order transitions

3. Three flavors: cubic terms rule the roost

4. Tetraquarks and the chiral transition

   a. two flavors: meh, so what

   b. three flavors: cannot be ignored: *two* chiral transitions?

Flavor symmetries

Consider a single, massive quark in QCD:

$$\mathcal{L} = \overline{q}\, \not{D} q + m\, \overline{q} q + \frac{1}{4}\, G_{\mu\nu}^2 \ , \ \not{D} = \gamma^\mu (\partial_\mu - igA_\mu)$$

In QCD, gluons generate the dynamics. But for this talk, we can ignore them, and with certain simple assumptions, concentrate on *global* symmetries (ind. of x_μ).
If there are N_f flavors of quarks,

$$\mathcal{L}_{qk} = \sum_{i=1}^{N_f} \overline{q}_i\, \not{D} q_i + m \overline{q}_i q_i$$

There is a symmetry of rotating quarks into one another,

$$q_i \to U_{ij} q_j \ , \ U^\dagger U = \mathbf{1}$$

The 2nd condition leaves the mass term invariant, so this is a symmetry of $U(N_f)$: unitary N_f x N_f matrices. For QCD, the relevant quarks are up, down, and strange, so $N_f = 2$ or 3.

By group theory, $U(N_f) = SU(N_f) \times U(1)$: the U(1) is quark number, $q_i \to e^{i\theta_V}\, q_i$
for $SU(N_f)$, $\det(U) = 1$.

Chiral symmetry

Dirac matrices anticommute, $\gamma_0\gamma_1$ -= - $\gamma_1\gamma_0$ ...
In 4-dim.'s, $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$ anti-commutes with any γ ($(\gamma_5)^2 = +1$)
Introduce chiral projectors, P_L and P_R,

$$P_{L,R} = \frac{1}{2}(1 \pm \gamma_5) \ , \ P_L^2 = P_L \ , \ P_R^2 = P_R \ , P_L P_R = P_R P_L = 0.$$

Introduce chiral fields,

$$q_L = P_L \psi \ , \ \overline{q}_L = \overline{q}\, P_R$$

Because of Lorentz symmetry,

$$\overline{q} = q^\dagger \gamma^0 \ , \ \overline{q}_{L,R} = \overline{q} P_{R,L}$$

The gauge term is chirally symmetric,

$$\overline{q}\, \not{D} q = \overline{q}\, \not{D}(P_L + P_R) q = \overline{q} P_R\, \not{D} P_L \psi + \overline{q} P_L\, \not{D} P_R q = \overline{q}_L\, \not{D} q_L + \overline{q}_R\, \not{D} q_R$$

while the mass term is not

$$\overline{q} q = \overline{q}(P_L + P_R) q = \overline{q}_R q_L + \overline{q}_L q_R$$

Chiral symmetry…

Now let there be N_f flavors. If massless, we have a larger symmetry:

$$SU(N_f)_L \times SU(N_f)_R : \; q_L = U_L q_L \; , \; q_R = U_R q_R$$

Two phases: L & R rotate the same, ~ θ_V ; or opposite ~ θ_A: $\quad q_{L,R} \to e^{\pm i\theta_A} q_{L,R}$

Thus the full chiral symmetry includes U(1) x $U_A(1)$.
U(1) is quark number, ignore at T = 0, μ = 0 (μ = quark chemical potential)

If m = 0, start with full $SU_L(N_f)$ x SU_R (N_f) x U(1) x $U_A(1)$ symmetry.
If nonzero m generated dynamically, spontaneously broken to $SU_V(N_f)$ x U(1).

However, because of the axial anomaly, $U_A(1)$ is broken *dynamically*,

$$\partial_\mu J_5^\mu = \partial_\mu \left(\overline{q}\gamma_5\gamma_\mu q\right) = \frac{3g^2}{16\pi^2} \, \mathrm{tr}\, G_{\mu\nu}\widetilde{G}_{\mu\nu}$$

Not obvious. g = QCD coupling. Quantum fluctuations: instantons, dyons…
contribute to the "breaking" of $U_A(1)$.

Two flavors, simplest

Start with simplest, two flavors. Take Pauli matrices in flavor t^a, a = 1, 2, 3:

$$\phi^0 \sim \overline{q}q \ , \ \phi^a \sim \overline{q}t^a\gamma_5 q$$

The ϕ^0 ~ "sigma", ϕ^a are pions. (why just these…?). Let $\vec{\phi} = (\phi^0, \phi^a)$

Swindle to deal with SU(2) x SU(2) ~ O(4)

$$\mathcal{V}(\vec{\phi}) = -\frac{m^2}{2}\vec{\phi}^2 + \frac{\lambda}{4}(\vec{\phi}^2)^2$$

Expand about $\langle\phi^0\rangle = \sigma_0 \ , \ \langle\phi^a\rangle = 0$

$$\frac{\partial\mathcal{V}}{\partial\sigma} = 0 \Longrightarrow \sigma_0 = 0 \ \text{ or } \ \frac{m^2}{\lambda}$$

Expanding to quadratic order, $m_\sigma^2 = 2\mu^2 \ , \ m_\pi^2 = 0$

Massless pions! If we add a term ~ h σ, then

$$m_\sigma^2 = 2\mu^2 + O(h) \ , \ m_\pi^2 \sim h$$

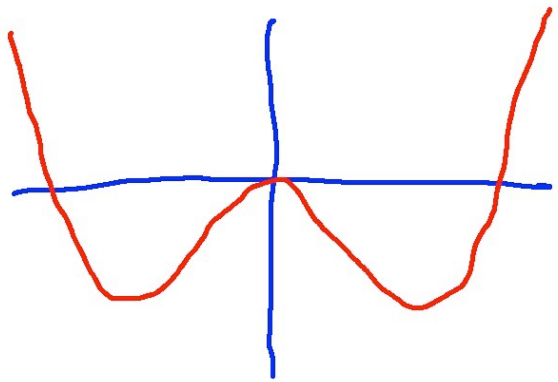
So background field h ~ "bare" quark mass, gives pions a mass

Chiral phase transition, massless pions

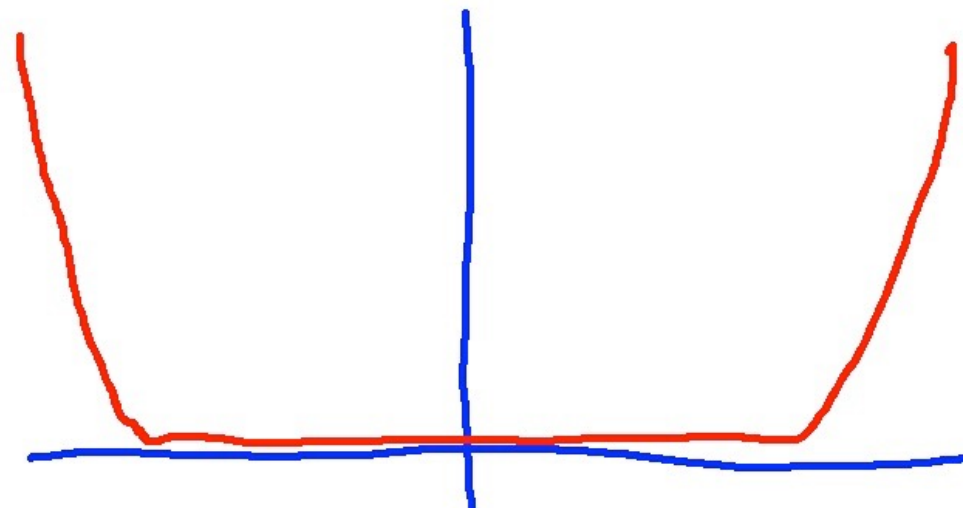
Model variation in T = temperature:
m^2 goes from negative to positive values:

$$\mathcal{V}(\vec{\phi}) = -\frac{m^2}{2}\vec{\phi}^{\,2} + \frac{\lambda}{4}(\vec{\phi}^{\,2})^2$$

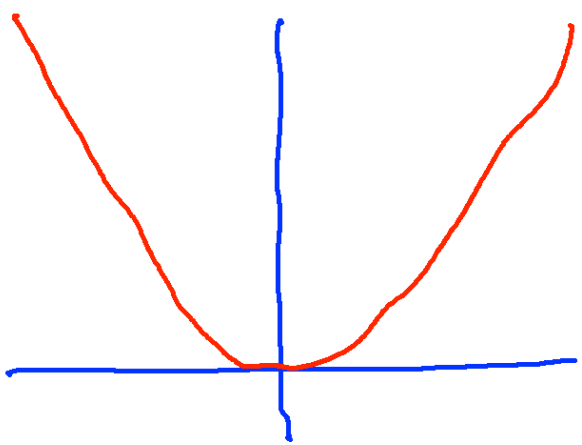
$T = 0$: $m^2 < 0$, $\langle\phi\rangle \neq 0$



$T = T_\chi$: $m^2 = 0$, $\langle\phi\rangle = 0 \Downarrow$



$T \gg \sigma_0$: $m^2 > 0$, $\langle\phi\rangle = 0$



Chiral transition *second* order, $m^2 = 0$ at T_χ.
Universality class O(4)

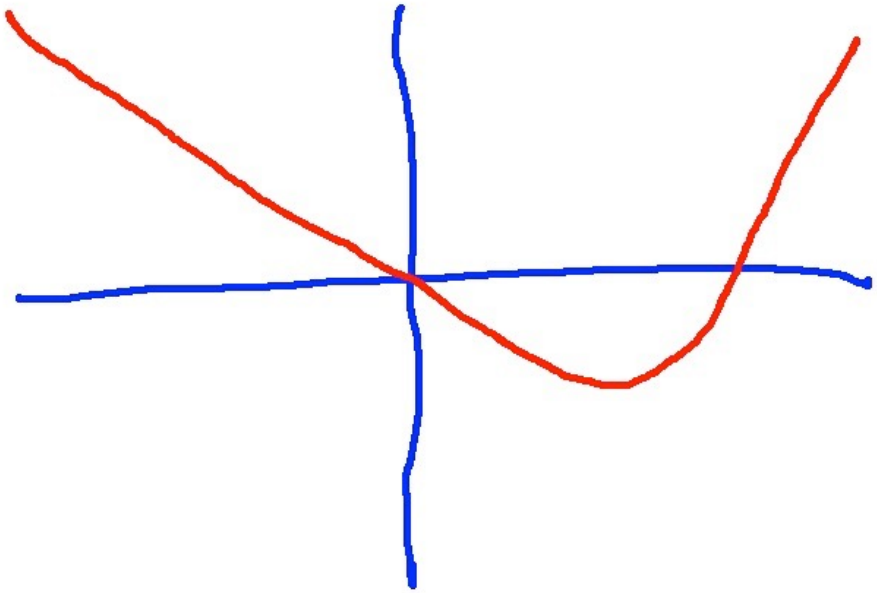
Chiral phase transition, massive pions

Only change is that the potential includes a linear term,

$$\mathcal{V}(\vec{\phi}) = -\frac{m^2}{2}\vec{\phi}^{\,2} + \frac{\lambda}{4}(\vec{\phi}^{\,2})^2 - h\,\phi^0$$

Even when m^2 vanishes,
the potential is *not* flat.

With a linear term, one can *never* flatten
the potential for *any* value of the mass
and coupling.



$\langle\phi\rangle \neq 0$ at *any* temperature: $\langle\phi\rangle$ is large at $T = 0$, and $\langle\phi\rangle \sim 1/T^2$ at high T (hm…)

Chiral transition is *crossover*: $\langle\phi\rangle$ varies smoothly from large to small value.
Also, no large change in entropy, etc.

Two flavors, complete

Now take ϕ^a to be a *complex* valued field:

$$\phi^0 \sim \bar{q}q \ , \ i\bar{q}\gamma_5 q \ , \ \phi^a \sim \bar{q}t^a\gamma_5 q \ , \ i\bar{q}t^a q$$

ϕ^0: isosinglet σ ($m_\sigma \sim 500$ MeV ?) and η ($m \sim 550$);
ϕ^a: isotriplet of π's ($m \sim 140$) and a_0's ($m \sim 980$).

Terms invariant under O(8):

$$\mathcal{V}(\vec{\phi}) = -\frac{m^2}{2}\vec{\phi}^* \cdot \vec{\phi} + \frac{\lambda}{4}(\vec{\phi}^* \cdot \vec{\phi})^2$$

Invariant under O(4) x O(2):

$$-\lambda_2 \ (\vec{\phi}^*)^2 \vec{\phi}^2$$

There are also terms invariant *just* under O(4). Simplest is a mass term,

$$\mathcal{V}_A(\vec{\phi}) = -m_A^2 \left((\vec{\phi}^*)^2 + \vec{\phi}^2 \right)$$

Plus other quartic terms invariant just under O(4).

Anomaly for two flavors

Simple understanding of the anomaly. Explictly, $\phi = (\sigma + i\eta, \pi^a + i\, a_0^a)$:

$$\mathcal{V}_A = m_A^2(-\sigma^2 + \eta^2 - \vec{\pi}^2 + \vec{a}_0^2)$$

Anomaly necessary to break the $U_A(1)$ symmetry, and ensure a heavy η.

One can add a term $\sim h\,\sigma$ and compute masses. A bit involved, one finds

$$m_\eta^2 - m_\pi^2 = m_{a_0}^2 - m_\sigma^2$$

Moral: the anomaly pushes the η mass *up*, and the σ mass *down*.

Phase transition: if anomaly large at T_χ, the η and a_0 decouple, don't really matter.

If anomaly small at T_χ, *might* change the nature of the chiral transition.
Unlikely, the anomaly has to become *very* small.

Chiral symmetry, more carefully

For 3 flavors of massless quarks coupled to a gauge field,

$$\mathcal{L}^{qk} = \overline{q}\, \not{D}\, q = \overline{q}_L \not{D}\, q_L + \overline{q}_R \not{D}\, q_R \quad , \quad q_{L,R} = \frac{1 \pm \gamma_5}{2}\, q$$

Classically, global flavor symmetry of $SU(3)_L$ x $SU(3)_R$ x $U(1)_A$,

$$q_L \to \mathrm{e}^{-i\alpha/2}\, U_L\, q_L \quad , \quad q_R \to \mathrm{e}^{+i\alpha/2}\, U_R\, q_R$$

Simplest order parameter for χ symmetry breaking (χSB'g): $\Phi^{ab} = \overline{q}_L^{bA}\, q_R^{aA}$

a,b… = flavor. A,B… = color

$$\Phi \to \mathrm{e}^{+i\alpha}\, U_R\, \Phi\, U_L^\dagger$$

Quantum mechanically, axial $U(1)_A$ is broken by instantons +…. to $Z(3)_A$ at T=0

't Hooft instanton vertex is invariant under $Z(3)_A$:

$$\det \Phi \to \mathrm{e}^{3i\alpha}\, \det \Phi$$

As $T \to \infty$, $U(1)_A$ approximately restored as $\sim 1/T^7$ [9].

Sigma models for χ symmetry

Need to classify the possible terms by symmetry
Generally, Φ transforms under G_{cl}= $SU(3)_L$ x $SU(3)_R$ x $U(1)_A$
Best to consider a special case:

$$\Phi \to e^{+i\theta_A}\, U_R\, \Phi\, U_L^\dagger$$

$$\Phi = \begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & \phi_3 \end{pmatrix}$$

Φ is a complex 3x3 matrix, so ϕ_1, ϕ_2, & ϕ_3 are complex as well.
Simplest terms are constructed from powers of:

$$\mathrm{tr}\,\Phi^\dagger\Phi = |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2$$

$$(\mathrm{tr}\,\Phi^\dagger\Phi)^2 = (|\phi_1|^2)^2 + 2|\phi_1|^2|\phi_2|^2 + (|\phi_2|^2)^2 + \ldots$$

These are invariant under O(6). For the the full Φ, this becomes O(18):

$$\mathcal{V}_{O(18)} = m^2\,\mathrm{tr}\,\Phi^\dagger\Phi + \lambda_{O(18)}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2$$

And more terms

There is one other quartic term

$$\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2 = (|\phi_1|^2)^2 + (|\phi_2|^2)^2 + (|\phi_3|^2)^2$$

No cross terms, so clearly independent from the other quartic term, ~ O(18).

One last term needed:

$$\mathcal{V}_A = \kappa(\det\Phi + \det\Phi^\dagger) = \kappa(\phi_1\phi_2\phi_3 + \phi_1^*\phi_2^*\phi_3^*)$$

Invariant under $SU(3)_L$ x $SU(3)_R$: $\det\Phi \to \det(U_R\,\Phi\,U_L^\dagger) = \det\Phi$

But not under $U(1)_A$! However, if we choose $\Phi \to e^{2\pi i/3}\Phi$

Then this term is invariant under $Z(3)_A$, as $\det\Phi \to (e^{2\pi i/3})^3 \det\Phi = \det\Phi$.
This represents the axial anomaly: 't Hooft

All the fields

$$\mathcal{V}_{eff} = m^2 \,\mathrm{tr}\, \Phi^\dagger \Phi - \kappa(\det \Phi + \mathrm{c.c.}) + \lambda \,\mathrm{tr}\left(\Phi^\dagger \Phi\right)^2 + \lambda_{O(18)} \left(\mathrm{tr}\, \Phi^\dagger \Phi\right)^2$$

Φ complex 3x3 matrix, so 18 degrees of freedom.

Half are $J^P = 0^-$: one singlet, η' , and 8 fields: 3 π's, 4 K's, and a η

Half are $J^P = 0^+$: one singlet, σ_0 and 8 fields: 3 a_0's, 4 $\varkappa$'s, and a σ_8

Take simplest case: explicit SU(3) symmetry, no background field, $m^2 < 0$:

$$\langle \Phi \rangle = \phi_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Spontaneous breaking of symmetry => 8 Goldstone Bosons (GB),

$m_\pi = m_K = m_\eta = 0$. Axial anomaly splits the would be 9th GB, $m_{\eta'}{}^2 \sim \varkappa\, \phi_0 \neq 0$

All $J^P = 0^+$ fields are massive

With massive pions

Now add a term to make the GB's massive:

$$\mathcal{V}_{eff} \to \mathcal{V}_{eff} - \mathrm{tr}\, H\Phi$$

Let H be SU(3) symmetric,

$$H = h \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Symmetry broken, but still SU(3) symmetric. So only four masses, all nonzero:

mass of $\pi = K = \eta \neq$ mass η'. mass of $a_0 = \varkappa = \sigma_8 \neq$ singlet σ_0.

But the masses satisfy a simple relation (like two flavors, 't Hooft '86)

$$m_{\eta'}^2 - m_\pi^2 = m_{a_0}^2 - m_\sigma^2$$

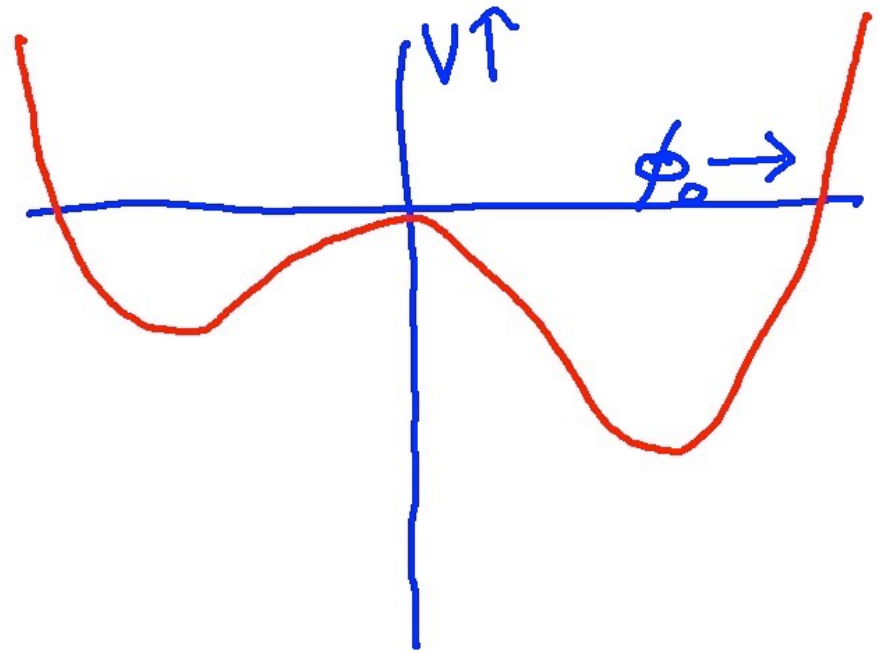
The anomaly moves η' *up* from the π, but also moves singlet σ *down* from the a_0!

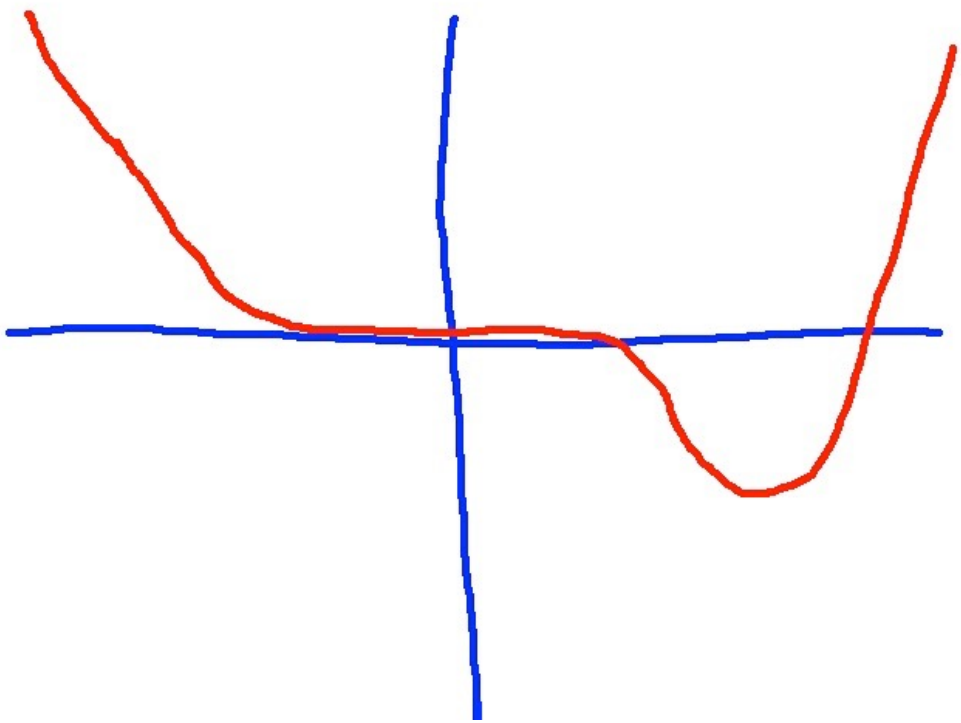
Chiral transition, three flavors

Cubic terms
very important

$$\mathcal{V}_{eff}(\phi_0) = 3\,m^2\,\phi_0^2 - 2\kappa\,\phi_0^2 + 3(\lambda + 3\,\lambda_{O(18)})\phi_0^4$$

At T = 0: $m^2 < 0$, $\langle\phi\rangle \neq 0$,
looks like $\varkappa = 0$: =>

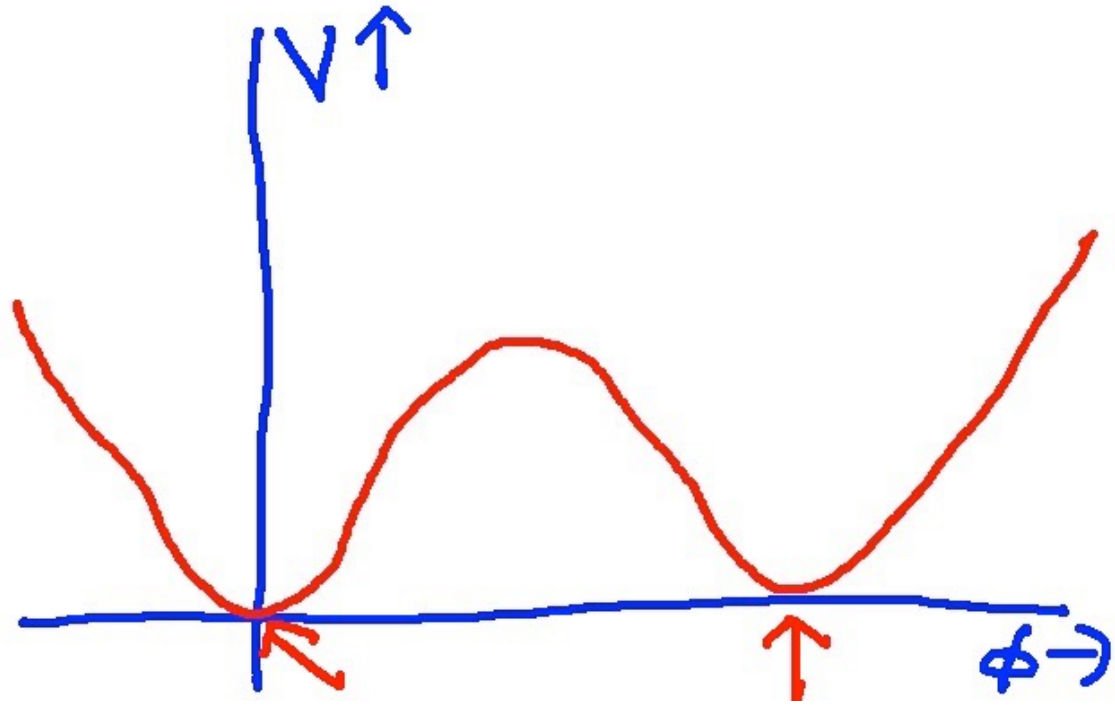




<= Would be critical point, $m^2 = 0$:
because of the cubic term,
can*not* flatten the potential

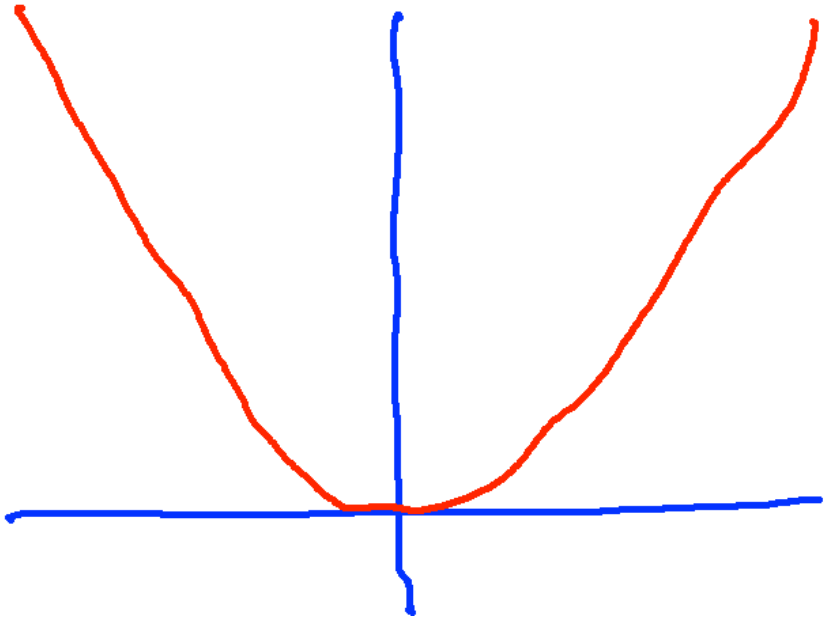
First order transition

Transition occurs for $m^2 > 0$:
barrier between the two vacua,
so *first* order transition at T_χ. =>



$\varkappa$ is $Z(3)_A$ invariant, not $U(1)_A$ inv.
So $\varkappa$ *must* vanish as $T \to \infty$.
Is it big or small at T_χ?

At high temperature, again
large, positive m^2 dominates all,
$\langle\phi\rangle = 0$



QCD and 2+1 flavors

In QCD, $m_{up} \sim 5$ MeV, $m_{down} \sim 10$ MeV, $m_{strange} \sim 100$ MeV.

Because of confinement, and the axial anomaly, can take $m_{ud} = (m_u + m_d)/2$

$$\mathcal{V}_h = -\,\mathrm{tr}\, H(\Phi + \Phi^\dagger) \qquad\qquad H = h \begin{pmatrix} m_{ud} & 0 & 0 \\ 0 & m_{ud} & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$m_\pi \sim 140$, $m_K \sim 495$, $m_\eta \sim 540$, $m_{\eta'} \sim 960$. $\varkappa \sim 4500$. MeV

Can show $m_\pi^2 \sim m_{ud}$, $m_K^2 \sim m_s$, so $m_{ud}/m_s \sim 1/27$.

$<\phi_{ud}> = f_\pi /2 \sim 46$, $<\phi_s> = f_K /2 \sim 56$.

$$\langle \Phi \rangle = \begin{pmatrix} \phi_{ud} & 0 & 0 \\ 0 & \phi_{ud} & 0 \\ 0 & 0 & \phi_s \end{pmatrix}$$

Discuss other masses later.

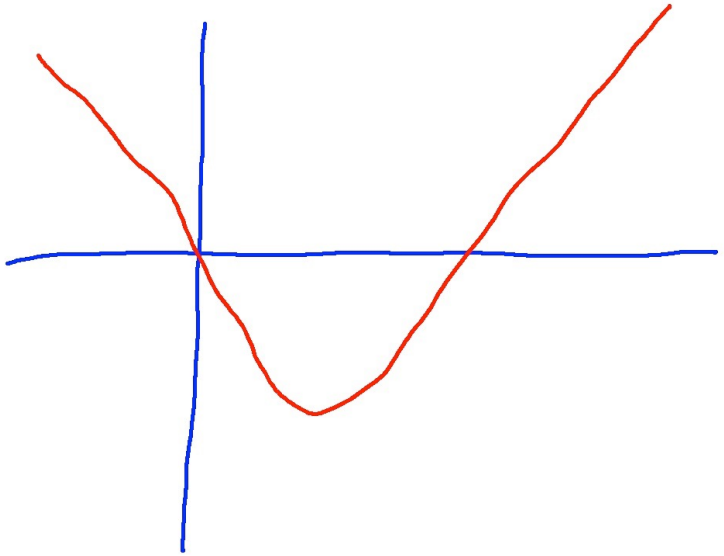
Find $\lambda \sim 50 >> \lambda_{O(18)} \sim 1$.

Background field: big or small?

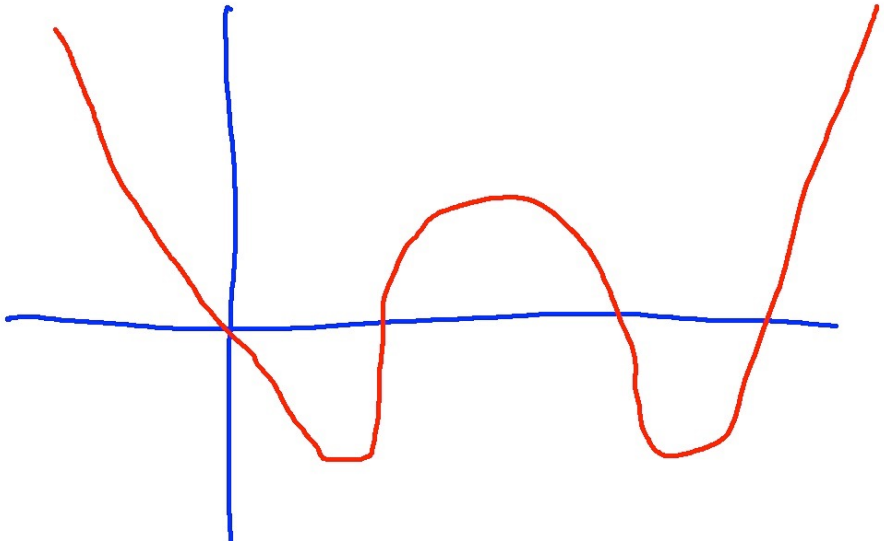
Schematically, the potential with both a background field, and a cubic term:

$$\mathcal{V} = -h\,\phi + m^2\,\phi^2 - \kappa\,\phi^3 + \lambda\,\phi^4$$

For large h, smooth crossover.
Look for peak in chiral susceptibility:



For small h, true first order transition.

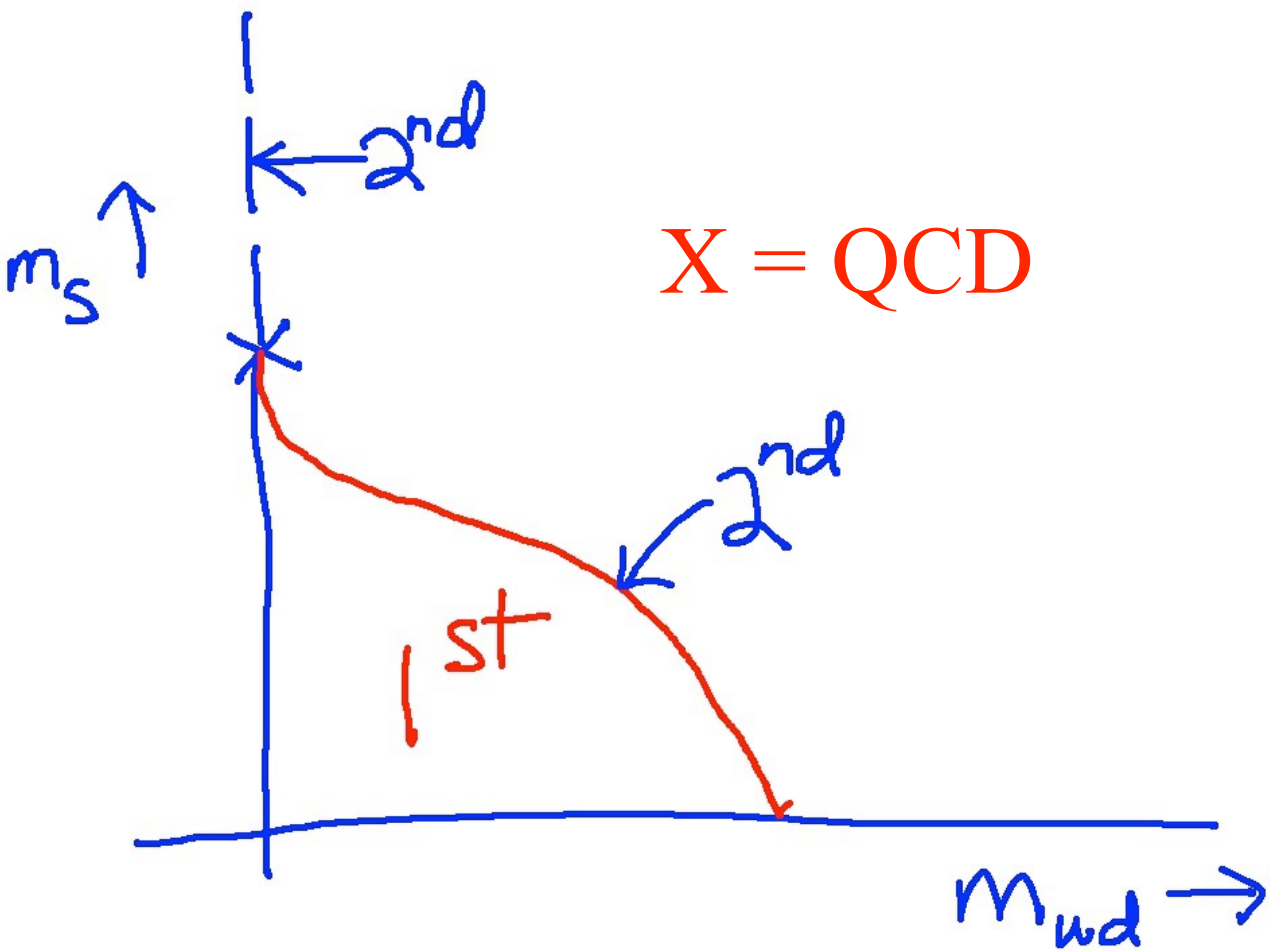


For QCD, small h!

Cubic term drives the transition first order.

For small enough masses, *must* be 1st order $\langle \det \Phi \rangle = \phi_{ud}^2 \, \phi_s$

But *very* sensitive to m_{ud}/m_s : "Columbia" phase diagram



Lattice:
$T_\chi \sim 154 \pm 10$ MeV

Crossover.

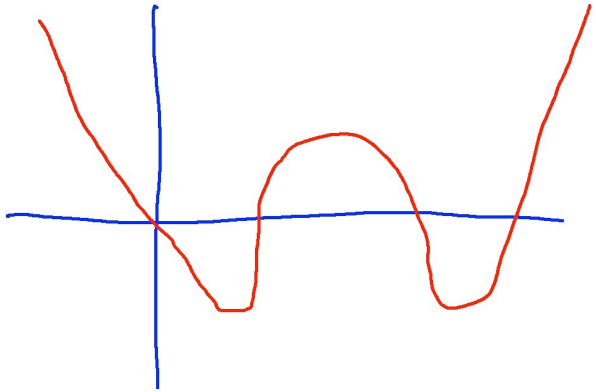
No 1st order transition until at least $m_\pi \sim 50$ MeV

Bazavov et al, 1701.04325

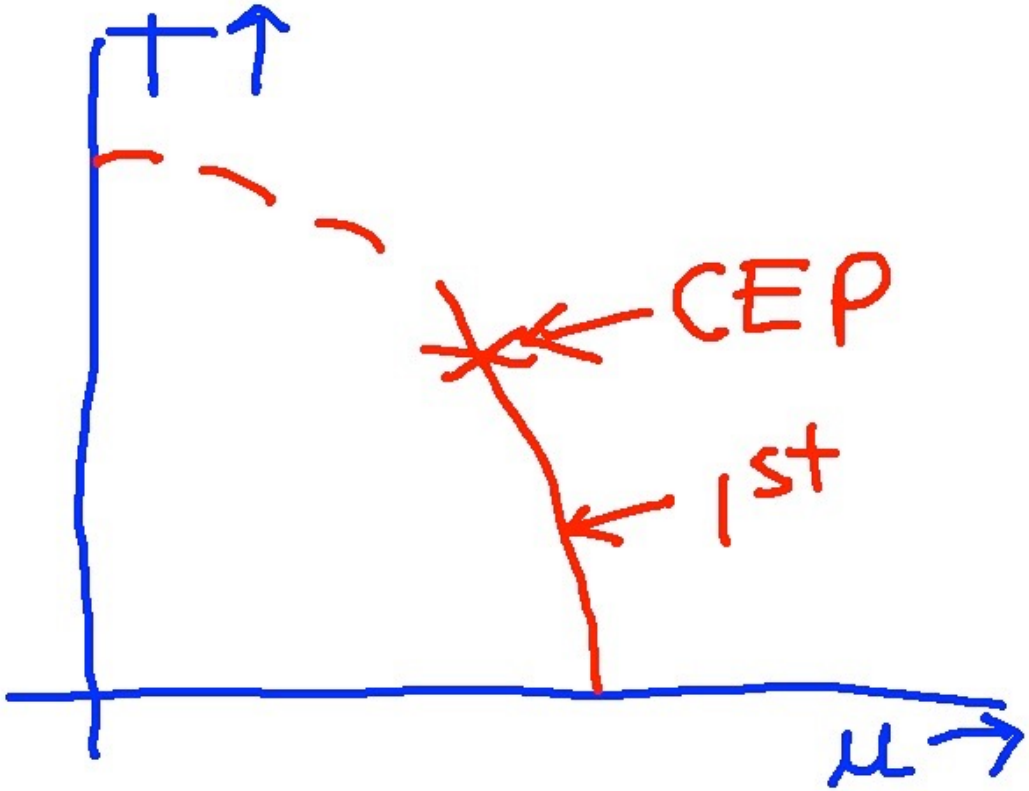
Critical endpoint?

Lattice result for nonzero T, μ = 0.
Quark loops generate a negative quartic term, stabilized by a positive sextic term.
This can drive the chiral transition first order.

$$\mathcal{V} = -h\,\phi + m^2\,\phi^2 - \kappa\,\phi^3 + (\lambda - \lambda_{qk})\,\phi^4 + \lambda_6\,\phi^6$$



Possible to have crossover turn first order at a Critical EndPoint
Rajagopal, Shuryak, & Stephanov '99, '00



Swept under the rug…

For 2nd order transitions, can classify transitions according to their *universality class:* depends only upon spatial dimension (=3) & *symmetry*

For two flavors, assumed quartic coupling positive, so 2nd order transition, O(4)

If anomaly small by T_χ, the symmetry goes from O(4) -> O(4) x O(2)

In 4 - ε dimensions, O(4) x O(2) "fluctuation induced" : flows to *negative* λ

Recently: conformal solutions indicate may be *new* fixed point for O(4) x O(2):

Nakayama & Ohtsuki, 1404.0489, 1407.6195

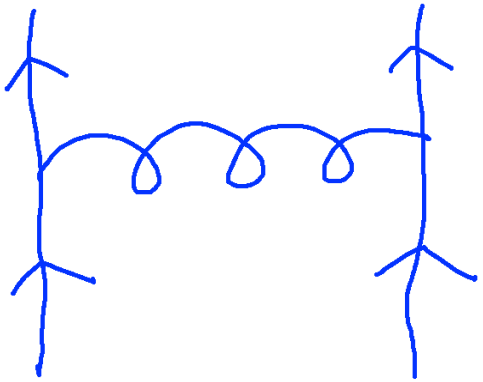
Have *no* good models for the relation between anomaly and T_χ:

Fejos, 1506.07399; Heller & Mitter, 1512.05241; Fejos & Hosaka, 1604.05982

lattice suggests anomaly *non*zero at T_χ

Diquarks and tetraquarks for two flavors

Jaffe '79: most attractive channel for
quark-quark scattering is antisym. in *both* flavor and color.



Color: 3 x 3 = $\underline{3}$ (antisym) + 6 (sym)

Two flavors: 2 x 2 = 1 (antisym) + 3 (sym)

For two flavors diquark is a color triplet, flavor *singlet*,

$$\chi_L^A = \epsilon^{ABC}\ \epsilon^{ab}\ (q_L^{aB})^T\ \mathcal{C}^{-1}\ q_L^{bC}$$

(A, B, C = color; a, b = flavor) Also χ_R. χ_L and χ_R *singlets* under $Z(2)_A$.

One complex valued tetraquark field:

$$\zeta = (\chi_R^A)^*\ \chi_L^A$$

Sigma models and tetraquarks for two flavors

The tetraquark field ζ is a *singlet* under flavor and $Z(2)_A$.

Split complex ζ into its real and imaginary parts, ζ_r and ζ_i.

QCD is even under parity, so only even powers of ζ_i can appears, forget ζ_i.

But *any* powers of ζ_r can!

$$\mathcal{V}^A_{\zeta_r} = h_r\,\zeta_r + m_r^2\,\zeta_r^2 + \kappa_r\,\zeta_r^3 + \lambda_r\,\zeta_r^4$$

Hence $\langle \zeta_r \rangle$ is *always* nonzero!

Couplings to ϕ start with $U(1)_A$ inv.: $\mathcal{V}_{\zeta\phi} = \kappa\,\zeta\,\phi^* \cdot \phi + \ldots$

The tetraquark ζ_r is just a massive field with a v.e.v. : *boring!*

Tetraquarks for three flavors

Three flavors: 3 x 3 = $\underline{3}$ + 6 .
Diquark field flavor *anti-triplet*, $\underline{3}$

$$\chi_L^{aA} = \epsilon^{abc}\, \epsilon^{ABC}\, (q_L^{bB})^T\, \mathcal{C}^{-1}\, q_L^{cC}$$

LR tetraquark field ζ transforms *identically* to Φ under $G_\chi = SU(3)_L \times SU(3)_R$

$$\zeta^{ab} = (\chi_R^{aA})^*\, \chi_L^{bA}$$

Under $U(1)_A$, Φ has charge +1, ζ charge -2.

Since ζ & Φ in *same* representation of G_χ, *direct* mixing term. $Z(3)_A$ invariant:
Black, Fariborz, Schechter ph/9808415 + ...;
't Hooft, Isidori, Maiani, Polosa 0801.2288 + ...

$$\mathcal{V}^A_{\zeta\Phi,2} = \widetilde{m}^2\, \mathrm{tr}\left(\zeta^\dagger \Phi + \Phi^\dagger \zeta\right)$$

An extra *dozen* couplings.
e.g., $U(1)_A$ inv. cubic term:

$$\mathcal{V}^\infty_{\zeta\Phi,3} = \kappa_\infty\, \epsilon^{abc}\, \epsilon^{a'b'c'}\, \left(\zeta^{aa'}\, \Phi^{bb'}\, \Phi^{cc'} + \mathrm{c.c.}\right)$$

"Mirror" model, T = 0

Spectrum : $\Phi = \pi, K, \eta, \eta'; a_0, \kappa, \sigma_8, \sigma_0$; $\zeta = \widetilde{\pi}, \widetilde{K}, \widetilde{\eta}, \widetilde{\eta}'; \widetilde{a}_0, \widetilde{\kappa}, \widetilde{\sigma}_8, \widetilde{\sigma}_0$.

General model has 20 couplings:

Fariborz, Jora, & Schechter: ph/0506170; 0707.0843; 0801.2552. Pelaez, 1510.00653

Instead study "mirror" model, where Φ and ζ start with *identical* couplings

$$\mathcal{V}_\Phi = m^2 \, \mathrm{tr}\left(\Phi^\dagger \Phi\right) - \kappa \left(\det \Phi + \mathrm{c.c.}\right) + \lambda \, \mathrm{tr}\left(\Phi^+ \Phi\right)^2$$

$$\mathcal{V}_\zeta = m^2 \, \mathrm{tr}\left(\zeta^\dagger \zeta\right) - \kappa \left(\det \zeta + \mathrm{c.c.}\right) + \lambda \, \mathrm{tr}\left(\zeta^+ \zeta\right)^2$$

Assume only $\zeta\Phi$ coupling is mass term: $\mathcal{V}^A_{\zeta\Phi,2} = \widetilde{m}^2 \, \mathrm{tr}\left(\zeta^\dagger \Phi + \Phi^\dagger \zeta\right)$

Simple, because mass *only* mixes: $\pi \leftrightarrow \widetilde{\pi}$, $K \leftrightarrow \widetilde{K} \ldots$

Spectrum of the mirror model

In the chiral limit, the mass eigenstates: (need to assume $\widetilde{m}^2 < 0$)

$$\pi, \widetilde{\pi} = 0\,,\ -2\,\widetilde{m}^2 \ \ ;\ \ \eta', \widetilde{\eta}' = 3\,\kappa\,\phi\,,\ 3\,\kappa\,\phi - \ 2\,\widetilde{m}^2$$

$$a_0, \widetilde{a}_0 = m^2 + \kappa\,\phi + 6\,\lambda\,\phi^2 \pm \widetilde{m}^2 \ \ ;\ \ \sigma_0, \widetilde{\sigma} = m^2 - 2\,\kappa\,\phi + 6\,\lambda\,\phi^2 \pm \widetilde{m}^2$$

All states are mixtures of Φ and ζ. Of course 8 Goldstone bosons.
Satisfy *generalized* 't Hooft relation

$$m^2_{\eta'} + m^2_{\widetilde{\eta}'} - m^2_{\pi} - m^2_{\widetilde{\pi}} = m^2_{a_0} + m^2_{\widetilde{a}_0} - m^2_{\sigma} - m^2_{\widetilde{\sigma}}$$

Since *every* multiplet is doubled, this can easily be satisfied (unlike if just one).

Even with same couplings, all masses are *split* by the mixing term.

At nonzero T, the thermal masses of the Φ and ζ can*not* be equal!

With tetraquarks, *maybe* two chiral transitions

In chiral limit, *may* have have *two* chiral phase transitions. =>

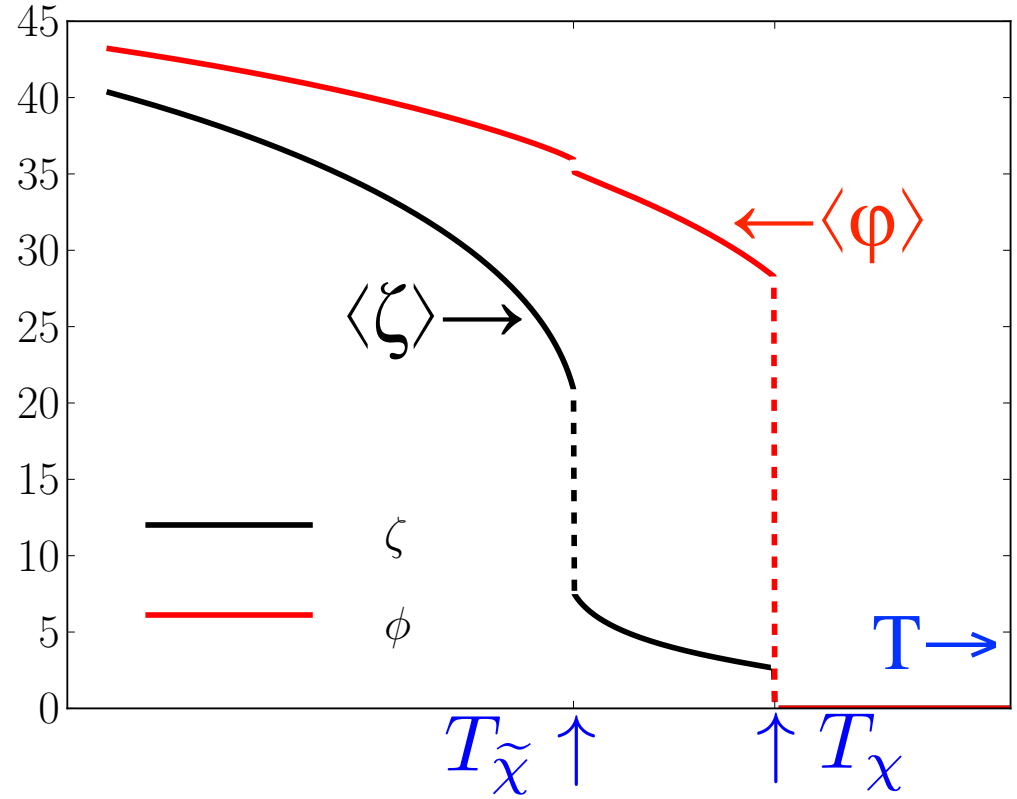
At first, both jump, remain nonzero.

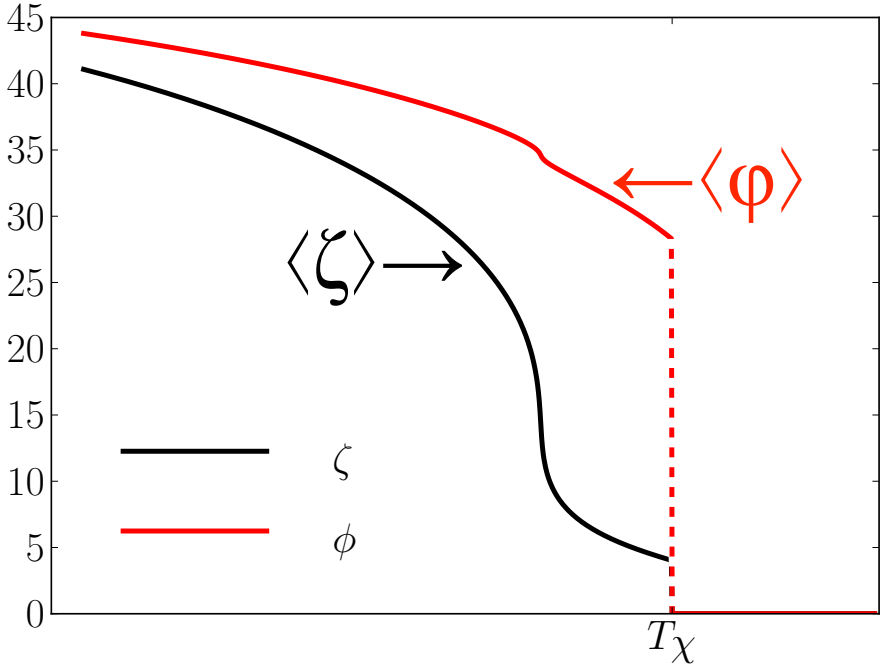
At second, both jump to zero.

$$m_\phi^2(T) = 3\,T^2 + m^2$$

$$m_\zeta^2(T) = 5\,T^2 + m^2$$

$$\widetilde{m}^2 = -(100)^2$$





<= Also possible to have single chiral phase transition, tetraquark crossover

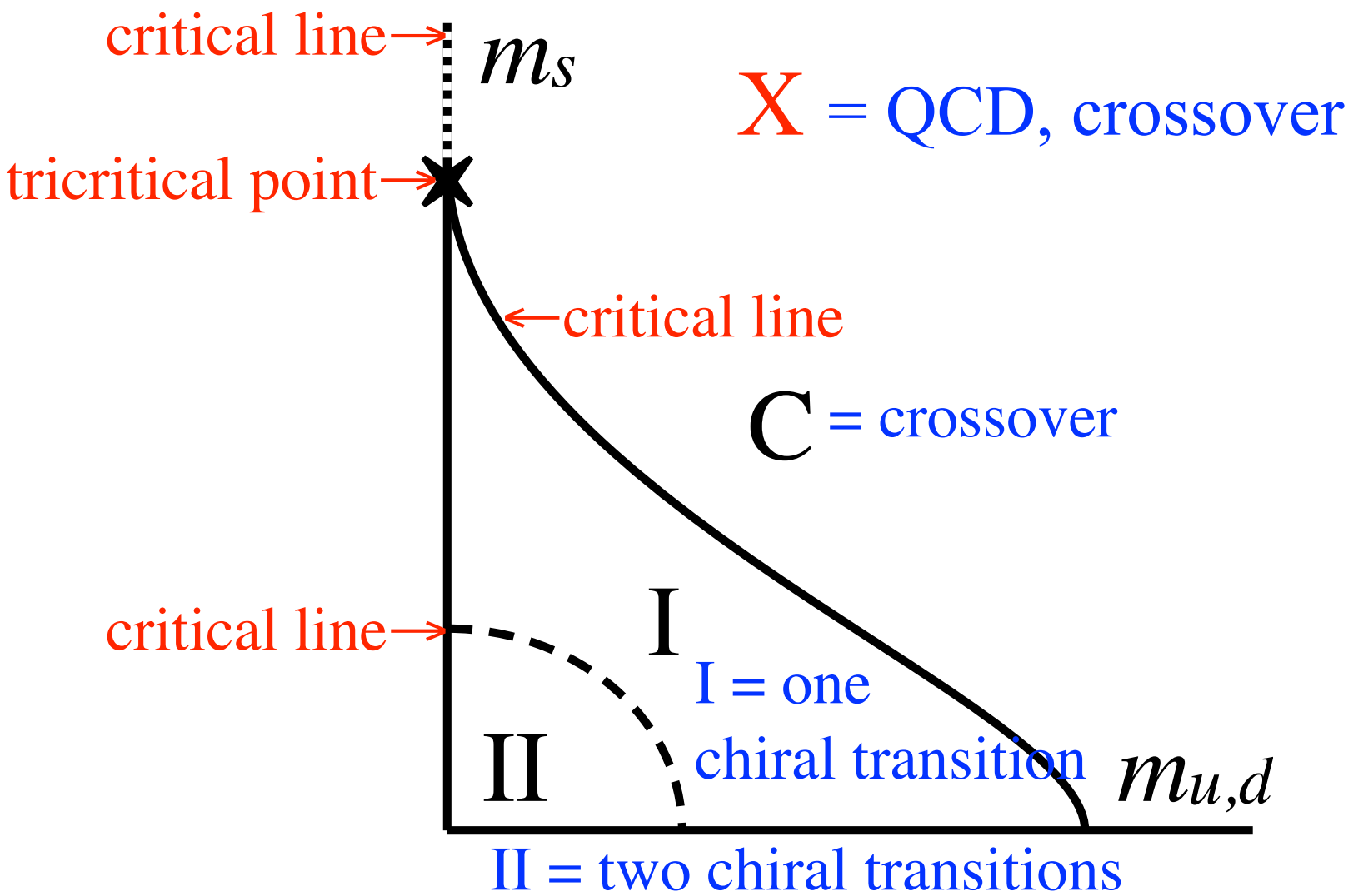
$$\widetilde{m}^2 = -(120)^2$$

”Columbia” phase diagram for light quarks

Lattice: chiral transition crossover in QCD

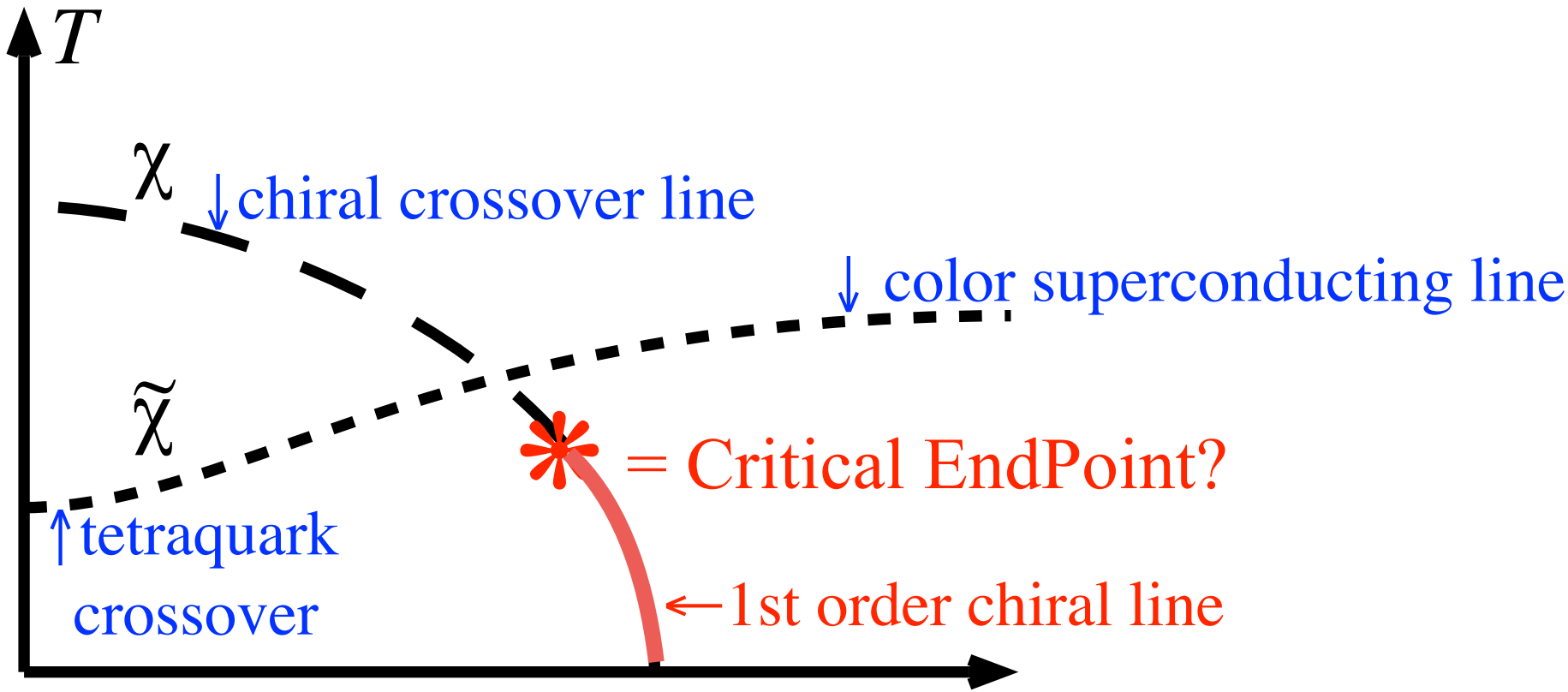
If two chiral phase transitions for three massless flavors, persists for nonzero mass

Implies new phase diagram in the plane of $m_u = m_d$ versus m_s:



Tetraquarks in the plane of T and μ

Diquark fields are *identical* to the order parameters for color superconductivity.
Tetraquark condensate = gauge invariant square of CS condensate. Suggests:



Line for chiral crossover *might* end, meet line for *first* order chiral transition at *Critical EndPoint* (CEP). Massless σ at CEP. Rajagopal, Shuryak & Stephanov, '99

In *effective* models, to find the CEP, *must* include tetraquarks: need the *right* σ!

Using the lattice to fix the σ model?

Briceno, Dudek, Edwards, & Wilson (Hadron Spectrum Coll.), 1607.05900

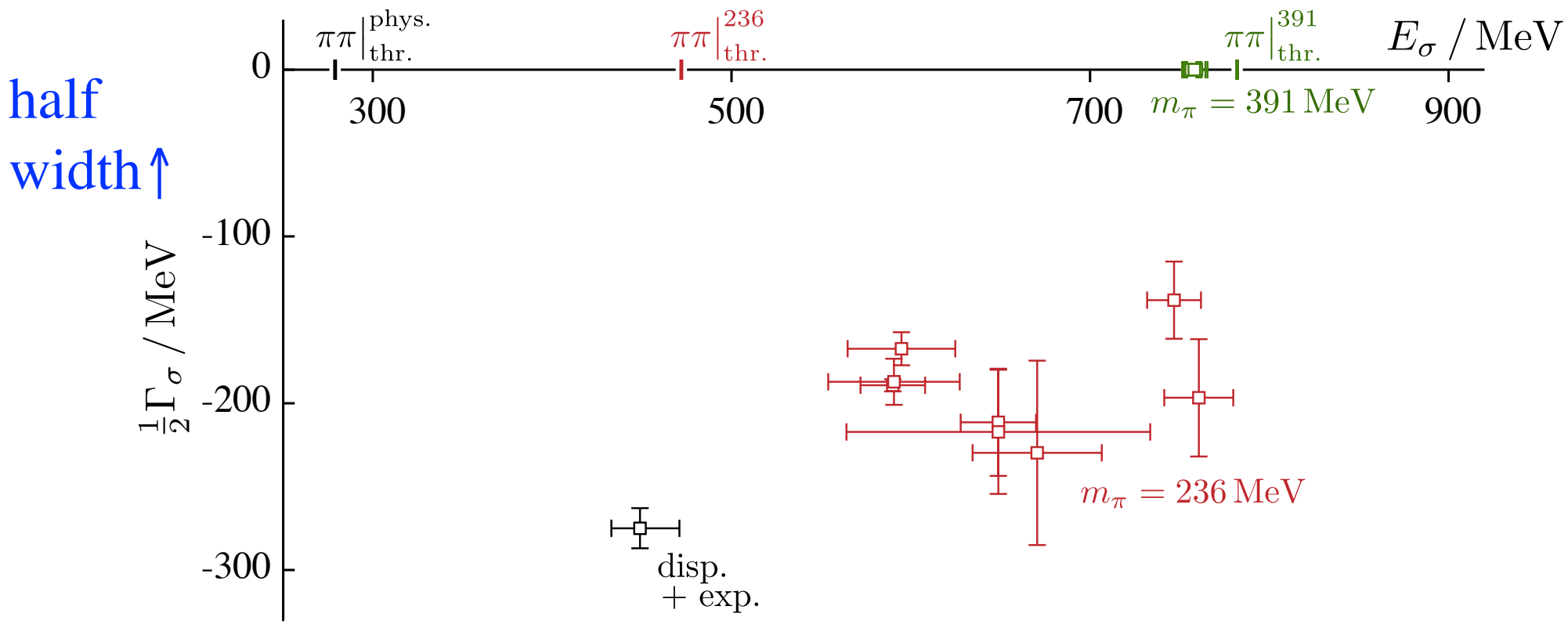
First study of the σ meson on the lattice with light, dynamical quarks

m_π = 391 MeV: σ bound state resonance just below ππ threshold

m_π = 236 MeV: σ broad resonance, well above ππ threshold

Perhaps: fix the parameters of σ model plus tetraquarks from the lattice

Very difficult, but of *direct* relevance for the CEP.



Four flavors, three colors: *hexa*quarks

Diquark 2-index antisymmetric tensor:

$$\chi_L^{(ab)A} = \epsilon^{abcd}\,\epsilon^{ABC}\,(q_L^{cB})^T\,\mathcal{C}^{-1}\,q_L^{dC}$$

So LR tetraquark is same:

$$\zeta^{(ab),(cd)} = \left(\chi_R^{(ab)A}\right)^\dagger\,\chi_L^{(cd)A}$$

Tetraquark couples to usual Φ through cubic, quadratic terms, *so what*.

Instead, consider *tri*quark field:

$$\chi_L^a = \epsilon^{abcd}\,\epsilon^{ABC}\,q_L^{bA}\,(q_L^{cB})^T\,\mathcal{C}^{-1}\,q_L^{dC}$$

Triquark is a color singlet, fundamental rep. in flavor.
Hence a LR *hexa*quark field is just like the usual Φ,
and mixes *directly* with it.

$$\xi^{ab} = (\chi_R^a)^\dagger\,\chi_L^b$$

Analysis for general numbers of flavors and colors is *not* trivial.
Like color superconductivity.