String theory: hot or cold?

Steve Gubser, Princeton University

Based on work with A. Nellore, S. Pufu, F. Rocha, and A. Yarom

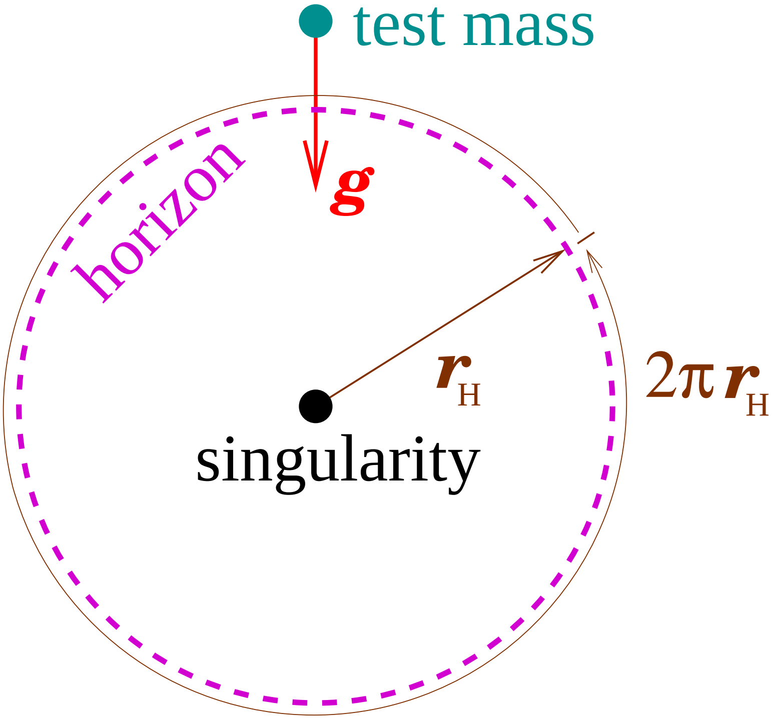
Brookhaven National Laboratory, 3-10-09

Contents

1. Black holes

Black holes have a temperature [Hawking 1975]:

$$T = \frac{g}{2\pi}\frac{\hbar}{ck_B} = \frac{g}{9.8\,\mathrm{m/s^2}}\,3.9\times 10^{-19}\,\mathrm{K}\,. \qquad (1)$$



g is the surface gravity at the horizon:

$$g = \frac{G_N M}{r_H^2} \qquad r_H = \frac{2G_N M}{c^2}\,. \qquad (2)$$

Putting everything together,

$$T = \frac{\hbar c^3/k_B}{8\pi G_N M}\,. \qquad (3)$$

If $M = 3M_{\rm sun}$, then

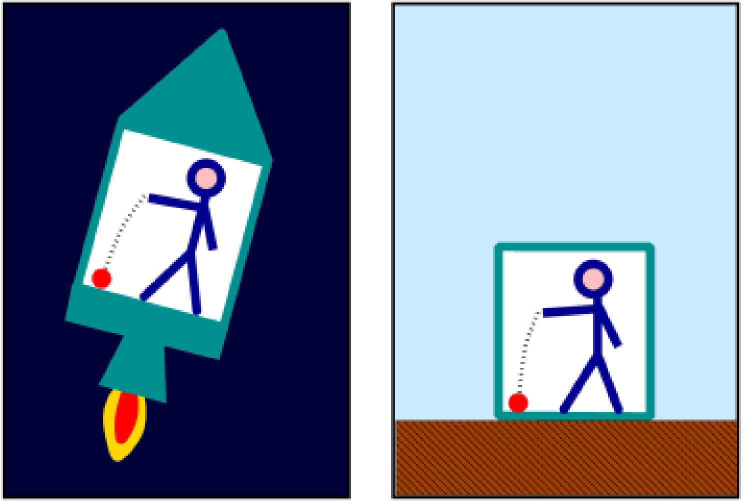
$$T \approx 2\times 10^{-8}\,\mathrm{K}\,. \qquad (4)$$

Hereafter I set $\boxed{\hbar = c = k_B = 1.}$

General Relativity says that constant gravity is like constant acceleration. So we expect

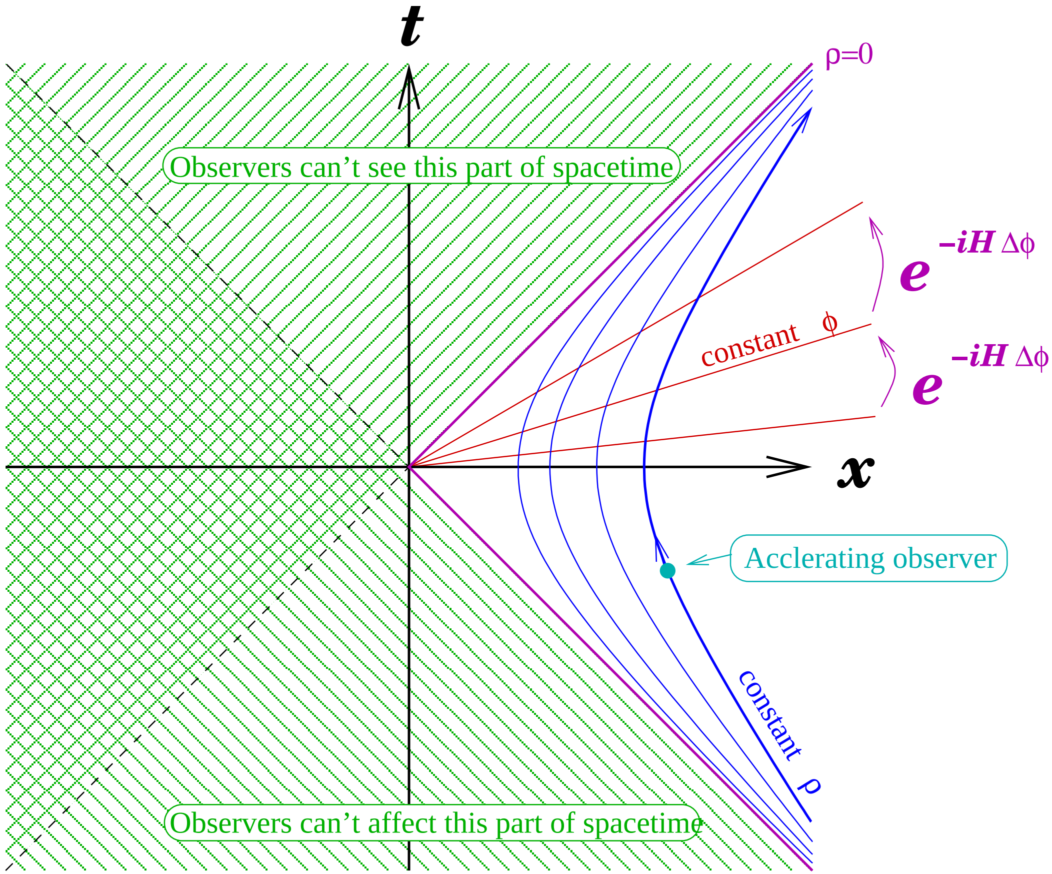
$$T = \frac{a}{2\pi} \tag{5}$$

for observers with constant (proper) acceleration a. Can we understand where (5) comes from?



http://upload.wikimedia.org/wikipedia/commons/9/95/Elevator_gravity2.png

Constant proper acceleration: $x = \rho \cosh \phi \qquad t = \rho \sinh \phi \qquad a = 1/\rho.$

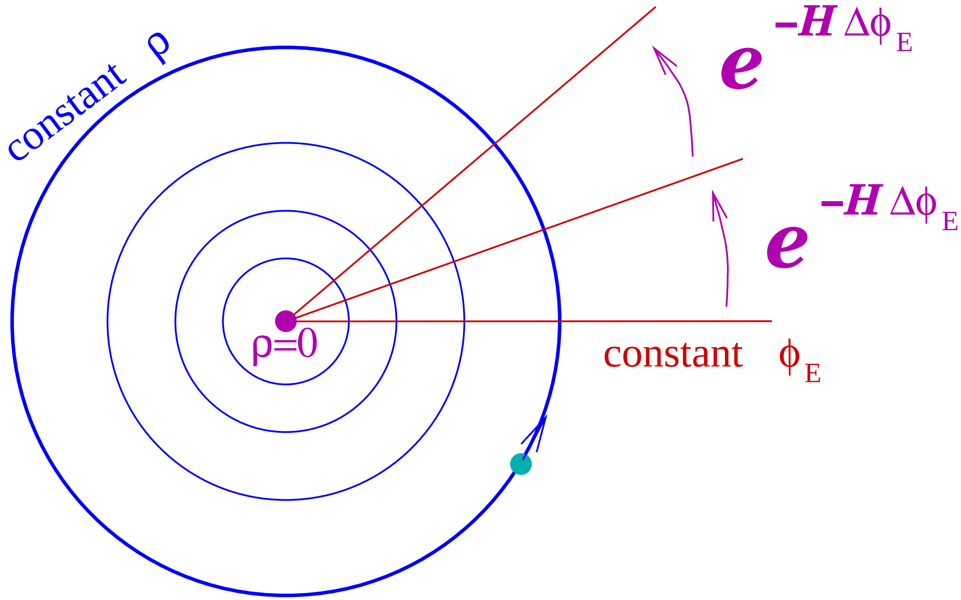


$$ds^2 = -dt^2 + dx^2 = -\rho^2 d\phi^2 + d\rho^2 \,. \qquad (6)$$

A dimensionless Hamiltonian H steps us in increments of ϕ.

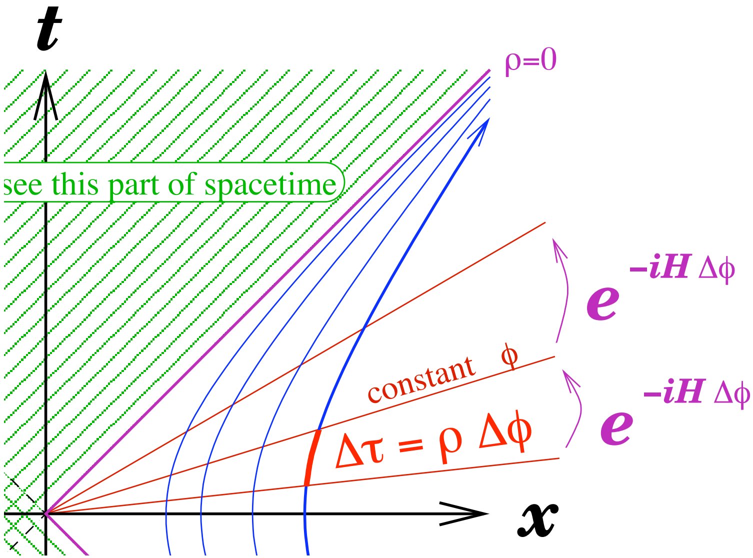
To continue to Euclidean signature, set $\phi_E = i\phi$. Then $ds_E^2 = \rho^2 d\phi_E^2 + d\rho^2$.

After Wick rotation



To avoid a singularity at $\rho = 0$, we should identify $\phi_E \sim \phi_E + 2\pi$, which means $T = 1/2\pi$ with respect to dimensionless "time" ϕ.

Proper time τ measured by an accelerating observer at fixed ρ is not ϕ but $\rho\phi$.

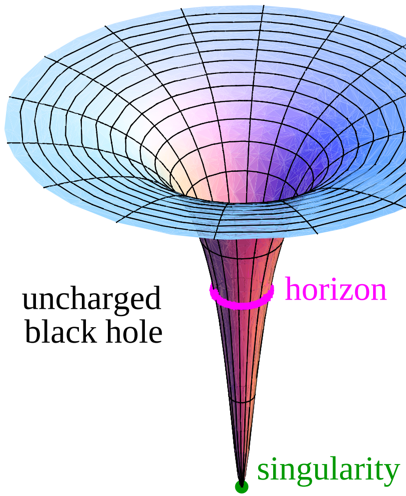


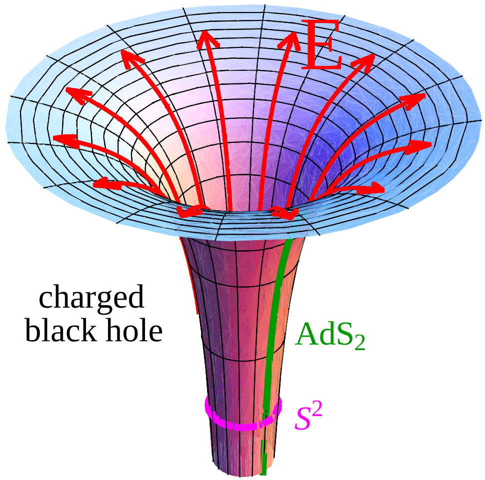
So dimensionful temperature is

$$T = \frac{1}{2\pi\rho} = \frac{a}{2\pi} \, . \tag{7}$$

2. D3-branes

Most black holes in string theory are variants of the *electrically charged* black hole, which has zero temperature if $Q = M$ in Planck units.

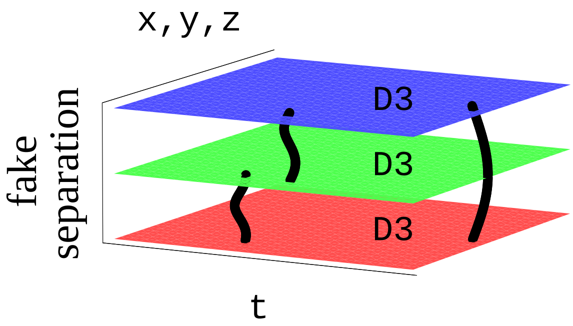




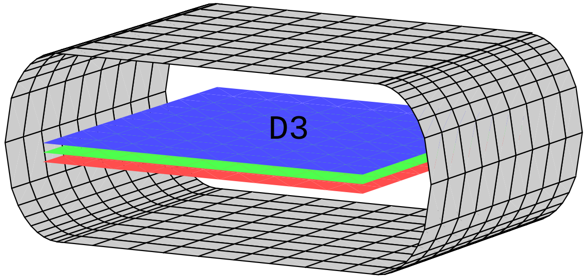
The horizon of the charged black hole is infinitely far down the throat, which has asymptotically constant size. And there is no singularity!

Some of the objects in string theory bend spacetime into similar shapes—but with more dimensions. D3-branes are my favorite example:

D-brane representation: Strings can run from one D3-brane to another.

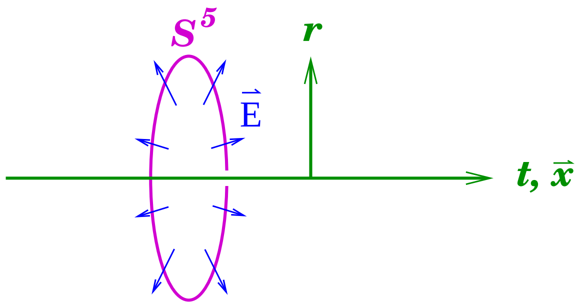


Geometrical representation: D3-branes infinitely far down the throat of a "black brane."



The black brane description works best when there are many branes (say, $N \gg 1$ D3-branes) on top of one another.

Another view of the black brane geometry: S^5 surrounds the D3's; t and $\vec{x}$ are along the D3's.



D3-branes are my favorite because the strings running between them act as gluons in $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang-Mills theory (SYM):

$$\mathcal{L} = -\frac{1}{2g_{YM}^2} \operatorname{tr} F_{\mu\nu}^2 + \text{(superpartners)}\,. \tag{8}$$

This is like the lagrangian of QCD, except that the quarks of QCD are replaced by the superpartners (4 Majorana adjoint fermions plus 6 real adjoint scalars).

The geometry near D3-branes is AdS_5–Schwarzschild $\times S^5$:

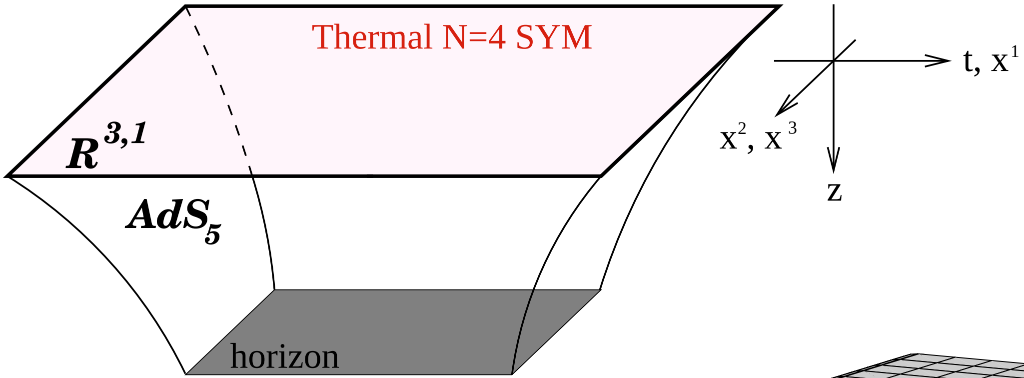
$$ds^2 = \frac{L^2}{z^2}\left(-h dt^2 + d\vec{x}^2 + \frac{dz^2}{h}\right) + L^2 d\Omega_5^2 \qquad \text{where } h = 1 - \frac{z^4}{z_H^4}\,. \tag{9}$$

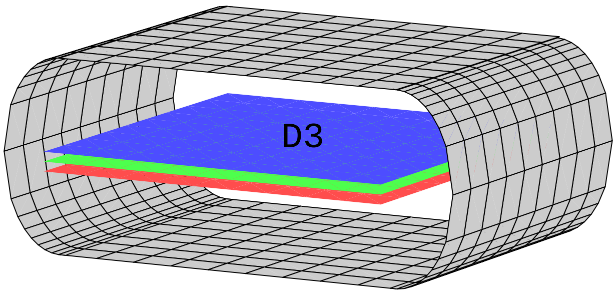
From black hole calculations (e.g. $T = g/2\pi$),

$$T = \frac{1}{\pi z_H} \qquad\qquad \frac{\epsilon}{T^4} = \frac{3\pi^2}{8} N^2 = \frac{3}{4}\frac{\epsilon_{\rm free}}{T^4} \tag{10}$$

Black hole description is valid when $N \gg 1$ and $g_{YM}^2 N \gg 1$.

AdS/CFT maps a thermal bath to a black hole in five or ten dimensions.

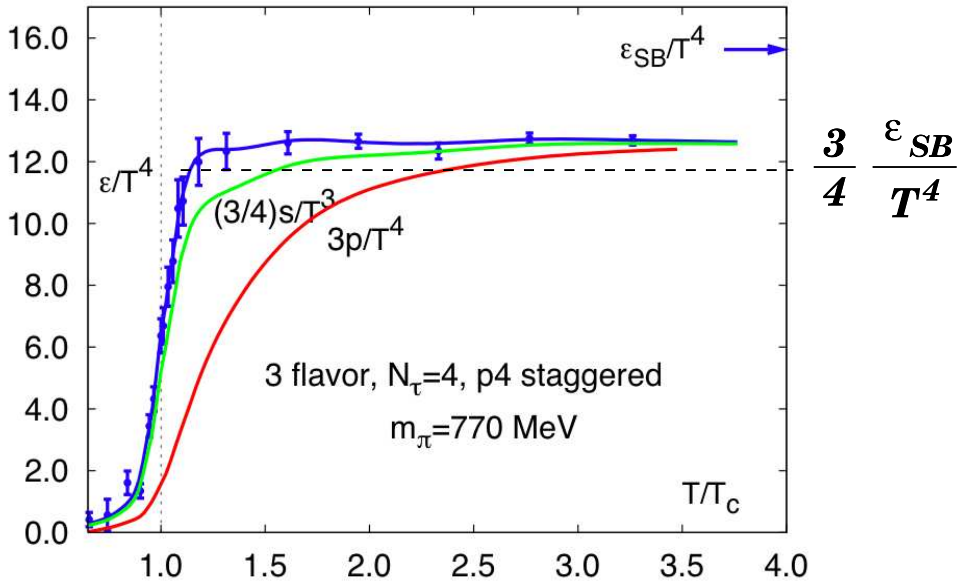




Could finite temperature QCD be thought of in a similar way? If $\alpha_s \equiv \frac{g_s^2}{4\pi} \approx 1/2$, then

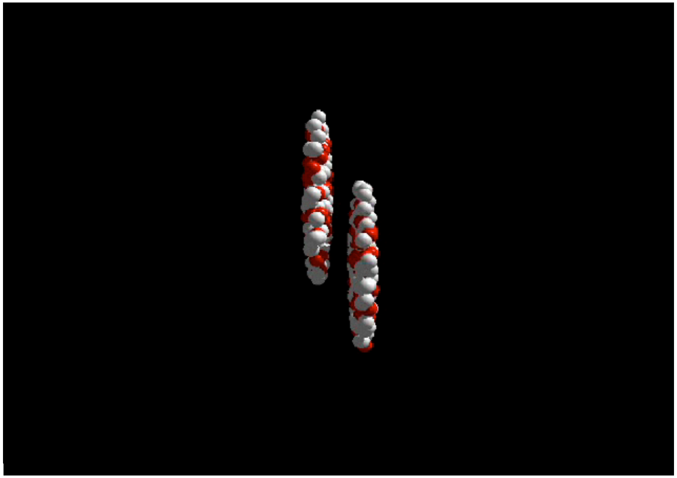
$$N = 3 \overset{?}{\gg} 1 \qquad\qquad g_s^2 N = 12\pi\alpha_s \approx 19 \gg 1 \,. \tag{11}$$

Also, lattice shows $\epsilon/\epsilon_{\rm free} < 1$ for $T > T_c \approx 170\,{\rm MeV}$.

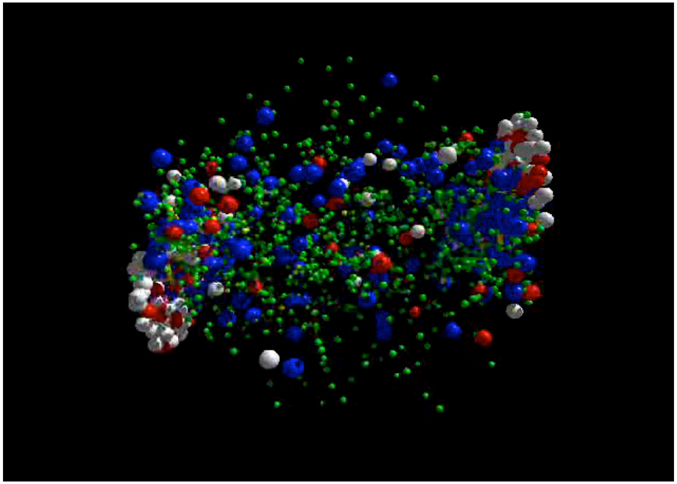


Equation of state of QCD from lattice simulations. SB refers to the free theory. From [Karsch 2002].

So, maybe we can at least make some approximate comparisons between strongly coupled SYM and deconfined QCD.



BEFORE collision



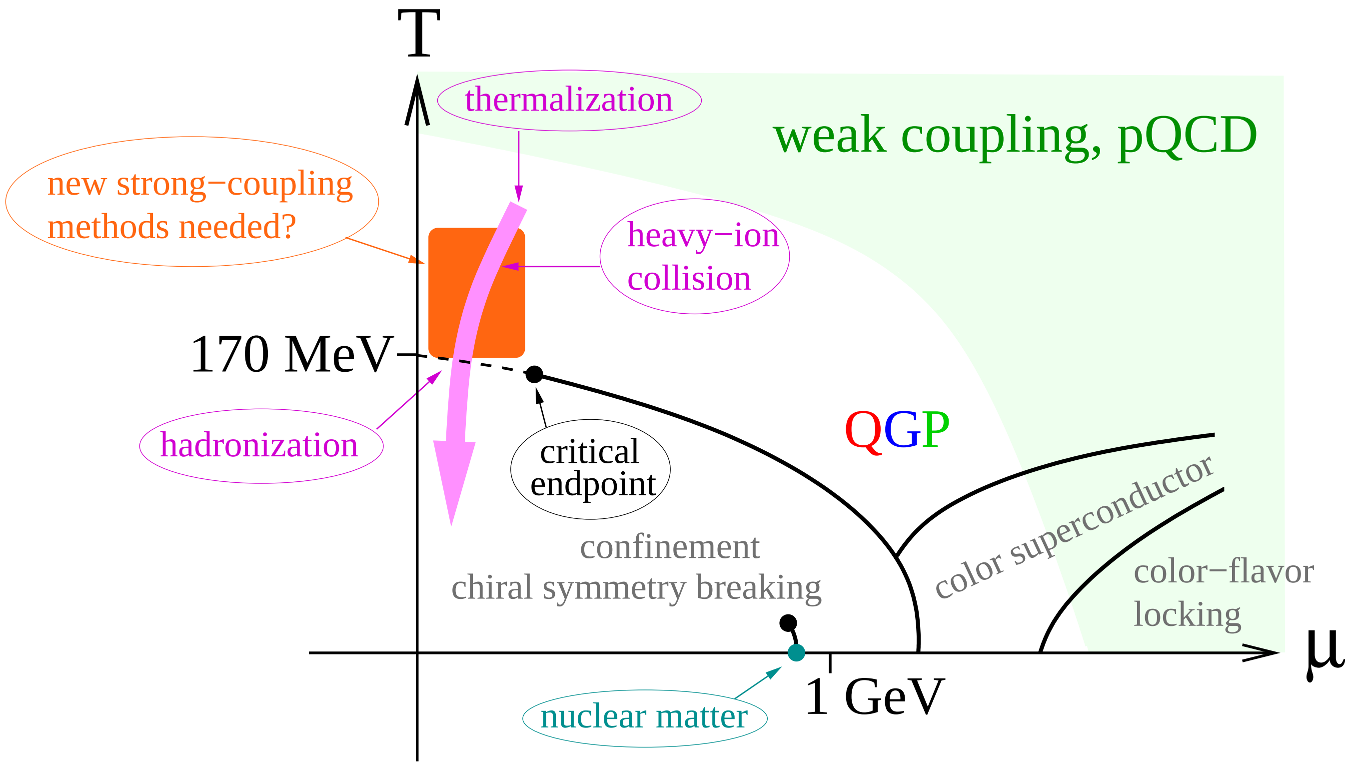
AFTER collision [UrQMD]

3. Heavy ion collisions

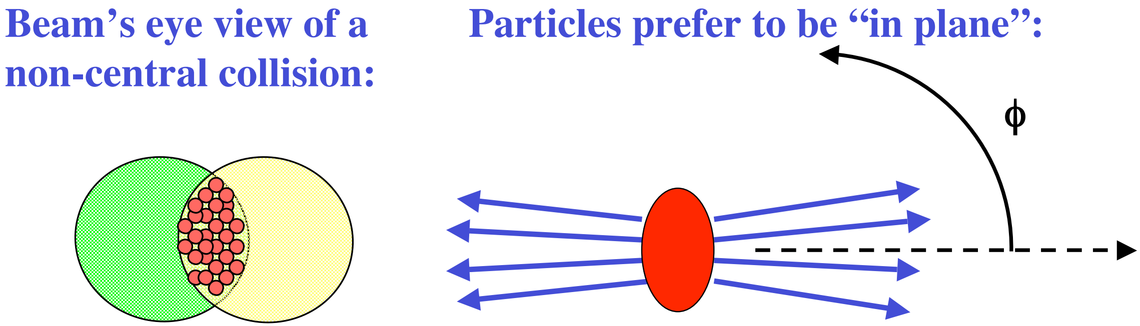
The Relativistic Heavy Ion Collider (RHIC) collides gold on gold, so 394 nucleons total.

- Total center of mass energy is about $39\,\mathrm{TeV}$.
- Roughly 5000 charged particles emerge.
- *What happens during the collision?*

Nuclei probably melt into a plasma of quarks and gluons (the QGP). But the interactions among the quarks and gluons may be hard to describe perturbatively.



A heavy-ion collision traverses the QCD phase diagram. See e.g. [Rajagopal 2000].



Cartoon of elliptic flow. From [Baker 2001].

Measurements of anisotropies of particle production around the beamline fit well to a hydrodynamic treatment of the expanding QGP, provided viscosity is small:

$$0 \lesssim \frac{\eta}{s} \lesssim 0.2 \,, \tag{12}$$

in accord with a calculation for D3-branes at large N and $g_{YM}^2 N$:

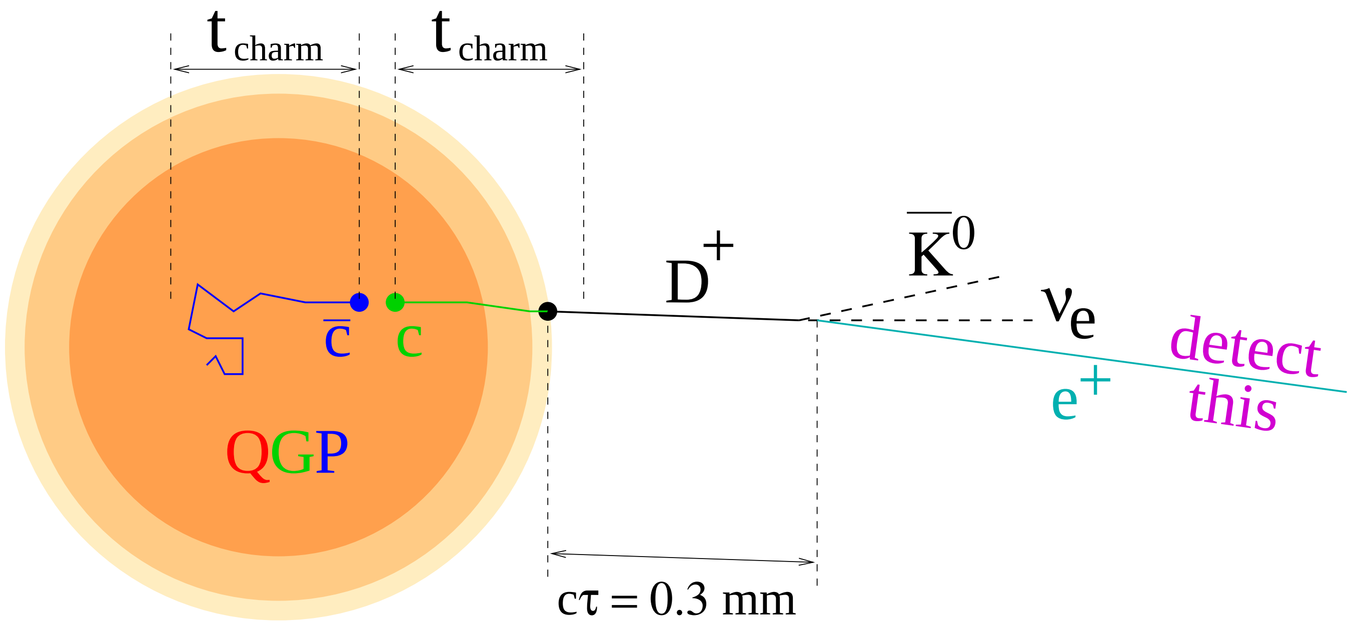
$$\frac{\eta}{s} = \frac{1}{4\pi} \,. \tag{13}$$

It's hard to see how to satisfactorily account for $\eta/s \ll 1$ starting from pQCD.

The favorable comparison of a strong coupling calculation (13) with the data-driven bound (12) is now cited as a reason to believe the QGP is strongly coupled.

4. Heavy quarks in the QGP

If a heavy quark (c or b) is produced inside the QGP, does it escape? It depends on its relaxation time t_{charm}:



STAR and PHENIX can detect the e^+, which carries roughly $1/2$ the momentum of the c quark when it escapes.

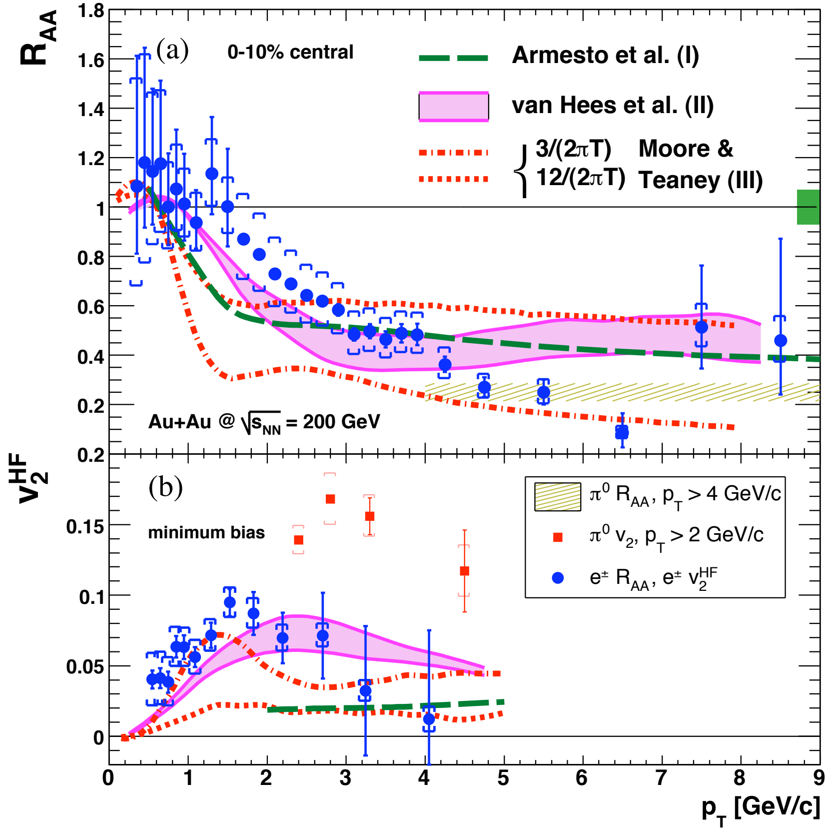
Early pQCD estimates indicated $t_{\rm charm} \sim 10\,{\rm fm}$ [van Hees and Rapp 2005]. So it was a surprise that data is more consistent with $t_{\rm charm} = 3-6\,{\rm fm}$.

R_{AA} is, roughly, the fraction of charm quarks that escape with a given p_T.

v_2 measures how much charm quarks respond to anisotropic expansion of QGP.

Big v_2 and small R_{AA} go together.

Red dotted line: $t_c \approx 8\,{\rm fm}$.
Red dash-dot line: $t_c \approx 2\,{\rm fm}$.
Pink region: $t_c \approx 4.5\,{\rm fm}$.

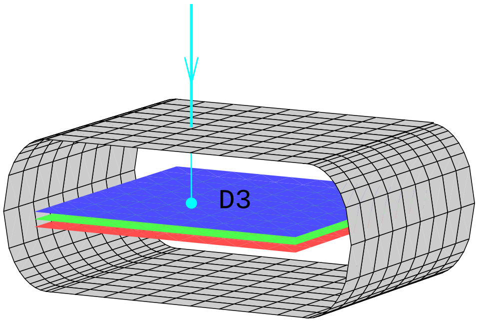


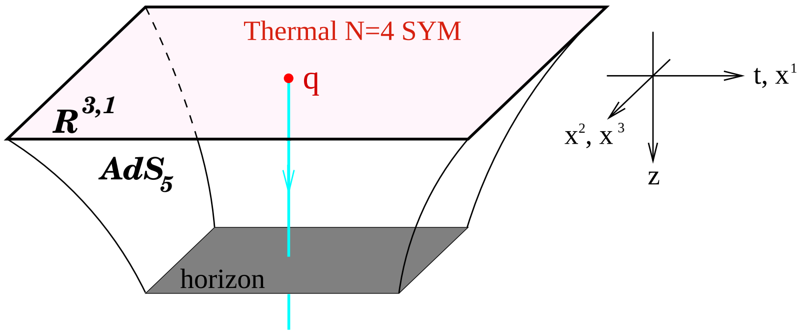
R_{AA} and v_2 for heavy quarks. p_T is for a non-photonic electron. From [Adare et al. 2006].

5. The trailing string

In D3-brane picture, a heavy quark is represented as a long string ending on a brane:

Two views of a stationary string ending on a D3-brane.

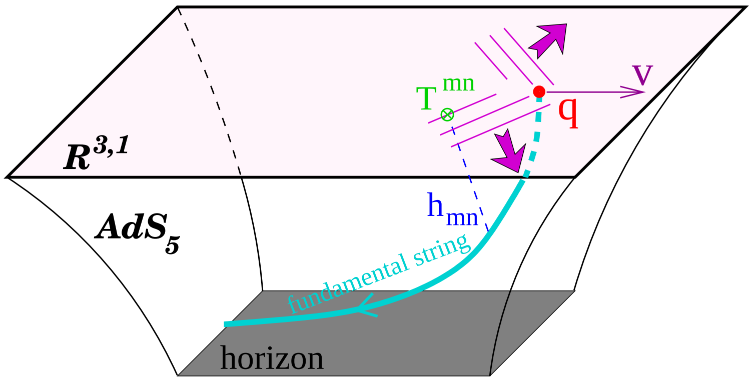




A string trails out behind a moving quark

Horizon is "sticky" because infinite red-shift prevents string from moving where it crosses the horizon.

So when string moves, it trails out behind the quark.



Classical shape of the string leads directly to

$$\frac{dp}{dt} = F_{\text{drag}} - \frac{\pi\sqrt{\lambda}}{2} T^2 \frac{v}{\sqrt{1-v^2}} \approx -\frac{\pi\sqrt{\lambda}}{2} T^2 \frac{p}{m} \,. \tag{14}$$

So momentum falls off exponentially:

$$\frac{dp}{dt} = -\frac{p}{t_{\text{quark}}} \qquad \text{where} \quad t_{\text{quark}} = \frac{2}{\pi\sqrt{\lambda}} \frac{m}{T^2} \,. \tag{15}$$

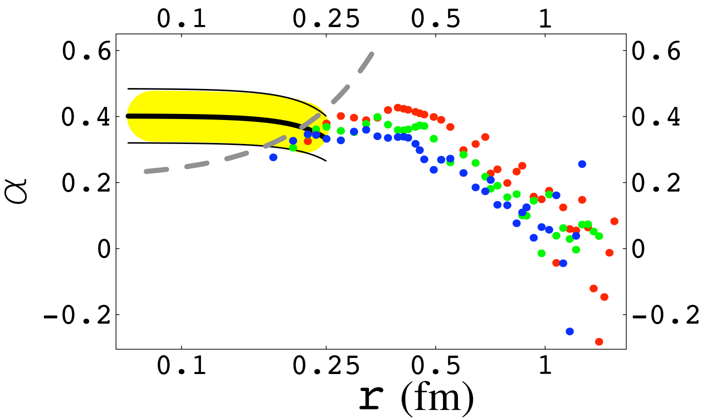
Now, how do we relate t_{quark} to t_{charm}?

1. There are $3\times$ as many degrees of freedom in $\mathcal{N} = 4$ SYM as in QCD. Surely F_{drag} should be somehow rescaled as a result.
2. $\lambda = g_{YM}^2 N$ is a free parameter in SYM. In QCD, $\lambda = 12\pi\alpha_s$ runs with energy scale. How do we match one to the other?

A proposal:

1. Compare QCD and SYM at fixed *energy density*, not temperature.
2. Deduce a value for λ by comparison with lattice data on q-$\bar{q}$ potential.

$T_{\rm SYM} \approx 190\,{\rm MeV} \;\leftrightarrow\; T_{\rm QCD} = 250\,{\rm MeV}$



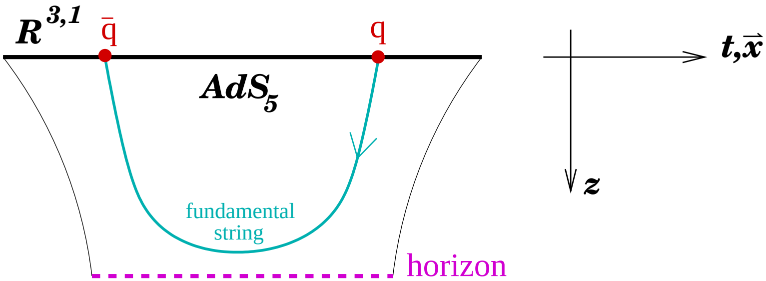
$\alpha_{q\bar{q}}$ compared between QCD and SYM

Lattice studies of q and $\bar{q}$ at fixed separation r lead to the *definition*

$$\alpha_{q\bar{q}}(r,T) \equiv \frac{3}{4} r^2 \frac{\partial F_{q\bar{q}}}{\partial r} \,.$$

Lattice data is from [Kaczmarek and Zantow 2005], for

$$T \approx 209,\; 233,\; 255\,{\rm MeV}\,.$$



q-$\bar{q}$ potential as computed from string theory

Similar calculation in string theory at $T=0$ gives

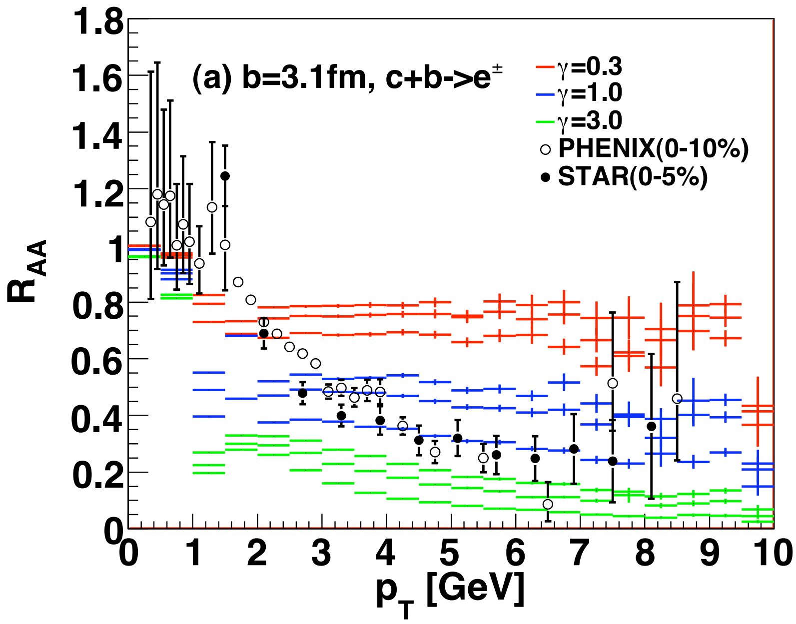
$$\alpha_{\rm SYM} \equiv \frac{3}{4} r^2 \frac{\partial V_{q\bar{q}}}{\partial r} = \frac{3\pi^2\sqrt{\lambda}}{\Gamma(1/4)^4} \,.$$

$T \neq 0$ results in some Debye screening.

The upshot: $\lambda \approx 5.5$, leading to a prediction from the trailing string,

$$\boxed{t_{\rm charm} \approx 2.1\,{\rm fm}} \qquad \text{for} \quad T_{\rm QCD} = 250\,{\rm MeV}\,. \tag{16}$$

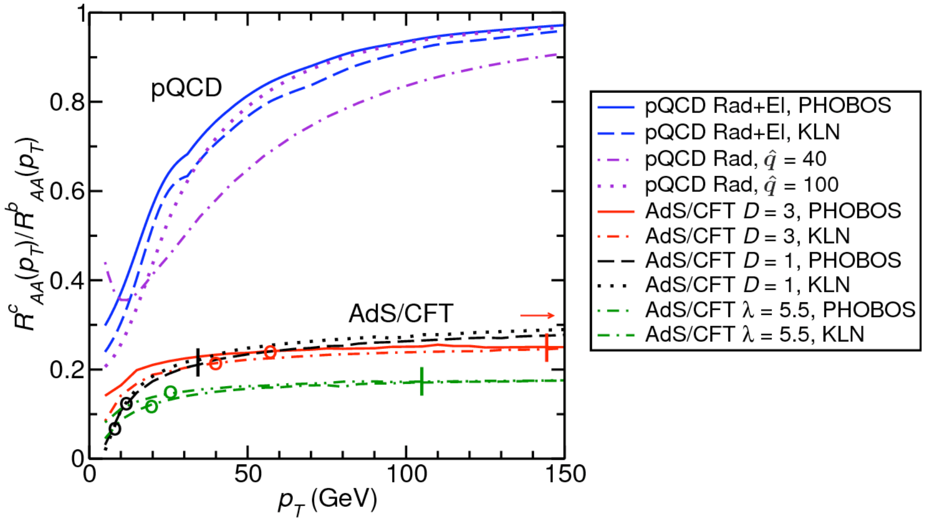
Maybe a little on the small side... But a recent study [Akamatsu et al. 2008] indicates data may be (almost) consistent with it.



$t_{\rm charm} \propto 1/\gamma$, and $\gamma \approx 2$ is the trailing string prediction. ($\gamma \neq 1/\sqrt{1-v^2}$.)

A distinctive difference [Horowitz and Gyulassy 2007] between pQCD and AdS/CFT predictions from RHIC to LHC energies comes from

$$R_{AA}^{cb} \equiv \frac{R_{AA}^b}{R_{AA}^c} \sim \begin{cases} \frac{t_{\text{bottom}}}{t_{\text{charm}}} \approx \frac{m_{\text{charm}}}{m_{\text{bottom}}} & \text{for AdS/CFT} \\ 1 - p_{cb}/p_T & \text{for pQCD, } p_{cb} \propto \hat{q} L^2 \end{cases} \tag{17}$$

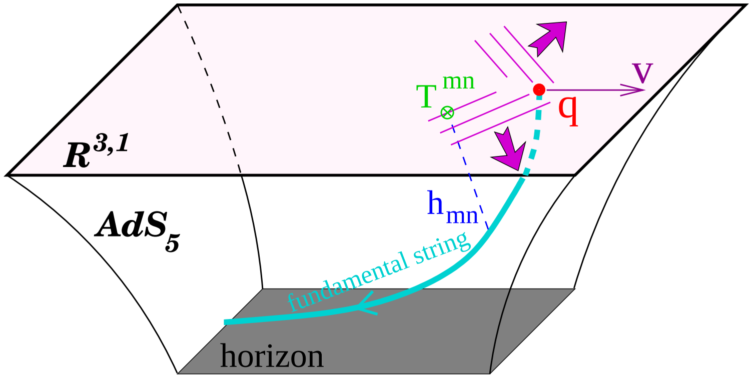


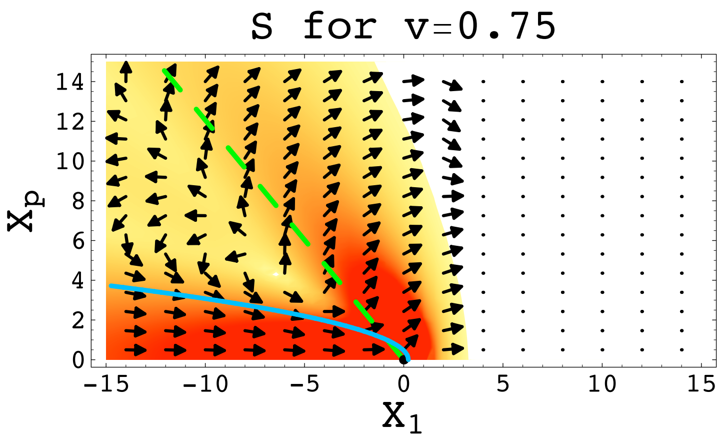
- Different curves correspond to different initial conditions.
- Initial conditions affect R_{AA}^c and R_{AA}^b about the same.
- So R_{AA}^{cb} is a fairly clean distinguishing observable.

But beware uncertainty on the limits of validity of AdS/CFT.

Gravitons emitted by the trailing string tell us where the lost energy goes.

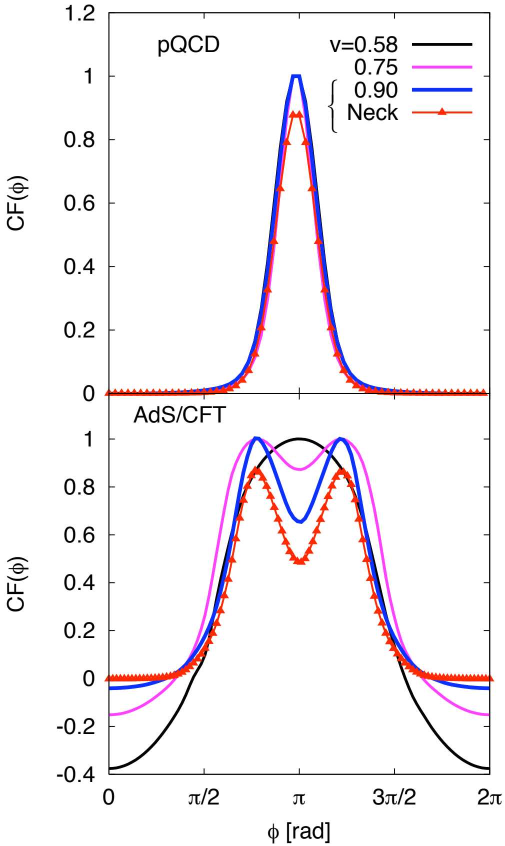
We can, for example, compute the Poynting vector $S^m = \langle T^{0m} \rangle$.



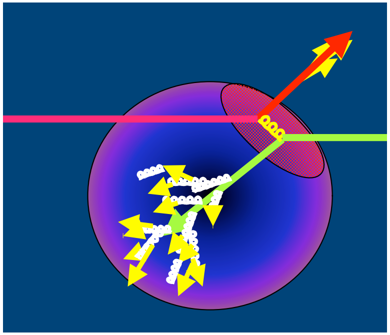


Quark is at $X_1 = X_p = 0$.

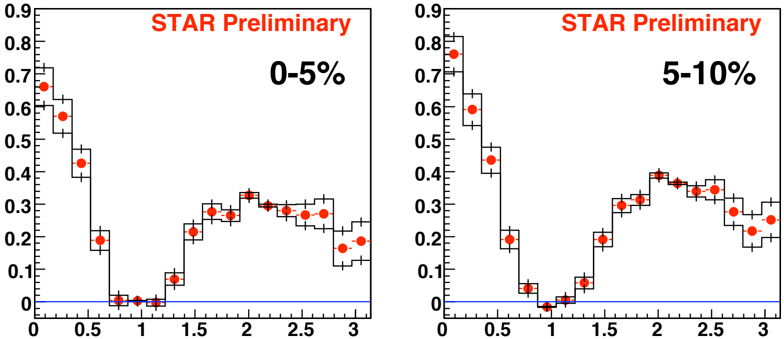
- A sonic boom appears when $v > 1/\sqrt{3}$.
- A diffusion wake is always present.
- String theory calculation extends to non-hydrodynamical regime close to the quark.
- $X = 1$ corresponds to about $0.25\,\mathrm{fm}$.



Near-quark regime of $\vec{S}$ from trailing string leads to something like jet-splitting [Betz et al. 2008].



A hard process and strong interaction with the medium, from [Jacak 2006].



Hadron two-point functions from [Ulery 2008] show "jet-splitting."

6. Superconducting black holes

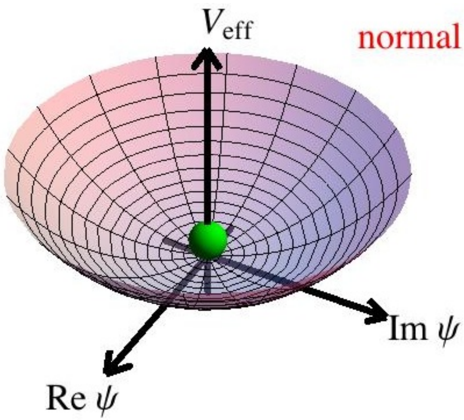
Is it now open season in string theory on all strongly coupled problems?

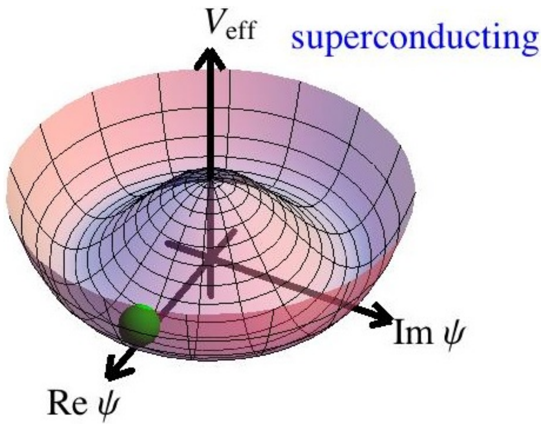
Condensed matter offers some good ones: high T_c superconductors, cold atoms, graphene....

Let's start simply. Can we persuade a black hole to superconduct?

The Abelian Higgs model is the simplest tool for understanding superconductors:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - |(\partial_\mu - iqA_\mu)\psi|^2 - V_{\text{eff}}(|\psi|, T) \tag{18}$$





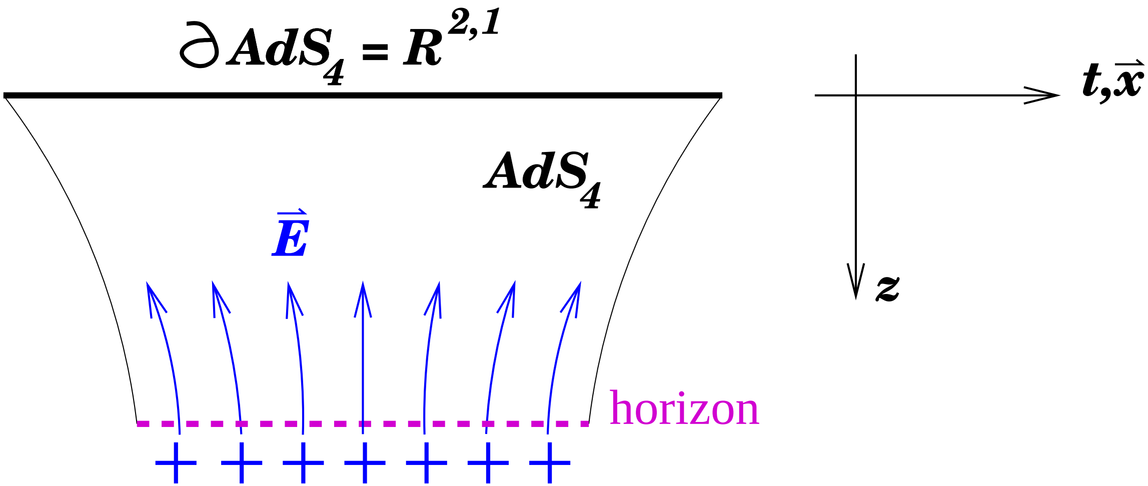
Do we get something distinctively new by coupling this model to gravity?

$$\mathcal{L} = R - \frac{1}{4}F_{\mu\nu}^2 - |(\partial_\mu - iqA_\mu)\psi|^2 - V(|\psi|)\,. \tag{19}$$

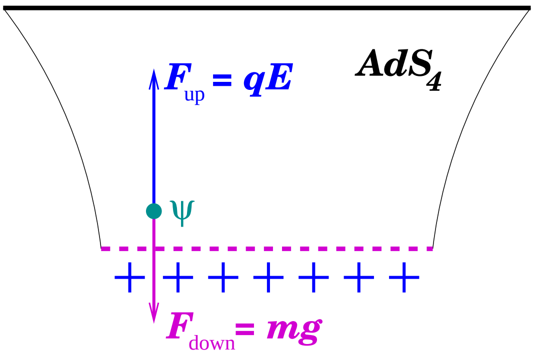
It's unnatural to tune extremum of V to 0, so let's *assume*

$$V = -\frac{6}{L^2} + m^2|\psi|^2\,. \tag{20}$$

Normal state corresponds to $\psi = 0$: a charged black hole in AdS_4:



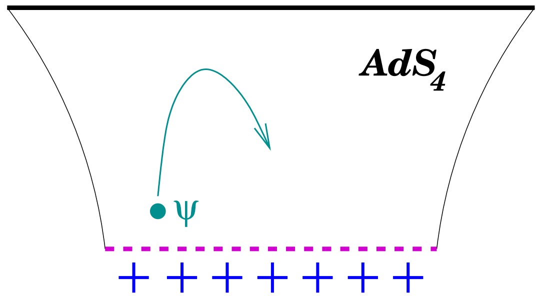
Below some temperature, quanta of ψ are driven upward from horizon: recall $T = g/2\pi$.



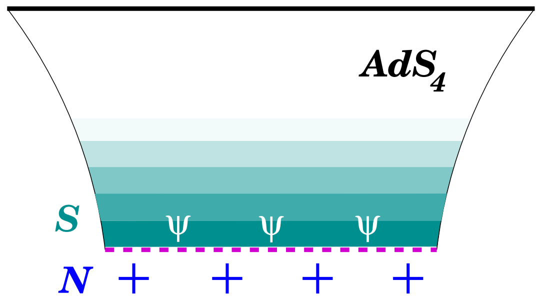
Condensate spontaneously breaks $U(1)$ gauge symmetry, so this is a superconductor: s-wave since ψ is a scalar.

Some fraction of charge remains in "normal" state, behind the horizon.

ψ quanta can never escape from AdS_4, so they fall back toward horizon.



Expected end state has an "atmosphere" of ψ quanta condensed above the horizon.

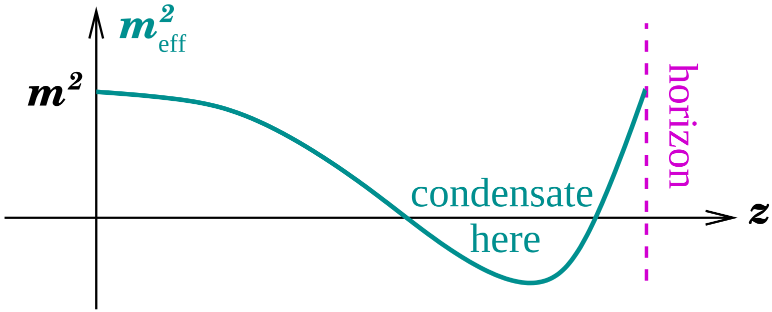


Condensate can be calculated in a simple way if we ignore its back-reaction on the metric and gauge field:

$$\mathcal{L}_\psi = -|(\partial_\mu - iqA_\mu)\psi|^2 - m^2|\psi|^2 = -g^{rr}|\partial_r\psi|^2 - m^2_{\text{eff}}|\psi|^2 \qquad (21)$$

where Φ is electrostatic potential and m^2_{eff} is the effective mass of ψ:

$$m^2_{\text{eff}} = m^2 + g^{tt}q^2\Phi^2 \,. \qquad (22)$$

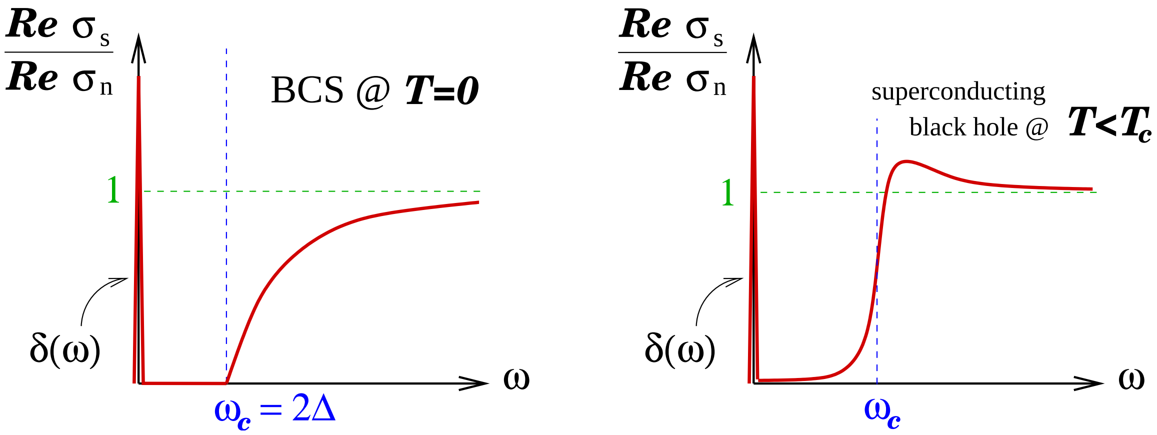


The condensate tends to form where $m^2_{\text{eff}} < 0$.

We never put T into the lagrangian, as one does in Landau theory. Instead, T crept into m^2_{eff} indirectly, through electrostatic potential in the extra dimension.

One of the hallmarks of BCS conductivity is a characteristic shape of conductance,

$$\sigma(\omega) \equiv J(\omega)/E(\omega)\,. \tag{23}$$

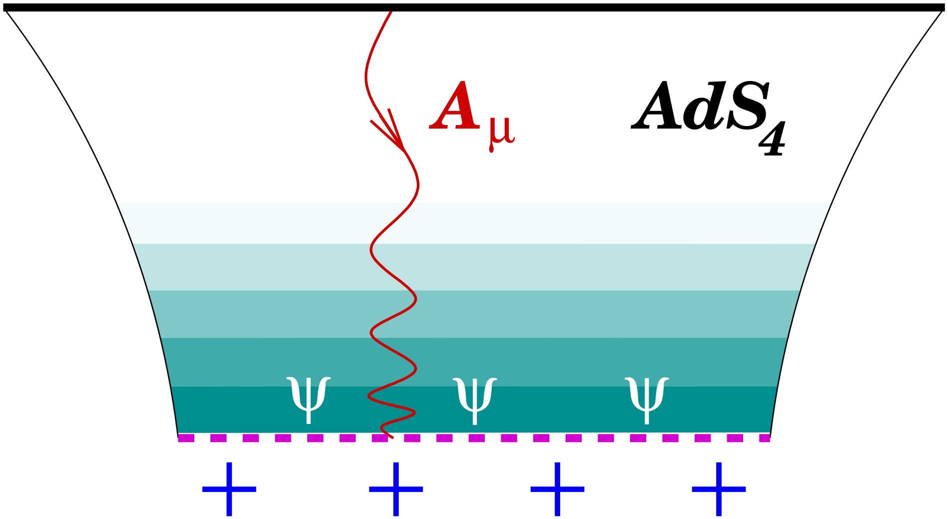


$\mathrm{Re}\,\sigma(\omega)$ quantifies how well a superconductor absorbs photons of frequency ω. If $\omega > 2\Delta$, the photon can break a Cooper pair into two electrons. BCS says $\omega_c \approx 3.5T_c$.

Contribution $\sim \delta(\omega)$ signals infinite DC conductance.

To calculate $\mathrm{Re}\,\sigma$ for the black hole, we can "shine" photons down on it.

If $\omega \gtrsim \omega_c$, the photon can penetrate through the superconducting atmosphere and fall into the black hole. ω_c/T_c seems to be non-universal.



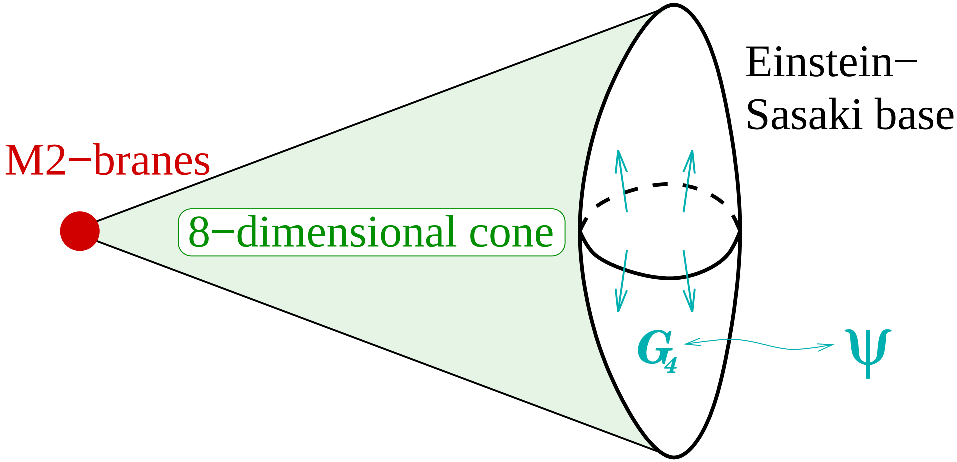
Roughly speaking, the equation to solve is

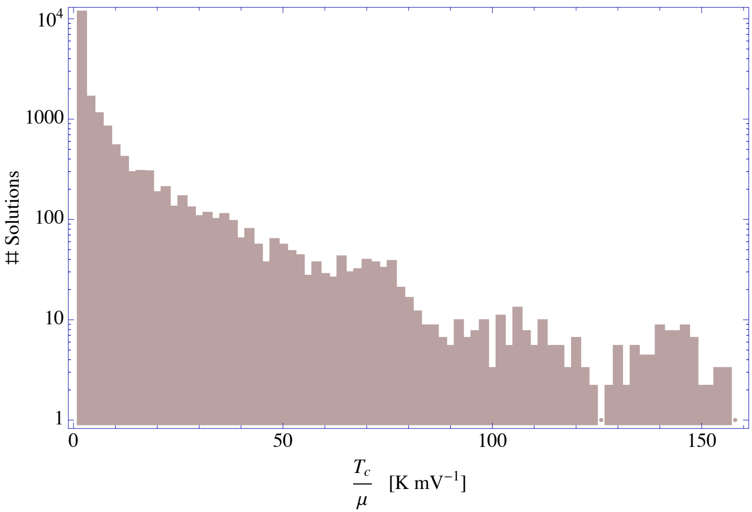
$$(\Box - m_\gamma^2)A_\mu = 0 \qquad \text{where } m_\gamma^2 = 2q^2|\psi(z)|^2\,, \tag{24}$$

$\mathrm{Re}\,\sigma$ relates to the probability for the photon to penetrate into the black hole.

Many embedding of Abelian Higgs in AdS_4 exist in string theory:

- M2-branes create AdS_4 geometry.
- Magnetic G_4 flux gives scalar field.
- Choice of shape of cone controls T_c.





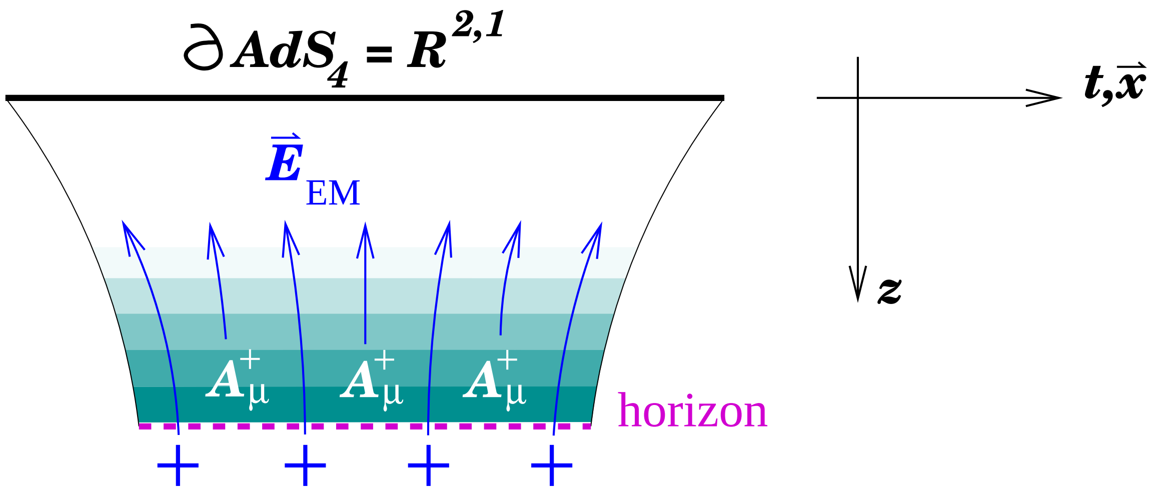
- T_c/μ is a dimensionless number: $11\,\mathrm{K} \approx 1\,\mathrm{meV}$.
- Units evoke high T_c materials.
- Boundary of AdS_4 is 2+1-dimensional, like a cuprate layer.

From [Denef and Hartnoll 2009].

If we start with an $SU(2)$ theory,

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{2g_{YM}^2} \operatorname{tr} F_{\mu\nu}^2 \,, \tag{25}$$

We naturally get a p-wave condensate (because A_μ^+ has spin 1) that spontaneously breaks $U(1)_{\rm EM} \subset SU(2)$ as well as the $SO(2)$ of rotations in $\vec{x}$ directions.



- Nodes in the gap are a little funny—infinitely narrow in some sense.
- $p + ip$ is possible, but disfavored by A^4 terms inside F^2.
- d-wave hasn't been as much studied in this framework.

7. Conclusions

Finite-temperature methods in string theory, based on black hole horizons, have matured to the point where we can make meaningful contact with experiment.

- Contact with heavy-ion collisions is based on a comparison of $\mathcal{N} = 4$ super-Yang-Mills theory to QCD.
  - Tricky, but at least we have gluons on both sides.
- Shear viscosity, drag force on heavy quarks, and jet-splitting provide fairly impressive points of contact between theory and data.

- Comparisons with low-temperature physics are more loosely motivated.
- Superconducting black holes are just one of several interesting attempts: There's also discussion relevant to superfluids and cold atoms.

8. Partial guide to stringy literature

On the temperature of black holes:

- [Hawking 1975], [Unruh 1976]

On D-branes and AdS/CFT

- [Polchinski 1995] (D-branes), [Witten 1996] (gauge theory)
- [Maldacena 1998; Gubser et al. 1998; Witten 1998] (AdS/CFT)

On applications to heavy ions:

- [Policastro et al. 2001; Kovtun et al. 2005] (η/s)
- [Herzog et al. 2006; Gubser 2006] (trailing string); [Casalderrey-Solana and Teaney 2006] (diffusion)
- [Friess et al. 2006; Yarom 2007; Gubser et al. 2007; Chesler and Yaffe 2007] (booms and wakes)

On superconducting black holes:

- [Herzog et al. 2007; Gubser and Rocha 2008] (Quantum criticality and AdS_4)
- [Gubser 2005 2008; Hartnoll et al. 2008; Denef and Hartnoll 2009] (Abelian Higgs model in AdS)
- [Gubser and Pufu 2008] (p-wave black holes)

On superfluids and cold atoms:

- [Son 2008; Adams et al. 2008; Kachru et al. 2008] (non-relativistic conformal symmetry, Lifshitz-like fixed points)
- [Herzog et al. 2008; Basu et al. 2008]

References

Allan Adams, Koushik Balasubramanian, and John McGreevy. "Hot Spacetimes for Cold Atoms". *JHEP*, 11:059, 2008, 0807.1111. doi: 10.1088/1126-6708/2008/11/059.

A. Adare et al. "Energy Loss and Flow of Heavy Quarks in Au+Au Collisions at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$". 2006, nucl-ex/0611018.

Yukinao Akamatsu, Tetsuo Hatsuda, and Tetsufumi Hirano. "Heavy Quark Diffusion with Relativistic Langevin Dynamics in the Quark-Gluon Fluid". 2008, 0809.1499.

Mark D. Baker. "The latest Results from PHOBOS: Systematics of Charged Particle Production through $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$", 2001. PowerPoint talk, available at http://www.phobos.bnl.gov/Presentations/paris01/.

Pallab Basu, Anindya Mukherjee, and Hsien-Hang Shieh. "Supercurrent: Vector Hair for an AdS Black Hole". 2008, 0809.4494.

Barbara Betz, Miklos Gyulassy, Jorge Noronha, and Giorgio Torrieri. "Anomalous Conical Di-jet Correlations in pQCD vs AdS/CFT". 2008, 0807.4526.

Jorge Casalderrey-Solana and Derek Teaney. "Heavy quark diffusion in strongly coupled N = 4 Yang Mills". 2006, hep-ph/0605199.

Paul M. Chesler and Laurence G. Yaffe. "The stress-energy tensor of a quark moving through a strongly-coupled N=4 supersymmetric Yang-Mills plasma: comparing hydrodynamics and AdS/CFT". 2007, 0712.0050.

Frederik Denef and Sean A. Hartnoll. "Landscape of superconducting membranes". 2009, 0901.1160.

Joshua J. Friess, Steven S. Gubser, Georgios Michalogiorgakis, and Silviu S. Pufu. "The stress tensor of a quark moving through N = 4 thermal plasma". 2006, hep-th/0607022.

S. S. Gubser, Igor R. Klebanov, and Alexander M. Polyakov. "Gauge theory correlators from non-critical string theory". *Phys. Lett.*, B428:105–114, 1998, hep-th/9802109.

Steven S. Gubser. "Phase transitions near black hole horizons". *Class. Quant. Grav.*, 22:5121–5144, 2005, hep-th/0505189.

Steven S. Gubser. "Drag force in AdS/CFT". 2006, hep-th/0605182.

Steven S. Gubser. "Breaking an Abelian gauge symmetry near a black hole horizon". 2008, 0801.2977.

Steven S. Gubser and Silviu S. Pufu. "The gravity dual of a p-wave superconductor". *JHEP*, 11:033, 2008, 0805.2960. doi: 10.1088/1126-6708/2008/11/033.

Steven S. Gubser and Fabio D. Rocha. "The gravity dual to a quantum critical point with spontaneous symmetry breaking". 2008, 0807.1737.

Steven S. Gubser, Silviu S. Pufu, and Amos Yarom. "Sonic booms and diffusion wakes generated by a heavy quark in thermal AdS/CFT". 2007, arXiv:0706.4307 [hep-th].

Sean A. Hartnoll, Christopher P. Herzog, and Gary T. Horowitz. "Building a Holographic Superconductor". *Phys. Rev. Lett.*, 101:031601, 2008, 0803.3295. doi: 10.1103/PhysRevLett.101.031601.

S. W. Hawking. "Particle Creation by Black Holes". *Commun. Math. Phys.*, 43:199–220, 1975. doi: 10.1007/BF02345020.

C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe. "Energy loss of a heavy quark moving through N = 4 supersymmetric Yang-Mills plasma". 2006, hep-th/0605158.

C. P. Herzog, P. K. Kovtun, and D. T. Son. "Holographic model of superfluidity". 2008, 0809.4870.

Christopher P. Herzog, Pavel Kovtun, Subir Sachdev, and Dam Thanh Son. "Quantum critical transport, duality, and M-theory". *Phys. Rev.*, D75:085020, 2007, hep-th/0701036. doi: 10.1103/PhysRevD.75.085020.

W. A. Horowitz and M. Gyulassy. "Testing AdS/CFT Deviations from pQCD Heavy Quark Energy Loss with Pb+Pb at LHC". 2007, arXiv:0706.2336 [nucl-th].

Barbara Jacak, 2006. "Plasma physics of the quark gluon plasma", Talk given in Boulder, May 2006. Available at http://www4.rcf.bnl.gov/~steinber/boulder2006/.

Shamit Kachru, Xiao Liu, and Michael Mulligan. "Gravity Duals of Lifshitz-like Fixed Points". *Phys. Rev.*, D78:106005, 2008, 0808.1725. doi: 10.1103/PhysRevD.78.106005.

Olaf Kaczmarek and Felix Zantow. "Static quark anti-quark interactions in zero and finite temperature QCD. I: Heavy quark free energies, running coupling and quarkonium binding". *Phys. Rev.*, D71: 114510, 2005, hep-lat/0503017.

Frithjof Karsch. "Lattice QCD at high temperature and density". *Lect. Notes Phys.*, 583:209–249, 2002, hep-lat/0106019.

P. Kovtun, D. T. Son, and A. O. Starinets. "Viscosity in strongly interacting quantum field theories from black hole physics". *Phys. Rev. Lett.*, 94:111601, 2005, hep-th/0405231.

Juan M. Maldacena. "The large N limit of superconformal field theories and supergravity". *Adv. Theor. Math. Phys.*, 2:231–252, 1998, hep-th/9711200.

Joseph Polchinski. "Dirichlet-Branes and Ramond-Ramond Charges". *Phys. Rev. Lett.*, 75:4724–4727, 1995, hep-th/9510017. doi: 10.1103/PhysRevLett.75.4724.

G. Policastro, D. T. Son, and A. O. Starinets. "The shear viscosity of strongly coupled N = 4 supersymmetric Yang-Mills plasma". *Phys. Rev. Lett.*, 87:081601, 2001, hep-th/0104066.

Krishna Rajagopal. “The phases of QCD in heavy ion collisions and compact stars”. *Acta Phys. Polon.*, B31:3021, 2000, hep-ph/0009058.

D. T. Son. “Toward an AdS/cold atoms correspondence: a geometric realization of the Schroedinger symmetry”. *Phys. Rev.*, D78:046003, 2008, 0804.3972. doi: 10.1103/PhysRevD.78.046003.

Jason Glyndwr Ulery. “Two- and Three-Particle Jet-Like Correlations”. 2008, 0801.4904.

W. G. Unruh. “Notes on black hole evaporation”. *Phys. Rev.*, D14:870, 1976. doi: 10.1103/PhysRevD.14.870.

UrQMD. The full MPEG can be found at http://www.bnl.gov/RHIC/images/movies/Au-Au_200GeV.mpeg, where it is attributed to the UrQMD group at Frankfurt, see http://www.physik.uni-frankfurt.de/ ~urqmd/.

Hendrik van Hees and Ralf Rapp. “Thermalization of heavy quarks in the quark-gluon plasma”. *Phys. Rev.*, C71:034907, 2005, nucl-th/0412015.

Edward Witten. “Bound states of strings and p-branes”. *Nucl. Phys.*, B460:335–350, 1996, hep-th/9510135. doi: 10.1016/0550-3213(95)00610-9.

Edward Witten. “Anti-de Sitter space and holography”. *Adv. Theor. Math. Phys.*, 2:253–291, 1998, hep-th/9802150.

Amos Yarom. “The high momentum behavior of a quark wake”. *Phys. Rev.*, D75:125010, 2007, hep-th/0702164.